Project 3

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Question 1

Part A)

i: the objective function is MINIMIZE 10P1W1 +15P1W2 +11P2W1 +8P2W2 +13P3W1 +8P3W2 +9P3W3 +14P4W2 +8P4W3 + 5W1R1 +6W1R2 +7W1R3 +10W1R4 +12W2R3 +8W2R4 +10W2R5 +14W2R6 +14W3R4 +12W3R5 +12W3R6 +6W3R7
where
each coefficient is the cost of that arc or edge of the graphic
and a variable of the form PmWn would mean the arc connecting Plant m to Warehouse n.

the contraints are:

```
!SUPPLY CONTRAINTS
P1W1+P1W2<150
P2W1+P2W2<450
P3W1+P3W2+P3W3<250
P4W2+P4W3<150

!DEMAND CONSTRAINTS
W1R1>100
W1R2>150
W1R3+W2R3>100
W1R4+W2R4+W3R4>200
W2R5+W3R5>200
W2R6+W3R6>150
W3R7>100

!DISTRIBUTION BALANCING CONTRAINTS
P1W1+P2W1+P3W1-W1R1-W1R2-W1R3-W1R4=0
P1W2+P2W2+P3W2+P4W2-W2R3-W2R4-W2R5-W2R6=0
P3W3+P4W3-W3R4-W3R5-W3R6-W3R7=0

!NON-NEGATIVITY CONTRAINTS
P1W1>0
P1W2>0
P2W1>0
P2W2>0
P3W1>0
P3W2>0
P3W3>0
P4W2>0
P4W3>0
W1R1>0
W1R2>0
W1R3>0
W1R4>0
```

```
W2R3>0
W2R4>0
W2R5>0
W2R6>0
W3R4>0
W3R5>0
W3R6>0
W3R7>0
```

ii: this is the program and report for the optimal solution
I've written out the optimal solution in (iii) below.

This is the LINDO program:

```
 MIN    10P1W1  +15P1W2         +
        11P2W1  +8P2W2          +
        13P3W1  +8P3W2  +9P3W3  +
                14P4W2  +8P4W3  +

        5W1R1   +6W1R2  +7W1R3  +10W1R4 +
                        12W2R3  +8W2R4  +10W2R5 +14W2R6 +
                                14W3R4  +12W3R5 +12W3R6 +6W3R7

ST
        !SUPPLY CONTRAINTS
        P1W1+P1W2<150
        P2W1+P2W2<450
        P3W1+P3W2+P3W3<250
        P4W2+P4W3<150

        !DEMAND CONSTRAINTS
        W1R1>100
        W1R2>150
        W1R3+W2R3>100
        W1R4+W2R4+W3R4>200
        W2R5+W3R5>200
        W2R6+W3R6>150
        W3R7>100

        !DISTRIBUTION BALANCING CONTRAINTS
        P1W1+P2W1+P3W1-W1R1-W1R2-W1R3-W1R4=0
        P1W2+P2W2+P3W2+P4W2-W2R3-W2R4-W2R5-W2R6=0
        P3W3+P4W3-W3R4-W3R5-W3R6-W3R7=0

        !NON-NEGATIVITY CONTRAINTS
        P1W1>0
        P1W2>0
        P2W1>0
```

```
        P2W2>0
        P3W1>0
        P3W2>0
        P3W3>0
        P4W2>0
        P4W3>0
        W1R1>0
        W1R2>0
        W1R3>0
        W1R4>0
        W2R3>0
        W2R4>0
        W2R5>0
        W2R6>0
        W3R4>0
        W3R5>0
        W3R6>0
        W3R7>0

END
```

This is the LINDO report:

```
 LP OPTIMUM FOUND AT STEP      13

        OBJECTIVE FUNCTION VALUE

        1)       17100.00

  VARIABLE        VALUE          REDUCED COST
      P1W1       150.000000          0.000000
      P1W2         0.000000          8.000000
      P2W1       200.000000          0.000000
      P2W2       250.000000          0.000000
      P3W1         0.000000          2.000000
      P3W2       150.000000          0.000000
      P3W3       100.000000          0.000000
      P4W2         0.000000          7.000000
      P4W3       150.000000          0.000000
      W1R1       100.000000          0.000000
      W1R2       150.000000          0.000000
      W1R3       100.000000          0.000000
      W1R4         0.000000          5.000000
      W2R3         0.000000          2.000000
      W2R4       200.000000          0.000000
      W2R5       200.000000          0.000000
      W2R6         0.000000          1.000000
      W3R4         0.000000          7.000000
```

W3R5	0.000000	3.000000
W3R6	150.000000	0.000000
W3R7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-16.000000
10)	0.000000	-18.000000
11)	0.000000	-21.000000
12)	0.000000	-15.000000
13)	0.000000	-11.000000
14)	0.000000	-8.000000
15)	0.000000	-9.000000
16)	150.000000	0.000000
17)	0.000000	0.000000
18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	200.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000
36)	100.000000	0.000000

NO. ITERATIONS= 13

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
P1W1	10.000000	1.000000	INFINITY
P1W2	15.000000	INFINITY	8.000000
P2W1	11.000000	2.000000	1.000000
P2W2	8.000000	5.000000	0.000000
P3W1	13.000000	INFINITY	2.000000
P3W2	8.000000	0.000000	1.000000
P3W3	9.000000	1.000000	1.000000
P4W2	14.000000	INFINITY	7.000000
P4W3	8.000000	1.000000	INFINITY
W1R1	5.000000	INFINITY	16.000000
W1R2	6.000000	INFINITY	17.000000
W1R3	7.000000	2.000000	18.000000
W1R4	10.000000	INFINITY	5.000000
W2R3	12.000000	INFINITY	2.000000
W2R4	8.000000	5.000000	16.000000
W2R5	10.000000	3.000000	18.000000
W2R6	14.000000	INFINITY	1.000000
W3R4	14.000000	INFINITY	7.000000
W3R5	12.000000	INFINITY	3.000000
W3R6	12.000000	1.000000	21.000000
W3R7	6.000000	INFINITY	15.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	150.000000	200.000000	0.000000
3	450.000000	INFINITY	0.000000
4	250.000000	250.000000	0.000000
5	150.000000	100.000000	0.000000
6	100.000000	0.000000	100.000000
7	150.000000	0.000000	150.000000
8	100.000000	0.000000	100.000000
9	200.000000	0.000000	200.000000
10	200.000000	0.000000	200.000000
11	150.000000	0.000000	100.000000
12	100.000000	0.000000	100.000000
13	0.000000	0.000000	200.000000
14	0.000000	0.000000	250.000000
15	0.000000	0.000000	100.000000
16	0.000000	150.000000	INFINITY
17	0.000000	0.000000	INFINITY
18	0.000000	200.000000	INFINITY
19	0.000000	250.000000	INFINITY
20	0.000000	0.000000	INFINITY
21	0.000000	150.000000	INFINITY
22	0.000000	100.000000	INFINITY

```
     23        0.000000        0.000000          INFINITY
     24        0.000000      150.000000          INFINITY
     25        0.000000      100.000000          INFINITY
     26        0.000000      150.000000          INFINITY
     27        0.000000      100.000000          INFINITY
     28        0.000000        0.000000          INFINITY
     29        0.000000        0.000000          INFINITY
     30        0.000000      200.000000          INFINITY
     31        0.000000      200.000000          INFINITY
     32        0.000000        0.000000          INFINITY
     33        0.000000        0.000000          INFINITY
     34        0.000000        0.000000          INFINITY
     35        0.000000      150.000000          INFINITY
     36        0.000000      100.000000          INFINITY
```

iii. the optimal solution for shipping is
ship 150 from P1 to W1
ship 200 from P2 to W1
ship 250 from P2 to W2
ship 150 from P3 to W2
ship 100 from P3 to W3
ship 150 from P4 to W3
ship 100 from W1 to R1
ship 150 from W1 to R2
ship 100 from W1 to R3
ship 200 from W2 to R4
ship 200 from W2 to R5
ship 150 from W3 to R6
ship 100 from W3 to R7
for a total cost of 17,100

Part B)
Q1: there is no feasible solution for the problem (when eliminating Warehouse 2) without making adjustments.
However if we are able to increase production at Plant 4 from 150 to 450, we have an optimal solution of given in the following report with a cost of 18,400. Alternatively, we could operate Plant 3 at 300 for a cost of 18,700.

Report with Plant 4 operating at 450:

```
 LP OPTIMUM FOUND AT STEP      1

        OBJECTIVE FUNCTION VALUE

        1)      18400.00
```

VARIABLE	VALUE	REDUCED COST
P1W1	150.000000	0.000000
P2W1	400.000000	0.000000
P3W1	0.000000	2.000000
P3W3	0.000000	1.000000
P4W3	450.000000	0.000000
W1R1	100.000000	0.000000
W1R2	150.000000	0.000000
W1R3	100.000000	0.000000
W1R4	200.000000	0.000000
W3R4	0.000000	1.000000
W3R5	200.000000	0.000000
W3R6	150.000000	0.000000
W3R7	100.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	50.000000	0.000000
4)	250.000000	0.000000
5)	550.000000	0.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-21.000000
10)	0.000000	-20.000000
11)	0.000000	-20.000000
12)	0.000000	-14.000000
13)	0.000000	-11.000000
14)	0.000000	-8.000000
15)	150.000000	0.000000
16)	400.000000	0.000000
17)	0.000000	0.000000
18)	0.000000	0.000000
19)	450.000000	0.000000
20)	100.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	200.000000	0.000000
24)	0.000000	0.000000
25)	200.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000

NO. ITERATIONS= 1

RANGES IN WHICH THE BASIS IS UNCHANGED:

OBJ COEFFICIENT RANGES

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
P1W1	10.000000	1.000000	INFINITY
P2W1	11.000000	1.000000	1.000000
P3W1	13.000000	INFINITY	2.000000
P3W3	9.000000	INFINITY	1.000000
P4W3	8.000000	1.000000	1.000000
W1R1	5.000000	INFINITY	16.000000
W1R2	6.000000	INFINITY	17.000000
W1R3	7.000000	INFINITY	18.000000
W1R4	10.000000	1.000000	21.000000
W3R4	14.000000	INFINITY	1.000000
W3R5	12.000000	INFINITY	20.000000
W3R6	12.000000	INFINITY	20.000000
W3R7	6.000000	INFINITY	14.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	150.000000	400.000000	50.000000
3	450.000000	INFINITY	50.000000
4	250.000000	INFINITY	250.000000
5	1000.000000	INFINITY	550.000000
6	100.000000	50.000000	100.000000
7	150.000000	50.000000	150.000000
8	100.000000	50.000000	100.000000
9	200.000000	50.000000	200.000000
10	200.000000	550.000000	200.000000
11	150.000000	550.000000	150.000000
12	100.000000	550.000000	100.000000
13	0.000000	50.000000	400.000000
14	0.000000	550.000000	450.000000
15	0.000000	150.000000	INFINITY
16	0.000000	400.000000	INFINITY
17	0.000000	0.000000	INFINITY
18	0.000000	0.000000	INFINITY
19	0.000000	450.000000	INFINITY
20	0.000000	100.000000	INFINITY
21	0.000000	150.000000	INFINITY
22	0.000000	100.000000	INFINITY
23	0.000000	200.000000	INFINITY
24	0.000000	0.000000	INFINITY
25	0.000000	200.000000	INFINITY
26	0.000000	150.000000	INFINITY
27	0.000000	100.000000	INFINITY

This is the LINDO program:

```
MIN     10P1W1                  +
        11P2W1                  +
        13P3W1          +9P3W3  +
                        8P4W3   +

        5W1R1   +6W1R2  +7W1R3  +10W1R4 +
                                14W3R4  +12W3R5 +12W3R6 +6W3R7

ST
        !SUPPLY CONTRAINTS
        P1W1<150
        P2W1<450
        P3W1+P3W3<250 !if increased from 250 to 300 we have a feasible solution
        P4W3<150 !if increased from 150 to 450 we have a feasible solution

        !DEMAND CONSTRAINTS
        W1R1>100
        W1R2>150
        W1R3>100
        W1R4+W3R4>200
        W3R5>200   !lower demand from 200 to 150 for feasible solution
        W3R6>150    !lower demand from 150 to 100 for feasible solution
        W3R7>100    !lower demand from 100 to 50 for feasible solution

        !DISTRIBUTION BALANCING CONTRAINTS
        P1W1+P2W1+P3W1-W1R1-W1R2-W1R3-W1R4=0
        P3W3+P4W3-W3R4-W3R5-W3R6-W3R7=0

        !NON-NEGATIVITY CONTRAINTS
        P1W1>0
        P2W1>0
        P3W1>0
        P3W3>0
        P4W3>0
        W1R1>0
        W1R2>0
        W1R3>0
        W1R4>0
        W3R4>0
        W3R5>0
        W3R6>0
        W3R7>0

END
```

Q2: no there is not a feasible solution when Warehouse 2 is eliminated unless either demand or supply is adjusted. This is because supply from the heaviest producers, P2 and P3, but especially P2, had difficulty routing to demand at R5, R6, and R7 because of lack of routes eliminated by eliminating Warehouse 2.

Part C)

Yes, it is feasible with a cost of 18,300 as seen in the following report.
The optimal solution, as seen in the report is:
ship 150 from P1 to W1
ship 350 from P2 to W1
ship 100 from P2 to W2
ship 250 from P3 to W3
ship 150 from P4 to W3
ship 100 from W1 to R1
ship 150 from W1 to R2
ship 100 from W1 to R3
ship 150 from W1 to R4
ship 50 from W2 to R4
ship 50 from W2 to R5
ship 150 from W3 to R5
ship 150 from W3 to R6
ship 100 from W3 to R7

```
 LP OPTIMUM FOUND AT STEP     15

        OBJECTIVE FUNCTION VALUE

        1)      18300.00

  VARIABLE        VALUE          REDUCED COST
      P1W1       150.000000          0.000000
      P1W2         0.000000          8.000000
      P2W1       350.000000          0.000000
      P2W2       100.000000          0.000000
      P3W1         0.000000          4.000000
      P3W2         0.000000          2.000000
      P3W3       250.000000          0.000000
      P4W2         0.000000          9.000000
      P4W3       150.000000          0.000000
      W1R1       100.000000          0.000000
      W1R2       150.000000          0.000000
      W1R3       100.000000          0.000000
      W1R4       150.000000          0.000000
      W2R3         0.000000          7.000000
      W2R4        50.000000          0.000000
      W2R5        50.000000          0.000000
      W2R6         0.000000          4.000000
      W3R4         0.000000          4.000000
      W3R5       150.000000          0.000000
```

```
     W3R6       150.000000          0.000000
     W3R7       100.000000          0.000000


      ROW   SLACK OR SURPLUS     DUAL PRICES
       2)         0.000000          1.000000
       3)         0.000000          0.000000
       4)         0.000000          2.000000
       5)         0.000000          3.000000
       6)         0.000000        -16.000000
       7)         0.000000        -17.000000
       8)         0.000000        -18.000000
       9)         0.000000        -21.000000
      10)         0.000000        -23.000000
      11)         0.000000        -23.000000
      12)         0.000000        -17.000000
      13)         0.000000        -11.000000
      14)         0.000000        -13.000000
      15)         0.000000        -11.000000
      16)         0.000000          5.000000
      17)       150.000000          0.000000
      18)         0.000000          0.000000
      19)       350.000000          0.000000
      20)       100.000000          0.000000
      21)         0.000000          0.000000
      22)         0.000000          0.000000
      23)       250.000000          0.000000
      24)         0.000000          0.000000
      25)       150.000000          0.000000
      26)       100.000000          0.000000
      27)       150.000000          0.000000
      28)       100.000000          0.000000
      29)       150.000000          0.000000
      30)         0.000000          0.000000
      31)        50.000000          0.000000
      32)        50.000000          0.000000
      33)         0.000000          0.000000
      34)         0.000000          0.000000
      35)       150.000000          0.000000
      36)       150.000000          0.000000
      37)       100.000000          0.000000

 NO. ITERATIONS=      15


 RANGES IN WHICH THE BASIS IS UNCHANGED:

                           OBJ COEFFICIENT RANGES
```

VARIABLE	CURRENT COEF	ALLOWABLE INCREASE	ALLOWABLE DECREASE
P1W1	10.000000	1.000000	INFINITY
P1W2	15.000000	INFINITY	8.000000
P2W1	11.000000	INFINITY	1.000000
P2W2	8.000000	2.000000	INFINITY
P3W1	13.000000	INFINITY	4.000000
P3W2	8.000000	INFINITY	2.000000
P3W3	9.000000	2.000000	INFINITY
P4W2	14.000000	INFINITY	9.000000
P4W3	8.000000	3.000000	INFINITY
W1R1	5.000000	INFINITY	16.000000
W1R2	6.000000	INFINITY	17.000000
W1R3	7.000000	7.000000	18.000000
W1R4	10.000000	INFINITY	2.000000
W2R3	12.000000	INFINITY	7.000000
W2R4	8.000000	2.000000	INFINITY
W2R5	10.000000	4.000000	2.000000
W2R6	14.000000	INFINITY	4.000000
W3R4	14.000000	INFINITY	4.000000
W3R5	12.000000	2.000000	4.000000
W3R6	12.000000	4.000000	23.000000
W3R7	6.000000	INFINITY	17.000000

RIGHTHAND SIDE RANGES

ROW	CURRENT RHS	ALLOWABLE INCREASE	ALLOWABLE DECREASE
2	150.000000	350.000000	0.000000
3	450.000000	INFINITY	0.000000
4	250.000000	50.000000	0.000000
5	150.000000	50.000000	0.000000
6	100.000000	0.000000	100.000000
7	150.000000	0.000000	150.000000
8	100.000000	0.000000	100.000000
9	200.000000	0.000000	150.000000
10	200.000000	0.000000	50.000000
11	150.000000	0.000000	50.000000
12	100.000000	0.000000	50.000000
13	0.000000	0.000000	350.000000
14	0.000000	0.000000	150.000000
15	0.000000	0.000000	50.000000
16	100.000000	150.000000	50.000000
17	0.000000	150.000000	INFINITY
18	0.000000	0.000000	INFINITY
19	0.000000	350.000000	INFINITY
20	0.000000	100.000000	INFINITY
21	0.000000	0.000000	INFINITY
22	0.000000	0.000000	INFINITY

23	0.000000	250.000000	INFINITY
24	0.000000	0.000000	INFINITY
25	0.000000	150.000000	INFINITY
26	0.000000	100.000000	INFINITY
27	0.000000	150.000000	INFINITY
28	0.000000	100.000000	INFINITY
29	0.000000	150.000000	INFINITY
30	0.000000	0.000000	INFINITY
31	0.000000	50.000000	INFINITY
32	0.000000	50.000000	INFINITY
33	0.000000	0.000000	INFINITY
34	0.000000	0.000000	INFINITY
35	0.000000	150.000000	INFINITY
36	0.000000	150.000000	INFINITY
37	0.000000	100.000000	INFINITY

This is the LINDO program:

```
 MIN    10P1W1  +15P1W2         +
        11P2W1  +8P2W2          +
        13P3W1  +8P3W2  +9P3W3  +
                14P4W2  +8P4W3  +

        5W1R1   +6W1R2  +7W1R3  +10W1R4 +
                        12W2R3  +8W2R4  +10W2R5 +14W2R6 +
                                14W3R4  +12W3R5 +12W3R6 +6W3R7

ST
        !SUPPLY CONTRAINTS
        P1W1+P1W2<150
        P2W1+P2W2<450
        P3W1+P3W2+P3W3<250
        P4W2+P4W3<150

        !DEMAND CONSTRAINTS
        W1R1>100
        W1R2>150
        W1R3+W2R3>100
        W1R4+W2R4+W3R4>200
        W2R5+W3R5>200
        W2R6+W3R6>150
        W3R7>100

        !DISTRIBUTION BALANCING CONTRAINTS
        P1W1+P2W1+P3W1-W1R1-W1R2-W1R3-W1R4=0
        P1W2+P2W2+P3W2+P4W2-W2R3-W2R4-W2R5-W2R6=0
```

```
P3W3+P4W3-W3R4-W3R5-W3R6-W3R7=0

!LIMIT WAREHOUSE 2
P1W2+P2W2+P3W2+P4W2<100

!NON-NEGATIVITY CONTRAINTS
P1W1>0
P1W2>0
P2W1>0
P2W2>0
P3W1>0
P3W2>0
P3W3>0
P4W2>0
P4W3>0
W1R1>0
W1R2>0
W1R3>0
W1R4>0
W2R3>0
W2R4>0
W2R5>0
W2R6>0
W3R4>0
W3R5>0
W3R6>0
W3R7>0

END6
```

Part D)

This is the generalized solution.
For the objective function we must have $\sum_{u,v \epsilon E} a(u,v)f_{u,v}$ where (u,v) is an edge in the set of edges, and $a(u,v)f_{u,v}$ is the cost for the edge.
We must also have the following constraints:

Supply constraints needed are each an edge in the graph from supply vertices to warehouse vertices. Let us denote supply vertices as u and warehouse vertices as w and the graph (half of the graph E) as S and with s_u being the supply amount associated with source u.
Then we have $(\sum_{u,w \epsilon S} f_{u,w}) < s_u$ for each $u \epsilon S$

Demand constraints needed are each an edge in the graph from warehouse vertices to demand vertices. Let us denote warehouse vertices as warehouse vertices as w and demand vertices as v and the graph (half of the graph E) as D and with d_v being the demand amount associated with retailer v.
Then we have $(\sum_{w,v \epsilon D} f_{w,v}) > d_v$ for each $v \epsilon D$

Non-negative supply constraints needed are of the form $f_{u,w} > 0$ for each $u, w \epsilon S$.

Also we need non-negative demand constraints of the form $f_{w,v} > 0$ for each $w, v \epsilon D$.

Distrbution flow constraints: we need to insure that distribution flows through the warehouses are balanced–that inflows match outflows. Therefore we need constraints of the form $(\sum_{u,w\epsilon S} f_{u,w}) - (\sum_{w,v\epsilon D} f_{w,v}) = 0$ for each $u\epsilon S$, $v\epsilon D$ which is connecting through $w\epsilon E$

Question 2

To solve both questions, I wrote a single Mathematica script. It is included at the end of this section.

Part A - Minimum Calories

item	amount (100g)
tomato	5.21×10^{-8}
lettuce	5.85×10^{-1}
spinach	9.15×10^{-8}
carrot	2.60×10^{-8}
sunflower	3.08×10^{-9}
tofu	8.78×10^{-1}
chickpea	9.98×10^{-9}
oil	1.44×10^{-9}
Total calories	114.75
Total cost	$2.33

Part B - Minimum Cost

item	amount (100g)
tomato	4.41×10^{-7}
lettuce	1.12×10^{-6}
spinach	0.832
carrot	1.08×10^{-6}
sunflower	0.096
tofu	1.51×10^{-6}
chickpea	1.152
oil	8.78×10^{-8}
Total calories:	278.49
Total cost	$1.55

Part C

As we can see from question A and B's results, the lowest calorie salad and the cheapest salad, vary significantly. If Veronica wants to sell the lower calorie salad, and make a 3 dollar profit, she'll have to make a few changes.

Out most expensive piece on the menu is smoked tofu. Additionally, it's one of the highest of both sodium and protein. By reducing tofu, and increasing spinach and chickpeas, we can keep a similar calorie range, while reducing the budget.

Code

```
cost[tomato_, lettuce_, spinach_, carrot_, sunflower_, tofu_, chickpea_, oil_] :=
1*tomato + .75*lettuce + .5*spinach + .5*carrot +
.45*sunflower + 2.15*tofu + .95*chickpea + 2*oil

energy[tomato_, lettuce_, spinach_, carrot_, sunflower_, tofu_, chickpea_, oil_] :=
21*tomato + 16*lettuce +
40*spinach + 41*carrot + 585*sunflower + 120*tofu + 164*chickpea + 884*oil

protein[tomato_, lettuce_, spinach_, carrot_, sunflower_, tofu_, chickpea_, oil_] :=
.85*tomato + 1.62*lettuce +
2.86*spinach + 0.93*carrot + 23.4*sunflower + 16.00*tofu + 9.0*chickpea + 0*oil

fat[tomato_, lettuce_, spinach_, carrot_, sunflower_, tofu_, chickpea_, oil_] :=
0.33*tomato + 0.20*lettuce + 0.39*spinach + 0.24*carrot + 48.7*sunflower +
5*tofu + 2.6*chickpea + 100.00*oil

carbs[tomato_, lettuce_, spinach_, carrot_, sunflower_, tofu_, chickpea_, oil_] :=
4.64*tomato + 2.37*lettuce + 3.63*spinach + 9.58*carrot + 15.00*sunflower +
3.00*tofu + 27.0*chickpea + 0*oil

sodium[tomato_, lettuce_, spinach_, carrot_, sunflower_, tofu_, chickpea_, oil_] :=
.009*tomato + .028*lettuce + .065*spinach + .069*carrot + .0038*sunflower +
.120*tofu + .078*chickpea + 0*oil

dcal = Minimize[{energy[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil],
    tomato >= 0
    && lettuce >= 0
    && spinach >= 0
    && carrot >= 0
    && sunflower >= 0
    && tofu >= 0
    && chickpea >= 0
    && oil >= 0
    && protein[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] >= 15
    && 2 <= fat[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] <= 8
    && sodium[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] <= .2
    && carbs[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] >= 4
    && ((lettuce + spinach)/(tomato+lettuce+spinach+carrot+sunflower+tofu+chickpea+oil)) >= .40
    },
    {tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil}
]

dcost = Minimize[{cost[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil],
    tomato >= 0
    && lettuce >= 0
```

```
    && spinach >= 0
    && carrot >= 0
    && sunflower >= 0
    && tofu >= 0
    && chickpea >= 0
    && oil >= 0
    && protein[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] >= 15
    && 2 <= fat[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] <= 8
    && sodium[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] <= .2
    && carbs[tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil] >= 4
    && ((lettuce + spinach)/(tomato+lettuce+spinach+carrot+sunflower+tofu+chickpea+oil)) >= .40
    },
    {tomato,lettuce,spinach,carrot,sunflower,tofu,chickpea,oil}
]

Export["cal.csv", dcal, "csv"]
Export["cost.csv", dcost, "csv"]
```

1 Part A

One alternative to the least squares line is the Least Absolute Deviations (LAD). Formulate a linear program whose optimal solution minimizes the sum of the absolute deviations of the data from the line. That is formulate

$$min \sum_{i=1}^{n} |y_i - (a_1 x_i + a_0)|$$

as an LP and solve for the a_0 and a_1 that minimize the sum of absolute deviations.

1.1 i: Write the linear program for the general problem written as an objective and set of constraints

The goal is to minimize $min \sum_{i=1}^{n} |y_i - (a_1 x_i + a_0)|$. In order to create an objective, we drop the sum and set it equal to z_i for all values $i = 1, ..., n$. We can reduce that by dropping the absolute values and set it as an inequality.

$$-z_i \leq y_i - (a_1 x_i + a_0) \leq z_i$$

After that it gets simplified down to the following objectives and constraints.

$$y_i - (a_1 x_i + a_0) \leq z_i \text{ for all values } i = 1, ..., n$$

$$y_i - (a_1 x_i + a_0) \geq -z_i \text{ for all values } i = 1, ..., n$$

1.2 ii: Use the linear program to find the LAD regression line for the data set $(x, y) = (1,5), (1,3), (2,13), (3,8), (4,10), (5,14), (6,18)$ What was the sum of absolute deviations?

The absolute deviation is calculated by taking the least squares values for y and finding the difference between that and the calculated actual value of y using the data. See the chart below. The trendline has an equation of $y = 2.315x + 2.875$

Table 1: Part A (ii)

x	y: Data Points	Trendline	Differences	Squared
1	5	3.93	1.07	1.15
1	3	3.93	0.93	0.87
2	13	5.99	7.01	49
3	8	8.07	0.07	0.01
4	10	10.14	0.14	0.02
5	14	12.21	1.79	3.2
6	18	14.29	3.72	13.84

Based on the chart above, the sum of the absolute deviations is 14.73.

1.3 iii: Plot the points and graph your LAD line and the least squares line. Comment.

The value for point 2 appears to be an outlier. The value of the data point at x = 2 falls outside the line of best fit the most.

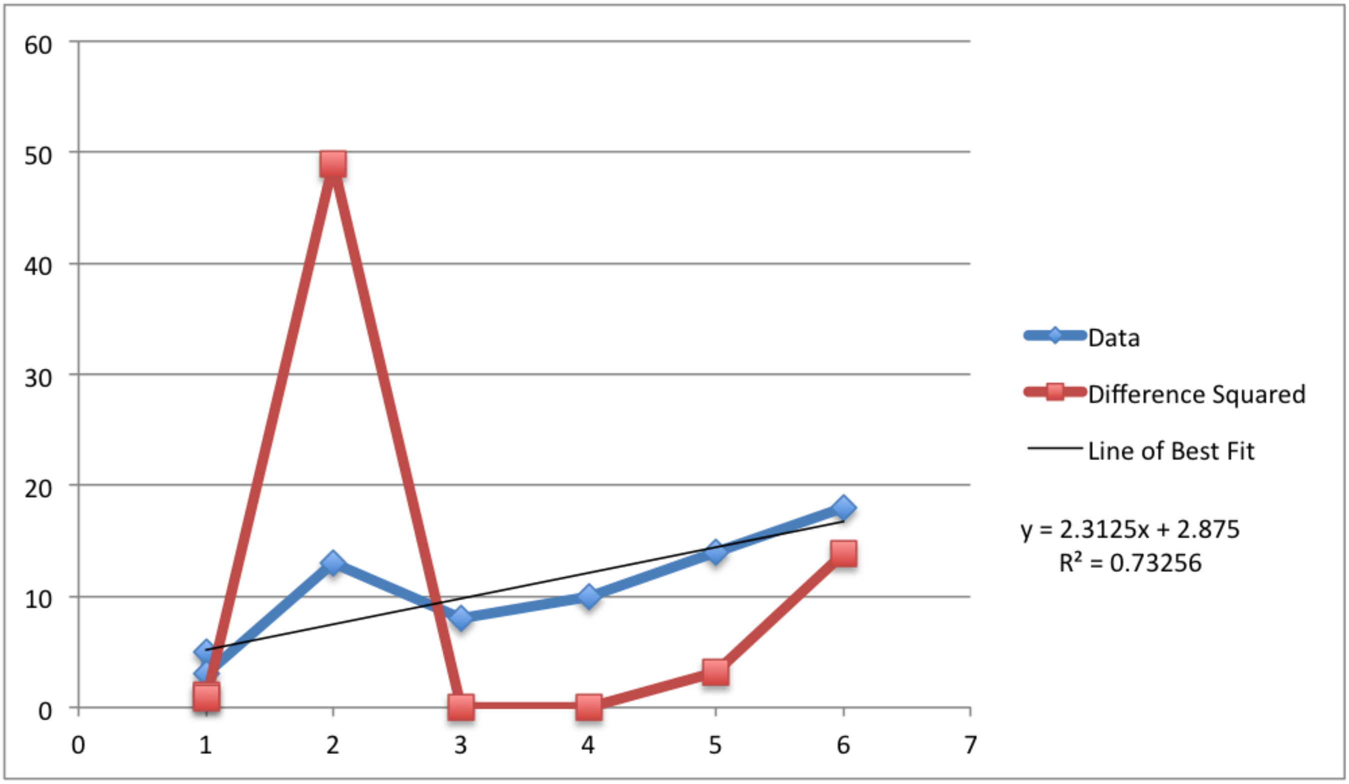

Figure 1: example caption

2 Part B

Another alternative to the least squares method is to find a line that minimizes of the maximum absolute deviation (MMAD). That is formulate

$$min\ max_i\ |y_i - (a_1x_i + a_0)|$$

as an LP.

2.1 i: Write the linear program for the general problem written as an objective and set of constraints

Following the same procedures as in Part A, set the equation equal to z and try to minimize z for all values $i = 1, ..., n$. The resulting equations are:

$$y_i - (a_1x_i + a_0) \geq z_i \text{ for all values } i = 1, ..., n$$

$$y_i - (a_1x_i + a_0) \leq -z_i \text{ for all values } i = 1, ..., n$$

It is important to note that these are opposite of the solutions as found in part A.

2.2 ii: Use the linear program to find the MMAD regression line for the data set $(x, y) = (1,5), (1,3), (2,13), (3,8), (4,10), (5,14), (6,18)$ What was the min of the max absolute deviations?

Minimize z subject to $z \geq max_i\ |y_i - (a_1x_i + a_0)|$

2.3 iii: Plot the points and graph the MMAD line and the least squares line. Compare.

2.4 iv: Can you create a data set for which all three methods of regression (least squares, LAD, MMAD) compute the same line.

The only set that could have the same result is the empty set or a set of zero values. All three methods use different methodologies to calculate the line of best fit. They all use either minimization, maximization, or a combination there of, and as such, there will be minute differences between the calculations of the regression.

3 Part C

Multiple Linear Regression. Generalize the simple linear regression model to allow for two independent variables (x_1 and x_2). ?? =??$_2$??$_2$+??$_1$??$_1$+??$_0$, The model is called multiple linear not because the result is a line but because all variables are 1st degree. Extend the techniques from Part A to find the least absolute deviations regression equation. Use linear programming to fit a LAD multiple linear regression model to the data set below:

x_1	x_2	y
1	1	5
1	2	9
2	2	12
0	1	3
0	0	0
1	3	11

Solving for a_0, a_1, and a_2 using a system of equations and the values in the table above. Using the above values, a_0 must be 0. It is the only way x_1 and x_2 could be zero if y is 0. The result is that a_2 equals 3. The final value, a_1, is 2 or 3 depending on the data points you use to calculate them. Using LAD, we minimize the values. making $a_1 = 2$.
The final estimation is $y = 3 \times x_2 + 2 \times x_1$.