Homework #4 (version 2)

ME 3250 Section 2

due 20 October 2023

1 Taylor–Green flow [16 points]

In 1937 Taylor and Green published an exact solution of the equations of incompressible-fluid motion. For two-dimensional flow, the velocity field is:

$$u(t, x, y) = e^{-\nu t} \sin x \, \cos y, \tag{1}$$

$$v(t, x, y) = -e^{-\nu t} \cos x \, \sin y. \tag{2}$$

a. Is the flow steady or unsteady?

b. Find the fluid acceleration $\vec{a}(t, x, y)$.

c. Find the general equation of the streamlines in the flow.

d. Find the equation for the streamline that passes by location $x = 1$ and $y = 1$.

e. Do the streamlines change with time? Is your answer consistent with your answer in question a? Please, briefly explain.

2 DSP Gate (again) [16 Points]

The engineers at Dive-by-the-seat-of-your-pants Gate (DSP Gate) are designing a new underwater propulsion system for their submersibles. A container is initially filled with 1.4 kg of Krypton gas. The nozzle exit has area $A = 0.2\ \mathrm{m}^2$ and the gas at the nozzle exit has density $\rho = 3.5\ \mathrm{kg/m}^3$. The uniform velocity of the gas at the nozzle exit is $V(t) = \frac{2}{(t+1)^2}\ \mathrm{m\,s}^{-1}$, where t is time.

a. At the nozzle exit, is the flow of Krypton gas steady or unsteady?

b. What is the mass flow rate at the nozzle exit?

c. What is the mass of the gas remaining in the container as a function of time?

d. How long will it take for the gas to run out?

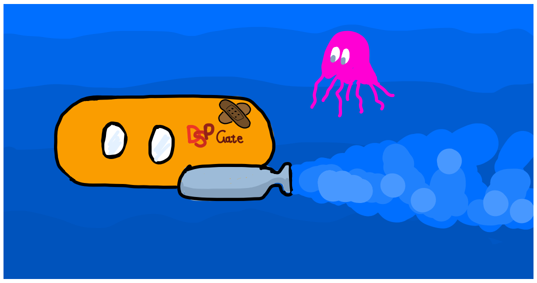

Figure 1: Artist's illustration of DSP Gate submersible with active propulsion system.

3 Control volume choice [16 Points]

We will compute the anchoring force of Example 5.10 of the textbook using a different shape of control volume. Please see the class notes for the control volume shape and results. Here, we will use the values for the velocity and area we used in class: $V_1 = 10$ m/s, $A_1 = A_2 = 2\ \mathrm{m}^2$, and density $\rho = 1000\ \mathrm{kg/m}^3$. We found in class that the force is $\vec{F} = -\rho V_1^2 A_1 (1 - \cos\theta)\ \hat{\imath} + \rho V_1^2 A_1 \sin\theta\ \hat{\jmath}$. Show that the anchoring force $\vec{F}$ is the same for both control volumes: the one we used in class and that of Figure 2. In your solution, clearly indicate were the two calculations differ.

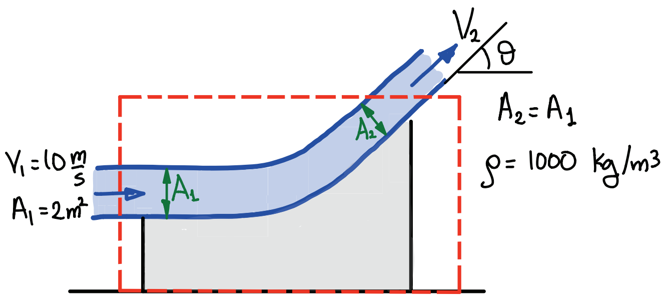

Figure 2: Another control volume for the problem we solved in class.

4 Bees (again) [16 Points]

The bees are planing another swarm on an airplane. This time, they are looking to improve their aerodynamic performance. Thus, they rented time in a wind tunnel to perform testing. A bee is placed in a circular wind tunnel of radius $R = 0.5$ m, see figure. At the wind tunnel test section inflow, the velocity is uniform $V_1 = 10\ \mathrm{m\,s^{-1}}$ and the gage pressure is $p_1 = 200$ Pa. At the test section outflow the velocity profile is no longer uniform and it is given by

$$V_2(r) = a(1 - \cos 2\pi r), \tag{3}$$

where a is an unknown (for now) parameter, and the gage pressure is $p_2 = 150$ Pa. All velocity vectors at Stations 1 and 2 are along the axial direction.

a. Find the value of the parameter a.

b. Find the drag force on the bee. The drag force is the resistance force on the bee from the flow in the wind tunnel. It is the force we need to apply on the support to keep the bee in place. We will neglect the support's resistance.

Assume steady incompressible flow and neglect the friction with the wind tunnel walls. Use $\rho = 1.2\ \mathrm{kg\,m^{-3}}$ for the density of air.

Helpful hits: You will need to work with cylindrical coordinates because the problem has cylindrical symmetry. At the wind tunnel exit the velocity depends on the coordinate r. Please be careful how you treat this in the integrals.

Also: this is a "real" problem. You can use this method to estimate the drag force on objects. No bees were harmed in the making of this problem.

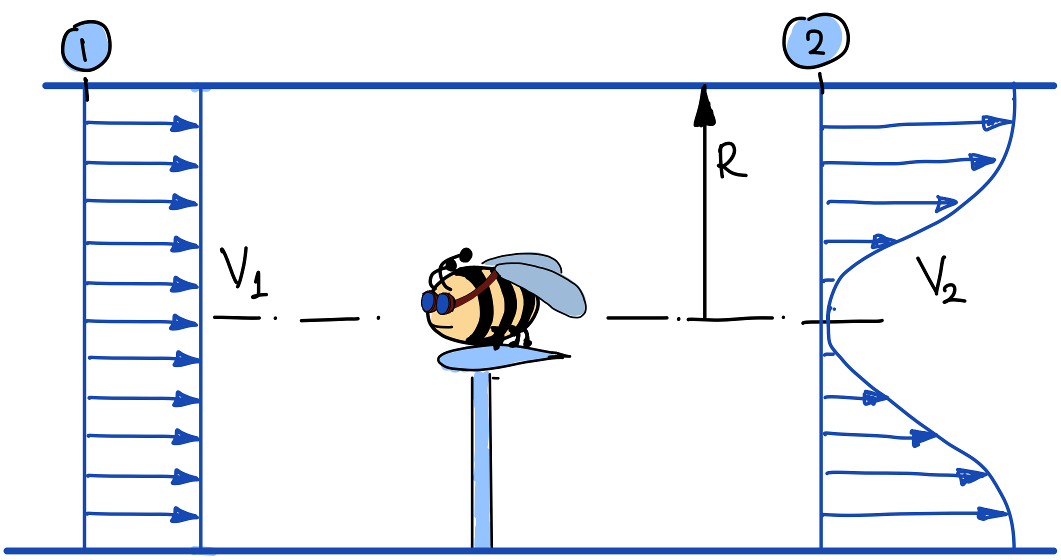

Figure 3: Bee in a wind tunnel.