

Dynamics

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% Symbolic Expressions
syms t1 t2 t3 t4 t5 t6 a2 a3 d2 d3 d4
syms c1 c2 c3 Ix1 Ix2 Ix3 Iy1 Iy2 Iy3 Iz1 Iz2 Iz3 m1 m2 m3 m4;

DH = [
    0      0      0      t1;    %alpha, a, d, theta
    -pi/2   0      d2      t2;
    0      a2      d3      t3;
    pi/2   a3      d4      0;
]

T_01 = transformationMatrix(DH(1,:));
T_12 = transformationMatrix(DH(2,:));
T_23 = transformationMatrix(DH(3,:));
T_34 = transformationMatrix(DH(4,:));

T_04 = T_01*T_12*T_23*T_34;
T_04 = simplify(T_04);

[R_01, P_01] = tr2rt(T_01); R_10 = transpose(R_01);
[R_12, P_12] = tr2rt(T_12); R_21 = transpose(R_12);
[R_23, P_23] = tr2rt(T_23); R_32 = transpose(R_23);
[R_34, P_34] = tr2rt(T_34); R_43 = transpose(R_34);
[R_04, P_04] = tr2rt(T_04); R_40 = transpose(R_04);

PC1 = [0; d2/2 ; 0];
PC2 = [a2/2; 0 ; 0];
PC3 = [0 ; -d4/2; 0];

IC1 = (1/12)*m1*(d2^2)*[1 0 0; 0 0 0; 0 0 1];
IC2 = (1/12)*m2*(a2^2)*[0 0 0; 0 1 0; 0 0 1];
IC3 = (1/12)*m3*(d4^2)*[1 0 0; 0 0 0; 0 0 1]...
    + (m4*(d4/2)^2)* [1 0 0; 0 0 0; 0 0 1];

syms f4x f4y f4z n4x n4y n4z g dt1 dt2 dt3 ddt1 ddt2 ddt3 ;

f4 = [f4x; f4y; f4z];
n4 = [n4x; n4y; n4z];

w0 = zeros(3,1);
wd0 = zeros(3,1);

v0 = zeros(3,1);
vd0 = [0 ; 0 ; -g];

% Inward Iteration

% i = 0
w1 = R_10 * w0 + dt1*R_01(1:3,3);
wd1 = R_10 * wd0 + R_10 * cross(w0, dt1*R_01(1:3,3)) + ddt1*R_01(1:3,3);

vd1 = R_10 * (cross(wd0, P_01) + cross(w0, cross(w0, P_01))) + vd0;
vcd1 = cross(wd1,PC1) + cross(w1,cross(w1,PC1)) + vd1;

F1 = m1 * vcd1 ;
N1 = IC1 * wd1 + cross(w1,IC1*w1);

% i = 1
w2 = R_21 * w1 + dt2*R_12(1:3,3);
wd2 = R_21 * wd1 + R_21 * cross(w1, dt2*R_12(1:3,3)) + ddt2*R_12(1:3,3);

vd2 = R_21 * (cross(wd1, P_12) + cross(w1, cross(w1, P_12))) + vd1;
vcd2 = cross(wd2,PC2) + cross(w2,cross(w2,PC2)) + vd2;

F2 = m2 * vcd2 ;
N2 = IC2 * wd2 + cross(w2,IC2*w2);

% i = 3
w3 = R_32 * w2 + dt3*R_23(1:3,3);
wd3 = R_32 * wd2 + R_32 * cross(w2, dt3*R_23(1:3,3)) + ddt3*R_23(1:3,3);

vd3 = R_32 * (cross(wd2, P_23) + cross(w2, cross(w2, P_23))) + vd2;
vcd3 = cross(wd3,PC3) + cross(w3,cross(w3,PC3)) + vd3;

F3 = (m3+m4) * vcd2;
N3 = IC3 * wd3 + cross(w3,IC3*w3);

% Outward iteration
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% i = 3
f3 = R_34 * f4 + F3;
n3 = N3 + R_34*n4 + cross(PC3, F3) + cross(P_34, R_34*f4);
f3 = simplify(f3)
n3 = simplify(n3)

% i = 2
f2 = R_23 * f3 + F2;
n2 = N2 + R_23*n3 + cross(PC2, F2) + cross(P_23, R_23*f3);
f2 = simplify(f2)
n2 = simplify(n2)

% i = 1
f1 = R_12 * f2 + F1;
n1 = N1 + R_12*n2 + cross(PC1, F1) + cross(P_12, R_12*f2);
f1 = simplify(f1)
n1 = simplify(n1)

tau1 = n1(3);
tau2 = n2(3);
tau3 = n3(3);

TAU = [tau1; tau2; tau3]

M11 = subs(tau1, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
M12 = subs(tau1, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
M13 = subs(tau1, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])

M21 = subs(tau2, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
M22 = subs(tau2, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
M23 = subs(tau2, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])

M31 = subs(tau3, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
M32 = subs(tau3, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
M33 = subs(tau3, [ddt1, ddt2, ddt3, dt1, dt2, dt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])

V1F1 = subs(tau1, [ddt1, ddt2, ddt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
V2F2 = subs(tau2, [ddt1, ddt2, ddt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])
V3F3 = subs(tau3, [ddt1, ddt2, ddt3, g, f4x, f4y, f4z, n4x, n4y, n4z], [0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0])

G1 = subs(tau1, [ddt1, ddt2, ddt3, dt1, dt2, dt3], [0, 0, 0, 0, 0, 0])
G2 = subs(tau2, [ddt1, ddt2, ddt3, dt1, dt2, dt3], [0, 0, 0, 0, 0, 0])
G3 = subs(tau3, [ddt1, ddt2, ddt3, dt1, dt2, dt3], [0, 0, 0, 0, 0, 0])

function [T] = transformationMatrix(DH_row)

T = [cos(DH_row(4))          -sin(DH_row(4))          0          DH_row(2);
     sin(DH_row(4))*cos(DH_row(1))  cos(DH_row(4))*cos(DH_row(1))  -sin(DH_row(1))  -sin(DH_row(1))*DH_row(3);
     sin(DH_row(4))*sin(DH_row(1))  cos(DH_row(4))*sin(DH_row(1))  cos(DH_row(1))  cos(DH_row(1))*DH_row(3);
     0          0          0          1];

end

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DH =

[ 0, 0, 0, t1]
[-pi/2, 0, d2, t2]
[ 0, a2, d3, t3]
[ pi/2, a3, d4, 0]

f3 =

f4x - (m3 + m4)*((a2*(dt2 - dt1*cos(t2))^2)/2 - g*sin(t2) + d2*ddt1*cos(t2))
(m3 + m4)*(g*cos(t2) + d2*ddt1*sin(t2) - (a2*dt1*sin(t2)*(dt2 - dt1*cos(t2)))/2) - f4z
f4y - ((a2*(ddt2 - ddt1*cos(t2) + dt1*dt2*sin(t2)))/2 + d2*dt1^2)*(m3 + m4)

n3 =

n4x - d4*f4y + (d4^2*(m3 + 3*m4)*(sin(t3)*(ddt2 - ddt1*cos(t2) + dt1*dt2*sin(t2)) - cos(t3)*(ddt1*sin(t2) + dt1*dt2*cos(t2)) + dt3*cos(t3)*(dt2 - dt1*cos(t2)) + dt1
n4y - a3*f4z + d4*f4x + (d4^2*ddt3*(m3 + 3*m4))/12 -

f2 =

sin(t3)*(f4z - (m3 + m4)*(g*cos(t2) + d2*ddt1*sin(t2) - (a2*dt1*sin(t2)*(dt2 - dt1*cos(t2)))/2)) + cos(t3)*(f4x - (m3 + m4)*((a2*(dt2 - dt1*cos(t2))^2)/2 -
sin(t3)*(f4x - (m3 + m4)*((a2*(dt2 - dt1*cos(t2))^2)/2 - g*sin(t2) + d2*ddt1*cos(t2))) - cos(t3)*(f4z - (m3 + m4)*(g*cos(t2) + d2*ddt1*sin(t2) - (a2*dt1*sin(t2)*(dt
f4y - ((a2*(ddt2 - ddt1*cos(t2) + dt1*dt2*s

n2 =

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$$\sin(t_3)*(n_4x - d_4*f_4y + (d_4^2*(m_3 + 3*m_4)*(\sin(t_3)*(ddt_2 - ddt_1*\cos(t_2) + dt_1*dt_2*\sin(t_2)) - \cos(t_3)*(ddt_1*\sin(t_2) + dt_1*dt_2*\cos(t_2)) + dt_3*\cos(t_3)*(dt_2 - dt_1*\cos(t_2))$$

$$f_1 =$$

$$\cos(t_2)*(\sin(t_3)*(f_4z - (m_3 + m_4)*(g*\cos(t_2) + d_2*ddt_1*\sin(t_2) - (a_2*dt_1*\sin(t_2)*(dt_2 - dt_1*\cos(t_2))))/2)) + \cos(t_3)*(f_4x - (m_3 + m_4)*((a_2*(dt_2 - dt_1*\cos(t_2))^2)/2 - f_4z*\cos(t_2 + t_3) - g*m_2 - g*m_1 - f_4x*\sin(t_2 + t_3) - g*m_3*\cos(t_3) - g*m_4*\cos(t_3) - (a_2*dt_1^2*m_3*\sin(t_2 - t_3))$$

$$n_1 =$$

$$n_4x*\cos(t_2 + t_3) + n_4z*\sin(t_2 + t_3) + a_2*f_4y*\sin(t_2) - (d_2*g*m_1)/2 - d_2*g*m_2 - d_4*f_4y*\cos(t_2 + t_3) + d_2*f_4z*\cos(t_2 + t_3) + d_3*f_4z*\cos(t_2 + t_3) + a_3*f_4y*\sin(t_2 + t_3) + n_4z*\cos(t_2 + t_3) - n_4x*\sin(t_2 + t_3) + a_3*f_4y*\cos(t_2 + t_3) - d_2*f_4x*\cos(t_2 + t_3) - d_3*f_4x*\cos(t_2 + t_3) + d_4*f_4y$$

$$TAU =$$

$$n_4z*\cos(t_2 + t_3) - n_4x*\sin(t_2 + t_3) + a_3*f_4y*\cos(t_2 + t_3) - d_2*f_4x*\cos(t_2 + t_3) - d_3*f_4x*\cos(t_2 + t_3) + d_4*f_4y*\sin(t_2 + t_3) - d_2*f_4z*\sin(t_2 + t_3) - d_3*f_4z*\sin(t_2 + t_3)$$

$$M_{11} =$$

$$(a_2^2*m_2)/6 + (a_2^2*m_3)/4 + (a_2^2*m_4)/4 + (d_2^2*m_1)/3 + d_2^2*m_2 + (d_4^2*m_3)/24 + (d_4^2*m_4)/8 + d_2^2*m_3*\cos(t_3) + d_2^2*m_4*\cos(t_3) + (a_2^2*m_2*\cos(2*t_2))/6 + (a_2^2*m_3*\cos(2*t_3))/6 + (a_2^2*m_4*\cos(2*t_4))/6$$

$$M_{12} =$$

$$(d_4^2*m_3*\cos(t_2 + 2*t_3))/24 - (a_2^2*m_3*\cos(t_2))/2 - (a_2^2*m_4*\cos(t_2))/2 - (d_4^2*m_3*\cos(t_2))/24 - (d_4^2*m_4*\cos(t_2))/8 - (a_2^2*m_2*\cos(t_2))/3 + (d_4^2*m_4*\cos(t_2 + 2*t_3))/24$$

$$M_{13} =$$

$$0$$

$$M_{21} =$$

$$(a_2*d_2*m_2*\sin(t_2))/2 - (d_2*d_4*\cos(t_2)*(m_3 + m_4))/2 - a_2*(d_2*\cos(t_2)*\sin(t_3)*(m_3 + m_4) - d_2*\cos(t_3)*\sin(t_2)*(m_3 + m_4))$$

$$M_{22} =$$

$$0$$

$$M_{23} =$$

$$(d_4^2*(m_3 + 3*m_4))/12$$

$$M_{31} =$$

$$-(d_2*d_4*\cos(t_2)*(m_3 + m_4))/2$$

$$M_{32} =$$

$$0$$

$$M_{33} =$$

$$(d_4^2*(m_3 + 3*m_4))/12$$

$$V_{1F1} =$$

$$(a_2*d_2*dt_1^2*m_3*\cos(t_2 + t_3))/4 + (a_2*d_2*dt_1^2*m_4*\cos(t_2 + t_3))/4 + (a_2*d_2*dt_2^2*m_3*\cos(t_2 + t_3))/2 + (a_2*d_3*dt_1^2*m_3*\cos(t_2 + t_3))/4 + (a_2*d_2*dt_2^2*m_4*\cos(t_2 + t_3))/4$$

$$V_{2F2} =$$

$$- a_2*((a_2*\sin(t_3)*(m_3 + m_4)*(dt_2 - dt_1*\cos(t_2))^2)/2 + (a_2*dt_1*\cos(t_3)*\sin(t_2)*(m_3 + m_4)*(dt_2 - dt_1*\cos(t_2)))/2) - (d_4^2*(m_3 + 3*m_4)*((\sin(2*t_2 + 2*t_3)*dt_1^2)/2 - \sin(2*t_2 + 2*t_3)*dt_1*dt_2 + (\sin(2*t_3)*dt_2^2)/2))/12 - (a_2*d_4*(m_3 + m_4)*(dt_2 - dt_1*\cos(t_2))^2)/4$$

$$V_{3F3} =$$

$$- (d_4^2*(m_3 + 3*m_4)*((\sin(2*t_2 + 2*t_3)*dt_1^2)/2 - \sin(2*t_2 + 2*t_3)*dt_1*dt_2 + (\sin(2*t_3)*dt_2^2)/2))/12 - (a_2*d_4*(m_3 + m_4)*(dt_2 - dt_1*\cos(t_2))^2)/4$$

$$G_1 =$$

$$n4z*\cos(t2 + t3) - n4x*\sin(t2 + t3) + a3*f4y*\cos(t2 + t3) - d2*f4x*\cos(t2 + t3) - d3*f4x*\cos(t2 + t3) + d4*f4y*\sin(t2 + t3) - d2*f4z*\sin(t2 + t3) - d3*f4z*\sin(t2 +$$

$$G2 =$$

$$n4y - a3*f4z + d4*f4x - a2*(\cos(t3)*(f4z - g*\cos(t2)*(m3 + m4)) - \sin(t3)*(f4x + g*\sin(t2)*(m3 + m4))) + (d4*g*\sin(t2)*(m3 + m4))/2 + (a2*g*m2*\cos(t2))/2$$

$$G3 =$$

$$n4y - a3*f4z + d4*f4x + (d4*g*\sin(t2)*(m3 + m4))/2$$