Extrapolation of buoy spectra using WSRA mss

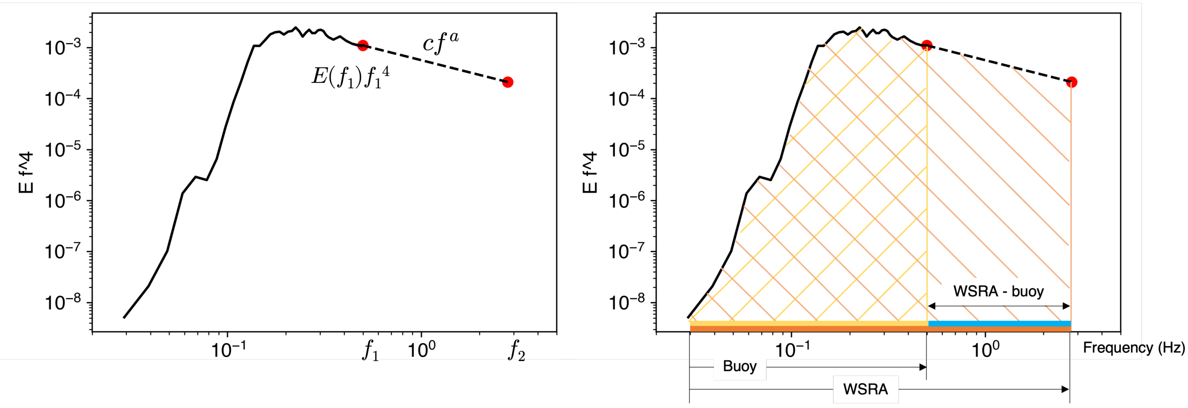

Figure 2: Variables in the slope extrapolation problem (left) and the regions which define the mean square slopes (right). The colored bars represent the frequency extent of each instrument, and their difference.

Our goal is to use knowledge of the WSRA mean square slope to extrapolate buoy spectra when both instruments are close to one another.
Let $E(f)$ denote the buoy-measured wave energy density spectrum over the frequencies, f. In integral form, mean square slope is proportional to the area beneath an Ef^4 spectrum:

$$\text{mss} = \frac{(2\pi)^4}{g^2} \int_{f_1}^{f_2} E(f) f^4 \mathrm{d}f \tag{2}$$

Now let mss_{buoy} represent the mean square slope from the integrated buoy spectrum, which covers scales from $-\infty$ to 6 m, and let mss_{WSRA} be the quasi-specular WSRA mean square slope estimate which covers $-\infty$ to 20 cm. The difference mean square slope, $\text{mss}_\delta = \text{mss}_{\text{WSRA}} - \text{mss}_{\text{buoy}}$, then covers 6 m down to 20 cm.

We seek an Ef^4 spectrum of the form cf^a which covers this range of wavelengths but in the frequency domain, starting at the end of the buoy spectrum, f_1, and extending until the frequency of the smallest WSRA wavelength, f_2. Here c is a constant and a is an unknown exponent.

The integral of cf^a should be equal to the area determined by the mean square slope

$$\begin{aligned} \int_{f_1}^{f_2} cf^a df &= \frac{g^2}{(2\pi)^4} \text{mss}_\delta \\ \frac{c}{a+1}\left(f_2^{a+1} - f_1^{a+1}\right) &= \frac{g^2}{(2\pi)^4} \text{mss}_\delta \end{aligned} \tag{3}$$

Since the spectrum must be continuous at f_1, then

$$E(f_1) f_1^4 = c f_1^a \tag{4}$$

such that

$$c = E(f_1) f_1^{(4-a)} \tag{5}$$

Inserting (5) into (6) results in an equation with one unknown, the exponent a, which can be solved for by finding the root of

$$\frac{E(f_1)}{a+1} f_1^{(4-a)} \left(f_2^{a+1} - f_1^{a+1}\right) - \frac{g^2}{(2\pi)^4} \text{mss}_\delta = 0 \tag{6}$$

Once a is found, the form of the regular energy spectrum over this frequency extent is then

$$E_\delta(f) = cf^n \tag{7}$$

where $n = a - 4$. If estimated properly, than the following should be approximately true

$$\frac{(2\pi)^4}{g^2} \int_{f_1}^{f_2} E_\delta(f) f^4 \mathrm{d}f = \mathrm{mss}_\delta \tag{8}$$