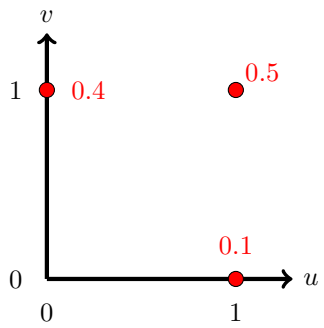
Let X and Y be binary random variables on sample space $S = \{a, b, c, d\}$, defined by

$$X(u) = \begin{cases} 1 & \text{if } u \in \{a, b, c\} \\ 0 & \text{else} \end{cases} \qquad Y(u) = \begin{cases} 1 & \text{if } u \in \{b, c, d\} \\ 0 & \text{else} \end{cases}$$

and whose joint probability mass function $p_{X,Y}(u, v)$ is shown below:



If $P(\{b\}) = 0.2$, then which of the following must be true?

(a) $P(\{a\}) = 0.1, P(\{c\}) = 0.3, P(\{d\}) = 0.4$

(b) $P(\{a\}) = 0.1, P(\{c\}) = 0.4, P(\{d\}) = 0.3$

(c) $P(\{a\}) = 0.3, P(\{c\}) = 0.1, P(\{d\}) = 0.4$

(d) $P(\{a\}) = 0.3, P(\{c\}) = 0.4, P(\{d\}) = 0.1$

(e) $P(\{a\}) = 0.4, P(\{c\}) = 0.3, P(\{d\}) = 0.1$

(f) $P(\{a\}) = 0.4, P(\{c\}) = 0.1, P(\{d\}) = 0.3$

(g) $P(\{a\}) = 0$

(h) $P(\{c\}) = 0$

(i) $P(\{d\}) = 0$

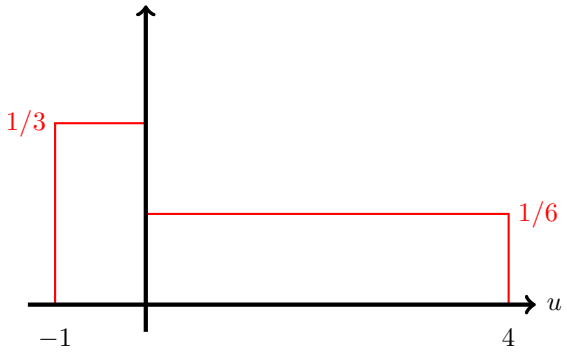
(j) None of these

Let X be a continuous random variable which is uniform on $[-1, 2]$ and let $Y = g(X)$, where

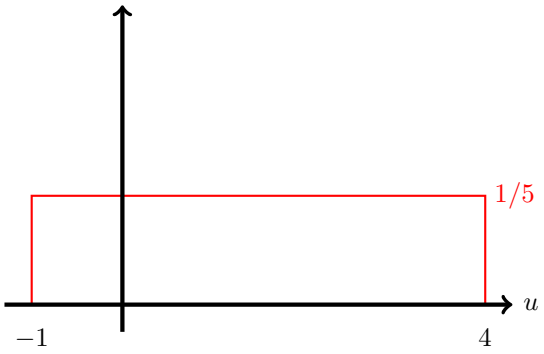
$$g(u) = \begin{cases} u & \text{if } u \leq 0 \\ 2u & \text{if } u > 0. \end{cases}$$

Which of the following is the probability density function $f_Y(u)$?
(Note: the red numbers represent heights)

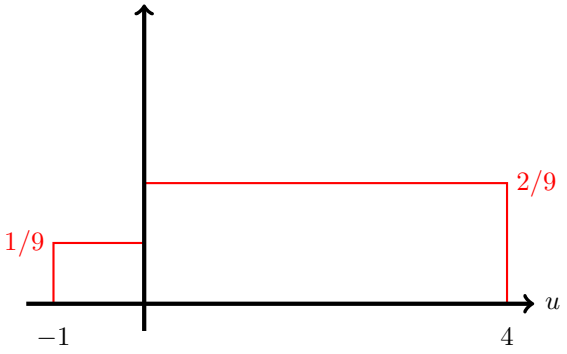
(a)



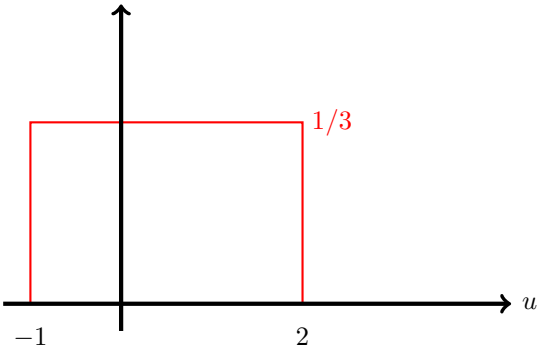
(b)



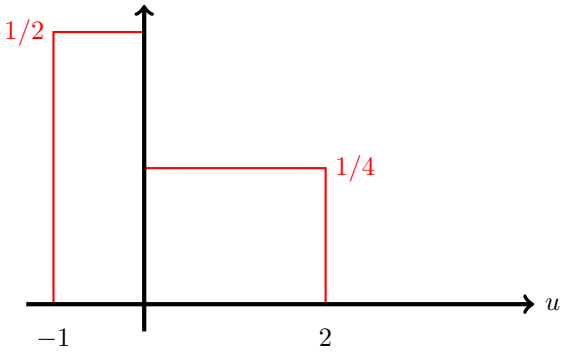
(c)



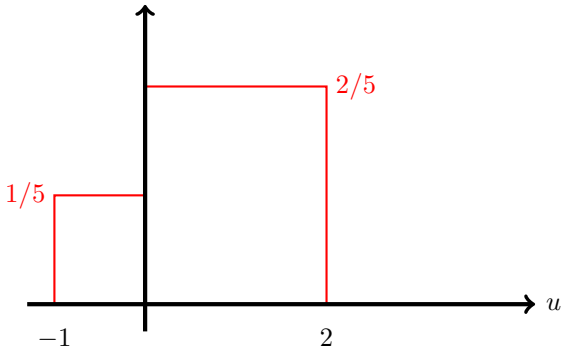
(d)



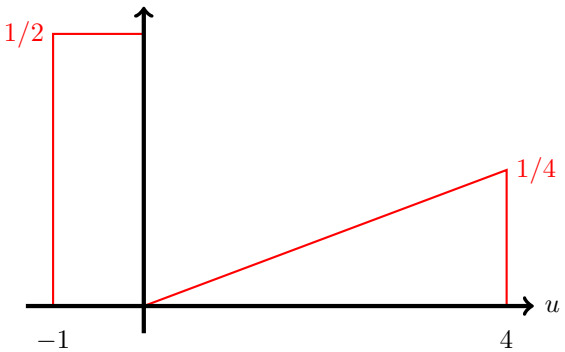
(e)



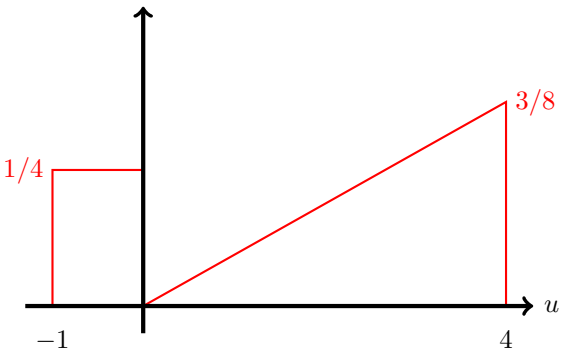
(f)



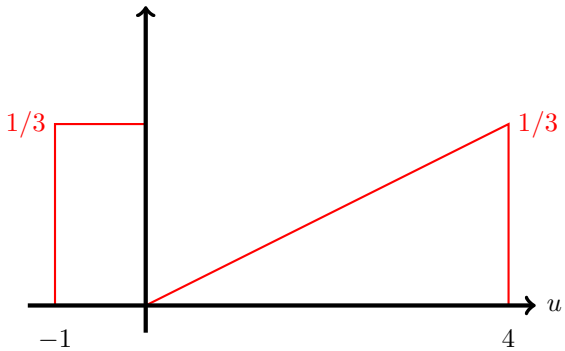
(g)



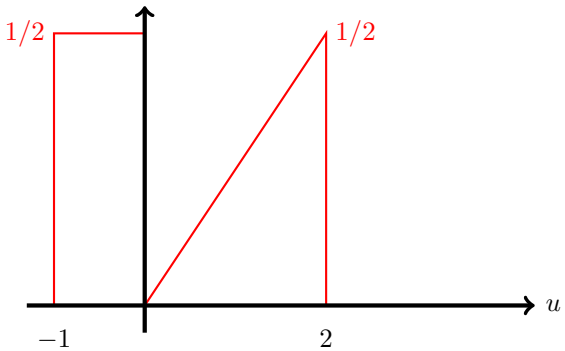
(h)



(i)



(j)



(a) (a)

(b) (b)

(c) (c)

(d) (d)

(e) (e)

(f) (f)

(g) (g)

(h) (h)

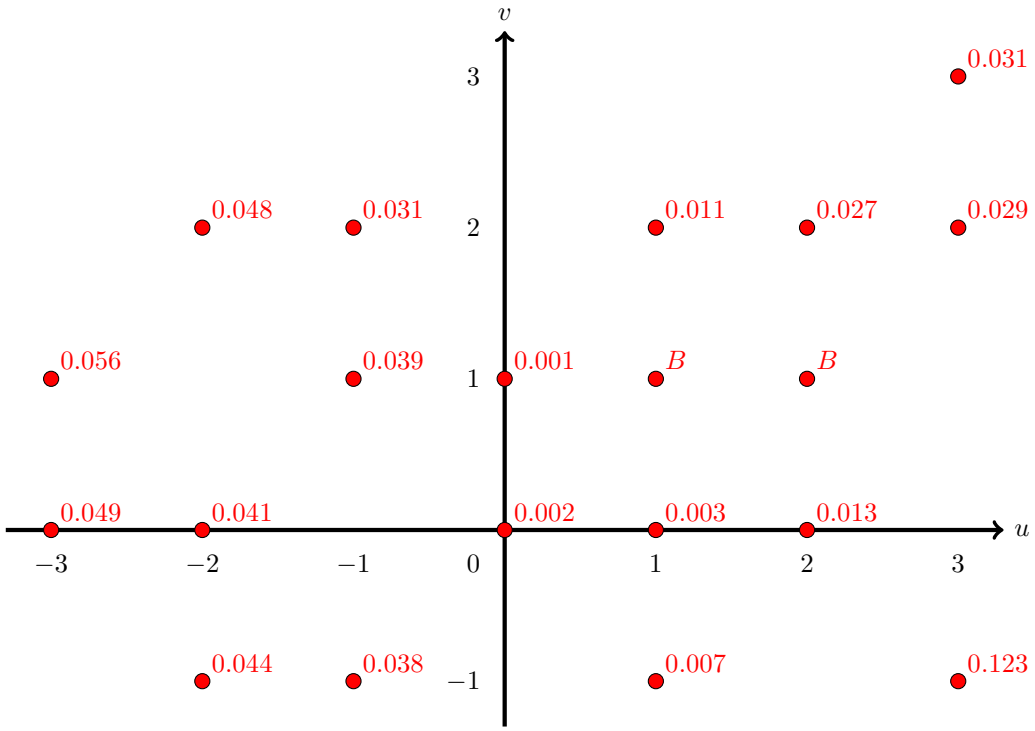
(i) (i)

(j) (j)

(k) None of these

If X and Y are discrete random variables whose joint probability mass function $p_{X,Y}(u,v)$ is shown in red below (where B is a positive constant), then what is

$$F_{X,Y}\left(\sqrt{5},\sqrt{7}\right) + P\left((X-3)^2 + (Y-2.5)^2 < 4/9\right)?$$



(a) 0.877

(b) 0.123

(c) 0.031

(d) 0.060

(e) 0.040

(f) 0.969

(g) 0.183

(h) 0.817

(i) 1

(j) 0

(k) None of these