$P(E^c) = 1 - P(E)$

Demorgan's: $\overline{(E \cup F)} = \overline{E} \cap \overline{F}$

$\overline{(E \cap F)} = \overline{E} \cup \overline{F}$

$C = \frac{n!}{r!(n-r)!}$

Classical : $\frac{\text{desired}}{\text{total}}$

$0 \leq P(E) \leq 1$, $P(S) = 1$

disjoint: $P(E_1 \cup E_2) = P(E_1) + P(E_2)$

Conditional : new info shrinks sample space

$P(E|F)$: probability of E given F

$0 \leq P(E|F) \leq 1$, $P(S|F) = 1$

disjoint: $P((E_1 \cup E_2)|F) = \sum_n P(E_n|F)$

$$P(E|F) = \frac{P(EF)}{P(F)}$$

$$P(EF) = P(E|F)P(F) = P(F|E)P(E)$$

Partition Rule

$$P(B) = P(B|A)P(A) + P(B|A^c)P(A^c)$$

$$P(B) = \sum P(B|A_n)P(A_n)$$

$\longrightarrow$ conditional

$$P(B|E) = P(B|A_1E)P(A_1|E) + \ldots + P(B|A_nE)P(A_n|E)$$

Chain Rule: things happening in a row

$$P(A_1 A_2 A_3) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 A_2)$$

$$\rightarrow P(A_1 A_2 \ldots A_n) = P(A_1) \cdot P(A_2 | A_1) \cdot P(A_3 | A_1 A_2) \cdots$$
$$P(A_n | A_1 A_2 \cdots A_{n-1})$$

Bayes Formula:

$$P(A_i | B) = \frac{P(B | A_i) P(A_i)}{\sum_{j=1}^{n} P(B | A_j) P(A_j)}$$

flipped: $P(B | A_i) \longleftrightarrow P(A_i | B)$

$$(\text{prob of cause } A) = \frac{\text{prob of result given cause } A}{\text{total prob of result from all causes}}$$

Events E, F are <u>independent</u> if $P(EF) = P(E)P(F)$

$$P(F) \neq 0 \longrightarrow P(E|F) = P(E)$$

$\rightarrow$ statistical independence: information, not physical interaction

E is ind. of F $\rightarrow$ learning that F has occurred gives no new info to help predict if E will occur

if E, F are ind., then $\longleftrightarrow$ 1) E, F^c are ind.
E^c, F are ind.
E^c, F^c are ind.

(if knowing F happened doesn't help predict E, then knowing F didn't happen doesn't help predict E either)

$$P(EF^c) = P(E)\,P(F^c|E)$$

disjoint events are not independent

independence of variables implies independence of events

Events A, B, C are independent if

1) $A \perp\!\!\!\perp B$
2) $B \perp\!\!\!\perp C$
3) $A \perp\!\!\!\perp C$
4) $P(ABC) = P(A)P(B)P(C)$

$P(ABC) = P(A)P(B)P(C)$ does not imply pairwise independence

Conditional Independence

$A \perp\!\!\!\perp B \mid C$ (conditionally independent given C) if

$P(AB|C) = P(A|C)\,P(B|C)$ assuming $P(C) \neq 0$

- if we know C has happened, learning B gives no info on A

$$A \perp\!\!\!\perp B \mid C \longleftrightarrow P(A|BC) = P(A|C)$$

$P(AB) < P(A)P(B) \rightarrow$ repulsion ($P(AB) = 0 \rightarrow$ disjoint)

$P(AB) = P(A)P(B) \rightarrow$ independence

$P(AB) < P(A)P(B) \rightarrow$ attraction

geometric series ; $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$, $|x|<1$

Independent trials

P of getting K occurences of E in n trials:

$$P(\text{exactly } k \text{ successes}) = \binom{n}{k} P(E)^k (1 - P(E))^{n-k}$$

$$\binom{n}{k} = \binom{n}{n-k}$$

if rolling 5 dice, choosing 2 as success is the same as choosing 3 as failures

$$\sum_{k=0}^{n} \binom{n}{k} p^k (1-p)^{n-k} = 1$$

(binomial theorem)

- if you add up the probability of getting 0, 1, 2, ... n successes, it must total to 1

→ binomial distribution function

- probability of getting k successes in n independent trials, where each trial has the same probability of success p

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Random Variables

function that maps outcomes in a sample space to a real number

$$X : S \rightarrow R$$