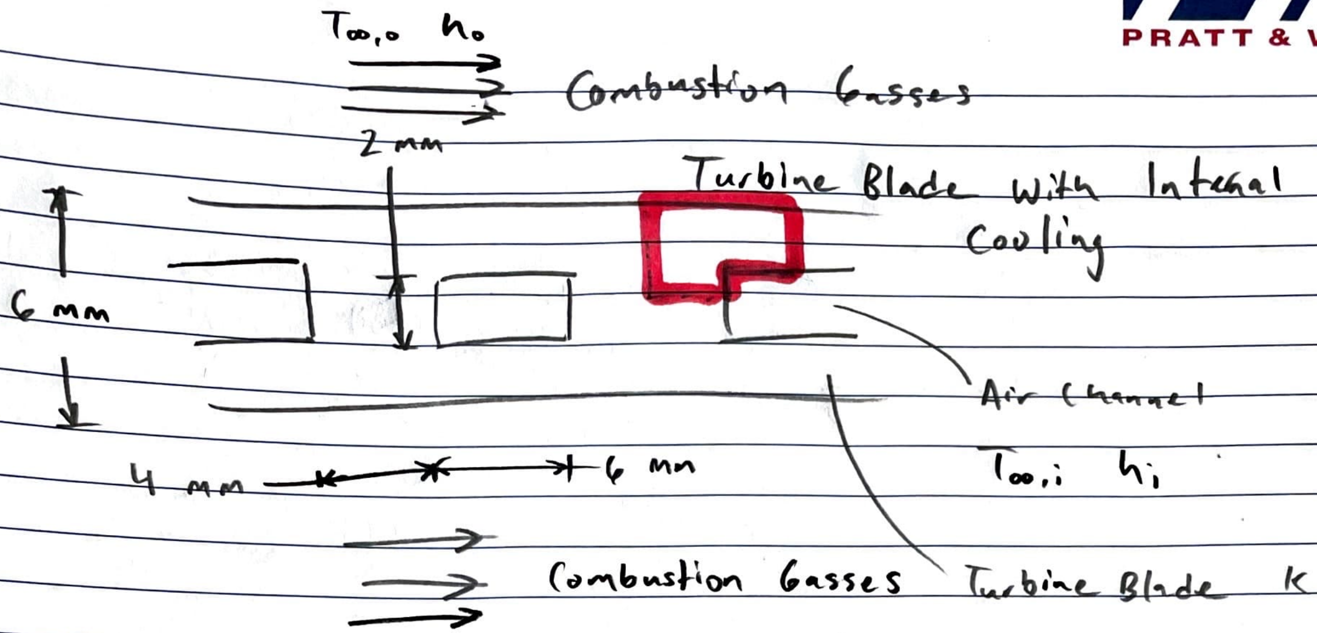
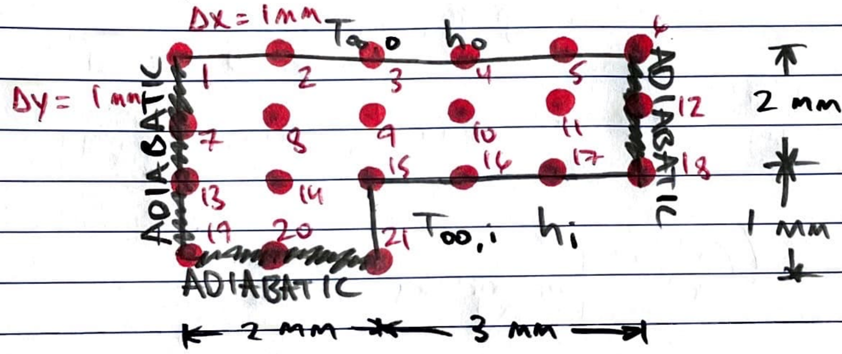


$h_o = 1000 \frac{W}{m^2 \cdot K}$, $T_{\infty,o} = 1700$ K

$k = 25 \frac{W}{m \cdot K}$

$h_i = 200 \frac{W}{m^2 \cdot K}$, $T_{\infty,i} = 400$ K

Using symmetry, we can reduce our domain to



Applying $-k \nabla^2 T = q$ in a finite difference scheme for each point

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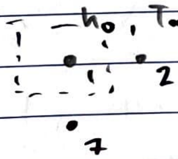
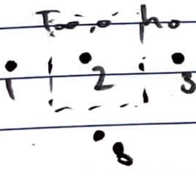
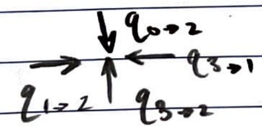


$$(T_2 + T_7) + \frac{2 h_o \Delta x}{k} T_{\infty,o} - 2\left(\frac{h_o \Delta x}{k} + 2\right) T_1 = 0$$

$$\rightarrow \quad T_2 + T_7 - \left(2 + \frac{h_o \Delta x}{k}\right) T_1 = -\frac{h_o \Delta x}{k} T_{\infty,o}$$





$$q_{1\to2} = \Delta y \Delta z\, k \left(\frac{T_1 - T_2}{\Delta x}\right)$$

$$q_{8\to2} = \Delta x \Delta z\, k \left(\frac{T_8 - T_2}{\Delta y}\right)$$

$$q_{0\to2} = \Delta x \Delta z\, h_o \left(\frac{T_0 - T_2}{\Delta y}\right) \qquad q_{3\to2} = \Delta x \Delta z\, k \left(\frac{T_3 - T_2}{\Delta x}\right)$$

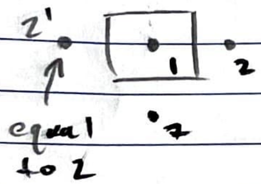
$$q_{9\to15} + q_{14\to15} + q_{16\to15} + q_{21\to15} + q_{i\to15} = 0$$

$$\left[\cancel{\Delta x}\, k \left(\frac{T_9 - T_{15}}{\cancel{\Delta y}}\right)\right] + \left[\cancel{\Delta y}\, k\left(\frac{T_{14}-T_{15}}{\cancel{\Delta x}}\right)\right] + \left[\left(\frac{\Delta x + \cancel{\Delta y}}{2}\right) h_i (T_i - T_{15})\right]$$

$$\left[\left(\frac{\cancel{\Delta y}}{2}\right) k \left(\frac{T_{16}-T_{15}}{\cancel{\Delta x}}\right)\right] + \left[\left(\frac{\cancel{\Delta x}}{2}\right)k\left(\frac{T_{21}-T_{15}}{\cancel{\Delta y}}\right)\right] +$$

$$\Delta x = \Delta y \rightarrow kT_9 + kT_{14} + \frac{k}{2}T_{16} + \frac{k}{2}T_{21}$$

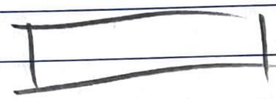
$$-(3k + \Delta x h_i)T_{15} = -\Delta x h_i T_i$$



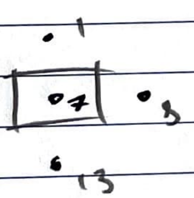
$\cancel{q_{2'\to1}} =$

$$kT_{2'} + kT_7 + kT_2 + (h_o \Delta x)T_o - (3k + \Delta x h_o)T_1 = 0$$

$$kT_7 + 2kT_2 + (h_o\Delta x)T_o - (3k+\Delta x h_o)T_1 = 0$$



$$\rightarrow q''_x = -k\frac{dT}{dx}$$



$$T_1 - 4T_7 + 2T_8 + T_{13} = 0 \quad \left(\frac{W}{m\cdot K}\right)\left(\frac{K}{m}\right) = \left(\frac{W}{m^2}\right)$$

$$q = A q''$$

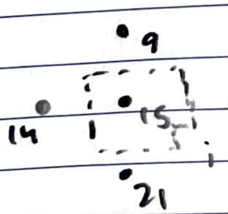
$$q_{1\to2} + q_{8\to2} + q_{3\to2} + q_{o\to2} = 0$$

$$\left(\cancel{\Delta y}\,\cancel{\Delta z}\,k\left(\frac{T_1-T_2}{\cancel{\Delta x}}\right)\right) + \left(\Delta x\,\cancel{\Delta z}\,k\left(\frac{T_8-T_2}{\cancel{\Delta y}}\right)\right) + \left(\cancel{\Delta x}\,\cancel{\Delta z}\,k\left(\frac{T_3-T_2}{\cancel{\Delta x}}\right)\right)$$

$$+ \left(\Delta x\,\cancel{\Delta z}\,h_o\left(T_o - T_2\right)\right)$$

$$\Delta y = \Delta x \rightarrow kT_1 + kT_8 + kT_3 + (h_o\Delta x)T_o$$

$$- (3k + \Delta x h_o)T_2 = 0$$



$$q_{i\to15} = \left(\frac{\Delta x}{2}+\frac{\Delta y}{2}\right)h_i(T_i - T_{15})$$

$$q_{14\to15} = \Delta y\, k\left(\frac{T_{14}-T_{15}}{\Delta x}\right), \quad q_{9\to15} = \Delta x\, k\left(\frac{T_9-T_{15}}{\Delta y}\right)$$

$$q_{21\to15} = \left(\frac{\Delta x}{2}\right)k\left(\frac{T_{21}-T_{15}}{\Delta y}\right), \quad q_{16\to15} = \left(\frac{\Delta y}{2}\right)k\left(\frac{T_{16}-T_{15}}{\Delta x}\right)$$

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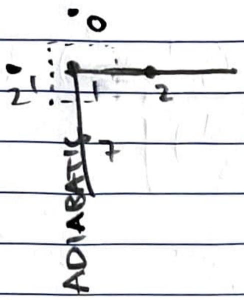


$$q_{0\to1} + q_{2'\to1} + q_{2\to1} + q_{7\to1} = 0$$

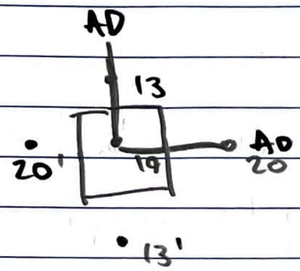
$$\left[\left(\Delta x + \cancel{2}\frac{\Delta y}{\cancel{2}}\right) h_0 \left(T_0 - T_1\right)\right]$$

$$+ \left[\frac{\cancel{\Delta y}}{2} k \left(\frac{T_{2'} - T_1}{\cancel{\Delta x}}\right)\right] + \left[\frac{\cancel{\Delta y}}{2} k \left(\frac{T_2 - T_1}{\cancel{\Delta x}}\right)\right]$$

$$+ \left[\cancel{\Delta x}\, k \left(\frac{T_7 - T_1}{\cancel{\Delta y}}\right)\right]$$

$\Delta x = \Delta y$ $\longrightarrow$

$$\cancel{2} \Delta x h_0 T_0 + k T_2 + k T_7 - (2k + \cancel{2} \Delta x h_0) T_1 = 0$$



$$q_{13\to19} + q_{13'\to19} + q_{20\to19} + q_{20'\to19}$$

$$\cancel{2}\left(\cancel{\Delta x}\, \cancel{k} \left(\frac{T_{13} - T_{19}}{\cancel{\Delta y}}\right)\right) + \cancel{2}\left(\cancel{\Delta y}\, \cancel{k} \left(\frac{T_{20} - T_{19}}{\cancel{\Delta x}}\right)\right)$$

$$T_{13} + T_{20} - 2T_{19} = 0$$

Node	Equation
1	$-(2k+\Delta x h_o)T_1 + (2k)T_2 + kT_7 = -h_o \Delta x T_o$
2	$kT_1 - (2k+\Delta x h_o)T_2 + kT_3 + 2kT_8 = -h_o \Delta x T_o$
3	$kT_2 - (2k+\Delta x h_o)T_3 + kT_4 + 2kT_9 = -h_o \Delta x T_o$
4	$kT_3 - (2k+\Delta x h_o)T_4 + kT_5 + 2kT_{10} = -h_o \Delta x T_o$
5	$kT_4 - (2k+\Delta x h_o)T_5 + kT_6 + 2kT_{11} = -h_o \Delta x T_o$
6	$(2k)T_5 - (2k+\Delta x h_o)T_6 + kT_{12} = -h_o \Delta x T_o$
7	$T_1 - 4T_7 + 2T_8 + T_{13} = 0$
8	$T_2 + T_7 - 4T_8 + T_9 + T_{14} = 0$
9	$T_3 + T_8 - 4T_9 + T_{10} + T_{15} = 0$
10	$T_4 + T_9 - 4T_{10} + T_{11} + T_{16} = 0$
11	$T_5 + T_{10} - 4T_{11} + T_{12} + T_{17} = 0$
12	$T_6 + 2T_{11} - 4T_{12} + T_{18} = 0$
13	$T_7 - 4T_{13} + 2T_{14} + T_{19} = 0$
14	$T_8 + T_{13} - 4T_{14} + T_{15} + T_{20} = 0$
15	$kT_9 + kT_{14} - (3k+\Delta x h_i)T_{15} + \frac{k}{2}T_{16} + \frac{k}{2}T_{21} = -\Delta x h_i T_i$
16	$kT_{10} + \frac{k}{2}T_{15} - (2k+\Delta x h_i)T_{16} + \frac{k}{2}T_{17} = -\Delta x h_i T_i$
17	$kT_{11} + \frac{k}{2}T_{16} - (2k+\Delta x h_i)T_{17} + \frac{k}{2}T_{18} = -\Delta x h_i T_i$
18	$kT_{12} + kT_{17} - (2k+\Delta x h_i)T_{18} = -\Delta x h_i T_i$
19	$T_{13} + T_{20} - 2T_{19} = 0$
20	$2T_{14} + T_{19} - 4T_{20} + T_{21} = 0$
21	$kT_{15} + kT_{20} - (2k+\Delta x h_i)T_{21} = -h_i \Delta x T_i$

$$c_1 = -(2k + \Delta x h_o)$$
$$c_2 = -(3k + \Delta x h_i)$$
$$c_3 = -(2k + \Delta x h_i)$$