Experiment 5: BLOOD FLOW IN ARTERIES

Aim: We aim at finding blood velocity in the arteries by solving the Wommersley Equations in terms of Bessel function*. Visualizing the blood flow velocity in the arteries for different values of angular frequency versus Fractional radius for the first four harmonics from a sinusoidal pressure gradient.

*Bessel function is a solution of Bessel equations.

Problem Statement: The blood consists of formed elements which are Red Blood Cells (RBCs), White Blood Cells (WBCs) and platelets.

The following features are considered in order to model the blood flow:

1. The flow is pulsatile, with a time history containing major frequency components up to the eighth harmonic of the heart period.
2. The arteries are elastic and tapered tubes.
3. The geometry of the arteries is complex and includes tapered, curved and branching tubes.
4. In small arteries, the viscosity depends upon vessel radius and shear rate.

Viscosity (μ): measure of resistance to gradual deformation by shear stress.

$$\gamma = \frac{\mu}{\rho}$$

Where,

γ is kinematic viscosity and

ρ is density.

Pulsatile: flow with periodic variations.

Wommersley Equations:

The simplest model for pulsatile flow was developed by Wommersley (1955) for a fully developed oscillatory flow of an incompressible fluid in a rigid, straight circular cylinder.

The problem is defined for a sinusoidal pressure gradient composed from sinuses and cosinuses.

The equation for the motion of a viscous liquid in laminar flow in a tube of circular cross-section with radius R in its general form for an incompressible liquid is:

$$\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{1}{\mu}\frac{\partial \rho}{\partial z}=\frac{\rho}{\mu}\frac{\partial w}{\partial t}$$

The form of the pressure gradient is taken as a simple harmonic motion and written in complex form:

$$\frac{\partial \rho}{\partial z}=A^{*}e^{i\omega t}$$

With this substitution $w=ue^{i\omega t}$ we obtain:

$$u''^{e^{i\omega t}}+\frac{1}{r}u'^{e^{i\omega t}}+\frac{1}{\mu}A^{*}e^{i\omega t}=$$

$$\frac{d^2 u}{dr^2}+\frac{1}{r}\frac{du}{dr}-\frac{i\omega t}{\mu}u=-\frac{A^{*}}{\mu}$$

This is a form of Bessel's equations and its solution by the method of separation of variables, appropriate to the boundary conditions can be written as:

$$u(r)=\frac{A^{*}}{i\omega p}\left[1-\frac{J_0\left(r\sqrt{(\omega\rho/\mu)i^{3/2}}\right)}{J_0\left(R\sqrt{(\omega\rho/\mu)i^{3/2}}\right)}\right]$$

Where an expression of the for $J_0(xi^{3/2})$ is a Bessel function of the first kind of order zero and its complex argument. The quantity $R\sqrt{(\omega\rho/\mu)}$ is a non-dimensional parameter that characterizes kinematic similarities in the liquid motion and it is written as the symbolα. The radius is also made non-dimensional by substituting the fractional radius $y={}^{r}/_{R}$. The solution for velocity ω is then:

$$w(r)=\frac{A^{*}R^2}{i\omega\alpha^2}\left[1-\frac{J_0(\alpha i^{3/2})}{J_0(\alpha i^{3/2})}\right]*e^{i\omega t}$$

MATLAB code:

```
clc;
clear all;
clf;
y=-1:0.01:1;
A=1,R=1,m=2,a=3.34;
subplot(2,2,1)
I0=besselj(0,i*sqrt(i)*y *a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5);hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
xlabel('y')
ylabel('velocity')
title('presure gradient for alpha=3.34')
A=0.4, R=0.5,m=1, a=4.72;
subplot(2,2,2)
I0=besselj(0,i*sqrt(i)*y *a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5);hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);

xlabel('y')
ylabel('velocity')
title('presure gradient for alpha=4.72')
A=0.4, R=0.5, m=1,a=5.78;
subplot(2,2,3)
I0=besselj(0,i*sqrt(i)*y *a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5);hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
xlabel('y')
ylabel('velocity')
title('presure gradient for alpha=5.78')
A=0.4, R=0.5, m=1, a=6.67;
subplot(2,2,4)
```

```
I0=besselj(0,i*sqrt(i)*y *a);
I1=besselj(0,i*sqrt(i)*a);
W1=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*0);
W2=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/12);
W3=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/6);
W4=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/4);
W5=A*R^2/(i*m*a^2)*(1-I0/I1)*exp(i*pi/3);
plot(y,W1,'k','linewidth',2.5);hold on
plot(y,W2,'m','linewidth',2.5);hold on
plot(y,W3,'b','linewidth',2.5);hold on
plot(y,W4,'r','linewidth',2.5);hold on
plot(y,W5,'g','linewidth',2.5);
xlabel('y')
ylabel('velocity')
title('presure gradient for alpha=6.67')
```

Output:

```
A =

    1


R =

    1


m =

    2

Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 14)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 15)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 16)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 17)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 18)

A =

   0.4000


R =

   0.5000


m =
```

```
     1

Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 31)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 32)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 33)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 34)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 35)

A =

   0.4000


R =

   0.5000


m =

     1

Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 49)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 50)
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> In Experiment_5 (line 51)
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> In Experiment_5 (line 52)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 53)

A =

   0.4000


R =

   0.5000


m =

     1

Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 66)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 67)
Warning: Imaginary parts of complex X and/or Y arguments ignored
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> In Experiment_5 (line 68)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 69)
Warning: Imaginary parts of complex X and/or Y arguments ignored
> In Experiment_5 (line 70)
```

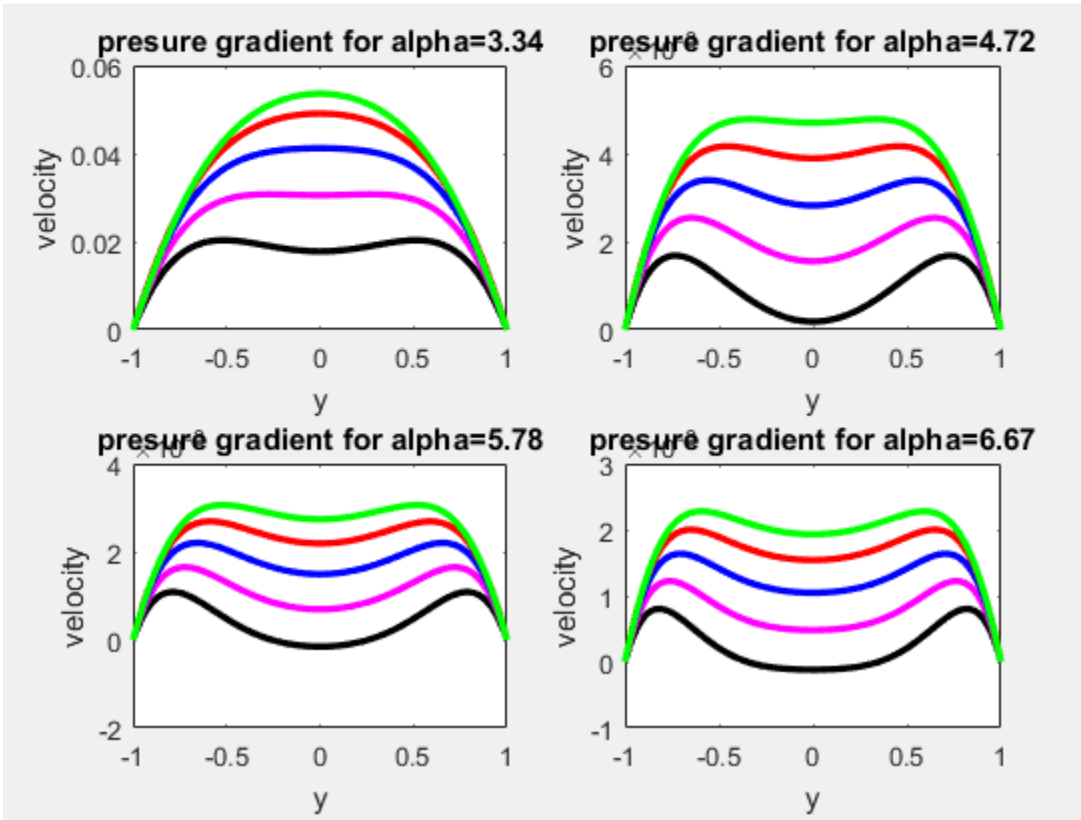

DATE 18/9/17

EXPERIMENT - 5

$A=1, R=1, m=2$

25/9/17

25/9/17

EXPERIMENT - 6

$$P_0(x)\frac{d^2y}{dx^2} + P_1(x)\frac{dy}{dx} + P_2'(x)y = 0$$

$$y'' + \frac{b(x)}{x}y' + \frac{c(x)}{x^2}y = 0 \quad --(2)$$

$$y(x) = x^r \sum_{m=0}^{\infty} a_m x^m \quad - (3)$$

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EXPERIMENT - 7

$$\begin{bmatrix} M_s & 0 \\ 0 & M_{us} \end{bmatrix} \begin{bmatrix} \frac{d^2y_s}{dt^2} \\ \frac{d^2y_{us}}{dt^2} \end{bmatrix} + \begin{bmatrix} C_s & -C_s \\ -C_s & C_s \end{bmatrix} \begin{bmatrix} \frac{dy_s}{dt} \\ \frac{dy_{us}}{dt} \end{bmatrix}$$

$$+ \begin{bmatrix} K_s & -K_s \\ -K_s & K_s + K_t \end{bmatrix} \begin{bmatrix} y_s \\ y_{us} \end{bmatrix} = \begin{bmatrix} 0 \\ K_{t}u \end{bmatrix}$$

30/10/17