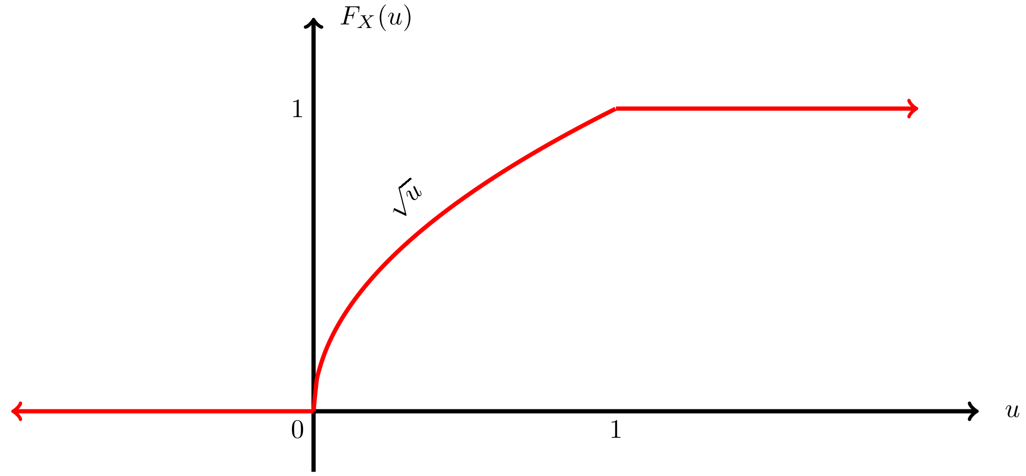
Let X be a random variable, whose cummulative distribution function equals $\sqrt{u}$ in the interval $[0, 1]$, as shown below. What is the probability that $X + 2$ lies in the interval $[0, 2.25]$, and $81X^2$ is greater than 1 ?



(a) 1/6

(b) 1/3

(c) 2/3

(d) 1/2

(e) 1/4

(f) 1/9

(g) 5/9

(h) 5/36

(i) 3/4

(j) 0

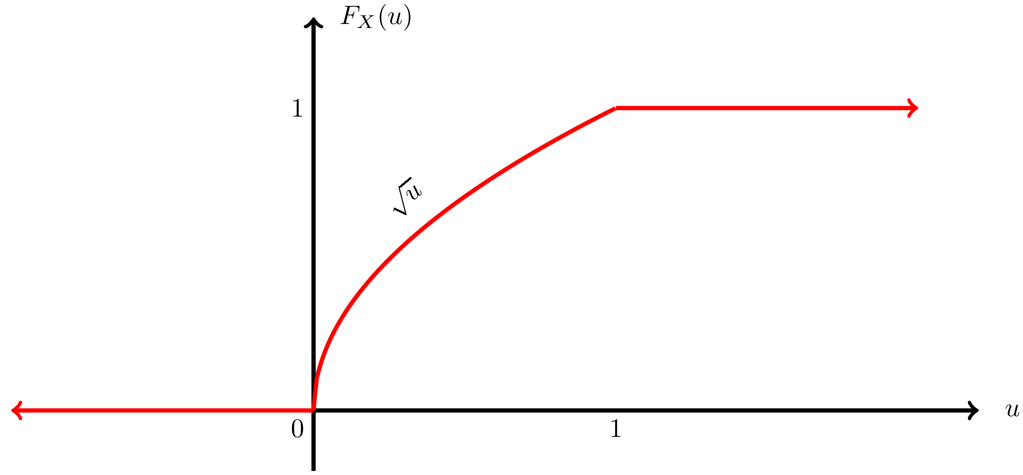
(k) 1

(l) None of these

Solution:

$$\begin{aligned}
P(X+2\in[0,2.25],\ 81X^2>1) &= P(1/9<X<1/4)\\
&= F_X(1/4)-F_X(1/9)\\
&= \sqrt{1/4}-\sqrt{1/9}\\
&= 1/2-1/3\\
&= 1/6.
\end{aligned}$$

Let X be a random variable, whose cummulative distribution function equals $\sqrt{u}$ in the interval $[0, 1]$, as shown below. If $Y = 6 - 9X$, then what is the CDF of Y evaluated at 5 ?



(a) 2/3

(b) 1/3

(c) $\sqrt{5}$

(d) $1/\sqrt{5}$

(e) 1/15

(f) 1/9

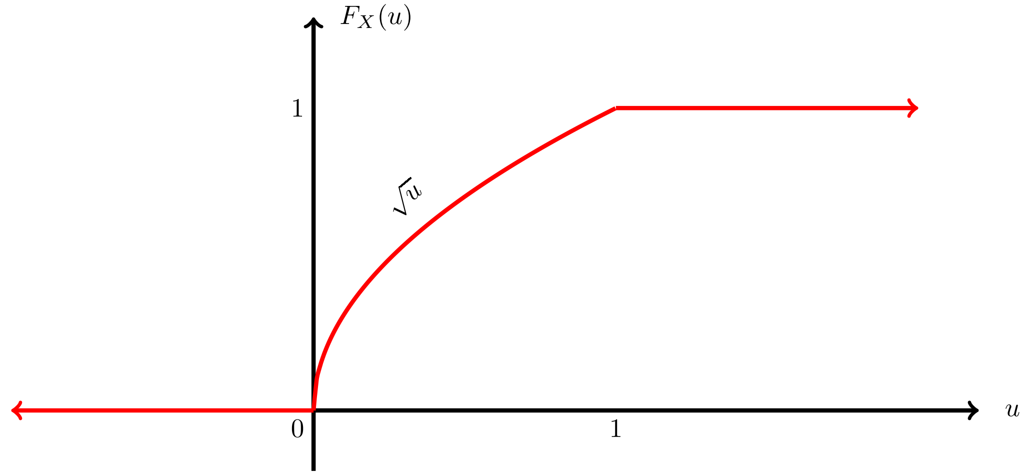
(g) 0

(h) 1

(i) 1/2

(j) None of these

Let X be a random variable, whose cummulative distribution function equals $\sqrt{u}$ in the interval $[0, 1]$, as shown below. If $Y = 7 - 16X$, then what is the CDF of Y evaluated at 6 ?



(a) 3/4

(b) 1/4

(c) $\sqrt{6}$

(d) $1/\sqrt{6}$

(e) 1/23

(f) 1/16

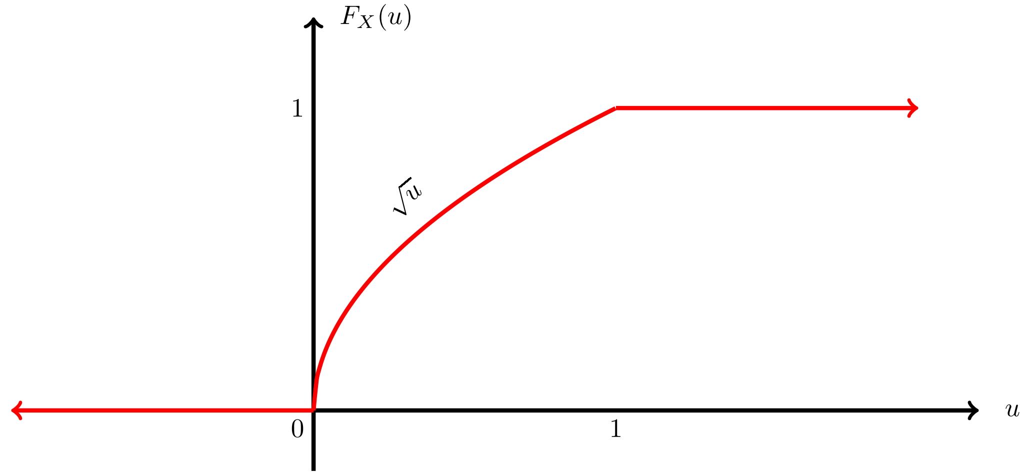
(g) 0

(h) 1

(i) 1/2

(j) None of these

Let X be a random variable, whose cummulative distribution function equals $\sqrt{u}$ in the interval $[0, 1]$, as shown below. If $Y = 8 - 25X$, then what is the CDF of Y evaluated at 7 ?



(a) 4/5

(b) 1/5

(c) $\sqrt{7}$

(d) $1/\sqrt{7}$

(e) 1/33

(f) 1/25

(g) 0

(h) 1

(i) 1/2

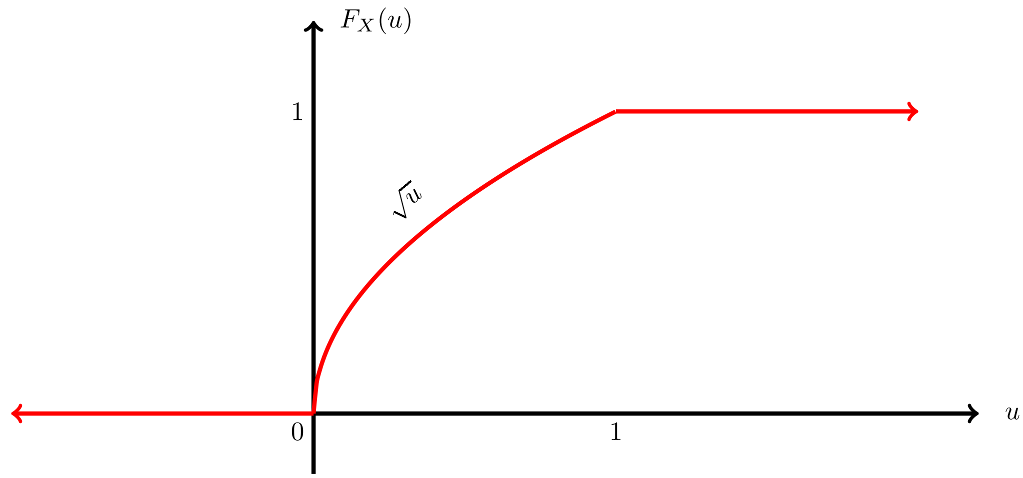
(j) None of these

Solution:
If $A > 0$, then

$$\begin{aligned} F_Y(u) &= P(B - Ax \le u) \\ &= P(x \ge (B-u)/A) \\ &= 1 - F_X((B-u)/A) \\ &= 1 - \sqrt{(B-u)/A}. \end{aligned}$$

This requires $(B-u)/A \in [0,1]$.
If $(B-u)/A = 1/k^2$, then $F_Y(u) = (k-1)/k$.

Let X be a random variable, whose cummulative distribution function equals $\sqrt{u}$ in the interval $[0, 1]$, as shown below. What is the variance of X ?



(a) 4/45

(b) 2/15

(c) 1/5

(d) 1/3

(e) 1/9

(f) 1/4

(g) 3/4

(h) 0

(i) 1

(j) 1/2

(k) None of these

Solution:

$$
\begin{aligned}
f_X(u) &= \frac{d}{du}u^{1/2} = \frac{1}{2\sqrt{u}} \\
E[X^n] &= \int_0^1 \frac{u^n du}{2\sqrt{u}} \\
&= \frac{1}{2}\int_0^1 u^{n-(1/2)} du \\
&= \frac{1}{2n+1} u^{n+(1/2)}\Big|_0^1 \\
&= \frac{1}{2n+1} \\
\sigma_X^2 &= E[X^2] - (E[X])^2 = (1/5) - (1/3)^2 \\
&= 4/45
\end{aligned}
$$