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(a) $$\sum_{x=0}^{N-1} e^{-2\pi i k x/N} = \frac{1-e^{-2\pi i k}}{1-e^{-2\pi i k/N}}$$

Since $\sum_{x=0}^{N-1} e^{-2\pi i k x/N}$ is a geo series, we can evaluate it by taking the sum from $x=0$ to $x=\infty$ & subtracting the sum from $x=N$ to $x=\infty$:

$$\sum_{x=0}^{N-1} e^{-2\pi i k x/N} = \sum_{x=0}^{\infty} e^{-2\pi i k x/N} - \sum_{x=N}^{\infty} e^{-2\pi i k x/N}$$

let $x' = x + N$:

$$\Rightarrow = \sum_{x=0}^{\infty} e^{-2\pi i k x/N} - \underbrace{\sum_{x'=0}^{\infty} e^{-2\pi i k (x'-N)/N}}_{e^{2\pi i k} \sum_{x=0}^{\infty} e^{-2\pi i k x/N}}$$

$$\Rightarrow = \left(1 - e^{-2\pi i k}\right) \underbrace{\sum_{x=0}^{\infty} e^{-2\pi i k x/N}}_{\text{geo series} \rightarrow \sum_{k=0}^{\infty} r^k = \frac{1}{1-r}} = \boxed{\frac{1-e^{-2\pi i k}}{1-e^{-2\pi i k/N}}}$$

b) $\lim_{k \to 0} \sum_{x=0}^{N-1} e^{-2\pi i k x/N} \rightarrow \sum_{x=0}^{N-1} 1 = N$

$k = mN, \quad m \in \mathbb{Z}$

$\sum_{x=0}^{N-1} e^{-2\pi i x m}$