## Practical introduction to time integration

The concept:

ODE 
$$\frac{du}{dt} = f(u)$$
 initial condition  $u(t=0) = u_0$   
Solution: function  $u(t)$ 

on the computer we will get discrete  $u(n \Delta t)$ 

on the computer we know up because it is the initial condition

## Runge-Kutta Method 3

We will use the standard third-order Runge-Kutta method (the same method we discussed in class) to numerically integrate a couple of ODEs. To advance the solution u from time t to  $t + \Delta t$ , three sub-steps, are taken. If the solution at time t is  $u_n$  the following three steps are taken to advance the solution to  $u_{n+1}$ at  $t + \Delta t$ :

$$u_{n+\alpha} = u_n + \Delta t \frac{1}{2} f(u_n)$$

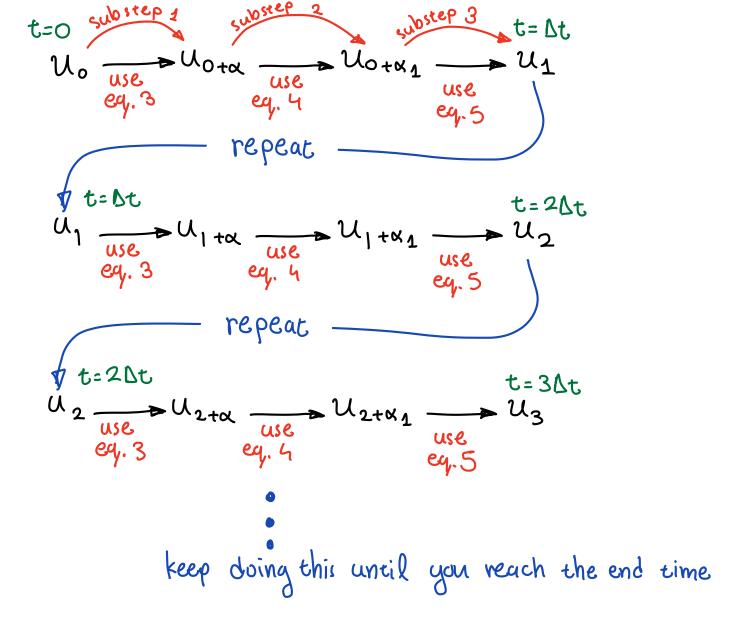
$$u_{n+\alpha_1} = u_n + \Delta t \left( -f(u_n) + 2f(u_{n+\alpha}) \right)$$
Substep 1

(3)

(4)

$$u_{n+\alpha_1} = u_n + \Delta t \left( -f(u_n) + 2f(u_{n+\alpha}) \right)$$
 Substep 2 (4)

$$u_{n+1} = u_n + \Delta t \left( \frac{1}{6} f(u_n) + \frac{2}{3} f(u_{n+\alpha}) + \frac{1}{6} f(u_{n+\alpha_1}) \right)$$
 Substee 3 (5)



```
function [u, time]=rkexample(dt, tend)
                                                                                                                                            2 space for variables
                      np=length(time);
                                                                                                                                                                                                 — initial condition
                      u=zeros(size(time));
                      u(1)=1;
                      for n=1:np-1
                                            r0=rhs(u(n));
       u1=update(dt, [1/2 0 0], u(n), r0, 0, 0);
r1=rhs(u1); - - - - - (Un+a)
Un+a, u2=update(dt, [-1 2 0], u(n), r0, r1, 0);
                                                                                                                                                                                                                                                                                                                                        5 Subscep 2
     r2=rhs(u2); - f(Untas)
Unn u(n+1)=update(dt, [1/6 2/3 1/6], u(n), r0, r1, r2); - Subscep 3
                      end
                                           function u=update(dt, a, u, rhs1, rhs2, rhs3) \frac{1}{2} RK substep:
u=u+dt*(a(1)*rhs1+a(2)*rhs2+a(3)*rhs3);
end
\frac{1}{2}
\frac{
                                             function r=rhs(u)
                                                                r=u;
                                             end
```

end