

**Context from Theorem 7.1:**

*Suppose that  $\{a_k\}_{k=1}^{\infty}$  is a sequence of monotonically decreasing non-negative numbers.*

**Problem 7.12(b):**

*Let  $s_n$  be the  $n$ -th partial sum of  $\sum a_k$  and  $t_m$  be the  $m$ -th partial sum of  $\sum_{\ell=0}^{\infty} 2^{\ell} a_{2^{\ell}}$ . Use the inequality above to prove that  $s_{2^{m+1}-1} \leq t_m$  for all  $m \in \mathbb{N}$ .*

**Counter-example:** Let  $\{a_k\}_{k=1}^{\infty}$  be such that

$$\begin{aligned} a_1 &= 3, \\ a_2 &= 2, \\ a_3 &= 1, \end{aligned}$$

and  $a_k = 0$  for all  $k > 3$ . Clearly,  $\{a_k\}_{k=1}^{\infty}$  is monotonically decreasing, and all its terms are nonnegative. Let  $m = 1 \in \mathbb{N}$ . Then,

$$\begin{aligned} s_{2^{m+1}-1} &= s_{2^2-1} = s_3 = a_1 + a_2 + a_3 = 3 + 2 + 1 = 6, \\ t_m &= t_1 = 2^1 a_{2^1} = 2a_2 = 4, \end{aligned}$$

so  $s_{2^{m+1}-1} > t_m$  in this case.  $\square$

**Proposition:** Perhaps the statement should read:

*Let  $s_n$  be the  $n$ -th partial sum of  $\sum a_k$  and  $t_m$  be the  $m$ -th partial sum of  $\sum_{\ell=0}^{\infty} 2^{\ell} a_{2^{\ell}}$ . Use the inequality above to prove that  $s_{2^{m+1}-1} \leq t_m$  for all  $m \in \mathbb{N}$ .*

Maybe this was the original intention, but it seems odd for the one series to start at 1 and the other to start at 0.