

If A and B are independent events with probabilities $1/3$ and $1/7$, respectively, then what is the probability of the event $A - B$?

(a) $2/7$

(b) 0

(c) $1/21$

(d) $4/21$

(e) $4/7$

(f) $1/3$

(g) $3/7$

(h) $6/7$

(i) $2/21$

(j) $1/4$

(k) $1/6$

(l) None of these

$$A \perp\!\!\!\perp B \rightarrow P(A - B) = P(A)(1 - P(B)) \\ = \frac{1}{3} \cdot \frac{6}{7} = \frac{6}{21} = \frac{2}{7}$$

Suppose three fair 6-sided dice are tossed. One die is red, another is blue, and the third one is green. What is the probability that the sum of the red and green dice values is less than 4, given the blue die value is larger than 3?

(a) $\frac{1}{12}$

(b) $\frac{1}{6}$

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

(e) $\frac{2}{3}$

(f) $\frac{1}{18}$

(g) $\frac{1}{36}$

(h) $\frac{1}{2}$

(i) $\frac{3}{4}$

(j) $\frac{5}{6}$

(k) $\frac{5}{36}$

(l) None of these

$$E = R + G < 4$$

$$|\mathcal{S}_E| = 36$$

$$F = B > 3$$

$$|\mathcal{S}_F| = 6$$

~~check $E \perp\!\!\!\perp F$; $P(E|F) = P(E)$~~

$$\frac{P(EF)}{P(F)} = P(E)$$

$$\frac{\frac{1}{2}}{\frac{1}{6}} = \frac{3}{36}$$

~~assume $E \perp\!\!\!\perp F$ (E only depends on R, G , F only on B)~~

$$\rightarrow P(E) = P(E|F)$$

$$= \left(\frac{1}{12}\right)$$

We perform an experiment such that for each positive integer n , the number n is produced with probability 2^{-n} . A random variable X then converts the outcome n into the fraction $\frac{1}{2^n}$. What is the probability that $\frac{3X}{X^2+1}$ times $X^4 + 2X^2 + e^X$ is positive?

(a) $1/4$

(b) $3/4$

(c) $1/8$

(d) $7/8$

(e) $1/3$

(f) $2/3$

(g) $1/2$

(h) $1/6$

(i) $5/6$

(j) 1

(k) 0

(l) None of these

$$P\left(\frac{3X}{X^2+1} \times X^4 + 2X^2 + e^X > 0\right) ?$$

$$\text{let } Y = \frac{3X}{X^2+1} \times X^4 + 2X^2 + e^X$$

$$Y > 0 \text{ when } X > 0$$

$$\rightarrow \frac{1}{2^n} > 0$$

$$\rightarrow 2^n - 5 > 0$$

$$\rightarrow n > 2.5$$

$$n \in \mathbb{Z} \rightarrow n \geq 3$$

$$P = 2^{-n} = \left(\frac{1}{2}\right)^n$$

$$\text{CDF : } F_n(u) = 1 - \left(1 - \frac{1}{2}\right)^{2u}$$

$$P(n \geq 3) = 1 - P(n \leq 2)$$

$$= 1 - P(n \leq 2)$$

$$= 1 - 1 - \left(1 - \frac{1}{2}\right)^{22}$$

$$= 1 - \frac{3}{4}$$

$$= \frac{1}{4}$$