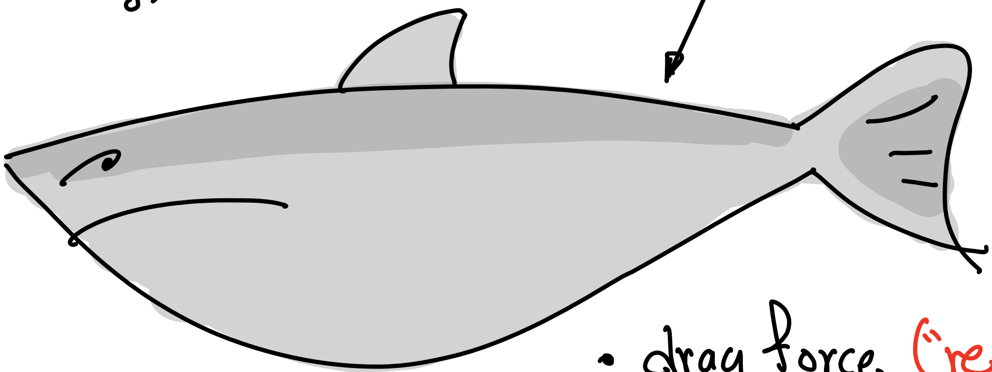
Discrete Functions:

$\vec{u}(t,x,y,z)$

shark likes to chase fish



- drag force ("resistance")

How to compute drag force from $\vec{u}$?

$$\text{Drag} = \Big(\underbrace{\underbrace{\oint_S \underset{\substack{\uparrow \\ \text{stress tensor}}}{T_{ij}}\ \underset{\substack{\uparrow \\ \text{surface normal}}}{n_j}\ dA}_{\text{force on } dA}}_{\text{integral of force over surface}}\Big) \cdot \underset{\substack{\uparrow \\ \text{unit vector in direction of motion}}}{\hat{u}}$$

use to project force on $\hat{u}$ to get Drag

<u>Notation</u> : $u_i = u(x_i) = u(i\,\Delta x)$ $\quad i = 0, 1, 2, 3, \ldots$

Δx is constant

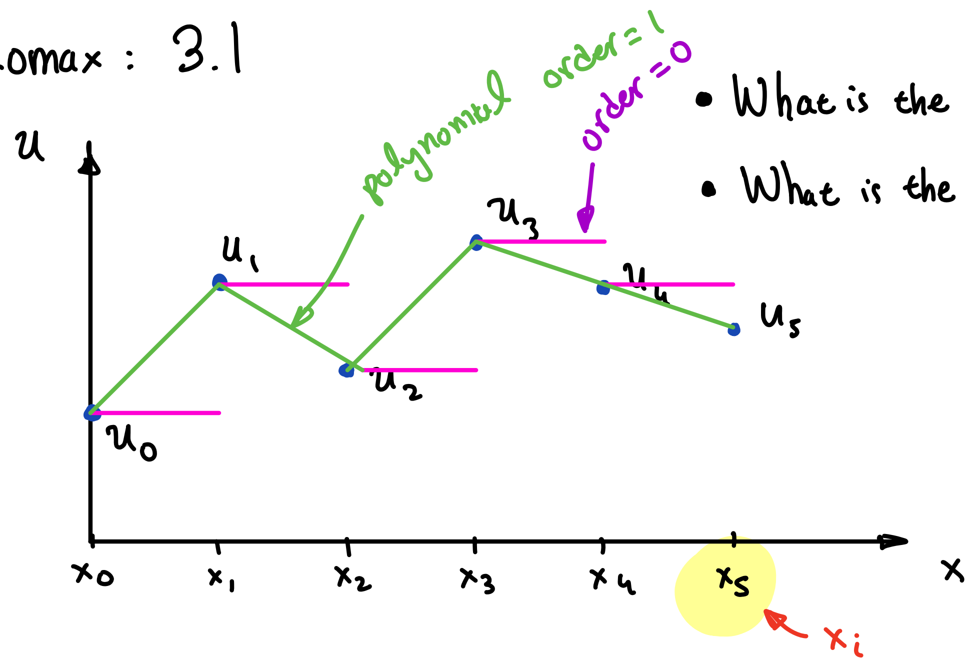
Δx: grid spacing in x

similarly: u^n ← time index

$u^n = u(t_n) = u(n\,\Delta t)$

Δt: spacing in time

Lomax : 3.1



- What is the integral of u?
- What is the derivative?

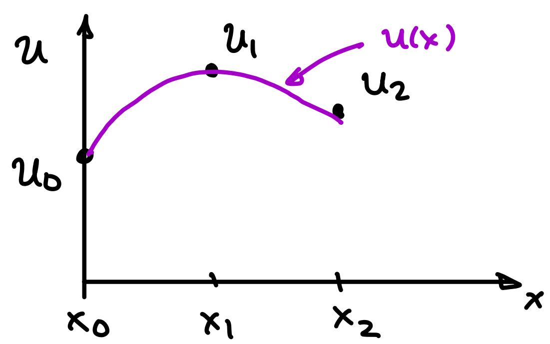
Polynomial Interpolation: (Lomax 3.4.3)

Lagrange polynomial: $u(x) = \sum_{k=0}^{K} a_k(x)\, u_k$ K+1 data points

discrete points

$$a_k = \prod_{\substack{0 \le m \le K \\ m \ne k}} \frac{x_m - x}{x_m - x_k} \quad \leftarrow \text{polynomial}$$

Example: $K=2 \Rightarrow 3$ points



eq. 3.43 Lomax

$$u(x) = \frac{(x_1 - x)(x_2 - x)}{(x_1 - x_0)(x_2 - x_0)} u_0 + \frac{(x_0 - x)(x_2 - x)}{(x_0 - x_1)(x_2 - x_1)} u_1 + \frac{(x_0 - x)(x_1 - x)}{(x_0 - x_2)(x_1 - x_2)} u_2$$

Hermite Polynomials: function <u>and</u> its derivative at x_k

$$u(x) = \sum_k a_k(x)\, u_k + \sum b_k(x) \left.\frac{\partial u}{\partial x}\right|_k$$

Lagrange

Chebyshev polynomials:

$T_0(x) = 1$ on $-1 \leq x \leq 1$

extrema are $-1, 1$

$T_1(x) = x$

$T_{n+1}(x) = 2x\, T_n(x) - T_{n-1}(x)$

what is more important are the roots: $x_k = \cos\left(\frac{2k-1}{2n}\pi\right)$, $k = 1, \dots, n$