

Simultaneous Estimation of Normalized Source Dead-Layer and Detector Dead-Layer in a Double-Sided Strip Detector (Draft 1)

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Abstract

1 Introduction

2 Procedure

Here we describe the technique used to analyze the data. A detailed procedure of how the data was collected can be found elsewhere.

2.1 The Model

In this section, we will progress through an increasingly complicated set of scenarios which lead to the model upon which we will regress to determine the detector dead-layer depth.

Suppose a charged particle is emitted by a decaying radioactive source and subsequently enters an ionizing radiation detector where the charge from scattered particles is measured, and finally histogrammed as a channel via digitizer. We assume that there exists a linear relationship between the energy of the particle as it enters the part of the detector responsible for ionization of charge.¹ In practice, energy loss occurs before the charged particle enters the part of the detector in which charge is actually measured. Two main effects contribute to this loss: energy loss as the particle travels through an inactive part of the detector, known as the **detector dead-layer**, and as the particle travels through the source it is emitted from. In particular, if the source and detec-

tor are flat and both have small dimensions compared to their distance of separation, then all charged particles leave the source at a constant angle ϕ and enter the detector another constant angle θ . If the source and detector respectively have orthogonal distances d_S and d_D and stopping power functions $S_S(E)$ and $S_D(E)$, then we expect the relationship

$$E_{\text{measured}} = E_0 - \int_0^{d_S \sec(\phi)} S_S(E_S(x)) dx - \int_0^{d_D \sec(\theta)} S_D(E_D(x)) dx \quad (1)$$

where in both cases, $E(x)$ is determined implicitly as the inverse of invertible function $x(E) = \int_{E_0}^E dE/S(E)$. Note that such an inverse exists because $x(E)$ increases monotonically with E since $S(E) > 0$.

If the distances d_S and d_D are both small, then Equation 1 can be simplified considerably by assuming the S is constant over the region of integration, yielding

$$E_{\text{measured}} = E_0 - d_S S_S(E_0) \sec(\phi) - d_D S_D(E_0 - u \sec(\phi)) \sec \theta \quad (2)$$

Hence the energy measured is linear in $\sec(\theta)$ and approximately linear in $\sec(\phi)$.

Now in a two-sided strip-detector, each strip is held at the same voltage and shares the same relationship between channel (e.g. of the peak in the

¹In general, a polynomial relationship could also be used with equal difficulty in the implementation discussed later.

spectrum or the μ parameters of the Bortels function, to be discussed shortly) and energy. Moreover, because the detector has two-sides with strips running orthogonal, we can experimentally determine which pixel an incident particle hit. We can independently determine the angles θ_{ij} and ϕ_{ij} given adequate information about the relative geometry of the detector and a source. Equation 2 holds separately for each pixel, so if we observe the extract the position of each peak p_{ij} , we expect that there exist A_i and B_i for each strip such that we have the relationship:

$$A_i p_{ij} + B_i = E_0 - d_D S_D(E_0 - m \sec(\phi_{ij})) \sec \theta_{ij} - m \sec(\phi_{ij}), \quad (3)$$

where we have relabeled the constant $d_S S_{S(E_0)} \equiv m$. We call m the **source dead-layer** and d_D the **detector dead-layer**; note that the detector dead-layer has units of distance while the source dead-layer has units of energy.

Now suppose that multiple alpha peaks are observed in the spectrum, known to originate with energy E_{0k} . Then Equation 3 holds separately for each peak with a different $m \rightarrow m_k$ for each peak, since the stopping power of one or multiple sources varies with energy. However, the linear calibration of each strip is independent of k , as are the coordinates of each pixel and hence θ_{ij} and ϕ_{ij} as well. Thus, we expect the relationship:

$$A_i p_{ijk} + B_i = E_{0k} - d_D S_D(E_{0k} - m_k \sec(\phi_{ij})) \sec \theta_{ij} - m_k \sec(\phi_{ij}) \quad (4)$$

We now add the final complication of multiple datasets, labeled by l . Our data was obtained by setting the detector voltage to 0, adjusting sources, and then turning the detector voltage back on. When the sources are moved, we will obtain new angles θ_{ijl} and ϕ_{ijl} , still independent of the peak index. We cannot assume that the linear calibration of each strip remains the same after resetting the detector, so the linear calibration coefficients are indexed as A_{il} and B_{il} . The peak positions shift between datasets, so they are indexed as p_{ijkl} . However, m_k and E_{0k} are properties of the sources which are unchanged between different datasets. Applying Equation 4 to each dataset

and isolating E_{0k} , we obtain the model

$$\begin{aligned} E_{0k} &= A_{il} p_{ijkl} + B_{il} + m_k \sec(\phi_{ijl}) \\ &+ d_D S_D(E_{0k} - m_k \sec(\phi_{ijl})) \sec \theta_{ijl} \\ &\equiv M(A_{il}, B_{il}, d_D, m_k; p_{ijkl}, \phi_{ijl}, \theta_{ijl}) \end{aligned} \quad (5)$$

where $(A_{il}, B_{il}, d_D, m_k)$ are parameters and $(p_{ijkl}, \phi_{ijl}, \theta_{ijl})$ is one data point, both defined for each $ijkl$. If our data is consistent with such a model, then minimizing the quantity

$$\chi^2 = \sum_{ijkl} \frac{(E_{0k} - M)^2}{\Delta(E_{0k} - M)^2} \quad (6)$$

with respect to the parameters should converge with $\tilde{\chi}^2 \equiv \chi^2 / (N - D) \sim 1$, where N is the total number of data-tuples $(p_{ijkl}, \phi_{ijl}, \theta_{ijl})$ and D is the number of degrees of freedom in the fit. In addition, the detector manufacturer has given an estimate $d_D = 100 \pm 25$ nm, so we expect a convergent regression to yield a d_D in this vicinity.

2.2 The Dataset

We obtained four independent datasets between whose collection the voltage of the detector was reset. Hence, $l \in 1 \dots 4$. The strip detectors used have 32 strips running orthogonally on either side, so $i, j \in 1 \dots 32$. Finally, in each dataset, the detector recorded spectra of three distinct sources, of which two peaks were used. This gives a total of $2 \cdot 3 = 6$ peaks, so $k \in 1 \dots 6$. All of the sources were parallel to the detector except in one dataset, in which one of the sources was rotated relative to the plane of the detector; this particular source allows decoupling of the source and detector dead-layers.

It is worth noting that the regression in Equation 6 could, in principle, be performed if there were only one independent dataset (l fixed) for spectra with only one peak ($k = 1$) as long as the source contributing the one peak was rotated relative to the detector so that $\theta_{ij} \neq \phi_{ij}$. In this sense, our regression is highly constrained, having far more data than is reasonably necessary to achieve a good fit.

3 Analysis

3.1 Fitting

3.2 Peak Extraction

3.3 Computation of θ_{ijl} , ϕ_{ijl}

3.4 Simultaneous Calibration

Now that the data $(p_{ijkl}, \theta_{ijl}, \phi_{ijl})$ have been obtained for all i, j, k, l , we may proceed to minimize Equation 6 as a function of the parameters. In order to approximate the stopping power of the detector, we took a

We once again used the `lmfit` Python library with the Levenberg-Marquardt least-squares algorithm.[2] When all four available datasets were combined, we obtained a good fit with

4 Conclusion

5 Appendix

5.1 Alternate Models

References

- [1] Leslie Lamport, *L^AT_EX: a document preparation system*, Addison Wesley, Massachusetts, 2nd edition, 1994.
- [2] Newville, Matthew et al.. (2014). LMFIT: Non-Linear Least-Square Minimization and Curve-Fitting for Python. Zenodo. 10.5281/zenodo.11813