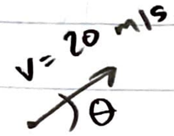
Euler Project 307: Firecracker Jacob Ivanov 03/10/2022




$v_x = 20\cos(\theta)$
$v_y = 20\sin(\theta)$

$a_y = 0 = (4.81)(t) = 20\sin(\theta)$ # t @ $y_{max}$

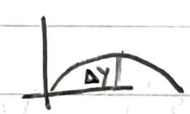
$\cancel{v_f^2} = v_0^2 + 2a\Delta y$

$\frac{v_0^2}{2a} = \Delta y = \frac{(20\sin(\theta))^2}{2(9.81)}$ , $\Delta x = (20\cos(\theta))\left(\frac{20\sin(\theta)}{9.81}\right)$

# Above is the coordinate of the y-max point for
# A particle with a given initial angle.

$x = \frac{400}{9.81}\cos(\theta)\sin(\theta) \Rightarrow \frac{dx}{d\theta} = 0$ @ $\theta = 45°$

$t_{turn} = 2\sqrt{\frac{2\Delta y}{g}}$




$\Delta x = (20\cos(\theta))(t)$

$\Delta y = (20\sin(\theta))(t) + \left(\frac{-9.81}{2}\right)(t)^2$

$\Delta y = \frac{1}{2}(9.81)(t)^2$

$t = \sqrt{\frac{2\Delta y}{9.81}} = \sqrt{\frac{200}{9.81}} = 4.51$

$t = \cancel{\tfrac{1}{4}}\sqrt{\frac{\cancel{2}\left(\frac{(20\sin(\theta))^2}{\cancel{2}(9.81)}\right)}{9.81}} + \sqrt{\frac{200 + \frac{(20\sin(\theta))^2}{2(9.81)}}{9.81}}$

$\Delta x(\theta) = t\,(20\cos(\theta))$

Since $x_{max} = (t)(20\cos(\theta))$, we need to find the max theta

$$\cancel{t(\theta) = \sqrt{\frac{(20\sin(\theta))^2}{9.81^2}} + \sqrt{\frac{200}{9.81}}} = \frac{20\sin(\theta)}{9.81} + \sqrt{\frac{200}{9.81}}$$

$$t(\theta) = \left(\frac{20\sin(\theta)}{9.81} + \sqrt{\frac{2\left(100 + \frac{(20\sin(\theta))^2}{2(9.81)}\right)}{9.81}}\right)$$

$$y = 100 + 20\sin(\theta)t + \frac{1}{2}(-9.81)(t)^2$$

$$x = 20\cos(\theta)t \Rightarrow \theta = \arccos\left(\frac{x}{20t}\right)$$

$$y = 100 + \cancel{20}\sin(\theta)\left(\frac{x}{\cancel{20}\cos(\theta)}\right) + \frac{1}{2}(-9.81)\left(\frac{x}{20\cos(\theta)}\right)^2$$

$$= 100 + \tan(\theta)x - \frac{9.81}{800\cos^2(\theta)}x^2$$

$$x = 20\cos(\theta)$$

$$\theta = \cos^{-1}\left(\frac{x}{20}\right)$$

$$\cancel{y = 100 + \tan\left(\cos^{-1}\left(\frac{x}{20}\right)\right)x - \frac{9.81x^2}{800\cos^2\left(\cos^{-1}\left(\frac{x}{20}\right)\right)}}$$

$$\cancel{y = 100 + 20\sin(\theta) - \frac{9.81(20\cos(\theta))^2}{800\cos^2(\theta)}}$$

$$\cancel{= 100 + 20\sin(\theta) - \frac{9.81}{2}}$$

$$y_{max} = f(x, t, \theta)(90 - \theta)$$

$$y_{max} = 100 + \frac{(20\sin(\theta))^2}{2(9.81)} = 100 + \frac{200}{9.81}\sin^2(\theta)$$

$$\frac{dy_{max}}{d\theta} = \frac{200}{9.81}(2\cos(\theta)\sin(\theta)) = \frac{400}{9.81}\cos(\theta)\sin(\theta)$$

$$\frac{dy_{max}}{d\theta} = 0 = \cancel{\frac{400}{9.81}}\cos(\theta)\sin(\theta)$$

$\cos(\theta) = 0$ @ $\theta = 90°$, $\sin(\theta) = 0$ @ $\theta = 0°$

$$\cancel{y_{max} = 100 + \frac{200}{9.81}\sin^2\left(\arccos\left(\frac{x}{20t}\right)\right)}$$

$$y(x,\theta) = 100 + \tan(\theta)x - \frac{9.81x^2}{800\cos^2(\theta)}$$

$$\frac{\partial y(x,\theta)}{\partial\theta} = x\sec^2(\theta) - \frac{9.81x^2\sin(\theta)}{400\cos^3(\theta)}$$

$$y = 100 + \frac{200}{9.81} - \left(\frac{9.81}{800}\right)x^2 = x = \pm\sqrt{\frac{100+\frac{200}{9.81}}{\frac{9.81}{800}}} = \pm x_1$$

$$\cancel{V = \pi\int_0^{x_1}\left(100 + \frac{200}{9.81} - \left(\frac{9.81}{800}\right)x^2\right)^2 dx}$$

$$\sqrt{\frac{y - 100 - \frac{200}{9.81}}{-\left(\frac{9.81}{800}\right)}} = x$$

$$V = \pi\int_0^{100+\frac{200}{9.81}}\left(\frac{y-100-\frac{200}{9.81}}{-\left(\frac{9.81}{800}\right)}\right)dy = 1854532.8455$$