BLOOD FLOW IN ARTERIES

Contents

AIM AND PROBLEM STATEMENT

- We aim at finding blood velocity in the arteries by solving the Wommersley Equations in terms of Bessel function.
- Visualizing the blood flow velocity in the arteries for different values of angular frequency verses Fractional radius for the first four harmonics from a sinusoidal pressure gradient.

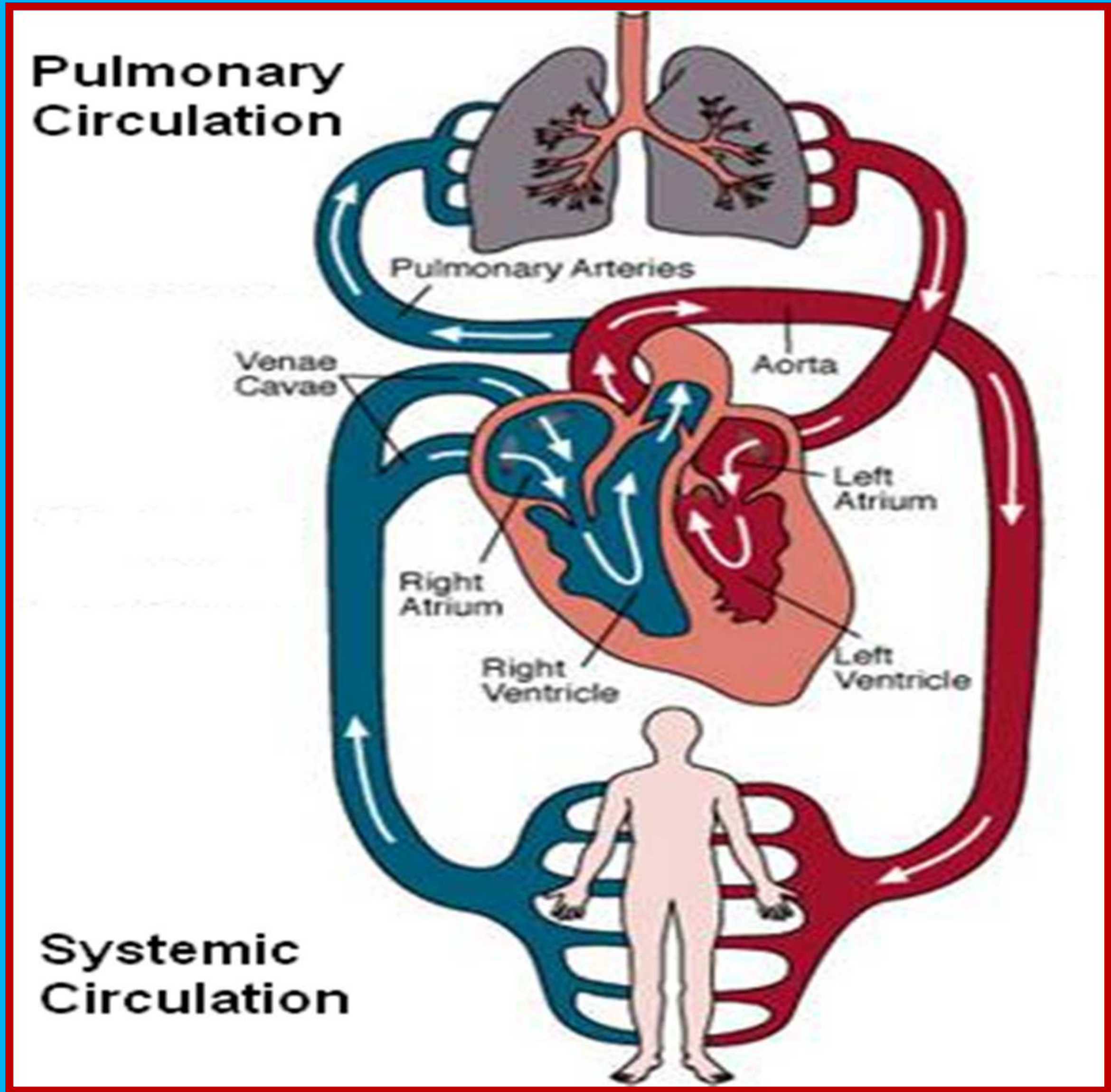
BLOOD FLOW

- The study of the behavior of blood flow in the blood vessels provides understanding on connection between flow and the development of diseases such as atherosclerosis, thrombosis, aneurysms etc. and how the flow dynamics is changed under these conditions.

- The understanding of the flow dynamics past prosthetic devices such as heart valves, vascular grafts and artificial hearts will help improving the design of the implants.
- The functioning of several extra-corporeal flow devices such as blood oxygenators and dialysis machines, which are commonly used in modern medicine, can be improved if blood flow behavior through the devices is well understood

CARDIOVASCULAR PHYSIOLOGY

- The cardiovascular system includes the heart, blood and blood vessels of the systemic and pulmonary circulation.
- The flow of blood from the left ventricle into the aorta then to the peripheral regions of the body and back to the right atrium is defined as the systemic circulation.
- The arteries and arterioles carry the oxygenated blood to the capillaries in the tissues and the deoxygenated blood returns to the right atrium through the venules and the veins.
- Blood flow from the right ventricle into the lungs and back to the left atrium is defined as the pulmonary circulation.


Pulmonary Circulation
Pulmonary Arteries
Venae Cavae
Aorta
Left Atrium
Right Atrium
Right Ventricle
Left Ventricle
Systemic Circulation

	mean diameter [mm]	number of vessels
Aorta	19 - 4.5	1
Arteries	4 – 0.15	110.000
Arterioles	0.05	2.7 ·106
Capillaries	0.008	2.8 ·109

- Blood receives oxygen in the lungs and nutrients in the intestine and delivers them to the cells in all parts of the body.
- The circulating blood also removes cellular wastes and carbon dioxide from the cells for excetion through the kidneys and the lung. It maintains the visceral organs (brain, kidney, liver…) at a constant temperature by convecting the heat generated and dissipating the same through transfer across the skin.
- It stabilizes the body temperature and pH. Fundamental requirements of the circulatory system are to provide adequate blood flow without interruption and to regulate blood flow according to the various demands of the body.
- The contracting heart supplies the energy required to maintain the blood flow through the vessels.
- The pressure gradient developed between the arterial and the venous end of the circulation is the driving force causing blood flow through the vessels.

PHYSICAL PROPERTIES OF BLOOD

- The whole blood consists of formed elements that are suspensed in plasma.
- The plasma is a dilute electrolyte solution containing about 8% by weight of proteins. About 45% by volume of whole blood consist of formed elements and about 55% of plasma in the normal human blood.
- The formed elements of blood are red blood cells (95%), white blood cells (0.13%) and platelets (4.9%). The diameter of red blood cell is about 8.5 μm at the thickest portion and about 1 μm at the thinnest portion. Its membrane is flexible and the cell can pass through capillaries of diameter as small as 5 μm assuming a bent shape.

BLOOD FLOW IN ARTERIES

- The aorta and arteries have a low resistance to blood flow compared with the arterioles and capillaries. When the ventricle contracts, a volume of blood is rapidly ejected into the arterial vessels.
- Since the outflow to the arteriole is relatively slow because of their high resistance to flow, the arteries are inflated to accommodate the extra blood volume.
- During diastole, the elastic recoil of the arteries forces the blood out into the arterioles. Thus, the elastic properties of the arteries help to convert the pulsatile flow of blood from the heart into a more continuous flow through the rest of the circulation. Hemodynamic is a term used to describe the mechanisms that affect the dynamics of blood circulation.

An accurate model of blood flow in the arteries would include the following realistic features

1. The flow is pulsatile, with a time history containing major frequency components up to the eighth harmonic of the heart period.

2. The arteries are elastic and tapered tubes.

3. The geometry of the arteries is complex and includes tapered, curved, and branching tubes.

4. In small arteries, the viscosity depends upon vessel radius and shear rate.

WOMERSLEY EQUATIONS

- Blood flow in the large arteries is driven by the heart, and accordingly it is a pulsating flow.
- The simplest model for pulsatile flow was developed by Wommersley (1955) for a fully developed oscillatory flow of an incompressible fluid in a rigid, straight circular cylinder.
- The problem is defined for a sinusoidal pressure gradient composed from sinuses and cosinuses,

The equation for the motion of a viscous liquid in laminar flow in a tube of circular crosssection with radius R in its general form for an incompressible liquid is

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{\mu}\frac{\partial p}{\partial z}=\frac{\rho}{\mu}\frac{\partial w}{\partial t}$$

Following common convention, the axis of the tube is taken as the z axis and the velocity in the direction of that axis is w (the velocities in the x and y axes for a rigid tube are both zero). The coefficient of viscosity is μ and the density of the liquid is ρ. The form of the pressure gradient is taken as a simple harmonic motion and written in complex form.

$$\frac{\partial p}{\partial z} = A^* e^{i\omega t}$$

With this substitution and $w = ue^{i\omega t}$ we obtain

$$\frac{d^2 u}{dr^2} + \frac{1}{r}\frac{du}{dr} - \frac{i\omega\rho}{\mu}u = -\frac{A^*}{\mu}$$

This is a form of Bessel's equation, and its solution by the method of separation of variables, appropriate to the boundary conditions, can be written as

$$u(r)=\frac{A^*}{i\omega\rho}\left[1-\frac{J_0\left(r\sqrt{(\omega\rho/\mu)}i^{3/2}\right)}{J_0\left(R\sqrt{(\omega\rho/\mu)}i^{3/2}\right)}\right]$$

where an expression of the form is $J_0\left(xi^{3/2}\right)$ a Bessel function of the first kind of order zero and complex argument. The quantity $R\sqrt{(\omega\rho/\mu)}$ is a non-dimensional parameter that characterizes kinematic similarities in the liquid motion and it is written as the symbol α .The radius is also made non-dimensional by substituting the fractional radius $y=r/R$.The solution for the velocity w is then

α

$$w(r) = \frac{A^* R^2}{i\omega\alpha^2}\left[1 - \frac{J_0\left(\alpha y i^{3/2}\right)}{J_0\left(\alpha i^{3/2}\right)}\right] * e^{i\omega t}$$

Where α is the Womersley number

The velocity profiles for the first four harmonics resulting from the pressure gradient *Cos wt* which oscillates sinusoidally shown in the figure for α =3.34,4.72,5.78 and 6.67. When α is large, the velocity profile becomes blunt.

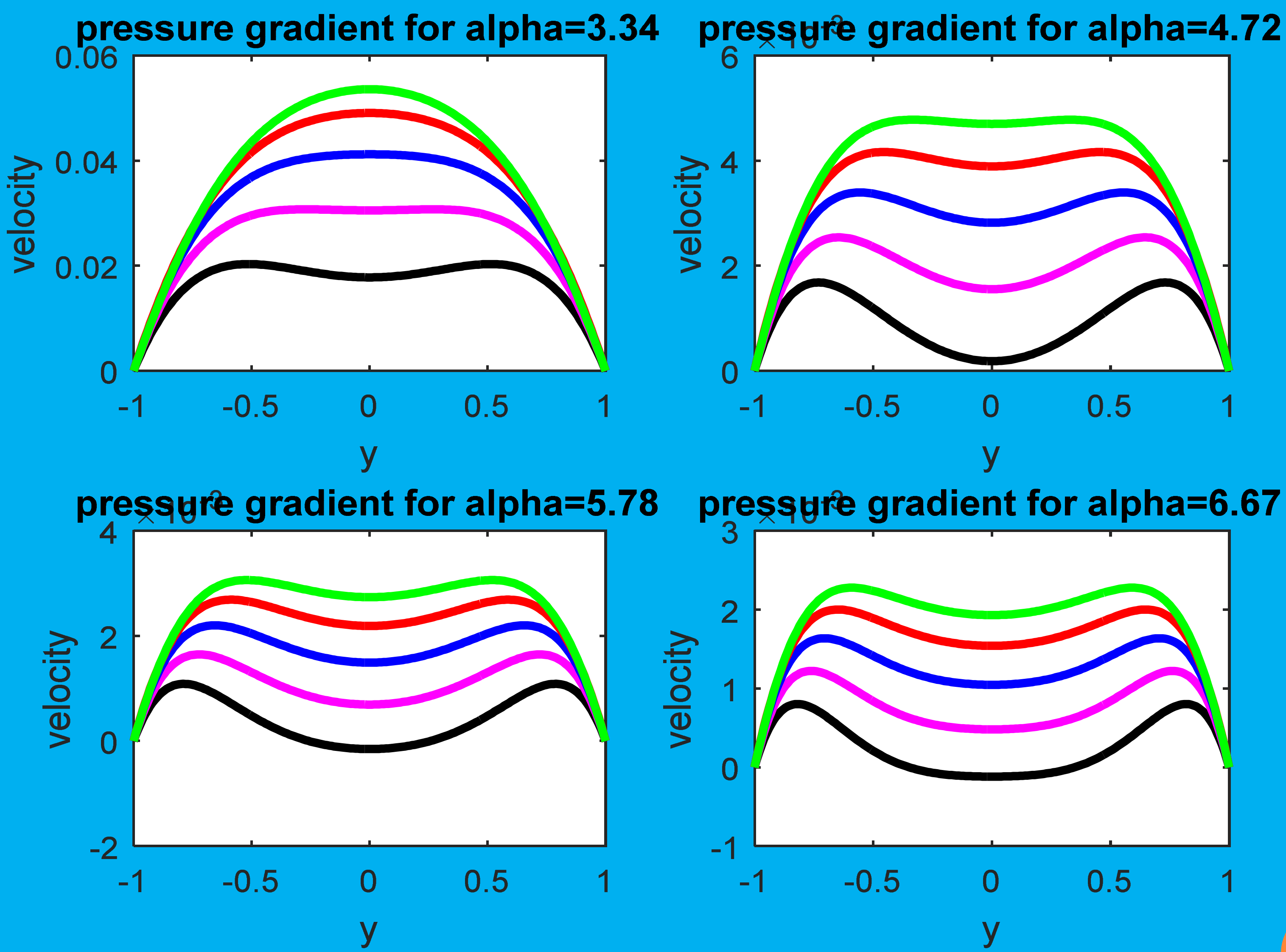
MATLAB CODE

```
% Bessel Functions of First Kind
clc;clear all;clf;
% Modified Bessel Functions of Zero Order
y=-1:0.01:1;
A=1,R=1,m=2,a=3.34;
subplot(2,2,1)
I0=besselj(0,i*sqrt(i)*y*a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5); hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
```

```
xlabel('y');
ylabel('velocity')
title('pressure gradient for alpha=3.34')
A=0.4,R=0.5,m=1,a=4.72;
subplot(2,2,2)
I0=besselj(0,i*sqrt(i)*y*a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5); hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
```

```
xlabel('y')
ylabel('velocity')
title('pressure gradient for alpha=4.72')
A=0.4,R=0.5,m=1,a=5.78;
subplot(2,2,3)
I0=besselj(0,i*sqrt(i)*y*a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5); hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
xlabel('y');
ylabel('velocity')
title('pressure gradient for alpha=5.78')
A=0.4,R=0.5,m=1,a=6.67;
subplot(2,2,4)
```

```
I0=besselj(0,i*sqrt(i)*y*a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5); hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
xlabel('y')
ylabel('velocity')
title('pressure gradient for alpha=6.67')
```


pressure gradient for alpha=3.34
pressure gradient for alpha=4.72
pressure gradient for alpha=5.78
pressure gradient for alpha=6.67
velocity
y
0.06
0.04
0.02
0
-1
-0.5
0
0.5
1