

One piece of fruit is chosen randomly from a box containing 3 apples, 4 bananas, and 5 cherries. Each piece of fruit is equally likely. The random variables X and Y are defined as follows:

$$\begin{aligned} X(\text{apple}) &= 0 \\ X(\text{banana}) &= 1 \\ X(\text{cherry}) &= -1 \end{aligned}$$

$$\begin{aligned} Y(\text{apple}) &= 1 \\ Y(\text{banana}) &= 1 \\ Y(\text{cherry}) &= 0 \end{aligned} \quad \rightarrow \text{add } P(a) + P(b)$$

What is the value of the CDF difference $F_X(\frac{\sqrt{2}}{2}) - F_Y(1)$?

$$|S| = 3 + 4 + 5 = 12$$

(a) $-1/3$

(b) $1/3$

(c) $1/4$

(d) $-1/4$

(e) $5/12$

(f) $-5/12$

(g) 0

(h) 1

(i) -1

(j) $7/12$

(k) $3/4$

(l) $2/3$

(m) None of these

$$P(X=0) = \frac{3}{12}$$

$$P(Y=1) = \frac{7}{12}$$

$$P(X=1) = \frac{4}{12}$$

$$P(Y=0) = \frac{5}{12}$$

$$P(X=-1) = \frac{5}{12}$$

$$F_X(u) = P(X \leq u)$$

$$F_X\left(\frac{\sqrt{2}}{2}\right) = P\left(X \leq \frac{\sqrt{2}}{2}\right) \quad \frac{\sqrt{2}}{2} \approx .7 \quad F_Y(1) = P(Y \leq 1)$$

$$= P(X=0) + P(X=-1)$$

$$= P(Y=1) + P(Y=0)$$

$$= \frac{8}{12} = \frac{2}{3}$$

$$= 1$$

$$\rightarrow \frac{2}{3} - 1 = \left(-\frac{1}{3}\right)$$

One piece of fruit is chosen randomly from a box containing 3 apples, 4 bananas, and 5 cherries. Each piece of fruit is equally likely. The random variables X and Y are defined as follows:

$$\begin{array}{ll} X(\text{apple}) = 0 & Y(\text{apple}) = 1 \\ X(\text{banana}) = 1 & Y(\text{banana}) = 1 \\ X(\text{cherry}) = -1 & Y(\text{cherry}) = 0 \end{array}$$

What is the probability that $(X^2 - 2XY)^2$ is positive?

(a) $3/4$

(b) $1/4$

(c) $1/3$

(d) $2/3$

(e) $5/12$

(f) $7/12$

(g) 0

(h) 1

(i) $1/2$

(j) $1/12$

(k) $3/8$

(l) None of these

$$P(X=0) = \frac{3}{12} \quad P(Y=1) = \frac{7}{12}$$

$$P(X=1) = \frac{4}{12} \quad P(Y=0) = \frac{5}{12}$$

$$P(X=-1) = \frac{5}{12}$$

$$F_X(u) = P(X \leq u)$$

$$(X^2 - 2XY)^2 = (X(X - 2Y))^2 > 0$$

$$P((X(X - 2Y))^2 > 0) = P(X(X - 2Y) \neq 0)$$

→ run for each outcome

$$a \quad X=0, Y=1 : 0(0-2 \cdot 1) = 0 \quad X$$

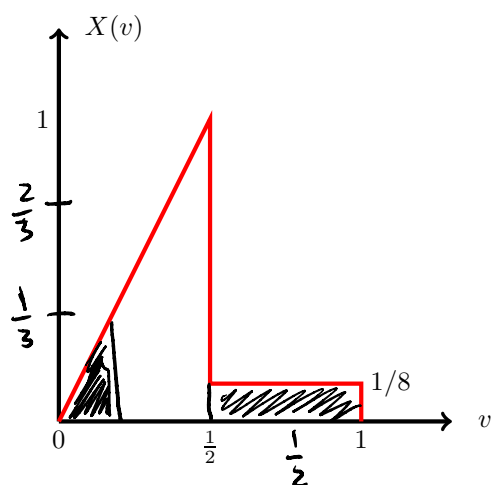
$$b \quad X=1, Y=1 : 1(1-2 \cdot 1) = -1 \quad \checkmark$$

$$c \quad X=-1, Y=0 : 1(1-2 \cdot 0) = 1 \quad \checkmark$$

$$P(X(X - 2Y) \neq 0) = P(b) + P(c)$$

$$= \frac{4}{12} + \frac{5}{12} = \frac{9}{12} = \frac{3}{4}$$

An experiment has sample space $S = [0, 1]$ and $P([a, b]) = b - a$ whenever $0 \leq a \leq b \leq 1$. A random variable X on S is defined as shown in the graph below. What is the CDF value $F_X(1/3)$?



$$F_X(u) = P(X \leq u)$$

$$P[a, b] = b - a \rightarrow \text{length on } v \text{ axis} = P$$

(a) $2/3$

(b) $1/6$

(c) $1/3$

(d) $1/2$

(e) $7/24$

(f) $11/24$

(g) $1/8$

(h) $1/4$

(i) $3/8$

(j) $5/6$

(k) $5/24$

(l) None of these

$$x(v) = 2v, \quad v < \frac{1}{2}$$

$$2v \leq \frac{1}{3}$$

$$\rightarrow v \leq \frac{1}{6}$$

$$\frac{1}{6} + \frac{1}{2} = \frac{2}{3}$$