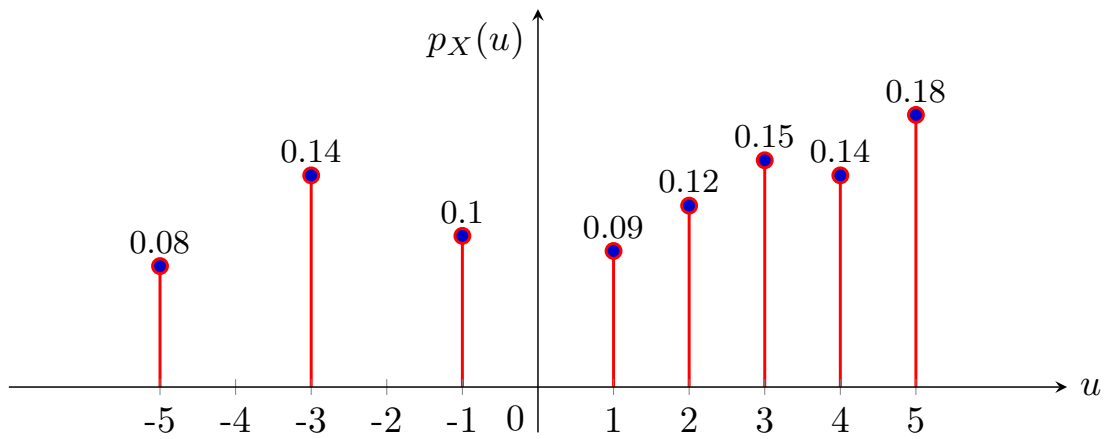
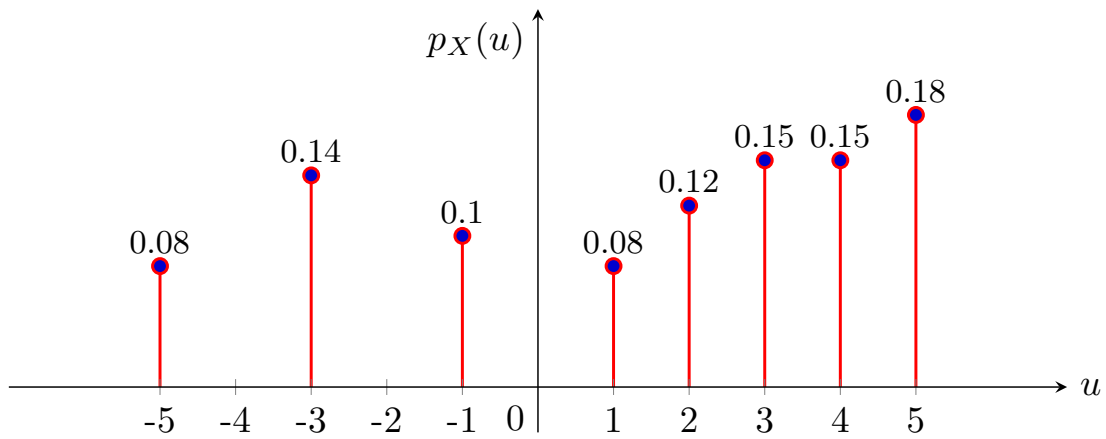


Let  $X$  be a discrete random variable, whose probability mass function is shown below. What is the probability that  $X$  is 2 or 3, given that  $2|X - 1|$  is less than 3 ?



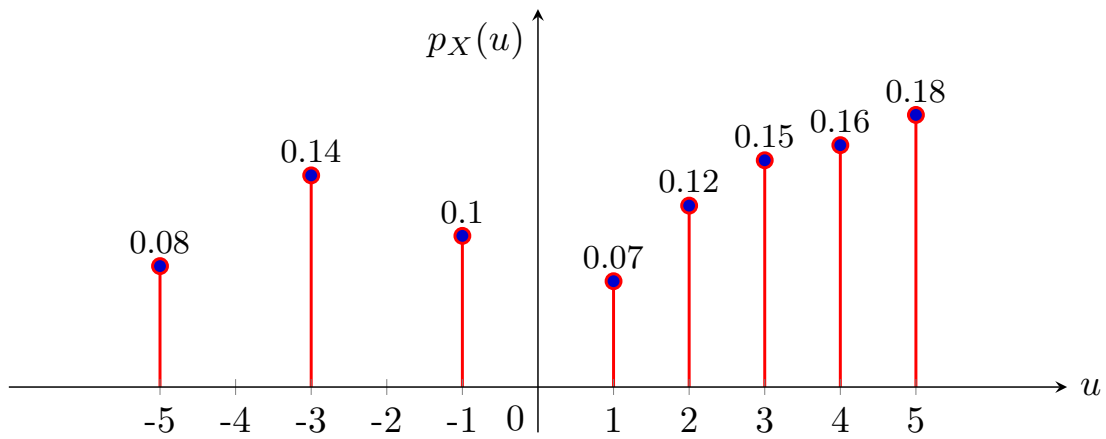
- (a)  $4/7$
- (b)  $3/7$
- (c)  $1/4$
- (d)  $9/46$
- (e)  $0.21$
- (f)  $0.27$
- (g)  $0$
- (h)  $1$
- (i)  $1/2$
- (j) None of these

Let  $X$  be a discrete random variable, whose probability mass function is shown below. What is the probability that  $X$  is 2 or 3, given that  $2|X - 1|$  is less than 3 ?



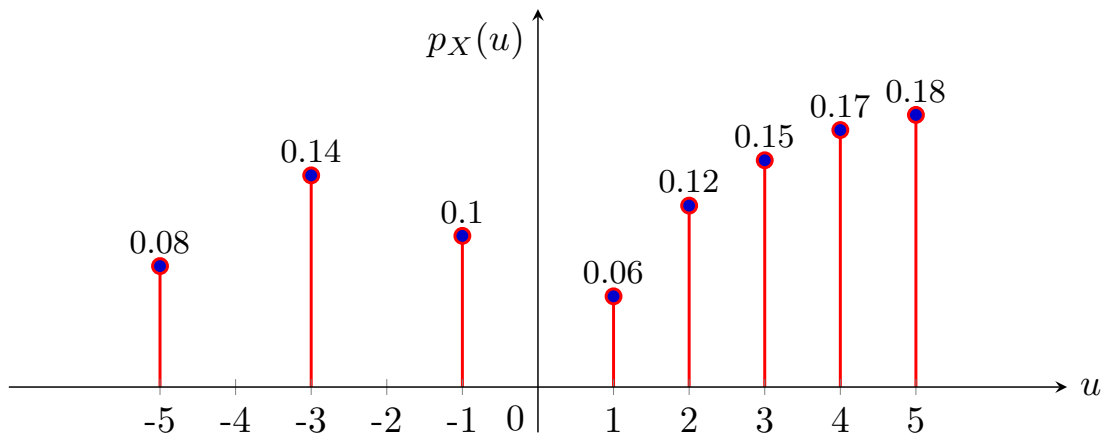
- (a)  $3/5$
- (b)  $2/5$
- (c)  $8/35$
- (d)  $8/45$
- (e)  $0.2$
- (f)  $0.27$
- (g)  $0$
- (h)  $1$
- (i)  $1/2$
- (j) None of these

Let  $X$  be a discrete random variable, whose probability mass function is shown below. What is the probability that  $X$  is 2 or 3, given that  $2|X - 1|$  is less than 3 ?



- (a) 12/19
- (b) 7/19
- (c) 7/34
- (d) 7/44
- (e) 0.19
- (f) 0.27
- (g) 0
- (h) 1
- (i) 1/2
- (j) None of these

Let  $X$  be a discrete random variable, whose probability mass function is shown below. What is the probability that  $X$  is 2 or 3, given that  $2|X - 1|$  is less than 3 ?



- (a)  $2/3$
- (b)  $1/3$
- (c)  $2/11$
- (d)  $6/43$
- (e) 0.18
- (f) 0.27
- (g) 0
- (h) 1
- (i)  $1/2$
- (j) None of these

**Solution:**

$$P(X \in \{2, 3\} \mid 2|X - 1| < 3) = P(X \in \{2, 3\} \mid P(|X - 1| < 3/2)) = P(X \in \{2, 3\} \mid X \in \{1, 2\}) = \frac{p_X(2)}{p_X(1) + p_X(2)}$$

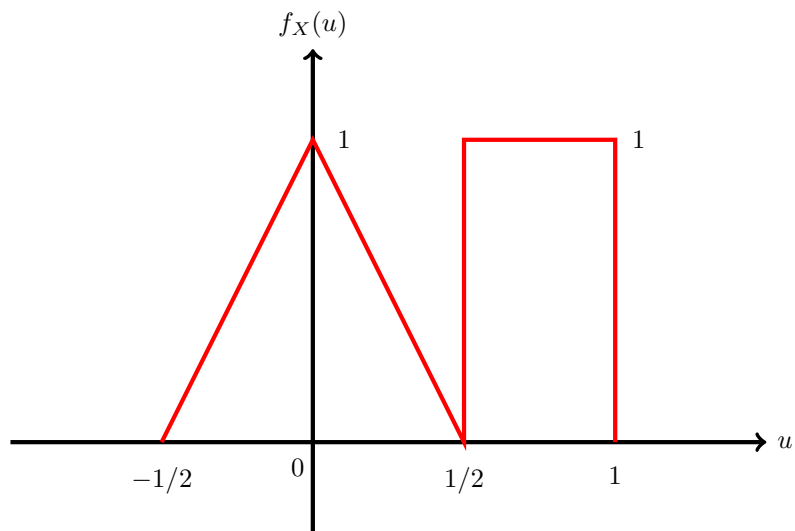
Suppose you flip a fair coin three times and  $X$  is the number of Heads you get divided by the sum of the numbers of Heads and Tails you get. What is the expected value of  $X$  ?

- (a)  $1/2$
- (b)  $5/6$
- (c)  $3/2$
- (d)  $1/3$
- (e)  $1/6$
- (f)  $2/3$
- (g)  $0$
- (h)  $1$
- (i)  $2$
- (j)  $3$
- (k) None of these

**Solution:** Solution:

$$E[X] = \left(\frac{0}{3} \cdot \frac{1}{8}\right) + \left(\frac{1}{3} \cdot \frac{3}{8}\right) + \left(\frac{2}{3} \cdot \frac{3}{8}\right) + \left(\frac{3}{3} \cdot \frac{1}{8}\right) = \frac{3+6+3}{24} = \frac{1}{2}.$$

Let  $X$  be a continuous random variable whose probability density function is shown below in red. What is the expected value of  $X$  ?



- (a)  $3/8$
- (b)  $5/8$
- (c)  $1/8$
- (d)  $7/8$
- (e)  $1/2$
- (f)  $1/4$
- (g)  $1/9$
- (h)  $3/4$
- (i)  $1/16$
- (j)  $-1/2$
- (k)  $1$
- (l)  $0$
- (m) None of these



**Solution:** Since  $uf_X(u)$  is an odd function of  $u$  in the interval  $[-1/2, 1/2]$ , we get

$$E[X] = \underbrace{\int_{-1/2}^{1/2} uf_X(u)du}_0 + \int_{1/2}^1 udu = (u^2/2)\Big|_{1/2}^1 = 3/8.$$