Suppose we flip a biased coin with $P(\text{Heads}) = 0.2$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 2. What is the probability the output is 2, given the coin comes up Heads?

(a) 1/6

(b) 1/2

(c) 5/6

(d) 1/3

(e) 0.2

(f) 0.8

(g) 0.9

(h) 1

(i) 0

(j) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 0.4$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 3. What is the probability the output is 3, given the coin comes up Heads?

(a) 1/6

(b) 1/2

(c) 5/6

(d) 1/3

(e) 0.4

(f) 0.6

(g) 0.8

(h) 1

(i) 0

(j) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 0.6$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 4. What is the probability the output is 4, given the coin comes up Heads?

(a) 1/6

(b) 1/2

(c) 5/6

(d) 1/3

(e) 0.6

(f) 0.4

(g) 0.7

(h) 1

(i) 0

(j) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 0.7$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 5. What is the probability the output is 5, given the coin comes up Heads?

(a) 1/6

(b) 1/2

(c) 5/6

(d) 1/3

(e) 0.7

(f) 0.3

(g) 0.65

(h) 1

(i) 0

(j) None of these

Solution: 1/6

Suppose we flip a biased coin with $P(\text{Heads}) = 0.2$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 2. What is the probability the coin comes up Heads, given the output is 1 ?

(a) 1

(b) 1/6

(c) 1/2

(d) 5/6

(e) 1/3

(f) 0.2

(g) 0.8

(h) 0.9

(i) 0

(j) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 0.4$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 3. What is the probability the coin comes up Heads, given the output is 1 ?

(a) 1

(b) 1/6

(c) 1/2

(d) 5/6

(e) 1/3

(f) 0.4

(g) 0.6

(h) 0.8

(i) 0

(j) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 0.6$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 4. What is the probability the coin comes up Heads, given the output is 1 ?

(a) 1

(b) 1/6

(c) 1/2

(d) 5/6

(e) 1/3

(f) 0.6

(g) 0.4

(h) 0.7

(i) 0

(j) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 0.7$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 5. What is the probability the coin comes up Heads, given the output is 1 ?

(a) 1

(b) 1/6

(c) 1/2

(d) 5/6

(e) 1/3

(f) 0.7

(g) 0.3

(h) 0.65

(i) 0

(j) None of these

Solution: 1.

Suppose we flip a biased coin with $P(\text{Heads}) = 3/8$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 2. What is the probability the output is either 1 or 2 ?

(a) 3/4

(b) 3/8

(c) 1/4

(d) 1/8

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 5/6$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 3. What is the probability the output is either 1 or 3 ?

(a) 4/9

(b) 2/9

(c) 5/9

(d) 5/18

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 3/10$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 4. What is the probability the output is either 1 or 4 ?

(a) 4/5

(b) 2/5

(c) 1/5

(d) 1/10

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Suppose we flip a biased coin with $P(\text{Heads}) = 5/8$. If we get Heads, then we roll a fair 6-sided die and observe the output, and if we get Tails, then we just declare the output to be 5. What is the probability the output is either 1 or 5 ?

(a) 7/12

(b) 7/24

(c) 5/12

(d) 5/24

(e) 1/6

(f) 5/6

(g) 1/3

(h) 2/3

(i) 1

(j) 0

(k) None of these

Solution: Let E and F be the events that the output is 1 and 5, respectively. Let G be the event the coin is Heads. Then

$$
\begin{aligned}
P(E|G^c) &= 0 \\
P(E \cup F) &= P(E) + P(F) \\
&= \; P(E|G)P(G) + P(E|G^c)P(G^c) \\
&\quad + P(F|G)P(G) + P(F|G^c)P(G^c) \\
&= (1/6)P(G) + 0 \\
&\quad + (1/6) \cdot P(G) + 1 \cdot (1 - P(G)) \\
&= 1 - (2/3)P(G)
\end{aligned}
$$