ME 5311 ASSIGNMENT 4 JACOB IVANOV

1. Implicit Centered-Difference Stability

With the following Implicit Scheme, we can conduct a von Neumman Stability Analysis

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + \alpha \frac{u_{i+1}^{n+1} - u_{i-1}^{i+1}}{2\Delta x} = 0 \tag{1}$$

By replacing every u_i^n term with $g^n e^{\underline{i}i\theta}$, we can obtain the following:

$$\frac{g^{n+1}e^{\underline{i}i\theta} - g^n e^{\underline{i}i\theta}}{\Delta t} + \alpha \frac{g^{n+1}e^{\underline{i}(i+1)\theta} - g^{n+1}e^{\underline{i}(i-1)\theta}}{2\Delta x} = 0 \tag{2}$$

Which by factoring like terms, we can find:

$$e^{\underline{i}i\theta}\left[\frac{g^n(g-1)}{\Delta t} + \frac{\alpha g^n g}{2\Delta x}\left(e^{\underline{i}\theta} - e^{-\underline{i}\theta}\right)\right] = 0 \tag{3}$$

By using Euler's Formula, we can expand the exponentials. We can also divide through by $e^{\underline{i}i\theta}$ and g^n to obtain the following:

$$\frac{g-1}{\Delta t} + \frac{\alpha g \underline{i}}{\Delta x}\sin(\theta) = 0 \tag{4}$$

We can then isolate g:

$$g\left[\frac{1}{\Delta t} + \frac{\alpha \underline{i}\sin(\theta)}{\Delta x}\right] = \frac{1}{\Delta t} \tag{5}$$

By solving for g, canceling out a Δt term, and multiplying by the complex conjugate over itself, we can find the following:

$$g = \frac{\Delta x^2}{\Delta x^2 + \Delta t^2\alpha^2\sin^2(\theta)} - \underline{i}\frac{\Delta x \Delta t \alpha \sin(\theta)}{\Delta x^2 + \Delta t^2\alpha^2\sin^2(\theta)} \tag{6}$$

Which if we can the norm, and simplify:

$$|g|^2 = \frac{\Delta x}{\Delta x^2 + \Delta t^2\alpha^2\sin^2(\theta)} \tag{7}$$

It can be seen that for any $\Delta x > 1$, $|g|^2 > 1$. This does not seem to be correct.