Suppose random variables X and Y have joint probability density function $f_{X,Y}(u, v)$ which equals e^{-u} whenever $0 < v < u$, and equals zero elsewhere. What is the probability that X is less than one, given that Y is greater than one ?

(a) 0

(b) 1/2

(c) 1/3

(d) 1/4

(e) $1/e$

(f) $2/e$

(g) $e/(1 + e)$

(h) $(e - 1)/(e + 1)$

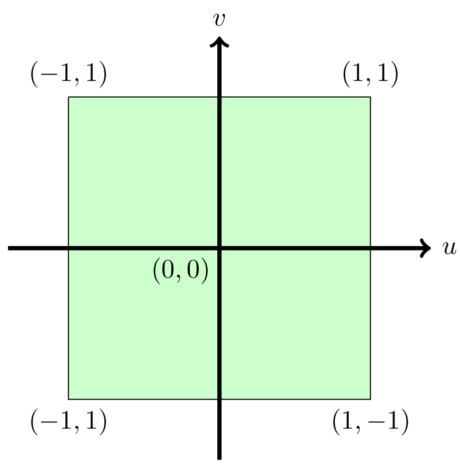
(i) $1/e^2$

(j) 1

(k) None of these

Solution:
The joint pdf if zero in the region $\{(u,v) : u < 1,\ v > 1\}$ so the probability is zero.

Suppose X and Y are random variables whose joint probability density function is constant in the set $\{(u,v) : u^2 + v^2 < 2\}$ and is zero elsewhere. What is the probability that the point (X,Y) lies inside the green square shown below?



(a) $2/\pi$

(b) $2/\pi^2$

(c) $\pi/6$

(d) $\pi/4$

(e) $1/(2\pi)$

(f) $1/(4\pi)$

(g) $1/2$

(h) $1/3$

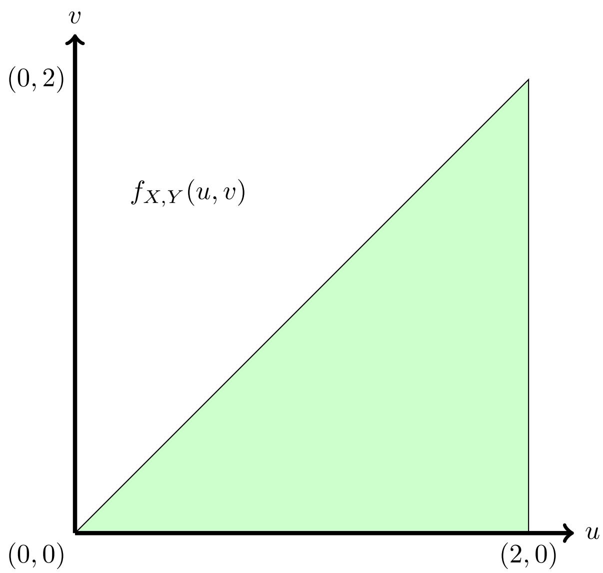
(i) $1/4$

(j) 1

(k) None of these

Solution:
The square is inscribed in the circle and $f_{X,Y}(u,v) = 1/2\pi$. The probability that (X,Y) lies in the square is $\int_{-1}^{1}\int_{-1}^{1}(1/2\pi)dudv = 4/2\pi = 2/\pi$.

Suppose the joint probability density function $f_{X,Y}(u,v)$ of random variables X and Y equals $uv/2$ in the green triangle shown below, and equals zero elsewhere. What is the probability that X is less than one ?



(a) 1/16

(b) 1/32

(c) 1/2

(d) 1/12

(e) 1/4

(f) 1/6

(g) 1/64

(h) 1/3

(i) 1/24

(j) 1/8

(k) 1

(l) None of these

Solution:
$P(X < 1) = \int_0^1 \int_0^u (uv/2) dv du = (1/4) \int_0^1 u^3 du = 1/16$