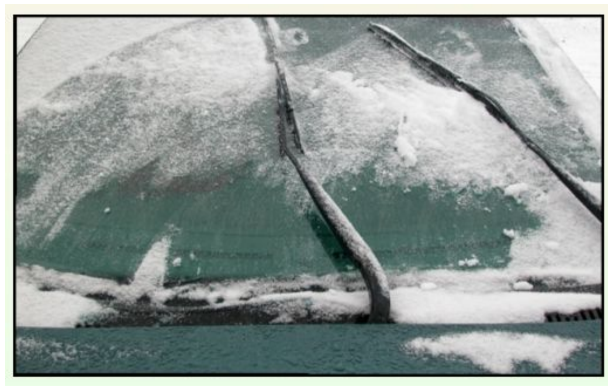
Experiment #04 - FUNCTIONALITY OF A WINDSHIELD WIPERS

Aim: Modelling and analyzing the working mechanism of the **windshield wipers** by solving governing equations.



To solve and visualize the solutions of **linear differential equations** using **dsolve** function.

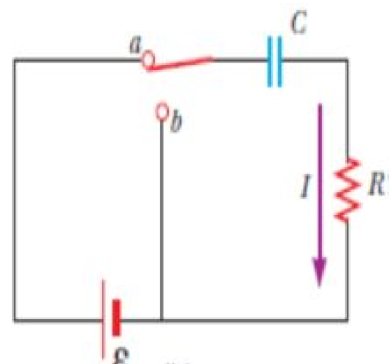
Functionality:
The operating of such wipers depends on the charging and discharging of a capacitor. The wipers are part of an *RC*-circuit whose time constant can be varied by selecting different values of *R* (resistor) through a multiposition switch. As the voltage across the capacitor increases, the capacitor reaches a point at which is discharges and triggers the wipers;another cycle begins. Can be divided into two parts:

- Charging a capacitor in an *RC* circuit
- Discharging a capacitor in an *RC* circuit

Mathematical background:
Kirchhoff's voltage law:

$\sum V = 0$ (loop)

Sum of the voltage drops across the closed circuit is 0.
Following figures show a simple series RC circuit:



(b)

To analyze the circuit quantitatively, apply Kirchhoff's loop rule to the circuit after the switch is thrown to position. Traversing the loop clockwise gives:

$$E - \frac{q}{C} - IR = 0$$

Where, $\frac{q}{C}$ is the potential difference across the capacitor and potential difference across R resistor.

$R\frac{dq}{dt} + \frac{q}{C} = E \ldots(1)$

$ye^{\int pdt} = \int Qe^{\int pdt}dt + C \ldots(*)$

$\frac{dq}{dx} + py = Q \ldots(2)$

$\frac{dq}{dt} + \frac{1}{RC}q = \frac{E}{R} \ldots(3)$

$p = \frac{1}{RC}, Q = \frac{E}{R}$

$qe^{\int pdt} = \int Qe^{\int pdt}dt + C \ldots(4)$

Now solving (3) in (4)

$qe^{\int \frac{1}{RC}dt} = \int \frac{E}{R}e^{\int \frac{1}{RC}dt}dt + A$

$qe^{\frac{t}{RC}} = \frac{E}{R}e^{\frac{t}{RC}}RC + A$

$q = EC + Ae^{-\frac{t}{RC}}$

Now, $\tau = RC$, time constant
When, $t = 0, q = 0$,
$A =- EC$
$q = EC(1 - e^{-\frac{t}{\tau}})$

Using, $E - \frac{q}{C} - IR = 0$
When, $t = 0, q = 0 \Rightarrow i = \frac{E}{R}$, initial current
When, $t = 0, i = 0 \Rightarrow q = EC = Q_{max}$, maximum charge

(i.e.) $q = EC(1 - e^{-\frac{t}{\tau}}) \Rightarrow Q_{max} = EC(1 - e^{-\frac{t}{\tau}}\ ) \ldots (5)$

Using the above equation,
$i = \frac{dq}{dt} = \frac{E}{R} e^{-\frac{t}{\tau}} \ldots (6)$

From (5), as $t \to 0, q \to 0$
As $t \to \infty, q \to Q_{max}$

From (6), as $t \to 0, i \to \frac{E}{R}$,initial current
And as $t \to \infty, i \to 0$

Discharging a capacitor in an RC circuit
We eliminate the *emf* from equation (1), we obtain the appropriate loop equation for the circuit:

$$- \frac{q}{C} - iR = 0$$

Then $- R\frac{dq}{dt} = \frac{q}{C}$ and solving the differential equation, we will get the charge as a function of time for a discharging capacitor.

$$q(t) = Q(e^{\frac{-t}{\tau}})$$

Differentiation the equation above, we will get the current as a function of time for a discharging capacitor

$$i(t) = \frac{Q}{RC} \text{ ,initial current}$$

Matlab syntax

`dsolve(eqn,cond)`	Solves the ordinary differential equation with the initial or boundary condition
`ezplot(f)`	Plots the expression *f(x)* over the default domain $-2\pi < x < 2\pi$, where *f(x)* is not an implicit function of only one variable.

Exercises:

Q: An uncharged capacitor and a resistor are connected in series to a battery as shown in the circuit, where $E = 12.0\text{V}$, $C = 5.00\,\mu F$ and $R = 8.00 \times 10^5\Omega$. The switch is thrown to position *a*. Using MATLAB find the time constant of the circuit, the maximum charge on the capacitor and the maximum current in the circuit. Plot the charge and the current in the circuit as functions of time.

MATLAB code:

```
clc
clear all
clf
syms t
R=input('Resistor: ')
C=input('Capacitor: ')
E=input('emf: ')
tu = R*C%time constant
qmax = E*C%maximum charge
Imax = E/R%maximum current
charge = dsolve('Dq+q/4=1.5000e-05','q(0)=0','t')
current = diff(charge)
figure
ezplot(charge,[0,tu])
figure
ezplot(current,[0,tu])
```

Output:

```
Resistor: 8*(10^5)

R =

      800000

Capacitor: 5*(10^-6)

C =

   5.0000e-06

emf: 12

E =

    12


tu =

    4.0000


qmax =

   6.0000e-05


Imax =

   1.5000e-05


charge =

3/50000 - (3*exp(-t/4))/50000


current =

(3*exp(-t/4))/200000
```

Figure 1:

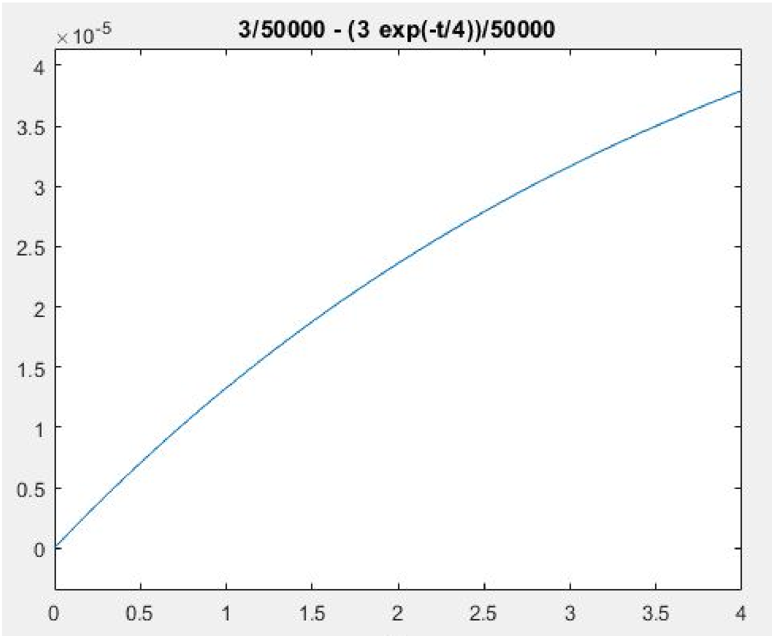
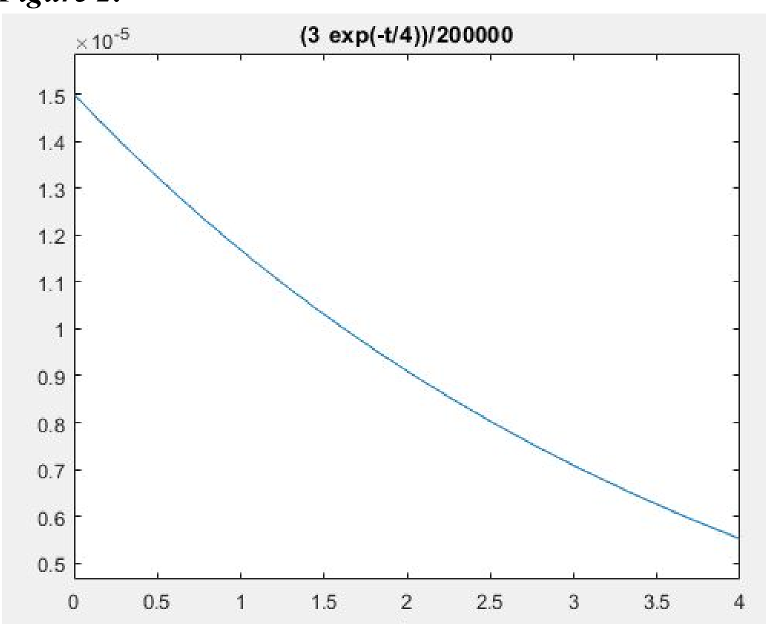

3/50000 - (3 exp(-t/4))/50000
×10^-5
4
3.5
3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
3.5
4

Figure 2:


(3 exp(-t/4))/200000
×10^-5
1.5
1.4
1.3
1.2
1.1
1
0.9
0.8
0.7
0.6
0.5
0
0.5
1
1.5
2
2.5
3
3.5
4
t

Q: An uncharged capacitor and a resistor are connected in series to a battery as shown in the circuit, where $E = 14.0\text{V}$, $C = 10.00\,\mu F$ and $R = 6.00 \times 10^5\Omega$. The switch is thrown to position a. Using MATLAB find the time constant of the circuit, the maximum charge on the capacitor and the maximum current in the circuit. Plot the charge and the current in the circuit as functions of time.

A:

```
Resistor: 6*(10^5)

R =

      600000

Capacitor: 10*(10^-6)

C =

   1.0000e-05

emf: 14

E =

    14


tu =

    6.0000


qmax =

   1.4000e-04


Imax =

   2.3333e-05


charge =

3/50000 - (3*exp(-t/4))/50000


current =

(3*exp(-t/4))/200000
```

Figure 3:

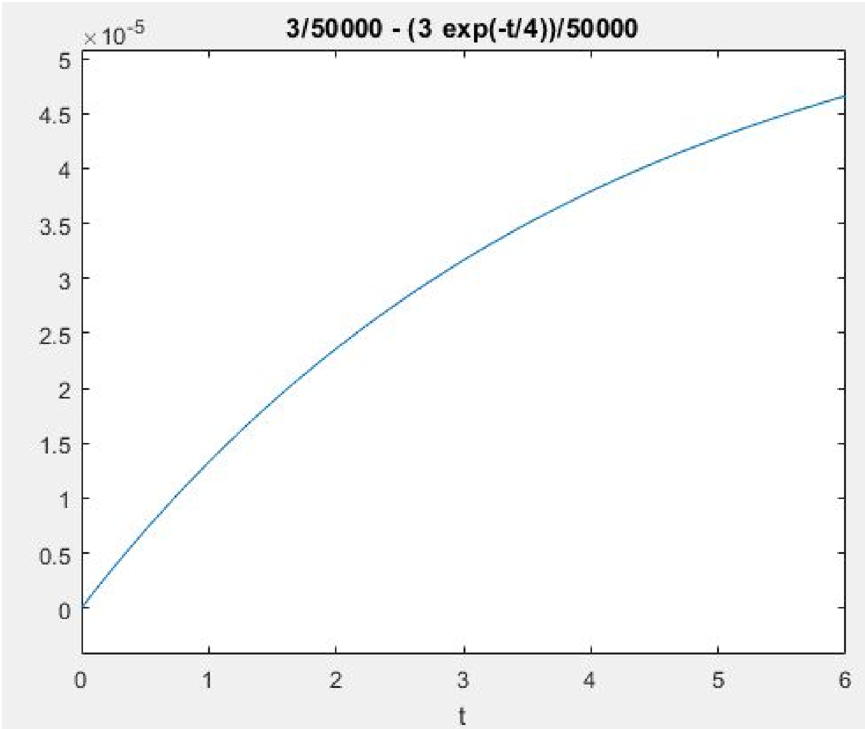
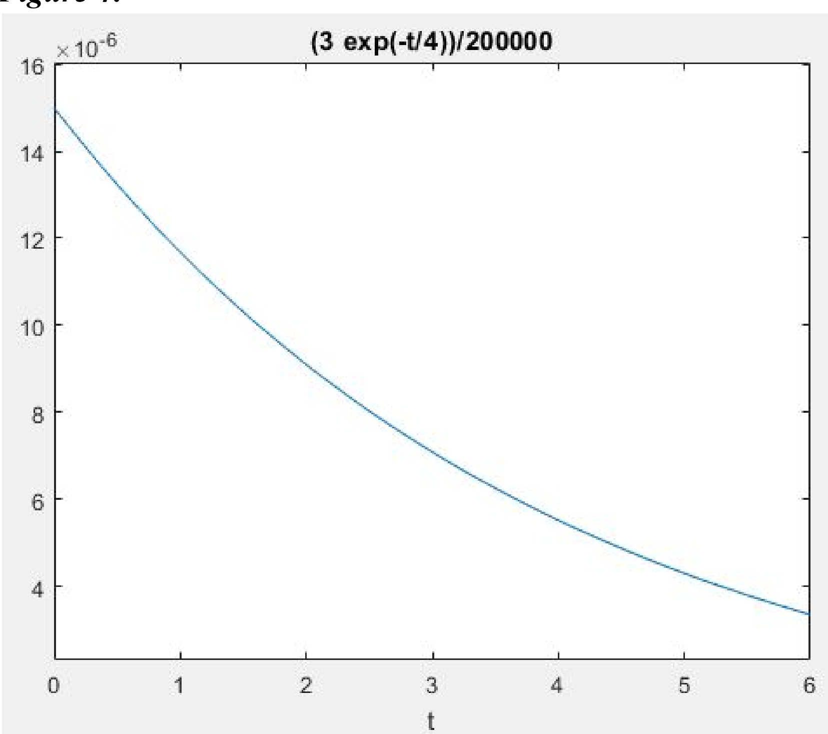


Figure 4:



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Using MATLAB, $\frac{d^2y}{dt^2} - 4\frac{dy}{dt} - 5y = e^{2t}$, $\left.\frac{dy}{dt}\right|_{t=0}$; $y(0)=2$

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$e - \frac{q}{c} - IR = 0$

$R\frac{dq}{dt} + \frac{q}{c} = e$

$ye^{\int p\,dt} = \int Q e^{\int p\,dt}\,dt + c$

$\cancel{R\frac{dq}{dt}} = Q\int e^{\int p\,dt}\,dt + c$ $\quad \frac{dq}{dx} + py = Q$ — (2)

$\Rightarrow$

$= Q\frac{e^{\int p\,dt}}{\int p\,dt} + c$ (3) $- \frac{dq}{dt} + \frac{1}{Rc}q = \frac{e}{R}$

$y\cancel{e^{\int p\,dt}} = \frac{Q\,e^{\int p\,dt}}{\int p\,dt} + c$ $\quad P = \frac{1}{Rc}$; $Q = \frac{E}{R}$

$\Rightarrow y\int p\,dt = Q + c$

$\Rightarrow y = \frac{Q + c}{\int p\,dt}$ $\quad qe^{\int p\,dt} = \int Q e^{\int p\,dt} + C$ — (4)

$\Rightarrow qe^{\int\frac{dt}{RC}} = \int \frac{E}{R} e^{\int\frac{dt}{RC}}\,dt + A$

$\Rightarrow qe^{t/RC} = \int \frac{E}{R} e^{t/RC}\,dt + A$

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$\Rightarrow q e^{t/RC} = \frac{E}{R} \cdot RC \cdot e^{t/RC} + A$

$\Rightarrow$ ~~$q e^{t/R}$~~ $q = EC + A e^{-t/RC}$

$\tau = RC$, time constant

when $t=0$, $q=0$

$A = -EC$

$q = EC(1-e^{-t/\tau})$

$E - \frac{q}{C} - iR = 0$

when $t=0$, $q=0$, $\Rightarrow i = \frac{E}{R}$, initial current

$t=0$, $i=0$, $\Rightarrow q = EC$, maximum charge

Q_{max}

$i = \frac{dq}{dt} = \frac{E}{R} e^{-t/\tau}$

✓ 25/9/17