Modeling Bubbles in Solution With ODEs

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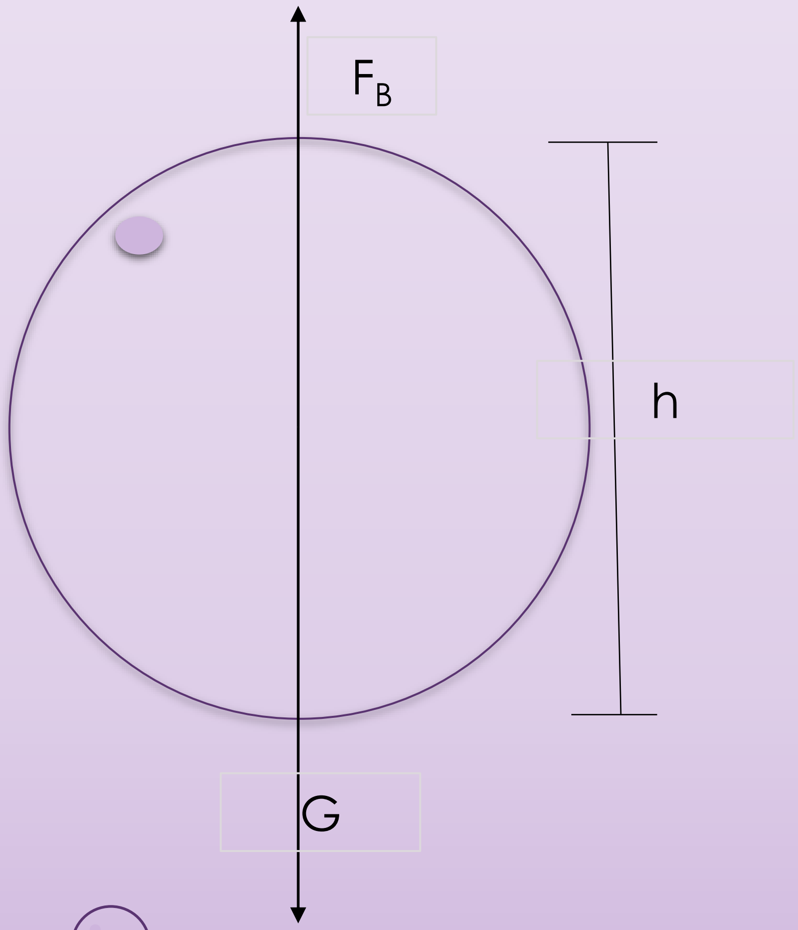
Why Bubbles?

Goal: Does a bubble rise in a solution proportionally to the surface area of the bubble? Is Archimedes principle reality?

Archimedes' Principle

The force of Buoyancy is equal to the amount of liquid the bubble displaces



F_B =ρGha

F_B –the Force of Buoyancy

ρ-density of liquid
G-gravity (9.8 m/s^2)
h-length of floating object
a-surface area

Meet the System

The change in Radius: $$\frac{dA(t)}{dt} = \varepsilon\left(4.13\pi A(t)^2\right)F(t)$$ ε - Concentration of Gas in solution

The change in Force of Buoyancy: $$\frac{dF(t)}{dt} = \beta A(t)^2 \mathrm{H}(\mathrm{t})$$ β - Density of Liquid

The change in Height: $$\frac{dH(t)}{dt} = \alpha e^{F(t)}\left(1 - \frac{H(t)}{n}\right)$$ α- Type of Gas
n-height of container

Assumptions:

- The solution is special where all parameters equal one
- As surface area increases as the height increases
- The bubble forms at the bottom of the glass
- The solution is not being disturbed
- No Ice
- Bubbles do not oscillate at top of glass

Analysis:

- Linearization – unsuccessful (3 repeated eigenvalues of zero) at only eq. point (origin)

Went to graphical analysis!

- DE plots
- 3d plots
- Vector Fields
- Euler's Method
- Phase-portrait

★ Sometimes a Taylor Series expansion of $e^{F(t)}$ was used

$$e^{F(t)} = 1 + F(t) + \frac{F(t)^2}{2!} + \frac{F(t)^3}{3!}$$

Directly solving the System:

$$A(t) = \frac{1}{\int -\frac{648738883\, F(t)}{50000000}\, \mathrm{d}t + \_C1}$$

$$F(t) = \int A(t)^2\, H(t)\, \mathrm{d}t + \_C1$$

$$H(t) = \left(\int \mathrm{e}^{F(t) + \frac{\left(\int \mathrm{e}^{F(t)}\, \mathrm{d}t\right)}{22}}\, \mathrm{d}t + \_C1\right) \mathrm{e}^{\int -\frac{\mathrm{e}^{F(t)}}{22}\, \mathrm{d}t}$$

- Can't directly solve
- Try another way

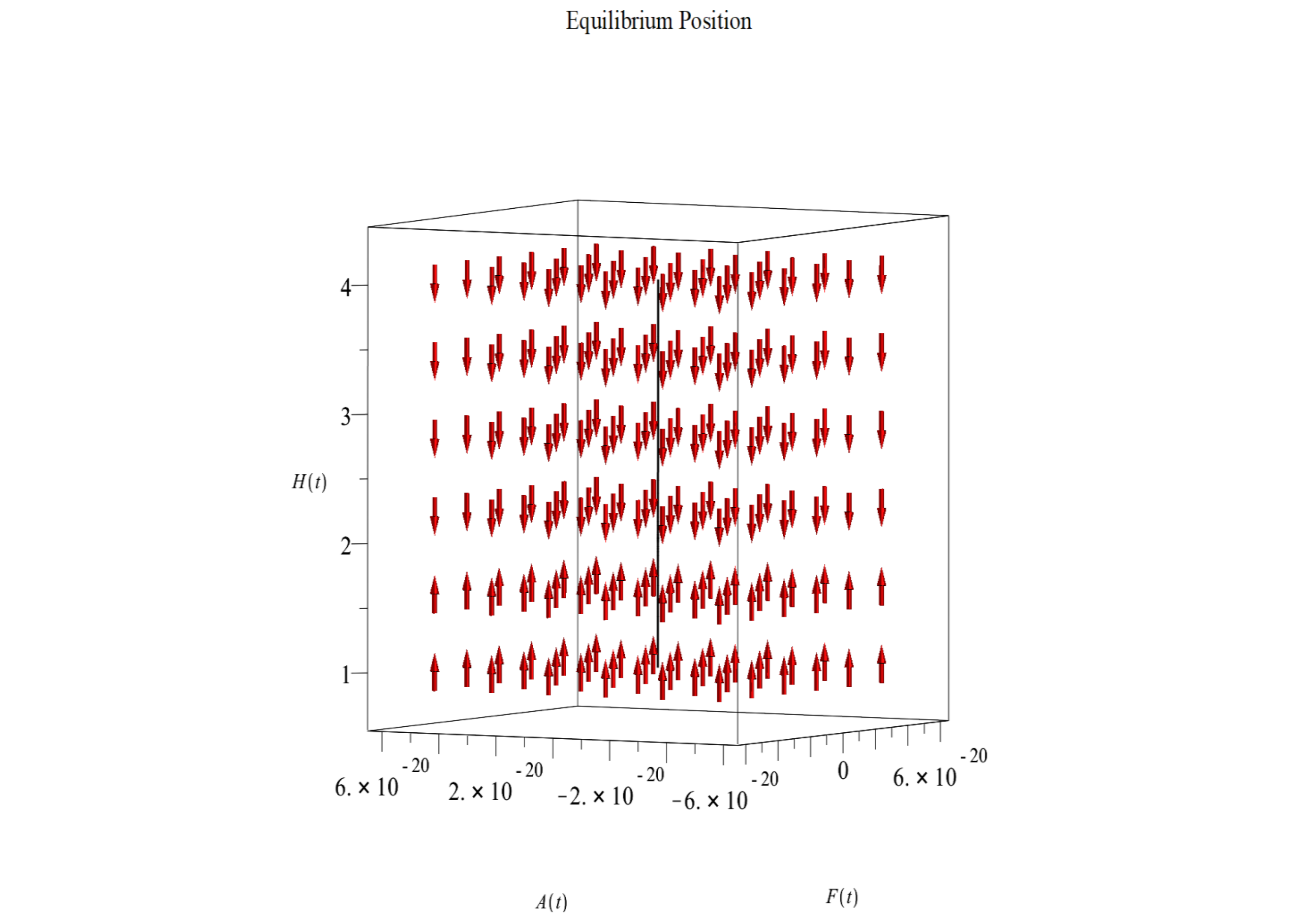
Jacobian and Linearization:

Jacobian Pre-plugging in Equilibrium values (With approximation):

$$\begin{matrix} 12.97477766\,A^2 & 0 & 25.94955532\,A\,F \\ 0 & 1.0\,A^2 & 2.0\,A\,H \\ 1.0\,(1+F)\left(1-\frac{H}{22}\right) & -0.04545454546 - 0.04545454546\,F - 0.02272727273\,F^2 & 0 \end{matrix}$$

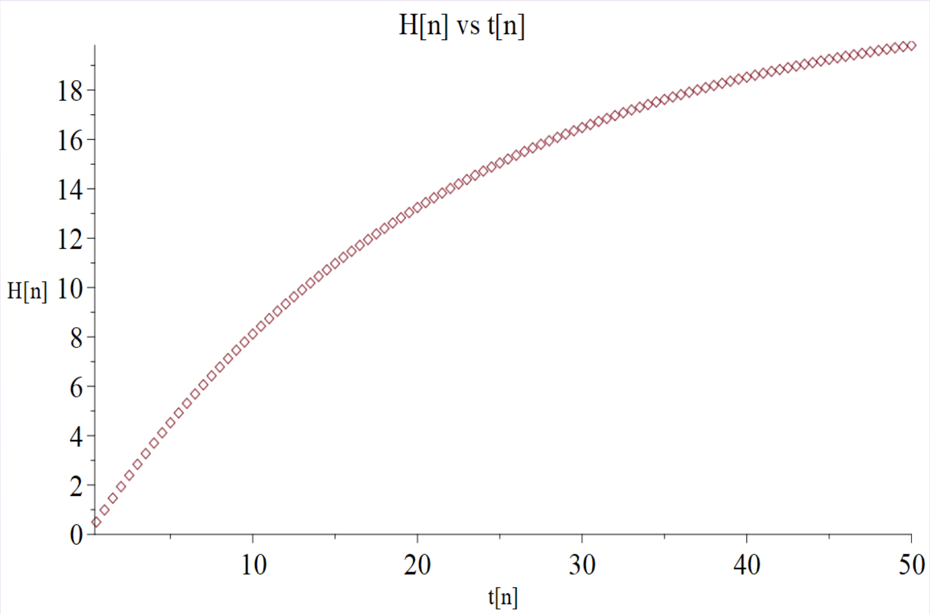
Equilibrium point:

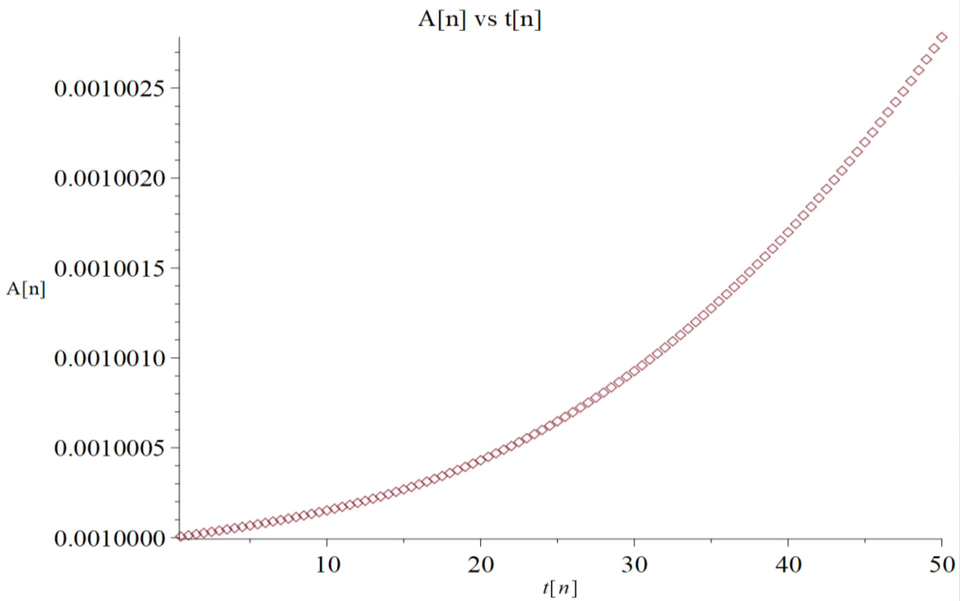
When A(t), F(t) = 0 and H(t)= n (sink)

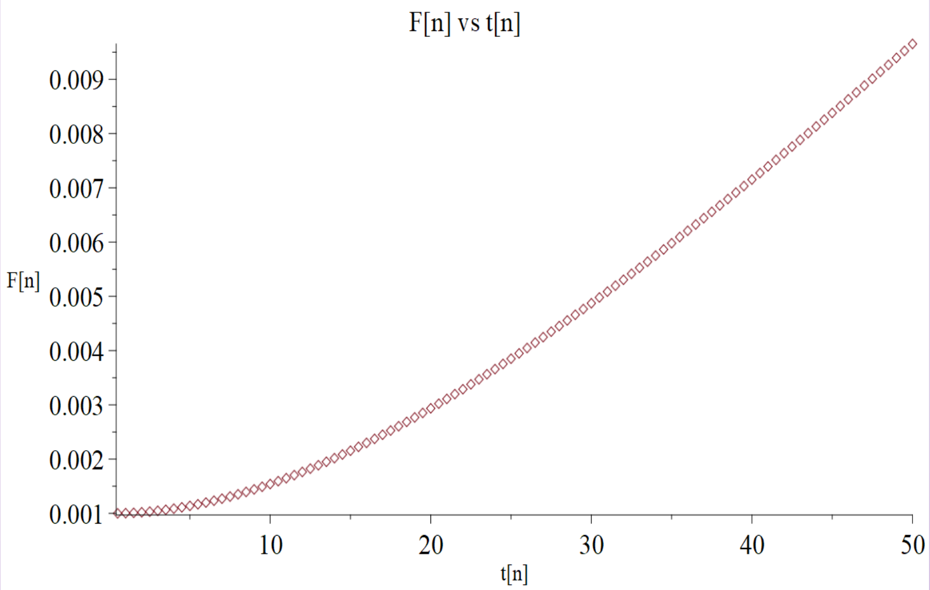


Euler's method

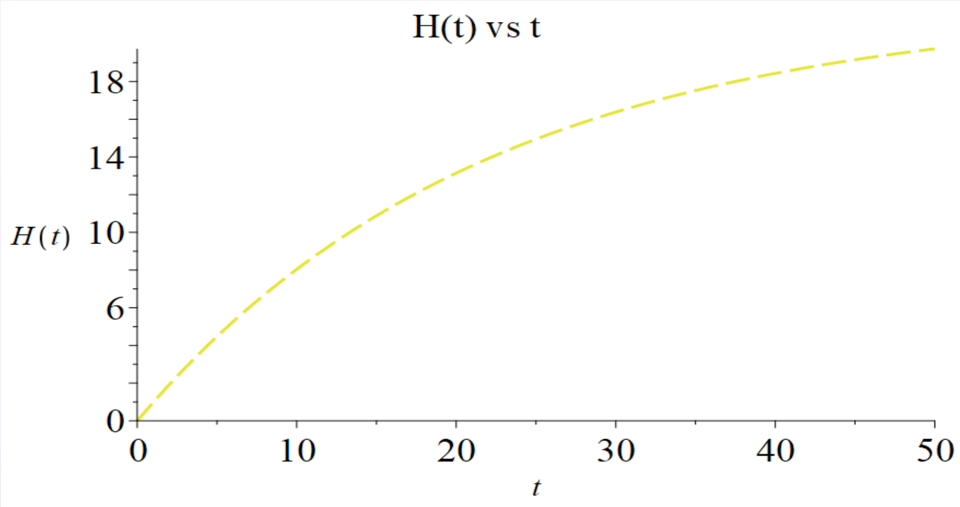
Numerically solving using Euler's method

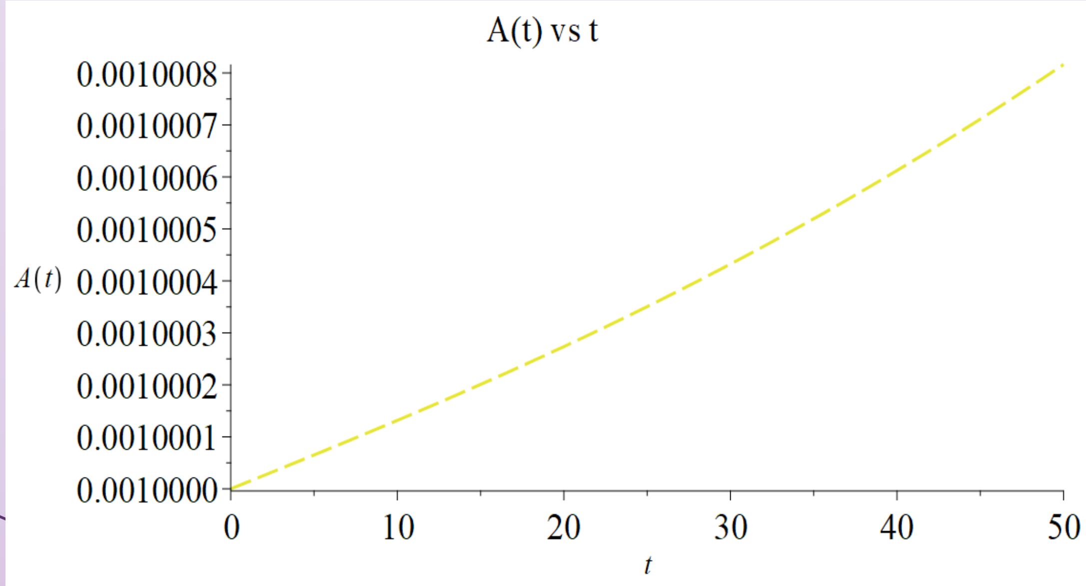


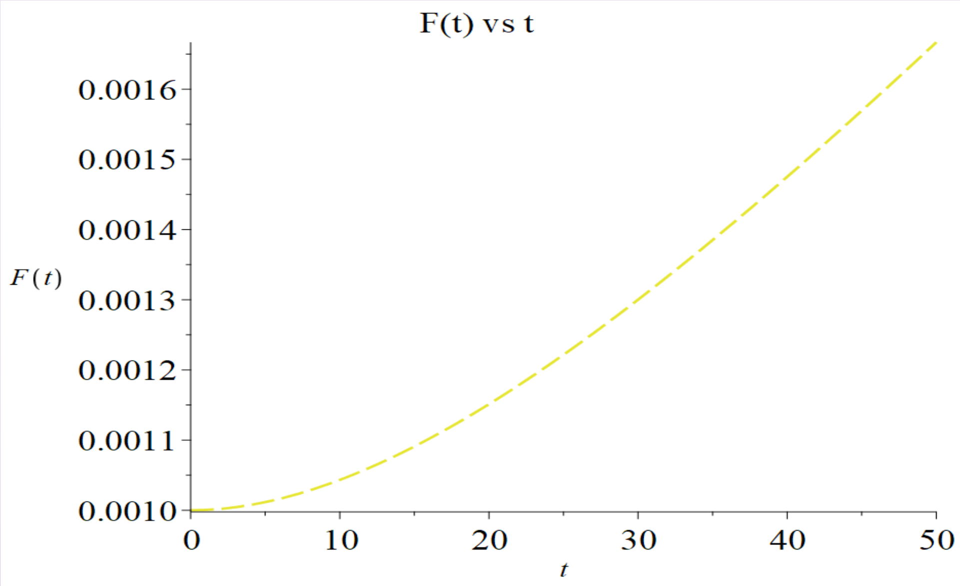




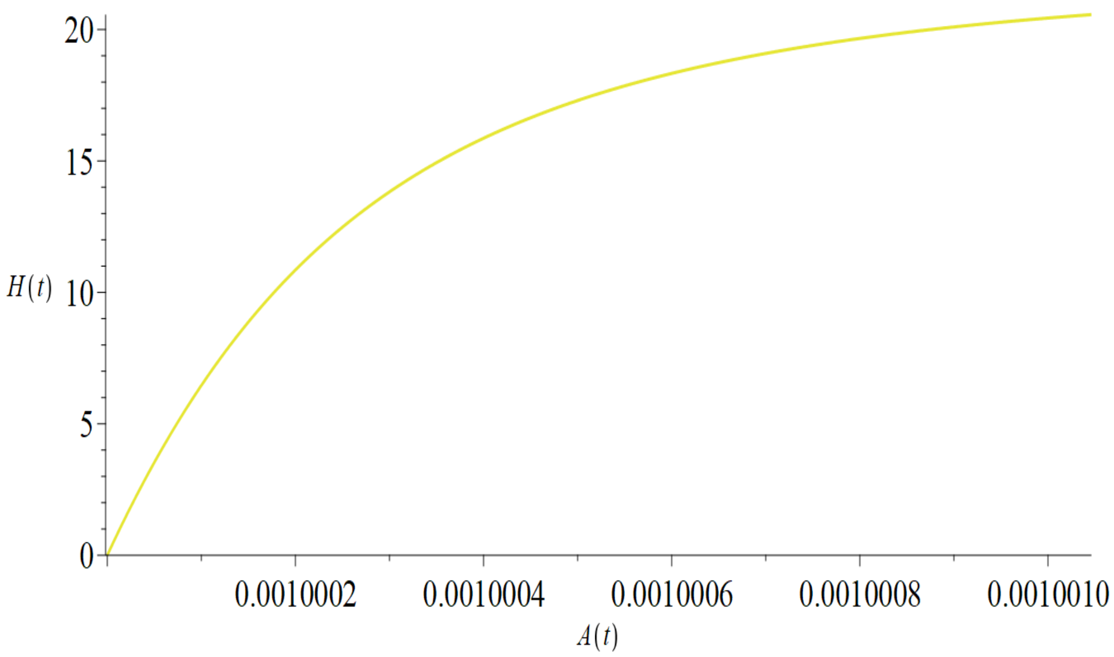
DE plots

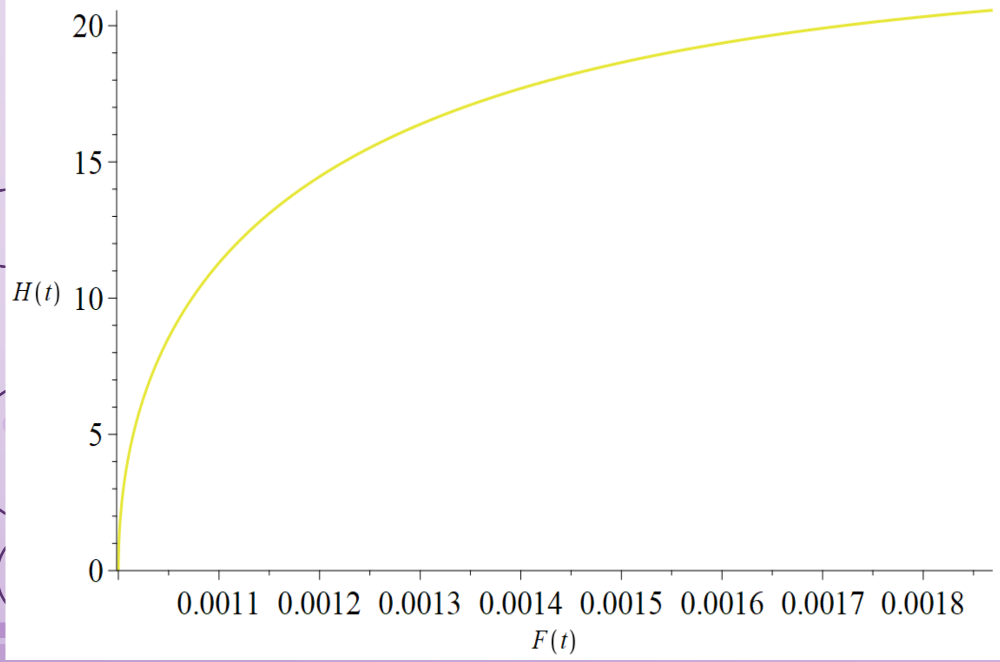

H(t) vs t
18
14
H(t) 10
6
0
0 10 20 30 40 50
t

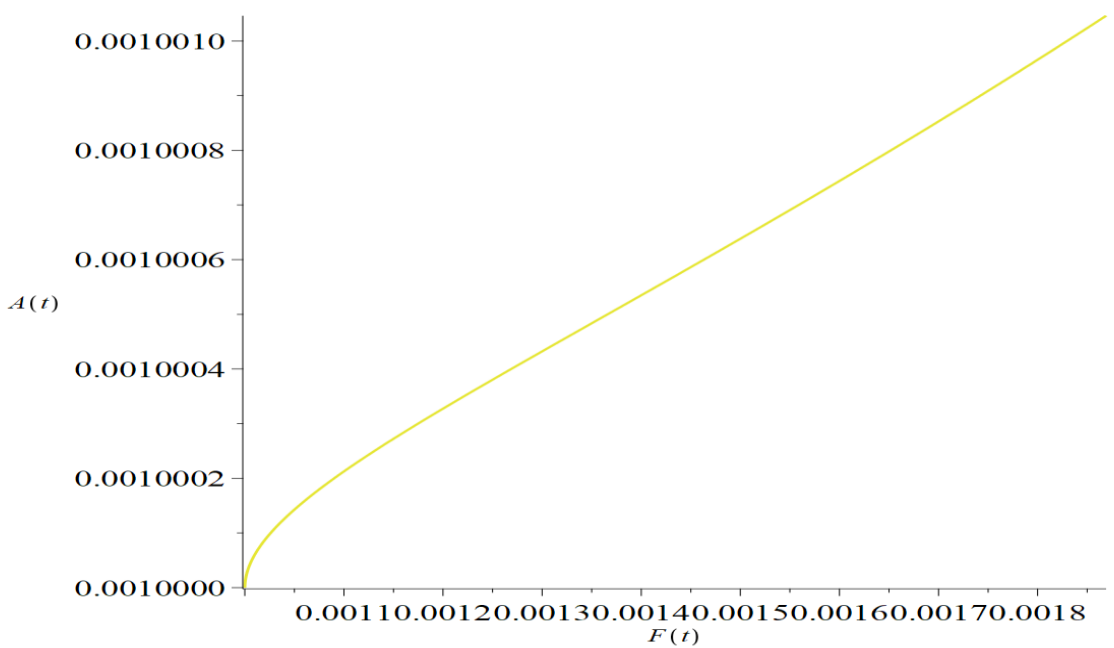

A(t) vs t
0.0010008
0.0010007
0.0010006
0.0010005
A(t) 0.0010004
0.0010003
0.0010002
0.0010001
0.0010000
0 10 20 30 40 50
t


F(t) vs t
0.0016
0.0015
0.0014
F(t) 0.0013
0.0012
0.0011
0.0010
0 10 20 30 40 50
t

Phase Portraits

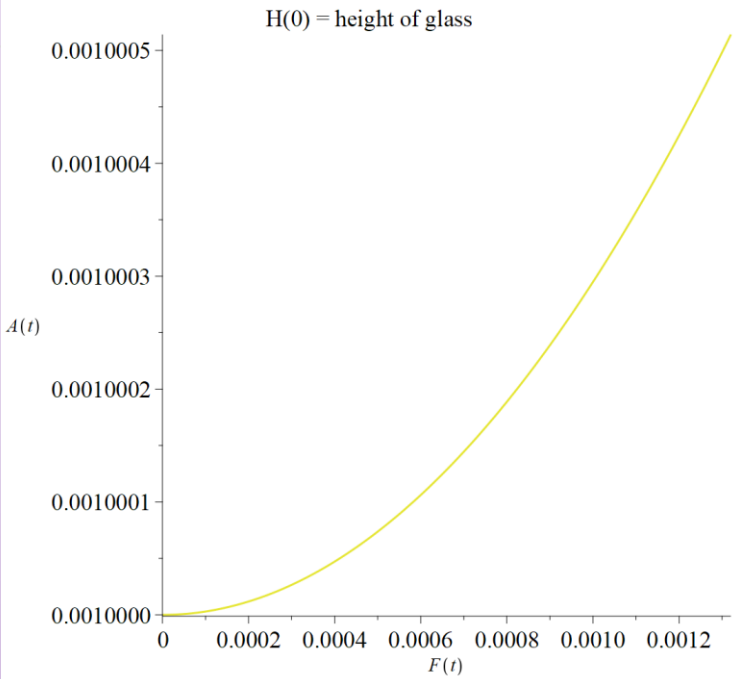






Where can improvements be made?

1. Oscillation of Bubble at top of glass
2. Bubbles do not always form at bottom of solution
3. What happens when you are disturbing the system

4. 

Does this answer if Archimedes' Principle holds true?

Are There any Questions