ME 5311: Week 13

Today

Introduction to the computation of turbulent flows

- Primary Goal:
  - Understand what you are getting into…
- More “academic” goals:
  - Understand fundamental difficulty of modeling and simulation of multi-scale physics
  - Understand the “output” of CFD computations
- This is *not* a presentation on turbulence physics or flows!

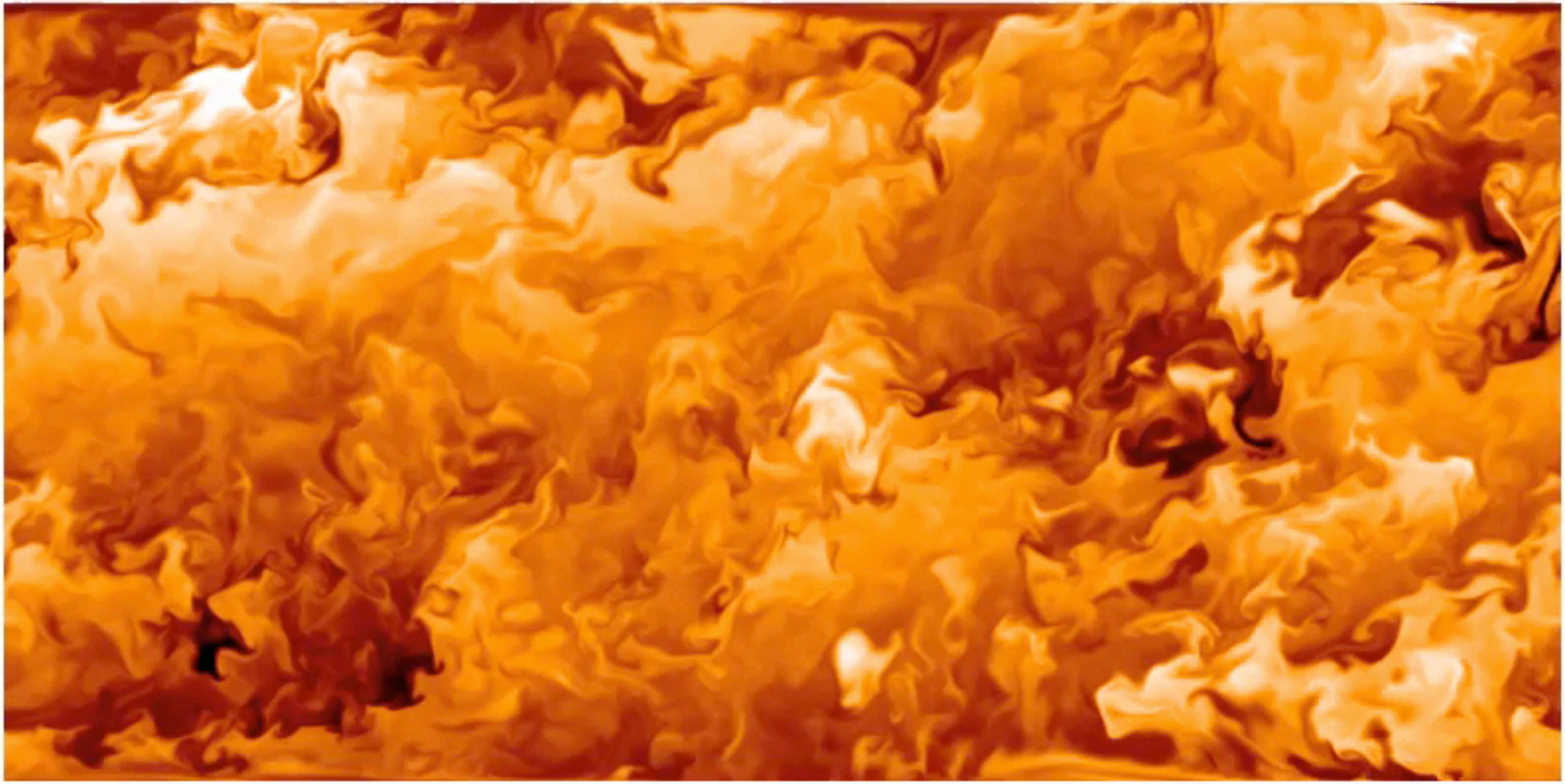
Next Week

Introduction to parallel computing

Donuts in class (I am not sure we should be eating in this room — don’t tell anyone)

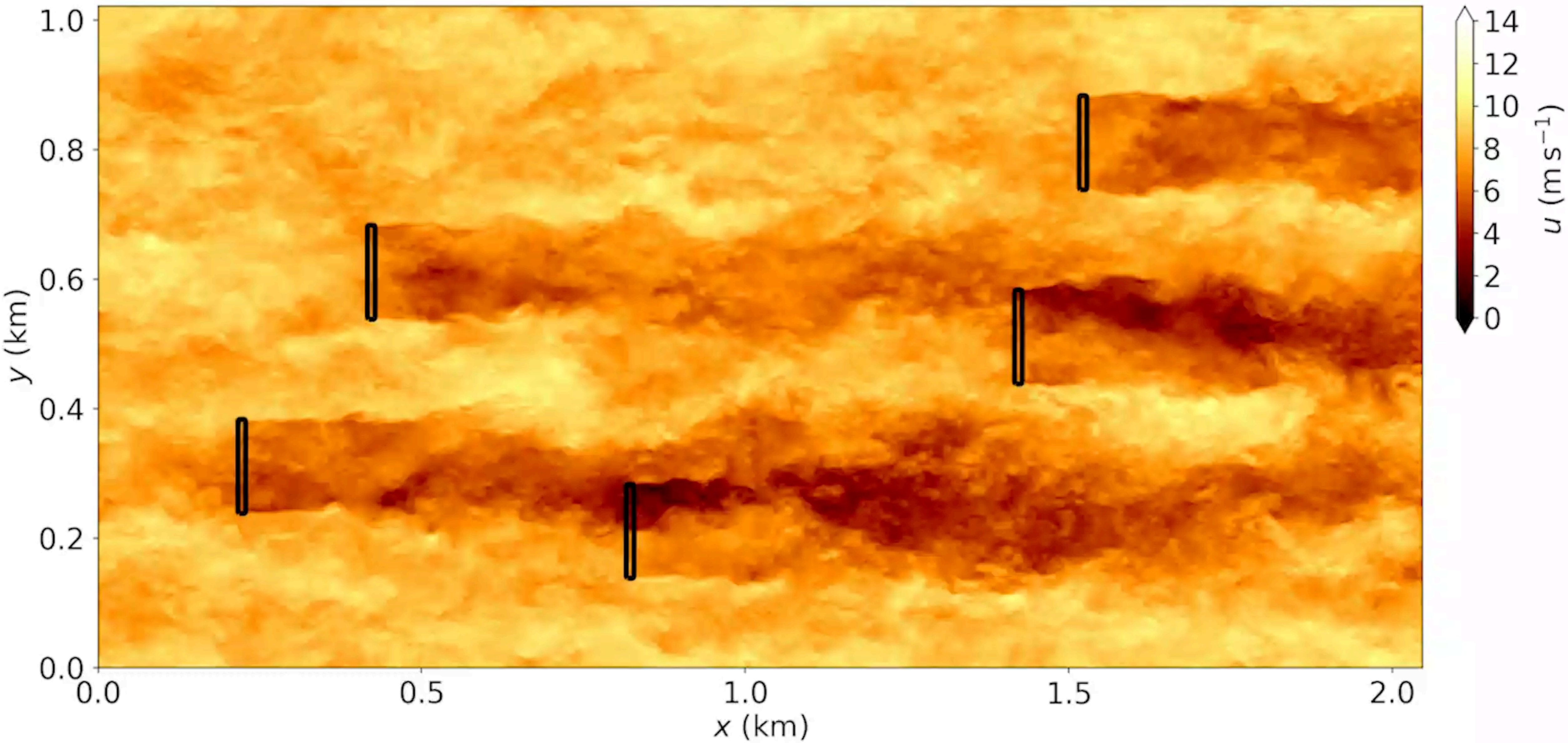

ugust 2011
Barents Sea
Norway
50 km

Passive scalar mixing in sheared turbulence

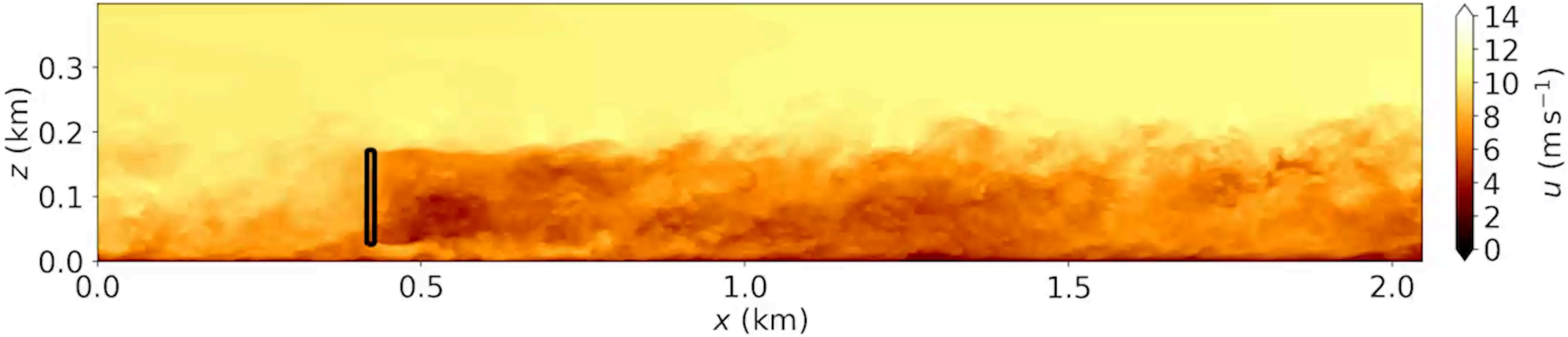


Wind farm large-eddy simulation

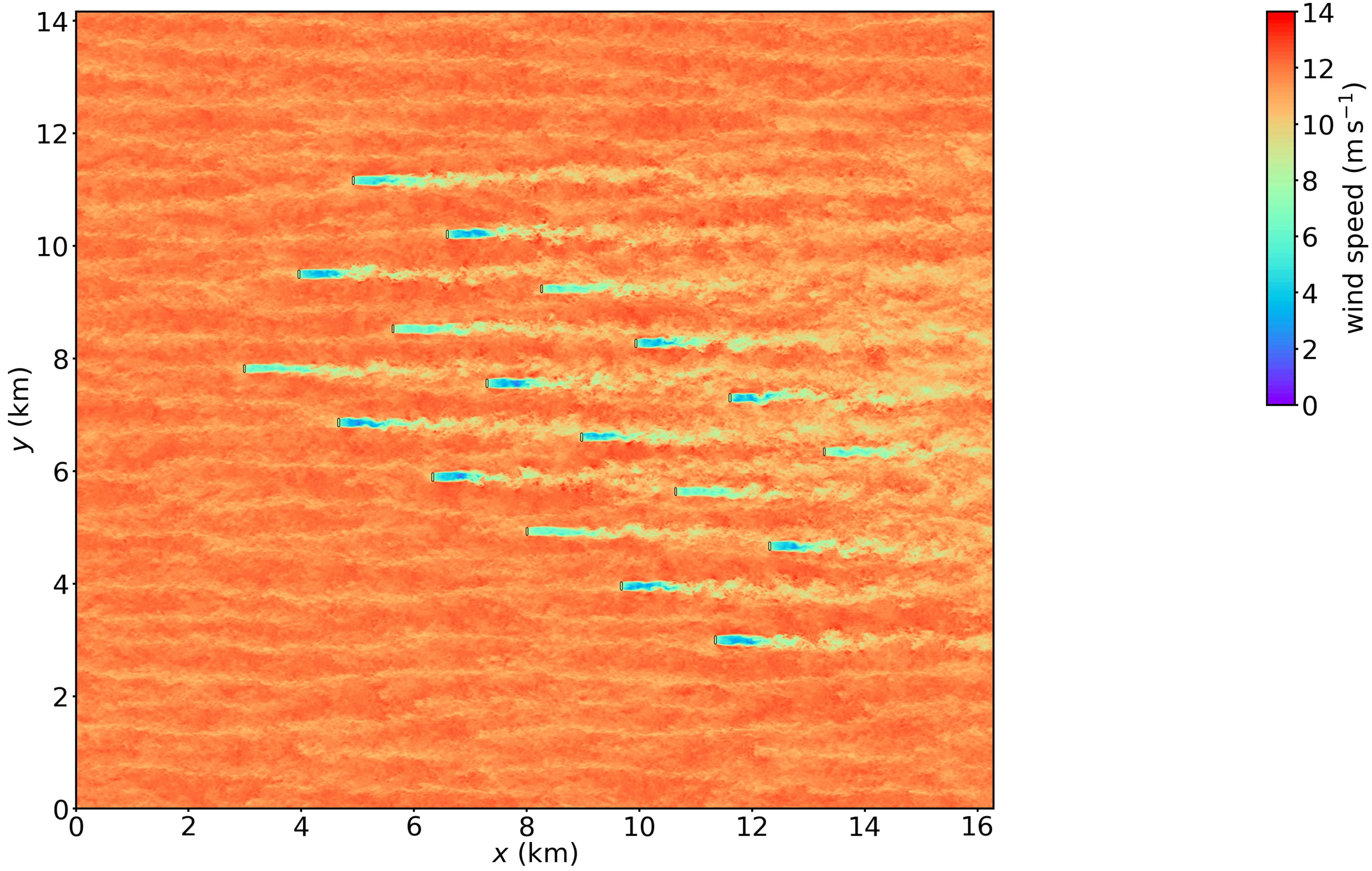
- Actuator disc model: captures volume-mean momentum extraction from turbines
- Regions of low wind speed downstream of turbines show reduction of remaining kinetic energy in the atmosphere



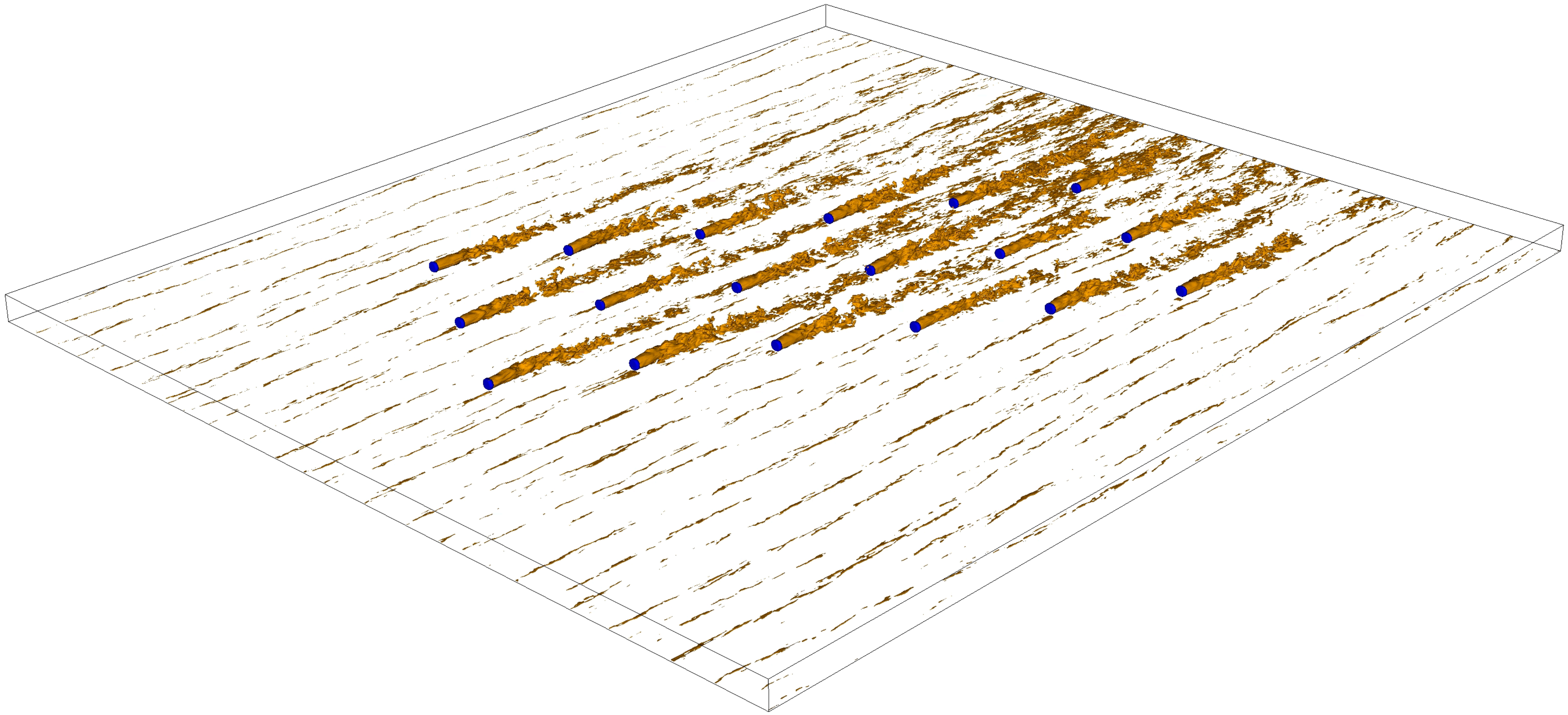
Wind farm large-eddy simulation

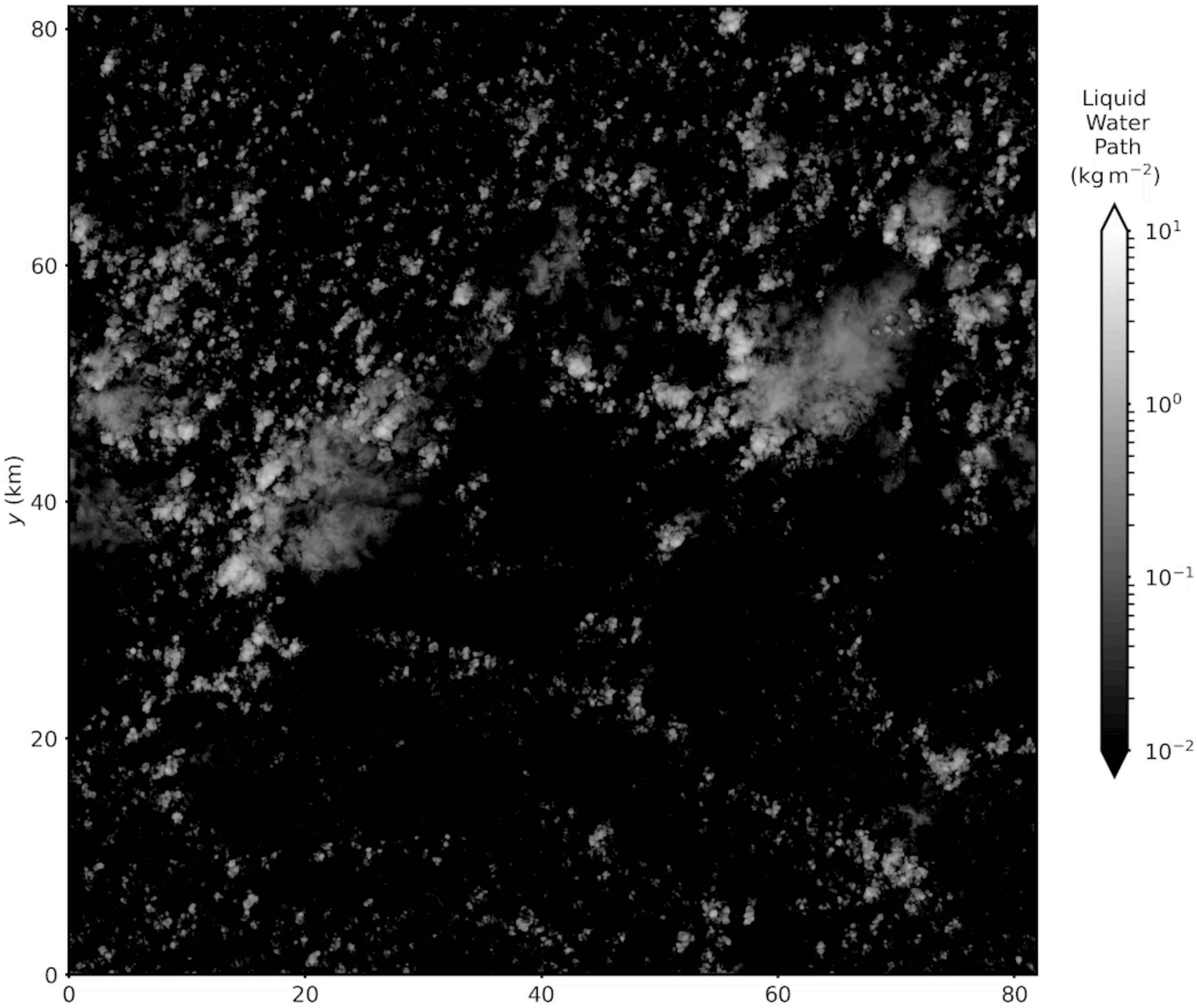


Wind farm large-eddy simulation



Wind farm large-eddy simulation





Liquid Water Path (kg m⁻²)
10^1
10^0
10^-1
10^-2
y (km)
x (km)
0
20
40
60
80

Atmospheric turbulence

Atmospheric turbulence

A definition of turbulence?

- Definition from Merriam-Webster:
- Turbulence (noun): the quality or state of being turbulent: such as

  a: great commotion or agitation • emotional turbulence (not what we want)

  b: irregular atmospheric motion especially when characterized by up-and-down currents (not true, only applies to atmospheric motions)

  c: departure in a fluid from a smooth flow (only partially true, this is more of a definition of unsteady flow)

Requirements for a turbulent flow

- Unsteady flow
- Must have vorticity
- Turbulence is a three-dimensional phenomenon
  - Two-dimensional variants exist on paper, but all physical flows are three-dimensional
- Motions at different scales

Randomness

- Turbulent flows are "random" in the sense that the value velocity field at $u(t, x, y, z)$ is neither certain or "impossible" — subject to the physical constraints

- Are the equations deterministic?

  - Yes, they do not have random coefficients

- What causes randomness?

  - Sensitivity to initial and boundary conditions

  - Small perturbations tend to amplify…

How are small-scale motions created?

- Because of non-linearity
- Convection term is non linear: $u\dfrac{\partial u}{\partial x}$
- take “single mode” $u(x) = \sin(x)$
- Repeat: u(x) = sin(2x)
- Observations:
  - new wavelengths are created
  - the new wavelengths are shorter (finer scales) than the initial
  - The new wavelengths have smaller amplitudes
- How does the creation of perpetually smaller scales terminate?

$$u\frac{\partial u}{\partial x} = \sin(x)\cos(x) = \frac{1}{2}sin(2x)$$

$$u\frac{\partial u}{\partial x} = \frac{1}{2}\sin(2x)\cos(2x) = \frac{1}{4}sin(4x)$$

Energy transfer in fluid: "triadic interactions"

$$\widehat{\frac{\partial}{\partial x} uv} = +ik\, \widehat{uv}$$

$$= +ik \sum_p \hat{u}\, e^{ipx} \sum_q \hat{v}_q e^{iqx}$$

$$= +ik \sum_p \sum_q \hat{u}_p \hat{v}_q \langle e^{i(p+q)x} e^{-ikx} \rangle$$

$$= ik \sum_p \sum_q \hat{u}_p \hat{v}_q \delta_{k,p+q}$$

$$= ik \sum_{p+q=k} \hat{u}_p \hat{v}_{k-p}$$

$\underbrace{k = p + q}_{\text{wavenumber triad}}$

$$= ik \sum_{p+q=k} \hat{u}_p \hat{u}_q$$

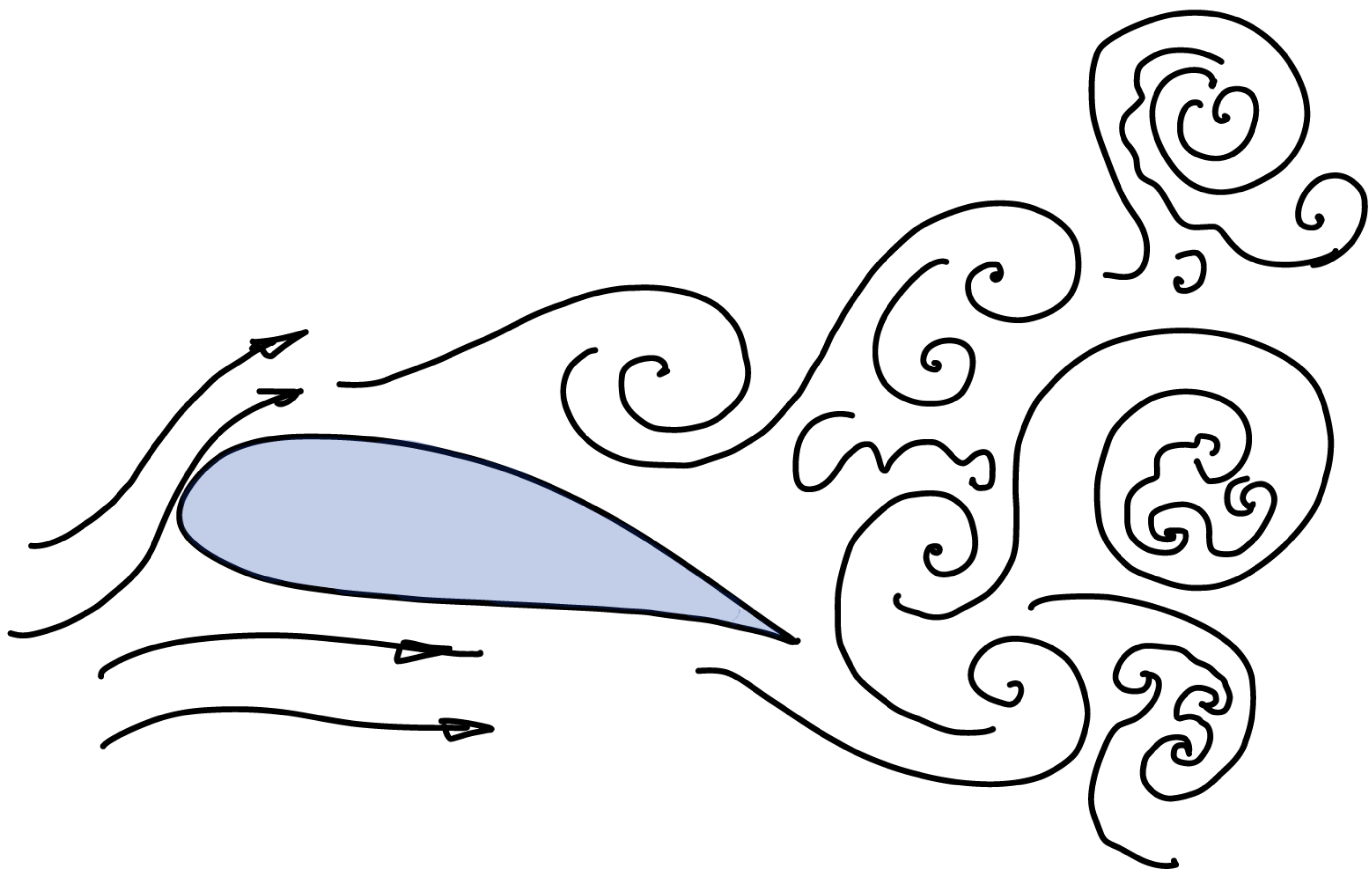
Non local

Non-linearity and turbulence

- In practice the problem is more complex: some modes grow and some decay
- Various flow instabilities can cause an “initially” smooth flow to become irregular: a flow state called turbulent
- Simulation of turbulence is a primary challenge for CDF today

Image: NASA

The range of spatial scales: kinetic energy cascade



Richardson cascade (1922, p. 66)

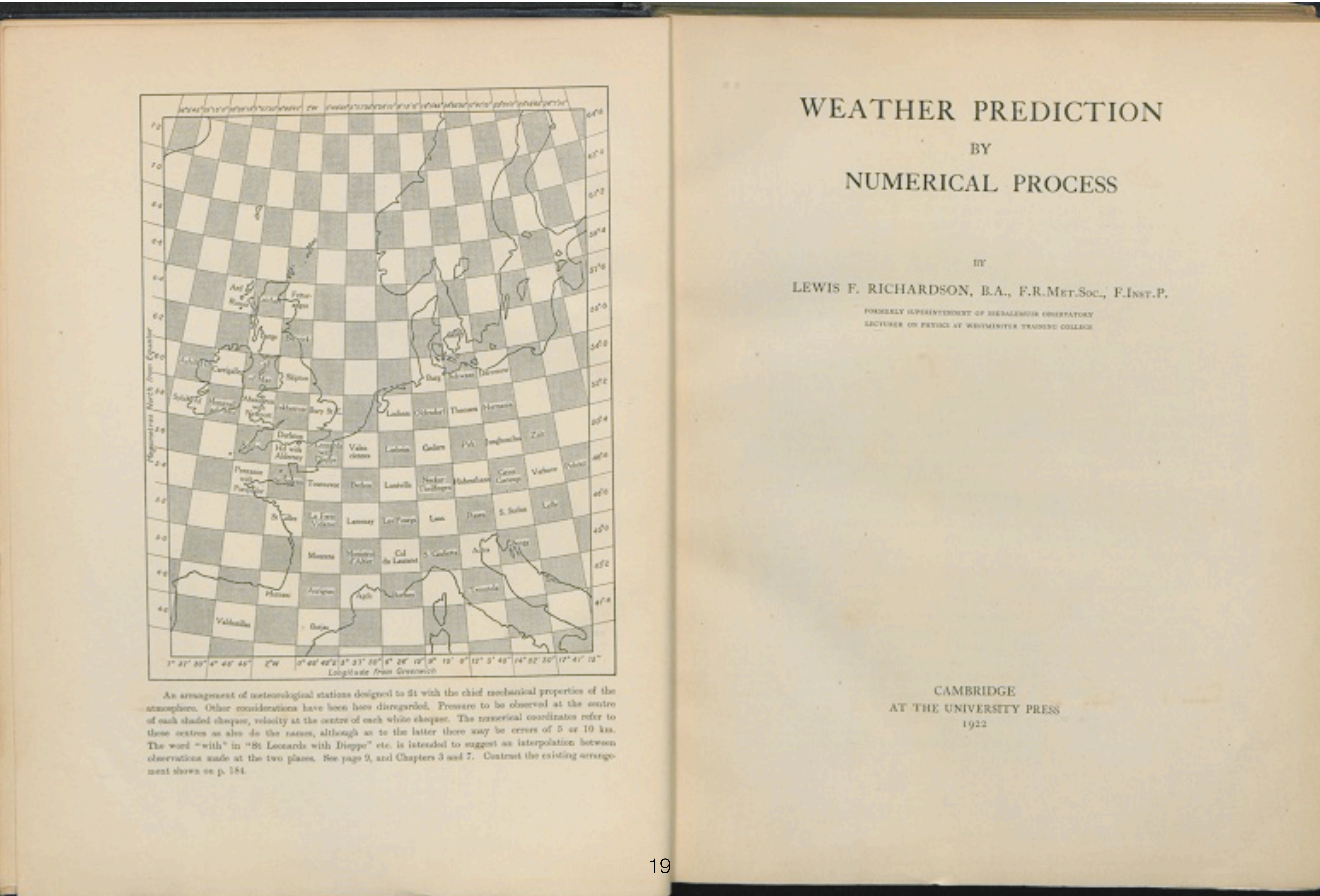
Big whirls have little whirls

that feed on their velocity,

And little whirls have lesser whirls

and so on to viscosity

1922: First attempt
Richardson envisions numerical weather prediction



An arrangement of meteorological stations designed to fit with the chief mechanical properties of the atmosphere. Other considerations have been here disregarded. Pressure to be observed at the centre of each shaded chequer, velocity at the centre of each white chequer. The numerical coordinates refer to these centres as also do the names, although as to the latter there may be errors of 5 or 10 km. The word "with" in "St Leonards with Dieppe" etc. is intended to suggest an interpolation between observations made at the two places. See page 9, and Chapters 3 and 7. Contrast the existing arrangement shown on p. 184.

WEATHER PREDICTION

BY

NUMERICAL PROCESS

BY

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LECTURER ON PHYSICS AT WESTMINSTER TRAINING COLLEGE

CAMBRIDGE

AT THE UNIVERSITY PRESS

1922

Range of scales

Reynolds number : $Re = \frac{UL}{\nu}$

Smallest flow scale : Kolmogorov scale: $\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4}$

Kinetic energy dissipation rate

$\varepsilon \sim \frac{U^3}{L}$ scaling for ε

$$\frac{\eta}{L} = \left(\frac{\nu^3}{\varepsilon L^4}\right)^{1/4} = \left(\frac{\nu^3}{U^3 L^3}\right)^{1/4} = Re^{-3/4}$$

Scale separation : $\frac{\text{large}}{\text{small}} = \frac{L}{\eta} = \left(\frac{L^4 \varepsilon}{\nu^3}\right)^{1/4} = \left(\frac{L^3 U^3}{\nu^3}\right)^{1/4} = Re^{3/4}$

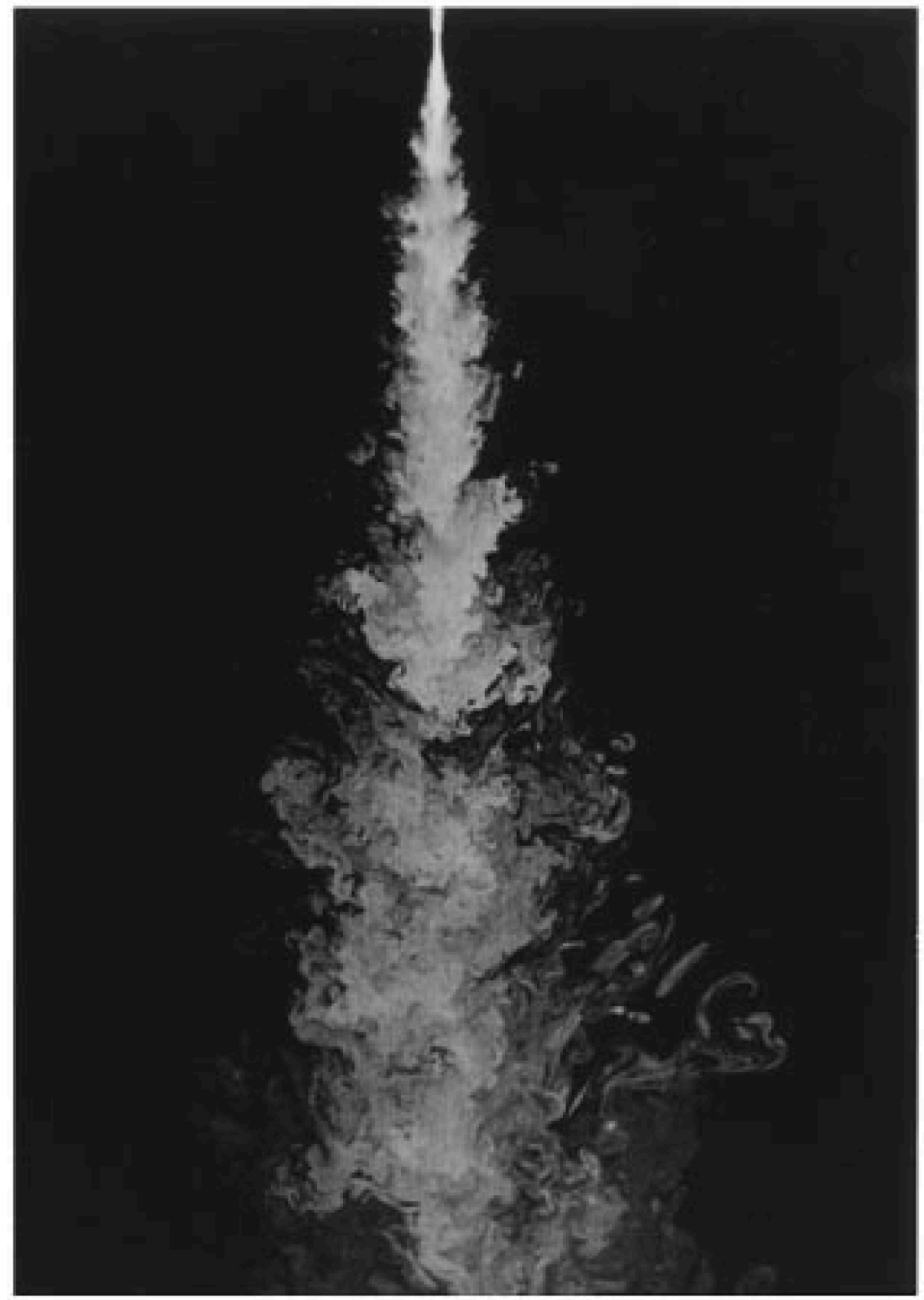
Reynolds number

- *Re* scale is logarithmic but there can be a few “special” thresholds:
  - Transition to unsteady flow
  - Transition to turbulence
  - Transition to fully-developed turbulence
    - Called the “mixing transition” (Dimotakis 2000)

Reynolds number: turbulent jet

$Re = 1.5 \times 10^3$

$Re = 10^4$



Dimotakis et al. (1983)

Example: Flow over 737 wing

737 wing: Span = 28 m

Chord = 4 m

Re = ?

U? Take $M = 0.7$ $\quad U = cM = \sqrt{\gamma R T}\, M$

Take altitude $\approx$ 10 km $\longrightarrow$ $T = 225\,K$

$\gamma = 1.4 \quad R = 287 \Rightarrow c \simeq 305\ m/s$

$U = 214\ m/s$

$\nu = 9.4 \times 10^{-6}$ @ $T = 225\,K$

$$Re = \frac{Ud}{\nu} = \frac{214 \times 4}{\nu} = 91 \times 10^{6}$$

Kolmogorov Scale: $\eta = d\, Re^{-3/4} = 4 \times 10^{-6}$ m

d ← chord

$\Delta x = \eta$

Computational domain: $(1.5 \text{ span}) \times (2 \times \text{Chord})^2$

$$\text{Total \# grid points} = \frac{\text{Comp. Domain Volume}}{\Delta x^3}$$

$$= 2 \times 10^{19}$$

Save 4 valirabes (u, v, w, p) + 2 RHS = 10

$$\text{Memory} = \underbrace{2 \times 10^{19}}_{\text{grid}} \times \underset{\substack{\uparrow \\ \text{variables}}}{10} \times 8 \text{ bytes} = 200 \text{ million TB}$$

Problems

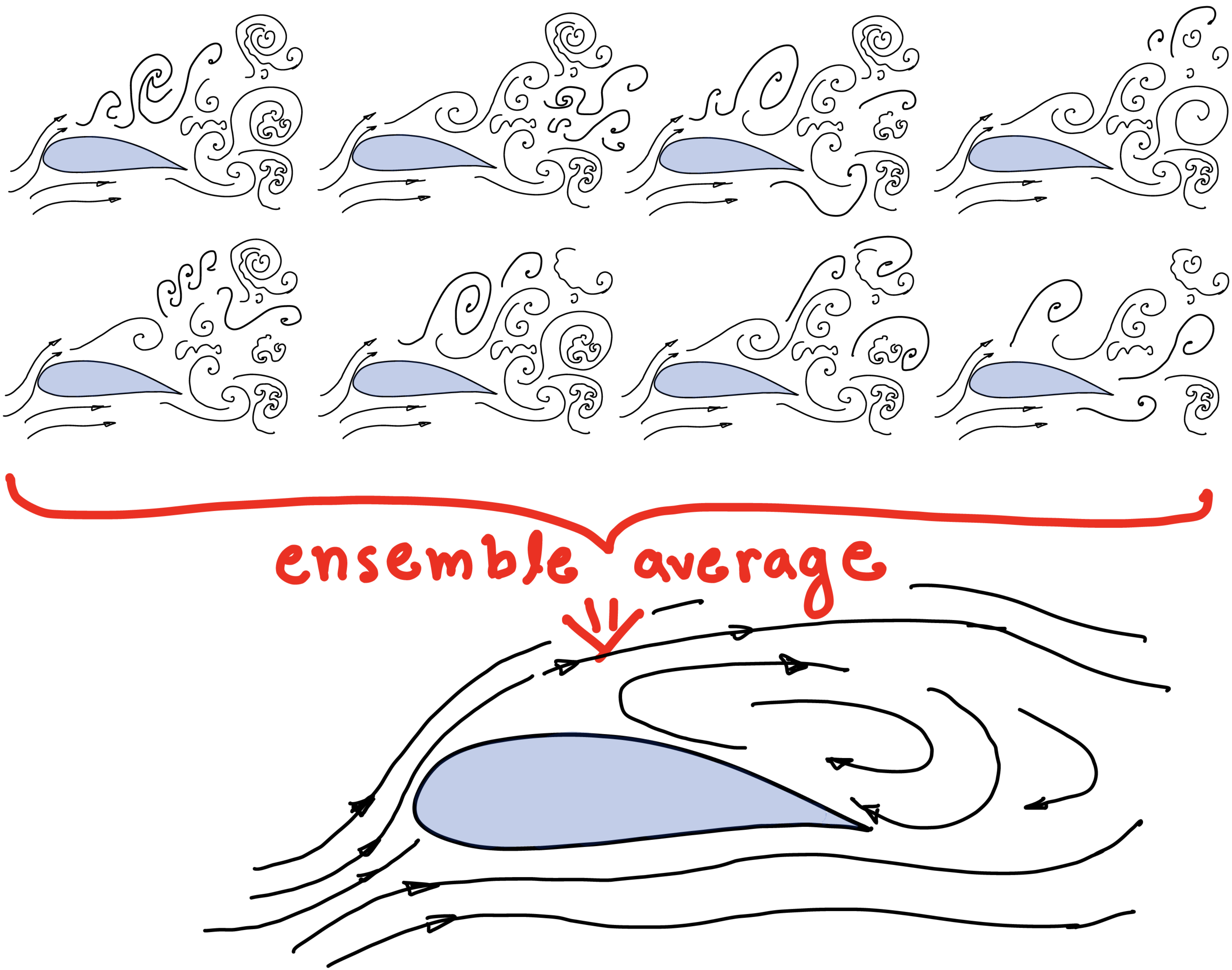
- Randomness
- Multiple scales

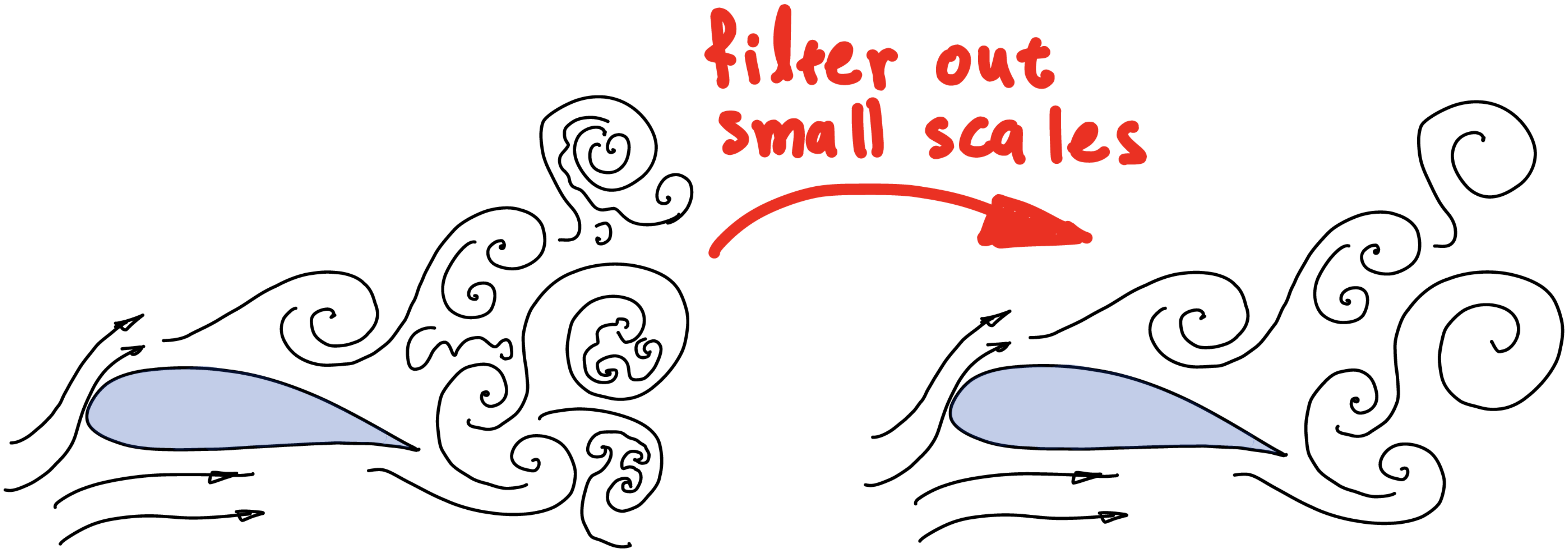
Possible solutions: randomness

- What is the meaning of $u(n, i, j, k)$?

Possible solutions: multiple scales

- Solution 1

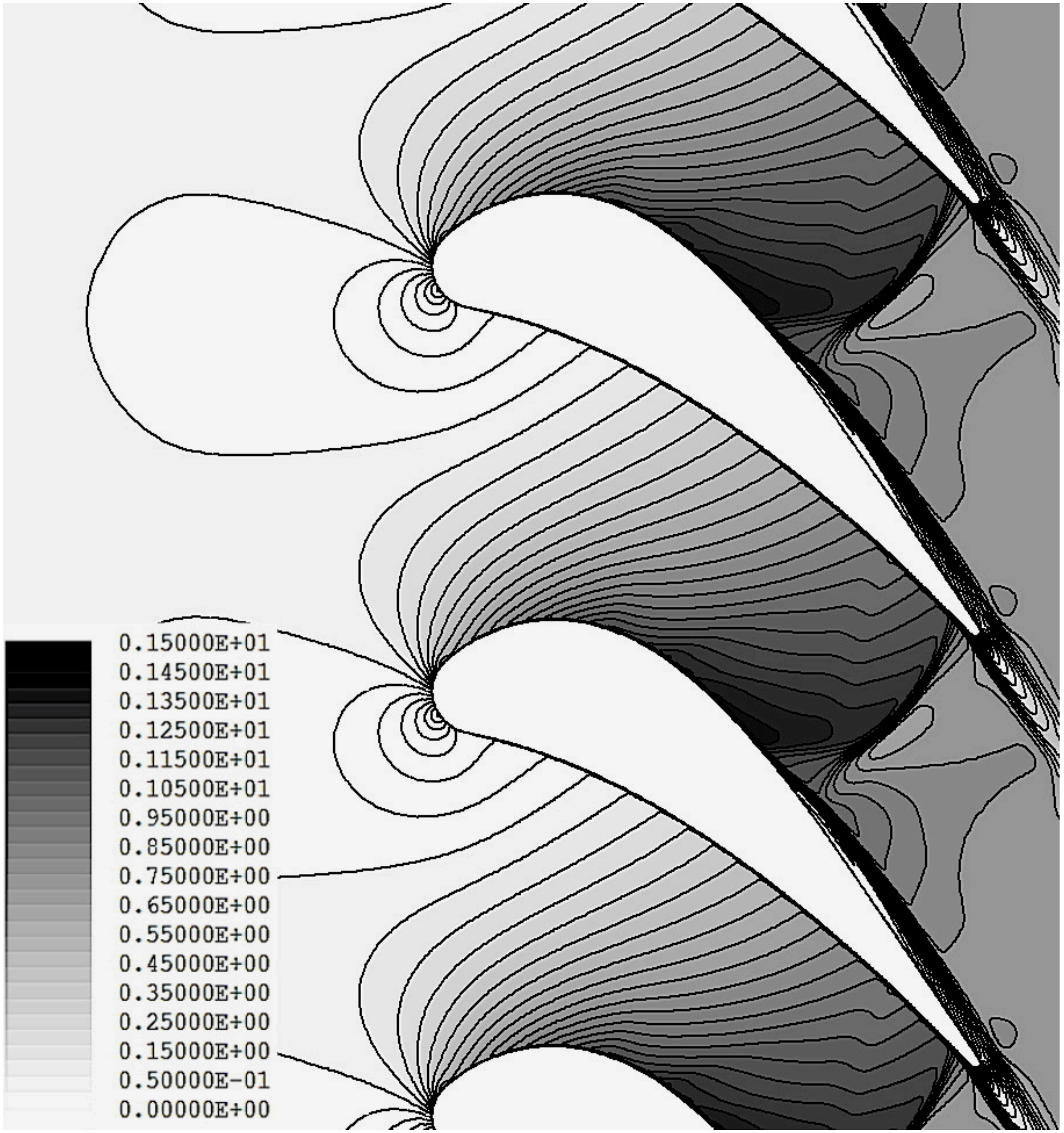

ensemble average



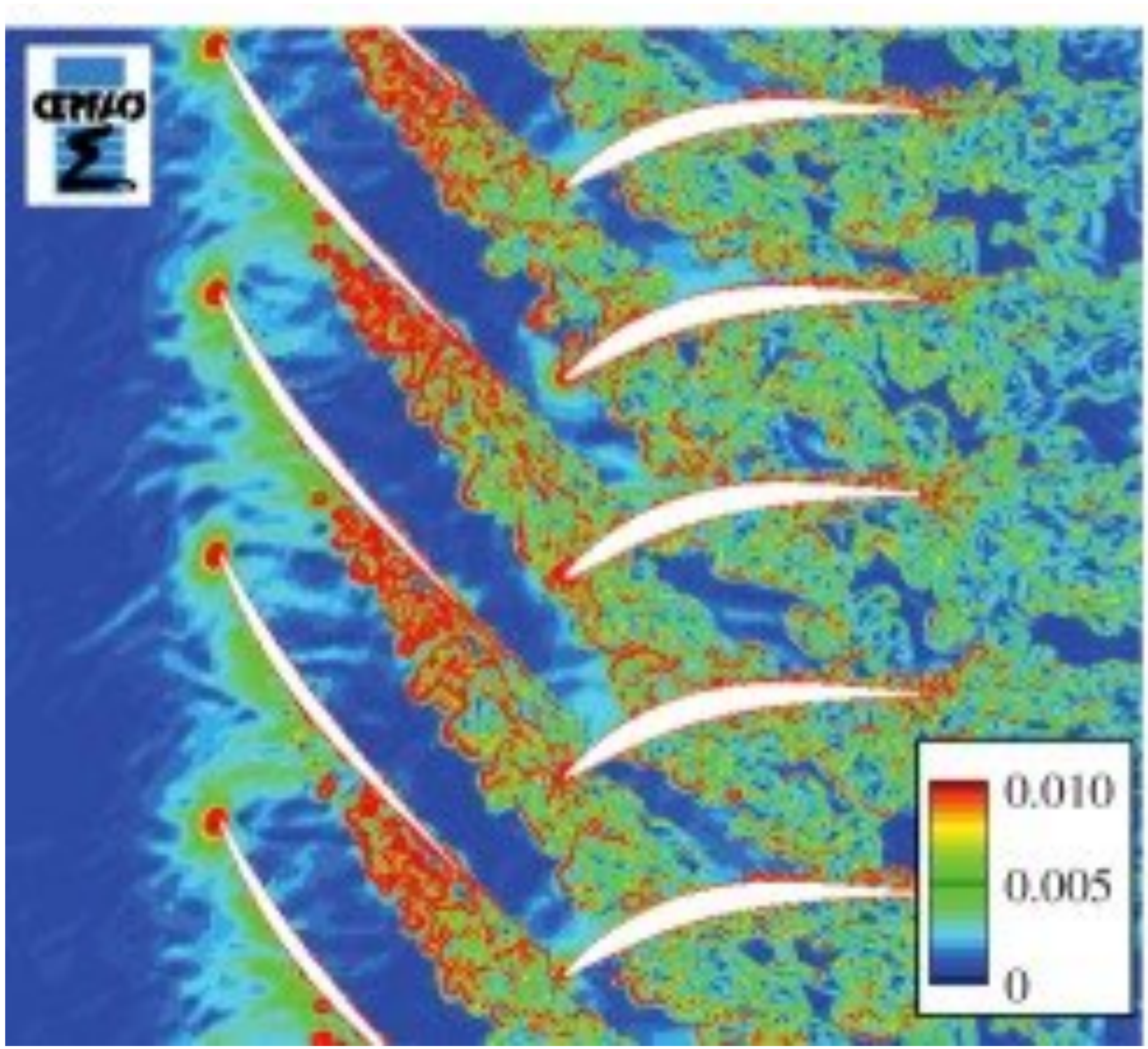
$\Delta x = 10^{-6}\,m \longrightarrow \Delta x = 10^{-1}\,m$

Where is the turbulence?

Mach number in a turbine stage



Entropy production in a compressor stage



(Gourdain 2013)

All scales resolved → Direct Numerical Simulation (DNS)

- Most Accurate
- Really expensive!
- Research tool

Small scales are filtered → Large Eddy Simulation (LES)

- Pretty accurate
- Expensive but "doable"

Ensemble average → Reynolds Averaged Navier-Stokes (RANS)

- Less accurate
- Cheap

Formaly RANS and LES decompose fields in

$$u = \bar{u} + u'$$

the meaning of $\bar{\bullet}$ and $\bullet'$ is <u>different</u> in RANS and LES

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \Rightarrow \overline{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}} = 0 \Rightarrow \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0$$

$$\overline{\frac{\partial uu}{\partial x} + \frac{\partial uv}{\partial y}} = \frac{\partial \overline{uu}}{\partial x} + \frac{\partial \overline{uv}}{\partial y} =$$

$$\overline{uv} = \overline{(\bar{u}+u')(\bar{v}+v')} = \overline{\bar{u}\bar{v}} + \overline{\bar{u}v'} + \overline{\bar{v}u'} + \overline{u'v'}$$

$$= \bar{u}\bar{v} + \bar{u}\cancelto{0}{\overline{v'}} + \bar{v}\cancelto{0}{\overline{u'}} + \overline{u'v'}$$

$$= \bar{u}\bar{v} + \overline{u'v'}$$

$$\frac{\partial}{\partial x_j}\overline{u_i u_j} = \frac{\partial}{\partial x_j}\bar{u}_i\bar{u}_j + \frac{\partial}{\partial x_j}\overline{u_i' u_j'}$$

New term: $\tau_{ij} = \overline{u_i u_j} - \overline{u_i' u_j'}$

← we need expression for this
Turbulence model

τ_{ij} is the turbulent stress tensor
models the effects of unresolved motions
on the resolved fields

Turbulence modeling:

Solving: 1. mass

2. momentum + turbulent stresses

3. scalars (Temperature) + turbulent fluxes

Example: τ_{ij} in LES

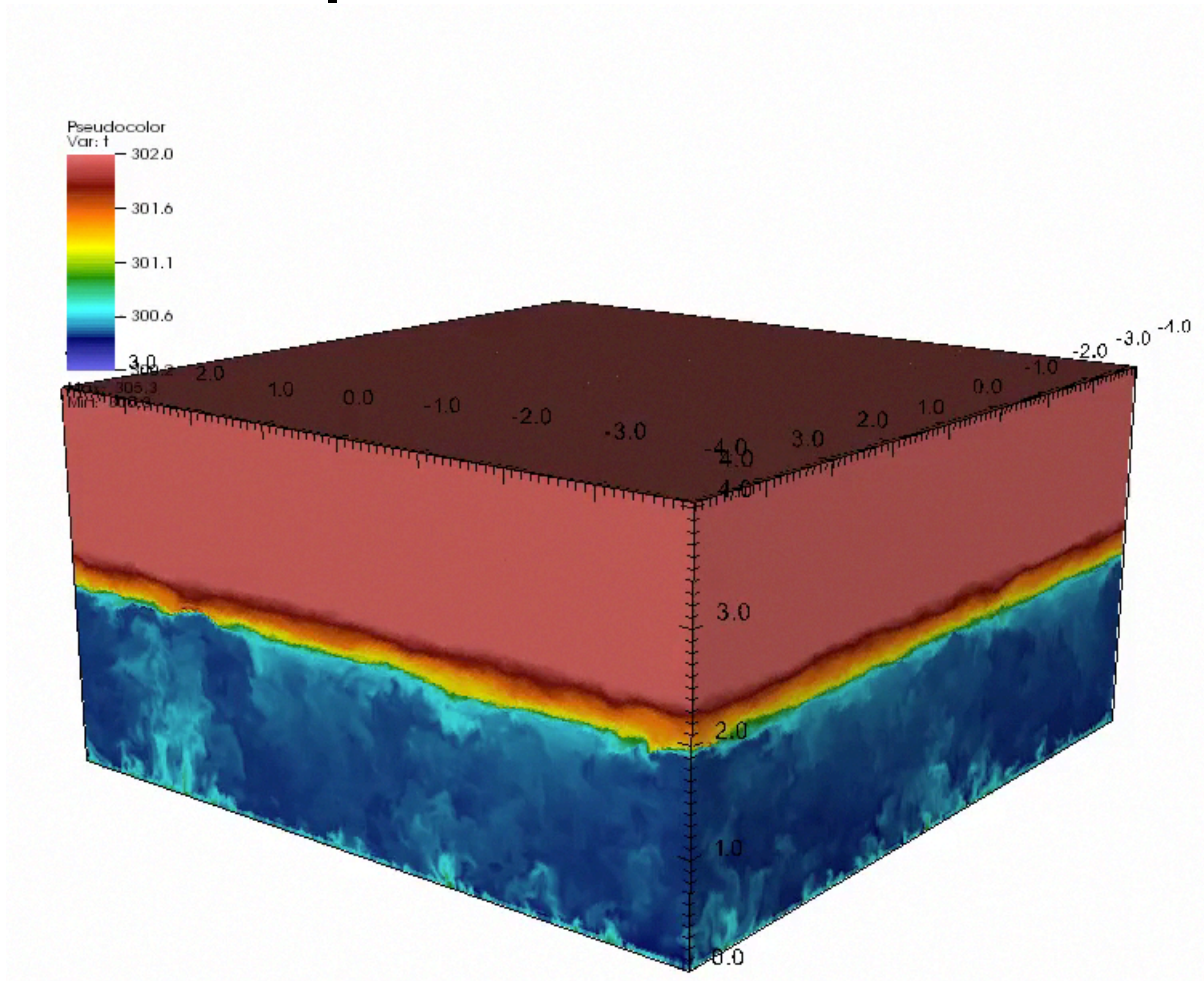
model using an "eddy" viscosity

$$\tau_{ij} = \mu_t \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right) = 2 \mu_t \bar{S}_{ij}$$

Smagorinsky model: $\mu_t = C_s^2 \Delta_x^2 \bar{S}_{ij} \bar{S}_{ij}$

$C_s \simeq 0.18$

LES example: turbulent convection



LES example: turbulent convection

- Check for grid convergence of flow statistics
  - No grid convergence – no prediction
- Comparison with observations of Lenschow et al. (1980)
  - Data can be “noisy” depending on the “quality” of the experimental conditions
- Use theory to scale the data!