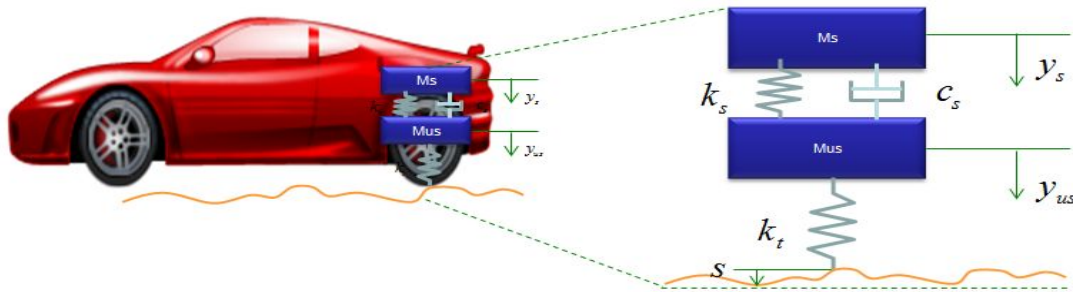


Experiment #07 - SPRUNG MASS DISPLACEMENT IN A QUARTER CAR (ONE WHEEL) MODEL

Aim: To write MATLAB code for system of second order differential equations of the form $X' + AX = 0$ using diagonalization. Determining the sprung mass displacement in a car suspension for one wheel. Solving a coupled system of ordinary differential equations derived from the mathematical model.

Mathematical Background:



The system shown in Figure. 1 is an quarter car system where,

M_s	is the sprung mass
M_{us}	is the unsprung mass
k_s	is the stiffness coefficient of the suspension
k_t	is the vertical stiffness of the tire
c_s	is the damping coefficient of the suspension
b_2	is the damping coefficient of the tire
y_2	the vertical displacement of sprung mass
y_{us}	is the vertical displacement of unsprung mass
s	is the road excitation

Sprung mass equation in a quarter car (one wheel):

$$M_s \frac{d^2 y_s}{dt^2} = -c_s \left(\frac{dy_s}{dt} - \frac{dy_{us}}{dt} \right) - k_s (y_s - y_{us}) = 0 \dots (1)$$

Unsprung mass equation in a quarter car (one wheel):

$$M_{us} \frac{d^2 y_{us}}{dt^2} = -c_s \left(\frac{dy_{us}}{dt} - \frac{dy_s}{dt} \right) - k_s (y_{us} - y_s) + k_t y_{us} = k_t s \dots (1)$$

Matrix form of above equations:

$$\begin{bmatrix} M_s & 0 \\ 0 & M_{us} \end{bmatrix} \begin{bmatrix} \frac{d^2 y_s}{dt^2} \\ \frac{d^2 y_{us}}{dt^2} \end{bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix} \begin{bmatrix} \frac{dy_s}{dt} \\ \frac{dy_{us}}{dt} \end{bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \begin{bmatrix} y_s \\ y_{us} \end{bmatrix} = \begin{bmatrix} 0 \\ k_t s \end{bmatrix}$$

$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{c}\dot{\mathbf{y}} + \mathbf{k}\mathbf{y} = \mathbf{f}$

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \frac{d^2}{dt^2} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} (k_1 + k_2) & -k_2 \\ -k_2 & (k_2 + k_3) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\mathbf{M} \frac{d^2 \mathbf{x}}{dt^2} + \mathbf{K} \mathbf{x} = 0$

MATLAB syntax

<code>eig(A)</code>	Returns a vector of the eigenvalues of matrix A
<code>solve(eq,x)</code>	Returns the set of all complex solutions of an equation or inequality <i>eq</i> with respect to <i>x</i> .

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$$\begin{bmatrix} M_s & 0 \\ 0 & M_{us} \end{bmatrix} \begin{bmatrix} \frac{d^2 y_s}{dt^2} \\ \frac{d^2 y_{us}}{dt^2} \end{bmatrix} + \begin{bmatrix} c_s & -c_s \\ -c_s & c_s \end{bmatrix} \begin{bmatrix} \frac{dy_s}{dt} \\ \frac{dy_{us}}{dt} \end{bmatrix} + \begin{bmatrix} k_s & -k_s \\ -k_s & k_s + k_t \end{bmatrix} \begin{bmatrix} y_s \\ y_{us} \end{bmatrix} = \begin{bmatrix} 0 \\ k_t s \end{bmatrix}$$

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