Experiment #10 - STABILITY ANALYSIS OF DIFFERENCE EQUATION SYSTEMS USING Z-TRANSFORMS

Aim: Finding Poles, Zeros and Gain using MATLAB.

Background:

- The MATLAB provides a number of functions to assist in the zero-pole-gain analysis of the system.
- The command [z p k] = tf2zpk(b,a) finds the zeros z, poles p, and gain k from the coefficients b and a of the transfer function H(z).
- The command zplane(b,a) plots the zeros and poles of the system along with a reference unit circle. A o is used for zeros and and x is used for poles. If all the x lies inside the unit circle, then the given system is stable.

inverse z-transform:

$$z\,x_n = X(z) = \sum_{n=0}^{\infty} x_n z^{-n}$$

pulse transfer function:

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{a_0 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_N z^{-N}}$$

$$= z^{-N-M} \frac{b_0 z^M + b_1 z^{M-1} + b_2 z^{M-2} + \dots + b_M}{a_0 z^N + a_1 z^{N-1} + a_2 z^{N-2} + \dots + a_N}$$

~~Poles, Zeros, and Gain~~

which is the ratio of the z-transform of the state-~~sequence~~ response $\{x_n\}$ to the z-transform of the input $\{u_n\}$, when all initial conditions are zero.

Poles, zeros and Gain

- If the numerator and denominator of the proper form of a transfer function are factored:

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- the roots of the numerator polynomial are called zeros.
- the roots of the denominator are called poles.
- the leading coefficient of numerator is gain.

A system is called stable if all its poles lie inside of a unit circle on the z-plane.

zplane [b, a], plots the zeros and poles of the system along with a reference unit circle

[z p k] = tf2zpk(b, a) finds the zero z, poles p, and gain k from the ~~of~~ coefficient b and a of the transfer function H(z).

eg:

$$H(z) = \frac{X(z)}{U(z)} = \frac{1 + 2z^{-1}}{1 + 0.2z^{-2}} = \frac{z^2 + 2z}{z^2 + 0.2z}$$

eg:

$$H(z) \quad \frac{18z^3}{18z^3 + 3z^2 - 4z - 1}$$

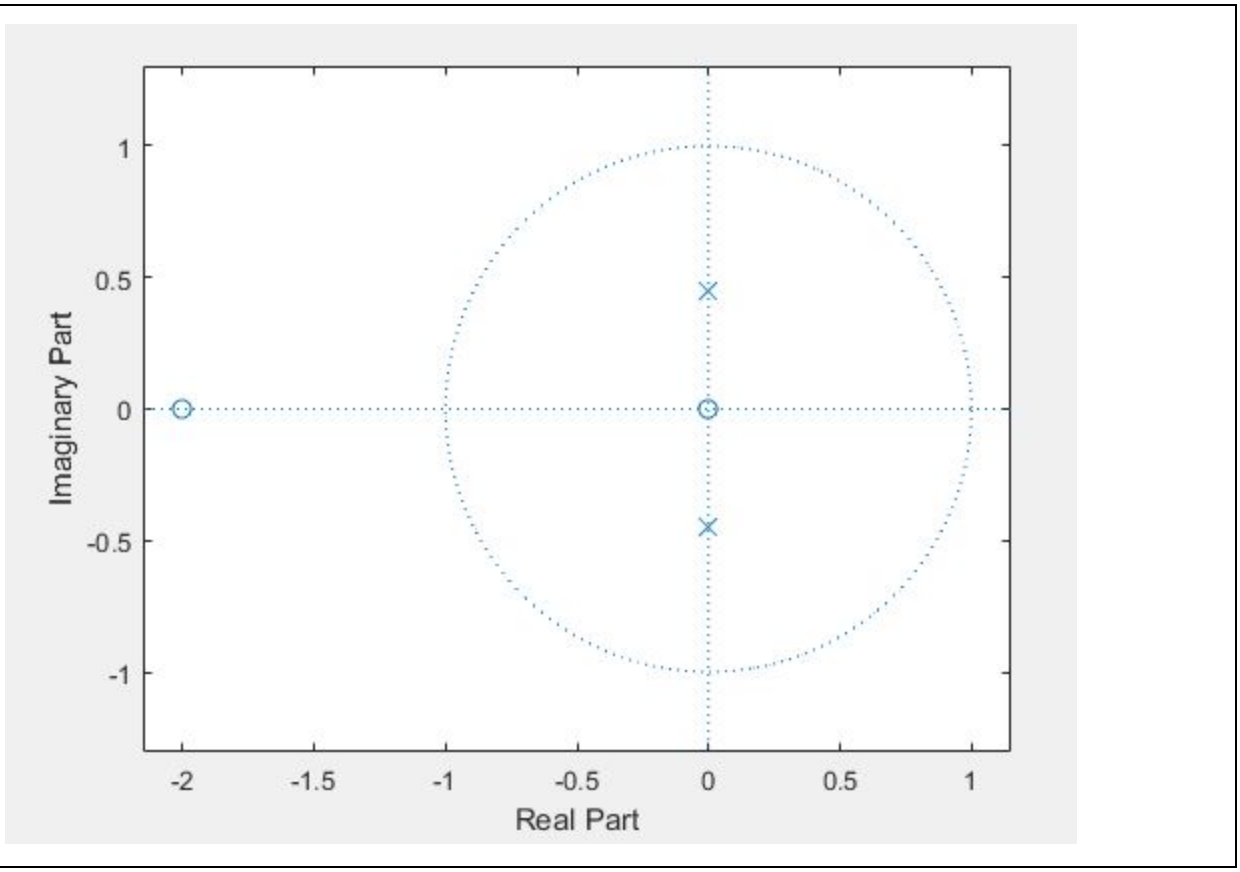
20/10/17

MATLAB code

```
clc
clear all

num = [1 2 0];
den = [1 0 0.2];

[z p k] = tf2zpk(num,den)
zplane(z,p)
```

Output

```
z =

     0
    -2


p =

   0.0000 + 0.4472i
   0.0000 - 0.4472i


k =

     1

>>
```


Imaginary Part
Real Part
1
0.5
0
-0.5
-1
-2
-1.5
-1
-0.5
0
0.5
1

MATLAB code

```
clc
clear all
num = [18 0 0 0];
den = [18 3 -4 -1];
[r,p,k] = residuez(num,den);
disp('Residues'); disp(r');
disp('Poles'); disp(p');
disp('Constants'); disp(k)
```

Output

```
Residues
    0.3600    0.2400    0.4000

Poles
    0.5000   -0.3333   -0.3333

Constants
     0

>>
```