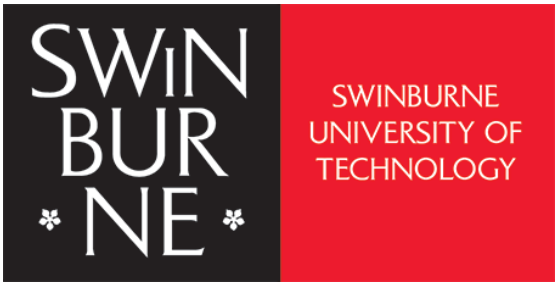
Tree-Based Search

Robot Navigation Problem



Jacob Milligan
100660682

Swinburne University of Technology
COS30019 - Introduction to AI

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Software Libraries and Code Repository

The code-base is version controlled via git and can be accessed on github and uses several external libraries. These include:

- Catch - A header-only unit-testing framework for C++
- SDL2 - Platform abstraction media library used in the visualizer for opening a window, handling input and rendering graphics
- SDL2_ttf - TrueType rendering library extension for SDL2 for drawing text
- Skyrocket.Path - The platform agnostic file-system component from my game framework, Skyrocket, for navigating the file-system independent of the operating system being used.

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1 Introduction

Tree and graph-based search algorithms are frequently used AI-related problem domains. This report explores six such search methods and their completeness, optimality, and efficiency as applied to the Robot Navigation Problem (RNP), a Task environment in which a simulated robot attempts to find an optimal route to a given *goal* node in an $N \times M$ grid of nodes, where $N = width$, $M = height$, both of which are non-zero. To demonstrate the properties of each search method, two research components were conducted - an interactive GUI visualizer showing how each method interacts with its search tree, and an analysis of their efficiency in terms of run-time and memory usage and how it relates to the use case for each method.

Glossary of Terms The acronyms and terminology referenced here as they relate to graph and tree based search are defined and described for the reader in section 9, **glossaries**. Links to these entries are found throughout the report, highlighted blue for reference.

2 Search Algorithms

Each of the following search methods were tested using a combination of static and dynamic tests as outlined in subsection 5.2.1, with that data used in this section.

2.1 Breadth-First Search (BFS)

Breadth-First search is an uninformed search method that uses a First-In, First-Out queue structure (FIFO) as its frontier, resulting in the shallowest node encountered expanded first. As a consequence of this expansion method, BFS is complete in all spaces as eventually every node in the search space will be expanded. Furthermore, BFS is technically optimal when the step cost of a node is non-decreasing and equivalent to it's depth (Russell and Norvig 2010, p. 82). However, while BFS is both complete and optimal it can be inefficient in terms of both memory and time. As BFS eventually analyses every node in the search space, larger grids can increase its memory consumption much more than other algorithms, as demonstrated in figure 1. Furthermore, it's runtime increases proportional the time it takes to scan it's list of neighbors (Cormen et al. 2009, p. 597), which will always be the amount of nodes that exist at depth d extending from the start, minus the ones explored.

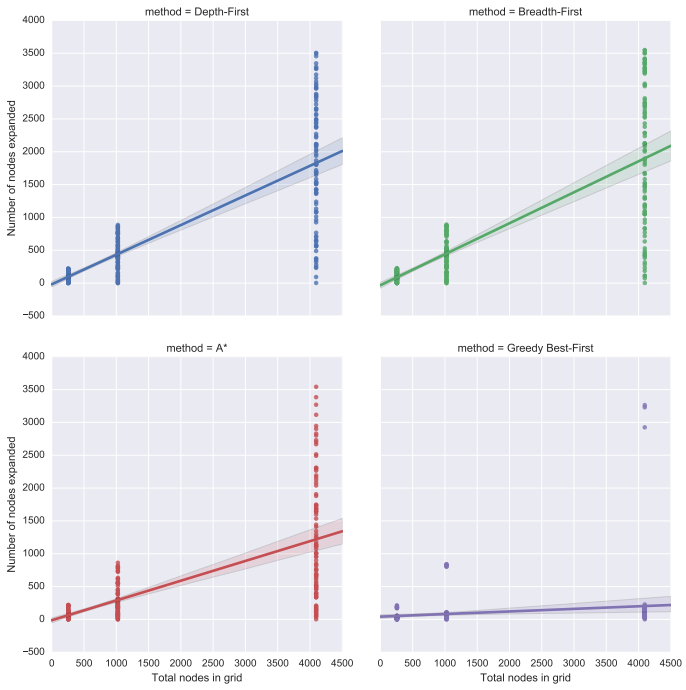


Figure 1: Linear relationship between size of RNP grid and the number of nodes expanded for each search algorithm

2.2 Depth-First Search (DFS)

Depth-First search is an uninformed search method that uses a Last-In, First-Out queue structure (LIFO) as its frontier. It will always follow the last-encountered node until it hits either a leaf node (a node with no children) or the goal. In theory this poses obvious advantages over BFS, namely in terms of its memory usage - as it only ever contains the current path and each of the path nodes immediate neighbors. However, DFS is non-optimal - for example, if it chooses to expand a node to its left (based a pre-defined order of expansion), it will follow that path even if the goal was located one node to the right. In the worst case, this could result in DFS following the longest possible path possible rather than a shorter alternative. Figure 2 highlights this problem, showing the difference in distribution of path lengths between BFS and DFS across 300 randomly generated grids. Furthermore, DFS is only complete in graph-based searches whereas the tree-based version is *incomplete* (Russell and Norvig 2010, p. 86). Both of these factors result in large runtime consequences, especially as the search space increases in size as demonstrated in section 5.2.

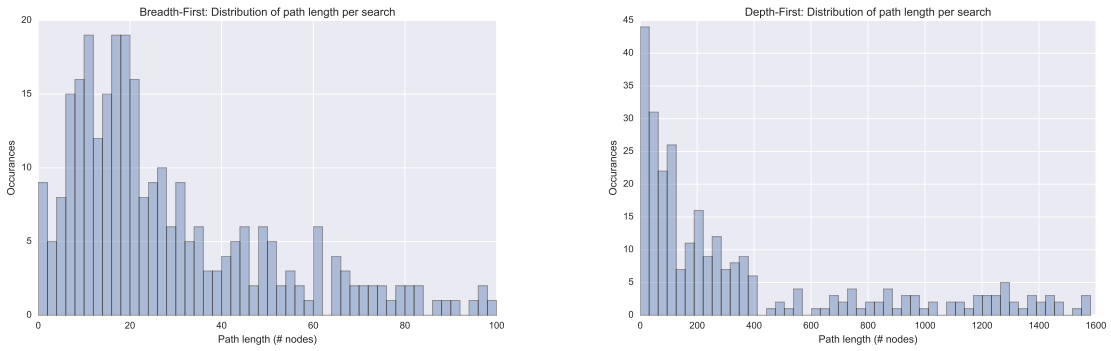
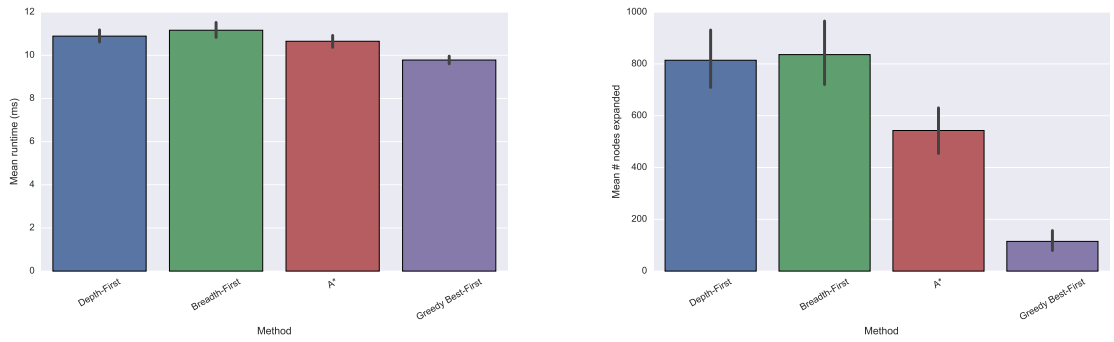


Figure 2: Comparison between the distributions of total path length for Breadth-First Search and Depth-First Search

2.3 Greedy Best-First Search (GBFS)

Greedy Best-First is an *informed* search method which, like all informed methods outlined in this report, uses a priority queue frontier ordered by lowest f value. GBFS expands whatever node in its frontier that is closest to the goal first, therefore it's evaluation is $f(n) = h(n)$. However, GBFS is not optimal nor is it complete (Jones 2008), this is because its heuristic is not *admissible*, often overestimating the quality of a node, for example in scenarios when the goal is behind a wall but unreachable, the node closest will be evaluated as the best neighbor when it isn't. However while not strictly optimal, often GBFS can return a *close to optimal path* while expanding less nodes in a faster time than other search methods as demonstrated in figure 3.



(a) Mean run-times

(b) Mean nodes expanded

Figure 3: The mean number of nodes expanded and mean run-time for the non-custom search methods

2.4 A* Search (A*)

A* is an informed search search method that stores its frontier in a priority queue structure sorted by best node first, as determined by the nodes *evaluation function* $f(n)$ which evaluates the cost of reaching a given node n. For A*, f is defined as the cost to reach n from the starting node along the current path - $g(n)$ and its heuristic function $h(n)$ which provides an estimation of the cost of reaching the goal. A requirement for A* to be correctly implemented is that h be admissible, that is it never overestimates the quality of a path or node. In the context of the RNP, as the environment is a 2D grid, an admissible heuristic can be achieved by simply measuring the distance from the current node using either euclidean distance, calculated by $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$, or using a 'city-block' distance (the amount of nodes traversed without diagonal movements) referred to as *manhattan* distance, calculated by $|x_1 - x_2| + |y_1 - y_2|$. A* is both complete and optimal (Hart, Nilsson, and Raphael 1968, p. 104), as well as efficient in terms of both time and space complexity. For this reason it's used as a comparison for all subsequent search methods described here.

2.5 Iterative-Deepening Depth-First Search (IDDFS)(CUS1)

Iterative Deepening Depth-First search is an improved version of regular depth-first search uninformed search which executes DFS until either the goal is encountered or some depth-limit ℓ is reached, signaling a cutoff upon which the search tree is cleared, increasing ℓ by 1 and running the algorithm again, running iterations of DFS until the goal is found. This method aims to combine the best qualities of DFS and BFS, taking the limited space requirements of DFS and combining it with the completeness and optimality of BFS.

2.6 Iterative-Deepening A* Search (IDA*)(CUS2)

Iterative Deepening A* is a search method that attempt to improve the memory requirements of A*. It is similar to IDDFS in its methodology, except for what is used as ℓ for the current iteration. A cutoff is reached when the current nodes f cost exceeds the current ℓ value, upon which ℓ is increased to be equal to that nodes f, the search tree cleared and another iteration begun. In a sample of 300 randomly

generated grids of differing sizes, as outlined in section 5.2, IDA* cut down the mean amount of nodes expanded by approximately 50 nodes.

However, while memory requirements are improved, on graph-based searches such as the RNP IDA*'s tendency to analyze the same state for different depths in a standard implementation showed an increase of approximately $2ms$ per search in the same tests. However, it's important to note that the implementation described in section 3 does not make as much effort to track repeated states as it could and IDA* is technically as efficient as A* given the right implementation (Korf 1985).

3 Implementation

3.1 Search Methods

Each search method is a derived form of the base `SearchMethod` class, implementing the `search()` and `get_child()` virtual methods.

BFS BFS was implemented iteratively, using a FIFO frontier, its pseudocode shown below.

Algorithm 1 BFS

```
1:  procedure SEARCH(env: a problem environment)
2:      n ← starting state, explored ← {}, frontier ← empty FIFO queue
3:      if n = goal then return solution
4:      add n to frontier
5:      while frontier ≠ empty do
6:          n ← first node in frontier
7:          for all valid child c with action a in env do
8:              if not explored then
9:                  add c to explored set
10:                 if c = goal then return solution
11:                 add to frontier
12:     return no solution found
```

DFS DFS is implemented in a very similar way to the pseudocode outlined above for BFS, however it uses a LIFO queue as it's frontier instead of a FIFO queue.

GBFS and A* GBFS and A* are again both implemented similarily to BFS and DFS but using a a priority queue as their frontier. Furthermore, when getting a valid child from the environment, both add their f cost to the child node before adding it to the frontier, to do this they override `SearchMethods`'s `get_child()` method. GBFS will simply assign as its f cost the distance to the goal using one of the distance functions outlined in section 2.4, while A* will sum both the distance and the child nodes parent f value to get $f(n) = g(n) + h(n)$.

IDDFS IDDFS is implemented recursively in the following manner using a specialized `IDDFSResults` struct wrapping a cutoff value and the solution if it exists in a single structure.

Algorithm 2 IDDFS implementation pseudocode

```
1:  procedure SEARCH(env: a problem environment)
2:      results ← empty results set, ℓ ← 1
3:      while results.cutoff do
4:          results ← DEPTH_LIMITED_SEARCH(env, ℓ)
5:          ℓ ← ℓ + 1
6:      return results
7:
8:  procedure DEPTH_LIMITED_SEARCH(env: a problem environment, ℓ)
9:      n ← starting node in env
10:     return RECURSIVE_DLS(n, env, ℓ)
11:
```

```
12: procedure RECURSIVE_DLS(env: problem environment, n: a node. d: the current depth)
13:     Add n to explored set
14:     if n = goal then return solution
15:     if d < 1 then return cutoff
16:     cutoff_occurred ← false, results ← empty results set
17:     for all children c of n with valid actions a in env do
18:         if c not in explored set then
19:             results ← RECURSIVE_DLS(c, env, d − 1)
20:             if results.cutoff then cutoff_occurred ← true
21:             else if solution found in results then return results
22:     return cutoff_occurred
```

IDA* IDA* is implemented almost identically to to IDDFS, however it keeps a temporary map of the current path to ensure that nodes in the same path aren't repeated, alongside using the last encountered f value higher than the current ℓ as the cutoff.

3.2 Frontier

Each search method utilized a different frontier container type. This is provided via the `Frontier<T>` template type which has FIFO (`Frontier<std::queue>`), LIFO (`Frontier<std::vector>`), and Priority Queue (`Frontier<std::priority_queue>`) template specializations which implement their methods in a container-specific way whilst providing a common interface for search methods.

3.3 Explored Set

The `ExploredSet` class provides an interface for adding and querying expanded nodes and denoting them as *explored*. This is implemented internally using an `std::unordered_map` and several helper functions with a high-level interface for dealing with nodes directly. The explored set also contains an `std::vector` of all the operations that occurred during the search algorithm for use in the visualizer (see 5.1) to use in displaying the search tree in order.

3.4 Environment

Finally, the environment class defines valid actions and generates new child nodes based on the current *state* - a point in the grid. It also defines a *goal test* for determining whether a given state is the goal.

4 Features and Known Bugs

4.1 CLI application

The CLI reads in test files from a relative path from the executables directory. Alongside the positional arguments required by the assignment spec, the CLI application also defines the following option flags:

- `-h | --help` - Displays a help message describing available commands
- `-v | --visualizer` - Opens the GUI visualizer
- `-s | --stats` - Displays extra stats about the algorithm such as largest frontier

The following search methods are available in both the CLI and the GUI visualizer:

- `BFS` - Breadth-First Search (BFS)
- `DFS` - Depth-First Search (DFS)
- `GBFS` - Greedy Best-First Search (GBFS)
- `AS` - A* Search (A*)
- `CUS1` - Iterative-Deepening Depth-First Search (IDDFS)
- `CUS2` - Iterative-Deepening A* Search (IDA*)

4.2 Known issues

- **A*** - A known bug exists where every so often A* will expand a few nodes more than it should, however this doesn't affect the completeness or optimality of the implementation
- **IDA*** - While the implementation here expands less nodes than A*, it does so by keeping track of explored nodes. This shouldn't be necessary for IDA* to work, however without that feature the algorithm seemed to take far too long to complete.
- **IDDFS** - IDDFS sometimes takes a very long time on large grids - an issue I was unable to resolve.

5 Research

The research component can be split into two components - the GUI **visualizer** and the search method **analysis** component.

5.1 Visualizer

To be able to both verify the behavior of each search algorithm and to demonstrate to others how each method affects its search tree, a GUI visualizer was developed as a research component. It can be opened via the same CLI application as the assignment component using the `./robonav -v` option. This will open the visualizer app - a grid with cells representing states in the search space - black cells are walls, the red cell is the start state, and the green cell is the goal.

Usage Both the start and end state can be moved by clicking and dragging, while walls can be placed and removed by clicking in empty spaces. The keys **1 - 6** are used to choose each search algorithm, **space** clears the current path and **enter** begins a search. To quit press the **escape** key.

Visuals The visualization of each search algorithm is not in real-time but in a time proportional to the amount of operations each one made. The visualizer displays each operation as it happened in order as denoted by the moving orange cell (the current node in the search tree), explored nodes are denoted as light blue which are added as the search tree expands. Finally, once the goal is found, the path taken to reach it is highlighted green.

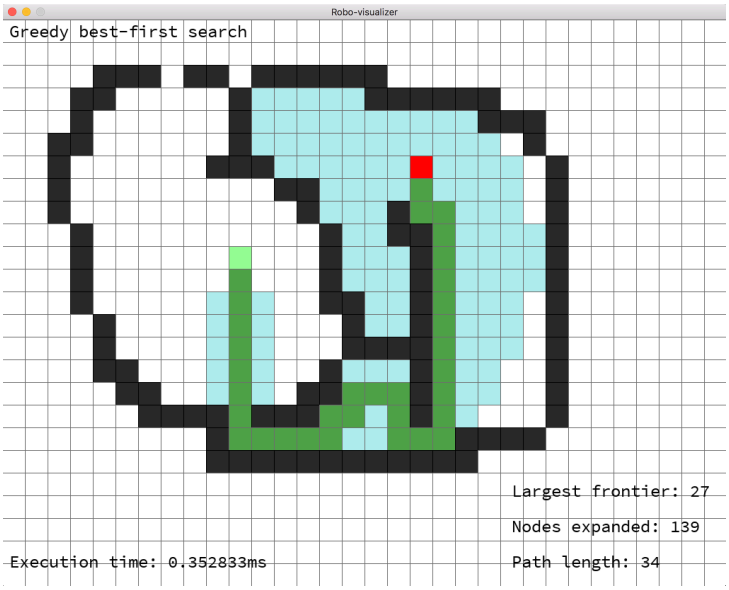


Figure 4: Screenshot of the GUI pathfinding visualizer

5.2 Search Method Analysis

While each search method has been discussed at a high level, this section aims to find the most effective search algorithm for the RNP. A method for random-generation of search problems with expected optimal paths was developed for the purpose of testing each search method across a variety of environments, with their time and memory usage recorded alongside the optimality of each solution.

5.2.1 Methodology

100 randomly generated search spaces were generated for grids of sizes 16×16, 32×32, and 64×64, 300 samples in total. Each search method was made to solve the sample recording their average run-time, number of nodes expanded, and other data of interest. Run-time and nodes expanded were chosen as data points for analysis as they demonstrate the time and space complexity of each search method, while path length was chosen to demonstrate optimality.

Search space generation Each cell in a grid was iterated over in a random order, with walls having a 15% chance of being placed in each cell. Starting and ending positions were placed randomly, ensuring that no cells collided. As A* has been demonstrated to be both *optimal* and *complete*, with the implementations correctness having been proven through static tests as outlined in on several pathfinding benchmarks, A* was used to calculate the optimal path as a benchmark for all subsequent tests.

5.2.2 Results

As outlined in table 1, A* and Greedy Best-First had the lowest mean run-times ($\mu = 10.65$, $\mu = 9.78$ respectively), while both iterative deepening methods had the highest. Furthermore the higher run-times had the highest standard deviations ($\sigma = 142.93$ and $\sigma = 6.22$).

	Run-time (ms)		
	Mean	Median	Std. Dev
method			
A*	10.65	9.97	2.46
Breadth-First	11.16	10.14	2.91
Depth-First	10.89	10.20	2.42
Greedy Best-First	9.78	9.56	1.54
Iterative-Deepening A*	13.18	11.04	6.22
Iterative-Deepening Depth-First	74.41	16.37	142.93

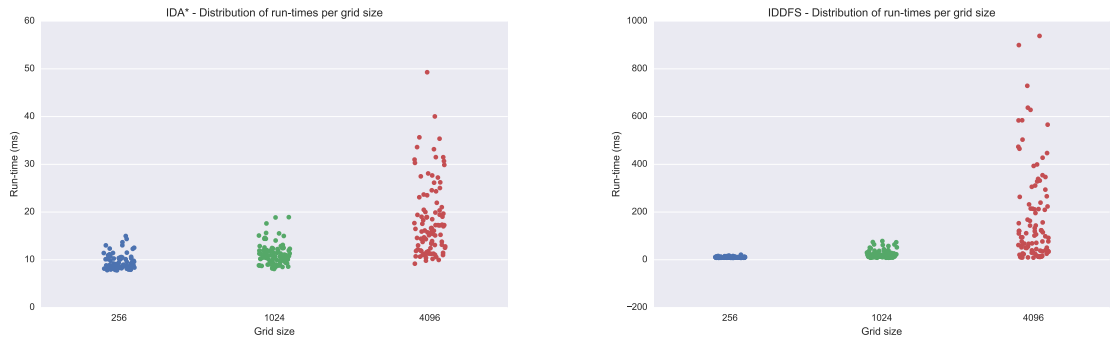
Table 1: Averages and standard deviation for the run-time of each search method on all grids

Table 2 describes the average memory usage for each method, where GBFS and IDA* expanded the lowest number of nodes on average ($\mu = 114.76$ and $\mu = 511.21$), while IDDFS expanded the highest number ($\mu = 19493.45$), with the highest standard deviation ($\sigma = 36364.81$).

	Number of nodes expanded		
	Mean	Median	Std. Dev
method			
A*	542.93	200.5	751.61
Breadth-First	835.93	397.0	1020.19
Depth-First	814.13	397.0	950.98
Greedy Best-First	114.76	49.0	336.36
Iterative-Deepening A*	511.21	177.0	736.04
Iterative-Deepening Depth-First	19493.45	2910.0	36364.81

Table 2: Averages and standard deviation for the amount of nodes expanded for each search method on all grids

Figure 5 shows the distribution of run-times for different grid sizes for both IDA* and IDDFS, demonstrating the locations of outliers for each method.



(a) IDA* (b) IDDFS

Figure 5: Distribution of run-times for each grid size for both IDA* and IDDFS

Each search method had the same average path length ($\mu = 27.94$), aside from GBFS which was slightly higher ($\mu = 30.05$). DFS is excluded from the results here as its average path length ($\mu = 372.64$) was, as expected, too large of an outlier to be of much use for comparisons. Figure 6 shows the linear relationship between the number of walls present in a grid and the number of nodes expanded per search. Both IDA* and IDDF are excluded from these results as neither method showed a significant relationship between these variables. Finally, while the number of solutions was recorded for each search method, the mean number of solutions found was the same for all methods ($\mu = 92.47$).

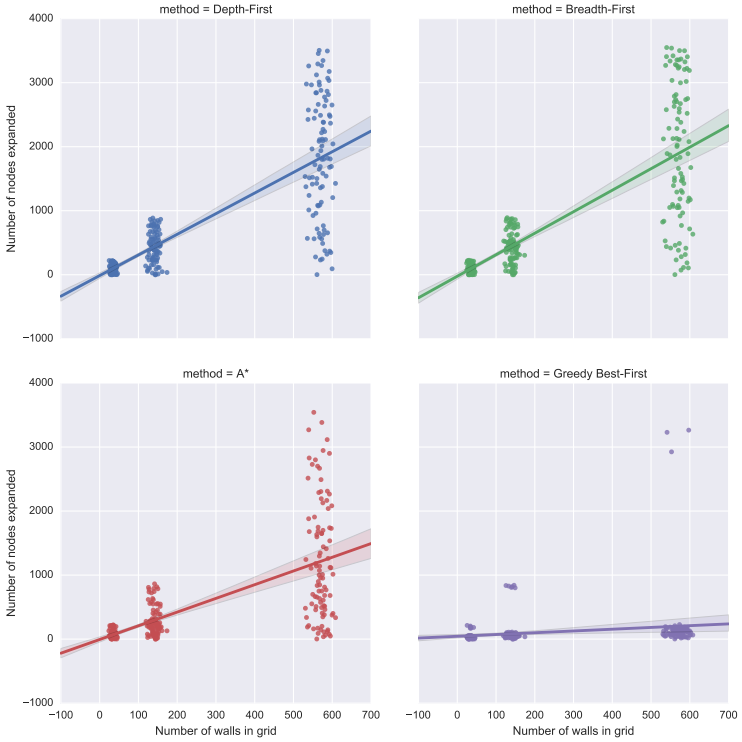


Figure 6: Linear relationship between number of walls in a grid and the number of nodes expanded

5.2.3 Discussion

As expected (discussed in section 2) each search method excluding DFS and GBFS proved *optimal* in all problems in which a solution existed. However GBFS, while non-optimal, had an average path length that proved not much longer than the optimal one with a lower standard deviation, finding solutions faster and expanding far fewer nodes than other methods. This indicates that, although it may not always find the optimal solution for the RNP, it reliably finds a *good-enough* solution, using much less resources in doing so and could be possibly suited for problems where any kind of kind of useful path, not necessarily the optimal one, is required in a short period of time such.

Of the **optimal** search methods, A* proved to have the most reliably efficient run-time whilst still remaining modest in terms of memory usage, demonstrating its usefulness as a general purpose search algorithm. Of note, is that IDA* expanded less nodes on average while still maintaining a modest run-time, this could be of use for computing environments with limited memory, however for environments where memory is plentiful the trade off in run-time may be too great to make IDA* worth utilizing. Finally, of the custom methods, IDDFS proved to be an outlier with both the average amount of nodes expanded and its average run-time quite high. However, as highlighted in figure 5, IDDFS had a few very high outliers on larger grids but showed otherwise to show a similar distribution to other methods which could be the cause of the high averages. This indicate that while IDDFS is certainly optimal and can be more efficient than other uninformed methods its high run-time σ may prove it to be too unreliable for problems such as the RNP and a simpler method such as BFS may be of better use.

6 Conclusion

6.1 Improvements

The slower run-time performance of IDDFS could see some improvements by storing a list of the nodes currently existing in the given path, allowing it to avoid circular paths. One particular area of interest for the performance of each informed search method would be to investigate different heuristics - in the implementation outlined here, manhattan distance was used for calculating each methods heuristic, however other methods exist that could affect how each one expands the nodes in their search tree and in what order.

6.2 Final Thoughts

Overall, for the RNP, A* is demonstrably the most complete, optimal, and efficient general purpose search algorithm, proving to have the lowest mean run-time and lowest number of nodes expanded amongst the methods considered optimal. For memory-limited environments, however IDA* may be of use where run-time is perhaps less important. Finally, amongst the *uninformed* search methods, BFS showed to be more effective over both DFS and IDDFS.

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Acronyms

A* A* Search.

BFS Breadth-First Search.

DFS Depth-First Search.

FIFO First-In, First-Out queue structure.

GBFS Greedy Best-First Search.

IDA* Iterative-Deepening A* Search.

IDDFS Iterative-Deepening Depth-First Search.

LIFO Last-In, First-Out queue structure.

Glossary

admissible An admissible heuristic is one that never *over-estimates* the cost to get to the goal, meaning it's estimation is **never** higher that the **lowest** possible cost to get to the goal.

complete A search algorithm is called **complete** when it's guaranteed to find a solution as long as one exists.

heuristic function A function, denoted as $h(n)$ which returns an *estimation* of the quality of a specific node n.

informed search Synonymous with *heuristic search*. A search algorithm that has domain-specific knowledge of the search space, such as the distance from the start or end states, that define a heuristic function function $f(n)$ for evaluating the quality of a given node n.

node A generalized element of different types data structures that represents a single value or object of interest. It will usually contain a reference or pointer to it's parent or adjacent nodes.

optimal A search algorithm is called **optimal** when it is guaranteed to find the solution with the lowest path cost that exists each time it's executed.

RNP The **Robot Navigation Problem (RNP)** is a Task environment in which a simulated robot attempts to find an optimal route to a given *goal* node in an $N \times M$ grid of nodes, where $N = width$, $M = height$ and both are non-zero.

task environment In the context of a search problem, the *task environment* is the space and problem for which an agent attempts to find a solution.