Experiment 7

Sprung Mass Displacement in a Quarter Car (One Wheel) Model

MAT2002–Lab

Application of Differential and Difference Equations

Aim

- To write MATLAB code for system of second order differential equations of the form $X'' + AX = 0$ using Diagonalization
- Determining the sprung mass displacement in a car suspension for one wheel. Solving a coupled system of ordinary differential equations derived from the mathematical model.

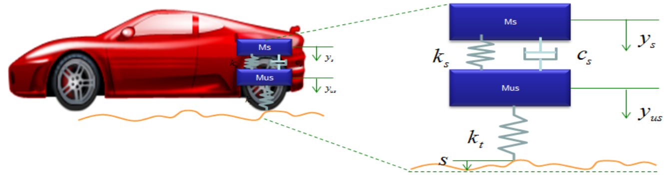


Figure.1

Mathematical form

Using the transformation $Y = PX$, the given system of differential equations can be reduced to uncoupled system as $X'' + AX = 0$, where D is the diagonal matrix with eigen values of A as diagonal elements and Q is the modal matrix corresponding to A.

MATLAB syntax used

- **eig**:*eig(A)* returns a vector of the eigenvalues of matrix A
- **solve**:*solve(eq, x)* returns the set of all complex solutions of an equation or inequality *eq* with respect to x.

Problem

Solve the following system of equations

$$y_1'' = -5y_1 - 2y_1$$

$$y_2'' = -2y_1 - 2y_2$$

Solution

- Given Differential Equation: $Y'' + AY = 0$ where $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 2 \\ 2 & 2 \end{bmatrix}$
- Considering $Y = PX$, $X'' = DX$ where P is modal matrix, D is diagonal matrix and $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$.
- For the given problem, $P = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 0 \\ 0 & 6 \end{bmatrix}$.
- Solving $X'' = DX$, $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} c_1 \cos t + c_2 \sin t \\ c_3 \cos \sqrt{6}t + c_4 \sin \sqrt{6}t \end{bmatrix}$.
- Solution: $Y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \begin{bmatrix} c_1 \cos t + c_2 \sin t \\ c_3 \cos \sqrt{6}t + c_4 \sin \sqrt{6}t \end{bmatrix}$

Solution by Matlab Code

```
clc
clear all
close all
syms x1(t) x2(t)
A = input('Enter the coefficient matrix A: ');
lambda = eig(A);
fprintf('eigen values of A are %f, %f\n\n',lambda);
for i=1:length(lambda)
    temp = null(A-lambda(i)*eye(size(A)),'r');
    P(:,i) = temp./min(temp);
end
```

Solution by Matlab Code

```
disp('The Modal Matrix is: ');
disp(P);
D = inv(P)*A*P;
X = [x1;x2];
Sol1 = dsolve(diff(x1,2) + D(1)*x1 == 0);
Sol2 = dsolve(diff(x2,2) + D(4)*x2 == 0);
disp('The solution of the system diff(X,2)+DX=0 is: ');
disp(Sol1);
disp(Sol2);
disp('The Solution of the given system is: ');
Y = P*[Sol1; Sol2]
```

Output of the Matlab Code

Output:

Y =

2*C5*cos(6ˆ(1/2)*t) + 2*C6*sin(6ˆ(1/2)*t) + C2*cos(t) + C3*sin(t)

C5*cos(6ˆ(1/2)*t) + C6*sin(6ˆ(1/2)*t) - 2*C2*cos(t) âĂŞ 2*C3*sin(t)

Exercise

Solve the following system of equations

1.
$$x_1'' = -2x_1 - 5x_1$$
$$x_2'' = -5x_1 - 2x_2$$

2.
$$x_1'' = -5x_1 - 2x_1$$
$$x_2'' = +2x_1 - 2x_2$$

Sprung Mass Displacement in a Quarter Car (One Wheel) Model

- The vehicle suspension system differ depending on the manufacturer which ensures a wide range of models. Whichever solution is adopted to design, a suspension system has the primary role to ensuring the safety function
- It is known that road unevenness produce oscillations of the vehicle wheels which will transmitted to their axles. It becomes clear that the role of the suspension system witch connect the axles to the car body is to reduce as much vibrations and shocks occurring in the operation. This causes, the necessity to using a suspension of a better quality.

Sprung Mass Displacement in a Quarter Car (One Wheel) Model

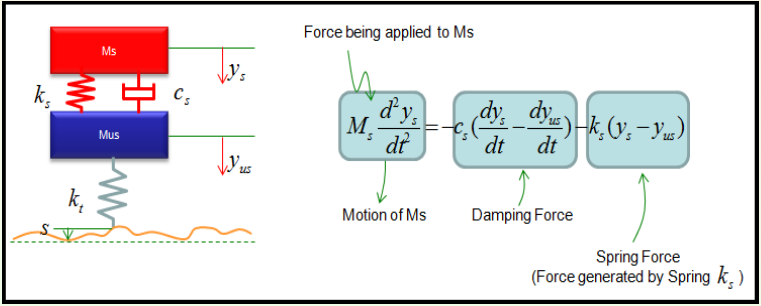
- A quality suspension must achieve a good behavior of the vehicle and a degree of comfort depending on the interaction with uneven road surface. When the vehicle is requested by uneven road profile, it should not be too large oscillations, and if this occurs, they must be removed as quickly. The design of a vehicle suspension is an issue that requires a series of calculations based on the purpose.
- Suspension systems are classified in the well-known terms of passive, semi-active, active and various in between systems. Typical features are the required energy and the characteristic frequency of the actuator. Passive system are the most common. So far, several models have been developed , such as quarter car, half car or full car suspensiony.

Sprung Mass Displacement in a Quarter Car (One Wheel) Model

The system shown in Figure.1 is an quarter car system where

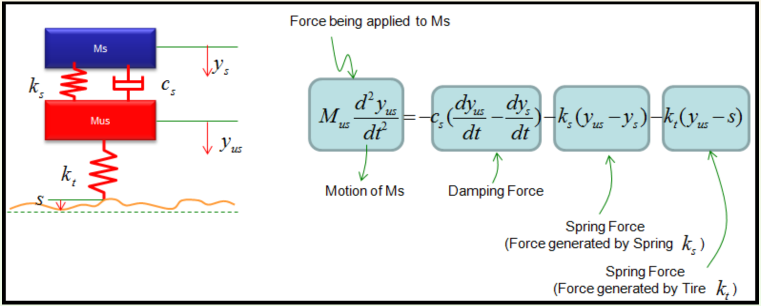
- M_s - is the sprung mass
- M_{us} - is the unsprung mass
- k_s - is the stiffness coefficient of the suspension
- k_t - is the vertical stiffness of the tire
- c_s - is the damping coefficient of the suspension
- b_2 - is the damping coefficient of the tire
- y_s - the vertical displacement of sprung mass
- y_{us} - is the vertical displacement of unsprung mass
- s - is the road excitation

Sprung Mass equation in a Quarter car (one Wheel)



$$M_s \frac{d^2 y_s}{dt^2} + c_s \left(\frac{d y_s}{dt} - \frac{d y_{us}}{dt} \right) + k_s (y_s - y_{us}) = 0 \qquad (1)$$

Unsprung Mass equation in a Quarter car (one Wheel)



$$M_{us}\frac{d^2y_{us}}{dt^2} + c_s\left(\frac{dy_{us}}{dt} - \frac{dy_s}{dt}\right) + k_s(y_{us} - y_s) + k_t y_{us} = k_t s \qquad (2)$$

The differential equation of the sprung and un sprung masses of the quarter-car model

$$M_s \frac{d^2 y_s}{dt^2} + c_s\left(\frac{dy_s}{dt} - \frac{dy_{us}}{dt}\right) + k_s(y_s - y_{us}) = 0$$

$$M_{us} \frac{d^2 y_{us}}{dt^2} + c_s\left(\frac{dy_{us}}{dt} - \frac{dy_s}{dt}\right) + k_s(y_{us} - y_s) + k_t y_{us} = k_t s$$

Matrix form of above equations

$$\begin{bmatrix} M_s & 0 \\ 0 & M_{us} \end{bmatrix} \begin{bmatrix} \dfrac{d^2 y_s}{dt^2} \\ \dfrac{d^2 y_{us}}{dt^2} \end{bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix} \begin{bmatrix} \dfrac{dy_s}{dt} \\ \dfrac{dy_{us}}{dt} \end{bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \begin{bmatrix} y_s \\ y_{us} \end{bmatrix} = \begin{bmatrix} 0 \\ k_t s \end{bmatrix}$$

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{c}\dot{\mathbf{y}} + \mathbf{k}\mathbf{y} = \mathbf{f}$$

Matrix form of above equations (cont.)

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\mathbf{M} \frac{d^2\mathbf{x}}{dt^2} + \mathbf{K}\mathbf{x} = 0$$

Exercise

Two particles each of mass m gm are suspended from two springs of stiffness k_1 and k_2 . After the system comes to rest , the lower mass if pulled downwards and released. Discuss their motion. If $m_1 = m_2 = 1$ and $k_1 = 3, k_2 = 2$, find the displacement.