Assignment 1

Due: Wednesday, February 1st, by the start of class

Markov Models

Consider a Markov model with a cycle length of one year. For people who are well, their annual probability of becoming ill is 0.20 and their annual probability of dying from other causes is 0.05. For any given year (i.e., *within* a cycle), if people who are well become ill, they face a 0.30 probability that they will die from the illness that year; in subsequent years, their annual probability of dying from illness or other causes is 0.10. People who are ill can never return to the WELL state. Assume that people are initially well.

(a) Write down a transition probability matrix for this Markov chain.

(b) How many cycles must elapse by the time $\geq$ 25% of the population is dead? *Show all hand calculations using matrix algebra* (you may use TreeAge software or Excel to check your calculations).

(c) For persons who are ill, what is the 5-year survival probability? [Hint: consider your starting population]

Consider the Markov model as described for parts (a)-(c) except that each year, people who are well face an *unknown* probability of becoming ill. A cohort has been followed for 5 years and the proportions of the cohort in the WELL, ILL, and DEAD states have been traced:

	Markov States		
Cycle	WELL	ILL	DEAD
0	1.0000	0.0000	0.0000
1	0.6500	0.2100	0.1400
2	0.4550	0.3028	0.2422
3	0.3412	0.3362	0.3226
4	0.2730	0.3384	0.3886
5	0.2320	0.3237	0.4443

(d) Use the Markov tracing to determine the annual probability of becoming ill. Does it remain constant over time?

Consider a different Markov model with the following three states: WELL, ILL, and DEAD. Suppose that a new medication has been discovered, which can reduce the probability that illness develops by 50%. **Tracing A** (below) reflects the health of a population that is receiving prophylaxis with this new medication. A new treatment has also been discovered, which can reduce the mortality among those who become ill by 50%. **Tracing B** represents the health of the population that receives the new treatment once they become ill. Because of a deadly interaction between the medication and the treatment, only one option can be pursued. Assume that the utility of WELL is 1.0, the utility of ILL is 1.0, and the utility of DEAD is 0.

New Medication (for WELL people)
Tracing A

Cycle	WELL	ILL	DEAD	Cycle Reward
0	1.0000	0.0000	0.0000	0.0000
1	0.8788	0.0375	0.0838	0.8726
2	0.7513	0.0627	0.1860	0.7383
3	0.6245	0.0783	0.2972	0.6071
4	0.5043	0.0860	0.4097	0.4857
5	0.3833	0.0934	0.5233	0.3735
6	0.2731	0.0949	0.6323	0.2744
7	0.1946	0.0814	0.7240	0.1962
8	0.1386	0.0650	0.7963	0.1379
9	0.0988	0.0499	0.8514	0.0958
10	0.0704	0.0373	0.8923	0.0661
11	0.0501	0.0274	0.9224	0.0454
12	0.0357	0.0200	0.9443	0.0310
13	0.0255	0.0145	0.9601	0.0212
14	0.0181	0.0104	0.9715	0.0144
15	0.0129	0.0075	0.9796	0.0098
16	0.0092	0.0054	0.9854	0.0067
17	0.0066	0.0038	0.9896	0.0045
End	0.0047	0.0027	0.9926	0.0000

New Treatment (for ILL people)
Tracing B

Cycle	WELL	ILL	DEAD	Cycle Reward
0	1.0000	0.0000	0.0000	0.0000
1	0.8075	0.1125	0.0800	0.8762
2	0.6137	0.2055	0.1808	0.7430
3	0.4373	0.2692	0.2935	0.6103
4	0.2908	0.3003	0.4089	0.4863
5	0.1657	0.3124	0.5218	0.3747
6	0.0787	0.2965	0.6248	0.2800
7	0.0374	0.2519	0.7107	0.2056
8	0.0178	0.2029	0.7793	0.1494
9	0.0084	0.1589	0.8327	0.1078
10	0.0040	0.1223	0.8737	0.0776
11	0.0019	0.0932	0.9049	0.0556
12	0.0009	0.0706	0.9285	0.0398
13	0.0004	0.0533	0.9462	0.0285
14	0.0002	0.0402	0.9596	0.0204
15	0.0001	0.0302	0.9697	0.0146
16	0.0000	0.0227	0.9773	0.0104
17	0.0000	0.0170	0.9830	0.0074
18	0.0000	0.0128	0.9872	0.0053
19	0.0000	0.0096	0.9904	0.0038
End	0.0000	0.0072	0.9928	0.0000

(e) Which strategy is preferred: new prophylaxis (Tracing A) or new treatment (Tracing B)?

(f) What factor(s) could change the result noted in (e)?

(g) (Optional) A constant discount rate is used in the analysis. Determine this discount rate *r* from the tracings.