$E = \{a, b, \ldots\}$

$P(E) = p(a) + p(b) + \ldots$

Suppose an experiment has sample space $S = \{a, b, c, d, e, f\}$ and define the events $U = \{a, b, d, e\}$, $V = \{c, e, f\}$, and $W = \{a, e\}$. If $P(U) + P(V) = 7/5$ and $P(W) = 1/2$, then what is $P(\{a\})$?

(a) 1/10

(b) 1/5

(c) 1/20

(d) 2/5

(e) 3/10

(f) 1/2

(g) 0

(h) 3/10

(i) 3/20

(j) 3/5

(k) 1/6

(l) None of these

$P(W) = \frac{1}{2} = P(a) + P(e)$

$\frac{7}{5} = P(\{a, b, d, e, c, e, f\})$

$\frac{7}{5} = 1 + p(e)$

$\rightarrow p(e) = \frac{2}{5}$

$\frac{1}{2} = p(a) + \frac{2}{5}$

$\rightarrow p(a) = \frac{1}{10}$

Suppose A, B, and C are three events in the sample space of an experiment such that

$$\begin{aligned} P(A \cup B \cup C) &= 0.9 \\ P(A \cup B) &= 0.7 \\ P(AB^cC^c) &= 0.4 \\ P(ABC) &= 0.25 \end{aligned}$$

What is the probability of the union of the events A^cB^cC and $A^cB^cC^c$?

(a) 0.3

(b) 0.1

(c) 0.2

(d) 0.4

(e) 0.5

(f) 0.6

(g) 0.7

(h) 0.15

(i) 0.75

(j) 0.35

(k) 0.65

(l) None of these

$\bar{A}\bar{B}C \cup \bar{A}\bar{B}\bar{C}$

$= \bar{A}\bar{B}(C \cup \bar{C})$

$= \bar{A}\bar{B}(1)$

$= \overline{A \cup B}$

$P(\overline{A \cup B}) = 1 - P(A \cup B)$

$= 1 - 0.7 = 0.3$

Suppose an experiment has an infinite sample space $S = \{a_1, a_2, a_3, \dots\}$ where $P(a_k) = 2^{-k}$ for each $k = 1, 2, 3, \dots$. For each k, define the infinite event $E_k = \{a_k, a_{k+1}, a_{k+2}, \dots\}$. What is the probability that the events E_2 and E_4^c both occur?

(a) 3/8

(b) 5/8

(c) 1/2

(d) 1/8

(e) 1/4

(f) 3/4

(g) 1/16

(h) 3/16

(i) 0

(j) 5/16

(k) 2/3

(l) None of these

$E_2 = \{a_2, a_3, a_4, \dots\}$

$E_4^c = \{a_1, a_2, a_3\}$

$P(E_2 \cap E_4^c) = P(\{a_2, a_3\})$

$= P(a_2) + P(a_3)$

$= 2^{-2} + 2^{-3}$

$= \frac{1}{4} + \frac{1}{8} = \frac{3}{8}$