“What is the purpose”? you may ask.   
“Grid”, I answer

[Go Grid, or go home]

Not to toot my own horn, but

[Jacob]

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[Master Thesis]

[Study programme]

[30/60 credits]

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Figure 1‑1. A very cool dude

Contents

[1 Theory 4](#_Toc94623805)

[1.1 Ionizing Radiation 4](#_Toc94623806)

[1.1.1 Photon interaction in matter 4](#_Toc94623810)

[1.1.2 Charged particle interaction in matter 8](#_Toc94623811)

[1.2 Dosimetry 12](#_Toc94623812)

[1.2.1 Quantities 12](#_Toc94623813)

[1.3 Cavity Theory 16](#_Toc94623814)

[2 Appendix A 19](#_Toc94623815)

**[2.1](#_Toc94623818)****[Compton Scattering](#_Toc94623818)** [19](#_Toc94623818)

[2.2 Mean free path 23](#_Toc94623819)

# Theory

## Ionizing Radiation

Radiation is transfer of energy. The main categories are non-ionizing and ionizing radiation. I.e., it either has enough energy to liberate an electron from the atom, or it doesn’t. Non-ionizing radiation consists of low-energy electromagnetic (EM) waves such as UV-light and microwaves. When we move further right in the EM spectrum, we get ionizing X-ray and particles. To clarify: The EM waves do not become particles, but we know from the discovery of the photo-electric effect that you might interpret EM waves as “showers” of photons (Einstein & Infeld, 1938). Together with neutrons, they make up a group called uncharged particles. They are highly penetrating because they need to interact directly with a target.

Charged particles are different, they consist of particles with either positive or negative charge. Some examples are protons (+), electrons (-) and -particles (+2). Their interaction probability is greater compared to uncharged particles. A photon must be in proximity of either a nucleus or an electron for an interaction to occur. A charged particle may interact at a distance. Their Coulomb field interacts with the Coulomb fields of other electrons, causing a “Continuous Slowing Down” (Attix, 1986, p. 160).

The path of a charged particle through a medium is highly dependent on the particle at hand. What is the charge, is it highly charged and how heavy it is?

As medical physicists, we’re primarily interested in ionizing radiation and using its properties to kill cancer cells. This is further examined in the radiobiology section (ref here).

### Photon interaction in matter

Photons are energy-carrying particles without mass traveling at the speed of light. They interact with the surrounding medium in several ways. The main interactions are Rayleigh Scattering, Photoelectric effect, Compton Scattering, pair/triplet production and photonuclear interactions. Which interaction you’ll have is highly dependent on the atomic number of the photon absorber and the energy of the incoming photon. The probability of interaction is defined as interaction cross-section , with the unit . For our study, we’re mostly interested in the photo-electric effect and Compton scattering.

Pair productions happen when a photon is annihilated in the presence of the Coulomb field of a nucleus. It creates an electron-positron pair. A less probable phenomenon is when a photon is annihilated near the Coulomb field of a bound electron. The same electron-positron pair is created, but the remaining energy causes ionization of the bound electron.   
The photon beam energy we use is in the kV range. But, in , pair production does not become the dominant interaction type until we reach MV energies. Photonuclear interactions also occur with MV energies. However, it is far less probable than pair production (Attix, 1986, p. 154). Rayleigh scattering is an interaction where the photon is deflected from its path, but the energy is conserved (Attix, 2008, p.153). Because there is no energy being imparted in the medium, we do not get any dose.   
Diagram

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Figure 1‑1. Photon interaction probability (defined as interaction cross-section [] as a function of atomic number Z and photon energy [MeV] The curves represent the area where two interactions have the same probability (Attix, 1986, p.125 ).

**Photo-electric effect**

Photo-electric effect is when an incident photon’s energy is absorbed by an electron bound to an atom If the energy is equal to or larger than the binding energy of the electron it will ionize the electron. The energy transferred from the photon to the electron depends on its initial energy and the electron’s binding energy (Attix, 1986, p. 139).

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If the energy of the photon beam is not sufficient, it will not help by increasing the photon flux. The reason being, that an electron may only absorb the energy of a single photon.   
The photon might liberate a tighter bound electron. A looser bound electron will fill the vacancy, and the energy difference between the energy levels is emitted as characteristic X-rays. Another possibility is that the energy difference is transferred directly to a valence electron, causing its liberation. This process is called the Auger effect.

The cross-section per atom for the photo-electric effect.

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Diagram

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Figure 1‑2. Illustration of the photoelectric effect. A bound electron absorbs all the energy of incoming a photon. If the energy is high enough, it will ionize the electrons (Attix, 1986, p.138). The kinetic energy of the electron is dependent on the initial energy of the photon and the binding energy of the electron . Recoil energy is given to the atom, but it is approximately .

**Compton scattering**

Diagram

Description automatically generatedThe scattering process differs from the photo-electric effect in that the electron orbiting the nucleus is assumed free, which results in an inelastic collision. It is illustrated in Figure 1‑3. The errors from this assumption have proved negligible (Attix, 1986,p. 125), as the errors don’t become substantial until we have a high atomic number Z and low initial energy . When these conditions are reached, the photoelectric become the dominating interaction type. Therefore we’re able to accept these errors.

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The second difference compared to the photo-electric effect is that the photon only transfers part of its energy. The energy of the scattered photon follows this expression, which is derived in Appendix A.

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We observe a strong correlation with the energy of the incident photon, and the scattering angle. And a maximum energy transfer to the electron for .   
The Compton cross section per electron was derived by Klein and Nishina. They improved on the existing theory of Thomson scattering. Thomson’s cross section was independent of incident photon energy, and assumed (Attix, 1986, p.130). This is correct for low energies. However, for energies above the difference in energy cannot be neglected. This is illustrated in **Error! Reference source not found.**.

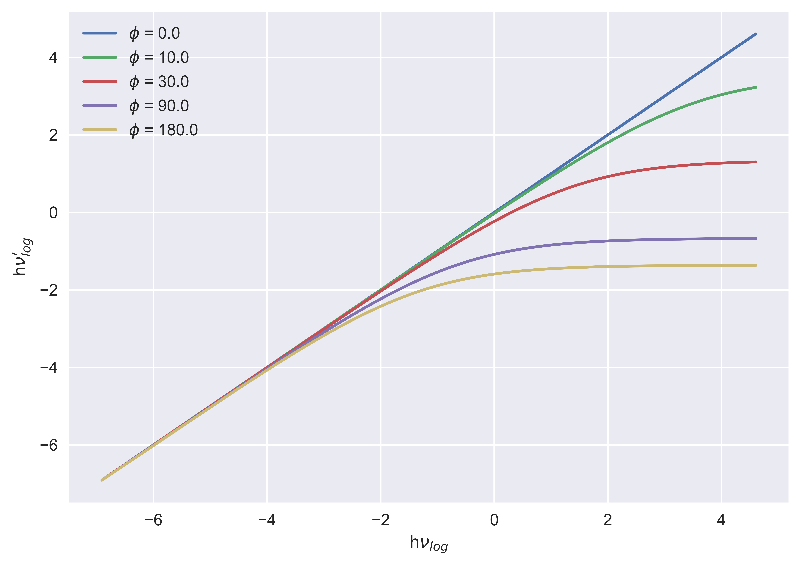


Figure 1‑4. Compton scatter energies in . is energy if incident photon, is energy of scattered photon.

The Klein-Nishina cross section is represented by the symbol .

Each interaction has its cross section. The photo-electric effect has , Compton scatter has pair production has and Rayleigh scattering has . In medical physics, you’re interested in all interactions that cause the ionization of biomolecules. Therefore, it is practical to sum each cross section to create a combined interaction variable. It is defined as the attenuation coefficient

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| It represents the combined probability of all interactions to explain how a photon is attenuated. However, it is more insightful to normalize it with density . This way, we get , which represents how a photon is attenuated in a medium. This is called the **mass attenuation coefficient**.  Now that we have the probability of interaction, we can include the fraction of kinetic energy transferred from the incident photon to a secondary electron within a volume of interest. This is called the **mass energy-transfer coefficient** (Attix, 2008).   |  |  | | --- | --- | |  | 1‑5 |   Where is the energy of the liberated electron after Compton scatter, is the energy of either K- or L-shell (highest binding energy) characteristic X-rays generated after the Auger effect. These X-rays carry energy away from the volume of interest and we therefore subtract their energy.  is the rest energy of the electron-positron pair created by pair production. We subtract because it’s the energy needed to create the electron-positron pair, leaving us with the kinetic energy.  The last expression we want is the **mass energy-absorption coefficient** (Attix, 2008). It represents the energy absorbed by the volume. It relates to the mass energy-transfer coefficient by   |  |  | | --- | --- | |  | 1‑6 |   Where g is the fraction of energy lost to radiative loss. Bremsstrahlung created by radiative transfer moves energy out of the system.  **Photon range**  Mean free path is the expected distance a photon will travel before interacting with the medium. In 2.2 we derived an expression for mean free path for a photon traversing a slab of material with infinitesimal area and thickness .   |  |  | | --- | --- | |  | 1‑7 |   The pathlength of the photon decreases with increasing attenuation. This is an important result, because it allows us to predict the path of the photon. It is especially useful when performing Monte Carlo simulations, which we’ll come back to in (ref here). | 1‑3 |

### Charged particle interaction in matter

A charged particle is a particle with either positive or negative net electrical charge, such as electrons (-), protons (+) and ions (an atom with a surplus of electrons or protons).

Because of their charge, the particles will interact with other charged particles through their Coulomb fields. The Coulomb force exerted from a charged particle upon another is defined by the equation

Where K is a constant, and is the charge of particle 1 and 2 and r is the distance between the particles. Both particles are in this case assumed stationary. The Coulomb fields enables the charged particles to interact at a distance. We will refer to this distance as the impact parameter. We have three main categories of interactions:  
Soft collisions, hard collisions and Coulomb interactions with the nucleus.

**Soft collisions**

Soft collisions are small Coulomb interactions between the Coulomb fields of an atom and a charged particle. The impact parameter is much larger than the atomic radius, and small amounts of energy are transferred to the orbiting electrons of an atom, mainly causing excitations. Even though the energy transfers are very small, the interaction is highly probable, therefore contributing to half of the energy loss of the charge particle (Attix, 1986, p.161).

**Hard collision**

Hard collisions happen when the impact parameter has the same order of magnitude as the atomic radius. The result is a significant kinetic energy transfer to an assumed free and stationary electron. These electrons are called -rays, which undergo the same charge particle interactions.

**Radiative transfer**

Radiative transfer, also known as bremsstrahlung, is a process where a charged particle interacts with the nucleus’s Coulomb field. The impact parameter must be much lower than the atomic radius for this to happen. The interaction mainly occurs with electrons and will therefore be the focus point (Attix, 1986, p.163). The radiative transfer refers to an inelastic collision between the nucleus and the electron. See Figure 1‑8. The electron with its negative charge is attracted to the nucleus’s positive charge, causing a deacceleration and deflection of the electron from its incident path. The decrease in kinetic energy is converted to a photon, thus conserving energy.   
The probability of radiative transfer is much lower compared to elastic scattering (2-3%) and is proportional to (Grieken & Markowicz, 1993, p.3), where q is the charge, Z is the atomic number of the atom, T is the kinetic energy of the electron and is the rest mass of the electron. With a larger Z, the atom has a higher proton count, resulting in the nucleus having a larger Coulomb field attracting the electron. The kinetic energy of the electron is important because the electron needs to penetrate the electron cloud surrounding the nucleus.

**Diagram, schematic

Description automatically generated**Radiative transfer is used when creating X-rays inside an X-ray tube, but we will come back to this in the section covering the X-ray tube (ref here).

Figure 1‑8. Illustration of radiative transfer, where an electron inelastically collides with an atom’s nucleus deflecting it from its path. The result is an emitted photon with energy equaling the energy loss of the electron (Hapugoda, 2017).

**Stopping Power**

Stopping power is how much energy we expect the charged particle to lose per unit length. It can be found by integrating differential energy loss per length over possible energy transfers.   
As discussed, the charged particle might lose its energy by colliding or by radiative transfer, we therefore separate these contributions

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Energy lost to radiative transfer does not contribute to dose, because of the larger range of the photons. Radiative stopping power is still important to accurately describe the range of the charged particle.   
Collision stopping power is split into two parts: for soft and hard collisions . As we Energy loss is dependent on the material it penetrates, we therefore introduce **mass stopping power** by dividing the stopping power by the material density .   
The total collision mass stopping power becomes

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Where is electrons per gram, is classical electron radius, , I is mean excitation potential in the medium and is shell correction. The collision stopping power assumes the electron’s velocity to be much greater than orbiting electrons in atoms. As the electrons slow down the assumption becomes untrue and the shell correction will account for this.

Mass collision stopping power is closely related to absorbed dose. When CPE is achieved absorbed dose is expressed as

is electron fluence in a radiation field (see 1.2.1).

Stopping power is useful because we can estimate the range of the charged particle, but we also need to know how much of that energy is absorbed by the medium. Linear Energy Transfer (LET) represents this quantity with the unit . It is also known as restricted stopping power (Attix, 1986, p.179). When high energy electrons experience hard collisions, they liberate secondary electrons. A cutoff energy is introduced, because some electrons might have high enough energy to escape the volume of interest. If none of the secondary electrons can escape, we have CPE (see 1.2.1) and

LET is especially important in radiobiology, because it measures how damaging a radiation type might be. We will come back to this in (ref here).

**CSDA**

If we assume that the electrons are continuously slowing down (i.e., neglecting fluctuations in energy loss) as they interact, we can integrate total mass stopping power to get an approximate range called the continuously slowing down approximation (CSDA) range

This lets us describe the path of secondary charged particles following an ionization event. It is different to **projected** range, as it describes the total range of the particle. **Projected** range is a measure of how deep into the medium the particle penetrates.

## Dosimetry

Taking any medicine requires knowledge about the correct dosage. Medicine is often administered through pills with carefully measured ingredients to give the right effect. Radiation does not have that advantage. Radiation dose is dependent on the energy, exposure time, distance from source, material of the absorber and quality (e.g., photons, protons or neutrons) of the radiation.   
Theoluteion has been to use the effects of radiation on different substances such as discoloring (film dosimeter), temperature change (calorimeter) and light emission (thermoluminescence). To understand all the intricacies of dose measurement we need to define some quantities.

### Quantities

**Ionizing Radiation Fields**We wish to find number of ionizations in a point P inside a field of ionizing radiation (Attix, 1986). A ray cannot interact with a point, we therefore define a finite area around the point. The incident angle of the ray might not be parallel to the area , we therefore need to account for all possible angles. This results in a sphere around P as seen in Figure 1‑12 with infinitesimal volume dV, mass dm and cross-sectional area da.   
The number of traversing rays per cross sectional area da is defined as **fluence** .

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If the number of rays differs over time, the fluence needs to be integrated over time to get the fluence rate.   
The energy of the rays is equally as important as the amount, e.g., in an X-ray tube you create bremsstrahlung. The beam is not monoenergetic and we must sum over all energies to get **energy fluence**  **.**

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The fluence is often dependent on solid angle , because we often have a case of a target placed at a distance away from a radiation source. Then the rays cannot move around the sphere and traverse from the back.

**Diagram

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Figure 1‑12. Ionizing radiation field defined as a sphere encapsulation a point P with infinitesimal volume dV, mass dm, with a cross sectional area da (Attix, 1986, p.6).

**KERMA**

With the energy fluence we have the energies traversing the sphere, but to get a dose we need the rays to interact and release energy. This is where KERMA comes in. Kinetic Energy Release per Mass. It describes the process where uncharged particles (photons and neutrons) transfer their energy to electrons in a defined volume (Attix, 1986). The electrons then traverse through a smaller volume (see Figure 1‑20). The energy transfer can be expressed with the expression

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Where is the energy transferred from uncharged radiation with energy into the volume minus the energy of the uncharged radiation leaving the volume without interacting.   
RL stands for radiative losses and represents interactions where charged particles generate photon energy after the initial ionization. If these photons leave the volume, it does not matter because we’re only interested in the energy transferred by the incident particles entering the volume.  
The final term is conversion of rest mass to energy or energy to rest mass e.g., pair production (see 1.1.1) where a photon annihilates creating an electron positron pair.   
With we can define KERMA

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For monoenergetic photons, KERMA is related to energy fluence using the expression

Where is the mass energy transfer coefficient (see 1.1.1), which represents the probability of the photons leaving energy in the volume.

Until now, we’ve neglected how the electrons have spent their energy after they’ve received it from the incident photons. Accounting for radiative loss gives net energy transfer represented by this expression

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represents the energy of the electrons, not lost to radiative transfer. We can now separate KERMA into two: collision KERMA and radiative KERMA

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We can relate and energy fluence to another known quantity: mass energy-absorption coefficient

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**Absorbed dose**

We know that dose is energy absorbed in a volume, which includes the charged particles. The energy can be represented by

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Dose then becomes

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The unit is still , but it is called Gray or Gy.  
  
**Charged Particle Equilibrium (CPE)**

If the amount of energy from charged particles entering the volume is equal to the energy leaving the volume. This situation is called **charged particle equilibrium** (CPE), and  
absorbed dose reduces to .

When CPE is achieved, we can easily find the dose ratio between two volumes. This is a practical metric because we often need to relate dose in one medium to another medium. We will come back to this in 1.3.   
CPE is not necessarily easy to achieve. If the volumes are near the source, we’ll have much higher fluence on the side closest to the source (Attix, 1986, p.72). This causes more ionizations closer to the surface of V, compared to the surface of v, and CPE fails.   
For larger photon energies, the range of the liberated charged particles will increase compared to the range of the photons. Therefore, charged particles ionized near the surface of V have larger ranges than the charged particles ionized near the surface of v. The generated charged particles will therefore enter volume v but doesn’t exit, and CPE fails.

Chart

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Figure 1‑20. Charged particle equilibrium visualized, where photon energy enters a volume transferring energy to charged particles (electrons in this case), that traverses a smaller volume . The electrons exiting are of same type and energy distribution as the electrons entering, and we have CPE.

**Transient Charged Particle Equilibrium (TCPE)**

CPE demands no significant attenuation of photons within the smaller volume in Figure 1‑20. Then dose becomes collision kerma . A more realistic assumption is that absorbed dose is proportional to collision kerma. The expression for dose becomes

where f is the dose contribution from attenuation of photons through the volume.

## Cavity Theory

When measuring dose, we use a dosimeter. A very popular dosimeter is the ionization chamber, which we’ll discuss in detail (ref here). A more basic description of an ion chamber is that it consists of a gas filled chamber connected to an electrometer. Radiation ionizes the gas, and the electrometer measures a charge proportional to absorbed dose.   
However, dose (a.k.a., energy absorbed in the medium) is not equal between mediums of different density. We therefore need to relate the dose absorbed by the gas to the medium we’re interested in.

**Bragg-Gray cavity**

We saw in 1.1.2 that absorbed dose might be written as

Where is electron fluence and is mass stopping power.In Bragg-Gray cavity theory, dose to water is related to dose to air by

Where is the mass stopping power ratio between water and air. We see that the electron fluence is assumed constant, but for this to be true two conditions needs to be fulfilled (Attix, 1986, p. 232):

1. Fluence of charged particles should not be perturbed
2. Only charged particles crossing the cavity contributes to dose.

Let’s assume you have a volume of water with an air-filled cavity inside, like in Figure 1‑21. Compared to water, air has less stopping power because of lower density. Less energy is therefore lost inside the cavity, and the energy of the electrons entering the cavity is different than the energy of the electrons leaving the cavity. To satisfy the first condition we therefore need a cavity so small that the energy difference becomes negligible.   
We also don’t want dose contribution from ionization of electrons caused by photons. This results in more energy being imparted in the cavity and the Bragg-Gray relation fails.

**­**

Graphical user interface

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Figure 1‑21. A volume with material w (e.g., water), with a cavity volume inside with air inside, and

**Bragg-Gray-Laurence**

The ideal Bragg-Gray scenario is not possible, as it would require an infinitesimally dimensioned dosimeter to not lose any charged particles to the cavity. It also demands the stopping power ratio to be independent on energy (Alm Carlsson, 2001). Looking at the expression for mass collision stopping power in

The theory was therefore improved by introducing the CSDA (see 1.1.2), which demands that we know the energy distribution of the electrons in the cavity (Alm Carlsson, 2001). Even though the charged particle fluence is perturbed by the cavity, we can still assume charged particle equilibrium (CPE, see 1.2.1), because the electrons don’t stop inside the volume. (er ikke dette en motstridelse? Fluence er jo antall, og CPE antar )

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# Appendix A

## **Compton Scattering**

Here we derive the energy of a photon scattered after interaction with a free electron at rest. We will use the four-momentum vector   , which consist of a time component and three space components .

First, we use conservation of energy.

The energy of the incident photon is . Because of the assumption of an electron at rest, we only have rest energy . The energy of the photon after the interaction is simply , and the energy of the electron after the interaction is unknown. We end up with this equation.

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Finding requires that we introduce conservation of momentum. We need to find the components of the four-momentum vector for the photon and the electron.

For the incident photon, the time component become . We only have momentum in the x-direction. Using the relation

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and the fact that the photon is massless, we get .

The electron at rest does not have momentum, but it does have rest energy. The time component become . Where is the rest mass of the electron.

We use trigonometry to find the spatial components of the momentum for the electron and photon after the interaction. Combining all the results, we get four four-momentum vectors:

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With all the components in place, we use conservation of momentum.

First, we separate the and , then we square both sides of the equation. For simplicity, we remove the vector sign above our four-vectors. We get this equation.

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The product of two four-vectors is , we see that becomes . And becomes . We use the dot product to find , , , and .

If we use equation 2‑2, we can exchange with . Then we end up with

Finally, we put all our calculations together and rewrite equation 2‑3 to get

solving for we get

inserting this expression into equation 2‑1

and solving for

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Which is the expression for the energy of the photon after the interaction, with scattering angle .

## Mean free path

Here we derive the mean free path of a photon.

Assume that you have incoming photons hitting a slab of material with infinitesimal area dA and width dx (see **Error! Reference source not found.**). The total probability of N photons hitting the slab, with an attenuation coefficient of is

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Where n is photons per unit volume

The probability of **not** interacting ­is of course , then number of photons after the slab becomes

Rearranging and inserting our expression for P we get

If we divide by and let approach , we get

Solving the differential equation, we get

where is the number of photons entering the slab. Now we have the fraction of photons that doesn’t interact in the slab. If we multiply Q with P, we get a binomial looking probability density function, which describes the probability of an interaction happening somewhere between and

Integrating over possible pathlength from to we get an expected pathlength of

Solving the integral using partial integration we get

Using L’Hôpital’s rule we see that

For the second term we get . This results in a mean free path of

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| Schematic  Description automatically generated with low confidence  Figure 2‑2. A thin slab of material with atoms that might interact with an incoming photon (“Mean Free Path,” 2021). | 2‑3 |
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