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# CME Homework Assignment 2 Computational Work

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## (20pts) Computation Team Work:

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Now consider the sensor localization problem on plane  $\mathbb{R}^2$  with two sensors  $x_1$  and  $x_2$  and three anchors  $a_1 = (1; 0)$ ,  $a_2 = (-1; 0)$  and  $a_3 = (0; 2)$ . Suppose we know the (Euclidean) distances from one sensor to  $a_1$  and  $a_2$ , denoted by  $d_{11}$  and  $d_{12}$ ; distances of the other to  $a_2$  and  $a_3$ , denoted by  $d_{22}$  and  $d_{23}$ ; and the distance between the two sensors, denoted by  $\hat{d}_{12}$ . Then, from the anchor and distance information we like locate the sensor positions  $x_1, x_2 \in \mathbb{R}^2$ .

Do the following numerical experimentations:

- Generate two sensor points anywhere and try the SOCP relaxation model

$$\begin{aligned} \|x_1 - a_i\|^2 &\leq d_{1i}^2, \quad i = 1, 2 \\ \|x_2 - a_i\|^2 &\leq d_{2i}^2, \quad i = 2, 3 \\ \|x_1 - x_2\|^2 &\leq \hat{d}_{12}^2. \end{aligned}$$

Did you find the correct locations? What have you observed?

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In order to examine the performance of the SOCP relaxation model for sensor localization in  $\mathbb{R}^2$ , let's examine some interesting possible cases. First, let us consider fixing the point  $x_1$  in the center of the convex hull of the anchors. That is, let  $x_1$  be fixed at  $[0, 1]$ , and examine the performance of the optimization for  $x_2$  points both within the convex hull and outside the hull.

### $x_1$ Fixed in Center of Convex Hull

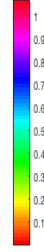
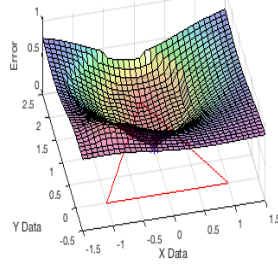
First observe that it is interesting to examine the optimization's performance in locating each of the sensors individually.

The subplots below show the errors of the SOCP relaxation for  $x_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$  and  $x_2$  varying across the meshgrid.

The first plot is the error in locating  $x_1$ , the second plot  $x_2$  and the third plot the total error. The blue dot indicates the fixed  $x_1$  point. Recall that  $x_1$  only has distance information about  $a_1$  and  $a_2$  whereas  $x_2$  has information on  $a_2, a_3$ .

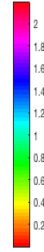
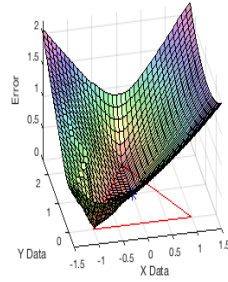
Minimum Error of (1.3464e-13) at  $X_2^* = (0.1, 5294)$

Magnitude of  $X_1$  Error with SOCP Relaxation  $X_1$  Fixed at  $(x = 0, y = 1)$  in Hull



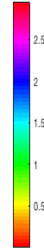
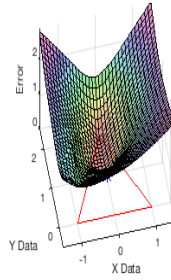
Minimum Error of (7.2371e-12) at  $X_2^* = (-0.088235, 1.3529)$

Magnitude of  $X_2$  Error with SOCP Relaxation  $X_1$  Fixed at  $(x = 0, y = 1)$  in Hull



Minimum Error of (9.7393e-12) at  $X_2^* = (-0.088235, 1.6176)$

Magnitude of Total Error with SOCP Relaxation  $X_1$  Fixed at  $(x = 0, y = 1)$  in Hull

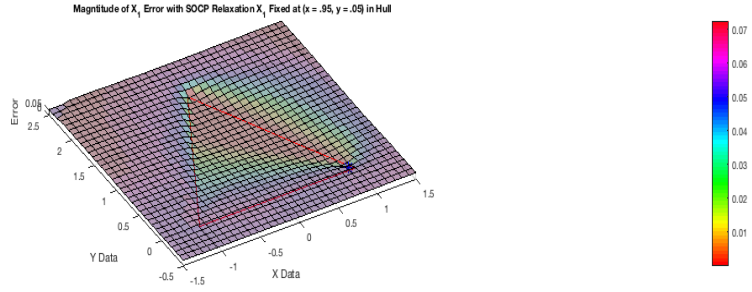


Observe that the error in locating  $x_1$  when  $x_2$  is close to  $a_3$  is very low and increases dramatically when  $x_2$  is closer to  $a_1$  or  $a_2$ , even within the convex hull. This intuitively makes sense, since when  $x_2$  is close to either anchors within the hull, it in effect is very easily to locate and could be loosely thought of as acting as an additional anchor point. When  $x_2$  outside the convex hull the error is uniformly large, which is expected in SOCP as per homework 1. Now for the error in locating  $x_2$ , we observe an interesting result. When  $x_2$  is along the vertex connecting  $a_2$  to  $a_3$  the optimization procedure is able to perform very well. However as  $x_2$  drifts towards  $a_1$  or outside the convex hull, the error increases dramatically. This is because we only have distance information for  $x_2$  with  $a_2, a_3$ . So in effect, when  $x_2$  is near  $a_1$ , the optimization is unable to find forces such that the total force on each anchor point is zero.

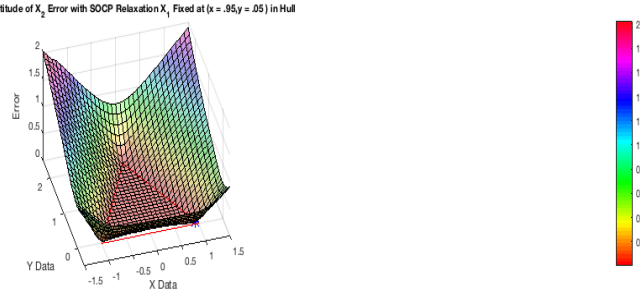
### $x_1$ Fixed near $a_1$

Now let us examine the performance of the optimization when  $x_1$  is fixed closed to  $a_1$ ,  $x_1 = \begin{bmatrix} .95 \\ .05 \end{bmatrix}$ .

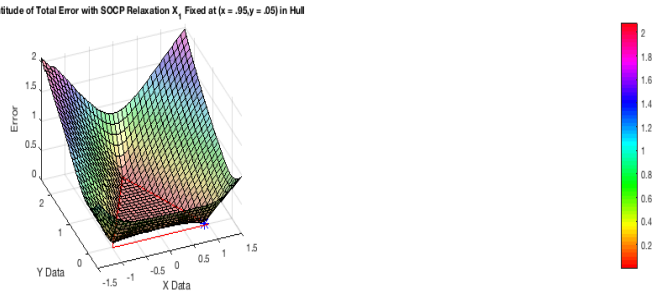
Minimum Error of (5.195e-13) at  $X_2$ : (0.52941,0.91178)



Minimum Error of (0.6573e-13) at  $X_2$ : (0.088235,1)



Minimum Error of (7.4849e-12) at  $X_2$ : (0.088235,1.3529)



The first thing to notice is that  $x_1$  is uniformly easy to locate when  $x_1$  is near the anchor point. Because of this,  $x_1$  does not create an extra layer of uncertainty in the location of  $x_2$ , and thus the results for the SOCP relaxation in  $\mathbb{R}^2$  with two sensors become very much similar to the problem of locating one sensor. The error is low within the convex hull and increases dramatically outside of it. This is due to the fact that when outside the convex hull, the forces acting upon the sensor points are unable to balance.

- Now try the SDP relaxation: find  $X = [x_1, x_2] \in \mathbb{R}^{2 \times 2}$  and

$$Z = \begin{pmatrix} I & X \\ X^T & Y \end{pmatrix} \in S^4$$

to meet the constraints in the standard form:

$$\begin{aligned}
 (1; 0; 0; 0)(1; 0; 0; 0)^T \bullet Z &= 1, \\
 (0; 1; 0; 0)(0; 1; 0; 0)^T \bullet Z &= 1, \\
 (1; 1; 0; 0)(1; 1; 0; 0)^T \bullet Z &= 2, \\
 (a_i; -1; 0)(a_i; -1; 0)^T \bullet Z &= d_{1i}^2, \quad i = 1, 2, \\
 (a_i; 0; -1)(a_i; 0; -1)^T \bullet Z &= d_{2i}^2, \quad i = 2, 3, \\
 (0; 0; 1; -1)(0; 0; 1; -1)^T \bullet Z &= \hat{d}_{12}^2, \\
 Z &\succeq 0 \in S^4.
 \end{aligned}$$

Did you find the correct locations? What have you observed? Can you conclude something?

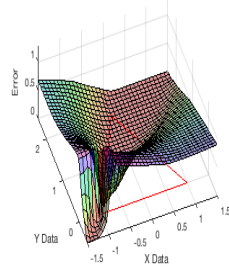
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### $x_1$ Fixed in Center of Convex Hull

Lets consider the same approach as in SDP and fix  $x_1$  in the center of the Hull and close to an anchor point.  
The

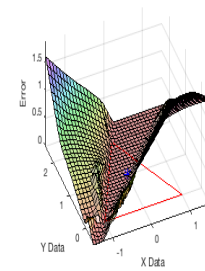
Minimum Error of  $(1.9795e-13)$  at  $X_2: (-0.088235, 1.4413)$

Magnitude of  $X_1$  Error with SDP Relaxation  $X_1$  Fixed at  $(x=0, y=1)$  in Hull



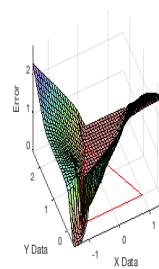
Minimum Error of  $(1.1618e-13)$  at  $X_2: (0.1, 3.529)$

Magnitude of  $X_2$  Error with SDP Relaxation  $X_1$  Fixed at  $(x=0, y=1)$  in Hull



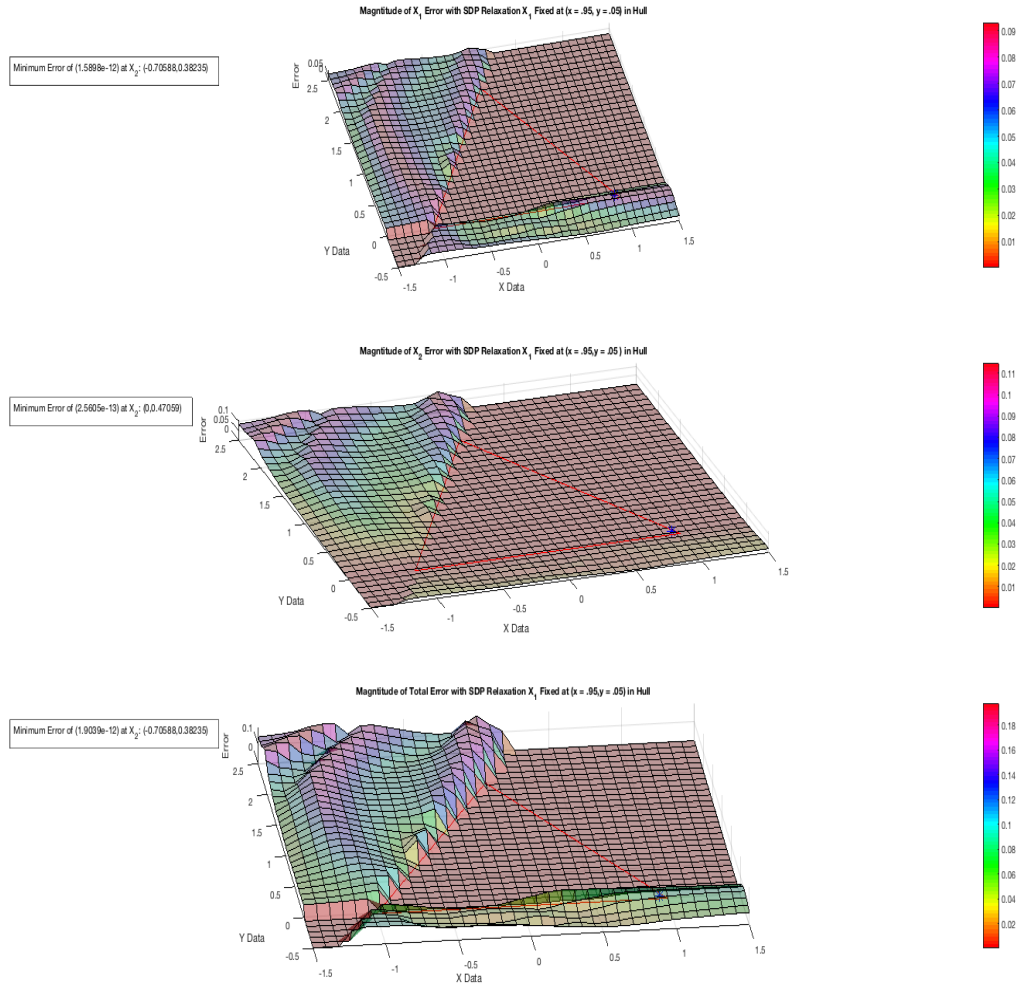
Minimum Error of  $(6.4166e-13)$  at  $X_2: (-0.44118, 0.73529)$

Magnitude of Total Error with SDP Relaxation  $X_1$  Fixed at  $(x=0, y=1)$  in Hull



Similar to SDP, when  $x_1$  is placed in the center of the convex hull, the error in locating  $x_1$  decreases as  $x_2$  approaches the vertex  $a_2, a_3$ , with the optimum  $x_2$  point being very close to  $x_2$ . The error increases when  $x_2$  drifts outside the convex Hull, but not as dramatically as with SOCP relaxation. The error in  $x_2$  similarly increases as  $x_2$  drifts away from the vertex connecting  $a_2$  and  $a_3$ . This is because the stress matrix in the SOCP relaxation is not of full rank. When the SDP has a unique solution along the vertex, the results are good, however, when  $x_2$  drifts away from the vertex  $a_2 \rightarrow a_3$ , the U is rank deficient.

### $x_1$ Fixed near $a_1$



Now when  $x_1$  is fixed near  $a_1$ , the solution is pretty good (both for  $x_2$  and  $x_1$ ) for  $x_2$  points both in and outside of the convex hull. The error increases slightly as  $x_2$  leaves the convex Hull, but not dramatically. Overall, though, the results for SDP when  $x_1$  is close to  $a_1$  are greatly superior to that of SOCP. Intuitively, the SDP relaxation allows for the internal forces to have two directions where SOCP only allows for one. When the stress matrix has full rank, the SDP relaxation has a larger possibility to balance the force in that

position and can thus perform better. However, when the dual stress matrix in SDP does not have full rank, the points are not localizable.

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1 clear all
2 close all
3
4
5 a = [1, -1, 0; 0, 0, 2];
6
7
8 x1 = [2*rand(25,1) + -1, 2*rand(25,1)];
9 x2 = [2*rand(25,1) + -1, 2*rand(25,1)];
10
11
12 dist_x1 = pdist2(x1, a(:,1:2)');
13 dist_x2 = pdist2(x2, a(:,2:3)');
14 dhat = pdist2(x1, x2);
15
16
17 %% Fix point in convex hull
18
19
20
21 %% Middle of HULL
22
23
24 clear all
25 close all
26
27
28 a = [1, -1, 0; 0, 0, 2];
29 %%
30
31 [X_1, Y_1] = meshgrid(linspace(-1.5, 1.5, 35)', linspace(-.5, 2.5, 35)');
32
33 Cx1_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
34 Cx2_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
35 Ctotal_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
36
37 Vx1_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
38 Vx2_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
39 Vtotal_error_SOCP = zeros(size(X_1,1), size(Y_1,1));
40
41
42
43 for j=1:size(X_1, 1)
44     j;
45     for l=1:size(Y_1,1)
46         l;
47         x1_center = [0,1];
48         dist_x1_center = pdist2(x1_center, a(:,1:2)');
49
50
51         x1_vertex = [.95,.05];
52         dist_x1_vertex = pdist2(x1_vertex, a(:,1:2)');
53
54
55         x2 = [X_1(j,l), Y_1(j,l)];
56         dist_x2 = pdist2(x2, a(:,2:3)');
57
58         dhat_center = pdist2(x1_center, x2);
59         dhat_vertex = pdist2(x1_vertex, x2);

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312         cvx_begin quiet
313         61         variables x_1(2) x_2(2)
314         minimize 0
315         63         subject to
316         for i = 1:2
317         65             norm(x_1 - a(:, i),2) <= dist_x1_center(i);
318             norm(x_2 - a(:, i+1),2) <= dist_x2(i);
319             norm(x_1 - x_2,2) <= dhat_center;
320         end
321         cvx_end
322
323         Cx1_error_SOCP(j,1) = norm(x_1 - x1_center');
324         Cx2_error_SOCP(j,1) = norm(x_2 - x2');
325         Ctotal_error_SOCP(j,1) = Cx1_error_SOCP(j,1) + Cx2_error_SOCP(j,1);
326         Cdistance(j,1) = dhat_center;
327
328         cvx_begin quiet
329         77         variables x_1(2) x_2(2)
330         minimize 0
331         79         subject to
332         for i = 1:2
333         81             norm(x_1 - a(:, i),2) <= dist_x1_vertex(i);
334             norm(x_2 - a(:, i+1),2) <= dist_x2(i);
335             norm(x_1 - x_2,2) <= dhat_vertex;
336         end
337         cvx_end
338
339         Vx1_error_SOCP(j,1) = norm(x_1 - x1_vertex');
340         Vx2_error_SOCP(j,1) = norm(x_2 - x2');
341         Vtotal_error_SOCP(j,1) = Vx1_error_SOCP(j,1) + Vx2_error_SOCP(j,1);
342         Vdistance(j,1) = dhat_vertex;
343
344     end
345 end
346
347 %%
348 figure ()
349 subplot (3,1,1)
350 surf(X_1, Y_1, Cx1_error_SOCP)
351 colormap hsv
352 alpha (.4)
353 colorbar
354 view(-30,30); camlight; axis image
355
356
357 title ('Magntitude of X_1 Error with SOCP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
358 xlabel('X Data')
359 ylabel('Y Data')
360 zlabel('Error')
361 [mx,k] = min(Cx1_error_SOCP(:));
362 [ix,jx] = ind2sub(size(Cx1_error_SOCP),k);
363 dim = [.10 .595 .3 .3];
364 str = strcat('Minimum Error of (', num2str(Cx1_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ', num2str(Y_1(ix,jx)),')');
365 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
366
367 hold on
368 tmp = a;
369 tmp(:,4) = tmp(:,1);
370 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')

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415
121 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
122 xlabel('X Data')
123 ylabel('Y Data')
124 hold off
125
126
127 subplot(3,1,2)
128 surf(X_1, Y_1, Cx2_error_SOCP)
129 colormap hsv
130 alpha(.4)
131 colorbar
132 view(-30,30); camlight; axis image
133
134
135 title('Magntitude of X_2 Error with SOCP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
136 xlabel('X Data')
137 ylabel('Y Data')
138 zlabel('Error')
139
140
141 [mx,k] = min(Cx2_error_SOCP(:));
142 [ix,jx] = ind2sub(size(Cx2_error_SOCP),k);
143 dim = [.10 .295 .3 .3];
144 str = strcat('Minimum Error of (', num2str(Cx2_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ', num2str(Y_1(ix,jx)),')');
145 annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);
146
147 hold on
148 tmp = a;
149 tmp(:,4) = tmp(:,1);
150 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
151 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
152 xlabel('X Data')
153 ylabel('Y Data')
154 hold off
155
156
157 subplot(3,1,3)
158 surf(X_1, Y_1, Ctotal_error_SOCP)
159 colormap hsv
160 alpha(.4)
161 colorbar
162 view(-30,30); camlight; axis image
163
164
165 title('Magntitude of Total Error with SOCP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
166 xlabel('X Data')
167 ylabel('Y Data')
168 zlabel('Error')
169
170
171 [mx,k] = min(Ctotal_error_SOCP(:));
172 [ix,jx] = ind2sub(size(Ctotal_error_SOCP),k);
173 dim = [.10 .009 .3 .3];
174 str = strcat('Minimum Error of (', num2str(Ctotal_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ', num2str(Y_1(ix,jx)),')');
175 annotation('textbox',dim,'String',str,'FitBoxToText','on', 'FontSize',10);
176
177 hold on
178 tmp = a;
179 tmp(:,4) = tmp(:,1);
180 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
181 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)

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416 xlabel('X Data')
417 ylabel('Y Data')
418 hold off
419
420
421 %
422 figure()
423 subplot(3,1,1)
424 surf(X_1, Y_1, Vx1_error_SOCP)
425 colormap hsv
426 alpha(.4)
427 colorbar
428 view(-30,30); camlight; axis image
429
430
431 title('Magnitude of X_1 Error with SOCP Relaxation X_1 Fixed at (x = .95, y = .05) in Hull', 'FontSize',
432 10)
433 xlabel('X Data')
434 ylabel('Y Data')
435 zlabel('Error')
436
437
438 [mx,k] = min(Vx1_error_SOCP(:));
439 [ix,jx] = ind2sub(size(Vx1_error_SOCP),k);
440 dim = [.10 .595 .3 .3];
441 str = strcat('Minimum Error of (', num2str(Vx1_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ' ',
442 ', ', num2str(Y_1(ix,jx)),')');
443 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
444
445 hold on
446 tmp = a;
447 tmp(:,4) = tmp(:,1);
448 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
449 plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
450 xlabel('X Data')
451 ylabel('Y Data')
452 hold off
453
454 subplot(3,1,2)
455
456 surf(X_1, Y_1, Vx2_error_SOCP)
457 colormap hsv
458 alpha(.4)
459 colorbar
460 view(-30,30); camlight; axis image
461
462
463 title('Magnitude of X_2 Error with SOCP Relaxation X_1 Fixed at (x = .95, y = .05 ) in Hull', 'FontSize',
464 10)
465 xlabel('X Data')
466 ylabel('Y Data')
467 zlabel('Error')
468 [mx,k] = min(Vx2_error_SOCP(:));
469 [ix,jx] = ind2sub(size(Vx2_error_SOCP),k);
470 dim = [.10 .295 .3 .3];
471 str = strcat('Minimum Error of (', num2str(Vx2_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ' ',
472 ', ', num2str(Y_1(ix,jx)),')');

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468 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
469
470 hold on
471 tmp = a;
472 tmp(:,4) = tmp(:,1);
473 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
474 plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
475 xlabel('X Data')
476 ylabel('Y Data')
477 hold off
478
479 subplot(3,1,3)
480 surf(X_1, Y_1, Vtotal_error_SOCP)
481 colormap hsv
482 alpha(.4)
483 colorbar
484 view(-30,30); camlight; axis image
485
486 title('Magntitude of Total Error with SOCP Relaxation X_1 Fixed at (x = .95,y = .05) in Hull','FontSize',
487 10)
488 xlabel('X Data')
489 ylabel('Y Data')
490 zlabel('Error')
491 [mx,k] = min(Vtotal_error_SOCP(:));
492 [ix,jx] = ind2sub(size(Vtotal_error_SOCP),k);
493 dim = [.1 .011 .3 .3];
494 str = strcat('Minimum Error of (', num2str(Vtotal_error_SOCP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx))
495 ,',', num2str(Y_1(ix,jx)),')');
496 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
497
498 hold on
499 tmp = a;
500 tmp(:,4) = tmp(:,1);
501 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
502 plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
503 xlabel('X Data')
504 ylabel('Y Data')
505 hold off
506
507 %%
508
509 clear all
510
511 a = [1, -1, 0; 0, 0, 2];
512
513 %%
514 [X_1, Y_1] = meshgrid(linspace(-1.5, 1.5,35)', linspace(-.5,2.5,35)');
515
516 Cx1_error_SDP = zeros(size(X_1,1), size(Y_1,1));
517 Cx2_error_SDP = zeros(size(X_1,1), size(Y_1,1));
518 Ctotal_error_SDP = zeros(size(X_1,1), size(Y_1,1));
519
520 Vx1_error_SDP = zeros(size(X_1,1), size(Y_1,1));
521 Vx2_error_SDP = zeros(size(X_1,1), size(Y_1,1));
522 Vtotal_error_SDP = zeros(size(X_1,1), size(Y_1,1));

```

```

520
521 A1 = [1; 0; 0; 0]; A2 = [0; 1; 0; 0]; A3 = [1; 1; 0; 0];
522 A = [A1, A2, A3];
523 a = [1, -1, 0; 0, 0, 2];
524 for j=1:size(X_1, 1)
525     j;
526     for l=1:size(Y_1,1)
527         x1_center = [0,1];
528         dist_x1_center = pdist2(x1_center, a(:,1:2)');
529
530         x1_vertex = [.95,.05];
531         dist_x1_vertex = pdist2(x1_vertex, a(:,1:2)');
532
533         x2 = [X_1(j,1), Y_1(j,1)];
534         dist_x2 = pdist2(x2, a(:,2:3)');
535
536         dhat_center = pdist2(x1_center, x2);
537         dhat_vertex = pdist2(x1_vertex, x2);
538
539         cvx_begin sdp quiet
540             variable Z(4,4) symmetric
541             minimize(0);
542             subject to
543
544                 sum(dot(A(:,1)*A(:,1)', Z)) == 1;
545                 sum(dot(A(:,2)*A(:,2)', Z)) == 1;
546                 sum(dot(A(:,3)*A(:,3)', Z)) == 2;
547
548                 for i = 1:2
549                     sum(dot([a(:, i); -1; 0] * [a(:, i); -1; 0]', Z)) ...
550                         == dist_x1_center(i)^2;
551                     sum(dot([a(:, i+1); 0; -1] * [a(:, i+1); 0; -1]', Z)) ...
552                         == dist_x2(i)^2;
553                 end
554
555                 sum(dot([0; 0; 1; -1] * [0; 0; 1; -1]', Z)) == dhat_center^2 ;
556
557                 Z >= 0;
558
559             cvx_end
560
561             x_1 = [Z(3,1), Z(3,2)]';
562             x_2 = [Z(4,1), Z(4,2)]';
563
564             Cx1_error_SDP(j,1) = norm(x_1 - x1_center');
565             Cx2_error_SDP(j,1) = norm(x_2 - x2');
566             Ctotal_error_SDP(j,1) = Cx1_error_SDP(j,1) + Cx2_error_SDP(j,1);
567             Cdistance(j,1) = dhat_center ;
568
569             cvx_begin sdp quiet
570                 variable Z(4,4) symmetric
571                 minimize(0);
572                 subject to
573
574                     sum(dot(A(:,1)*A(:,1)', Z)) == 1;
575                     sum(dot(A(:,2)*A(:,2)', Z)) == 1;

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572 sum(dot(A(:,3)*A(:,3)', Z)) == 2;
573
574 for i = 1:2
575     sum(dot([a(:, i); -1; 0] * [a(:, i); -1; 0]', Z)) ...
576         == dist_x1_vertex(i)^2;
577     sum(dot([a(:, i+1); 0; -1] * [a(:, i+1); 0; -1]', Z)) ...
578         == dist_x2(i)^2;
579 end
579
580 sum(dot([0; 0; 1; -1] * [0; 0; 1; -1]', Z)) == dhat_vertex^2 ;
581
582 Z >= 0;
583
584 cvx_end
585
586 x_1 = [Z(3,1), Z(3,2)]';
587 x_2 = [Z(4,1), Z(4,2)]';
588
589 if isnan(x_1(1)) || isnan(x_1(2))
590     disp('YOOx1')
591     x_1;
592 end
593
594 if isnan(x_1(1)) || isnan(x_1(2))
595     disp('YOOx2')
596     x_2;
597 end
598
599 Vx1_error_SDP(j,1) = norm(x_1 - x1_vertex');
600 Vx2_error_SDP(j,1) = norm(x_2 - x2');
601 Vtotal_error_SDP(j,1) = Vx1_error_SDP(j,1) + Vx2_error_SDP(j,1);
602 Vdistance(j,1) = dhat_vertex ;
603
604 end
605 end
606
607 %%
608 figure ()
609 subplot (3,1,1)
610
611 surf(X_1, Y_1, Cx1_error_SDP)
612 colormap hsv
613 alpha (.4)
614 colorbar
615 view(-30,30); camlight; axis image
616
617 title ('Magntitude of X_1 Error with SDP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
618 xlabel('X Data')
619 ylabel('Y Data')
620 zlabel('Error')
621
622 [mx,k] = min(Cx1_error_SDP(:));
623 [ix,jx] = ind2sub(size(Cx1_error_SDP),k);
624 dim = [.10 .605 .3 .3];
625 str = strcat('Minimum Error of (', num2str(Cx1_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ',
626             ', num2str(Y_1(ix,jx)),')');
627 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
628

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425 hold on
426 tmp = a;
427 tmp(:,4) = tmp(:,1);
428 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
429 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
430 xlabel('X Data')
431 ylabel('Y Data')
432 hold off
433
434
435 subplot(3,1,2)
436 surf(X_1, Y_1, Cx2_error_SDP)
437 colormap hsv
438 alpha(.4)
439 colorbar
440 view(-30,30); camlight; axis image
441
442 title('Magntitude of X_2 Error with SDP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
443 xlabel('X Data')
444 ylabel('Y Data')
445 zlabel('Error')
446
447 [mx,k] = min(Cx2_error_SDP(:));
448 [ix,jx] = ind2sub(size(Cx2_error_SDP),k);
449 dim = [.10 .295 .3 .3];
450 str = strcat('Minimum Error of (', num2str(Cx2_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ',
451             ', num2str(Y_1(ix,jx)),')');
452 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
453
454 hold on
455 tmp = a;
456 tmp(:,4) = tmp(:,1);
457 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
458 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
459 xlabel('X Data')
460 ylabel('Y Data')
461 hold off
462
463
464 subplot(3,1,3)
465 surf(X_1, Y_1, Ctotal_error_SDP)
466 colormap hsv
467 alpha(.4)
468 colorbar
469 view(-30,30); camlight; axis image
470
471
472 title('Magntitude of Total Error with SDP Relaxation X_1 Fixed at (x = 0, y = 1) in Hull', 'FontSize', 10)
473 xlabel('X Data')
474 ylabel('Y Data')
475 zlabel('Error')
476
477 [mx,k] = min(Ctotal_error_SDP(:));
478 [ix,jx] = ind2sub(size(Ctotal_error_SDP),k);
479 dim = [.10 .009 .3 .3];
480 str = strcat('Minimum Error of (', num2str(Ctotal_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ', ',
481             ', ', num2str(Y_1(ix,jx)),')');

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676 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
677
678 hold on
679 tmp = a;
680 tmp(:,4) = tmp(:,1);
681 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
682 plot3(x1_center(1), x1_center(2), 0, '-*b','MarkerSize',10)
683 xlabel('X Data')
684 ylabel('Y Data')
685 hold off
686 %
687 figure()
688 subplot(3,1,1)
689 surf(X_1, Y_1, Vx1_error_SDP)
690 colormap hsv
691 alpha(.4)
692 colorbar
693 view(-30,30); camlight; axis image
694
695 title('Magntitude of X_1 Error with SDP Relaxation X_1 Fixed at (x = .95, y = .05) in Hull', 'FontSize',
696 10)
697 xlabel('X Data')
698 ylabel('Y Data')
699 zlabel('Error')
700
701 [mx,k] = min(Vx1_error_SDP(:));
702 [ix,jx] = ind2sub(size(Vx1_error_SDP),k);
703 dim = [.10 .595 .3 .3];
704 str = strcat('Minimum Error of (', num2str(Vx1_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',
705 ', num2str(Y_1(ix,jx)),')');
706 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
707
708 hold on
709 tmp = a;
710 tmp(:,4) = tmp(:,1);
711 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
712 plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
713 xlabel('X Data')
714 ylabel('Y Data')
715 hold off
716
717 subplot(3,1,2)
718 surf(X_1, Y_1, Vx2_error_SDP)
719 colormap hsv
720 alpha(.4)
721 colorbar
722 view(-30,30); camlight; axis image
723
724 title('Magntitude of X_2 Error with SDP Relaxation X_1 Fixed at (x = .95, y = .05 ) in Hull', 'FontSize',
725 10)
726 xlabel('X Data')
727 ylabel('Y Data')
728 zlabel('Error')
729
730 [mx,k] = min(Vx2_error_SDP(:));
731 [ix,jx] = ind2sub(size(Vx2_error_SDP),k);

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541 dim = [.10 .295 .3 .3];
542 str = strcat('Minimum Error of (', num2str(Vx2_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',
543           ', num2str(Y_1(ix,jx)),')');
544 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
545 hold on
546 tmp = a;
547 tmp(:,4) = tmp(:,1);
548 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
549 plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
550 xlabel('X Data')
551 ylabel('Y Data')
552 hold off
553
554
555 subplot(3,1,3)
556 surf(X_1, Y_1, Vtotal_error_SDP)
557 colormap hsv
558 alpha(.4)
559 colorbar
560 view(-30,30); camlight; axis image
561
562
563 title('Magnitude of Total Error with SDP Relaxation X_1 Fixed at (x = .95,y = .05) in Hull','FontSize',
564       10)
565 xlabel('X Data')
566 ylabel('Y Data')
567 zlabel('Error')
568 [mx,k] = min(Vtotal_error_SDP(:));
569 [ix,jx] = ind2sub(size(Vtotal_error_SDP),k);
570 dim = [.10 .011 .3 .3];
571 str = strcat('Minimum Error of (', num2str(Vtotal_error_SDP(ix,jx)),')', ' at X_2: (', num2str(X_1(ix,jx)), ',
572           ', num2str(Y_1(ix,jx)),')');
573 annotation('textbox',dim,'String',str,'FitBoxToText','on','FontSize',10);
574
575 hold on
576 tmp = a;
577 tmp(:,4) = tmp(:,1);
578 plot3(tmp(1,:), tmp(2,:), zeros(size(tmp(2,:))), '-r')
579 plot3(x1_vertex(1), x1_vertex(2), 0, '-*b','MarkerSize',10)
580 xlabel('X Data')
581 ylabel('Y Data')
582 hold off

```