Suppose the probability a coin comes up Heads when flipped is 1/3. If you flip the coin three times, what is the probability it comes up Heads at least once? $\rightarrow p(1)+p(2)+p(3)$

(a) 19/27

(b) 8/27

(c) 1/27

(d) 26/27

(e) 1/9

(f) 1/3

(g) 2/9

(h) 7/8

(i) 1/8

(j) 3/8

(k) 2/3

(l) None of these

$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k} \qquad \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$p(x=1) = \binom{3}{1}\left(\frac{1}{3}\right)\left(1-\frac{1}{3}\right)^{3-1} \qquad \binom{3}{1} = \frac{6}{1(2)!} = 3$$

$$= 3\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)^2$$

$$= \frac{4}{9}$$

$$p(x=1) = \binom{3}{2}\left(\frac{1}{3}\right)^2\left(\frac{2}{3}\right)^1 \qquad \binom{3}{2} = \frac{6}{2(1)!} = 3$$

$$= \cancel{3}\cdot\frac{1}{9}\cdot\frac{2}{\cancel{3}}$$

$$= \frac{2}{9}$$

$$p(x=2) = \binom{3}{3}\left(\frac{1}{3}\right)^3\cdot 1$$

$$= \frac{1}{27}$$

$$p(x=1)+p(x=2)+p(x=3) = \frac{4}{9}+\frac{2}{9}+\frac{1}{27} = \frac{19}{27}$$

If A and B are independent events such that $P(AB) = 1/4$ and $P(A - B) = 1/8$, then what is the probability of the event $A^c B^c$?

(a) 5/24

(b) 7/24

(c) 1/24

(d) 1/8

(e) 1/4

(f) 1/6

(g) 3/8

(h) 5/8

(i) 1/12

(j) 1/3

(k) 1/2

(l) None of these

$$P(A) = P(AB) + P(A-B)$$
$$= \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$$

$$A \perp\!\!\!\perp B \rightarrow P(AB) = P(A)P(B)$$

$$\frac{1}{4} = \frac{3}{8} P(B)$$

$$\rightarrow P(B) = \frac{2}{3}$$

$$P(A^c B^c) = P(A^c) P(B^c)$$
$$= \left(\frac{5}{8}\right)\left(\frac{1}{3}\right) = \frac{5}{24}$$

Let A, B, C be events such that A and B are independent given C. If $P(A^cB^cC) = P(AB^cC^c) = P(A^cBC^c) = 1/8$, $P(AB^cC) = 1/6$, $P(AB) = 1/4$, and $P(ABC^c) = 0$, then what is $P(A^cBC)$?

(a) 3/16

(b) 3/8

(c) 1/16

(d) 1/32

(e) 3/32

(f) 1/8

(g) 1/4

(h) 5/8

(i) 5/32

(j) 1/24

(k) 5/24

(l) None of these

$A \perp\!\!\!\perp B \mid C$

$P(AB) = P(ABC) + P(ABC^c)$

$\frac{1}{4} = P(ABC) + 0$

$P(AC) = P(ABC) + P(AB^cC)$

$P(AC) = \frac{1}{4} + \frac{1}{6} = \frac{5}{12}$

$P(BC) = P(ABC) + P(A^cBC)$

$P(BC) = \frac{1}{4} + x$

$P(C) = P(ABC) + P(A^cBC) + P(AB^cC) + P(A^cB^cC)$

$= \frac{1}{4} + x + \frac{1}{6} + \frac{1}{8}$

$= \frac{13}{24} + x$

$A \perp\!\!\!\perp B \mid C \longleftrightarrow P(A \mid BC) = P(A \mid C)$

$$\frac{P(ABC)}{P(BC)} = \frac{P(AC)}{P(C)}$$

$$\frac{\frac{1}{4}}{\frac{1}{4} + x} = \frac{\frac{5}{12}}{\frac{13}{24} + x} \xrightarrow{\text{calculator}} \frac{3}{16}$$