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505368811 MAE C163B Project 4

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Equations of Motion -- Derivation

```
clear all; close all; clc

syms 11 12 t1 t2 t3 m1 m2 dt1 dt2 ddt1 ddt2 g f3x f3y n3z

L(1) = Link('revolute','d', 0, 'a', 0, 'alpha', 0 ,'modified');
L(2) = Link('revolute','d', 0, 'a', 11, 'alpha', 0 ,'modified');
L(3) = Link('revolute','d', 0, 'a', 12, 'alpha', 0 ,'modified');

RR = SerialLink(L, 'name', 'RR-Manipulator');
```

Newton-Euler Formulation

```
th = [t1 t2 0]
T_01 = RR.A([1], th);
T_12 = RR.A([2], th);
T_2T = RR.A([3], th);
T_0T = RR.A([1 2 3], th);
T_0T = simplify(T_0T)
[R_01, P_01] = tr2rt(T_01); R_10 = transpose(R_01);
[R_12, P_12] = tr2rt(T_12); R_21 = transpose(R_12);
[R_2T, P_2T] = tr2rt(T_2T); R_32 = transpose(R_2T);
[R_0T, P_0T] = tr2rt(T_0T);
R_0T = simplify(R_0T)
P_0T = simplify(P_0T)
PC1 = [11/2; 0; 0];
PC2 = [12/2; 0; 0];
IC1 = (1/12) * m1 * 11^2 * [0 0 0; 0 1 0; 0 0 1];
IC2 = (1/12) * m2 * 12^2 * [0 0 0; 0 1 0; 0 0 1];
f3 = [f3x; f3y; 0];
n3 = [0;0;n3z];
w0 = zeros(3,1);
wd0 = zeros(3,1);
v0 = zeros(3,1);
vd0 = [0; 0; -g];
% Inward Iteration
w1 = R_10 * w0 + dt1*R_01(1:3,3)
wd1 = R_10 * wd0 + R_10 * cross(w0, dt1*R_01(1:3,3)) + ddt1*R_01(1:3,3)
vd1 = R_10 * (cross(wd0, P_01) + cross(w0, cross(w0, P_01)) + vd0)
vcd1 = cross(wd1,PC1) + cross(w1,cross(w1,PC1)) + vd1
F1 = m1 * vcd1
N1 = IC1 * wd1 + cross(w1,IC1*w1)
w2 = R_21 * w1 + dt2*R_12(1:3,3)
wd2 = R_21 * wd1 + R_21 * cross(w1, dt2*R_12(1:3,3)) + ddt2*R_12(1:3,3)
vd2 = R_21 * (cross(wd1, P_12) + cross(w1, cross(w1, P_12)) + vd1)
vcd2 = cross(wd2,PC2) + cross(w2,cross(w2,PC2)) + vd2
F2 = m2 * vcd2
N2 = IC2 * wd2 + cross(w2,IC2*w2)
% Outward Iteration
% i = 2
f2 = R_2T * f3 + F2;
n2 = N2 + R_2T*n3 + cross(PC2, F2) + cross(P_2T, R_2T*f3);
```

```
f2 = simplify(f2)
n2 = simplify(n2)
f1 = R_12 * f2 + F1;
n1 = N1 + R_12*n2 + cross(PC1, F1) + cross(P_12, R_12*f2);
f1 = simplify(f1)
n1 = simplify(n1)
th =
[t1, t2, 0]
[\cos(t1 + t2), -\sin(t1 + t2), 0, 12*\cos(t1 + t2) + 11*\cos(t1)]
[\sin(t1 + t2), \cos(t1 + t2), 0, 12*\sin(t1 + t2) + 11*\sin(t1)]
                        0, 1,
          0,
                            0, 0,
                                                                1]
            0,
R_0T =
[\cos(t1 + t2), -\sin(t1 + t2), 0]
[\sin(t1 + t2), \cos(t1 + t2), 0]
          0,
                         0, 1]
P_0T =
l2*cos(t1 + t2) + l1*cos(t1)
l2*sin(t1 + t2) + l1*sin(t1)
w1 =
  0
dt1
wd1 =
   0
ddt1
vd1 =
 0
 0
vcd1 =
-(dt1^2*l1)/2
 (ddt1*l1)/2
          -g
F1 =
-(dt1^2*l1*m1)/2
 (ddt1*l1*m1)/2
           -g*m1
                 0
                 0
(ddt1*l1^2*m1)/12
w2 =
        0
        0
dt1 + dt2
```

wd2 =

```
9
                                                                                  a
 ddt1 + ddt2
vd2 =
 - 11*cos(t2)*dt1^2 + ddt1*l1*sin(t2)
                l1*sin(t2)*dt1^2 + ddt1*l1*cos(t2)
vcd2 =
ddt1*11*sin(t2) - (12*(dt1 + dt2)^2)/2 - dt1^2*11*cos(t2)
11*sin(t2)*dt1^2 + (12*(ddt1 + ddt2))/2 + ddt1*11*cos(t2)
                                                                                                                                                                                                                                                                                                                                                                                                                                                          -g
F2 =
 -m2*((12*(dt1 + dt2)^2)/2 - ddt1*11*sin(t2) + dt1^2*11*cos(t2))
       m2*(l1*sin(t2)*dt1^2 + (l2*(ddt1 + ddt2))/2 + ddt1*l1*cos(t2))
N2 =
                                                                                                                                                                                                          0
 (12^2m2^*(ddt1 + ddt2))/12
f2 =
 f3x - m2*((12*(dt1 + dt2)^2)/2 - ddt1*11*sin(t2) + dt1^2*11*cos(t2))
f3y + m2*(l1*sin(t2)*dt1^2 + (l2*(ddt1 + ddt2))/2 + ddt1*l1*cos(t2))
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                             -g*m2
 n2 =
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       (g*12*m2)/2
n3z \ + \ f3y*12 \ + \ (ddt1*12^2*m2)/3 \ + \ (ddt2*12^2*m2)/3 \ + \ (dt1^2*11*12*m2*sin(t2))/2 \ + \ (ddt1*11*12*m2*cos(t2))/2 \ + \ (ddt1*11*12*m2*cos(t2)
f1 =
\cos(t2)^*(\text{f3x - m2*}((12^*(\text{dt1 + dt2})^2)/2 - \text{ddt1*}11^*\sin(t2) + \text{dt1}^2*11^*\cos(t2))) - \sin(t2)^*(\text{f3y + m2*}(11^*\sin(t2)^*\text{dt1}^2 + (12^*(\text{ddt1 + dd2}))/2 + \text{ddt1*}11^*\cos(t2)))) - (\text{ddt1}^2)^*(\text{f3y + m2*}(11^*\sin(t2)^*)/2 + (12^*(\text{ddt1 + dd2}))/2 + (12^*(\text{ddt1 + dd2}))/2 + (12^*(\text{dd1} + \text{dd2}))/2 + (
       \sin(t2)*(f3x - m2*((12*(dt1 + dt2)^2)/2 - ddt1*11*\sin(t2) + dt1^2*11*\cos(t2))) + \cos(t2)*(f3y + m2*(11*\sin(t2)*dt1^2 + (12*(ddt1 + ddt2))/2 + ddt1*11*\cos(t2))) + (12*(ddt1 + ddt2))/2 + (12*(ddt
n1 =
n3z + f3y*12 + f3y*11*cos(t2) + f3x*11*sin(t2) + (ddt1*11^2*m1)/3 + ddt1*11^2*m2 + (ddt1*12^2*m2)/3 + (ddt2*12^2*m2)/3 - (dt2^2*11*12*m2*sin(t2))/2 + ddt1*11*2*m2*sin(t2))/2 + ddt1*11*2*m2*sin(t2)/3 + (ddt1*11^2*m2)/3 +
```

Design Trajectory

```
clear all; close all; clc

11 = 0.5;
12 = 0.5;

L(1) = Link('revolute','d', 0, 'a', 0, 'alpha', 0, 'modified');
L(2) = Link('revolute','d', 0, 'a', 11, 'alpha', 0, 'modified');
L(3) = Link('revolute','d', 0, 'a', 12, 'alpha', 0, 'modified');

RR = SerialLink(L, 'name', 'RR-Manipulator');

g = 0
    rho = 1000;
    r_outer = 0.1;
    r_inner = 0.005;
    m1 = rho*11*pi*(r_outer^2 - r_inner^2);
    m2 = rho*12*pi*(r_outer^2 - r_inner^2);
PC1 = [11/2; 0; 0];
PC2 = [12/2; 0; 0];

IX = 0.5*m1*(r_outer^2 + r_inner^2);
Iy = Ix/2 + (1/12)*m1*11^2; Iz = Ix/2 + + (1/12)*m2*12^2;
```

```
IC1 = [Ix 0 0; 0 Iy 0; 0 0 Iz]
IC2 = [Ix 0 0; 0 Iy 0; 0 0 Iz]
f3 = [-10;0;0];
n3 = [0;0;10];
w0 = zeros(3,1);
wd0 = zeros(3,1);
vd0 = [0; 0; 0];
t1_initial = -acos(0.45/0.5)
t2_{initial} = 2*acos(0.45/0.5)
t1_final = -acos(0.05/0.5)
t2_final = 2*acos(0.05/0.5)
N = 100;
t1 = linspace(t1_initial, t1_final, N+2)
t2 = linspace(t2_initial, t2_final, N+2)
totalTime = 4;
dt = 4/N;
dt1 = (diff(t1))/dt
dt2 = (diff(t2))/dt
ddt1 = (diff(t1,2))/dt^2
ddt2 = (diff(t2,2))/dt^2
for j=1:N
    th = [t1(j) t2(j) 0];
    T_01 = RR.A([1], th);
    T_12 = RR.A([2], th);
    T_2T = RR.A([3], th);
    T_0T = RR.A([1 2 3], th);
    [R_01, P_01] = tr2rt(T_01); R_10 = transpose(R_01);
    [R_12, P_12] = tr2rt(T_12); R_21 = transpose(R_12);
    [R_2T, P_2T] = tr2rt(T_2T); R_32 = transpose(R_2T);
    [R_0T, P_0T] = tr2rt(T_0T);
    x(j) = P_0T(1); y(j) = P_0T(2);
    % i = 0
    w1 = R_10 * w0 + dt1(j)*R_01(1:3,3);
    wd1 = R_10 * wd0 + R_10 * cross(w0, dt1(j)*R_01(1:3,3)) + ddt1(j)*R_01(1:3,3);
    vd1 = R_10 * (cross(wd0, P_01) + cross(w0, cross(w0, P_01)) + vd0);
    vcd1 = cross(wd1,PC1) + cross(w1,cross(w1,PC1)) + vd1;
    F1 = m1 * vcd1;
    N1 = IC1 * wd1 + cross(w1,IC1*w1);
    % i = 1
    w2 = R_21 * w1 + dt2(j)*R_12(1:3,3);
    wd2 = R_21 * wd1 + R_21 * cross(w1, dt2(j)*R_12(1:3,3)) + ddt2(j)*R_12(1:3,3);
    vd2 = R_21 * (cross(wd1, P_12) + cross(w1, cross(w1, P_12)) + vd1);
   vcd2 = cross(wd2,PC2) + cross(w2,cross(w2,PC2)) + vd2;
    F2 = m2 * vcd2;
    N2 = IC2 * wd2 + cross(w2,IC2*w2);
    \% i = 2
    f2 = R_2T * f3 + F2;
    n2(:,j) = N2 + R_2T*n3 + cross(PC2, F2) + cross(P_2T, R_2T*f3);
    f1 = R 12 * f2 + F1;
    n1(:,j) = N1 + R_12*n2(:,j) + cross(PC1, F1) + cross(P_12, R_12*f2);
time = linspace(0, totalTime, N);
% Plot Trajectory
figure(1)
title('Trajectory Animation')
xlabel('X-Direction (m)')
ylabel('Y-Direction (m)')
h = animatedline;
axis([0 1 -0.5 0.5])
for j=1:N
    addpoints(h,x(j),y(j));
    pause(0.05)
end
```

```
title('EE Position vs. Time')
ylabel('X and Y Plane (m)')
xlabel('Time (s)')
hold on
plot(time, y);
hold off
% Joint Torque
figure(3)
plot(time, n1);
title('Joint Torque vs. Time')
ylabel('Joint Torque (Nm)')
xlabel('Time (s)')
hold on
plot(time, n2);
legend('Joint 1','Joint 2')
g =
    0
IC1 =
    0.0785
                  0
                            0
             0.3657
                            0
        0
         0
                      0.3657
IC2 =
    0.0785
                  0
                            0
        0
             0.3657
                            0
                       0.3657
t1_initial =
   -0.4510
t2_initial =
    0.9021
t1_final =
   -1.4706
t2_final =
    2.9413
  Columns 1 through 7
   -0.4510 -0.4611 -0.4712 -0.4813 -0.4914 -0.5015 -0.5116
  Columns 8 through 14
   -0.5217 -0.5318 -0.5419 -0.5520 -0.5621 -0.5722 -0.5823
  Columns 15 through 21
   -0.5924 -0.6025 -0.6125 -0.6226 -0.6327 -0.6428 -0.6529
  Columns 22 through 28
   -0.6630 -0.6731 -0.6832 -0.6933 -0.7034 -0.7135 -0.7236
  Columns 29 through 35
   -0.7337 \quad -0.7438 \quad -0.7539 \quad -0.7640 \quad -0.7741 \quad -0.7842 \quad -0.7943
  Columns 36 through 42
   -0.8044 -0.8144 -0.8245 -0.8346 -0.8447 -0.8548 -0.8649
```

% End Effector Position

figure(2)
plot(time, x);

| Columns 43 | through | 49 | | | | |
|------------|-----------|---------|---------|---------|---------|---------|
| -0.8750 | -0.8851 | -0.8952 | -0.9053 | -0.9154 | -0.9255 | -0.9356 |
| Columns 50 | through | 56 | | | | |
| -0.9457 | -0.9558 | -0.9659 | -0.9760 | -0.9861 | -0.9962 | -1.0063 |
| Columns 57 | through | 63 | | | | |
| -1.0164 | -1.0264 | -1.0365 | -1.0466 | -1.0567 | -1.0668 | -1.0769 |
| Columns 64 | through | 70 | | | | |
| -1.0870 | -1.0971 | -1.1072 | -1.1173 | -1.1274 | -1.1375 | -1.1476 |
| Columns 71 | through | 77 | | | | |
| -1.1577 | -1.1678 | -1.1779 | -1.1880 | -1.1981 | -1.2082 | -1.2183 |
| Columns 78 | through | 84 | | | | |
| -1.2283 | -1.2384 | -1.2485 | -1.2586 | -1.2687 | -1.2788 | -1.2889 |
| Columns 85 | through | 91 | | | | |
| -1.2990 | -1.3091 | -1.3192 | -1.3293 | -1.3394 | -1.3495 | -1.3596 |
| Columns 92 | through | 98 | | | | |
| -1.3697 | -1.3798 | -1.3899 | -1.4000 | -1.4101 | -1.4202 | -1.4302 |
| Columns 99 | through | 102 | | | | |
| -1.4403 | -1.4504 | -1.4605 | -1.4706 | | | |
| t2 = | | | | | | |
| Columns 1 | through : | 7 | | | | |
| 0.9021 | 0.9222 | 0.9424 | 0.9626 | 0.9828 | 1.0030 | 1.0232 |
| Columns 8 | through : | 14 | | | | |
| 1.0434 | 1.0636 | 1.0838 | 1.1040 | 1.1241 | 1.1443 | 1.1645 |
| Columns 15 | through | 21 | | | | |
| 1.1847 | 1.2049 | 1.2251 | 1.2453 | 1.2655 | 1.2857 | 1.3059 |
| Columns 22 | through | 28 | | | | |
| 1.3260 | 1.3462 | 1.3664 | 1.3866 | 1.4068 | 1.4270 | 1.4472 |
| Columns 29 | through | 35 | | | | |
| 1.4674 | 1.4876 | 1.5078 | 1.5279 | 1.5481 | 1.5683 | 1.5885 |
| Columns 36 | through | 42 | | | | |
| 1.6087 | 1.6289 | 1.6491 | 1.6693 | 1.6895 | 1.7097 | 1.7298 |
| Columns 43 | through | 49 | | | | |
| 1.7500 | 1.7702 | 1.7904 | 1.8106 | 1.8308 | 1.8510 | 1.8712 |
| Columns 50 | through | 56 | | | | |
| 1.8914 | 1.9116 | 1.9318 | 1.9519 | 1.9721 | 1.9923 | 2.0125 |
| Columns 57 | through | 63 | | | | |
| 2.0327 | 2.0529 | 2.0731 | 2.0933 | 2.1135 | 2.1337 | 2.1538 |
| Columns 64 | through | 70 | | | | |
| 2.1740 | 2.1942 | 2.2144 | 2.2346 | 2.2548 | 2.2750 | 2.2952 |
| Columns 71 | through | 77 | | | | |
| 2.3154 | 2.3356 | 2.3557 | 2.3759 | 2.3961 | 2.4163 | 2.4365 |
| Columns 78 | through | 84 | | | | |
| 2.4567 | 2.4769 | 2.4971 | 2.5173 | 2.5375 | 2.5576 | 2.5778 |

| 2.5980 | 2.6182 | 2.6384 | 2.6586 | 2.6788 | 2.6990 | 2.7192 | | |
|----------------------|-----------|---------|---------|---------|---------|---------|--|--|
| Columns 92 | through | 98 | | | | | | |
| 2.7394 | 2.7595 | 2.7797 | 2.7999 | 2.8201 | 2.8403 | 2.8605 | | |
| Columns 99 | through | 102 | | | | | | |
| 2.8807 | 2.9009 | 2.9211 | 2.9413 | | | | | |
| dt1 = | | | | | | | | |
| Columns 1 | through 7 | 7 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 8 through 14 | | | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 15 | through | 21 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 22 | through | 28 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 29 | through | 35 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 36 | through | 42 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 43 | through | 49 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 50 | through | 56 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 57 | through | 63 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 64 | through | 70 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 71 | through | 77 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 78 | through | 84 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 85 | through | 91 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 92 | through | 98 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | -0.2524 | | |
| Columns 99 | through | 101 | | | | | | |
| -0.2524 | -0.2524 | -0.2524 | | | | | | |
| dt2 = | | | | | | | | |
| Columns 1 through 7 | | | | | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | | |
| Columns 8 | through 1 | 14 | | | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | | |
| Columns 15 | through | 21 | | | | | | |

Columns 85 through 91

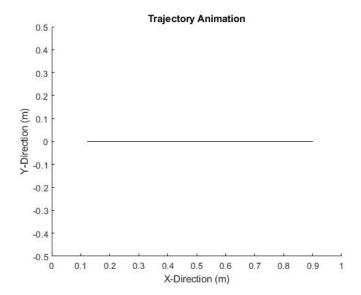
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
|------------|------------|---------|---------|---------|---------|---------|
| Columns 22 | through 2 | 28 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 29 | through 3 | 5 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 36 | through 4 | 12 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 43 | through 4 | 19 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 50 | through 5 | 66 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 57 | through 6 | 53 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 64 | through 7 | 70 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 71 | through 7 | 77 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 78 | through 8 | 34 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 85 | through 9 | 91 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 92 | through 9 | 8 | | | | |
| 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 | 0.5048 |
| Columns 99 | through 1 | .01 | | | | |
| 0.5048 | 0.5048 | 0.5048 | | | | |
| ddt1 = | | | | | | |
| 1.0e-12 * | | | | | | |
| Columns 1 | through 7 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columns 8 | through 14 | | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columns 15 | through 2 | 11 | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columns 22 | through 2 | 28 | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0.0694 |
| Columns 29 | through 3 | 35 | | | | |
| -0.1388 | 0.1388 | -0.1388 | 0.1388 | -0.1388 | 0.1388 | -0.1388 |
| Columns 36 | through 4 | 12 | | | | |
| 0.0694 | 0 | 0.0694 | -0.1388 | 0.1388 | -0.1388 | 0.1388 |
| Columns 43 | | | | | | |
| | | | 0.1388 | -0.1388 | 0.1388 | -0.1388 |
| Columns 50 | through 5 | 66 | | | | |
| | | | 0.0694 | 0.0694 | -0.1388 | 0.1388 |
| | | | | | | |

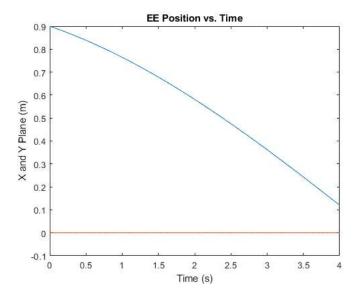
| Columns 57 | through | 63 | | | | |
|------------|-----------|---------|---------|---------|---------|---------|
| 0 | -0.1388 | 0 | 0.1388 | 0 | -0.1388 | 0 |
| Columns 64 | through | 70 | | | | |
| 0.1388 | 0 | -0.1388 | 0 | 0.1388 | 0 | -0.2776 |
| Columns 71 | through | 77 | | | | |
| 0.2776 | 0 | -0.1388 | 0.1388 | -0.1388 | 0.1388 | 0 |
| Columns 78 | through | 84 | | | | |
| -0.1388 | 0 | 0.1388 | 0 | -0.1388 | 0 | 0.1388 |
| Columns 85 | through | 91 | | | | |
| 0 | -0.1388 | 0 | 0.1388 | 0 | -0.1388 | 0 |
| Columns 92 | through | 98 | | | | |
| 0.1388 | 0 | -0.1388 | 0 | 0.1388 | 0 | -0.1388 |
| Columns 99 | through | 100 | | | | |
| 0 | 0.1388 | | | | | |
| ddt2 = | | | | | | |
| 1.0e-12 * | | | | | | |
| Columns 1 | through : | 7 | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columns 8 | through : | 14 | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columns 15 | through | 21 | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| Columns 22 | through | 28 | | | | |
| 0 | 0 | 0 | 0 | 0 | 0 | -0.1388 |
| Columns 29 | through | 35 | | | | |
| 0.2776 | -0.2776 | 0.2776 | -0.2776 | 0.2776 | -0.2776 | 0.2776 |
| Columns 36 | through | 42 | | | | |
| -0.1388 | 0 | -0.1388 | 0.2776 | -0.2776 | 0.2776 | -0.2776 |
| Columns 43 | through | 49 | | | | |
| 0.2776 | -0.2776 | 0.2776 | -0.2776 | 0.2776 | -0.2776 | 0.2776 |
| Columns 50 | through | 56 | | | | |
| -0.1388 | -0.1388 | 0.2776 | -0.1388 | -0.1388 | 0.2776 | -0.2776 |
| Columns 57 | through | 63 | | | | |
| 0 | 0.2776 | 0 | -0.2776 | 0 | 0.2776 | 0 |
| Columns 64 | through | 70 | | | | |
| -0.2776 | 0 | 0.2776 | 0 | -0.2776 | 0 | 0.5551 |
| Columns 71 | through | 77 | | | | |
| -0.5551 | 0 | 0.2776 | -0.2776 | 0.2776 | -0.2776 | 0 |
| Columns 78 | through | 84 | | | | |
| 0.2776 | 0 | -0.2776 | 0 | 0.2776 | 0 | -0.2776 |
| Columns 85 | through | 91 | | | | |
| 0 | 0.2776 | 0 | -0.2776 | 0 | 0.2776 | 0 |
| Columns 92 | through | 98 | | | | |

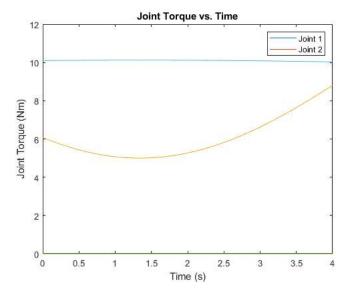
-0.2776 0 0.2776 0 -0.2776 0 0.2776

Columns 99 through 100

0 -0.2776







Inertia Tensor

```
clear all; close all; clc;
h = 0.1; l = 0.1; w = 0.1; r = 0.1; d=0.4;
I_cube = [h^2+1^2 0 0; 0 w^2+h^2 0; 0 0 1^2+h^2];
I_{cyl} = [(1/12)*(3*r^2+h^2) 0 0; 0 (1/12)*(3*r^2+h^2) 0; 0 0 0.5^r^2];
I_Acm = I_cube - I_cyl;
I_A = I_Acm + ([-d \ 0 \ 0]^*[-d; \ 0; \ 0]^*eye(3) - [d^2 \ 0 \ 0; \ 0 \ 0 \ 0])
% Body B
rB = 0.05; lB = 0.8;
I\_B = [0.5*rB^2 \ 0 \ 0; \ 0 \ (1/12)*(3*rB^2+1B^2) \ 0; \ 0 \ 0 \ (1/12)*(3*rB^2+1B^2)]
% Body C
I_Ccm = I_Acm;
I_C1 = I_Ccm + ([d 0 0]*[d; 0; 0]*eye(3) - [d^2 0 0; 0 0 0; 0 0 0])
Rotx = [1 \ 0 \ 0; \ 0 \ 1/2^0.5; \ 0 \ -1/2^0.5; \ 0 \ -1/2^0.5];
I_C = Rotx*I_C1*Rotx
% Total Inertia
syms mA mB mC
I = mA*I_A + mB*I_B + mC*I_C;
I = vpa(I, 4)
```

```
I_A =
   0.0167
                  0
                           0
        0
             0.1767
                           0
                 0
                     -0.6906
I_B =
   0.0013
             0.0540
                           0
        0
        0
                  0
                      0.0540
I_C1 =
   0.0167
                 0
                           0
             0.1767
        0
                           0
        0
                  0
                     -0.6906
I_C =
   0.0167
                 0
                           0
           -0.2569 -0.4336
            -0.4336 -0.2569
I =
[0.01667*mA + 0.00125*mB + 0.01667*mC,
                                 0, 0.1767*mA + 0.05396*mB - 0.2569*mC,
                                                                                             -0.4336*mC]
```

0,

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