Velocity-Dependence of Kinetic Friction

Jacob O'Bryant
Department of Physics, Edmonds Community College, Lynnwood WA 98036[*]
(Dated: 7 June 2013)

I present experimental work on the velocity-dependence of dry surface contact kinetic friction under constant-speed linear motion at relative surface speeds between $0.10\,\mathrm{cm\,s^{-1}}$ and $12.0\,\mathrm{cm\,s^{-1}}$. I find that the standard Coulomb model of velocity-independent friction between surfaces is incorrect over this domain and that the force of sliding kinetic friction is instead described by a power-law dependence on velocity.

PACS numbers: 62.20.Qp,07.10.Pz
Keywords: PHYS243SP13;kinetic friction;standard model;dependencies;normal force;velocity

I. INTRODUCTION

Some of the earliest formal experiments on sliding friction were conducted by Leonardo da Vinci. Using an apparatus that involved a block tied to a weight through a pulley, he concluded that the force of friction F_{friction} for smooth surfaces is equal to $\frac{1}{4}N$,[1] where N is the normal force. More exact experiments later showed that the coefficient of N varied depending on the materials of the sliding objects. Charles Coulomb would eventually conduct many more extensive experiments, and we now know the equation

$$F_{\text{friction}} = \mu N \tag{1}$$

as Coulomb's law of friction. In a culmination of a year-long study of friction, an attempt is made to discover deviations from this model.

Coulomb's law is the standard model of friction as taught in textbooks; however, it is known to be just a rough approximation in certain situations. While it was previously shown that the force of friction is independent of the sliding velocity as Coulomb's law implies, these experiments only covered a velocity range of 1 to $4\,\mathrm{m\,s^{-1}}$.[4] Theoretical analysis indicates that for low velocities, F_{friction} follows a logarithmic model with respect to velocity and that for very low velocities, the relationship is linear.[2] It is thus hypothesized that by studying kinetic friction at velocities of under $1\,\mathrm{m\,s^{-1}}$, this logarithmic behavior will be observed.

In addition, previous experimentation has indicated that the relationship between F_{friction} and N may not be quite linear. Based on this, it is hypothesized that F_{friction} is not linearly proportional to N but instead follows a power or exponential model.

II. EXPERIMENT

Previous experiments have calculated the force of friction by sending a block into motion and then measuring the acceleration.[3,4] Unfortunately, it was sometimes difficult to make accurate calculations due to the dynamic nature of the experiments. In order to remedy this, an apparatus was constructed that keeps the block stationary and directly measures F_{friction}. This also makes it convenient to place weights on top of the block so that readings can be taken for different values of N.

A. Apparatus

1. Materials

- Thin, black plastic
- Several text books
- 2 wooden dowels
- 4 large rubber washers
- Duct tape
- Bendable wire
- Metal poles and clamps
- Force sensor and LabQuest console
- Fishing line
- Wooden block
- Motor and variable power source
- Section of rubber tube
- Hose clamp
- Weights
- Paper ruler

* me@jacobobryant.com

2. Construction

The basic function of the apparatus is to create a conveyor belt of sorts for the block to rest on. As the belt moves, friction acts between it and the block. The block was attached to the force meter to hold it in place and to measure F_{friction}.

The apparatus was constructed on top of a desk. First, the motor was attached to a pole on one of the tripod bases. The purpose of the motor is to rotate one of the wooden dowels which causes the belt to move. A short piece of rubber tubing was placed over the protruding end of the motor and the wooden dowel. A hose clamp fastened the tubing to the motor. The other end of the dowel was suspended in a loop of wire attached to the back of the desk with duct tape. The dowel was rotated at variable speeds by switching on the motor's power source.

FIG. 1. An early version of the apparatus.

Thin, nonelastic fabric was initially used as the material for the belt (see figure 1), but the edges constantly frayed and caught which made experimentation infeasible. A thin sheet of stiff plastic, cut to 18 cm by 3 m, replaced the fabric. Both ends of the strip of plastic were attached to the dowels with duct tape.

The other dowel was placed about half a meter away from the first. The table clamp was fastened onto the desk with the pole sticking up. A metal clamp fastened onto the pole acted as a holder for one end of the second dowel. The other end was placed in a loop of wire at the back of the desk in a similar manner to the first dowel. At this point the belt could be spooled around one of the dowels by turning on the motor.

Several text books were stacked underneath the belt to provide a surface for the block to rest on. They were stacked slightly higher than the dowels to make the belt taut. The force meter was placed above the second dowel by clamping it to the pole of the second tripod base which was placed just behind the dowel. The block was placed on the stack of textbooks and was attached to the force meter with the fishing line. Bowline knots were used to create loops in either end of the line. The rubber washers were placed on the wooden dowels to keep the belt spooling up in the same position instead of sliding off to the side.

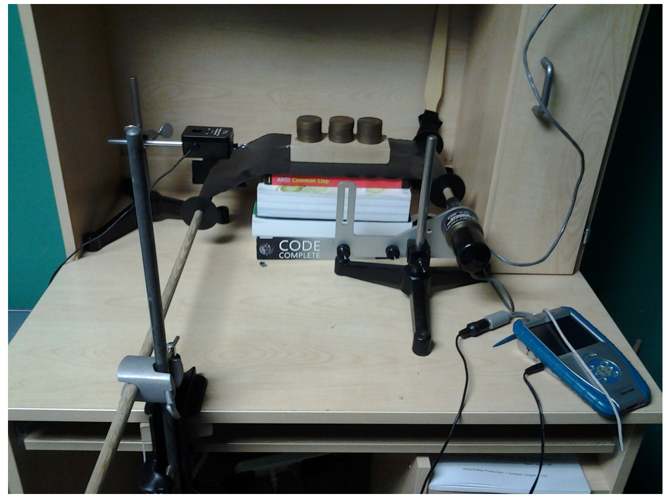

FIG. 2. The completed apparatus.

In order to measure the sliding velocity, a piece of paper with marks every centimeter was taped onto the text book next to the belt. Small pieces of tape were placed on the belt every 10 cm, so the velocity could be calculated by measuring the time it takes for the tape to travel across the paper ruler.

B. Procedure

First, the belt must be spooled around the dowel not attached to the motor. By reversing the wires connected from the motor to the power source, the direction in which the motor spins can be changed. As the motor spins counterclockwise, the left dowel can be easily rotated manually by spinning it with one's fingers. After the belt has been completely spooled around the left dowel, the wires in the power source are reversed and motor is turned on. This makes the other dowel rotate and begin to pull the belt across the text books which pulls the block away from the force meter until the end of the fishing line is reached. The force of friction will then be directly measured by the force meter. The sliding velocity can be adjusted by varying the power input level, and weights can easily be placed on top of the block to get different values of N. Before taking each new run of data, the conveyor belt must be reset to ensure that the block is sliding over the same section of plastic.

III. DATA AND ANALYSIS

First, data was collected for ten different values of N. Three runs were taken for the block without any extra weight, then three more runs were taken for each 100 gram weight added for a total of 30 runs. Figure 3 shows the results. The data points were calculated by taking the average value of F_{friction} for each run.

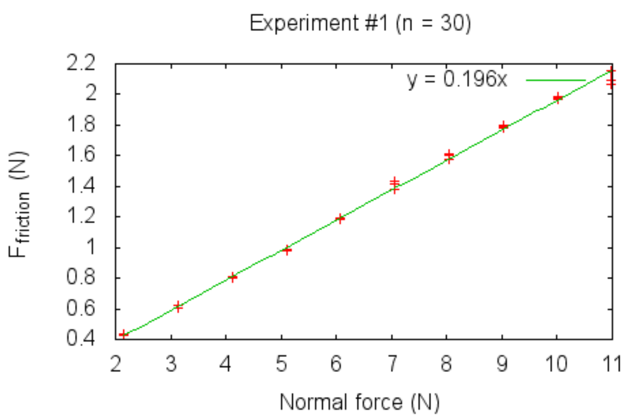

FIG. 3. F_{friction} vs. N

The data show a very strong proportionality between F_{friction} and N, so the hypothesis that the Coulomb model is inaccurate in this respect can be dismissed.

More data was collected in the same manner, but this time the normal force was kept constant and the sliding velocity was increased after every three runs. 41 runs were taken in all.

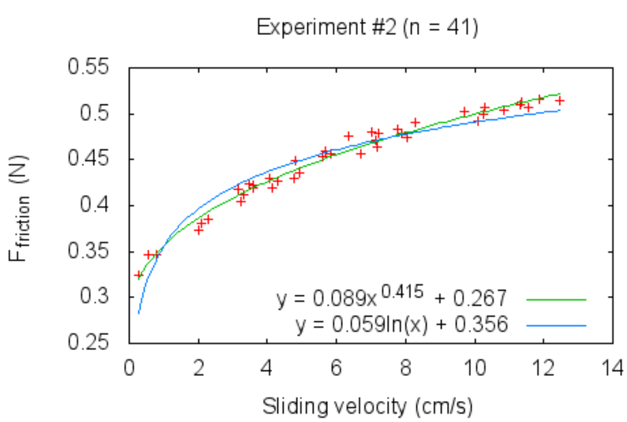

FIG. 4. F_{friction} vs. velocity

It is quite apparent that for this domain, friction does exhibit a significant dependence on velocity. Two best-fit curves are shown: a logarithmic model and a power model. Although Braun[2] models friction with a logarithmic curve, the power-law curve appears to be more accurate. It is possible that the domain over which the data were collected extends farther than the domain over which the logarithmic model applies, so perhaps fitting a narrower subset of the data with a logarithmic curve would yield a better fit.

IV. CONCLUSIONS

As hypothesized, there is a clear dependence of kinetic dry friction on the sliding velocity. There is no deviation from Coulomb's law in regard to the normal force; however, this was only of minor interest.

V. FUTURE WORK

Confirming that the velocity dependence exists is only the first step in analyzing the inaccuracies of Coulomb's law. First, more data need to be collected over a larger domain. Examining larger velocities would indicate where the behavior of friction switches from following a logarithmic model to being velocity-independent. Taking data over smaller velocities could indicate where the linear model comes into effect. The apparatus would have to be modified to be capable of producing sliding velocities outside the domain studied in this paper.

Second, further analysis of the theoretical model needs to be done. In order to actually predict what F_{friction} should be given sliding velocity, certain parameters about the system need to be estimated. This could be a difficult task, especially since the model itself is quite complex. But after doing so, the relationship between friction and temperature could be studied because the theoretical model takes that into account as well.

In addition, a better apparatus would greatly facilitate more data collection. The current apparatus has a major drawback in that it must be reset before each run. A looped belt would allow the apparatus to run continuously. This approach would introduce some challenges, for instance, how to prevent the belt from slipping across the dowels (or whatever else is used to rotate the belt) since it could no longer be duct taped in place.

ACKNOWLEDGMENTS

JO is thankful to Tom Fleming for his help in the initial design of the apparatus and in writing the abstract. He is also thankful to Mike O'Bryant for his ideas about further development of the apparatus.

Appendix A: Source Files

Several Python scripts were used to help analyze the data. splitdata.py takes the data from Vernier Logger Lite (exported to a plain text file) and outputs the actual numerical data only to separate files, one run per file. The analyze_n.py scripts were then used to read the data and output the data for F_{friction} with respect to N and velocity, respectively. datainput.py contains the code, common to both analyze_n.py scripts, used for reading the data outputted from splitdata.py. The graph_n.sh files contain the gnuplot scripts used to generate figures 3 and 4.

1. splitdata.py

```
#!/usr/bin/python3
'''
Splits the data from a LoggerLite data file (exported as a plain text file)
into separate files, stripping the non-numerical header information.
'''
from argparse import ArgumentParser
import os
import sys

HEADER_LINES = 7

parser = ArgumentParser(description="Split data into multiple files")
parser.add_argument("infile", help="the input file")
parser.add_argument("outdir", help="the output directory")
args = parser.parse_args()

if not os.path.exists(args.outdir):
    os.makedirs(args.outdir)
with open(args.infile, "r") as infile:
    # The first element is header stuff.
    runs = infile.read().split("\n\n")[1:]

i = 0
width = len(str(len(runs)))
for run in runs:
    if len(run) == 0:
        sys.stderr.write("omitting empty run: " + str(i) + "\n")
        continue
    outfile = "{0}/{1:0{2}}.dat".format(args.outdir, i, width)
    with open(outfile, "w") as outfile:
        outfile.write("\n".join(run.split("\n")[:-HEADER_LINES]) + "\n")
    i += 1
```

2. dataio.py

```
#!/usr/bin/python3
import os

def mean_force():
    '''Returns a list of the average forces of each run. The data directory
    must be in the current directory.
    '''
    n = len(os.listdir('data'))
    width = len(str(n))
    timeStart = 0.5
    timeLength = 5
    data = []
    for i in range(n):
```

```
        infile = open("data/{0:0{1}}.dat".format(i, width), 'r')
        average = 0
        k = 0
        for line in infile:
            t = float(line.split()[0])
            if t < timeStart:
                continue
            elif t >= timeStart + timeLength:
                break
            k += 1
            frictionForce = float(line.split()[1])
            average = average * ((k - 1) / k) + frictionForce / k
        infile.close()
        data.append(average)
    return data
```

3. analyze_1.py

```
#!/usr/bin/python3
'''Outputs the force of friction with respect to the normal force to stdout.'''
import os
import dataio

frictionForce = dataio.mean_force()
g = 9.80665
mass = 0.21987
massIncrement = 0.1
runsPerMass = 3
data = []
for i in range(len(frictionForce)):
    normalForce = g * (mass + massIncrement * (i // runsPerMass))
    print(str(normalForce).ljust(20) + str(frictionForce[i]))
```

4. graph_1.sh

```
#!/bin/bash
if [ -f fit.log ]; then
    rm fit.log
fi
gnuplot << EOF
set term png enhanced size 480,320
set style data points
set output 'friction_vs_load.png'
set title 'Experiment #1 (n = 30)'
set xlabel 'Normal force (N)'
set ylabel 'F_{friction} (N)'
f(x) = a * x
fit f(x) 'data.dat' using 1:2 via a
plot 'data.dat' using 1:2 title '', \
    f(x) title sprintf("y = %.3fx", a)
EOF
```

5. analyze_2.py

```
#!/usr/bin/python3
'''Outputs the force of friction with respect to sliding velocity to stdout.'''
import os
import dataio

with open('velocity.txt', 'r') as infile:
    # velocity.txt contains the time it took for a point on the belt to travel
    # a certain distance.  The distance is given by the the first column in
    # lines that have two columns, and it applies to all the subsequent columns
    # until a new distance is specified.
    velocities = []
    for line in infile:
        words = [float(word) for word in line.split()]
        if len(words) == 2:
            distance, time = words
        else:
            time, = words
        velocities.append(distance / time)

frictionForce = dataio.mean_force()
for v, f in zip(velocities, frictionForce):
    print(str(v).ljust(20) + str(f))
```

6. graph_2.sh

```
#!/bin/bash
if [ -f fit.log ]; then
    rm fit.log
fi
gnuplot << EOF
set term png enhanced size 480,320
set style data points
set key bottom
set output 'friction_vs_velocity.png'
set title 'Experiment #2 (n = 41)'
set xlabel 'Sliding velocity (cm/s)'
set ylabel 'F_{friction} (N)'
f(x) = a * x ** b + c
g(x) = d * log(x) + e
fit f(x) 'data.dat' using 1:2 via a, b, c
fit g(x) 'data.dat' using 1:2 via d, e
plot 'data.dat' using 1:2 title '', \
    f(x) title sprintf("y = %.3fx^{%.3f} + %.3f", a, b, c), \
    g(x) title sprintf("y = %.3fln(x) + %.3f", d, e)
EOF
```


[1] Persson, B. N. J.. "Historical Note." *Sliding friction: physical principles and applications*. Berlin: Springer, 1998. 11. Print.

[2] Braun, O., and M. Peyrard. *Dependence Of Kinetic Friction On Velocity: Master Equation Approach.* Physical Review E 83.4 (2011): 4. Web.

[3] O'Bryant, Jacob. *Verifying the Standard Model of Kinetic Friction.* `http://jacobobryant.com/stuff/papers.html` 2012.

[4] O'Bryant, Jacob. *Dependencies of Kinetic Friction.* `http://jacobobryant.com/stuff/papers.html`. 2013.