Exam 1

Jacob Plaza

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Question 1:

As predictive models get more and more complex the bias gets smaller and smaller. This is a good thing. Explain then why we don’t simply always use the most complex, lowest bias model possible.

You don’t always use the most complex, lowest bias model possible because of the bias-variance tradeoff. The more complex model you have, the lower the bias will be, but the higher the variance will be in exchange. Variance refers to how much f-hat would change if we estimated it using a different training set, and ideally you would not want your f-hat to change much between training sets. This is so that you have a generalizable model that applies to a variety of situations, and isn’t trained specifically just to represent your training data. So, you want to minimize both the variance and the bias, not just a model with super lower variance and high bias or low bias and high variance.

Question 2:

Best subset regression checks all possible combinations of variables in the model. Why don’t we just always use all subsets regression to choose our model?

First of all, best subsets regression suffers from computational limitations; the number of possible models that you would have to consider grows rapidly as p increases, which means that it quickly becomes computationally infeasible. Also, when p is large, that means you have a large search space, which means you have a higher chance of finding models with a high variance. So your models will look good on training data, but are not actually generalizable. A large search space can lead to overfitting and high covariance of coefficient estimates.

Furthermore, best subsests regression might not do very well in high dimensional datasets. You’re going to have to use enormous computational power, and its probably going to overfit the data.

Question 3:

K-fold cross validation is generally considered the best way to do cross-validation. Given an example of any scenario where you might not want to use k-fold cross validation and instead use a different method for cross validation.

When the size of the dataset is small, its better to use LOOCV. So an example of this would be analyzing survey data from a small data science class of 15 or so people. You wouldn’t have many observations, and so LOOCV would be preferable since it’ll use more training samples than K-fold would.

Question 4:

Explain what leave-one-out cross validation (LOOCV) is and briefly explain one reason why we prefer k-fold cross validation where k is smaller than n.

With leave one out cross validation, you split the set of observations into two parts, but instead of having two subsets which are of comparative size, only one observation is used for the validation set, and everything else is in the training set. You then fit a model on the training observations, and you make a prediction for the test error of the excluded observation. You then repeat that process by selecting a different observation point for the validation data, and you continue to train the model on your training set. The LOOCV estimate ends up being the average of the n test error estimates. LOOCV can be preferred for bias reduction. Each training set contains n-1 observations, which is pretty close to the number in the full data set. So, we might prefer it in situations where we want to reduce bias as much as possible.

Question 5

Stepwise Selection:

```
install.packages("leaps")
```

```
## Installing package into '/home/bultok/R/x86_64-pc-linux-gnu-library/4.0'
## (as 'lib' is unspecified)
```

```
library(leaps)
library(MASS)
load(url("https://github.com/gjm112/LoyolaTeaching/blob/main/data_exam1_2022.RData?raw=TRUE"))

lower <- lm(y ~ 1, data = data_exam1)
upper <- lm(y ~ ., data = data_exam1)
a <- lm(y ~ ., data = data_exam1)
```

First, I did a step AIC procedure to try to see if I could find a model with the lowest AIC. But since this data is high-dimensional, the AIC became negative infinity, since there are more features than observations; stepwise was producing very overfit functions. So we don't know if the coefficients included are actually relevant to predicting y, or if there are not problems with multi-collinearity.

I ommitted the code for the stepwise procedure from the code chunk since it produced a ridiculous amount of output (obviously)

But, this is the code that I used after the above lines:

stewpise = stepAIC(lower, scope = list(lower = lower,upper = upper),direction = “both”)

The model with the lowest AIC is:

y ~ X959 + X889 + X881 + X462 + X907 + X562 + X68 + X804 + X169 + X418 + X769 + X442 + X625 + X395 + X181 + X280 + X315 + X140 + X389 + X65 + X153 + X771 + X26 + X634 + X597 + X428 + X410 + X164 + X847 + X940 + X966 + X293 + X183 + X463 + X592 + X806 + X211 + X848 + X964 + X677 + X916 + X55 + X48 + X777 + X110 + X535 + X837 + X397 + X102 + X54 + X532 + X674 + X887 + X30 + X491 + X262 + X158 + X696 + X879 + X295 + X201 + X388 + X852 + X993 + X274 + X725 + X358 + X383 + X166 + X32 + X118 + X464 + X97 + X391 + X666 + X273 + X987 + X61 + X834 + X765 + X117 + X139 + X768 + X733 + X64 + X606 + X453 + X698 + X78 + X414 + X830 + X434 + X382 + X537 + X416 + X150 + X564 + X47 + X1

I decided to just do a forward stepwise procedure, since forward stepwise is more flexibile and would hopefully give a more accurate model:

```
regfit.fwd = regsubsets(y ~., data = data_exam1, nvmax = 19, method = "forward")
```

```
## Warning in leaps.setup(x, y, wt = wt, nbest = nbest, nvmax = nvmax, force.in =
## force.in, : 901 linear dependencies found
```

```
coef(regfit.fwd, 19)
```

```
## (Intercept)         X68        X140        X169        X181        X280
##   5.8289399   1.0617298   0.9240698  -1.6552823   1.1484161  -0.8223481
##        X315        X389        X395        X418        X442        X462
##  -0.8518664  -0.6025580   1.4072209   1.0267707  -1.5976523   1.1450267
##        X562        X625        X769        X804        X881        X889
##   1.3537767  -1.1168536   0.8748100  -1.3859445  -2.2934293  -2.0182441
##        X907        X959
##  -1.3137278   1.3363291
```

The best model from forward stepwise selection with 19 variables is

(Intercept) X68 X140 X169 X181 X280 X315 5.8289399 1.0617298 0.9240698 -1.6552823 1.1484161 -0.8223481 -0.8518664 X389 X395 X418 X442 X462 X562 X625 -0.6025580 1.4072209 1.0267707 -1.5976523 1.1450267 1.3537767 -1.1168536 X769 X804 X881 X889 X907 X959 0.8748100 -1.3859445 -2.2934293 -2.0182441 -1.3137278 1.33632

```
library(glmnet)
```

```
## Loading required package: Matrix
```

```
## Loaded glmnet 4.1-4
```

```
x = model.matrix(y ~., data_exam1)[, -1]
y = data_exam1$y
```

```
set.seed(1)
train = sample(1:nrow(x), nrow(x) / 2)
test = (-train)
y.test = y[test]
```

Lasso Regression

```
set.seed(1)
cv.out = cv.glmnet(x[train, ], y[train], alpha = 1)
lam = cv.out$lambda.min
lam
```

```
## [1] 1.315547
```

the model is 5.60620407 + X91 (0.13176002) + X94 (0.66938879) +X190 (-0.04044894) + X299 (-0.19255332) + X381(-0.11070250) + X397(0.64652487) + X528 (-0.032056650) + X783(-0.008343261) + X889(-0.538795882) + X959(0.097651340)

```
lasso.mod = glmnet(x[train, ], y[train], alpha = 1, lambda = 1.315547)
lass.coef = predict(lasso.mod, type = "coefficients", s=lam)
lass.coef[lass.coef[,1]!=0]
```

```
## <sparse>[ <logic> ] : .M.sub.i.logical() maybe inefficient
```

```
##  [1]  5.606204066  0.131760018  0.669388785 -0.040448939 -0.192553318
##  [6] -0.110702495  0.646524868 -0.032056650 -0.008343261 -0.538795882
## [11]  0.097651340
```

```
lass.coef
```

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## 1001 x 1 sparse Matrix of class "dgCMatrix"
##                       s1
## (Intercept)  5.606204066
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## X783       -0.008343261
## X784        .
## X785        .
## X786        .
## X787        .
## X788        .
## X789        .
## X790        .
## X791        .
## X792        .
## X793        .
## X794        .
## X795        .
## X796        .
## X797        .
## X798        .
## X799        .
## X800        .
## X801        .
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## X806        .
## X807        .
## X808        .
## X809        .
## X810        .
## X811        .
## X812        .
## X813        .
```

```
## X814         .
## X815         .
## X816         .
## X817         .
## X818         .
## X819         .
## X820         .
## X821         .
## X822         .
## X823         .
## X824         .
## X825         .
## X826         .
## X827         .
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## X829         .
## X830         .
## X831         .
## X832         .
## X833         .
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## X856         .
## X857         .
## X858         .
## X859         .
## X860         .
## X861         .
## X862         .
## X863         .
## X864         .
```

```
## X865         .
## X866         .
## X867         .
## X868         .
## X869         .
## X870         .
## X871         .
## X872         .
## X873         .
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## X880         .
## X881         .
## X882         .
## X883         .
## X884         .
## X885         .
## X886         .
## X887         .
## X888         .
## X889        -0.538795882
## X890         .
## X891         .
## X892         .
## X893         .
## X894         .
## X895         .
## X896         .
## X897         .
## X898         .
## X899         .
## X900         .
## X901         .
## X902         .
## X903         .
## X904         .
## X905         .
## X906         .
## X907         .
## X908         .
## X909         .
## X910         .
## X911         .
## X912         .
## X913         .
## X914         .
## X915         .
```

```
## X916        .
## X917        .
## X918        .
## X919        .
## X920        .
## X921        .
## X922        .
## X923        .
## X924        .
## X925        .
## X926        .
## X927        .
## X928        .
## X929        .
## X930        .
## X931        .
## X932        .
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## X935        .
## X936        .
## X937        .
## X938        .
## X939        .
## X940        .
## X941        .
## X942        .
## X943        .
## X944        .
## X945        .
## X946        .
## X947        .
## X948        .
## X949        .
## X950        .
## X951        .
## X952        .
## X953        .
## X954        .
## X955        .
## X956        .
## X957        .
## X958        .
## X959        0.097651340
## X960        .
## X961        .
## X962        .
## X963        .
## X964        .
## X965        .
## X966        .
```

```
## X967         .
## X968         .
## X969         .
## X970         .
## X971         .
## X972         .
## X973         .
## X974         .
## X975         .
## X976         .
## X977         .
## X978         .
## X979         .
## X980         .
## X981         .
## X982         .
## X983         .
## X984         .
## X985         .
## X986         .
## X987         .
## X988         .
## X989         .
## X990         .
## X991         .
## X992         .
## X993         .
## X994         .
## X995         .
## X996         .
## X997         .
## X998         .
## X999         .
## X1000        .
```

```
lasso.pred <- predict(lasso.mod, s = lam, newx = x[test,])
mean((lasso.pred - y.test)^2)
```

```
## [1] 21.8452
```

Ridge Regression

I did cross-validation to find the best lambda value, finding it as 1.315547.

```
set.seed(1)
cv.out1 = cv.glmnet(x[train, ], y[train], alpha = 0)
lam1 = cv.out$lambda.min
lam1
```

```
## [1] 1.315547
```

```
ridge.mod = glmnet(x, y, alpha = 0, lambda = 1.315547)
dim(coef(ridge.mod))
```

```
## [1] 1001    1
```

```
ridge.mod$lambda[1]
```

```
## [1] 1.315547
```

```
coef(ridge.mod)[, 1]
```

```
##   (Intercept)            X1            X2            X3            X4 
##  6.089391e+00 -4.560122e-02  2.548536e-03 -7.094993e-02  5.122055e-02 
##            X5            X6            X7            X8            X9 
## -2.821592e-02  3.147503e-02 -1.308064e-02 -2.573540e-02  1.276530e-04 
##           X10           X11           X12           X13           X14 
## -4.537072e-02  1.854451e-02 -3.597661e-02 -7.233811e-02 -1.037137e-03 
##           X15           X16           X17           X18           X19 
##  2.669354e-02 -2.084842e-02 -5.795093e-02 -7.152850e-02 -5.144660e-02 
##           X20           X21           X22           X23           X24 
## -4.955320e-02  4.145706e-03 -3.598434e-03  2.701516e-04  4.805684e-02 
##           X25           X26           X27           X28           X29 
##  1.248141e-02  2.177975e-02 -4.695262e-02  3.606428e-02 -6.874100e-02 
##           X30           X31           X32           X33           X34 
## -4.117462e-03  3.131608e-02  7.788934e-02  1.621698e-02  2.694962e-02 
##           X35           X36           X37           X38           X39 
##  2.473994e-02 -1.194042e-03 -2.367175e-02  3.963763e-02  5.513972e-02 
##           X40           X41           X42           X43           X44 
## -3.579352e-02  6.343920e-02  3.325401e-02 -5.040209e-02  3.166122e-02 
##           X45           X46           X47           X48           X49 
## -4.720694e-02  7.067574e-03 -7.817491e-02 -7.050879e-02  4.911324e-02 
##           X50           X51           X52           X53           X54 
##  1.844625e-02 -6.903010e-02  1.092693e-02 -5.772206e-02  1.341286e-02 
##           X55           X56           X57           X58           X59 
## -1.036963e-02 -6.651975e-02  3.381721e-02 -8.748316e-02  1.819786e-02 
##           X60           X61           X62           X63           X64 
##  5.424168e-02  1.771153e-02 -6.267707e-02 -4.008766e-02 -1.928326e-03 
##           X65           X66           X67           X68           X69 
## -4.482660e-02 -3.780057e-02  4.232880e-02  1.030632e-01 -5.868373e-02 
##           X70           X71           X72           X73           X74 
## -6.219004e-03 -4.473851e-02  1.545461e-02  2.758604e-02 -4.116162e-02 
##           X75           X76           X77           X78           X79 
## -3.716117e-02  7.219002e-03 -6.943233e-02  9.253062e-02 -4.025142e-02 
##           X80           X81           X82           X83           X84 
##  5.866104e-02  3.389455e-03 -3.455803e-03  1.608868e-02  3.014186e-02 
##           X85           X86           X87           X88           X89 
##  3.041363e-02 -9.413927e-02 -8.629784e-03  3.182163e-02 -3.445998e-02 
##           X90           X91           X92           X93           X94 
##  1.720817e-03  9.367182e-02 -6.379659e-02  5.235304e-02  1.141558e-01 
##           X95           X96           X97           X98           X99 
##  4.550485e-03 -8.285211e-02  2.501391e-02  5.762270e-03 -4.197631e-02 
##          X100          X101          X102          X103          X104 
## -3.311424e-02  7.785670e-02 -1.563571e-02 -4.656809e-02  2.994331e-02 
##          X105          X106          X107          X108          X109 
##  8.396094e-02 -7.101964e-03 -1.903076e-03  3.175431e-02 -2.917621e-02 
##          X110          X111          X112          X113          X114 
## -2.318703e-02  1.186889e-02  5.088147e-02  5.859377e-03  4.204426e-02 
##          X115          X116          X117          X118          X119 
## -6.790972e-02 -1.945834e-02 -4.628877e-03 -5.212763e-02 -1.357669e-02 
##          X120          X121          X122          X123          X124 
## -1.293385e-02 -1.674505e-02  2.276261e-02  6.327264e-02 -7.620998e-02 
##          X125          X126          X127          X128          X129 
```

```
## -3.097635e-03  8.243850e-03 -4.415282e-02  2.625118e-02 -3.851222e-02
##          X130          X131          X132          X133          X134
##  2.340332e-02 -5.892964e-02  9.570762e-03  1.719826e-02 -1.613259e-02
##          X135          X136          X137          X138          X139
## -6.690973e-02 -5.757253e-02  1.015115e-02 -3.672560e-02 -6.817290e-03
##          X140          X141          X142          X143          X144
##  5.468010e-03  4.461670e-02 -1.779961e-02 -1.017862e-02 -7.660082e-02
##          X145          X146          X147          X148          X149
##  5.313811e-02 -2.115336e-02 -2.499626e-02  9.657225e-02 -2.665327e-02
##          X150          X151          X152          X153          X154
##  2.721252e-02  2.500314e-02 -5.050785e-02 -3.701475e-02  6.873843e-02
##          X155          X156          X157          X158          X159
##  9.979130e-03  2.914388e-02 -1.762852e-02  2.031218e-02 -4.802763e-02
##          X160          X161          X162          X163          X164
## -4.892060e-02  6.181094e-03 -1.879357e-02 -3.805240e-03 -5.343638e-03
##          X165          X166          X167          X168          X169
##  9.051201e-02  4.925104e-02 -3.716513e-03 -2.969698e-02 -1.134576e-01
##          X170          X171          X172          X173          X174
## -2.328530e-02  4.403674e-02 -1.525679e-02 -2.520025e-02  2.641348e-02
##          X175          X176          X177          X178          X179
##  4.068104e-02  7.038430e-02  1.022942e-01 -2.761564e-02 -8.300210e-03
##          X180          X181          X182          X183          X184
##  3.671236e-02  2.037429e-02  5.921763e-02 -1.616852e-02 -3.255575e-02
##          X185          X186          X187          X188          X189
## -3.253981e-02  8.478193e-02 -8.667688e-02  6.552816e-03 -8.321407e-02
##          X190          X191          X192          X193          X194
## -1.229873e-01  7.326831e-02  2.635266e-02 -2.378197e-02 -5.577467e-03
##          X195          X196          X197          X198          X199
##  4.627540e-03  1.488198e-01 -1.073895e-01 -9.652723e-02 -2.398745e-02
##          X200          X201          X202          X203          X204
##  6.261051e-02 -6.944357e-02 -8.534394e-02 -6.239260e-02 -1.097053e-03
##          X205          X206          X207          X208          X209
## -1.362429e-02  7.511317e-02  2.608948e-02  1.056504e-02 -1.248420e-01
##          X210          X211          X212          X213          X214
##  1.694475e-02 -2.505236e-03 -5.146092e-02 -6.664386e-02  2.618997e-02
##          X215          X216          X217          X218          X219
## -6.282810e-02 -5.141056e-02 -2.109464e-02 -3.290813e-02  4.643093e-02
##          X220          X221          X222          X223          X224
## -9.306864e-03  6.850454e-03 -3.608217e-02  8.844661e-02 -4.624285e-02
##          X225          X226          X227          X228          X229
##  6.724184e-03 -1.900792e-02  2.596397e-02  5.163254e-02  4.436549e-02
##          X230          X231          X232          X233          X234
## -5.260988e-02  2.244141e-02  8.109473e-02 -9.347921e-02 -6.427242e-02
##          X235          X236          X237          X238          X239
##  2.367690e-02  6.218407e-03  4.610679e-02  5.583538e-03  9.081019e-02
##          X240          X241          X242          X243          X244
## -6.601586e-02 -2.304738e-02 -2.567316e-02 -3.523254e-03  6.464611e-02
##          X245          X246          X247          X248          X249
##  5.004898e-02  1.287796e-02 -2.498644e-02  2.435091e-02  6.063777e-02
##          X250          X251          X252          X253          X254
## -4.997905e-02 -6.016275e-02  2.688482e-02  6.133215e-02  2.040690e-02
```

```
##           X255          X256          X257          X258          X259 
##   5.081023e-02  1.049660e-01 -2.118347e-02  7.433740e-03  3.648414e-02 
##           X260          X261          X262          X263          X264 
##   1.068442e-02 -6.176403e-03  2.992119e-02  7.482477e-03 -4.159417e-02 
##           X265          X266          X267          X268          X269 
##   4.499439e-02  1.052220e-01 -1.676508e-02 -2.210922e-02 -1.926806e-02 
##           X270          X271          X272          X273          X274 
##   2.232968e-02 -1.266976e-03  1.727994e-02 -1.366974e-02  1.610937e-02 
##           X275          X276          X277          X278          X279 
##   3.264451e-02 -1.810868e-02  1.251296e-01  4.271185e-02  5.163134e-02 
##           X280          X281          X282          X283          X284 
##  -7.847923e-03 -3.222281e-02 -4.128684e-02 -6.186629e-03 -5.114668e-02 
##           X285          X286          X287          X288          X289 
##  -3.864616e-02 -5.120954e-02  2.175819e-02 -4.153327e-02  1.162589e-02 
##           X290          X291          X292          X293          X294 
##  -6.395672e-02  5.165058e-02  1.101114e-02  6.126477e-02  5.852834e-02 
##           X295          X296          X297          X298          X299 
##   1.037729e-01  6.041836e-03  2.251173e-02  9.458879e-02 -1.121838e-02 
##           X300          X301          X302          X303          X304 
##   3.165441e-02  2.382751e-02 -3.358140e-03  5.982766e-02 -2.696259e-03 
##           X305          X306          X307          X308          X309 
##   8.757765e-02 -4.070649e-02  2.260372e-02  2.168582e-02 -6.430610e-02 
##           X310          X311          X312          X313          X314 
##  -3.240309e-02 -3.414193e-02  4.144218e-03  9.302344e-03  1.330254e-02 
##           X315          X316          X317          X318          X319 
##   1.875170e-02  8.753185e-02  2.802896e-03  1.416944e-01  2.086394e-04 
##           X320          X321          X322          X323          X324 
##  -1.887014e-02  8.055645e-02 -1.245736e-02  6.789471e-02 -2.056609e-02 
##           X325          X326          X327          X328          X329 
##   7.893402e-03 -2.393104e-02  5.862184e-02  2.696305e-02 -3.789898e-02 
##           X330          X331          X332          X333          X334 
##  -1.001195e-02  7.401437e-02 -3.348837e-02 -1.657466e-02 -6.817829e-02 
##           X335          X336          X337          X338          X339 
##   1.725469e-02  2.960667e-02 -6.246169e-02 -6.074140e-02  2.616682e-02 
##           X340          X341          X342          X343          X344 
##  -3.659178e-02 -6.441300e-02 -1.456280e-02  2.677478e-02 -2.400717e-02 
##           X345          X346          X347          X348          X349 
##   1.769825e-02  4.380807e-02 -2.142301e-03 -1.553330e-02 -4.518999e-02 
##           X350          X351          X352          X353          X354 
##   1.559659e-03 -7.641393e-02  4.251790e-02 -6.677130e-02 -8.156290e-03 
##           X355          X356          X357          X358          X359 
##   2.140940e-02 -6.366646e-02 -3.312095e-02  1.523818e-01 -5.080567e-02 
##           X360          X361          X362          X363          X364 
##  -4.268811e-02  3.776534e-02  1.240630e-01 -6.995003e-03  8.846592e-03 
##           X365          X366          X367          X368          X369 
##  -1.905698e-03  4.942342e-02 -4.857954e-02 -1.365569e-01  2.744980e-02 
##           X370          X371          X372          X373          X374 
##  -5.170340e-02 -1.392541e-02 -8.745321e-02  2.760037e-02 -6.549281e-02 
##           X375          X376          X377          X378          X379 
##   8.559980e-03 -2.144151e-02  4.146362e-02  5.387358e-02 -1.212799e-01 
##           X380          X381          X382          X383          X384 
```

```
## -3.683541e-02 -3.588156e-02  5.531922e-02 -5.259996e-02  4.275551e-03
##           X385           X386           X387           X388           X389
##  9.249516e-02  3.846356e-02 -3.367228e-02  6.028533e-02  5.131490e-03
##           X390           X391           X392           X393           X394
## -1.435622e-02  2.733393e-02 -3.476768e-03 -3.425775e-02 -5.870483e-02
##           X395           X396           X397           X398           X399
##  1.613948e-02  4.932019e-02  1.272001e-01  7.135422e-02 -2.159864e-02
##           X400           X401           X402           X403           X404
##  6.981039e-02 -3.858001e-02 -3.491469e-02  3.877886e-03 -3.945877e-02
##           X405           X406           X407           X408           X409
## -1.501391e-02 -3.468984e-02 -1.563510e-02 -3.274572e-03  5.635707e-02
##           X410           X411           X412           X413           X414
##  1.767745e-02  2.755630e-02 -1.010898e-02 -3.569879e-02 -5.376168e-02
##           X415           X416           X417           X418           X419
##  1.699869e-02  2.530485e-02  2.367831e-02  3.718306e-02  1.232580e-04
##           X420           X421           X422           X423           X424
## -2.761780e-02  1.406012e-03 -7.273022e-02  6.172031e-02 -4.852892e-02
##           X425           X426           X427           X428           X429
##  5.675246e-02 -2.607580e-02  1.906802e-02 -7.569992e-02 -2.551512e-02
##           X430           X431           X432           X433           X434
##  7.972895e-02 -5.744972e-02 -3.638531e-02 -3.213540e-03 -5.560584e-02
##           X435           X436           X437           X438           X439
##  1.038786e-02  1.683723e-02 -3.690930e-02  6.781736e-02  4.868535e-02
##           X440           X441           X442           X443           X444
## -4.165488e-03 -1.110592e-03 -7.325837e-02 -3.261942e-03  4.723537e-02
##           X445           X446           X447           X448           X449
##  7.345883e-02  3.148358e-02  6.274631e-02 -2.219860e-02  2.207810e-02
##           X450           X451           X452           X453           X454
## -7.287929e-02 -4.126214e-02 -4.494981e-02  8.058441e-03  5.774020e-02
##           X455           X456           X457           X458           X459
##  1.034363e-02 -2.337060e-02  3.813593e-02  1.447165e-02  2.279942e-02
##           X460           X461           X462           X463           X464
##  4.852359e-02  3.703035e-02  1.254287e-01  8.837801e-02  2.093773e-03
##           X465           X466           X467           X468           X469
## -3.442027e-02 -3.647475e-02 -4.185995e-02 -1.974176e-02 -4.951807e-02
##           X470           X471           X472           X473           X474
##  5.376498e-03  1.690554e-02  4.602068e-05 -8.102650e-02 -2.411760e-02
##           X475           X476           X477           X478           X479
## -7.723992e-02 -2.202253e-03 -1.770913e-02  6.271741e-02 -4.240298e-02
##           X480           X481           X482           X483           X484
## -1.649026e-03  5.753745e-02  6.716665e-02 -1.032294e-01  2.742860e-02
##           X485           X486           X487           X488           X489
## -3.855934e-02 -2.815885e-02  8.483368e-03  1.590991e-02  1.078503e-02
##           X490           X491           X492           X493           X494
## -2.538859e-02  5.629614e-03  6.187885e-02  1.555117e-02  4.438323e-02
##           X495           X496           X497           X498           X499
## -2.064228e-02 -2.331412e-02  8.261330e-02 -5.330616e-02 -7.229864e-02
##           X500           X501           X502           X503           X504
##  1.315395e-01 -6.377850e-02 -5.798858e-03 -2.198205e-02 -2.686903e-02
##           X505           X506           X507           X508           X509
## -2.258608e-02  7.575239e-02  2.852933e-02  4.959634e-02 -6.738695e-03
```

```
##          X510          X511          X512          X513          X514
## -4.781362e-02 -7.015072e-02  2.899582e-02  7.041381e-02  7.508980e-03
##          X515          X516          X517          X518          X519
## -4.201484e-02  5.291336e-02  3.729307e-02 -5.783726e-02 -5.693410e-02
##          X520          X521          X522          X523          X524
##  8.497320e-02 -3.628182e-02  6.298211e-02 -3.668400e-02 -3.350182e-02
##          X525          X526          X527          X528          X529
## -2.293385e-02  6.051010e-02 -3.101182e-02 -3.300478e-02  2.821832e-02
##          X530          X531          X532          X533          X534
##  2.209218e-02 -2.838413e-02  5.135393e-02  1.976659e-02  6.583822e-02
##          X535          X536          X537          X538          X539
## -1.663232e-02  3.401808e-02 -1.597987e-02  7.305027e-02 -2.004515e-02
##          X540          X541          X542          X543          X544
## -3.039414e-02  9.764798e-03  3.622685e-02 -2.410946e-02  4.865095e-02
##          X545          X546          X547          X548          X549
##  3.119953e-02 -1.686832e-02  1.075649e-01 -1.610299e-02 -1.236257e-01
##          X550          X551          X552          X553          X554
##  6.693173e-02  9.583006e-03  1.270519e-02  6.120516e-02 -1.881596e-02
##          X555          X556          X557          X558          X559
## -4.684424e-02  4.403463e-02  1.000881e-01 -2.150870e-02 -3.374429e-02
##          X560          X561          X562          X563          X564
## -3.189695e-02 -2.355717e-03  1.300824e-01  8.550405e-02  2.619064e-02
##          X565          X566          X567          X568          X569
## -6.328449e-03  5.161452e-02 -9.933880e-03 -5.164999e-04 -6.470112e-03
##          X570          X571          X572          X573          X574
## -5.864460e-02 -2.240050e-02  7.508363e-02  3.889876e-03  4.102525e-03
##          X575          X576          X577          X578          X579
##  2.373619e-02  1.294633e-01 -7.556885e-03  2.778113e-02  1.123347e-01
##          X580          X581          X582          X583          X584
## -1.026992e-01  3.407560e-02  9.645412e-03  1.916149e-02 -4.485410e-02
##          X585          X586          X587          X588          X589
##  4.920429e-03 -3.752126e-02  2.649285e-02  4.983561e-02  2.800426e-02
##          X590          X591          X592          X593          X594
## -1.428237e-02 -3.844820e-02 -7.316392e-03  4.240627e-03 -8.911309e-03
##          X595          X596          X597          X598          X599
##  3.973491e-02  1.178701e-01 -7.720871e-03  4.038794e-02  6.728378e-02
##          X600          X601          X602          X603          X604
## -2.897725e-02  1.477915e-01 -3.191085e-02 -5.176530e-04  1.060767e-02
##          X605          X606          X607          X608          X609
##  1.305718e-02  3.899152e-02 -1.074894e-02  1.367337e-02 -3.648356e-02
##          X610          X611          X612          X613          X614
##  7.073940e-02  5.260232e-02 -3.243580e-02  2.977031e-03 -8.878343e-02
##          X615          X616          X617          X618          X619
##  4.826371e-02 -3.176779e-03 -4.204940e-02 -2.093391e-02  7.172025e-02
##          X620          X621          X622          X623          X624
## -1.175854e-02  1.365247e-01 -4.190125e-02 -5.648104e-03 -5.535918e-02
##          X625          X626          X627          X628          X629
## -7.030709e-02  4.632193e-02  7.673381e-02 -1.221974e-02  5.057213e-02
##          X630          X631          X632          X633          X634
##  7.518268e-02  6.957426e-02 -7.019493e-02  1.619386e-02 -2.168759e-02
##          X635          X636          X637          X638          X639
```

```
##  3.392340e-02  4.140015e-02 -1.254206e-02  2.524458e-02 -3.706777e-02
##          X640          X641          X642          X643          X644
##  3.390174e-03 -1.523154e-03 -3.459769e-02 -5.865275e-02  8.267947e-02
##          X645          X646          X647          X648          X649
## -6.467757e-02  4.758451e-03  4.822972e-03  7.105190e-02 -2.280398e-02
##          X650          X651          X652          X653          X654
##  6.723283e-03  5.277623e-02  1.495820e-02 -1.138255e-02  4.115144e-02
##          X655          X656          X657          X658          X659
##  4.761195e-02 -4.358562e-02  3.832315e-03  3.060249e-02  1.363510e-03
##          X660          X661          X662          X663          X664
##  5.804950e-02 -3.333648e-04 -1.614620e-02  2.896445e-02  2.069340e-04
##          X665          X666          X667          X668          X669
##  5.702779e-02  1.057120e-02  2.071905e-02 -9.718523e-03 -7.811483e-04
##          X670          X671          X672          X673          X674
## -8.850736e-02 -9.006361e-03 -3.301345e-02 -2.982653e-02 -1.944257e-02
##          X675          X676          X677          X678          X679
##  1.206554e-02 -5.255131e-02  5.657415e-02 -1.584078e-02  8.740665e-03
##          X680          X681          X682          X683          X684
## -4.450857e-02  1.467700e-02 -9.985254e-03  4.273509e-02 -1.789419e-02
##          X685          X686          X687          X688          X689
## -1.832097e-02 -1.848179e-02  3.331465e-02  5.533442e-03  9.735808e-03
##          X690          X691          X692          X693          X694
## -1.746752e-03  1.461180e-02 -2.249471e-02  4.014098e-02 -8.717310e-02
##          X695          X696          X697          X698          X699
## -4.716088e-02  6.981189e-02  3.851831e-03 -9.500737e-03 -5.110246e-02
##          X700          X701          X702          X703          X704
## -1.433139e-02 -3.919722e-02 -2.995089e-02  5.727983e-02  2.251607e-02
##          X705          X706          X707          X708          X709
## -4.498633e-02 -1.183621e-02  2.155812e-02  6.828155e-02 -5.528859e-03
##          X710          X711          X712          X713          X714
## -1.632253e-02  2.031787e-02 -9.833363e-02  4.971139e-02 -3.461545e-02
##          X715          X716          X717          X718          X719
## -2.265060e-02  3.526368e-02  6.387049e-02 -2.882719e-02  1.132460e-02
##          X720          X721          X722          X723          X724
##  2.169412e-02 -6.926310e-02  6.582650e-02  2.914810e-02 -3.876096e-02
##          X725          X726          X727          X728          X729
## -4.394038e-02 -4.923672e-03  2.039611e-02  4.270144e-02 -5.673195e-03
##          X730          X731          X732          X733          X734
## -3.673780e-02  5.415696e-03  5.372396e-02  9.420792e-03  1.398979e-02
##          X735          X736          X737          X738          X739
##  1.613242e-02 -6.673675e-03  9.473852e-03 -2.362723e-02 -6.911000e-03
##          X740          X741          X742          X743          X744
## -3.282014e-02 -1.854860e-02 -1.544703e-03  4.314837e-02  1.321840e-02
##          X745          X746          X747          X748          X749
## -6.900528e-04  2.181383e-02 -1.075672e-02 -1.353829e-02  1.535456e-02
##          X750          X751          X752          X753          X754
## -7.135722e-02 -5.381154e-02 -5.964982e-03 -6.992877e-03  1.352382e-02
##          X755          X756          X757          X758          X759
##  7.012827e-02 -2.224347e-02  1.712620e-02 -5.714053e-03 -1.910433e-02
##          X760          X761          X762          X763          X764
##  5.479634e-02  3.034145e-02 -2.868591e-02 -3.680679e-02 -1.961633e-02
```

```
##          X765          X766          X767          X768          X769 
##  3.576993e-03 -1.151127e-01 -1.876158e-02  3.380367e-03  5.893856e-02 
##          X770          X771          X772          X773          X774 
##  4.502666e-02  1.104148e-02  4.154110e-02  6.829509e-02 -3.357247e-02 
##          X775          X776          X777          X778          X779 
##  1.912706e-03  2.647066e-02  1.454531e-02  4.464222e-02  1.287837e-02 
##          X780          X781          X782          X783          X784 
##  3.494737e-02  1.265006e-02 -1.195310e-02 -5.407062e-02 -8.677286e-03 
##          X785          X786          X787          X788          X789 
## -1.368783e-02  5.774024e-02 -2.943051e-02 -2.535654e-02  5.419085e-02 
##          X790          X791          X792          X793          X794 
## -3.801427e-02 -4.584316e-02 -1.983026e-02  6.355585e-02 -6.106563e-02 
##          X795          X796          X797          X798          X799 
##  1.223160e-02 -1.534376e-03 -5.936168e-02 -1.843167e-02 -3.254771e-02 
##          X800          X801          X802          X803          X804 
##  9.116473e-03 -5.856663e-02 -4.267010e-03  2.513142e-02 -8.324084e-02 
##          X805          X806          X807          X808          X809 
##  1.441487e-02  8.789497e-04  1.650018e-02  8.915709e-02  1.263102e-02 
##          X810          X811          X812          X813          X814 
##  8.659973e-03  1.670368e-02 -4.776582e-02 -3.672885e-03  4.179946e-02 
##          X815          X816          X817          X818          X819 
##  1.069638e-03  1.642991e-02  2.379681e-02  3.779971e-02 -1.518080e-02 
##          X820          X821          X822          X823          X824 
## -6.703634e-02 -5.768249e-03 -2.958030e-02  1.004523e-02  3.139700e-02 
##          X825          X826          X827          X828          X829 
##  4.698676e-02  5.807272e-02  4.772272e-02 -5.795298e-02 -2.142131e-02 
##          X830          X831          X832          X833          X834 
## -2.176816e-02 -7.083455e-03  1.441852e-02 -1.488912e-02 -5.662829e-02 
##          X835          X836          X837          X838          X839 
## -5.055883e-02 -2.817138e-02  2.439850e-02 -1.562503e-02 -4.520425e-02 
##          X840          X841          X842          X843          X844 
##  4.573818e-03 -3.132320e-03 -1.111703e-02 -4.089783e-02  3.554434e-02 
##          X845          X846          X847          X848          X849 
## -4.695234e-02  2.212680e-02  1.620711e-02 -6.565560e-03 -5.749106e-02 
##          X850          X851          X852          X853          X854 
##  1.974847e-02  2.160327e-02 -1.627522e-02 -4.191646e-02 -7.023234e-02 
##          X855          X856          X857          X858          X859 
## -1.300940e-02 -3.918302e-02 -5.579642e-02  3.508213e-02  3.097069e-02 
##          X860          X861          X862          X863          X864 
##  6.012384e-02  5.007037e-02  7.283432e-03 -3.991054e-03 -3.432225e-02 
##          X865          X866          X867          X868          X869 
## -3.484013e-02  6.608920e-02 -7.884873e-02 -2.151452e-02 -5.454756e-03 
##          X870          X871          X872          X873          X874 
##  7.904581e-03 -1.041044e-02 -5.298208e-03 -1.884633e-02  1.296602e-02 
##          X875          X876          X877          X878          X879 
## -6.866586e-02  1.763339e-02 -4.771665e-02  6.554724e-03  1.971925e-03 
##          X880          X881          X882          X883          X884 
##  5.034486e-02 -1.507179e-01  4.118716e-02  1.219052e-03  2.390702e-02 
##          X885          X886          X887          X888          X889 
## -1.552242e-02 -3.218282e-02  8.738379e-02  5.080105e-02 -1.379932e-01 
##          X890          X891          X892          X893          X894 
```

```
## -4.219577e-02  8.351119e-02 -8.301334e-03  8.116669e-03  2.700451e-02
##          X895          X896          X897          X898          X899
##  6.486273e-02  3.574095e-02 -2.190381e-02  4.071088e-02  4.258393e-02
##          X900          X901          X902          X903          X904
## -6.186159e-02  1.200646e-01  1.567217e-02 -1.803865e-02  7.448683e-03
##          X905          X906          X907          X908          X909
##  8.357300e-02 -3.833535e-03 -1.570614e-01  6.623944e-02 -3.123559e-02
##          X910          X911          X912          X913          X914
##  7.753600e-02 -3.260707e-02 -1.050832e-02 -4.012580e-03 -6.948405e-03
##          X915          X916          X917          X918          X919
## -1.181122e-02  6.981476e-02  3.375249e-02 -3.382473e-03  6.522869e-02
##          X920          X921          X922          X923          X924
##  7.362210e-02  9.807710e-02  1.120671e-02  5.166859e-02 -2.208998e-02
##          X925          X926          X927          X928          X929
## -3.749099e-02 -9.995865e-03 -9.152830e-02 -4.861111e-02  3.709477e-02
##          X930          X931          X932          X933          X934
##  1.571583e-02 -3.921777e-02 -8.479789e-02 -3.780521e-03  7.861921e-02
##          X935          X936          X937          X938          X939
## -7.908303e-02 -3.035190e-03 -4.537824e-02 -5.436357e-02 -1.689915e-02
##          X940          X941          X942          X943          X944
##  3.468607e-02  4.148208e-03 -2.772144e-02 -4.366137e-02  5.162481e-02
##          X945          X946          X947          X948          X949
## -8.089292e-03 -1.514661e-03 -1.128415e-01 -3.859648e-02 -5.118973e-02
##          X950          X951          X952          X953          X954
## -3.726109e-02 -2.584366e-02  2.156953e-02 -2.219766e-02 -8.327907e-02
##          X955          X956          X957          X958          X959
## -5.164756e-03 -2.752255e-02  2.486825e-03 -8.100441e-02  1.312424e-01
##          X960          X961          X962          X963          X964
##  1.526268e-02  8.228648e-02 -9.675698e-03 -4.472605e-02 -2.468861e-02
##          X965          X966          X967          X968          X969
## -4.857841e-02  4.152457e-02  5.698161e-02  3.017360e-02  4.876449e-05
##          X970          X971          X972          X973          X974
## -4.636669e-02  1.559010e-02 -7.534974e-02 -4.779904e-03 -3.585161e-03
##          X975          X976          X977          X978          X979
##  4.657299e-02  3.682302e-02  1.307552e-02  4.155421e-02  1.868196e-02
##          X980          X981          X982          X983          X984
## -2.197971e-03  6.951517e-03 -3.537370e-02 -2.563264e-02 -3.283644e-03
##          X985          X986          X987          X988          X989
## -2.859790e-03 -3.856493e-02  8.267037e-02 -5.240554e-03 -3.067957e-02
##          X990          X991          X992          X993          X994
## -3.260888e-02 -1.421226e-02  8.845498e-03  5.306815e-02 -9.883008e-02
##          X995          X996          X997          X998          X999
##  8.922903e-02  4.347686e-03  2.628362e-02  9.635920e-03 -1.881914e-02
##         X1000
## -3.952854e-02
```

```
sqrt(sum(coef(ridge.mod)[, 1]^2))
```

```
## [1] 6.268897
```

```
ridge.pred <- predict(ridge.mod, s = lam, newx = x[test,])
mean((ridge.pred - y.test)^2)
```

```
## [1] 0.02245913
```

Which is the best model?

Since it has the least complexity, the lasso regression model would be best. Even though the test MSE of the lasso model is higher, being at 21.8542, and the ridge model having one of 0.02245913, I suspect that the ridge model likely only has such a small test MSE because it is still overfitting the data (it has 1,000 features after all!) So, we should go with the lasso model, so we may not havep to worry about as much of the potential covariance issues that can occur in high-dimensional datasets; it will hopefully be more generalizable to testing data.

Question 6

```
data(mtcars)
library(boot)
cor(mtcars$mpg, mtcars$wt, method = "spearman")
```

```
## [1] -0.886422
```

```
spearman.func = function(data, i) {
  cor(mtcars$mpg[i], mtcars$wt[i], method = "spearman")
  }

spearman.func (mtcars, 1:10)
```

```
## [1] -0.7339484
```

```
boot_strappin = boot(mtcars, statistic = spearman.func, R = 1000 )
boot_strappin
```

```
##
## ORDINARY NONPARAMETRIC BOOTSTRAP
##
##
## Call:
## boot(data = mtcars, statistic = spearman.func, R = 1000)
##
##
## Bootstrap Statistics :
##      original     bias    std. error
## t1* -0.886422 0.01711517  0.05326715
```

The bias is 0.01375834 and the standard error is 0.05087421. ### Question 7

```
library(class)
data("mtcars")
set.seed(1)
splitt = sort(sample(nrow(mtcars), nrow(mtcars)*.7))
train_mtcars <-mtcars[splitt,]
test_mtcars <-mtcars[-splitt,]
knn.mod = knn(train = train_mtcars, test = test_mtcars, cl=train_mtcars$gear, k = 1)
knn.mod
```

```
##  [1] 3 3 4 3 3 3 3 4 4 5
## Levels: 3 4 5
```

```
table(knn.mod, test_mtcars$gear)
```

```
##
## knn.mod 3 4 5
##       3 5 1 0
##       4 0 2 1
##       5 0 0 1
```

LOOCV to find the best k value:

```
nrow(train_mtcars)
```

```
## [1] 22
```

```
output = NULL
for (i in 1:nrow(train_mtcars)){
  valid = train_mtcars[i,]
  training2 = train_mtcars[-i,]
  knn1 = knn(train = valid, test = training2, cl = valid$gear, k = i)
  output[[i]] = knn1
}
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 2
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 3
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 4
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 5
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 6
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 7
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 8
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 9
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 10
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 11
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 12
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 13
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 14
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 15
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 16
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 17
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 18
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 19
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 20
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 21
## exceeds number 1 of patterns
```

```
## Warning in knn(train = valid, test = training2, cl = valid$gear, k = i): k = 22
## exceeds number 1 of patterns
```

```
output
```

```
## [[1]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[2]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[3]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[4]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[5]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[6]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[7]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[8]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[9]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[10]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[11]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[12]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[13]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
```

```
##
## [[14]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[15]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
##
## [[16]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[17]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[18]]
##  [1] 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3
## Levels: 3
##
## [[19]]
##  [1] 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
## Levels: 5
##
## [[20]]
##  [1] 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
## Levels: 5
##
## [[21]]
##  [1] 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5
## Levels: 5
##
## [[22]]
##  [1] 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
## Levels: 4
```