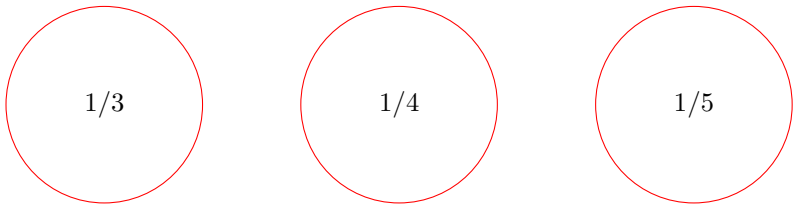
Suppose three biased coins are each flipped once. The probabilities of each coin getting Heads are shown below. Which of the following pairs of events are independent?




Check for overlap

if A → $C_1, C_2$ or $C_1$

B → $C_3$ or $C_2, C_3$

independent ✓

1 coin event & 2 coin event

→ (a)

(a) $\{\underline{H}\underline{H}H, \underline{H}\underline{H}T\}$ and $\{HH\underline{T}, H\underline{T}\underline{T}, T\underline{H}\underline{T}, T\underline{T}\underline{T}\}$

(b) $\{HHH\}$ and $\{TTT\}$

(c) $\{HHH\}$ and $\{TTT, TTH, THT, HTT\}$

(d) $\{HHT, HTH, THH, HHH\}$ and $\{TTT, TTH, THT, HTT\}$

(e) $\{TTT\}$ and $\{HHT, HTH\}$

(f) $\{HHH\}^c$ and $\{TTT, HHH\}^c$

(g) $\{HHH\}^c$ and $\{TTT\}^c$

(h) $\{HHH, TTT\}$ and $\{HHH, TTT\}$

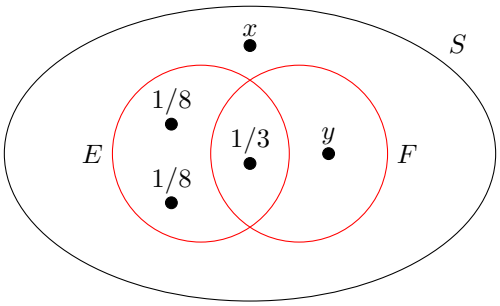
(i) $\{HHH\}$ and $\{HHH, HHT, HTH, THH, TTH, THT\}$

(j) $\{HHH\}$ and $\{HHH, TTT\}$

(k) $\{HHH, TTT\}$ and $\{HHH, TTT, THT, HTH\}$

(l) None of these

Suppose the sample space $S$ of an experiment is shown below, with each element labeled by its probability. Also shown as red circles are two events $E$ and $F$. If $E^c$ and $F^c$ are independent, then what is the value of $y$?




$E^c \perp\!\!\!\perp F^c \leftrightarrow E \perp\!\!\!\perp F$

(a) 5/21

(b) 5/7

(c) 1/12

(d) 1/3

(e) 2/3

(f) 1/24

(g) 1/8

(h) 3/8

(i) 5/12

(j) 5/24

(k) 1/4

(l) None of these

$$P(EF) = P(E)P(F)$$

$$\frac{1}{3} = \left(\frac{1}{8}+\frac{1}{8}+\frac{1}{3}\right)\left(\frac{1}{3}y\right)$$

$$\frac{1}{3} = \left(\frac{1}{4}+\frac{1}{3}\right)\left(\frac{1}{3}+y\right)$$

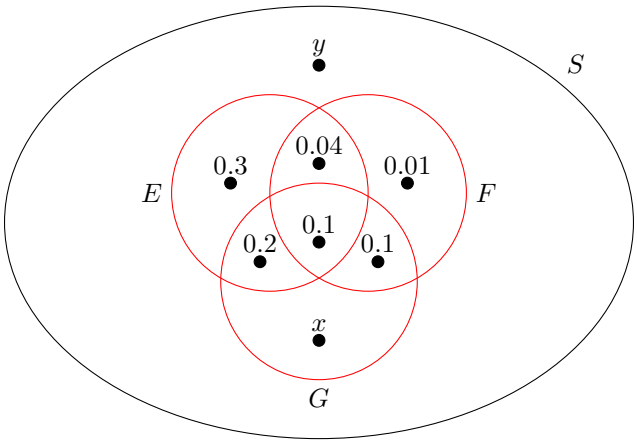
$$\frac{1}{3} = \frac{7}{12}\left(\frac{1}{3}+y\right)$$

$$\frac{12}{36} = \frac{7}{36} + \frac{7}{12}y$$

$$\frac{5}{36} = \frac{7}{12}y$$

$$\rightarrow y = \frac{60}{252} = \frac{30}{126} = \frac{15}{63} = \frac{5}{21}$$

Suppose the sample space $S$ of an experiment is shown below, with each element labeled by its probability. Also shown as red circles are three events $E$, $F$, and $G$. If $E$ and $F$ are independent given $G$, then what is the value of $x$?




$E \perp\!\!\!\perp F \mid G$

$P(E|FG) = P(E|G)$

$P(E|FG) = \frac{P(EFG)}{P(FG)} = \frac{0.1}{0.2} = \frac{1}{2}$

$P(E|G) = \frac{P(EG)}{P(G)} = \frac{0.3}{0.4+x}$

$\rightarrow \frac{1}{2} = \frac{0.3}{0.4+x}$

$0.4 + x = 0.6$

$\rightarrow x = 0.2 = \frac{1}{5}$

(a) 1/5

(b) 1/3

(c) 1/10

(d) 1/12

(e) 2/5

(f) 1/24

(g) 1/8

(h) 3/8

(i) 1/16

(j) 1/15

(k) 2/15

(l) None of these