A sample space $S = \{a, b, c, d, e, f\}$ has equally likely outcomes. Define a random variable $X$ by:

$$X(u) = \begin{cases} 1 & \text{if } u \in \{a, b, c\} \\ 3 & \text{if } u \in \{d, e\} \\ 100 & \text{if } u \in \{f\}. \end{cases}$$

and let $F$ denote its cumulative distribution function (CDF). What is $F(\pi)$ ?

(a) 5/6

(b) 1/6

(c) 1/3

(d) 1/5

(e) 1/2

(f) 2/3

(g) 1/100

(h) 4

(i) 1

(j) 0

(k) None of these

**Solution**:
$F(\pi) = P(X \leq \pi) = 1 - P(X = 100) = 1 - (1/6) = 5/6.$

You flip a biased coin (with $P(Heads) = 2/5$) twice and roll a fair die once. Let $X$ be the total number of Heads coming up on the two coin flips, and let $Y$ be the number that appears on the die. What is the probability that $X$ is larger than $Y$ ?

(a) 2/75

(b) 4/25

(c) 1/15

(d) 21/25

(e) 3/5

(f) 1

(g) 0

(h) 1/6

(i) 1/24

(j) None of these

You flip a biased coin (with $P(Heads) = 1/3$) twice and roll a fair die once. Let $X$ be the total number of Heads coming up on the two coin flips, and let $Y$ be the number that appears on the die. What is the probability that $X$ is larger than $Y$ ?

(a) 1/54

(b) 1/9

(c) 1/18

(d) 8/9

(e) 2/3

(f) 1

(g) 0

(h) 1/6

(i) 1/24

(j) None of these

You flip a biased coin (with $P(Heads) = 2/3$) twice and roll a fair die once. Let $X$ be the total number of Heads coming up on the two coin flips, and let $Y$ be the number that appears on the die. What is the probability that $X$ is larger than $Y$ ?

(a) 2/27

(b) 4/9

(c) 1/9

(d) 5/9

(e) 1/3

(f) 1

(g) 0

(h) 1/6

(i) 1/24

(j) None of these

You flip a biased coin (with $P(Heads) = 1/5$) twice and roll a fair die once. Let $X$ be the total number of Heads coming up on the two coin flips, and let $Y$ be the number that appears on the die. What is the probability that $X$ is larger than $Y$ ?

(a) 1/150

(b) 1/25

(c) 1/30

(d) 24/25

(e) 4/5

(f) 1

(g) 0

(h) 1/6

(i) 1/24

(j) None of these

**Solution**: Let $q = k/n$. Then, $P(X > Y) = P(X = 2, Y = 1) = P(X = 2)P(Y = 1) = q^2(1/6) = k^2/6n^2$.

Suppose we flip a fair coin three times. Define a random variable $X$ to equal one if we get exactly one Head in the three flips, and to be zero otherwise. Define $Y = 3X^2 + 2X - 1$. Let $F_Y$ be the CDF of $Y$. What is $F_Y(-1/2)$ ?

(a) 5/8

(b) 3/8

(c) 1/8

(d) 7/8

(e) 1/2

(f) 1/4

(g) 1/9

(h) 3/4

(i) 1/16

(j) $-1/2$

(k) 1

(l) 0

(m) None of these

**Solution**: $Y = -1$ if $X = 0$, and $Y = 4$ if $X = 1$, so $F_Y(-1/2) = P(Y = -1) = P(X = 0) = 5/8$.