Suppose the joint probability mass function $p_{X,Y}(u,v)$ of independent random variables X and Y is shown below. What is the value of A ?

(a) 7/30

(b) 23/30

(c) 0.7

(d) 0.49

(e) 0.02

(f) 0.9

(g) 0.8

(h) 1/2

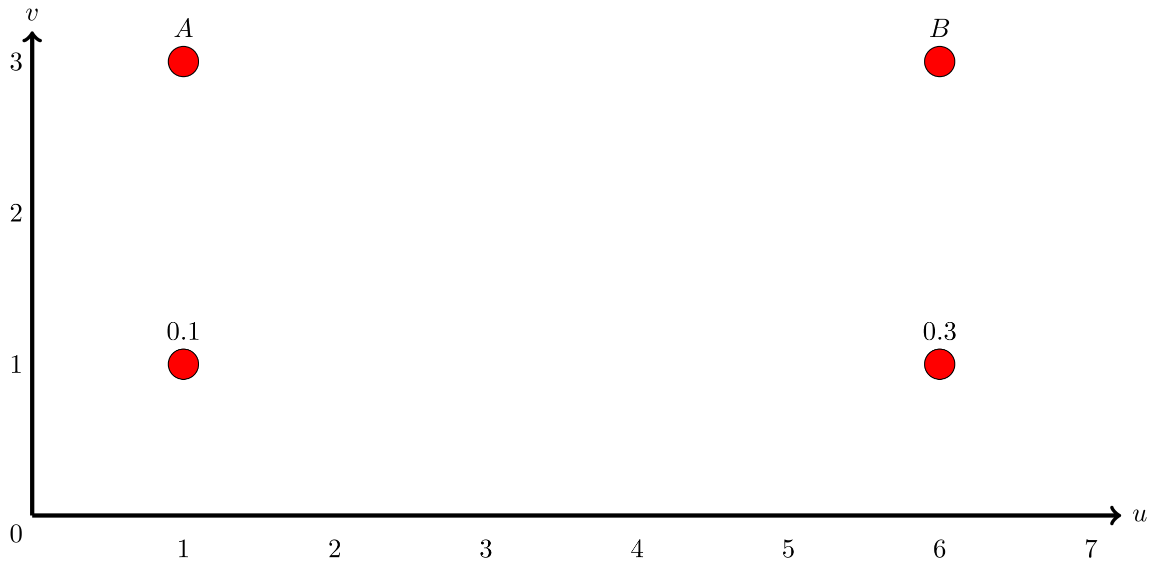
(i) 1/4

(j) 0

(k) 1

(l) None of these

Suppose the joint probability mass function $p_{X,Y}(u, v)$ of independent random variables X and Y is shown below. What is the value of A ?



(a) 3/20

(b) 17/20

(c) 0.6

(d) 0.36

(e) 0.03

(f) 0.9

(g) 0.7

(h) 1/2

(i) 1/4

(j) 0

(k) 1

(l) None of these

Solution:
Let $C = p_{X,Y}(1,1)$ and $D = p_{X,Y}(6,1)$. Then, by independence, $C = (A+C)(C+D)$, so $A = \frac{C}{C+D} - C$.

If random variables X and Y have joint probability density function $f_{X,Y}(u,v) = Ce^{2v(2-v)-u^2}$, where C is a constant, then what is the sum of their variances?

(a) $3/4$

(b) $3/2$

(c) $4/3$

(d) 6

(e) 3

(f) 2

(g) $\sqrt{3}/2$

(h) 5

(i) 4

(j) 1

(k) None of these

Solution:

$$\begin{aligned} f_{X,Y}(u,v) &= Ce^{2v(2-v)-u^2} \\ &= Ce^2 e^{-u^2} e^{-2(v-1)^2} \\ &= Ce^2 \cdot e^{-\frac{1}{2}\left(\frac{u^2}{1/2}\right)} \cdot e^{-\frac{1}{2}\left(\frac{(v-1)^2}{1/4}\right)} \\ &\therefore\ \sigma_X^2 + \sigma_Y^2 = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}. \end{aligned}$$

Suppose X and Y are independent random variables that are uniform on the intervals $[2, 10]$ and $[1, 12]$, respectively. What is the probability that $\max(X, Y)$ is less than 5 ?

(a) 3/22

(b) 35/88

(c) 21/88

(d) 7/22

(e) 3/8

(f) 5/8

(g) 7/11

(h) 0

(i) 1

(j) None of these

Suppose X and Y are independent random variables that are uniform on the intervals $[1, 10]$ and $[3, 11]$, respectively. What is the probability that $\max(X, Y)$ is less than 5 ?

(a) 1/9

(b) 5/12

(c) 1/3

(d) 1/6

(e) 4/9

(f) 5/9

(g) 3/4

(h) 0

(i) 1

(j) None of these

Suppose X and Y are independent random variables that are uniform on the intervals $[1, 11]$ and $[3, 9]$, respectively. What is the probability that $\max(X, Y)$ is less than 4 ?

(a) 1/20

(b) 7/12

(c) 1/4

(d) 1/12

(e) 3/10

(f) 7/10

(g) 5/6

(h) 0

(i) 1

(j) None of these

Solution:
If X is uniform on $[A, B]$ and Y is uniform on $[C, D]$, then

$$P(\max X, Y < t) = P(X < t, Y < t) = F_{X,Y}(t,t) = F_X(t)F_Y(t) = \frac{t-A}{B-A} \cdot \frac{t-C}{D-C}.$$