Experiment-8

Vertical deflection in swimming pool diving board

Aim

Finding the vertical deflection in a cantilever beam subjected to variable load and material properties and visualization of it.

Methodology

Solving governing equation of vertical deflection in swimming pool diving board using Laplace transform.

Laplace Transform

The Laplace Transform of a function $f(t)$,defined for all real numbers $t > 0$ is the function $F(s)$ defined by,

$$L[f(t)] = \int_0^\infty e^{-st} f(t)dt = F(s) \tag{1}$$

Inverse Laplace Transform

The inverse Laplace Transform of $F(s)$ is defined by

$$f(t) = L^{-1}[F(s)] = \int_0^\infty e^{-st} F(s)dt \tag{2}$$

List of MATLAB Commands used

Command	Purpose
laplace(f)	Returns the Laplace transform of f using the default independent variable t and the default transformation variable s.
laplace(f, *transVar*)	Uses the specified transformation variable *transVar* instead of s.
laplace(f, *var*, *transVar*)	Uses the specified independent variable *var* and transformation variable *transVar* instead of t and s respectively.

List of MATLAB Commands used

Command	Purpose
ilaplace(F)	Returns the inverse Laplace transform of F using the default independent variable s for the default transformation variable t. If F does not contain s, *ilaplace* uses *symvar*.
ilaplace(F, *transVar*)	Uses the specified transformation variable *transVar* instead of t.

List of MATLAB Commands used

Command	Purpose
$ilaplace(F, var, transVar)$	Uses the specified independent variable *var* and transformation variable *transVar* instead of s and t respectively.
$heaviside(t-a)$	To input the heaviside's unit step function $H(t-a)$.
$dirac(t-a)$	To input the dirac delta function $\delta(t-a)$.
laplace(diff(f(t), t), t, s)	s*laplace(f(t), t, s) - f(0)

Example 1

Write the MATLAB code which computes the Laplace Transform of

$$f(t)=\begin{cases} t^2, & t < 2, \\ t-1, & 2 < t < 3 \\ 7, t > 3. \end{cases}$$

MATLAB code

```
clear all
clc
syms t
f=input('Enter the function in terms of t:');
F=laplace(f );
F=simplify(F)
```

Command window

Enter the function in terms of t:

$t^2 * (heaviside(t-0) - heaviside(t-2)) + (t-1) * (heaviside(t-2) - heaviside(t-3)) + 7 * (heaviside(t-3))$

$Y = 5/(s * exp(3 * s)) - 3/(s * exp(2 * s)) - 3/(s^2 * exp(2 * s)) - 1/(s^2 * exp(3 * s)) - 2/(s^3 * exp(2 * s)) + 2/s^3$

Example 2

Solve $y'' + 2y' + 10y = 1 + 5(t - 5), y(0) = 1, y'(0) = 2$

MATLAB Code

```
clc
clear all
syms t s Y
y2=diff(sym('y(t)'),2);
y1=diff(sym('y(t)'),1);
y0=sym('y(t)');
a = input('The Coefficient of D2y = ');
b = input('The Coefficient of Dy = ');
c = input('The Coefficient of y = ');
nh = input('Enter the non homogenous part = ');
eqn=a*y2+b*y1+c*y0-nh;
LTY=laplace(eqn,t,s);
```

continued

```
if (a==0)
d = input('The initial value at 0 is ');
 LTY=subs(LTY,'laplace(y(t), t, s)','y(0)',Y,d)
else
d = input('The initial value at 0 is ');
e = input('The initial value at 0 is ');
LTY=subs(LTY,'laplace(y(t), t, s)','y(0)','D(y)(0)',Y,d,e)
end
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,t))
```

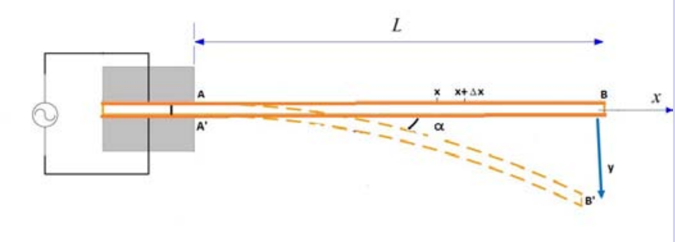
Command window

The Coefficient of D2y = 1
The Coefficient of Dy = 2
The Coefficient of y = 10
Enter the non homogenous part = 1 + 5*dirac(t-5)
The initial value at 0 is 1
The initial value at 0 is 2

$LTY = 10 * Y - s - 5/exp(5 * s) + 2 * Y * s + Y * s^2 - 1/s - 4$

$y = (cos(3 * t) - sin(3 * t)/3)/exp(t) - (cos(3 * t) + sin(3 * t)/3)/(10 * exp(t)) + (4 * sin(3 * t))/(3 * exp(t)) + (5 * heaviside(t - 5) * exp(5 - t) * sin(3 * t - 15))/3 + 1/10$

Mathematical Modelling



The following assumptions are undertaken in order to derive a differential equation of elastic curve for the loaded beam

- Stress is proportional to strain. Thus, the equation is valid only for beams that are not stressed beyond the elastic limit.
- The curvature is always small.
- Any deflection resulting from the shear deformation of the material or shear stresses is neglected.

- For the deflected shape of the beam, the slope α at any point is defined as $\tan\alpha = \frac{dy}{dx}$. Assuming $\tan\alpha = \alpha$ we can write $\alpha = \frac{dy}{dx}$.
- The curvature of a plane curve at a point can be expressed as

$$\frac{1}{\rho} = \frac{\frac{d^2y}{dx^2}}{\left(1 + \frac{dy}{dx}^2\right)^{\frac{3}{2}}}$$

- In the elastic curve of beam $\frac{dy}{dx}$ is very small so we can neglect its higher order terms. $\frac{1}{\rho} = \frac{d^2y}{dx^2}$

- From the theory of elasticity, if x is the distance of the section from the left end of the beam then $\dfrac{1}{\rho} = \dfrac{M(x)}{EI}$

  where M-Bending moment

  E-Modulus of Elasticity

  I-Moment of inertia of the cross section.
- $\dfrac{d^2y}{dx^2} = \dfrac{-M(x)}{EI}$- is the governing equation for an elastic curve.
- When a beam supports a distributive load $w(x)$ then $\dfrac{dM}{dx} = V$(Shear force) and , $\dfrac{dV}{dx} = -w$ Therefore $\dfrac{d^4y}{dx^4} = \dfrac{w(x)}{EI}$

Vertical deflection in a swimming pool diving board subjected to a distributed load can be seen as the deflection in a cantilever beam of length L subjected to a distributed load w(x) which is the solution of the differential equation

$$\frac{d^4y}{dx^4} = \frac{w(x)}{EI}$$

subjected to the boundary conditions
$y(0) = y'(0) = y''(L) = y'''(L) = 0.$
Note: $y''(L) = 0$ because there is no bending moment and $y'''(L) = 0$ because there is no shear at that point.

Problem

Find the deflection in a cantilever beam subjected to the following conditions $y(0) = y'(0) = y''(L) = y'''(L) = 0$ by taking $L = 3$, $E = 2.1 * 10^{11} N/mm^2$, $I = 4.5 * 10^{-11} mm^4$ and $w(x) = x$.

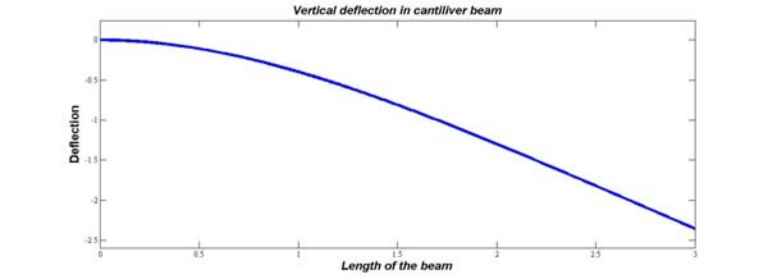
Matlab Code

```
clc
clear all
syms x s C D Y
y4=diff(sym('y(x)'),4);
y0=sym('y(x)');
L=input('Enter the length of the beam:');
E=input('Enter Modulus of elasticity:');
I=input('Enter Moment of inertia of the cross section:');
w=input('Enter distributive load w(x):');
eqn=E*I*y4-w;
LTY=laplace(eqn,x,s);
a=input('Enter y(0):');
b=input('Enter Dy(0):');
c =input('Enter D2y(L):');
d = input('Enter D3y(L):');
```

```
LTY=subs(LTY,{'laplace(y(x),x,s)','y(0)','D(y)(0)','D(D((y)))(0)',
                    'D(D(D((y))))(0)'},{Y, a, b, C, D})
eq=collect(LTY,Y);
Y=simplify(solve(eq,Y));
y=simplify(ilaplace(Y,s,x));
eq1=subs(diff(y,x,2),x,L);
eq2=subs(diff(y,x,3),x,L);
[C, D]=solve(eq1, eq2)
gen=subs(y);
def=subs(def,heaviside(x-L),0)
ezplot(-def,[0,L])
title('Vertical deflection in cantilever beam')
xlabel('Length of the beam')
ylabel('Deflection')
```

Command window

Enter the length of the beam: 3
Enter Modulus of elasticity: $2.1 * 10^{11}$
Enter Moment of inertia of the cross section: $4.5 * 10^{-11}$
Enter distributive load w(x): x*(heaviside(x-L)-heaviside(x))
Enter y(0):0
Enter Dy(0):0
Enter D2y(L):0
Enter D3y(L):0
LTY
$=-\frac{189}{20}*D-\frac{189}{20}*s*C+(1-(3*s+1)*exp(-3*s))/s^2+\frac{189}{20}*s^4*Y$
C $=-20/21$
D $=10/21$
def $=5/63*x^3-10/21*x^2-1/1134*x^5$



Vertical deflection in cantiliver beam
Deflection
Length of the beam
0
-0.5
-1
-1.5
-2
-2.5
0
0.5
1
1.5
2
2.5
3

Exercise

Find the deflection in a cantilever beam subjected to the following conditions $y(0) = y'(0) = y''(L) = y'''(L) = 0$ by taking $L = 2$, $E = 2.1 * 10^{11} N/mm^2$, $I = 4.5 * 10^{-11} mm^4$ and

$$w(x) = \begin{cases} x, & x < 1, \\ x - L, & 1 < x < L. \end{cases}$$

References

- http://web.mst.edu/~mecmovie/chap11/m11_01_propped_cant_cc.swf
- www.me.berkeley.edu/~lwlin/me128/BeamDeflection.pdf
- http://www.mathalino.com/reviewer/mechanics-and-strength-of-materials/chapter-6-beam-deflections
- http://www.engineersedge.com/beam_calc_menu.shtml