ME 5311 Semester Project: Extra Credit Part 1

due March 27

The goal of this extra credit assignment is to test two key elements of the two-dimensional Poisson equation numerical solution. The due date for this extra credit part is in one week from when it is assigned (not two weeks like Project Part 1).

1 Tridiagonal matrix solver

Consider the one-dimensional Poisson equation

$$\frac{\mathrm{d}^2 p}{\mathrm{d}y^2} = -16\pi^2 \sin 4\pi y \tag{1}$$

for $y \in [0, 1]$ and boundary conditions $p(0) = 0$ and $p(1) = 0$. The exact solution is $p(y) = \sin 4\pi y$. Discretize on a grid $y_j = j\,\Delta y$ with constant Δy for $j = 0, 1, \ldots N$, and use the second-order accurate finite difference approximations for the derivative

$$\left.\frac{\mathrm{d}^2 p}{\mathrm{d}y^2}\right|_j \approx \frac{p_{j-1} - 2\,p_j + p_{j+1}}{\Delta y^2}. \tag{2}$$

Use the provided tridiagonal solver to find the solution p_j and show that the convergence rate is second order.

2 Fourier method

Consider the one-dimensional Poisson equation

$$\frac{\mathrm{d}^2 p}{\mathrm{d}x^2} = -36\pi^2 \cos 6\pi x \tag{3}$$

for $x \in [0, 2]$ and periodic boundary conditions. The exact solution is $p(x) = \sin 4\pi y$. Discretize on a grid $x_i = (i - \frac{1}{2})\Delta x$ for $i = 1, 2, \ldots M$, and use the second-order accurate finite difference approximations for the derivative

$$\left.\frac{\mathrm{d}^2 p}{\mathrm{d}x^2}\right|_i \approx \frac{p_{i-1} - 2\,p_i + p_{i+1}}{\Delta x^2}. \tag{4}$$

Use the Fourier method we discussed in class to find the solution p_i and show that the convergence rate is second order.