Suppose an experiment has sample space $S = \{a, b, c, d, e, f, g, h\}$ with equiprobable outcomes. How many events in S occur when the experiment is conducted once?

(a) 128

(b) 1

(c) 256

(d) 2

(e) $\binom{8}{2}$

(f) $\binom{8}{1}\binom{8}{7}$

(g) 0

(h) 8

(i) 7

(j) None of these

Solution: $2^7 = 128$.

Suppose an experiment has sample space $S = \{a, b, c, d, e, f, g, h\}$ with equiprobable outcomes. Define the following three events:

$$R = \{a, b, c, d, e\}$$
$$T = S - \{e, h\}$$
$$Q = \{d, g, h\}.$$

Which of the following events is independent of the event RT ?

(a) TQ

(b) $\{a, c\}$

(c) $Q^c R^c$

(d) $R^c Q$

(e) $\{a, b, c\}$

(f) $\{a, b\}$

(g) T^c

(h) $\{a\}$

(i) R^c

(j) None of these

Suppose an experiment has sample space $S = \{a, b, c, d, e, f, g, h\}$ with equiprobable outcomes. Define the following three events:

$$T = \{a, b, c, d, e\}$$
$$R = S - \{e, h\}$$
$$Q = \{d, g, h\}.$$

Which of the following events is independent of the event TR ?

(a) RQ

(b) $\{a, c\}$

(c) $Q^c T^c$

(d) $T^c Q$

(e) $\{a, b, c\}$

(f) $\{a, b\}$

(g) R^c

(h) $\{a\}$

(i) T^c

(j) None of these

Suppose an experiment has sample space $S = \{a, b, c, d, e, f, g, h\}$ with equiprobable outcomes. Define the following three events:

$$
\begin{aligned}
R &= \{a, b, c, d, e\} \\
Q &= S - \{e, h\} \\
T &= \{d, g, h\}.
\end{aligned}
$$

Which of the following events is independent of the event RQ ?

(a) QT

(b) $\{a, c\}$

(c) $T^c R^c$

(d) $R^c T$

(e) $\{a, b, c\}$

(f) $\{a, b\}$

(g) Q^c

(h) $\{a\}$

(i) R^c

(j) None of these

Suppose an experiment has sample space $S = \{a, b, c, d, e, f, g, h\}$ with equiprobable outcomes. Define the following three events:

$$T = \{a, b, c, d, e\}$$
$$Q = S - \{e, h\}$$
$$R = \{d, g, h\}.$$

Which of the following events is independent of the event TQ ?

(a) QR

(b) $\{a, c\}$

(c) $R^c T^c$

(d) $T^c R$

(e) $\{a, b, c\}$

(f) $\{a, b\}$

(g) Q^c

(h) $\{a\}$

(i) T^c

(j) None of these

Solution: Let

$$
\begin{aligned}
Q &= \{a, b, c, d, e\} \\
R &= S - \{e, h\} \\
T &= \{d, g, h\}.
\end{aligned}
$$

Then, $P(QR) = P(\{a, b, c, d\}) = 1/2$ and $P(QR|RT) = P(QRT)/P(RT) = P(\{d\})/P(\{d, g\}) = 1/2 = P(QR)$, so QR is independent of QR. All the other answers are nonempty and either subsets of, or disjoint from QR.

Suppose we flip a biased coin with $P(\text{Heads}) = 3/4$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 3/7

(b) 4/7

(c) 1/7

(d) 3/4

(e) 1/4

(f) 9/16

(g) 7/16

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 2/5$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 2/7

(b) 5/7

(c) 1/7

(d) 2/5

(e) 3/5

(f) 4/25

(g) 21/25

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 4/5$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) $4/9$

(b) $5/9$

(c) $1/9$

(d) $4/5$

(e) $1/5$

(f) $16/25$

(g) $9/25$

(h) $1/3$

(i) $1/2$

(j) 1

(k) 0

(l) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 5/6$ twice. What is the probability the second flip is Heads, given that at least one of the flips is Tails?

(a) 5/11

(b) 6/11

(c) 1/11

(d) 5/6

(e) 1/6

(f) 25/36

(g) 11/36

(h) 1/3

(i) 1/2

(j) 1

(k) 0

(l) None of these

Solution: Let E be the event the first flip is heads and let F be the event the second flip is Heads. Then,

$$\begin{aligned} P(F|(EF)^c) &= \frac{P(F(EF)^c)}{P((EF)^c)} \\ &= \frac{P(F(E^c \cup F^c))}{1 - P(EF)} \\ &= \frac{P(FE^c)}{1 - q^2} \\ &= \frac{q(1-q)}{1-q^2} \\ &= \frac{q}{1+q} \end{aligned}$$

If $q = k/n$, then $P(F|(EF)^c) = \frac{k/n}{1+(k/n)} = \frac{k}{k+n}$.