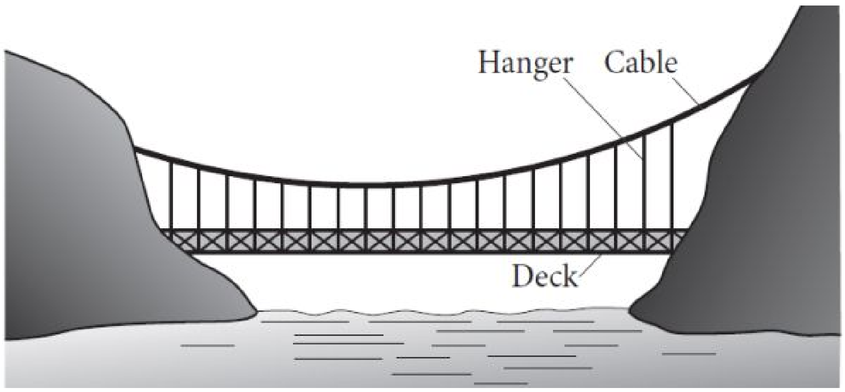
Experiment #03 - DETERMINING THE SHAPE OF A SUSPENSION BRIDGE

Aim: A suspension bridge consists of the main cable, the hangers, and the deck as shown in the figure below:

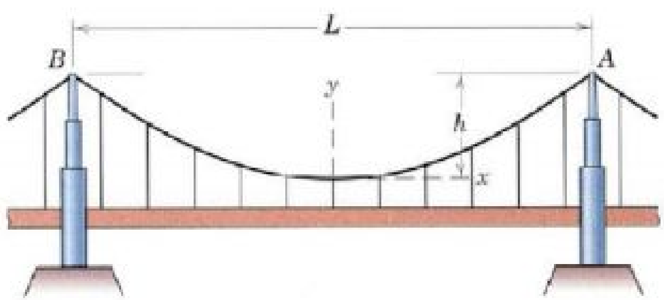


The self-weight of the deck and the loads applied on the deck and transferred to the cable through the hangers.

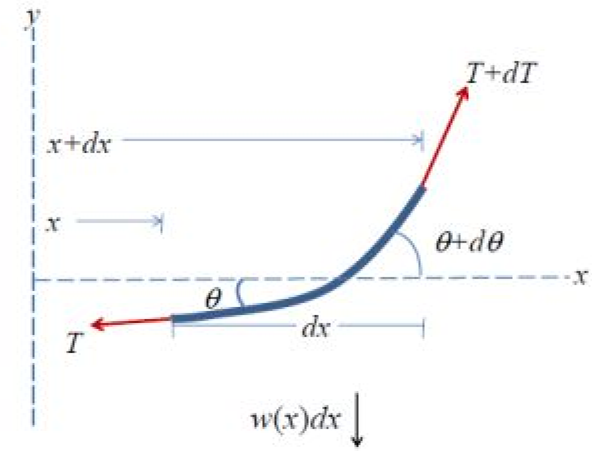
The **purpose** of this experiment is to determine the shape of the cable subject to different load functions and tension to the cable.

Problem Statement:

Punalar suspension bridge, Punalar, Kerala in 1887 is one of India's many suspension bridge.



The vertical distance between the highest and the lowest points of the cable is called Sag. While the horizontal distance between two supports A and B is called Span.



We assume that the cable is loaded by the distributed vertical external load $w(x)$. Now we consider the figure above to derive the equation of the cable:

$sin\theta = \frac{y_1}{T} \Rightarrow y_1 = T sin\theta$
$cos\theta = \frac{x}{T} \Rightarrow x = T cos\theta$
$sin(\theta + d\theta) = \frac{y_2}{T+dT} \Rightarrow y_2 = (T + dT) sin(\theta + d\theta)$
$cos(\theta + d\theta) = \frac{x+dx}{T+dT} \Rightarrow x + dx = (T + dT) cos(\theta + d\theta)$

For static equilibrium, the sum of the forces must be 0, which is, $\sum F_y = 0$

i.e. $-y_1 + y_2 - w(x)dx = 0$
$\Rightarrow - T sinT + (T + dT) sin(\theta + d\theta) - w(x)dx = 0$
$\Rightarrow - T sinT + (T + dT)[sin\theta cosd\theta + cos\theta sind\theta] = w(x)dx$

Assuming that $d\theta \to 0$,
$\Rightarrow sind\theta \to d\theta$
$\Rightarrow cosd\theta \to 1$

Therefore, this reduces to,
$- T sinT + TdT[sin\theta + cos\theta d\theta] = w(x)dx$
$\Rightarrow T cos\theta d\theta + dT sin\theta + dT cos\theta = w(x)dx$

$\sum F_y = 0$, gives,

$d(T sin\theta) = w(x)dx \ldots(1)$

$\Rightarrow T cos\theta = (T + dT)[cos(\theta + d\theta)]$
$\Rightarrow T cos\theta = (T + dT)[cos\theta cos d\theta - sin\theta sin d\theta]$
$\Rightarrow T cos\theta = (T + dT)[cos\theta - sin\theta d\theta]$

i.e., $T sin\theta d\theta - cos\theta dt + sin\theta d\theta dT = 0$
$\Rightarrow d(T cos\theta) = 0$

$\Rightarrow T cos\theta = T_H$, where T_H is a constant.
$\Rightarrow T = \frac{T_H}{cos\theta}$ …(2)

Using (2) in (1):
$d(T_H tan\theta) = w(x)dx$
$\Rightarrow T_H = \frac{d^2y}{dx^2} = w(x)dx$
(i.e.) $\frac{d^2y}{dx^2} = \frac{w(x)dx}{T_H}$

Mathematical background:

Partial integral using the method of variation of parameters:

$P.I. = u(x)y_1(x) + v(x)y_2(x)$

where,

$u(x) = -\int \frac{y_2(x) \cdot f(x)}{w} dx$

$v(x) = \int \frac{y_1(x) \cdot f(x)}{w} dx$

where,

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

Complementary function: $Ae^{m_1 x} + Be^{m_1 x}$
$(A + Bx)e^{m_1 x}$
$y_1(x) = e^{m_1 x}$
$y_2(x) = e^{m_1 x}$

Consider the second order linear ODE with constant coefficient in the following form:

$$(aD^2 + bD + c)y \;=\; f(x) \,...(1)$$

The solution of (1) is

$$y \;=\; C.F. + P.I.$$

Let us assume the particular integral is of the form $y_p = u(x)y_1(x) + v(x)y_2(x)$

where,

$$u(x) \;=\; -\int \frac{y_2(x) \cdot f(x)}{w} dx$$

$$v(x) \;=\; \int \frac{y_1(x) \cdot f(x)}{w} dx$$

Questions:

Matlab Code

```
% Program for solving differential equation of the form % ay"+by'+cy=f(x), for a, b and c as constants.

clear all;
close all;
clc;
syms A B x m

p=input('Enter the coefficients [a,b,c]: ');
f=input('Enter the RHS function f(x): ');
a=p(1);
b=p(2);
c=p(3);
disc=b^2-4*a*c;
m=subs(solve('a*m^2+b*m+c') );
if(disc>0)
    CF= A*exp(m(1)*x)+B*exp(m(2)*x);
    u=exp(m (1)*x);
    v=exp(m (2)*x);
elseif (disc==0)
    CF=(A+B*x)*exp(m (1)*x);
    u=exp(m (1)*x);
    v=x*exp(m (1)*x);
else
    alfa=real(m (1));
    beta=imag(m (1));
    CF=exp(alfa*x)*(A*cos(beta*x)+B*sin(beta*x));
    u=exp(alfa*x)*cos(beta*x);
    v=exp(alfa*x)*sin(beta*x);
end

% Method of variation of parameters.
```

```
f1=f/a;
jac=u*diff(v,x)-diff(u,x)*v; %Jacobian of u and v
P=int(-v*f1/jac,x);
Q=int(u*f1/jac,x);
PI=P*u+Q*v;
y_gen=CF+PI;
dy_gen=diff(y_gen);
cond=input('Enter the initial conditions x0, y(x0) and Dy(x0): ');
eq1=(subs(y_gen,x,cond(1))-cond(2));
eq2=(subs(dy_gen,x,cond(1))-cond(3));
[A B]=solve(eq1,eq2);
y=subs(CF+PI)
```

Example 1:

```
Enter the coefficients [a,b,c]: [1 0 1]
Enter the RHS function f(x): sec(x)
Warning: Support of character vectors that are not valid variable names or define a number
will be removed in a future
release. To create symbolic expressions, first create symbolic variables and then use
operations on them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In solve>getEqns (line 406)
  In solve (line 226)
  In experiment_3 (line 12)
Warning: Do not specify equations and variables as character vectors. Instead, create
symbolic variables with syms.
> In solve>getEqns (line 446)
  In solve (line 226)
  In experiment_3 (line 12)
Enter the initial conditions x0, y(x0) and Dy(x0): [0 0 1]

y =

sin(x) - (log(tan(x)^2 + 1)*cos(x))/2 + x*sin(x)
```

Example 2:

Code

```
% Program for solving differential equation of the form % ay"+by'+cy=f(x), for a, b and c as
constants.

clear all;
```

```
close all;
clc;

syms A B x m
p=input('Enter the coefficients [a,b,c]: ');
w=input('Enter the RHS function w(x): ');
T_h = input('Enter T_h: ');
f = w/T_h;

a=p(1);
b=p(2);
c=p(3);
disc=b^2-4*a*c;
m=subs(solve('a*m^2+b*m+c') );
if(disc>0)
    CF= A*exp(m(1)*x)+B*exp(m(2)*x);
    u=exp(m (1)*x);
    v=exp(m (2)*x);
elseif (disc==0)
    CF=(A+B*x)*exp(m (1)*x);
    u=exp(m (1)*x);
    v=x*exp(m (1)*x);
else
    alfa=real(m (1));
    beta=imag(m (1));
    CF=exp(alfa*x)*(A*cos(beta*x)+B*sin(beta*x));
    u=exp(alfa*x)*cos(beta*x);
    v=exp(alfa*x)*sin(beta*x);
end

% Method of variation of parameters.
f1=f/a;
jac=u*diff(v,x)-diff(u,x)*v; %Jacobian of u and v
P=int(-v*f1/jac,x);
Q=int(u*f1/jac,x);
PI=P*u+Q*v;
y_gen=CF+PI;
dy_gen=diff(y_gen);
cond=input('Enter the initial conditions x0, y(x0) and Dy(x0): ');
eq1=(subs(y_gen,x,cond(1))-cond(2));
eq2=(subs(dy_gen,x,cond(1))-cond(3));
[A B]=solve(eq1,eq2);
y=subs(CF+PI)
```

Output

```
Enter the coefficients [a,b,c]: [1 0 0]
Enter the RHS function w(x): x
Enter T_h: 1
Warning: Support of character vectors that are not valid variable names or define a number
will be removed in a future release.
To create symbolic expressions, first create symbolic variables and then use operations on
them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In solve>getEqns (line 406)
```

```
  In solve (line 226)
  In experiment_3 (line 17)
Warning: Do not specify equations and variables as character vectors. Instead, create
symbolic variables with syms.
> In solve>getEqns (line 446)
  In solve (line 226)
  In experiment_3 (line 17)
Enter the initial conditions x0, y(x0) and Dy(x0): [0 0 1]

y =

x^3/6 + x
```

Example 2:

Using MATLAB code, solve the equation $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} = e^{2t}$, $\frac{dy}{dt}$ at $t = 0$ and $y(0) = 2$.

```
Enter the coefficients [a,b,c]: [1 -4 -5]
Enter the RHS function f(x): exp(2*x)
Warning: Support of character vectors that are not valid variable names or define a number
will be removed in a future release.
To create symbolic expressions, first create symbolic variables and then use operations on
them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In solve>getEqns (line 406)
  In solve (line 226)
  In experiment_3 (line 14)
Warning: Do not specify equations and variables as character vectors. Instead, create
symbolic variables with syms.
> In solve>getEqns (line 446)
  In solve (line 226)
  In experiment_3 (line 14)
Enter the initial conditions x0, y(x0) and Dy(x0): [0 2 2]

y =

(25*exp(-x))/18 - exp(2*x)/9 + (13*exp(5*x))/18
```

Attestation

EXPERIMENT-3

DATE __/__/__

P.I. $= u(x)y_1(x) + v(x)y_2(x)$

where,

$$u(x) = -\int \frac{y_2(x)\cdot f(x)}{w}\,dx$$

$$v(x) = \int \frac{y_1(x)\cdot f(x)}{w}\,dx,$$

where,

$$w = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

C.F $Ae^{m_1 x} + Be^{m_2 x}$

$m_1 = m_2$

$(A + Bx)e^{m_1 x}$

$y_1(x) = e^{m_1 x}$

$y_2(x) = e^{m_2 x}$

Consider the second order linear ODE with Constant coefficient in the following form:

$$(aD^2 + bD + c)y = f(x) \quad \text{—} ①$$

The solution of ① is

$$y = C.F. + P.I.$$

Let us assume that particular integral is of the form $y_P = u(x)y_1(x) + v(x)y_2(x)$

where $u(x) = -\int \frac{y_2(x)f(x)}{y_1 y_2' - y_1' y_2}\,dx$ and