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Program: FindClosestPoints.java

To find theoretical worst-case running time this algorithm needs to be split up into three separate parts: quick sort, merge sort, and finding closest pair.

Quick sort theoretical worst case happens when one of the two subarrays are empty and the size of the other array is 1 less than the size of the subarray being partitioned. This forms $C_{worst}(n) = (n+1) + n + \cdots + 3 = \frac{(n+1)(n+2)}{2} - 3$, which makes $C_{worst}(n) \in \Theta(n^2)$.

Merge sort theoretical worst case can be found from $C_{worst}(n) =$

$$\begin{cases} 2C_{worst}\left(\frac{n}{2}\right) + n - 1, for\ n > 1 \\ \quad\quad 0\ , for\ n = 1 \end{cases}$$

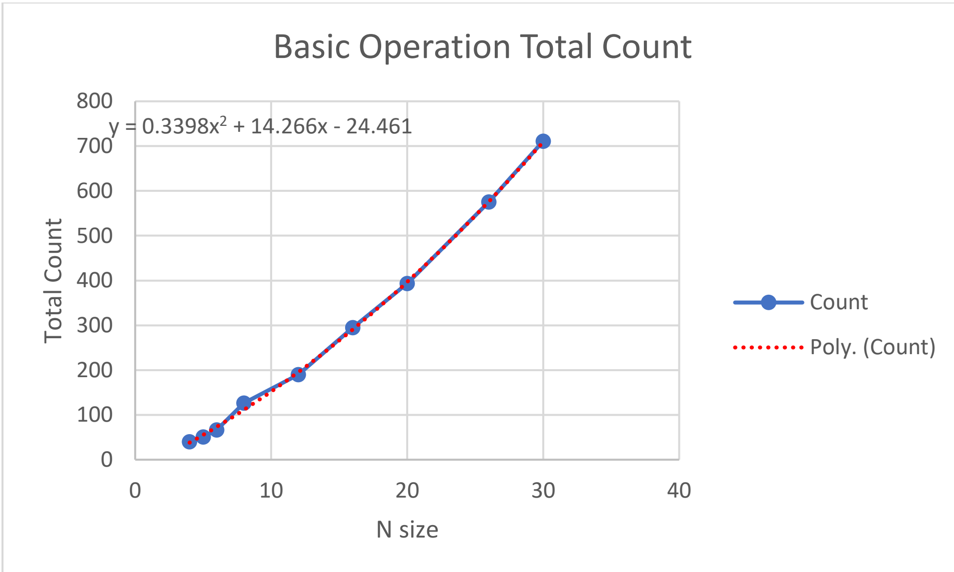
The 2C(n/2) comes from the recursive call in the to separate the points, while the n-1 comes from the merging back together. Using the master theorem C_{worst} = n*logn – n -1 ∈ Θ(n logn)

The closet pair theoretical worst case can be found using the master theorem as well. To find the class of efficiency of f(n), the operations before and after the recurrence relation need to be examined. The two sorting are shown above. The splitting of each array takes ∈Θ(n) time. The only other operation is the checking of the strip. This worst case could take $\Theta(n^2)$, however it is found that realistically only 5 points will be on this line so the class is in Θ(nlogn). This gives you T(n) = 2T(n/2) + f(n), if $n = 2^k$. $f(n) \in\Theta(n^2)$, due to the worst cause of the pre and post algorithm being $\in \Theta\ (n^2)$. This means $t(n) \in \Theta(n^2)$.
Due to Quick sort having a worst case of $\Theta(n^2)$. This algorithm will run at a **$C(n) \in \Theta(n^2)$**.

The following is the basic operation count for the algorithm. The number of points is the input size. The basic operations for each algorithm were counted, along with array copies that were needed for those operations.

points	basic operation count
4	40
5	51
6	66
8	126
12	190
16	295
20	393
26	575
30	711

Graphing the count results and applying a line a best fit proves that the class of efficiency is $\in\Theta(n^2)$.


Basic Operation Total Count
y = 0.3398x2 + 14.266x - 24.461
Total Count
N size
0
100
200
300
400
500
600
700
800
0
10
20
30
40
Count
Poly. (Count)