#1

$$f(x) = \sum_n \frac{f^{(n)}(a)(x-a)^n}{n!}$$

$$f(x \pm dx) = f \pm dx\, f' + \frac{dx^2}{2} f'' \pm \frac{dx^3}{3!} f''' + \frac{dx^4}{4!} f^{(4)} \pm \frac{dx^5}{5!} f^{(5)} + \dots$$

$$f(x \pm 2dx) = f \pm 2dx\, f' + 2dx^2 f'' \pm \frac{8dx^3}{8!} f''' + \frac{16}{4!} dx^4 f^{(4)} + \frac{2^5 dx^5}{5!} f^{(5)} + \dots$$

$$f'(x) \approx \frac{f(x+dx) - f(x-dx)}{2dx}$$

$$\approx \frac{1}{2dx}\Big[\cancel{f} + dx f' + \cancel{\frac{dx^2}{2} f''} + \frac{dx^3}{3!} f''' + \cancel{\frac{dx^4}{4!} f^{(4)}} + \frac{dx^5}{5!} f^{(5)}$$

$$- \cancel{f} + dx f' - \cancel{\frac{dx^2}{2} f''} + \frac{dx^3}{3!} f^{(3)} - \cancel{\frac{dx^4}{4!} f^{(4)}} + f^{(5)}\frac{dx^5}{5!}\Big]$$

$$\approx f' + \frac{dx^2}{3!} f^{(3)} + \cancel{\frac{dx^4}{5!} f^{(5)}} + \dots$$

use centered deriv w/ $x \pm 2dx$

$$f'(x) \approx 2f' + \frac{8}{8!} dx^2 f''' + \cancel{\frac{2^5}{5!} dx^4 f^{(5)}} \dots$$

Combine: $$f' = \underbrace{f' + \frac{dx^2}{2 \cdot 3!} f^{(3)} - \frac{8}{2 \cdot 8!} dx^2 f^{(3)}}_{\text{error term}} = f' + \underbrace{\frac{839}{10080} dx^2 f^{(3)}}_{\text{error term}}$$

b) Following what I did in (a)
the optimal dx should be the same
as derived in class, except
now a constant is multiplied

$$\Rightarrow \quad dx = \left(\frac{\epsilon f}{f'''} \; \frac{10080}{839} \right)^{1/3}$$

I'm suspecting my new approx. in (a)
is wrong.

using $\quad \epsilon = 10^{-7}$ &

$$\frac{d^3}{dx^3} e^x = e^x$$

also note $\quad \dfrac{d^3}{dx^3} e^{\frac{x}{100}} = \dfrac{e^{x/100}}{10^6}$