

SPECTRAL INTERPOLATION FOR TURBULENT FLUID FLOW

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ABOUT ME

- I am a Junior Mechanical Engineering student at the University of Connecticut.
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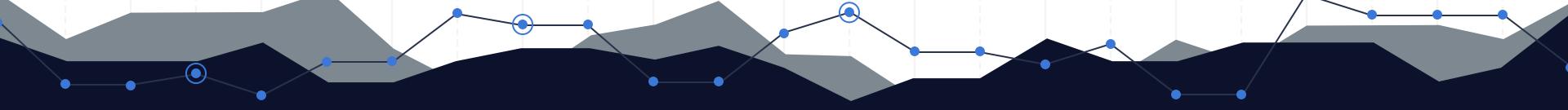
1D SPECTRAL INTERPOLATION

- Apply Discrete Fourier Transform (DFT) to sequence.
- Reconstruct sinusoids at desired position to obtain interpolation.

$$F[k] = \sum_{k=0}^{N-1} (f[n]e^{-\frac{2\pi kx}{N}i})$$

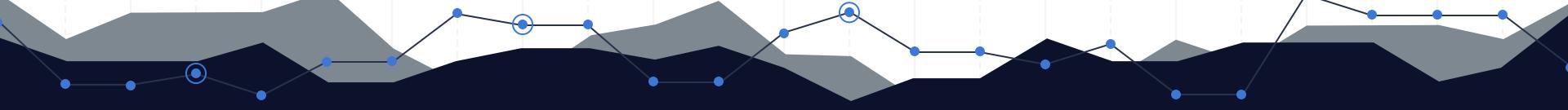
$$f(x) = \frac{1}{N} \sum_{k=0}^{N-1} (F[k]e^{\frac{2\pi kx}{N}i})$$

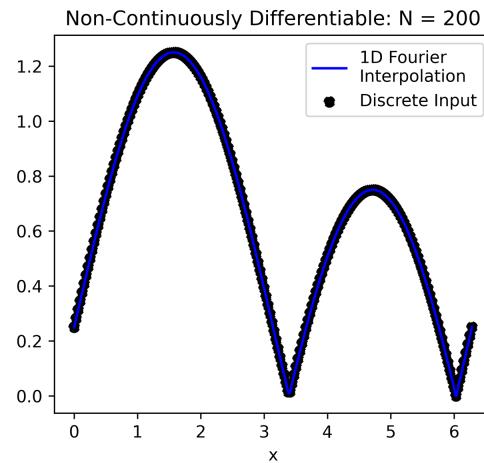
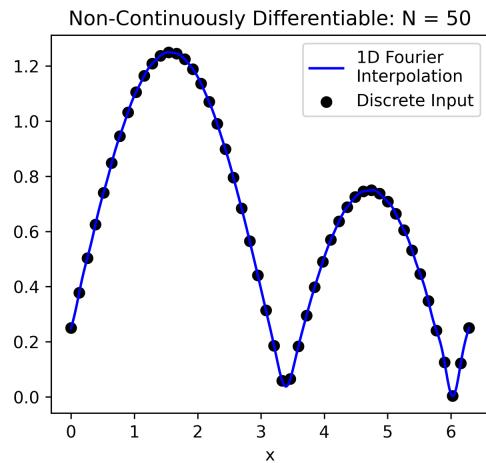
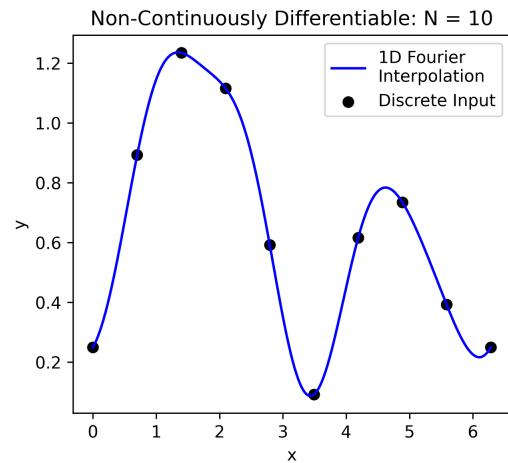
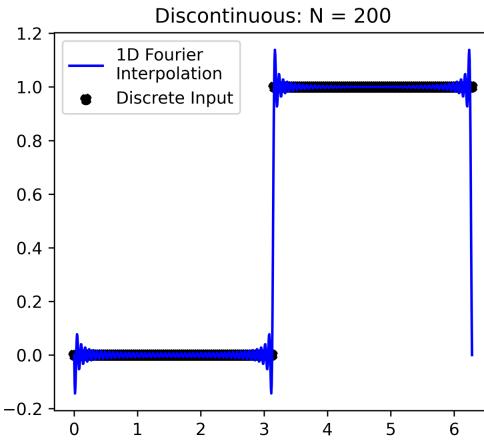
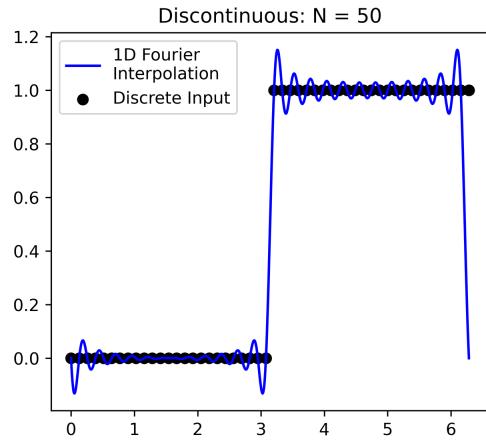
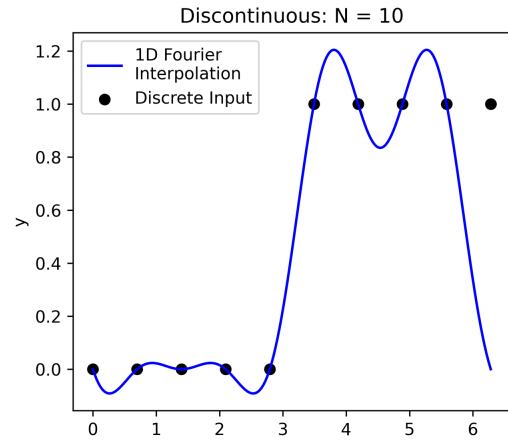
$$\text{inter1D}(n) = \frac{1}{N-1} \left[\text{Re}(F[0]) + 2 \sum_{k=1}^{\lfloor \frac{N-1}{2} \rfloor} \left(\text{Re}(F[k])\cos \frac{2k\pi n}{N} - \text{Im}(F[k])\sin \frac{2k\pi n}{N} \right) \right]$$



INTERPOLATION ASSUMPTIONS

- Discrete sequence must be N -periodic. Any periodic discontinuities will nullify the advantages a spectral interpolation has over linear interpolation.
- To avoid Gibbs Phenomenon, discrete sequence must converge to a continuously differentiable function as the number of elements within the sequence increases to infinity.

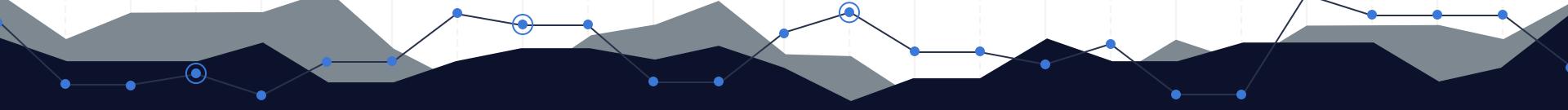




3D SPECTRAL INTERPOLATION

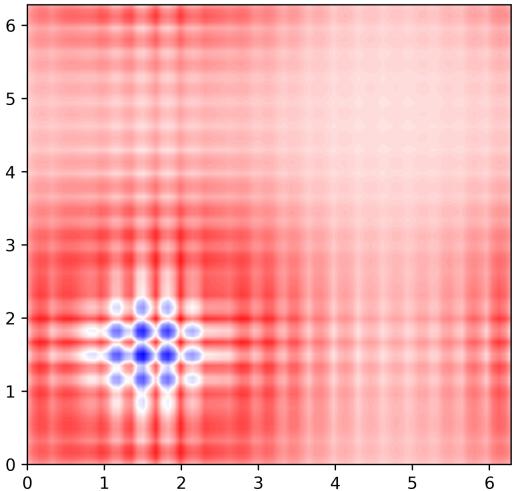
Consider a discrete right-hand coordinate grid with dimensions $N_i \times N_j \times N_k$. To obtain an interpolation at the coordinate (p_i, p_j, p_k) :

- At each k index, complete a 1D interpolation at $(p_i)_{j,k}$ for each j index.
- At each k index, complete a 1D interpolation at $(p_i, p_j)_k$ within the previously interpolated $(p_i)_{j,k}$ values.
- Complete a 1D interpolation at (p_i, p_j, p_k) within the previously interpolated $(p_i, p_j)_k$ values.

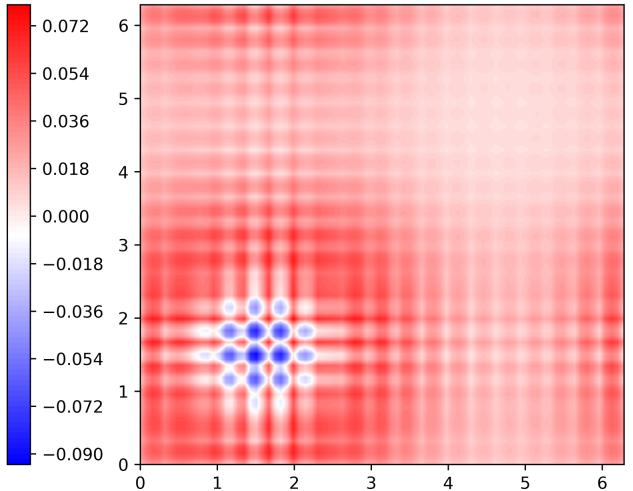


Function: $\rho(x, y, z) = e^{\sin(x) + \sin(y) + \sin(z)}$, Ni = 20, Nj = 20, Nk = 20

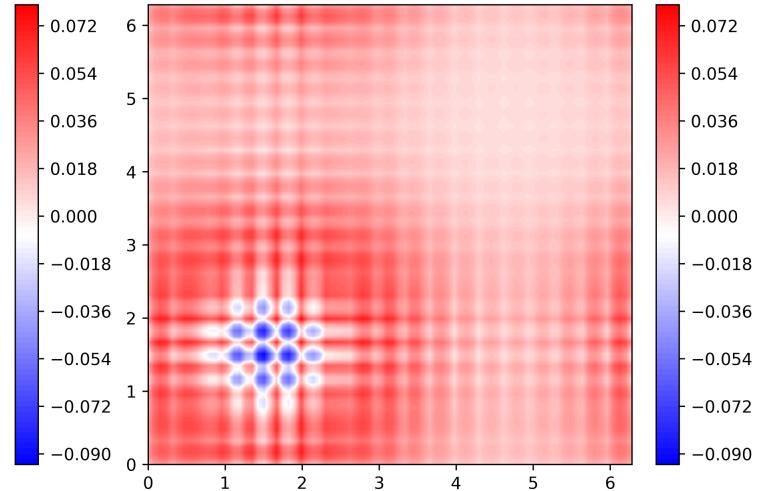
x = 3.1599 xyz Interpolation Error



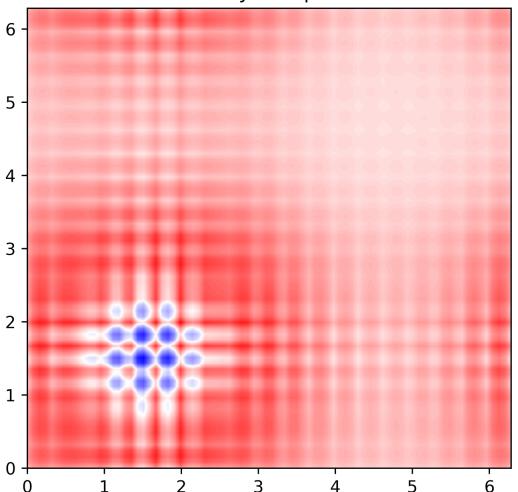
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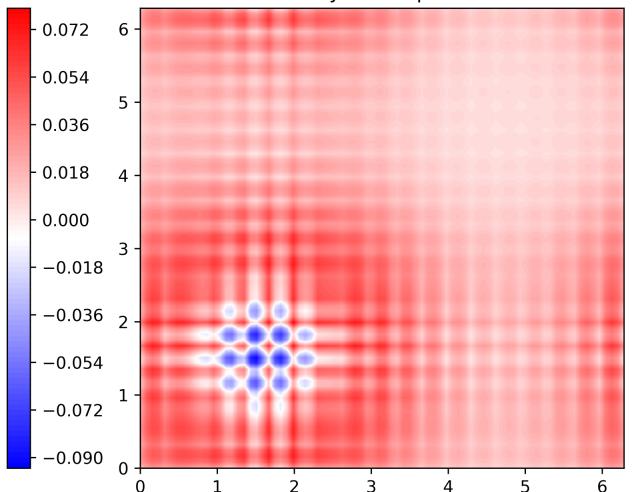
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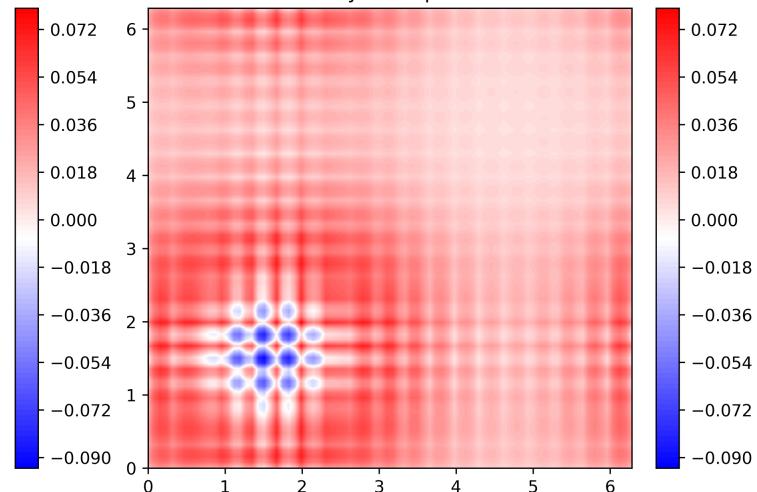
x = 3.1599 xzy Interpolation Error



x = 3.1599 yzx Interpolation Error



x = 3.1599 zyx Interpolation Error



3D INTERPOLATION CONVERGENCE

- Error arrays were generated by calculating the signed interpolation error to the known analytical function.
- A grid-normalized L2-Norm was completed on the error arrays.

$$\|e\|_2 = \sqrt{\frac{1}{N^3} \sum_i^N \sum_j^N \sum_k^N |e_{i,j,k}|^2}$$

$$\|e\|_2 = \sqrt{e_{i,j,k}^2} = e_{i,j,k} \approx 2.2 * 10^{-16}$$

