ME 5311 ASSIGNMENT 2 JACOB IVANOV

Simpson's Rule for numerical integration can be derived from Lagrange's Interpolating Polynomials by constructing the Lagrange Polynomial for 3 points:

$$\begin{aligned} f(x) = &+ \frac{f_0(x_1-x)(x_2-x)}{(x_1-x_0)(x_2-x_0)} \\ &+ \frac{f_1(x_0-x)(x_2-x)}{(x_0-x_1)(x_2-x_1)} \\ &+ \frac{f_2(x_0-x)(x_1-x)}{(x_0-x_2)(x_1-x_2)} \end{aligned} \tag{1}$$

Integrating $f(x)$ on the domain of $[x_0, x_2]$ is algebraically intense, but otherwise straightforward. Wolfram Mathematica was utilized to make those algebraic simplifications. The console can be seen below:

```
In[38]:= ClearAll

Out[38]= ClearAll

In[39]:= f[x_] := (x1 - x) (x2 - x) f0 / ((x1 - x0) (x2 - x0)) + (x0 - x) (x2 - x) f1 / ((x0 - x1) (x2 - x1)) +
           (x0 - x) (x1 - x) f2 / ((x0 - x2) (x1 - x2))

In[46]:= result = FullSimplify[Integrate[f[x], {x, x0, x2}]]
```

Out[46]=
$$-\frac{(x0-x2)\left(f1\,(x0-x2)^2+f0\,(x1-x2)\,(2\,x0-3\,x1+x2)-f2\,(x0-x1)\,(x0-3\,x1+2\,x2)\right)}{6\,(x0-x1)\,(x1-x2)}$$

```
In[59]:= Expand[result] /. {x0 - x1 → h, x1 - x2 → -h, x0 - x2 → -2 h}
```

Out[59]=
$$\frac{f1\,x0^3}{6\,h^2}-\frac{f2\,x0^3}{6\,h^2}+\frac{f0\,x0^2\,x1}{3\,h^2}+\frac{2\,f2\,x0^2\,x1}{3\,h^2}-\frac{f0\,x0\,x1^2}{2\,h^2}-\frac{f2\,x0\,x1^2}{2\,h^2}-\frac{f0\,x0^2\,x2}{3\,h^2}-\frac{f1\,x0^2\,x2}{2\,h^2}-\frac{f2\,x0^2\,x2}{6\,h^2}+\frac{f0\,x0\,x1\,x2}{3\,h^2}-\frac{f2\,x0\,x1\,x2}{3\,h^2}+\frac{f0\,x1^2\,x2}{2\,h^2}+\frac{f2\,x1^2\,x2}{2\,h^2}+\frac{f0\,x0\,x2^2}{6\,h^2}+\frac{f1\,x0\,x2^2}{2\,h^2}+\frac{f2\,x0\,x2^2}{3\,h^2}-\frac{2\,f0\,x1\,x2^2}{3\,h^2}-\frac{f2\,x1\,x2^2}{3\,h^2}+\frac{f0\,x2^3}{6\,h^2}-\frac{f1\,x2^3}{6\,h^2}$$

```
In[60]:= FullSimplify[Expand[result] /. {x1 → x0 + h, x2 → x0 + 2 h}]
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Out[60]=
$$\frac{1}{3}\,(f0+4\,f1+f2)\,h$$

Simpson's Integration was implemented with C, and the numerical error for the integral $\int_0^\pi \sin(x)\,\mathrm{d}x = 2$ for a $\Delta x = \pi/16$ was 0.000016591047935499l5.

The error was calculated for a $\Delta x = \pi/2^n$ where $n \in \{2, 3, ..., 30\}$ and the convergence was found to be 4th order (up to $N \approx 10^4$, where floating point errors start building up). This can also be seen by plotting this on a loglog plot.