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# CME Homework Assignment 2 Computational Work

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## (20pts) Computation Team Work:

Now consider the sensor localization problem on plane $R^2$ with two sensors $x_1$ and $x_2$ and three anchors $a_1 = (1; 0)$, $a_2 = (-1; 0)$ and $a_3 = (0; 2)$. Suppose we know the (Euclidean) distances from one sensor to $a_1$ and $a_2$, denoted by $d_{11}$ and $d_{12}$; distances of the other to $a_2$ and $a_3$, denoted by $d_{22}$ and $d_{23}$; and the distance between the two sensors, denoted by $\hat{d}_{12}$. Then, from the anchor and distance information we like locate the sensor positions $x_1, x_2 \in R^2$?

Do the following numerical experimentations:

- Generate two sensor points anywhere and try the SOCP relaxation model

$$\begin{aligned} \|x_1 - a_i\|^2 &\leq d_{1i}^2,\ i = 1, 2 \\ \|x_2 - a_i\|^2 &\leq d_{1i}^2,\ i = 2, 3 \\ \|x_1 - x_2\|^2 &\leq \hat{d}_{12}^2. \end{aligned}$$

  Did you find the correct locations? What have you observed?

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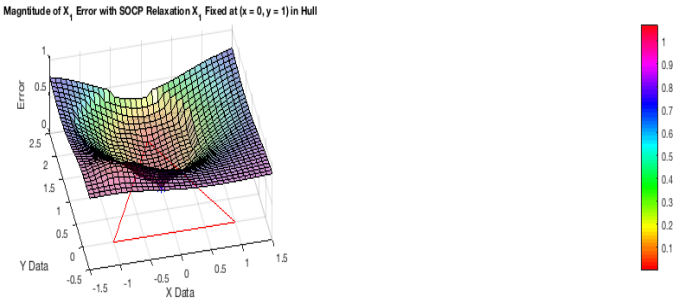
In order to examine the performance of the SOCP relaxation model for sensor localization in $\mathbb{R}^2$, lets examine some interesting possible cases. First, let us consider fixing the point $x_1$ in the center of the convex hull of the anchors. That is, let $x_1$ be fixed at $[0, 1]$, and examine the performance of the optimization for $x_2$ points both within the convex hull and outside the hull.

### $x_1$ Fixed in Center of Convex Hull

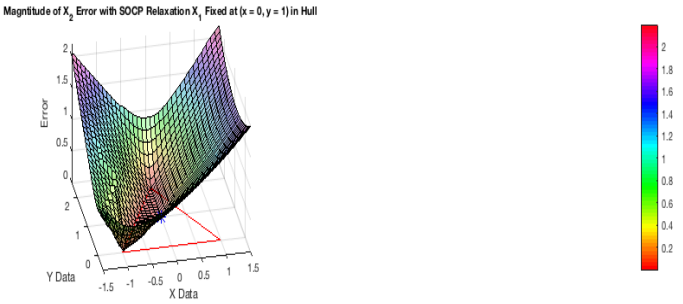
First observe that it is interesting to examine the optimization's performance in locating each of the sensors individually.

The subplots below show the errors of the SOCP relaxation for $x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $x_2$ varying across the meshgrid. The first plot is the error in locating $x_1$, the second plot $x_2$ and the third plot the total error. The blue dot indicates the fixed $x_1$ point. Recall that $x_1$ only has distance information about $a_1$ and $a_2$ whereas $x_2$ has information on $a_2, a_3$.

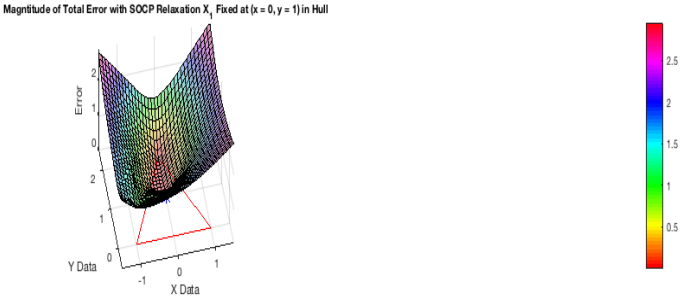
Minimum Error of (1.3464e-13) at $X_2$: (0,1.5294)




Minimum Error of (7.2371e-12) at $X_2$: (-0.088235,1.3529)



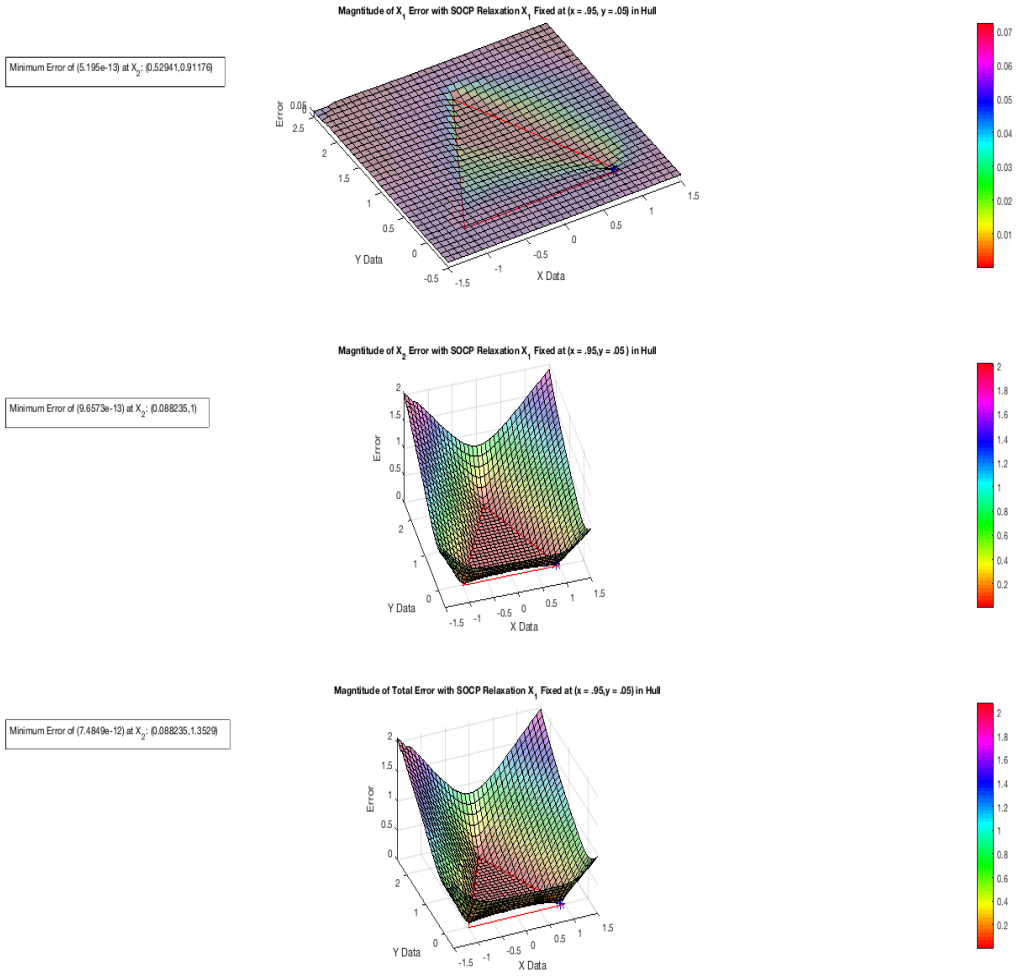

Minimum Error of (9.7393e-12) at $X_2$: (-0.088235,1.6176)




Observe that the error in locating $x_1$ when $x_2$ is close to $a_3$ is very low and increases dramatically when $x_2$ is closer to $a_1$ or $a_2$, even within the convex hull. This intuitively makes sense, since when $x_2$ is close to either anchors within the hull, it in effect is very easily to locate and could be loosely thought of as acting as an additional anchor point. When $x_2$ outside the convex hull the error is uniformly large, which is expected in SOCP as per homework 1. Now for the error in locating $x_2$, we observe an interesting result. When $x_2$ is along the vertex connecting $a_2$ to $a_3$ the optimization procedure is able to perform very well. However as $x_2$ drifts towards $a_1$ or outside the convex hull, the error increases dramatically. This is because we only have distance information for $x_2$ with $a_2, a_3$. So in effect, when $x_2$ is near $a_1$, the optimization is unable to find forces such that the total force on each anchor point is zero.

### $x_1$ **Fixed near** $a_1$

Now let us examine the performance of the optimization when $x_1$ is fixed closed to $a_1$, $x_1 = \begin{bmatrix} .95 \\ .05 \end{bmatrix}$.




The first thing to notice is that $x_1$ is uniformly easy to locate when $x_1$ is near the anchor point. Because of this, $x_1$ does not create an extra layer of uncertainty in the location of $x_2$, and thus the results for the SOCP relaxation in $\mathbb{R}^2$ with two sensors become very much similar to the problem of locating one sensor. The error is low within the convex hull and increases dramatically outside of it. This is due to the fact that when outside the convex hull, the forces acting upon the sensor points are unable to balance.

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- Now try the SDP relaxation: find $X = [x_1,\ x_2] \in R^{2\times 2}$ and

$$Z = \begin{pmatrix} I & X \\ X^T & Y \end{pmatrix} \in S^4$$

to meet the constraints in the standard form:

$$\begin{aligned}
(1;0;0;0)(1;0;0;0)^T \bullet Z &= 1,\\
(0;1;0;0)(0;1;0;0)^T \bullet Z &= 1,\\
(1;1;0;0)(1;1;0;0)^T \bullet Z &= 2,\\
(a_i;-1;0)(a_i;-1;0)^T \bullet Z &= d_{1i}^2,\ i=1,2,\\
(a_i;0;-1)(a_i;0;-1)^T \bullet Z &= d_{2i}^2,\ i=2,3,\\
(0;0;1;-1)(0;0;1;-1)^T \bullet Z &= \hat{d}_{12}^2,\\
Z &\succeq 0 \in S^4.
\end{aligned}$$

Did you find the correct locations? What have you observed? Can you conclude something?

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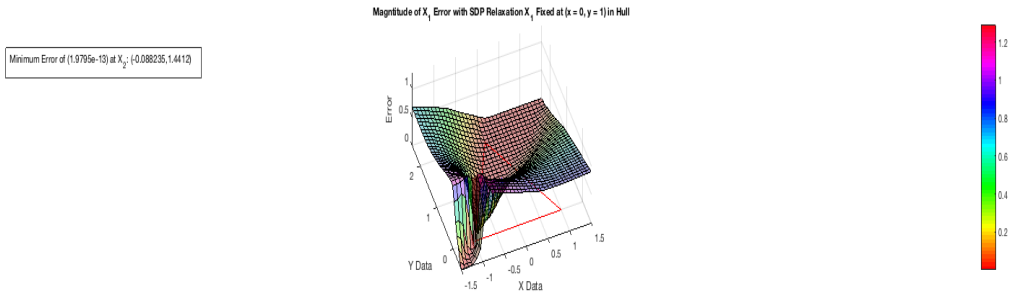
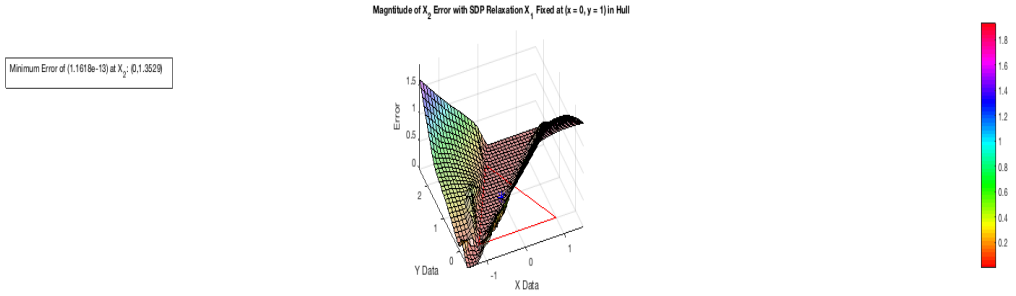
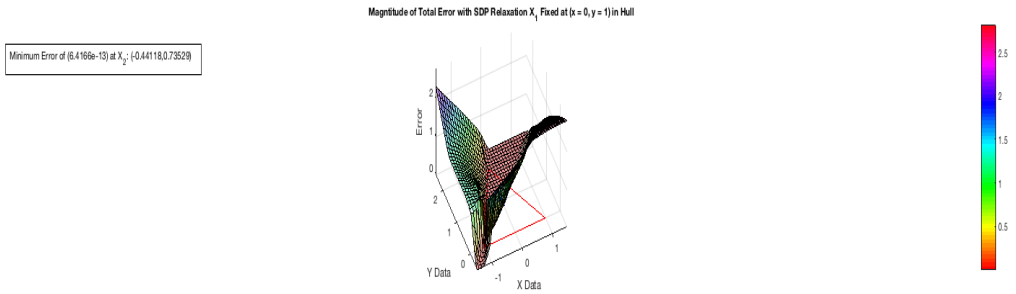
## $x_1$ Fixed in Center of Convex Hull

Lets consider the same approach as in SDP and fix $x_1$ in the center of the Hull and close to an anchor point. The

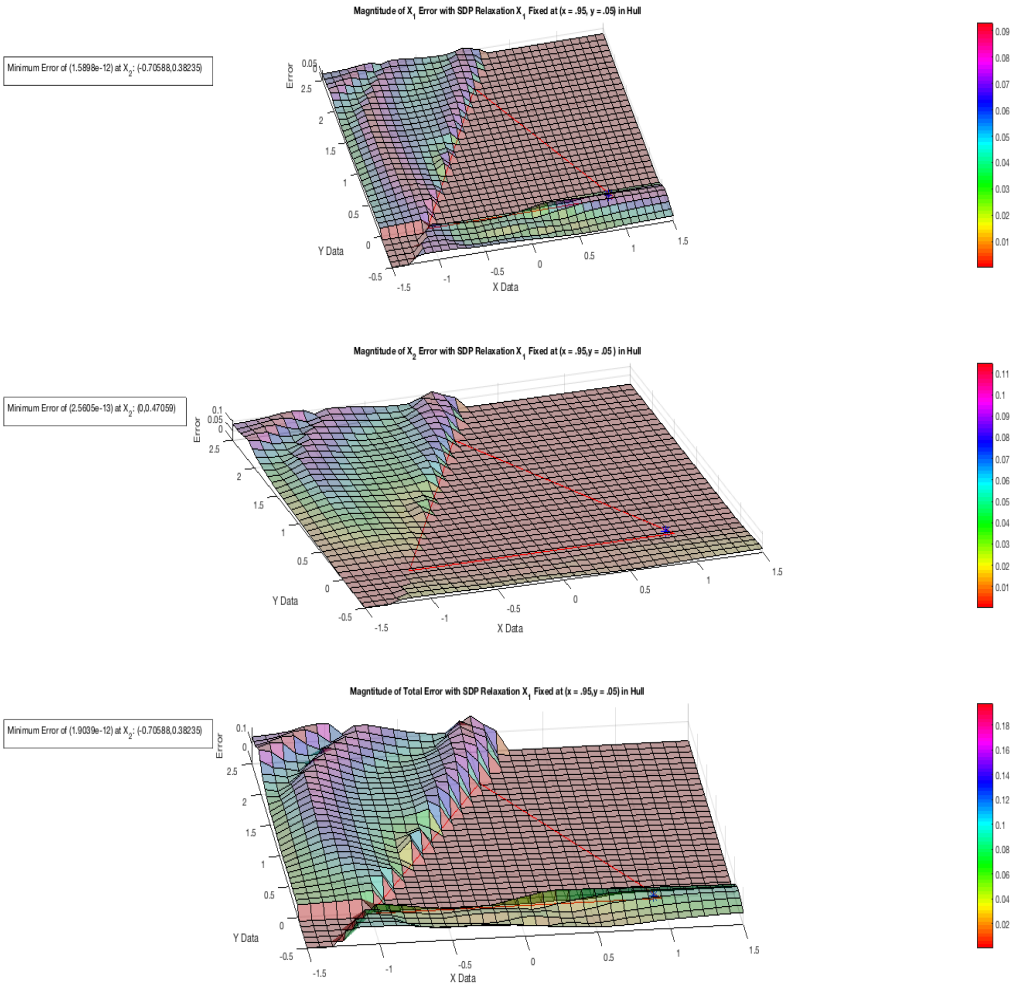


Similar to SDP, when $x_1$ is placed in the center of the convex hull, the error in locating $x_1$ decreases as $x_2$ approaches the vertex $a_2, a_3$, with the optimum $x_2$ point being very close to $x_2$ . The error increases when $x_2$ drifts outside the convex Hull, but not as dramatically as with SOCP relaxation. The error in $x_2$ similarly increases as $x_2$ drifts away from the vertex connecting $a_2$ and $a_3$. This is because the stress matrix in the SOCP relaxation is not of full rank. When the SDP has a unique solution along the vertex, the results are good, however, when $x_2$ drifts away from the vertex $a_2 \rightarrow a_3$, the U is rank deficient.

## $x_1$ Fixed near $a_1$




Now when $x_1$ is fixed near $a_1$, the solution is pretty good (both for $x_2$ and $x_1$) for $x_2$ points both in and outside of the convex hull. The error increases slightly as $x_2$ leaves the convex Hull, but not dramatically. Overall, though, the results for SDP when $x_1$ is close to $a_1$ are greatly superior to that of SOCP. Intuitively, the SDP relaxation allows for the internal forces to have two directions where SOCP only allows for one. When the stress matrix has full rank, the SDP relaxation has a larger possibility to balance the force in that

position and can thus perform better. Howevever, when the dual stress matrix in SDP does not have full rank, the points are not localizable.

```
clear all
close all


a = [1, -1, 0; 0, 0, 2];


x1 = [2*rand(25,1) + -1, 2*rand(25,1)];
x2 = [2*rand(25,1) + -1, 2*rand(25,1)];


dist_x1 = pdist2(x1, a(:,1:2)');
dist_x2 = pdist2(x2, a(:,2:3)');
dhat = pdist2(x1, x2);



%% Fix point in convex hull


%% Middle of HULL

clear all
close all


a = [1, -1, 0; 0, 0, 2];
%%

[X_1, Y_1] = meshgrid(linspace(-1.5, 1.5,35)', linspace(-.5,2.5,35)');

Cx1_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
Cx2_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
Ctotal_error_SOCP = zeros(size(X_1,1), size(Y_1,1));


Vx1_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
Vx2_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
Vtotal_error_SOCP = zeros(size(X_1,1), size(Y_1,1));



for j=1:size(X_1, 1)
    j;
    for l=1:size(Y_1,1)
        l;
        x1_center = [0,1];
        dist_x1_center = pdist2(x1_center, a(:,1:2)');

        x1_vertex = [.95,.05];
        dist_x1_vertex = pdist2(x1_vertex, a(:,1:2)');

        x2 = [X_1(j,1),Y_1(j,1)];
        dist_x2 = pdist2(x2, a(:,2:3)');

        dhat_center = pdist2(x1_center, x2);
        dhat_vertex = pdist2(x1_vertex, x2);

```

```
        cvx_begin quiet
            variables x_1(2) x_2(2)
            minimize 0
            subject to
            for i = 1:2
                norm(x_1 - a(:, i),2) <= dist_x1_center(i);
                norm(x_2 - a(:,i+1),2) <= dist_x2(i);
                norm(x_1 - x_2,2) <= dhat_center;
            end
        cvx_end

        Cx1_error_SOCP(j,1) = norm(x_1 - x1_center');
        Cx2_error_SOCP(j,1) = norm(x_2 - x2');
        Ctotal_error_SOCP(j,1) = Cx1_error_SOCP(j,1) + Cx2_error_SOCP(j,1);
        Cdistance(j,1) = dhat_center;

        cvx_begin quiet
            variables x_1(2) x_2(2)
            minimize 0
            subject to
            for i = 1:2
                norm(x_1 - a(:, i),2) <= dist_x1_vertex(i);
                norm(x_2 - a(:,i+1),2) <= dist_x2(i);
                norm(x_1 - x_2,2) <= dhat_vertex;
            end
        cvx_end

        Vx1_error_SOCP(j,1) = norm(x_1 - x1_vertex');
        Vx2_error_SOCP(j,1) = norm(x_2 - x2');
        Vtotal_error_SOCP(j,1) = Vx1_error_SOCP(j,1) + Vx2_error_SOCP(j,1);
        Vdistance(j,1) = dhat_vertex;


    end
end

%%
figure()
subplot(3,1,1)
surf(X_1, Y_1, Cx1_error_SOCP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image



 title('Magntitude of X_1 Error with SOCP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')
[mx,k] = min(Cx1_error_SOCP(:));
[ix,jx] = ind2sub(size(Cx1_error_SOCP),k);
dim = [.10 .595 .3 .3];
str = strcat('Minimum Error of (', num2str(Cx1_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), '
    ,', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
```

```
plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off


subplot(3,1,2)
surf(X_1, Y_1, Cx2_error_SOCP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image


title('Magntitude of X_2 Error with SOCP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')

[mx,k] = min(Cx2_error_SOCP(:));
[ix,jx] = ind2sub(size(Cx2_error_SOCP),k);
dim = [.10 .295 .3 .3];
str = strcat('Minimum Error of (', num2str(Cx2_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off


subplot(3,1,3)
surf(X_1, Y_1, Ctotal_error_SOCP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image



title('Magntitude of Total Error with SOCP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')

[mx,k] = min(Ctotal_error_SOCP(:));
[ix,jx] = ind2sub(size(Ctotal_error_SOCP),k);
dim = [.10 .009 .3 .3];
str = strcat('Minimum Error of (', num2str(Ctotal_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
```

```
xlabel('X Data')
ylabel('Y Data')
hold off


%
figure()
subplot(3,1,1)
surf(X_1, Y_1, Vx1_error_SOCP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image



title('Magntitude of X_1 Error with SOCP Relaxation X_1 Fixed at (x = .95, y = .05) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')



[mx,k] = min(Vx1_error_SOCP(:));
[ix,jx] = ind2sub(size(Vx1_error_SOCP),k);
dim = [.10 .595 .3 .3];
str = strcat('Minimum Error of (', num2str(Vx1_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off


subplot(3,1,2)


surf(X_1, Y_1, Vx2_error_SOCP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image


title('Magntitude of X_2 Error with SOCP Relaxation X_1 Fixed at (x = .95,y = .05 ) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')
[mx,k] = min(Vx2_error_SOCP(:));
[ix,jx] = ind2sub(size(Vx2_error_SOCP),k);
dim = [.10 .295 .3 .3];
str = strcat('Minimum Error of (', num2str(Vx2_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',', num2str(Y_1(ix,jx)),')');
```

```
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off


subplot(3,1,3)
surf(X_1, Y_1, Vtotal_error_SOCP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image


title('Magntitude of Total Error with SOCP Relaxation X_1 Fixed at (x = .95, y = .05) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')
[mx,k] = min(Vtotal_error_SOCP(:));
[ix,jx] = ind2sub(size(Vtotal_error_SOCP),k);
dim = [.1 .011 .3 .3];
str = strcat('Minimum Error of (', num2str(Vtotal_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);


hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off


%%

clear all

a = [1, -1, 0; 0, 0, 2];



%
[X_1, Y_1] = meshgrid(linspace(-1.5, 1.5,35)', linspace(-.5,2.5,35)');

Cx1_error_SDP = zeros(size(X_1,1), size(Y_1,1));
Cx2_error_SDP = zeros(size(X_1,1), size(Y_1,1));
Ctotal_error_SDP = zeros(size(X_1,1), size(Y_1,1));


Vx1_error_SDP = zeros(size(X_1,1), size(Y_1,1));
Vx2_error_SDP = zeros(size(X_1,1), size(Y_1,1));
Vtotal_error_SDP = zeros(size(X_1,1), size(Y_1,1));
```

```
A1 = [1; 0; 0; 0];    A2 = [0; 1; 0; 0];    A3 = [1; 1; 0; 0];
A = [A1, A2, A3];
a = [1, -1, 0; 0, 0, 2];
for j=1:size(X_1, 1)
    j;
    for l=1:size(Y_1,1)

        x1_center = [0,1];
        dist_x1_center = pdist2(x1_center, a(:,1:2)');

        x1_vertex = [.95,.05];
        dist_x1_vertex = pdist2(x1_vertex, a(:,1:2)');

        x2 = [X_1(j,1),Y_1(j,1)];
        dist_x2 = pdist2(x2, a(:,2:3)');

        dhat_center = pdist2(x1_center, x2);
        dhat_vertex = pdist2(x1_vertex, x2);

        cvx_begin sdp quiet
            variable Z(4,4) symmetric
            minimize(0);
            subject to

            sum(dot(A(:,1)*A(:,1)', Z)) == 1;
            sum(dot(A(:,2)*A(:,2)', Z)) == 1;
            sum(dot(A(:,3)*A(:,3)', Z)) == 2;

            for i = 1:2

                sum(dot([a(:, i); -1; 0] * [a(:, i); -1; 0]', Z)) ...
                    == dist_x1_center(i)^2;
                sum(dot([a(:, i+1); 0; -1] * [a(:, i+1); 0; -1]', Z)) ...
                    == dist_x2(i)^2;
            end

            sum(dot([0; 0; 1; -1] * [0; 0; 1; -1]', Z)) == dhat_center^2 ;

            Z >= 0;

        cvx_end

        x_1 = [Z(3,1), Z(3,2)]';
        x_2 = [Z(4,1), Z(4,2)]';



        Cx1_error_SDP(j,1) = norm(x_1 - x1_center');
        Cx2_error_SDP(j,1) = norm(x_2 - x2');
        Ctotal_error_SDP(j,1) = Cx1_error_SDP(j,1) + Cx2_error_SDP(j,1);
        Cdistance(j,1) = dhat_center ;


        cvx_begin sdp quiet
            variable Z(4,4) symmetric
            minimize(0);
            subject to

            sum(dot(A(:,1)*A(:,1)', Z)) == 1;
            sum(dot(A(:,2)*A(:,2)', Z)) == 1;
```

```
            sum(dot(A(:,3)*A(:,3)', Z)) == 2;

            for i = 1:2
                sum(dot([a(:, i); -1; 0] * [a(:, i); -1; 0]', Z)) ...
                    == dist_x1_vertex(i)^2;
                sum(dot([a(:, i+1); 0; -1] * [a(:, i+1); 0; -1]', Z)) ...
                    == dist_x2(i)^2;
            end

            sum(dot([0; 0; 1; -1] * [0; 0; 1; -1]', Z)) == dhat_vertex^2 ;

            Z >= 0;

        cvx_end


        x_1 = [Z(3,1), Z(3,2)]';
        x_2 = [Z(4,1), Z(4,2)]';



        if isnan(x_1(1)) || isnan(x_1(2))
            disp('YOOx1')
            x_1;
        end

        if isnan(x_1(1)) || isnan(x_1(2))
            disp('YOOx2')
            x_2;
        end

        Vx1_error_SDP(j,1) = norm(x_1 - x1_vertex');
        Vx2_error_SDP(j,1) = norm(x_2 - x2');
        Vtotal_error_SDP(j,1) = Vx1_error_SDP(j,1) + Vx2_error_SDP(j,1);
        Vdistance(j,1) = dhat_vertex ;

    end
end

%%
figure()
subplot(3,1,1)

surf(X_1, Y_1, Cx1_error_SDP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image


title('Magntitude of X_1 Error with SDP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')

[mx,k] = min(Cx1_error_SDP(:));
[ix,jx] = ind2sub(size(Cx1_error_SDP),k);
dim = [.10 .605 .3 .3];
str = strcat('Minimum Error of (', num2str(Cx1_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',
    ', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);
```

```
423
425 hold on
tmp = a;
427 tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
429 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
431 ylabel('Y Data')
hold off
433

435 subplot(3,1,2)
surf(X_1, Y_1, Cx2_error_SDP)
437 colormap hsv
alpha(.4)
439 colorbar
view(-30,30); camlight; axis image
441
title('Magntitude of X_2 Error with SDP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
443 xlabel('X Data')
ylabel('Y Data')
445 zlabel('Error')

447 [mx,k] = min(Cx2_error_SDP(:));
[ix,jx] = ind2sub(size(Cx2_error_SDP),k);
449 dim = [.10 .295 .3 .3];
str = strcat('Minimum Error of (', num2str(Cx2_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',
    ', num2str(Y_1(ix,jx)),')');
451 annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);

453 hold on
tmp = a;
455 tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
457 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
459 ylabel('Y Data')
hold off
461


463

subplot(3,1,3)
465 surf(X_1, Y_1, Ctotal_error_SDP)
colormap hsv
467 alpha(.4)
colorbar
469 view(-30,30); camlight; axis image


471


473 title('Magntitude of Total Error with SDP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
xlabel('X Data')
475 ylabel('Y Data')
zlabel('Error')
477

479 [mx,k] = min(Ctotal_error_SDP(:));
[ix,jx] = ind2sub(size(Ctotal_error_SDP),k);
dim = [.10 .009 .3 .3];
481 str = strcat('Minimum Error of (', num2str(Ctotal_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)),
    ',', num2str(Y_1(ix,jx)),')');
```

```
annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off
%
figure()
subplot(3,1,1)
surf(X_1, Y_1, Vx1_error_SDP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image


title('Magntitude of X_1 Error with SDP Relaxation X_1 Fixed at (x = .95, y = .05) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')


[mx,k] = min(Vx1_error_SDP(:));
[ix,jx] = ind2sub(size(Vx1_error_SDP),k);
dim = [.10 .595 .3 .3];
str = strcat('Minimum Error of (', num2str(Vx1_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off



subplot(3,1,2)
surf(X_1, Y_1, Vx2_error_SDP)
colormap hsv
alpha(.4)
colorbar
view(-30,30); camlight; axis image


title('Magntitude of X_2 Error with SDP Relaxation X_1 Fixed at (x = .95,y = .05 ) in Hull', 'FontSize', 10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')


[mx,k] = min(Vx2_error_SDP(:));
[ix,jx] = ind2sub(size(Vx2_error_SDP),k);
```

```
dim = [.10 .295 .3 .3];
str = strcat('Minimum Error of (', num2str(Vx2_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',
    ', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String', str,'FitBoxToText','on', 'FontSize',10);

hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off


subplot (3,1,3)
surf(X_1, Y_1, Vtotal_error_SDP)
colormap hsv
alpha (.4)
colorbar
view(-30,30); camlight; axis image


title ('Magntitude of Total Error with SDP Relaxation X_1 Fixed at (x = .95,y = .05) in Hull', 'FontSize',
    10)
xlabel('X Data')
ylabel('Y Data')
zlabel('Error')
[mx,k] = min(Vtotal_error_SDP(:));
[ix,jx] = ind2sub(size(Vtotal_error_SDP),k);
dim = [.10 .011 .3 .3];
str = strcat('Minimum Error of (', num2str(Vtotal_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)),
    ',', num2str(Y_1(ix,jx)),')');
annotation('textbox',dim,'String', str,'FitBoxToText','on', 'FontSize',10);


hold on
tmp = a;
tmp(:,4) = tmp(:,1);
plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
xlabel('X Data')
ylabel('Y Data')
hold off
```