Experiment #09 - POPULATION DYNAMICS

Aim: Analyzing and solving Difference equation for the population growth.

Motivation:

Differential equations arise while modeling physical phenomena in which the independent variable or space variable or both are continuous. Difference equation are the discrete analogues of Differential equation. For example:

1) In an experiment we may take measurements on some physical variable, say, temperature at equally spaced time intervals.
2) We may be interested in population growth of a certain species at discrete time intervals or
3) We may like to approximate differential equations by writing them in difference form.

Infact difference equations are essential for systems with discrete or digital data.

Problem Statement:

We divide this experiment in two parts

1) Formulation of population growth (Human population) by first order difference equation.
2) Formulation of population growth of rabbits in terms of second order difference equation (through Fibonacci numbers).

MATLAB Code

```
clc
clear all
syms z Y n positive
LHS=ztrans(sym('y(n+2)')-sym('y(n+1)')-sym('y(n)'),n,z);
RHS=ztrans(0,n,z)
newLHS=subs(LHS,{'ztrans(y(n),n,z)','y(0)','y(1)'},{Y,0,1});
Y=solve(newLHS-RHS,Y);
y=iztrans(Y,z,n)
```

Output

```
Warning: Support of character vectors that are not valid variable names or define a number
will be removed in a future
release. To create symbolic expressions, first create symbolic variables and then use
operations on them.
> In sym>convertExpression (line 1586)
```

```
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In experiment8 (line 4)
Warning: Support of character vectors that are not valid variable names or define a number
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  In sym (line 199)
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RHS =

0

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release. To create symbolic expressions, first create symbolic variables and then use
operations on them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In sym/subs>@(x)sym(x) (line 156)
  In sym/subs>normalize (line 156)
  In sym/subs>mupadsubs (line 147)
  In sym/subs (line 135)
  In experiment8 (line 6)
Warning: Support of character vectors that are not valid variable names or define a number
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operations on them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In sym/subs>@(x)sym(x) (line 156)
```

```
  In sym/subs>normalize (line 156)
  In sym/subs>mupadsubs (line 147)
  In sym/subs (line 135)
  In experiment8 (line 6)
Warning: Support of character vectors that are not valid variable names or define a number
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release. To create symbolic expressions, first create symbolic variables and then use
operations on them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In sym/subs>@(x)sym(x) (line 156)
  In sym/subs>normalize (line 156)
  In sym/subs>mupadsubs (line 147)
  In sym/subs (line 135)
  In experiment8 (line 6)
Warning: The solutions are valid under the following conditions: 0 < z^2 - z - 1.
 To include parameters and conditions in the solution, specify the 'ReturnConditions'
option.
> In solve>warnIfParams (line 508)
  In solve (line 357)
  In experiment8 (line 7)

y =

(2*(-1)^n*cos(n*(pi/2 + asinh(1/2)*1i)))/1i^n - (2*(-1)^(1 - n)*(-1)^n*5^(1/2)*(1/2 -
5^(1/2)/2)^(n - 1))/5 + (2*(-1)^(1 - n)*(-1)^n*5^(1/2)*(5^(1/2)/2 + 1/2)^(n - 1))/5

>>
```
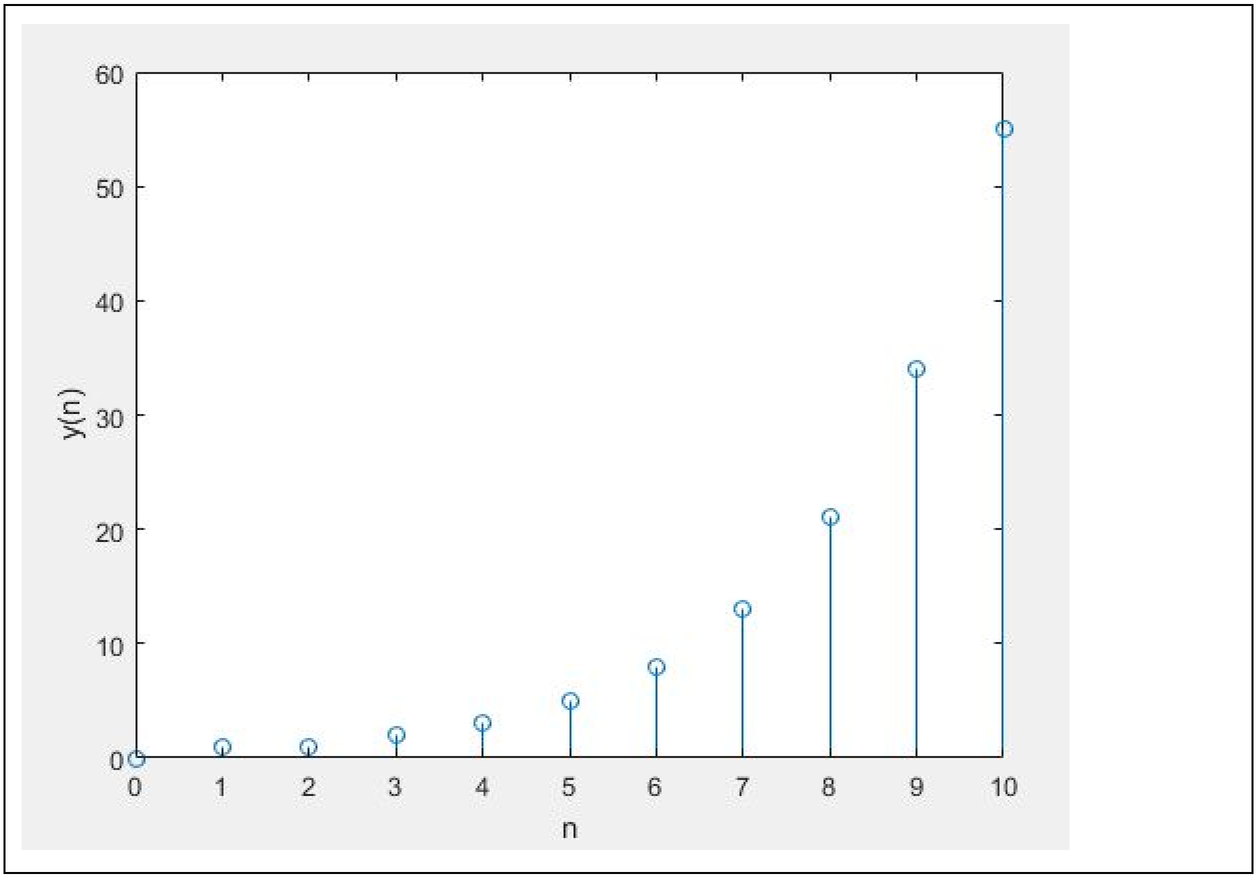

MATLAB code

```
clc
clear all
syms n k1 k2 m
assume(n,'integer')
a = input('Enter the coefficient of y(n+2): ');
b = input('Enter the coefficient of y(n+1): ');
c = input('Enter the coefficient of y(n): ');
g = input('Enter the non-homogeneous part: ');
r = subs(solve(a*m^2+b*m+c,m));
if imag(r)~=0
    rho = sqrt(real(r(1))^2 + imag(r(1))^2);
    theta = atan(abs(imag(r(1)))/real(r(1)));
```

```
    y1 = (rho^n)*cos(n*theta);
    y2 = (rho^n)*sin(n*theta);
elseif r(1)==r(2)
    y1 = r(1)^n;
    y2 = n*r(1)^n;
else
    y1 = r(1)^n;
    y2 = r(2)^n;
end
Co = det([y1, y2;subs(y1,n,n+1), subs(y2,n,n+1)]); %Casoratian of the solutions
y_c = k1*y1 + k2*y2;

disp('Complementary Solution is: ');
disp(y_c);
if(g ~= 0)
    y11 = subs(y1,n,n+1);
    y21 = subs(y2,n,n+1);
    Co1 = subs(Co,n,n+1);
    u1 = simplify(symsum(-g*y21/Co1,n))
    u2 = simplify(symsum(g*y11/Co1,n))
y_p = simplify(u1*y1+u2*y2);
    y = y_c + y_p;
else
    y = y_c;
end
check = input('If the given problem has initial conditions then enter 1 else enter 0: ');
if (check == 1)
    yval1 = input('Enter the initial condition at n = 0: ');
    yval2 = input('Enter the initial condition at n = 1: ');
    cond1 = strcat(char(subs(y,n,0)),'=',num2str(yval1));
    cond2 = strcat(char(subs(y,n,1)),'=',num2str(yval2));
    [k1,k2] = solve(cond1,cond2);
    y = subs(y);
end
disp('Complete Solution is: ')
disp(collect(collect(y,y1),y2))

if(check ~= 0)
nrange = 0:10;
    Y = subs(y,n,nrange);
    stem(nrange,Y);
    set(gca,'XTick',linspace(0,10,11))
xlabel('n');
ylabel('y(n)');
end
```

Output

```
Enter the coefficient of y(n+2):   1
Enter the coefficient of y(n+1): -1
Enter the coefficient of y(n): -1
Enter the non-homogeneous part: 0
Complementary Solution is:
k1*(1/2 - 5^(1/2)/2)^n + k2*(5^(1/2)/2 + 1/2)^n

If the given problem has initial conditions then enter 1 else enter 0: 1
Enter the initial condition at n = 0: 0
Enter the initial condition at n = 1: 1
Warning: Support of character vectors that are not valid variable names or define a number
will be removed in a future
release. To create symbolic expressions, first create symbolic variables and then use
operations on them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In solve>getEqns (line 406)
  In solve (line 226)
  In experiment9 (line 44)
Warning: Support of character vectors that are not valid variable names or define a number
will be removed in a future
release. To create symbolic expressions, first create symbolic variables and then use
operations on them.
> In sym>convertExpression (line 1586)
  In sym>convertChar (line 1491)
  In sym>tomupad (line 1243)
  In sym (line 199)
  In solve>getEqns (line 406)
  In solve (line 226)
  In experiment9 (line 44)
Warning: Do not specify equations and variables as character vectors. Instead, create
symbolic variables with syms.
> In solve>getEqns (line 446)
  In solve (line 226)
  In experiment9 (line 44)
Complete Solution is:
(5^(1/2)/5)*(5^(1/2)/2 + 1/2)^n + (-5^(1/2)/5)*(1/2 - 5^(1/2)/2)^n

>>
```



60
50
40
30
20
10
0
y(n)
0
1
2
3
4
5
6
7
8
9
10
n

EXPERIMENT – 9 30/10/17

POPULATION DYNAMICS

eg. for ~~differential~~ difference equations;

$$y_{n+2} + y_{n+1} \cancel{-y} - 2y_n = f(n)$$

I difference

$$\Delta u_n = u_{n+1} - u_n$$

II difference:

$$\begin{aligned}\Delta^2 u_n &= \Delta(\Delta u_n)\\ &= \Delta(u_{n+1} - u_n)\\ &= u_{n+2} - u_{n+1} - (u_{n+1} - u_n)\\ &= u_{n+2} - 2u_{n+1} + u_n\end{aligned}$$

30/10/17

DATE ___/___/___

Consider the following II order difference equation:

$$a y_{n+2} + b y_{n+1} + c y_n = 0, \ n \geq 0, \ a \neq 0$$

Let $y_n = \alpha^n$, $y_n = C.F.$

$$a\alpha^{n+2} + b\alpha^{n+1} + c\alpha^n = 0$$
$$\alpha^n(a\alpha^2 + b\alpha + c) = 0$$
$$\alpha^n \neq 0 \quad a\alpha^2 + b\alpha + c = 0$$

(i) roots are distinct, α_1, α_2

$$y_n = c_1\alpha^n + c_2\alpha_2^n$$

(ii) roots are equal

$$y_n = (c_1 + c_2 n)\alpha_1^n$$

(iii) roots are imaginary $(\alpha \pm i\beta)$

$$y_n = r^n[A\cos n\theta + B\sin n\theta]$$

where $r = \sqrt{\alpha^2 + \beta^2}$

$$\theta = \tan^{-1}\left(\frac{\beta}{\alpha}\right)$$

$$z(u_{n+1}) = z[U(z) - u_0]$$

$$z(u_{n+2}) = z^2[U(z) - u_0 - u_1 z^{-1}]$$

$$z(u_{n+3}) = z^3[U(z) - u_0 - u_1 z^{-2} - u_2 z^{-2}]$$

30/10/18

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Fibonacci sequence

0, 1, 1, 2, 3, 5, 8, 13 ...

The difference equation:

$y_{n+2} - y_{n+1} - y_n = 0$