Lab 1: Heat Engine

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Abstract
Engines have applications that require a large amount of mechanical work. However, the mechanical work that the engine has can be affected by the cyclical temperatures of the engine. This report describes the effect the cold reservoir temperature has on the mechanical work an engine provides. A piston experiment was performed with two different cold reservoir cycles: one at room temperature and one in ice cold water. The height changes in the piston were recorded in the cycle, and the process thermodynamic values and its uncertainties were calculated. The results were that the ice water useful work and efficiency was 43.41±1.44mJ and 0.00367±0.00013 and room temperature useful work was 29.38±1.40mJ and 0.003110±0.00015. It was concluded that while the hypothesis was correct, uncertainties in the net heat and net work were too large and a reassessment of the experiment was made.

Introduction
Engines have a variety of purposes as they allow homes to be heated, cars to be moved, and airplanes to fly. An engine, however, is not just simply a magical device that moves and powers everything though, it is done through the utilization of pistons. These pistons work by simply transmitting a force from a gas that is expanding in a cylinder to a crankshaft. The piston goes through a cycle of events to convert the energy given to it into work [1]. This process can be summarized as heat is inputted into the gas cylinder, which increases the volume in it and creates work. The heat is then removed from the cylinder and the pressure of the gas decreases. Work is then inputted back into the piston so that it goes back to its original state. An issue with these pistons though is determining how much work is done by the piston and what process can give the most work. The more work a single piston can give, the better the performance of the entire system. For these processes, there are two types of work: useful mechanical work and net thermodynamic work. The useful mechanical work is the total work that the piston does. In the case of the following lab, this is the work done to lift the mass. The net thermodynamic work is the total work done in the system. A ratio called thermal efficiency can be used to calculate how efficient the piston is. This value can be simplified to a ratio of the cold reservoir temperature and the hot reservoir temperature where the colder the reservoir temperature, the higher the thermal efficiency [2]. This helps determine how well the piston does when producing useful mechanical work. The following lab will be investigating useful mechanical work and net thermodynamic work done depending on the cycle a piston goes through. One cycle will be done in ice-cold water and another cycle will be done in room temperature water, both of which are assumed to be in quasi-equilibrium. Pressures were calculated using a FBD method and ideal gas law. The quantity to be measured is the height of the piston and uncertainties with it. Measurement uncertainties will go through propagation of error to ensure that the final work values are within a small range of error. These measurements will be used to find useful mechanical work and net thermodynamic work for both cycles and the two cycles will be compared. It is hypothesized that the ice-bath cycle will have a higher useful mechanical work done since it has a lower cold reservoir temperature.

Methods
Equipment used for this lab includes a PASCO TD-8572, 2 Pyrex beakers, a set of masses totaling 100g, a hotplate, an ice water bath, a digital caliper, and measuring tape. Height measurements of the piston will be done using the PASCO TD-8572 as it has a built-in graduated cylinder and the dimensions of the connecting hose and air cylinder were done with a measuring tape and digital caliper. In terms of environmental effects, the experiment should be done at room temperature as the temperature in the room can affect the final results. In terms of software, Excel and Python were used. Measurements (and their

zero-order uncertainty) were made on the connecting hose, air cylinder, and piston to calculate the system volume. A Pyrex beaker was filled with water and then boiled on the hot plate [100ºC] and served as the hot reservoir. The air cylinder connected to the piston was put into an ice water bath [0ºC] and served as the cold reservoir. The shutoff valve for the piston was opened to decompress the piston and return it to a pressure of zero. The piston was then manually moved to a height of around 20mm and the valve was then closed. The height of the piston was recorded; this height served as a starting point A for the entire piston cycle. The 100g mass was added to the piston platform, piston height was recorded and served as point B for the cycle. The air cylinder was then moved to the boiling water on the hot plate, the piston height was measured, and served as point C. The mass was then removed from the piston and another height measurement was taken for point D. Then the air cylinder was put back into the ice-water bath and the piston height was recorded, serving as point A'. This step is where everything goes back to point A because, in theory, the piston goes back to its original height. In reality, this does not happen which is why point A' exists. The entire process was then done two additional times for the ice-bath water and then redone for just room temperature water [20ºC]. In total, six trials were done for this experiment. With the piston heights, the volume was calculated by totaling the entire system's volume. So this includes the volume in the connecting hose, the air cylinder, and the volume at the specific piston height recorded. One method to calculate pressure was using ideal gas law. This was done using the molecular weight of air, the boiling, freezing, and room temperature of water, and the total volumes. The ideal gas law is simply:

$$PV = \frac{R^o}{M}T \tag{1}$$

where P is pressure, V is the total volume, M is the molecular weight, R^o is the universal gas constant, and T is temperature [3]. As ideal gas law was used, assumptions associated with the equation were also applied. The Z factor was assumed to be 1, as the environment is not near the critical point for air. If temperatures are the same between two states, the equation can be used to find the pressure or volume from one state to the next. Work was calculated using the following equation:

$$W = \int_1^2 PdV \tag{2}$$

where W is the work, P is pressure, and V is the total volume [3]. Depending on the state, the work equation changes. The four stages were then described and a p-V diagram was made for the ice-bath reservoir and the room temperature reservoir. The useful mechanical work and net thermodynamic work were calculated by summing the input work for useful mechanical work and all the calculated work for the net thermodynamic work. Inefficiencies in the system were found and uncertainties were calculated by using propagation of error, specifically sequential perturbation. The equations for this is as follows:

$$u_f = [D_1^{\ 2} + D_2^{\ 2} +\ldots + D_L^{\ 2}]^{\frac{1}{2}} \tag{3}$$

Where u_f is the final uncertainty and D_1, D_2, D_L, etc. are the disturbed values which are found by calculating the uncertainty of the functions by sequentially perturbing the input values by their uncertainties. This ensures that uncertainty does not have too big of an effect on the values calculated, otherwise, the experiment will need to be redone. The thermal efficiency is calculated using the equation:

$$\eta_{UW} = \frac{UW_{NET}}{Q_{B\rightarrow C}} \tag{4}$$

where UW_{NET} is the net useful mechanical work, $Q_{B\rightarrow C}$ is the heat added to the system for process B→C. From the calculated work and uncertainties, it is possible to determine if the ice reservoir or the room temperature reservoir gives the most work. Also note that another method of calculating the pressures was done using the free body diagram (FBD) method. This method involves using the following equation:

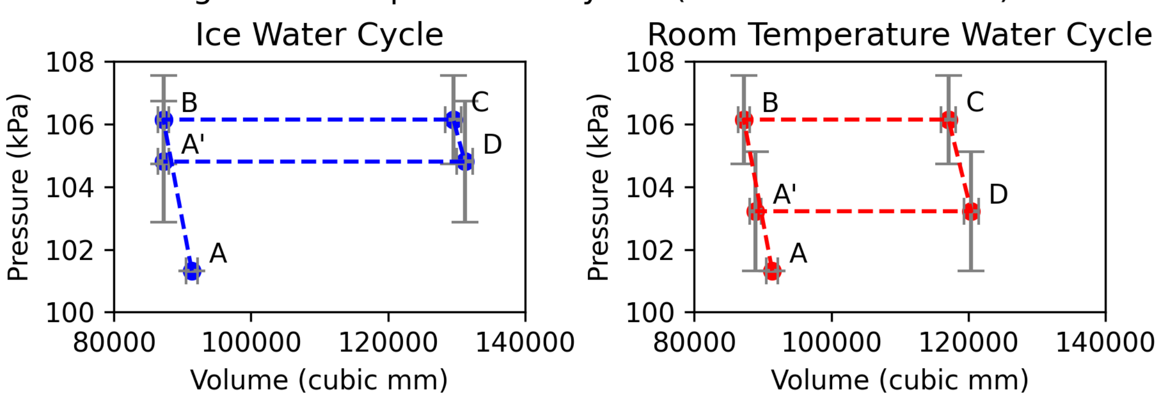
$$P = \frac{F}{A} + P_o \tag{5}$$

Where P_o is the atmospheric pressure, F is the gravitational force on the piston, A is the cross-sectional area on the piston and P is the final pressure. This method was used in hopes of seeing and solving the inaccuracies from the ideal gas method. From this, if the hypothesis is incorrect, then a revaluation of the experimental procedure and fundamental concepts of how a piston works should be completed.

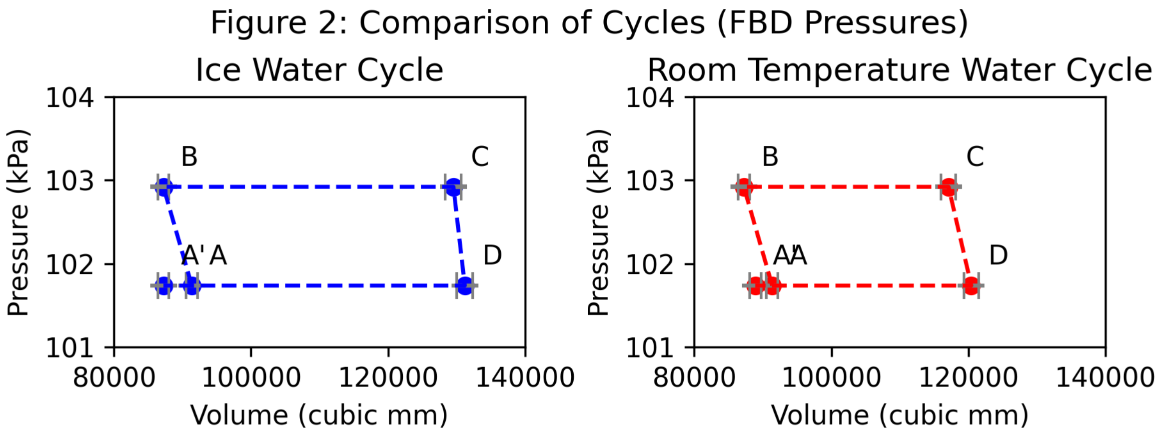
Results & Discussion

For this section, hand calculations completed for the lab are in the appendix and the ice water cold-reservoir cycle/room temperature water cold-reservoir cycle will be referred to as the IWC and RTWC, respectively. With piston heights recorded, the heights for each cycle were averaged. Total system volume at each cycle was calculated by approximating the air canister, air hose, and piston cylinder as three connected cylinders, and summing their volume. It was found that the uncertainty in total system volume at each cycle was dominated by the individual uncertainties of the air canister dimensions. For example, the volume at stage A can be found to be 91369.41±2877.16 or ±859.37 mm^3 including air canister dimension uncertainties, or not, respectively. Volumes at each cycle with both calculation methods can be found in the appendix. Two methods were used to calculate the pressures for each cycle. The first method utilizes the isothermal simplification of the ideal gas law, that is, Boyle's Law. However, this calculation does not account for lost system energy. This simplification results in inaccuracies, which will be discussed later. The second method uses the free body diagram (FBD) of the piston at each thermomechanical state as described in the Methods section. It should be noted that while the first method results in different pressures at the same stage between the IWC and RTWC cycles, the FBD method is consistent for both cycles. The tabulated pressures for each cycle are shown in the appendix in Table (1).

Figure 1: Comparison of Cycles (Ideal Gas Pressures)



As can be seen in Figure (1) above, cycle A' does not return to the same pressure as stage A in either the IWC or RTWC cycle. This is illogical, as it is in an identical thermomechanical state (100g mass removed, at 0°C). Furthermore, because the ideal gas pressures method uses the volumes at each state to sequentially calculate the pressure, we obtain uncertain values, which can be seen by the relative magnitudes of the error bars. As such, the FBD method will be used instead.



As can be seen from Figure (2) above, the FBD pressures calculation result in a much more logical similarity for states A and A'. It should also be noted that for Figures (2) and (3), it may make sense to plot process A→B and C→D as isothermal curves from theory, however, as no data was taken between the points, straight dashed lines have been added instead. It can be safely assumed that the working fluid air is a calorically perfect gas throughout both cycles, as the total temperature differential is relatively small. Since the air cylinder remains in the cold reservoir throughout process A→B, and the hot reservoir throughout process C→D, both processes are isothermal. The work for both processes can be easily calculated with Equation (5). As the internal energy will remain constant throughout both processes (due to constant temperature), the heat for both processes will be equal to the work. As the mechanical configurations of states A, D, and A' are identical (100g mass is removed from the piston) and states B, C are similarly identical (100g mass is added to the system), process B→C and D→A' can be approximated to be isobaric. As such, the work for each process can be easily calculated with Equation (5). The known temperature differential, density, and specific heat values can be used to find heats for each process. One additional thermodynamic quantity can be calculated, which will be referred to as the "useful work" and is obtained through the use of Equation (5). A comparison of thermodynamic process values calculated with both ideal gas and FBD pressure are tabulated in Tables (3), (4) in the appendix. It should be noted that these tables used FBD pressures, rather than ideal gas pressures. The sign convention used is that energy added to the system in the form of heat (Q) is positive and in the form of work (W) is negative.

Table 2: *Comparison of Net Thermodynamic Values (FBD Pressures) by Cycle*

Cycle	**Net Heat (mJ)**	**Net Work (mJ)**	**Net Useful Work (mJ)**
Ice Water Cycle (IWC)	-262.20±263.35	-380.97±296.61	43.41±1.44
Room Temp Water Cycle (RTWC)	-95.11±235.13	-132.37±285.46	29.38±1.40

It can be seen from Table (2) that the useful mechanical work for the IWC is greater than that of the RTWC. This is in line with the original hypothesis: that due to the larger temperature differential in the IWC compared to the RTWC, the IWC would have a greater output of useful mechanical work, even though the two cycles are otherwise identical. Due to the comparatively large uncertainty in the net combined work for both cycles, it is impossible to say with any real degree of confidence what the overall thermal efficiency is. However, if only the net useful work is considered, and the net thermodynamic work is ignored, then Equation (4) can be used to calculate the thermal efficiency. The useful work thermal efficiency for the IWC and RTWC cycles can be calculated by using Equation (4) to be 0.00367±0.00013 and 0.003110±0.00015, respectively. These are roughly equal, showing that the additional useful work for the IWC cycle is due to the additional heat transfer over process B→C, which is directly proportional to the change in temperature over that process, reiterating the original hypothesis.

Conclusion

It was found that the RTWC net useful work was 29.38±1.40 mJ and the IWC net useful work was 43.41±1.44 mJ. In terms of efficiency, the IWC thermal efficiency was 0.00367±0.00013 and the RTWC thermal efficiency was 0.003110±0.00015. The original hypothesis was correct in that IWC would have higher useful work than RTWC. So, in order to get more work out of a system, having a bigger difference between the hot and cold reservoir is needed. In terms of net useful work, the uncertainties did not widely impact the final results, but it did for the net heat and the net thermodynamic work. Unfortunately, there were many sources of error in this experiment of varying significance that were unaccounted for.

1. The radiative effect of the entire apparatus, which lost/gained heat to/from the ambient air and not just the cold/hot reservoirs. The process B→C, D→A' heats were therefore not technically proportional only to the difference in reservoir temperature. In reality, this had an insignificant effect. By insulating the apparatus, this source of error can be mitigated in further experiments.
2. The assumption that only the working fluid air absorbed/lost heat throughout the experiment. In reality, the entire experimental apparatus, from the air canister, hosing, and piston/cylinder assembly was heating/cooling. Though this has little effect on the thermodynamic work calculations, it likely caused us to severely underestimate the process B→C, D→A' heats. However, since the calculated heat is (in practice) directly proportional to the specific heats and masses, if one was to expand the analysis to include additional "working mass", it would only scale the useful work efficiencies. By increasing the working fluid in comparison to the apparatus mass, which can be accomplished by just increasing the system volume due to the Square-Cube Law, this source of error can be mitigated in any further experiments.
3. Any frictional force in the piston-cylinder assembly was ignored. With no readily available method to calculate the coefficient of friction of the sliding piston against the cylinder walls, it was assumed that it was negligible. This caused the calculated FBD pressures to be slightly underestimated. However, during the setup it was found that the piston cylinder freely moved with very little force, and so this was not an unreasonable assumption. In further experiments, this can be mitigated by calculating the coefficient of friction via basic physics principles.
4. It is possible that the piston/cylinder was not perfectly sealed, causing some degree of pressure loss due to leakage. This would make the quasi-equilibrium assumption invalid, however, each cycle stage was left undisturbed for about 30 seconds or so, and no significant height changes were observed. It is not believed this had a significant effect on further calculations. By speeding up the experiment, pressure leakage can be mitigated in further experiments.

From this, it should be said that the data for this report is limited to only the final useful work calculated and the thermal efficiencies. In terms of improvements to be made to the lab, one thing that could be done is adding a lubricant along the inside of the piston in order to decrease the frictional forces inside the piston-cylinder assembly. Also a validation test to ensure the piston was properly sealed should be conducted to ensure the piston being used is not damaged.

References

[1] *Piston*. Piston - Energy Education. (n.d.). Retrieved January 25, 2023, from https://energyeducation.ca/encyclopedia/Piston#:~:text=Pistons%20work%20by%20transferring%20the,known%20as%20a%20reciprocating%20engine

[2] *Thermal efficiency*. Thermal efficiency - Energy Education. (n.d.). Retrieved January 25, 2023, from https://energyeducation.ca/encyclopedia/Thermal_efficiency

[3] ME3264 – Lecture 4, ERROR PROPAGATION, SEQUENTIAL PERTURBATION [pdf]. Retrieved February 7th, 2023, from UConn ME3264 Blackboard

Appendix Tables

Table 1: Cycle *Pressures by Calculation Method*

Cycle	IWC Ideal Gas Pressure (Pa)	RTWC Ideal Gas Pressure (Pa)	FBD Pressure (Pa)
A	101325±0	101325±0	101738.89±7.54
B	106143.59±1410.69	106143.59±1410.69	102921.42±12.12
C	106143.59±1410.69	106143.59±1410.69	102921.42±12.12
D	104801.18±1926.62	103218.31±1905.13	101738.89±7.54
A'	104801.18±1926.62	103218.31±1905.13	101738.89±7.54

Table 3: *Thermodynamic Values (FBD Pressures), Ice Water Cycle*

Process	Heat (mJ)	Work (mJ)	Useful Work (mJ)
A→B (Isothermal)	-431.88±127.03	-431.88±127.03	-6.62±0.94
B→C (Isobaric)	11804.93±111.03	4354.44±147.38	67.54±0.98
C→D (Isothermal)	169.68±169.00	169.68±169.00	0.69±0.24
D→A' (Isobaric)	-11804.93±111.03	-4473.212±146.828	-18.20±0.40
NET	-262.20±263.35	-380.97±296.61	43.41±1.44

Table 4: *Thermodynamic Values (FBD Pressures), Room Temperature Water Cycle*

Process	Heat (mJ)	Work (mJ)	Useful Work (mJ)
A→B (Isothermal)	-431.88±127.03	-431.88±127.03	-6.62±0.94
B→C (Isobaric)	9443.94±88.82	3169.95±149.31	47.68±0.96
C→D (Isothermal)	336.77±152.88	336.77±152.88	1.37±0.24
D→A' (Isobaric)	-9443.94±88.82	-3207.209±140.301	-13.05±0.33
NET	-95.11±235.13	-132.37±285.46	29.38±1.40

Table 5: *Total System Volume (Ignoring Canister Dimension Uncertainties)*

Cycle State	IWC Volume (mm^3)	RTWC Volume (mm^3)
A	91369.413±859.369	91369.413±859.369
B	87221.529±826.769	87221.529±826.769
C	129529.946±1169.136	117086.294±1066.614
D	131189.100±1182.886	120404.602±1093.842
A'	87221.529±826.769	88880.683±839.774

Table 6: *Total System Volume (Including Canister Dimension Uncertainties)*

Cycle State	IWC Volume (mm^3)	RTWC Volume (mm^3)
A	91369.413±2877.164	91369.413±2877.164
B	87221.529±2876.142	87221.529±2876.142
C	129529.946±2897.157	117086.294±2888.549
D	131189.100±2898.456	120404.602±2890.648
A'	87221.529±2876.142	88880.683±2876.523

Table 7: *Thermodynamic Values (Ideal Gas Pressures), Ice Water Cycle*

Process	Heat (mJ)	Work (mJ)	Useful Work (mJ)
A→B (Isothermal)	-430.12±126.52	-430.12±126.52	-6.62±0.94
B→C (Isobaric)	11804.93±111.03	4490.77±163.29	67.54±0.98
C→D (Isothermal)	174.99±174.31	174.99±174.31	0.69±0.24
D→A' (Isobaric)	-11804.93±111.03	-4607.85±173.35	-18.20±0.40
NET	-255.13±266.55	-372.21±321.10	43.41±1.44

Table 8: *Thermodynamic Values (Ideal gas Pressures), Room Temperature Water Cycle*

Process	**Heat (mJ)**	**Work (mJ)**	**Useful Work (mJ)**
A→B (Isothermal)	-430.12±126.52	-430.12±126.52	-6.62±0.94
B→C (Isobaric)	9443.94±88.82	3169.95±149.31	47.68±0.96
C→D (Isothermal)	347.32±157.73	347.32±157.73	1.37±0.24
D→A' (Isobaric)	-9443.94±88.82	-3253.85±154.49	-13.05±0.33
NET	-82.80±238.04	-166.70±295.04	29.38±1.40

Table 9: *Other Measurements*

Hose Inner Diameter	3.3±0.0 mm
Hose Length	510±25 mm
Air Canister Diameter	37.5±0.5 mm
Air Canister Depth	60±2.0 mm
Piston Diameter	32.5±0.1 mm
Piston/Platform Mass	35.0±0.6 g

Appendix Raw Data

Table 10: *Experimental Raw Data*

Ice Water Cycle								
Cycle Point	Temperature (K)	Trial 1 Height (mm)	Trial 2 Height (mm)	Trial 3 Height (mm)	Average Height (mm)		Total Volume (Simple Uncertainties) (mm³)	Total Volume (All Uncertainties) (mm³)
A	273	25±0.5	25±0.5	25±0.5	25±0.5		91369.413±859.369	91369.413±2877.164
B	273	19±0.5	20±0.5	22±0.5	20±0.5		87221.529±826.769	87221.529±2876.142
C	373	69±0.5	71±0.5	72±0.5	71±0.5		129529.946±1169.136	129529.946±2897.157
D	373	71±0.5	74±0.5	75±0.5	73±0.5		131189.100±1182.886	131189.100±2898.456
A'	273	16±0.5	21±0.5	22±0.5	20±0.5		87221.529±826.769	87221.529±2876.142
Room-Temp Water Cycle								
Cycle Point	Temperature (K)	Trial 1 Height (mm)	Trial 2 Height (mm)	Trial 3 Height (mm)	Average Height (mm)		Total Volume (Simple Uncertainties) (mm³)	Total Volume (All Uncertainties) (mm³)
A	293	25±0.5	25±0.5	25±0.5	25±0.5		91369.413±859.369	91369.413±2877.164
B	293	21±0.5	20±0.5	20±0.5	20±0.5		87221.529±826.769	87221.529±2876.142
C	373	55±0.5	58±0.5	56±0.5	56±0.5		117086.294±1066.614	117086.294±2888.549
D	373	58±0.5	61±0.5	62±0.5	60±0.5		120404.602±1093.842	120404.602±2890.648
A'	293	22±0.5	22±0.5	21±0.5	22±0.5		88880.683±839.774	88880.683±2876.523

Appendix Calculations

$V_A = 91369\ mm^3 = 91.369 \cdot 10^{-6}\ m^3$
$V_B = 87221\ mm^3 = 87.221 \cdot 10^{-6}\ m^3$
$V_C = 129529\ mm^3 = 129.529 \cdot 10^{-6}\ m^3$
$V_D = 131189\ mm^3 = 131.189 \cdot 10^{-6}\ m^3$
$V_{A'} = 87221\ mm^3 = 87.221 \cdot 10^{-6}\ m^3$

$P_A = 101325\ Pa$
$P_B = 106144\ Pa$
$P_C = 106144\ Pa$
$P_D = 104800\ Pa$
$P_{A'} = 104800\ Pa$

A → B: Isothermal Compression

$$P_A V_A = P_B V_B \rightarrow P_B = P_A\left(\frac{V_A}{V_B}\right) = (101325\ Pa)\left(\frac{91.369 \cdot 10^{-6}\ \cancel{m^3}}{87.221 \cdot 10^{-6}\ \cancel{m^3}}\right) = 106144\ Pa$$

$$W_{A \to B} = P_A V_A \ln\left(\frac{V_B}{V_A}\right) = (101325\ Pa)(91.369 \cdot 10^{-6}\ m^3) \ln\left(\frac{87.221 \cdot 10^{-6}\ \cancel{m^3}}{91.369 \cdot 10^{-6}\ \cancel{m^3}}\right)$$

$$= -430.14\ mJ$$

$$\Delta U = U_B - U_A = U(T_B) - U(T_A) = 0 \therefore Q_{A \to B} = -430.14\ mJ$$

B → C: Isobaric Expansion

$$W_{B \to C} = P_B(V_C - V_B) = (106144\ Pa)(129.529 - 87.221) \cdot 10^{-6}\ m^3$$

$$= 4490.74\ mJ$$

$$Q_{B \to C} = mC_p(T_C - T_B) = V_A \rho_A C_p (T_C - T_B)$$

$$= (91.369 \cdot 10^{-6}\ \cancel{m^3})\left(1.292\ \frac{kg}{\cancel{m^3}}\right)\left(1.0\ \frac{kJ}{kg \cdot K}\right)(100 - 0)\ K = 11804.87\ mJ$$

C → D: Isothermal Expansion

$$P_D = P_C\left(\frac{V_C}{V_D}\right) = (106144\ Pa)\left(\frac{129.529 \cdot 10^{-6}\ \cancel{m^3}}{131.189 \cdot 10^{-6}\ \cancel{m^3}}\right) = 104800\ Pa$$

$$W_{C \to D} = P_C V_C \ln\left(\frac{V_D}{V_C}\right) = (106144\ Pa)(129.529 \cdot 10^{-6}\ m^3) \ln\left(\frac{131.189 \cdot 10^{-6}\ \cancel{m^3}}{129.529 \cdot 10^{-6}\ \cancel{m^3}}\right)$$

$$= 175.08\ mJ$$

$$Q_{C \to D} = 175.08\ mJ$$

D → A': Isobaric Compression

$$W_{D \to A'} = P_D(V_{A'} - V_D) = (104800\ Pa)(87.221 - 131.189) \cdot 10^{-6}\ m^3$$

$$= -4607.85\ mJ$$

$$Q_{D \to A'} = mC_p(T_{A'} - T_D) = V_A \rho_A C_p (T_{A'} - T_D) = -11804.87\ mJ$$

$V_A = 91.369 \cdot 10^{-6}\ m^3$

$V_B = 87.221 \cdot 10^{-6}\ m^3$

$V_C = 117.086 \cdot 10^{-6}\ m^3$

$V_D = 120.404 \cdot 10^{-6}\ m^3$

$V_{A'} = 88.880 \cdot 10^{-6}\ m^3$

$P_A = 101325\ Pa$

$P_B = 106144\ Pa$

$P_C = 106144\ Pa$

$P_D = 103218\ Pa$

$P_{A'} = 103218\ Pa$

$W_{A\to B} = -430.14\ mJ$

$Q_{A\to B} = -430.14\ mJ$

$W_{C\to D} = 347.29\ mJ$

$Q_{C\to D} = 347.29\ mJ$

$W_{B\to C} = 3169.99\ mJ$

$Q_{B\to C} = 9443.10\ mJ$

$W_{D\to A'} = -3253.87\ mJ$

$Q_{D\to A'} = -9443.90\ mJ$

IWC

$UW_{A\to B} = \left(\frac{135}{1000}\ kg\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{20-25}{1000}\ m\right) = -6.62\ mJ$

$UW_{B\to C} = \left(\frac{135}{1000}\ kg\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{71-20}{1000}\ m\right) = 67.54\ mJ$

$UW_{C\to D} = \left(\frac{35}{1000}\ kg\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{73-71}{1000}\ m\right) = 0.69\ mJ$

$UW_{D\to A'} = \left(\frac{35}{1000}\ kg\right)\left(9.81\ \frac{m}{s^2}\right)\left(\frac{20-73}{1000}\ m\right) = -18.20\ mJ$

RWC

$UW_{A\to B} = -6.62\ mJ$

$UW_{B\to C} = 47.68\ mJ$

$UW_{C\to D} = 1.37\ mJ$

$UW_{D\to A'} = -13.05\ mJ$

Appendix Code

Volume Perturbation Analysis - Jacob Ivanov.py

```
import numpy as np

def total_V(meas_list):
  # D_can, h_can, D_hose, l_hose, D_pis, h_pis
  return np.pi * (meas_list[1] * (meas_list[0] / 2) ** 2 + meas_list[3] *
(meas_list[2] / 2) ** 2 + meas_list[5] * (meas_list[4] / 2) ** 2)

h_pis_IWC = [25, 20, 71, 73, 20] # mm
h_pis_RTWC = [25, 20, 56, 60, 22] # mm

u_h_pis = [0.5, 0.5, 0.5, 0.5, 0.5] # mm

for i in range(len(h_pis_IWC)):
  pos = ["A", "B", "C", "D", "A'"]
  # meas = [37.5, 60, 3.3, 510, 32.5, h_pis_IWC[i]] # mm
  meas = [37.5, 60, 3.3, 510, 32.5, h_pis_RTWC[i]] # mm
  u_meas = [0.5, 2.0, 0, 25, 0.1, u_h_pis[i]] # mm

  nom = total_V(meas)

  D = []
  for j in range(len(meas)):
     meas_pert = meas.copy()
     meas_pert[j] += u_meas[j]

     D.append(total_V(meas_pert) - nom)
  D = np.array(D)

  u_tV = np.linalg.norm(D)
  print("{0}: Total Volume {1:.3f}±{2:.3f}".format(pos[i], nom, u_tV))

'''
IWC
A: Total Volume 91369.413±2877.164
B: Total Volume 87221.529±2876.142
C: Total Volume 129529.946±2897.157
```

```
D: Total Volume 131189.100±2898.456
A': Total Volume 87221.529±2876.142

RTWC
A: Total Volume 91369.413±2877.164
B: Total Volume 87221.529±2876.142
C: Total Volume 117086.294±2888.549
D: Total Volume 120404.602±2890.648
A': Total Volume 88880.683±2876.523
'''
```

Thermodynamic Perturbation Analysis - Jacob Ivanov.py

```
import numpy as np

def total_V(meas_list):
  # D_can, h_can, D_hose, l_hose, D_pis, h_pis
  return np.pi * (meas_list[1] * (meas_list[0] / 2) ** 2 + meas_list[3] *
(meas_list[2] / 2) ** 2 + meas_list[5] * (meas_list[4] / 2) ** 2)

h_pis_IWC = [25, 20, 71, 73, 20] # mm
h_pis_RTWC = [25, 20, 56, 60, 22] # mm

u_h_pis = [0.5, 0.5, 0.5, 0.5, 0.5] # mm

for i in range(len(h_pis_IWC)):
  pos = ["A", "B", "C", "D", "A'"]
  # meas = [37.5, 60, 3.3, 510, 32.5, h_pis_IWC[i]] # mm
  meas = [37.5, 60, 3.3, 510, 32.5, h_pis_RTWC[i]] # mm
  u_meas = [0, 0, 0, 25, 0.1, u_h_pis[i]] # mm

  nom = total_V(meas)

  D = []
  for j in range(len(meas)):
     meas_pert = meas.copy()
     meas_pert[j] += u_meas[j]

     D.append(total_V(meas_pert) - nom)
```

```
  D = np.array(D)

  u_tV = np.linalg.norm(D)
  print("{0}: Total Volume {1:.3f}±{2:.3f}".format(pos[i], nom, u_tV))

'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```

pB Uncertainty.py

```
import numpy as np

def pB(v):
  return 101325 * (v[0] / v[1])

v = [91369.413, 87221.529]
u_v = [859.369, 826.769]
D = []
nom = pB(v)
for i in range(len(u_v)):
  v_pert = v.copy()
  v_pert[i] += u_v[i]
  pB_pert = nom - pB(v_pert)
  D.append(pB_pert)
u_pB = np.linalg.norm(D)
print("Pressure B: {0:.3f}±{1:.3f}".format(nom, u_pB))
```

```
# Pressure B: 106143.585±1410.685 Pa
```

pD Uncertainty.py

```
import numpy as np

def pD(v):
  return v[0] * (v[1] / v[2])

# IWC
# v = [106143.585, 129529.946, 131189.100]
# u_v = [1410.685, 1169.136, 1182.886]

# RTWC
v = [106143.585, 117086.294, 120404.602]
u_v = [1410.685, 1066.614, 1093.842]

D = []
nom = pD(v)
for i in range(len(u_v)):
  v_pert = v.copy()
  v_pert[i] += u_v[i]
  pD_pert = nom - pD(v_pert)
  D.append(pD_pert)
u_pD = np.linalg.norm(D)
print("Pressure D: {0:.3f}±{1:.3f}".format(nom, u_pD))
# IWC: Pressure D: 104801.183±1926.619 Pa
# RTWC: Pressure D: 103218.306±1905.130 Pa

'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
```

```
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```

pV Cycles (FBD Pressure).py

```
import numpy as np
import matplotlib.pyplot as plt

fig, ax = plt.subplots(1, 2, figsize = (6.5, 3.5))
fig.tight_layout(pad = 3.5)
p_IWC = np.array([101738.89, 102921.42, 102921.42, 101738.89, 101738.89]) /
1000
up_IWC = np.array([7.54, 12.12, 12.12, 7.54, 7.54]) / 1000
v_IWC = np.array([91369.413, 87221.529, 129529.946, 131189.100, 87221.529])
uv_IWC = np.array([859.369, 826.769, 1169.136, 1182.886, 826.769])

p_RTWC = np.array([101738.89, 102921.42, 102921.42, 101738.89, 101738.89]) /
1000
up_RTWC = np.array([7.54, 12.12, 12.12, 7.54, 7.54]) / 1000
v_RTWC = np.array([91369.413, 87221.529, 117086.294, 120404.602, 88880.683])
uv_RTWC = np.array([859.369, 826.769, 1066.614, 1093.842, 839.774])

ax[0].scatter(v_IWC, p_IWC, color = "blue")
ax[0].errorbar(v_IWC, p_IWC, xerr = uv_IWC, yerr = up_IWC, ecolor = "grey",
capsize = 5, color = "blue", linestyle = "dashed")

ax[1].scatter(v_RTWC, p_RTWC, color = "red")
ax[1].errorbar(v_RTWC, p_RTWC, xerr = uv_RTWC, yerr = up_RTWC, ecolor = "grey",
capsize = 5, color = "red", linestyle = "dashed")

labels = ["A", "B", "C", "D", "A'"]
for i in range(len(labels)):
  ax[0].annotate(labels[i], (v_IWC[i] + 2500, p_IWC[i] + 0.25))
  ax[1].annotate(labels[i], (v_RTWC[i] + 2500, p_RTWC[i] + 0.25))



ax[0].set_ylim(101, 104)
ax[1].set_ylim(101, 104)
```

```
ax[0].set_xlim(80000, 140000)
ax[1].set_xlim(80000, 140000)

ax[0].set_ylabel("Pressure (kPa)")
ax[1].set_ylabel("Pressure (kPa)")
ax[0].set_xlabel("Volume (cubic mm)")
ax[1].set_xlabel("Volume (cubic mm)")

ax[0].set_title("Ice Water Cycle")
ax[1].set_title("Room Temperature Water Cycle")

fig.suptitle("Figure 3: Comparison of Cycles (FBD Pressures)")

plt.show()
fig.savefig('Figure 3.png', dpi = 300)
```

pV Cycles (Ideal Gas Pressure).py

```
import numpy as np
import matplotlib.pyplot as plt

fig, ax = plt.subplots(1, 2, figsize = (6.5, 3.5))
fig.tight_layout(pad = 3.5)
p_IWC = np.array([101325, 106143.585, 106143.585, 104801.183, 104801.183]) /
1000
up_IWC = np.array([0, 1410.685, 1410.685, 1926.619, 1926.619]) / 1000
v_IWC = np.array([91369.413, 87221.529, 129529.946, 131189.100, 87221.529])
uv_IWC = np.array([859.369, 826.769, 1169.136, 1182.886, 826.769])

p_RTWC = np.array([101325, 106143.585, 106143.585, 103218.306, 103218.306]) /
1000
up_RTWC = np.array([0, 1410.685, 1410.685, 1905.130, 1905.130]) / 1000
v_RTWC = np.array([91369.413, 87221.529, 117086.294, 120404.602, 88880.683])
uv_RTWC = np.array([859.369, 826.769, 1066.614, 1093.842, 839.774])

ax[0].scatter(v_IWC, p_IWC, color = "blue")
ax[0].errorbar(v_IWC, p_IWC, xerr = uv_IWC, yerr = up_IWC, ecolor = "grey",
capsize = 5, color = "blue", linestyle = "dashed")
```

```
ax[1].scatter(v_RTWC, p_RTWC, color = "red")
ax[1].errorbar(v_RTWC, p_RTWC, xerr = uv_RTWC, yerr = up_RTWC, ecolor = "grey",
capsize = 5, color = "red", linestyle = "dashed")

labels = ["A", "B", "C", "D", "A'"]
for i in range(len(labels)):
  ax[0].annotate(labels[i], (v_IWC[i] + 2500, p_IWC[i] + 0.25))
  ax[1].annotate(labels[i], (v_RTWC[i] + 2500, p_RTWC[i] + 0.25))


ax[0].set_ylim(100, 108)
ax[1].set_ylim(100, 108)
ax[0].set_xlim(80000, 140000)
ax[1].set_xlim(80000, 140000)

ax[0].set_ylabel("Pressure (kPa)")
ax[1].set_ylabel("Pressure (kPa)")
ax[0].set_xlabel("Volume (cubic mm)")
ax[1].set_xlabel("Volume (cubic mm)")

ax[0].set_title("Ice Water Cycle")
ax[1].set_title("Room Temperature Water Cycle")

fig.suptitle("Figure 2: Comparison of Cycles (Ideal Gas Pressures)")

plt.show()
fig.savefig('Figure 2.png', dpi = 300)
```

Q_B-C Uncertainty.py

```
import numpy as np

def Q_BC(meas):
  return meas[0] * 1e-6 * meas[1] * meas[2] * (meas[4] - meas[3]) # mJ

# IWC
# meas = [91369.413, 1.292, 1000, 0, 100]
# u_meas = [859.369, 0, 0, 0, 0]
```

```
# RTWC
meas = [91369.413, 1.292, 1000, 20, 100]
u_meas = [859.369, 0, 0, 0, 0]

D = []
nom = Q_BC(meas)

for i in range(len(u_meas)):
  meas_pert = meas.copy()
  meas_pert[i] += u_meas[i]
  Q_BC_pert = nom - Q_BC(meas_pert)
  D.append(Q_BC_pert)
u_Q_BC = np.linalg.norm(D)
print("Q_BC: {0:.3f}±{1:.3f}".format(nom, u_Q_BC))

# IWC Q_BC: 11804.928±111.030 mJ
# RTWC Q_BC: 9443.943±88.824 mJ

'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```

UW Uncertainty.py

```
import numpy as np


def UW(meas):
```

```
  return (meas[0] + meas[1]) / 1000 * 9.81 * (meas[3] - meas[2]) / 1000 * 1e3 #
mJ

meas = [0, 35, 60, 22]
u_meas = [0, 0.6, 0.5, 0.5]

D = []
nom = UW(meas)

for i in range(len(u_meas)):
  meas_pert = meas.copy()
  meas_pert[i] += u_meas[i]
  UW_pert = nom - UW(meas_pert)
  D.append(UW_pert)
u_UW = np.linalg.norm(D)
print("UW: {0:.3f}±{1:.3f}".format(nom, u_UW))

# IWC
# UW_AB: -6.622±0.937 mJ
# UW_BC: 67.542±0.983 mJ
# UW_CD: 0.687±0.243 mJ
# UW_DAp: -18.198±0.395 mJ

# RTWC
# UW_AB: 6.622±0.937 mJ
# UW_BC: 47.677±0.960 mJ
# UW_CD: 1.373±0.244 mJ
# UW_DAp: -13.047±0.330 mJ
```

W_A-B Uncertainty.py

```
import numpy as np

def W_AB(meas):
  return meas[0] * meas[1] * 1e-6 * np.log(meas[2] / meas[1]) # mJ

# IWC
# meas = [101325, 91369.413, 87221.529]
# u_meas = [0, 859.369, 826.769]
```

```
# RTWC
# meas = [101325, 91369.413, 87221.529]
# u_meas = [0, 859.369, 826.769]

# FBD IWC
meas = [101738.89, 91369.413, 87221.529]
u_meas = [7.54, 859.369, 826.769]

D = []
nom = W_AB(meas)

for i in range(len(u_meas)):
  meas_pert = meas.copy()
  meas_pert[i] += u_meas[i]
  W_AB_pert = nom - W_AB(meas_pert)
  D.append(W_AB_pert)
u_W_AB = np.linalg.norm(D)
print("W_AB: {0:.2f}±{1:.2f}".format(nom, u_W_AB))
# W_AB: -430.123±126.516 mJ, therefore,
# Q_AB: -430.123±126.516 mJ

# FBD
# W_AB: -431.88±127.03 mJ, therefore,
# Q_AB: -431.88±127.03

'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
```

```
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```

W_B-C Uncertainty.py

```
import numpy as np

def W_BC(meas):
  return meas[0] * (meas[2] - meas[1]) * 1e-6 # mJ

# IWC
# meas = [106143.585, 87221.529, 129529.946]
# u_meas = [1410.685, 826.769, 1169.136]

# RTWC
# meas = [106143.585, 87221.529, 117086.294]
# u_meas = [1410.685, 826.769, 1066.614]

# FBD
# IWC
# meas = [102921.42, 87221.529, 129529.946]
# u_meas = [12.12, 826.769, 1169.136]

# RTWC
meas = [106143.585, 87221.529, 117086.294]
u_meas = [1410.685, 826.769, 1066.614]

D = []
nom = W_BC(meas)

for i in range(len(u_meas)):
  meas_pert = meas.copy()
  meas_pert[i] += u_meas[i]
  W_BC_pert = nom - W_BC(meas_pert)
  D.append(W_BC_pert)
u_W_BC = np.linalg.norm(D)
print("W_BC: {0:.2f}±{1:.2f}".format(nom, u_W_BC))
```

```
# IWC W_BC: 4490.767±163.289 mJ
# RTWC W_BC: 3169.953±149.310 mJ

# FBD
# IWC W_BC: 4354.44±147.38
# RTWC W_BC: 3169.95±149.31

'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```

W_C-D Uncertainty.py

```
import numpy as np

def W_CD(meas):
  return meas[0] * meas[1] * 1e-6 * np.log(meas[2] / meas[1]) # mJ

# IWC
# meas = [106143.585, 129529.946, 131189.100]
# u_meas = [1410.685, 1169.136, 1182.886]

# RTWC
# meas = [106143.585, 117086.294, 120404.602]
# u_meas = [1410.685, 1066.614, 1093.842]

# FBD
```

```
# IWC
# meas = [102921.42, 129529.946, 131189.100]
# u_meas = [12.12, 1169.136, 1182.886]

# RTWC
meas = [102921.42, 117086.294, 120404.602]
u_meas = [12.12, 1066.614, 1093.842]

D = []
nom = W_CD(meas)

for i in range(len(u_meas)):
  meas_pert = meas.copy()
  meas_pert[i] += u_meas[i]
  W_CD_pert = nom - W_CD(meas_pert)
  D.append(W_CD_pert)
u_W_CD = np.linalg.norm(D)
print("W_CD: {0:.2f}±{1:.2f}".format(nom, u_W_CD))

# IWC
# W_CD: 174.990±174.309 mJ, therefore,
# Q_CD: 174.990±174.309 mJ
# RTWC
# W_CD: 347.318±157.729 mJ, therefore,
# Q_CD: 347.318±157.729 mJ

# FBD
# W_CD: 169.68±169.00 mJ, therefore,
# Q_CD: 169.68±169.00 mJ
# RTWC
# W_CD: 336.77±152.88 mJ, therefore,
# Q_CD: 336.77±152.88 mJ
'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
```

```
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```

W_D-Ap Uncertainty.py

```
import numpy as np

def W_DAp(meas):
  return meas[0] * (meas[2] - meas[1]) * 1e-6 # mJ

# IWC
# meas = [104801.183, 131189.100, 87221.529]
# u_meas = [1926.619, 1182.886, 826.769]

# RTWC
# meas = [103218.306, 120404.602, 88880.683]
# u_meas = [1905.130, 1093.842, 839.774]

# FBD
# IWC
# meas = [101738.89, 131189.100, 87221.529]
# u_meas = [7.54, 1182.886, 826.769]

# RTWC
meas = [101738.89, 120404.602, 88880.683]
u_meas = [7.54, 1093.842, 839.774]

D = []
nom = W_DAp(meas)

for i in range(len(u_meas)):
  meas_pert = meas.copy()
```

```
  meas_pert[i] += u_meas[i]
  W_DAp_pert = nom - W_DAp(meas_pert)
  D.append(W_DAp_pert)
u_W_DAp = np.linalg.norm(D)
print("W_DAp: {0:.3f}±{1:.3f}".format(nom, u_W_DAp))

# IWC W_DAp: -4607.853±173.353 mJ
# RTWC W_DAp: -3253.846±154.492 mJ

# FBD
# IWC W_DAp: -4473.212±146.828
# RTWC W_DAp: -3207.209±140.301

'''
IWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 129529.946±1169.136
D: Total Volume 131189.100±1182.886
A': Total Volume 87221.529±826.769

RTWC
A: Total Volume 91369.413±859.369
B: Total Volume 87221.529±826.769
C: Total Volume 117086.294±1066.614
D: Total Volume 120404.602±1093.842
A': Total Volume 88880.683±839.774
'''
```