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RATIONAL EXPECTATIONS AND THE THEORY OF PRICE MOVEMENTS[1]

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In order to explain fairly simply how expectations are formed, we advance the hypothesis that they are essentially the same as the predictions of the relevant economic theory. In particular, the hypothesis asserts that the economy generally does not waste information, and that expectations depend specifically on the structure of the entire system. Methods of analysis, which are appropriate under special conditions, are described in the context of an isolated market with a fixed production lag. The interpretative value of the hypothesis is illustrated by introducing commodity speculation into the system.

1. INTRODUCTION

THAT EXPECTATIONS of economic variables may be subject to error has, for some time, been recognized as an important part of most explanations of changes in the level of business activity. The "ex ante" analysis of the Stockholm School—although it has created its fair share of confusion—is a highly suggestive approach to short-run problems. It has undoubtedly been a major motivation for studies of business expectations and intentions data.

As a systematic theory of fluctuations in markets or in the economy, the approach is limited, however, because it does not include an explanation of the way expectations are formed. To make dynamic economic models complete, various expectations formulas have been used. There is, however, little evidence to suggest that the presumed relations bear a resemblance to the way the economy works.[2]

What kind of information is used and how it is put together to frame an estimate of future conditions is important to understand because the character of dynamic processes is typically very sensitive to the way expectations are influenced by the actual course of events. Furthermore, it is often necessary to make sensible predictions about the way expectations would change when either the amount of available information or the struc-

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[2] This comment also applies to dynamic theories in which expectations do not explicitly appear. See, for example, Arrow, Block, and Hurwicz [**3**, **4**].

ture of the system is changed. (This point is similar to the reason we are curious about demand functions, consumption functions, and the like, instead of only the reduced form "predictors" in a simultaneous equation system.) The area is important from a statistical standpoint as well, because parameter estimates are likely to be seriously biased towards zero if the wrong variable is used as the expectation.

The objective of this paper is to outline a theory of expectations and to show that the implications are—as a first approximation—consistent with the relevant data.

2. THE "RATIONAL EXPECTATIONS" HYPOTHESIS

Two major conclusions from studies of expectations data are the following:

1. Averages of expectations in an industry are more accurate than naive models and as accurate as elaborate equation systems, although there are considerable cross-sectional differences of opinion.

2. Reported expectations generally underestimate the extent of changes that actually take place.

In order to explain these phenomena, I should like to suggest that expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory.[3] At the risk of confusing this purely descriptive hypothesis with a pronouncement as to what firms ought to do, we call such expectations "rational." It is sometimes argued that the assumption of rationality in economics leads to theories inconsistent with, or inadequate to explain, observed phenomena, especially changes over time (e.g., Simon [**29**]). Our hypothesis is based on exactly the opposite point of view: that dynamic economic models do not assume enough rationality.

The hypothesis can be rephrased a little more precisely as follows: that expectations of firms (or, more generally, the subjective probability distribution of outcomes) tend to be distributed, for the same information set, about the prediction of the theory (or the "objective" probability distributions of outcomes).

The hypothesis asserts three things: (1) Information is scarce, and the economic system generally does not waste it. (2) The way expectations are formed depends specifically on the structure of the relevant system describing the economy. (3) A "public prediction," in the sense of Grunberg and Modigliani [**14**], will have no substantial effect on the operation of the economic system (unless it is based on inside information). This is not quite the same thing as stating that the marginal revenue product of economics is zero,

[3] We show in Section 5 that the hypothesis is consistent with these two phenomena.

because expectations of a single firm may still be subject to greater error than the theory.

It *does not* assert that the scratch work of entrepreneurs resembles the system of equations in any way; nor does it state that predictions of entrepreneurs are perfect or that their expectations are all the same.

For purposes of analysis, we shall use a specialized form of the hypothesis. In particular, we assume:

1. The random disturbances are normally distributed.
2. Certainty equivalents exist for the variables to be predicted.
3. The equations of the system, including the expectations formulas, are linear.

These assumptions are not quite so strong as may appear at first because any one of them virtually implies the other two.[4]

3. PRICE FLUCTUATIONS IN AN ISOLATED MARKET

We can best explain what the hypothesis is all about by starting the analysis in a rather simple setting: short-period price variations in an isolated market with a fixed production lag of a commodity which cannot be stored.[5] The market equations take the form

$$
\begin{aligned}
C_t &= -\beta p_t && \text{(Demand)}, \\
P_t &= \gamma p_t^e + u_t, && \text{(Supply)}, \\
P_t &= C_t && \text{(Market equilibrium)},
\end{aligned} \tag{3.1}
$$

where: P_t represents the number of units produced in a period lasting as long as the production lag,

C_t is the amount consumed,

p_t is the market price in the tth period,

p_t^e is the market price expected to prevail during the tth period on the basis of information available through the $(t-1)$'st period,

u_t is an error term—representing, say, variations in yields due to weather.

All the variables used are deviations from equilibrium values.

[4] As long as the variates have a finite variance, a linear regression function exists if and only if the variates are normally distributed. (See Allen [**2**] and Ferguson [**12**].) The certainty-equivalence property follows from the linearity of the derivative of the appropriate quadratic profit or utility function. (See Simon [**28**] and Theil [**32**].)

[5] It is possible to allow both short- and long-run supply relations on the basis of dynamic costs. (See Holt *et al.* [**17**, esp. Chapters 2-4, 19]). More difficult are the supply effects of changes in the number of firms. The relevance of the cost effects has been emphasized by Buchanan [**7**] and Akerman [**1**]. To include them at this point would, however, take us away from the main objective of the paper.

The quantity variables may be eliminated from (3.1) to give

$$p_t = -\frac{\gamma}{\beta} p_t^e - \frac{1}{\beta} u_t . \tag{3.2}$$

The error term is unknown at the time the production decisions are made, but it is known—and relevant—at the time the commodity is purchased in the market.

The prediction of the model is found by replacing the error term by its expected value, conditional on past events. If the errors have no serial correlation and $Eu_t = 0$, we obtain

$$Ep_t = -\frac{\gamma}{\beta} p_t^e . \tag{3.3}$$

If the prediction of the theory were substantially better than the expectations of the firms, then there would be opportunities for the "insider" to profit from the knowledge—by inventory speculation if possible, by operating a firm, or by selling a price forecasting service to the firms. The profit opportunities would no longer exist if the aggregate expectation of the firms is the same as the prediction of the theory:

$$Ep_t = p_t^e . \tag{3.4}$$

Referring to (3.3) we see that if $\gamma/\beta \neq -1$ the rationality assumption (3.4) implies that $p_t^e = 0$, or that the expected price equals the equilibrium price. As long as the disturbances occur only in the supply function, price and quantity movements from one period to the next would be entirely along the demand curve.

The problem we have been discussing so far is of little empirical interest, because the shocks were assumed to be completely unpredictable. For most markets it is desirable to allow for income effects in demand and alternative costs in supply, with the assumption that part of the shock variable may be predicted on the basis of prior information. By retracing our steps from (3.2), we see that the expected price would be

$$p_t^e = -\frac{1}{\beta + \gamma} Eu_t . \tag{3.5}$$

If the shock is observable, then the conditional expected value or its regression estimate may be found directly. If the shock is not observable, it must be estimated from the past history of variables that can be measured.

Expectations with Serially Correlated Disturbances. We shall write the u's as a linear combination of the past history of normally and independently distributed random variables ε_t with zero mean and variance σ^2:

$$u_t = \sum_{i=0}^{\infty} w_i \varepsilon_{t-i}, \qquad E\varepsilon_j = 0, \qquad E\varepsilon_i\varepsilon_j = \begin{cases} \sigma^2 \text{ if } i = j, \\ 0 \text{ if } i \neq j. \end{cases} \tag{3.6}$$

Any desired correlogram in the u's may be obtained by an appropriate choice of the weights w_i.

The price will be a linear function of the same independent disturbances; thus

$$p_t = \sum_{i=0}^{\infty} W_i \varepsilon_{t-i}. \tag{3.7}$$

The expected price given only information through the $(t-1)$'st period has the same form as that in (3.7), with the exception that ε_t is replaced by its expected value (which is zero). We therefore have

$$p_t^e = W_0 E\varepsilon_t + \sum_{i=1}^{\infty} W_i \varepsilon_{t-i} = \sum_{i=1}^{\infty} W_i \varepsilon_{t-i}. \tag{3.8}$$

If, in general, we let $p_{t,L}$ be the price expected in period $t+L$ on the basis of information available through the tth period, the formula becomes

$$p_{t-L,L} = \sum_{i=L}^{\infty} W_i \varepsilon_{t-i}. \tag{3.9}$$

Substituting for the price and the expected price into (3.1), which reflect the market equilibrium conditions, we obtain

$$W_0 \varepsilon_t + \left(1 + \frac{\gamma}{\beta}\right) \sum_{i=1}^{\infty} W_i \varepsilon_{t-i} = -\frac{1}{\beta} \sum_{i=0}^{\infty} w_i \varepsilon_{t-i}. \tag{3.10}$$

Equation (3.10) is an identity in the ε's; that is, it must hold whatever values of ε_j happen to occur. Therefore, the coefficients of the corresponding ε_j in the equation must be equal.

The weights W_i are therefore the following:

$$W_0 = -\frac{1}{\beta} w_0, \tag{3.11a}$$

$$W_i = -\frac{1}{\beta + \gamma} w_i \qquad (i = 1,2,3,\ldots). \tag{3.11b}$$

Equations (3.11) give the parameters of the relation between prices and price expectations functions in terms of the past history of independent shocks. The problem remains of writing the results in terms of the history of observable variables. We wish to find a relation of the form

$$p_t^e = \sum_{j=1}^{\infty} V_j p_{t-j}. \tag{3.12}$$

We solve for the weights V_j in terms of the weights W_j in the following manner. Substituting from (3.7) and (3.8), we obtain

$$\sum_{i=1}^{\infty} W_i \varepsilon_{t-i} = \sum_{j=1}^{\infty} V_j \sum_{i=0}^{\infty} W_i \varepsilon_{t-i-j} = \sum_{i=1}^{\infty} \left(\sum_{j=1}^{i} V_j W_{i-j} \right) \varepsilon_{t-i} . \tag{3.13}$$

Since the equality must hold for all shocks, the coefficients must satisfy the equations

$$W_i = \sum_{j=1}^{i} V_j W_{i-j} \qquad (i = 1,2,3,\ldots) . \tag{3.14}$$

This is a system of equations with a triangular structure, so that it may be solved successively for the coefficients V_1, V_2, $V_3, \ldots$.

If the disturbances are independently distributed, as we assumed before, then $w_0 = -1/\beta$ and all the others are zero. Equations (3.14) therefore imply

$$p_t^e = 0 , \tag{3.15a}$$

$$p_t = p_t^e + W_0 \varepsilon_t = -\frac{1}{\beta} \varepsilon_t . \tag{3.15b}$$

These are the results obtained before.

Suppose, at the other extreme, that an exogenous shock affects all future conditions of supply, instead of only the one period. This assumption would be appropriate if it represented how far technological change differed from its trend. Because u_t is the sum of all the past ε_j, $w_i = 1$ $(i = 0,1,2,\ldots)$. From (3.11),

$$W_0 = -1/\beta , \tag{3.16a}$$

$$W_i = -1/(\beta + \gamma) . \tag{3.16b}$$

From (3.14) it can be seen that the expected price is a geometrically weighted moving average of past prices:

$$p_t^e = \frac{\beta}{\gamma} \sum_{j=1}^{\infty} \left(\frac{\gamma}{\beta + \gamma} \right)^j p_{t-j} . \tag{3.17}$$

This prediction formula has been used by Nerlove [**26**] to estimate the supply elasticity of certain agricultural commodities. The only difference is that our analysis states that the "coefficient of adjustment" in the expectations formula should depend on the demand and the supply coefficients. The geometrically weighted moving average forecast is, in fact, optimal under slightly more general conditions (when the disturbance is composed of both permanent and transitory components). In that case the coefficient will depend on the relative variances of the two components as well as the supply and demand coefficients. (See [**24**].)

Deviations from Rationality. Certain imperfections and biases in the expectations may also be analyzed with the methods of this paper. Allowing for cross-sectional differences in expectations is a simple matter, because their aggregate effect is negligible as long as the deviation from the rational forecast for an individual firm is not strongly correlated with those of the others. Modifications are necessary only if the correlation of the errors is large and depends systematically on other explanatory variables. We shall examine the effect of over-discounting current information and of differences in the information possessed by various firms in the industry. Whether such biases in expectations are empirically important remains to be seen. I wish only to emphasize that the methods are flexible enough to handle them.

Let us consider first what happens when expectations consistently over- or under-discount the effect of current events. Equation (3.8), which gives the optimal price expectation, will then be replaced by

$$p_t^e = f_1 W_1 \varepsilon_{t-1} + \sum_{i=2}^{\infty} W_i \varepsilon_{t-i} . \tag{3.18}$$

In other words the weight attached to the most recent exogenous disturbance is multiplied by the factor f_1, which would be greater than unity if current information is over-discounted and less than unity if it is under-discounted.

If we use (3.18) for the expected price instead of (3.8) to explain market price movements, then (3.11) is replaced by

$$W_0 = -\frac{1}{\beta} w_0 , \tag{3.19a}$$

$$W_1 = -\frac{1}{\beta + f_1 \gamma} w_1 , \tag{3.19b}$$

$$W_i = -\frac{1}{\beta + \gamma} w_i \qquad (i = 2,3,4, \ldots) . \tag{3.19c}$$

The effect of the biased expectations on price movements depends on the statistical properties of the exogenous disturbances.

If the disturbances are independent (that is, $w_0 = 1$ and $w_i = 0$ for $i \geqslant 1$), the biased expectations have no effect. The reason is that successive observations provide no information about future fluctuations.

On the other hand, if all the disturbances are of a permanent type (that is, $w_0 = w_1 = \ldots = 1$), the properties of the expectations function are significantly affected. To illustrate the magnitude of the differences, the parameters of the function

$$p_t^e = \sum_{j=1}^{\infty} V_j p_{t-j}$$

are compared in Figure 3.1 for $\beta = 2\gamma$ and various values of f_1. If current information is under-discounted ($f_1 = 1/2$), the weight V_1 attached to the latest observed price is very high. With over-discounting ($f_1 = 2$), the weight for the first period is relatively low.

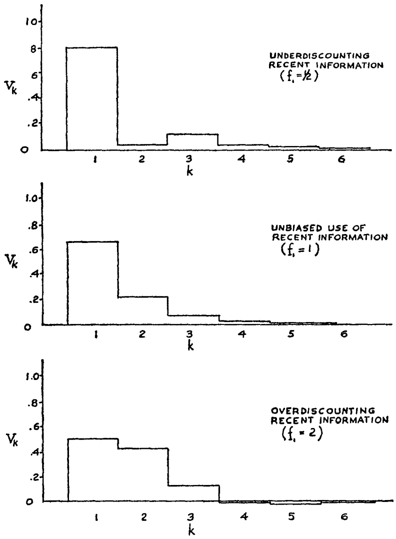

FIGURE 3.1.—Autoregression Coefficients of Expectations for Biased Use of Recent Information. ($w_0 = w_1 = \ldots = 1$).

The model above can be interpreted in another way. Suppose that some of the firms have access to later information than the others. That is, there is a lag of one period for some firms, which therefore form price expectations according to (3.8). The others, with a lag of two periods, can only use the following:

$$p_t^{e'} = \sum_{i=2}^{\infty} W_i \varepsilon_{t-i} \,. \tag{3.20}$$

Then the aggregate price expectations relation is the same as (3.18), if f_1 represents the fraction of the firms having a lag of only one period in obtaining market information (that is, the fraction of "insiders").

4. EFFECTS OF INVENTORY SPECULATION

Some of the most interesting questions involve the economic effects of inventory storage and speculation. We can examine the effect by adjoining to (3.1) an inventory demand equation depending on the difference between the expected future price and the current price. As we shall show, the price expectation with independent disturbances in the supply function then turns out to have the form

$$p_t^e = \lambda p_{t-1}, \tag{4.1}$$

where the parameter λ would be somewhere between zero and one, its value depending on the demand, supply, and inventory demand parameters.

Speculation with moderately well-informed price expectations reduces the variance of prices by spreading the effect of a market disturbance over several time periods, thereby allowing shocks partially to cancel one another out. Speculation is profitable, although no speculative opportunities remain. These propositions might appear obvious. Nevertheless, contrary views have been expressed in the literature.[6]

Before introducing inventories into the market conditions, we shall briefly examine the nature of speculative demand for a commodity.

Optimal Speculation. We shall assume for the time being that storage, interest, and transactions costs are negligible. An individual has an opportunity to purchase at a known price in the tth period for sale in the succeeding period. The future price is, however, unknown. If we let I_t represent the speculative inventory at the end of the tth period,[7] then the profit to be realized is

$$\pi_t = I_t(p_{t+1} - p_t). \tag{4.2}$$

Of course, the profit is unknown at the time the commitment is to be made. There is, however, the expectation of gain.

The individual demand for speculative inventories would presumably be based on reasoning of the following sort. The size of the commitment depends on the expectation of the utility of the profit. For a sufficiently small range of variation in profits, we can approximate the utility function by the first few terms of its Taylor's series expansion about the origin:

$$u_t = \phi(\pi_t) = \phi(0) + \phi'(0)\pi_t + \frac{1}{2}\phi''(0)\pi_t^2 + \dots. \tag{4.3}$$

The expected utility depends on the moments of the probability distribution of π:

$$Eu_t = \phi(0) + \phi'(0)E\pi_t + \frac{1}{2}\phi''(0)E\pi_t^2 + \dots. \tag{4.4}$$

[6] See Baumol [**5**]. His conclusions depend on a nonspeculative demand such that prices would be a pure sine function, which may always be forecast perfectly.

[7] Speculative inventories may be either positive or negative.

From (4.2) the first two moments may be found to be

$$E\pi_t = I_t(p^e_{t+1} - p_t)\,, \tag{4.5a}$$

$$E\pi_t^2 = I_t^2[\sigma^2_{t,1} + (p^e_{t+1} - p_t)^2]\,, \tag{4.5b}$$

where p^e_{t+1} is the conditional mean of the price in period $t+1$ (given all information through period t) and $\sigma^2_{t,1}$ is the conditional variance. The expected utility may therefore be written in terms of the inventory position as follows:

$$Eu_t = \phi(0) + \phi'(0)I_t(p^e_{t+1} - p_t) + \frac{1}{2}\phi''(0)I_t^2[\sigma^2_{t,1} + (p^e_{t+1} - p_t)^2] + \ldots. \tag{4.6}$$

The inventory therefore satisfies the condition

$$\frac{dEu}{dI_t} = \phi'(0)(p^e_{t+1} - p_t) + \phi''(0)I_t[\sigma^2_{t,1} + (p^e_{t+1} - p_t)^2] + \ldots = 0\,. \tag{4.7}$$

The inventory position would, to a first approximation, be given by

$$I_t = -\frac{\phi'(0)(p^e_{t+1} - p_t)}{\phi''(0)[\sigma^2_{t,1} + (p^e_{t+1} - p_t)^2]}\,. \tag{4.8}$$

If $\phi'(0) > 0$ and $\phi''(0) < 0$, the above expression is an increasing function of the expected change in prices (as long as it is moderate).

At this point we make two additional assumptions: (1) the conditional variance, $\sigma^2_{t,1}$, is independent of p^e_t, which is true if prices are normally distributed, and (2) the square of the expected price change is small relative to the variance. The latter assumption is reasonable because the original expansion of the utility function is valid only for small changes. Equation (4.8) may then be simplified to[8]

$$I_t = \alpha(p^e_{t+1} - p_t)\,, \tag{4.9}$$

where $\alpha = -\phi'(0)/\phi''(0)\sigma^2_{t,1}$.

Note that the coefficient α depends on the commodity in only one way: the variance of price forecasts. The aggregate demand would, in addition, depend on who holds the stocks as well as the size of the market. For some commodities, inventories are most easily held by the firms.[9] If an organized futures exchange exists for the commodity, a different population would

[8] This form of the demand for speculative inventories resembles that of Telser [**31**] and Kaldor [**20**].

[9] Meat, for example, is stored in the live animals or in any curing or ageing process.

be involved. In a few instances (in particular, durable goods), inventory accumulation on the part of households may be important.

The original assumptions may be relaxed, without affecting the results significantly, by introducing storage or interest costs. Margin requirements may, as well, limit the long or short position of an individual. Although such requirements may primarily limit cross-sectional differences in positions, they may also constrain the aggregate inventory. In this case, we might reasonably expect the aggregate demand function to be nonlinear with an upper "saturation" level for inventories. (A lower level would appear for aggregate inventories approaching zero.)

Because of its simplicity, however, we shall use (4.9) to represent inventory demand.

Market Adjustments. We are now in a position to modify the model of Section 3 to take account of inventory variations. The ingredients are the supply and demand equations used earlier, together with the inventory equation. We repeat the equations below (P_t represents production and C_t consumption during the tth period:

(4.10a) $$C_t = -\beta p_t \qquad \text{(Demand)},$$

(4.10b) $$P_t = \gamma p_t^e + u_t \qquad \text{(Supply)},$$

(4.10c) $$I_t = \alpha(p_{t+1}^e - p_t) \qquad \text{(Inventory speculation)}.$$

The market equilibrium conditions are

(4.11) $$C_t + I_t = P_t + I_{t-1}.$$

Substituting (4.10) into (4.11), the equilibrium can be expressed in terms of prices, price expectations, and the disturbance, thus

(4.12) $$-(\alpha+\beta)p_t + \alpha p_{t+1}^e = (\alpha+\gamma)p_t^e - \alpha p_{t-1} + u_t.$$

The conditions above may be used to find the weights of the regression functions for prices and price expectations in the same way as before. Substituting from (3.6), (3.7), and (3.8) into (4.12), we obtain

(4.13) $$\begin{aligned} &-(\alpha+\beta)\sum_{i=0}^{\infty} W_i \varepsilon_{t-i} + \alpha \sum_{i=1}^{\infty} W_i \varepsilon_{t+1-i} \\ &= (\alpha+\gamma)\sum_{i=1}^{\infty} W_i \varepsilon_{t-i} - \alpha \sum_{i=0}^{\infty} W_i \varepsilon_{t-1-i} + \sum_{i=0}^{\infty} w_i \varepsilon_{t-i}. \end{aligned}$$

In order that the above equation hold for all possible ε's, the corresponding coefficients must, as before, be equal. Therefore, the following system of equations must be satisfied:[10]

(4.14a) $$-(\alpha+\beta)W_0+\alpha W_1=w_0\,,$$

(4.14b) $$\alpha W_{i-1}-(2\alpha+\beta+\gamma)W_i+\alpha W_{i+1}=w_i \qquad (i=1,2,3,\ldots)\,.$$

Provided it exists, the solution of the homogeneous system would be of the form

(4.15) $$W_k=c\lambda_1^k\,,$$

where λ_1 is the smaller root of the characteristic equation

(4.16) $$\alpha-(2\alpha+\beta+\gamma)\lambda+\alpha\lambda^2=\alpha(1-\lambda)^2-(\beta+\gamma)\lambda=0\,.$$

λ_1 is plotted against positive values of $\alpha/(\beta+\gamma)$ in Figure 4.1.

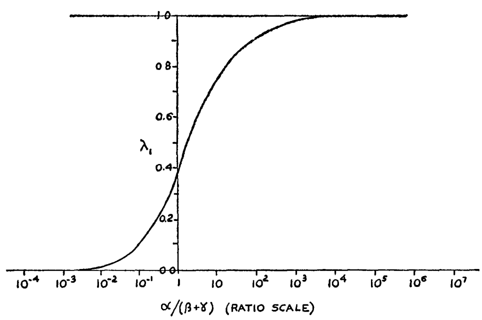

FIGURE 4.1.—Characteristic Root as a Function of $\alpha/(\beta+\gamma)$.

A unique, real, and bounded solution to (4.14) will exist if the roots of the characteristic equation are real. The roots occur in reciprocal pairs, so that if they are real and distinct exactly one will have an absolute value less than unity. For a bounded solution the coefficient of the larger root vanishes; the initial condition is then fitted to the coefficient of the smaller root.

The response of the price and quantity variables will be dynamically stable, therefore, if the roots of the characteristic equation are real. It is easy to see that they will be real if the following inequalities are satisfied:

(4.17a) $$\alpha>0\,,$$

(4.17b) $$\beta+\gamma>0\,.$$

The first condition requires that speculators act in the expectation of gain (rather than loss). The second is the condition for Walrasian stability. Hence an assumption about dynamic stability implies rather little about

[10] The same system appears in various contexts with embarrassing frequency. See Holt *et al.* [**17**] and Muth [**24**].

the demand and supply coefficients. It should be observed that (4.17) are not necessary conditions for stability. The system will also be stable if both inequalities in (4.17) are reversed (!) or if $0 > \alpha/(\beta+\gamma) > -1/4$. If $\alpha = 0$, there is no "linkage" from one period of time to another, so the system is dynamically stable for all values of $\beta+\gamma$.

Suppose, partly by way of illustration, that the exogenous disturbances affecting the market are independently distributed. Then we can let $w_0 = 1$ and $w_i = 0$ $(i \geqslant 1)$. The complementary function will therefore be the complete solution to the resulting difference equation. By substituting (4.15) into (4.14a), we evaluate the constant and find

$$W_k = -\frac{1}{(\alpha+\beta)-\alpha\lambda_1}\lambda_1^k . \tag{4.18}$$

The weights V_k may be found either from (3.14) or by noting that the resulting stochastic process is Markovian. At any rate, the weights are

$$V_k = \begin{cases} \lambda_1, & k = 1 , \\ 0, & k > 1 . \end{cases} \tag{4.19}$$

The expected price is therefore correlated with the previous price, and the rest of the price history conveys no extra information, i.e.,

$$p_t^e = \lambda_1 p_{t-1} , \tag{4.20}$$

where the parameter depends on the coefficients of demand, supply, and inventory speculation according to (4.16) and is between 0 and 1. If inventories are an important factor in short-run price determination, λ_1 will be very nearly unity so that the time series of prices has a high positive serial correlation.[11] If inventories are a negligible factor, λ_1 is close to zero and leads to the results of Section 3.

Effects of Inventory Speculation. Substituting the expected price, from (4.20), into (4.10), we obtain the following system to describe the operation of the market:

$$C_t = -\beta p_t , \tag{4.21a}$$

$$P_t = \gamma\lambda_1 p_{t-1} + \varepsilon_t , \tag{4.21b}$$

$$I_t = -\alpha(1-\lambda_1) p_t . \tag{4.21c}$$

The market conditions can be expressed in terms of supply and demand by including the inventory carryover with production and inventory carry-

[11] If the production and consumption flows are negligible compared with the speculative inventory level, the process approaches a random walk. This would apply to daily or weekly price movements of a commodity whose production lag is a year. Cf. Kendall [**22**].

TABLE 4.1

EFFECTS OF INVENTORY SPECULATION

Description	Symbol	General Formula	Approximation for Small α
1. Characteristic root	λ_1	[eq. (4.16)]	$\alpha/(\beta+\gamma)$
2. Standard deviation of prices	σ_p	$\lvert W_0\rvert(1-\lambda_1^2)^{-1/2}\sigma$	$\frac{1}{\beta}\left(1-\frac{\alpha}{\beta}\right)\sigma$
3. Standard deviation of expected price	σ_p^e	$\lambda_1\sigma_p$	$\frac{\alpha}{\beta(\beta+\gamma)}\sigma$
4. Standard deviation of output	σ_P	$(\sigma^2+\gamma^2\lambda_1^2\sigma_p^2)^{1/2}$	$\left[1+\frac{\alpha\gamma}{2\beta(\beta+\gamma)}\right]\sigma$
5. Mean producers' revenue	$EP_t p_t$	$\gamma\lambda_1^2\sigma_p^2+W_0\sigma^2$	$-\frac{1}{\beta}\left(1-\frac{\alpha}{\beta}\right)\sigma^2$
6. Mean speculators' revenue	$EI_t(p_{t+1}-p_t)$	$\alpha(1-\lambda_1)^2\sigma_p^2$	$\alpha\sigma^2$
7. Mean consumers' expenditure	$EC_t p_t$	$-\beta\sigma_p^2$	$-\frac{1}{\beta}\left(1-\frac{2\alpha}{\beta}\right)\sigma^2$

Notes: (1) σ is the standard deviation of the disturbance in the supply function (4.10b) with $w_0 = 1$ and $w_1 = w_2 = \ldots = 0$.
(2) $W_0 = -1/[\beta+\alpha(1-\lambda_1)]$.

forward with consumption; thus,

$$\text{(4.22)}\qquad \begin{aligned} Q_t &= C_t + I_t \qquad \text{(Demand)},\\ Q_t &= P_t + I_{t-1} \qquad \text{(Supply)}. \end{aligned}$$

Substituting from (4.21) we obtain the system:

$$\text{(4.23a)}\qquad Q_t = -[\beta + \alpha(1-\lambda_1)]p_t \qquad \text{(Demand)},$$

$$\text{(4.23b)}\qquad Q_t = [\gamma\lambda_1 - \alpha(1-\lambda_1)]p_{t-1} + \varepsilon_t \qquad \text{(Supply)}.$$

The coefficient in the supply equation is reduced while that of the demand equation is increased. The conclusions are not essentially different from those of Hooton [**18**]. The change is always enough to make the dynamic response stable.

If price expectations are in fact rational, we can make some statements about the economic effects of commodity speculation. (The relevant formulas are summarized in Table 4.1.) Speculation reduces the variance of prices by spreading the effect of a disturbance over several time periods. From Figure 4.2, however, we see that the effect is negligible if α is much less than the sum of β and γ. The standard deviation of expected prices first increases with α because speculation makes the time series more predictable and then decreases because of the small variability of actual prices. The variability of factor inputs and production follows roughly the same pattern (cf. Kaldor [**20**]).

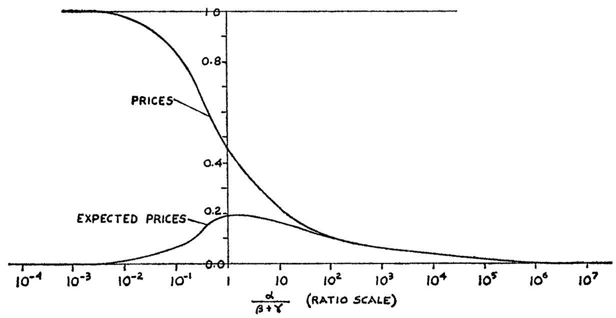

FIGURE 4.2.—Standard Deviation of Prices and Expected Prices as a Function of $\alpha/(\beta+\gamma)$ for $\beta = \gamma$.

In Figure 4.3 we see that mean income to speculators is always positive and has a maximum value slightly to the left of that for expected prices. Producers' revenue and consumers' expenditures both increase with α. Consumers' expenditures increase at first a little faster than the revenue of the producers. The effect of speculation on welfare is therefore not obvious.

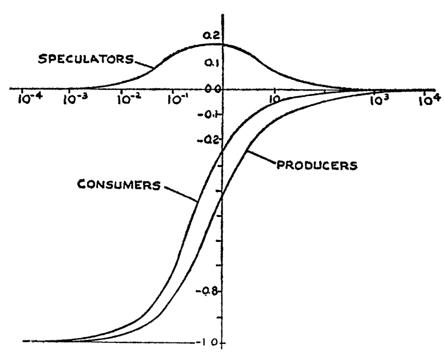

FIGURE 4.3.—Mean Income of Producers and Speculators, and Mean Expenditures of Consumers as a Function of $\alpha/(\beta+\gamma)$ for $\beta = \gamma$.

The variability of prices for various values of γ/β is plotted as a function of α/β in Figure 4.4. The general shape of the curve is not affected by values of γ/β differing by as much as a factor of 100. The larger the supply coefficient, however, the sharper is the cut-off in price variability.

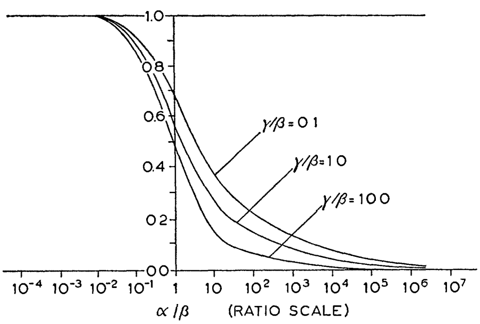

FIGURE 4.4—Standard Deviation of Prices for Various Values of γ/β as a Function of α/β.

5. RATIONALITY AND COBWEB THEOREMS

It is rather surprising that expectations have not previously been regarded as rational dynamic models, since rationality is assumed in all other aspects of entrepreneurial behavior. From a purely theoretical standpoint, there are good reasons for assuming rationality. First, it is a principle applicable to all dynamic problems (if true). Expectations in different markets and systems would not have to be treated in completely different ways. Second, if expectations were not moderately rational there would be opportunities for economists to make profits in commodity speculation, running a firm, or selling the information to present owners. Third, rationality is an assumption that can be modified. Systematic biases, incomplete or incorrect information, poor memory, etc., can be examined with analytical methods based on rationality.

The only real test, however, is whether theories involving rationality explain observed phenomena any better than alternative theories. In this section we shall therefore compare some of the empirical implications of the rational expectations hypothesis with those of the cobweb "theorem." The effects of rational expectations are particularly important because the cobweb theorem has often been regarded as one of the most successful attempts at dynamic economic theories (e.g., Goodwin [**13**]). Few students of agricultural problems or business cycles seem to take the cobweb theorem very seriously, however, but its implications do occasionally appear. For example, a major cause of price fluctuations in cattle and hog markets is sometimes believed to be the expectations of farmers themselves (Jesness [**19**]). Dean and Heady [**10**] have also suggested more extensive governmental forecasting and outlook services in order to offset an increasing tendency toward instability of hog prices due to a secular decrease in the elasticity of demand.

Implications of Cobweb Theorems. If the market equilibrium conditions of (3.1) are subjected to independent shocks in the supply function, the prediction of the theory would be

$$E(p_t \mid p_{t-1}, p_{t-2}, \ldots) = -\frac{\gamma}{\beta} p_t^e . \tag{5.1}$$

As a result, the prediction of the cobweb theory would ordinarily have the sign opposite to that of the firms. This, of course, has been known for a long time. Schultz noted that the hypothesis implies farmers do not learn from experience, but added: "Such a behavior is not to be ruled out as extremely improbable" [**27**, p. 78].

The various theories differ primarily in what is assumed about price expectations. The early contributors (through Ezekiel [**11**]) have assumed that the expected price is equal to the latest known price. That is,

$$p_t^e = p_{t-1} . \tag{5.2}$$

Goodwin [**13**] proposed the extrapolation formula,

$$p_t^e = p_{t-1} - \varrho(p_{t-1} - p_{t-2}) . \tag{5.3}$$

That is, a certain fraction of the latest change is added on to the latest observed price. Depending on the sign of ϱ, which should be between -1 and $+1$, we can get a greater variety of behavior. It is still the case, however, that farmers' expectations and the prediction of the model have the opposite sign.

A third expectations formula is much more recent. The adaptive expectations model, used by Nerlove [**25**], satisfies the following equation:

$$p_t^e = p_{t-1}^e + \eta(p_{t-1} - p_{t-1}^e) . \tag{5.4}$$

The forecast is changed by an amount proportional to the most recently observed forecast error. The solution of the difference equation gives the price expectation as a geometrically weighted moving average:

$$p_t^e = \eta \sum_{j=0}^{\infty} (1-\eta)^j p_{t-j} . \tag{5.5}$$

Certain properties of the cobweb models are compared with the rational model in Table 5.1 for shocks having no serial correlation. Such comparisons are a little treacherous because most real markets have significant income effects in demand, alternative costs in supply, and errors in both behavioral equations. To the extent that these effects introduce positive serial correlation in the residuals, the difference between the cobweb and rational models would be diminished. Subject to these qualifications, we shall compare the two kinds of models according to the properties of firms' expectations and the cyclical characteristics of commodity prices and output.

Expectations of Firms. There is some direct evidence concerning the quality of expectations of firms. Heady and Kaldor [**16**] have shown that, for the period studied, average expectations were considerably more

TABLE 5.1
PROPERTIES OF COBWEB MODELS

	Expectation p_t^e	Prediction $E(p_t \mid p_{t-1}, \ldots)$	Stability Conditions
(A) Classical (Schultz-Tinbergen-Ricci)	p_{t-1}	$-\frac{\gamma}{\beta} p_t^e$	$\gamma < \beta$
(B) Extrapolative (Goodwin)	$(1-\varrho)p_{t-1} + \varrho p_{t-2}$ $(-1 < \varrho < 1)$	$-\frac{\gamma}{\beta} p_t^e$	$\frac{\gamma}{\beta} < \begin{cases} \frac{1}{1-2\varrho}, & \varrho \leqslant \frac{1}{3} \\ \frac{1}{\varrho}, & \varrho \geqslant \frac{1}{3} \end{cases}$
(C) Adaptive (Nerlove)	$\eta \sum_{j=1}^{\infty} (1-\eta)^{j-1} p_{t-j}$ $(0 < \eta < 1)$	$-\frac{\gamma}{\beta} p_t^e$	$\frac{\gamma}{\beta} < \frac{2}{\eta} - 1$
(D) Rational	0	0	$\beta + \gamma \neq 0$
(E) Rational (with speculation)	$\lambda_1 p_{t-1}$ $(0 < \lambda_1 < 1)$	$\lambda_1 p_{t-1}$	$\alpha > 0$ $\beta + \gamma > 0$

Note: The disturbances are normally and independently distributed with a constant variance.

accurate than simple extrapolation, although there were substantial cross-sectional differences in expectations. Similar conclusions concerning the accuracy have been reached, for quite different expectational data, by Modigliani and Weingartner [**23**].

If often appears that reported expectations underestimate the extent of changes that actually take place. Several studies have tried to relate the two according to the equation:

$$p_t^e = b p_t + v_t', \tag{5.6}$$

where v_t' is a random disturbance. Estimated values of b are positive, but less than unity (see, e.g., Theil [**33**]). Such findings are clearly inconsistent with the cobweb theory, which ordinarily requires a negative coefficient. We shall show below that they are generally consistent with the rational expectations hypothesis.

Bossons and Modigliani [**6**] have pointed out that the size of the estimated coefficient, $\hat{b}$, may be explained by a regression effect. Its relevance may be seen quite clearly as follows. The rational expectations hypothesis states that, in the aggregate, the expected price is an unbiased predictor of the actual price. That is,

$$p_t = p_t^e + v_t\,, \qquad Ep_t^e v_t = 0\,, \qquad Ev_t = 0\,. \tag{5.7}$$

The probability limit of the least squares estimate of b in (5.6) would then be given by

$$\text{Plim}\ \hat{b} = (\text{Var}\ p^e)/(\text{Var}\ p) < 1\,. \tag{5.8}$$

Cycles. The evidence for the cobweb model lies in the quasi-periodic fluctuations in prices of a number of commodities. The hog cycle is perhaps the best known, but cattle and potatoes have sometimes been cited as others which obey the "theorem." The phase plot of quantity with current and lagged price also has the appearance which gives the cobweb cycle its name.

A dynamic system forced by random shocks typically responds, however, with cycles having a fairly stable period. This is true whether or not any characteristic roots are of the oscillatory type. Slutzky [**30**] and Yule [**34**] first showed that moving-average processes can lead to very regular cycles. A comparison of empirical cycle periods with the properties of the solution of a system of differential or difference equations can therefore be misleading whenever random shocks are present (Haavelmo [**15**]).

The length of the cycle under various hypotheses depends on how we measure the empirical cycle period. Two possibilities are: the interval

TABLE 5.2

CYCLICAL PROPERTIES OF COBWEB MODELS

	Serial Correlation Of Prices, r_1	Mean Interval Between Successive Upcrosses, L	Mean Interval Between Successive Peaks or Troughs, L'
(A) Classical	$r_1 = -\frac{\gamma}{\beta} < 0$		
(B) Extrapolative	$r_1 = \frac{-\gamma(1-\varrho)}{\beta + \gamma\varrho} < 0$	$2 \leqslant L \leqslant 4$	$2 \leqslant L' \leqslant 3$
(C) Adaptive	$-\frac{\eta\gamma}{\beta} \leqslant r_1 \leqslant 0$		
(D) Rational	$r_1 = 0$	$L = 4$	$L' = 3$
(E) Rational - with storage	$r_1 = \lambda_1 > 0$	$L > 4$	$3 \leqslant L' \leqslant 4$

Note: The disturbances are assumed to be normally and independently distributed with a constant variance. β and γ are both assumed to be positive.

between successive "upcrosses" of the time series (i.e., crossing the trend line from below), and the average interval between successive peaks or troughs. Both are given in Table 5.2, which summarizes the serial correlation of prices and mean cycle lengths for the various hypotheses.[12]

That the observed hog cycles were too long for the cobweb theorem was first observed in 1935 by Coase and Fowler [**8**, **9**]. The graph of cattle prices presented given by Ezekiel [**11**] as evidence for the cobweb theorem implies an extraordinarily long period of production (five to seven years). The interval between successive peaks for other commodities tends to be longer than three production periods. Comparisons of the cycle lengths should be interpreted cautiously because they do not allow for positive serial correlation of the exogenous disturbances. Nevertheless, they should not be construed as supporting the cobweb theorem.

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