wave period. Since ΔR is minimum for the chosen values of C_M and C_D, we

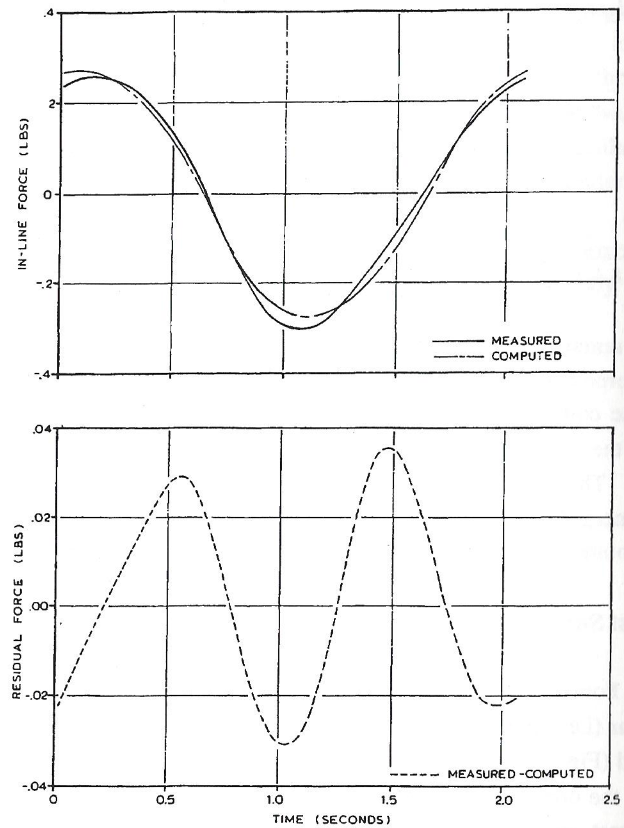

FIGURE 10.15
MEASURED FORCE VERSUS COMPUTED FORCE; RESIDUAL UNACCOUNTED FORCE

set to zero the derivatives of ΔR with respect to C_M and C_D respectively. This provides two equations in the two unknowns C_M and C_D. Solving for C_M and C_D from these equations, the following expressions are derived (the summations of the products in these expressions run from 1 to N, i.e. use one cycle of wave data).

$$C_M = \frac{\sum f_n T_n' \sum (T_n^D)^2 - \sum f_n T_n^D \sum T_n' T_n^D}{\sum (T_n')^2 \sum (T_n^D)^2 - \sum (T_n' T_n^D)^2} \tag{10.77}$$

$$C_D = \frac{\sum f_n T_n^D \sum (T_n^D)^2 - \sum f_n T_n' \sum T_n' T_n^D}{\sum (T_n')^2 \sum (T_n^D)^2 - \sum (T_n' T_n^D)^2} \tag{10.78}$$

For a sinusoidal description of the wave kinematics, either of the two methods can be used and should provide similar results.

10.8 FREE VIBRATION TESTS

Any test setup in the wave tank, whether for load tests or motion tests, may be treated as a spring mass system. Therefore, valuable information may be obtained from the free vibration of the system. For fixed structures, the vibration frequency determines if problem will be encountered from the dynamic amplification of the system (Section 7.5.7). For floating structures, information regarding the system natural period and damping may be determined from the vibration analysis. The data analysis is similar in both cases and is described below.

10.8.1 Low Frequency Hydrodynamic Coefficients

The magnitude of damping determines the extent of motions and corresponding mooring loads in a moored floating platform near its natural frequency (refer to Section 9.8). The free oscillation of the platform takes place at the natural frequency. In an experimental setup, this oscillation may be easily measured when the platform is disturbed from its equilibrium position. The platform returns to its equilibrium position and the duration of oscillation depends strictly on the damping of the system.

10.8.1.1 Linear System

The low-frequency hydrodynamic coefficients of the platform in still water can be determined from the recorded extinction curve. The equation of motion is described by a second-order differential equation having a single degree of freedom:

$$(M_0 + M_a)\ddot{x} + C\dot{x} + Kx = 0 \tag{10.79}$$

where x is the surge amplitude and dots represent first and second derivatives, and M_0 and K are the structure displacement (mass) and linear spring constant of the spring set, respectively. These quantities are measured directly. The quantities M_a and C are the added mass and linear damping coefficients, respectively. They are considered to be functions of the frequency of oscillation, ω_d. Note that in this case, ω_d is the damped natural frequency of the system. Values for M_a and C are determined in the following manner. By assuming a solution to Eq. 10.79 of the form $x=e^{st}$ and defining $M = M_0 + M_a$, the equation can be rewritten in the form

$$\left(s^2 + \frac{C}{M}s + \frac{K}{M}\right)e^{st} = 0 \tag{10.80}$$

and thus

$$s_{1,2} = -\frac{C}{2M} \pm \sqrt{\left(\frac{C}{2M}\right)^2 - \frac{K}{M}} \tag{10.81}$$

The damping factor, ζ, is defined as the ratio of the amount of damping C present in the system to that amount of damping, C_c, which will cause the part of the equation under the radical to go to zero. Therefore,

$$\frac{C_c}{2M} = \sqrt{\frac{K}{M}} = \omega_N \tag{10.82}$$

and

$$\frac{C}{2M} = \zeta\omega_N \tag{10.83}$$

In case of light damping, the radical is imaginary and Eq. 10.81 can be written as

$$s_{1,2} = \left[-\zeta \pm i\sqrt{1-\zeta^2}\right]\omega_N \tag{10.84}$$

The general solution to Eq. 10.79 is

$$x = x_0 \exp(-\zeta\omega_N t)\sin\left[\sqrt{1-\zeta^2}\,\omega_N t + \varepsilon\right] \tag{10.85}$$

in which x_0 is the magnitude of oscillation at t=0 and ε is its phase angle.

The solution of the equation of motion given by Eq. 10.85 represents harmonic oscillation values of subsequent amplitudes of oscillation in which the amplitudes decay exponentially. If two consecutive absolute values are given by $|x_k|$ and $|x_{k-1}|$, then the logarithmic decrement is defined as

$$\delta = \ln|x_k| - \ln|x_{k-1}| \tag{10.86}$$

which gives

$$\zeta = \frac{\delta}{\sqrt{\pi^2 + \delta^2}} \tag{10.87}$$

The logarithmic decrement may be quite accurately related to the damping factor simply by

$$\delta = \pi\zeta \tag{10.88}$$

For small values of ζ the error is small (for example, for $\zeta = 0.1$, the error is about 0.5 percent).

The term $x_0\exp(-\zeta\omega_N t)$ represents the curve that can be drawn through the succeeding peaks of the damped oscillation. Strictly speaking, the curve does not pass exactly through the peaks, but a small difference is usually neglected. If the natural logarithm of these peaks is taken, the quantity $\zeta\omega_N$ represents the slope, m, of the line that can be drawn through the converted values. The frequency of the damped motion, ω_d, is also obtained from Eq. 10.85, and thus we obtain two equations and two unknowns:

$$m = -\zeta\omega_N \tag{10.89}$$

$$\omega_d = \omega_N \sqrt{1-\zeta^2} \tag{10.90}$$

The terms on the left hand side of Eqs. 10.89 and 10.90 are obtained by fitting exponential curves to the decayed oscillation data (Fig. 10.16). Once the values of ω_N and ζ are known from the above equations, the added mass and damping coefficients are computed:

$$M_a = M - M_0 = \frac{K}{\omega_N^2} - M_0 \tag{10.91}$$

and

$$C = 2M\zeta\omega_N \tag{10.92}$$

Therefore, knowing the extinction curve for a moored floating structure, the damping of the system may be established by a simple analysis. This is illustrated by an example based on Fig. 10.16.

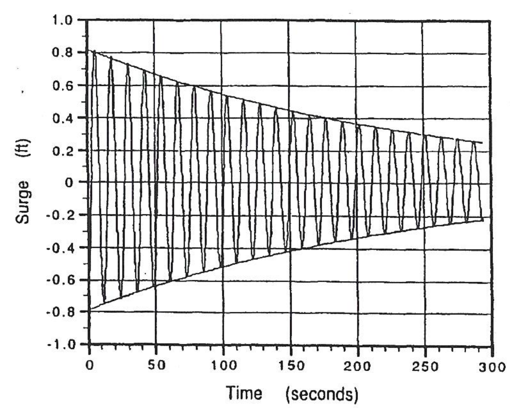

FIGURE 10.16
EXTINCTION TEST OF A TANKER IN SURGE

The extinction curve represents the free oscillation of a moored tanker. The displacement of the tanker is M_0 = 372.3 kg (25.5 slugs) and the spring constant K = 102 N/m (7.0 lbs/ft). The least square analysis described above gives an added mass coefficient and a damping factor of 0.049 and 0.008 respectively. The natural period between the positive peaks no. 2 and 3 in Fig. 10.16 is measured as 12.3 sec. Then,

$$M_a = 17.1\text{kg}(1.17\text{slug}) \tag{10.93}$$

and

$$C_a = \frac{M_a}{M_0} = 0.046 \tag{10.94}$$

Also, the amplitudes of peaks 2 and 3 are 0.235 and 0.22 m (0.77 and 0:72 ft) respectively. Therefore,

$$\delta = \frac{1}{2}(\ln 0.235 - \ln 0.22) = 0.03 \tag{10.95}$$

so that

$$\zeta = \frac{0.03}{\sqrt{\pi^2 + 0.03^2}} = 0.01 \tag{10.96}$$

This example shows that two peak values in the extinction curve determine the unknowns. In general, a least square fit of all data peaks in the extinction curve is desirable.

10.8.1.2 Nonlinear System

When nonlinear damping is present, the equation of motion for the damped free oscillation of a moored floating vessel, e.g., a semisubmersible in surge, is given by

$$M\ddot{x} + C\dot{x} + b_2|\dot{x}|\dot{x} + Kx = 0 \tag{10.97}$$

where M = total mass of the vessel in water, and b_2 = nonlinear damping coefficient. Since this equation is nonlinear, it is difficult to solve in a closed form. Therefore, the following simplification is made. On the assumption that each half cycle of the decayed oscillation is reasonably sinusoidal, the nonlinear term is linearized by a

Fourier series expansion as

$$|\dot{x}|\dot{x} = \frac{8}{3\pi}\omega_N x_k \dot{x} \tag{10.98}$$

where ω_N = frequency of oscillation corresponding to the natural frequency of the system, and x_k corresponds to the amplitude of the kth oscillation cycle. Upon substitution of Eq. 10.98 in Eq. 10.97, a linearized equation (with respect to time) is obtained

$$M\ddot{x} + C\dot{x} + \frac{8b_2}{3\pi}\omega_N x_k \dot{x} + Kx = 0 \tag{10.99}$$

Writing

$$C' = C + \frac{8b_2}{3\pi}\omega_N x_k \tag{10.100}$$

Eq. 10.99 becomes the familiar form as in Eq. 10.79 whose solution may be written in the form similar to Eq. 10.85 with ζ replaced by ζ', where ζ' is the damping factor including the linearized term,

$$\zeta' = \frac{C'}{2M\omega_N} \tag{10.101}$$

Then, using Eqs. 10.100, 10.101 and 10.88

$$\ln\frac{x_{k-1}}{x_{k+1}} = \frac{2\pi}{2M\omega_N}\left[C + \frac{8b_2}{3\pi}\omega_N x_k\right] = \frac{T_N}{2}\left[\frac{C}{M} + \frac{b_2}{M}\frac{16}{3T_N}x_k\right] \tag{10.102}$$

where T_N is the natural period of oscillation [Chakrabarti and Cotter (1990)]. In terms of the traditional damping factor, ζ, a more convenient nondimensional form may be written as

$$\frac{1}{2\pi}\ln\frac{x_{k-1}}{x_{k+1}} = \zeta + \frac{4}{3\pi}x_k\frac{b_2}{M} \tag{10.103}$$

Assuming that the nonlinear damping term may be represented by the Morison equation drag term,

$$b_2 = \frac{1}{2}\rho A C_D \tag{10.104}$$

where A = the projected area of the vessel in the direction of flow, Eq. 10.103 reduces to

$$\frac{1}{2\pi}\ln\frac{x_{k-1}}{x_{k+1}} = \zeta + \frac{2}{3\pi}\left(\frac{\rho A C_D}{M}\right)x_k \tag{10.105}$$

which is the equation of a straight line with the left-hand side representing the Y-axis and x_k the X-axis. Thus, knowing the peak values of the oscillation, the points (X,Y) from the measured data may be fitted to a straight line by the least square method (Fig. 9.24). Then, the quantities C_D and ζ may be obtained from the slope and intercept of the fitted line. It should be noted that for sufficient accuracy in these estimates, a reasonable number of peaks are required. However, for a highly damped system, the amplitude reduces to a small value rather quickly and the estimates in these cases are rough. Because waves introduce further damping in the system, the resulting traces may be difficult to analyze by the above method.

The added mass coefficient is computed from the measured natural period, T_N, the spring constant, K, and the displaced mass, M_0, using the formula

$$C_a = \left(\frac{KT_N^2}{4\pi^2} - M_0\right) / M_0 \tag{10.106}$$

10.8.2 Mechanical Oscillation

Consider the case of a floating structure model which is attached to a mechanical system similar to the ones described in Chapter 4. Consider also that the structure is forced to oscillate sinusoidally in a prescribed direction at a given amplitude and frequency, and the resulting forces are recorded. Assuming linear damping, the equation of motion due to sinusoidal oscillation has the form

$$M\ddot{x} + C\dot{x} + Kx = F_0 \sin \omega t \tag{10.107}$$