Closeness & Betweenness Centrality

Learning Goals

- Revisit the conceptualization of "centrality".
- Understand calculation of *closeness* centrality and *betweenness* centrality.
- Compare different measures of centrality.

When we say a *node* is "central,"
what do we mean conceptually?

Conceptualization

- "Everyone agrees, it seems, that centrality is an important structural attribute of networks. All concede that it is related to a high degree to other important group properties and processes. But there consensus ends." (Freeman, 1978/1979: 217)
- The type of measure we use depends on the substantive question of interest.
  - Various measures of centrality are correlated, but they operationalize different concepts.

Closeness Centrality

- How "close" is a node to other nodes?
  - In an undirected binary graph, *closeness centrality* measures how near a node is to the other nodes in the network.
    - This is based on the inverse of the distance of each actor to every other actor.
      - Terminology:
        - A **geodesic** is the shortest path between two nodes.
        - The **distance**, $d(n_i,n_j)$, is the length of the path between i and j.

Undirected Networks

Closeness Centrality: Undirected Binary Graphs

$$g$$

$$C_C(n_i) = [\sum_{j=1} d(n_i, n_j)]^{-1}$$

Closeness Centrality: Undirected Binary Graphs

$$g$$

$$C_C(n_i) = [\sum_{j=1} d(n_i, n_j)]^{-1}$$

How far is *i* from every node?

Closeness Centrality: Undirected Binary Graphs

$$g$$

$$C_C(n_i) = [\sum_{j=1} d(n_i, n_j)]^{-1}$$

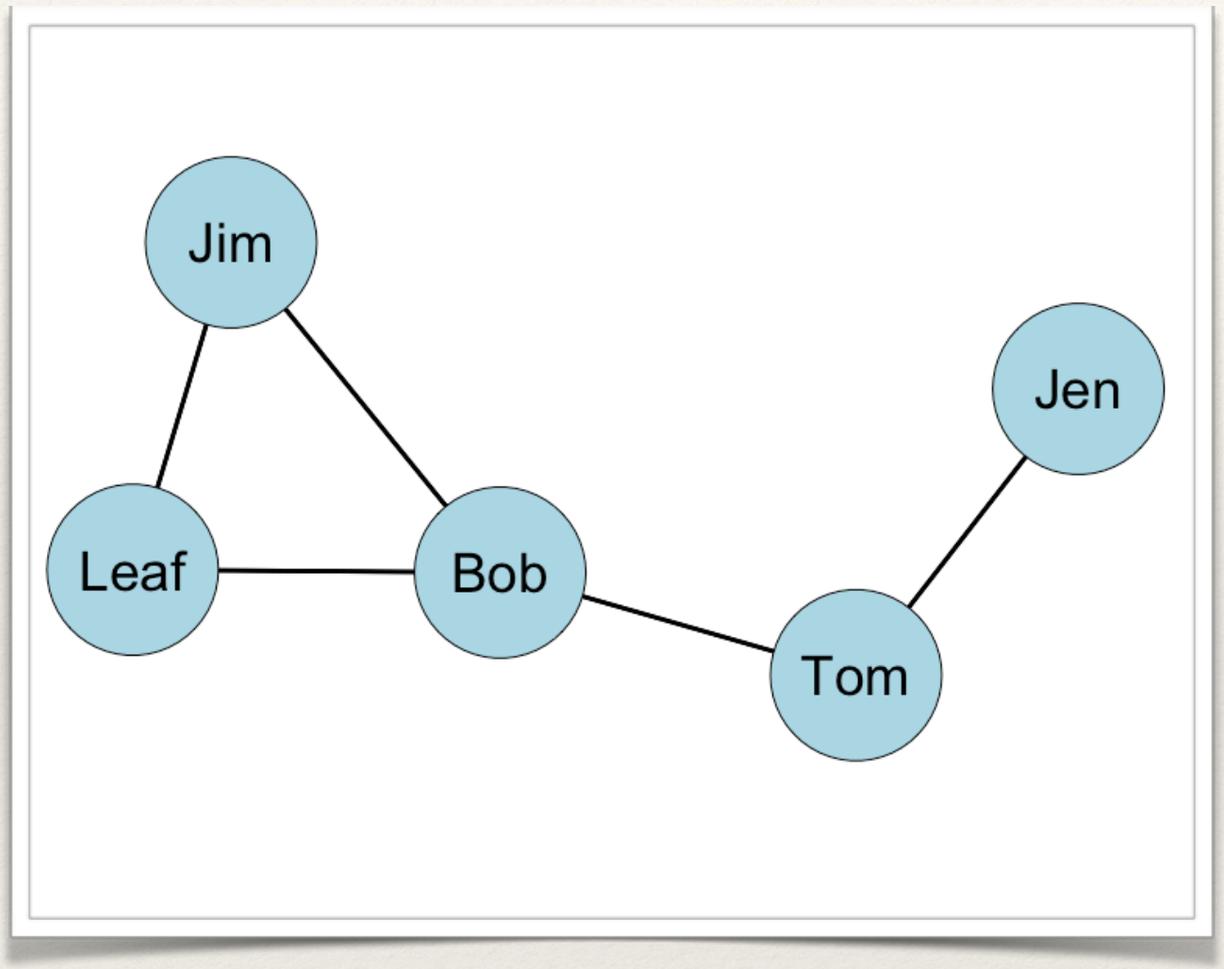
How far is *i* from every node?

Then take the inverse of the sum.

Closeness Centrality: Undirected Binary Graphs

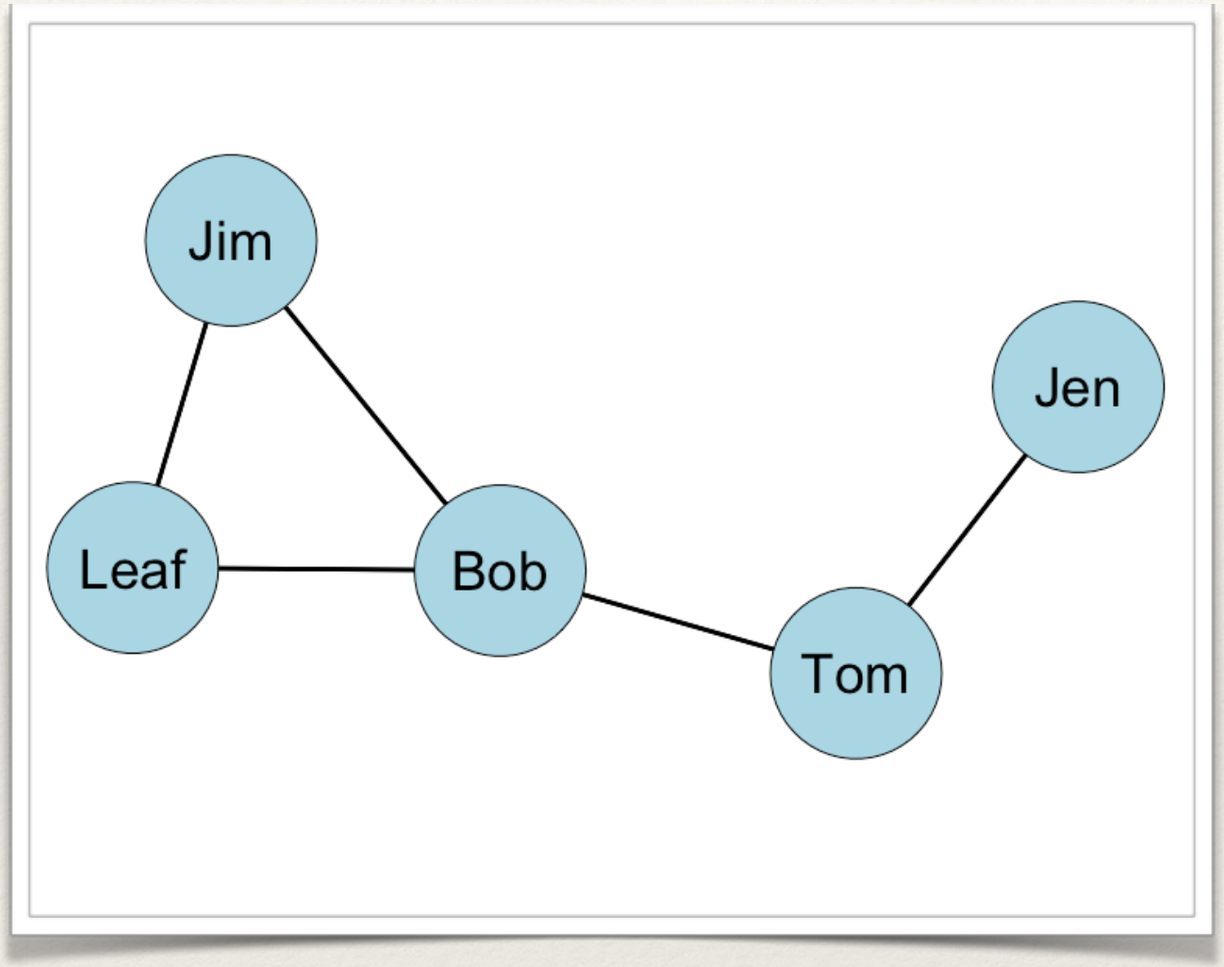
$$C_C(n_i) = [\sum_{j=1}^{g} d(n_i, n_j)]^{-1} = \frac{1}{[\sum_{j=1}^{g} d(n_i, n_j)]}$$

Example: Undirected, Binary Network



We want a matrix of the distances between each node.

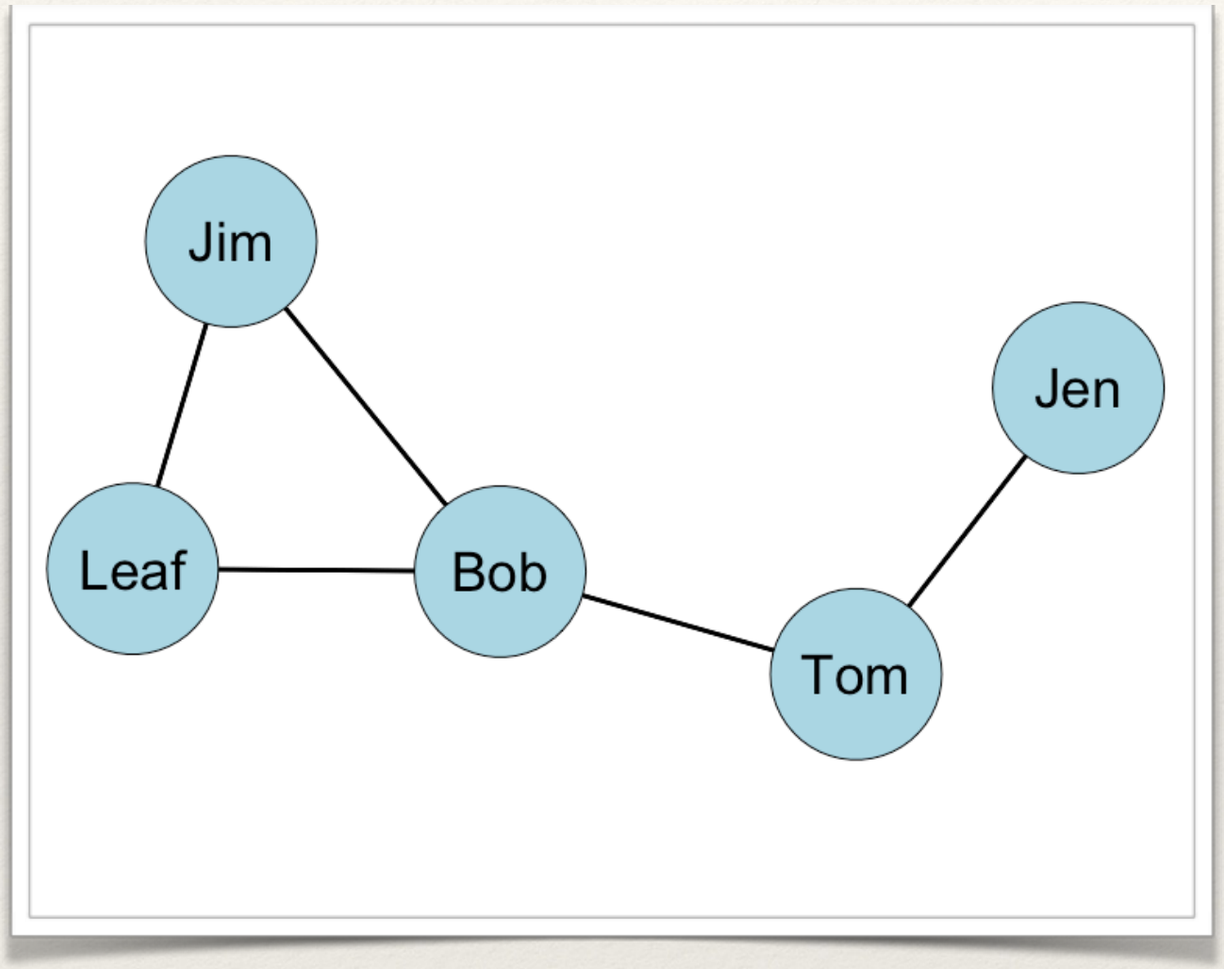
Example: Undirected, Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		?	?		
Tom					
Bob					
Leaf					
Jim					

How far is Jen from Tom? From Bob?

Example: Undirected, Binary Network

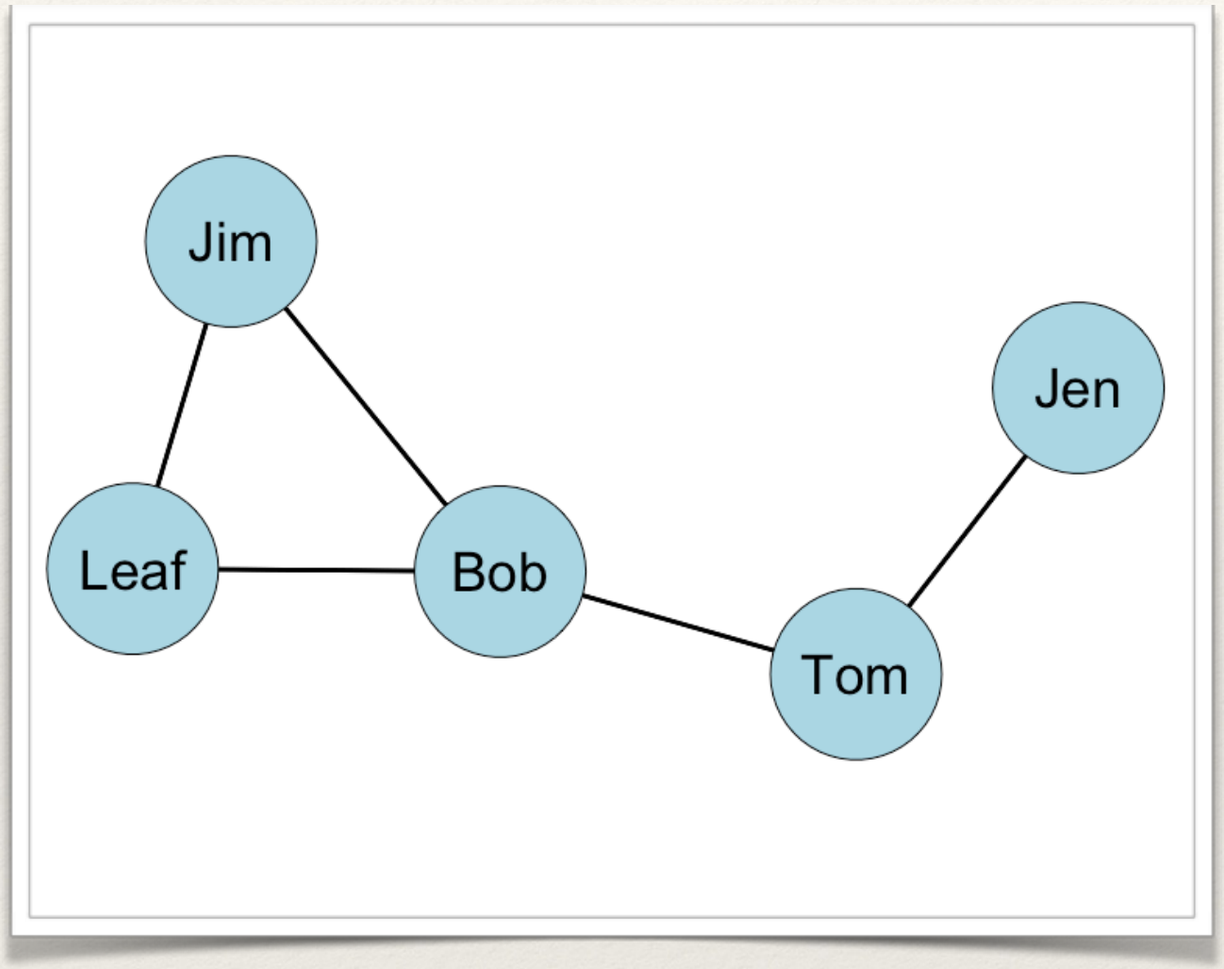


Distance Matrix

	Jen	Tom	Bob	Leaf	Jim
Jen		1			
Tom					
Bob					
Leaf					
Jim					

How far is Jen from Tom? From Bob?

Example: Undirected, Binary Network

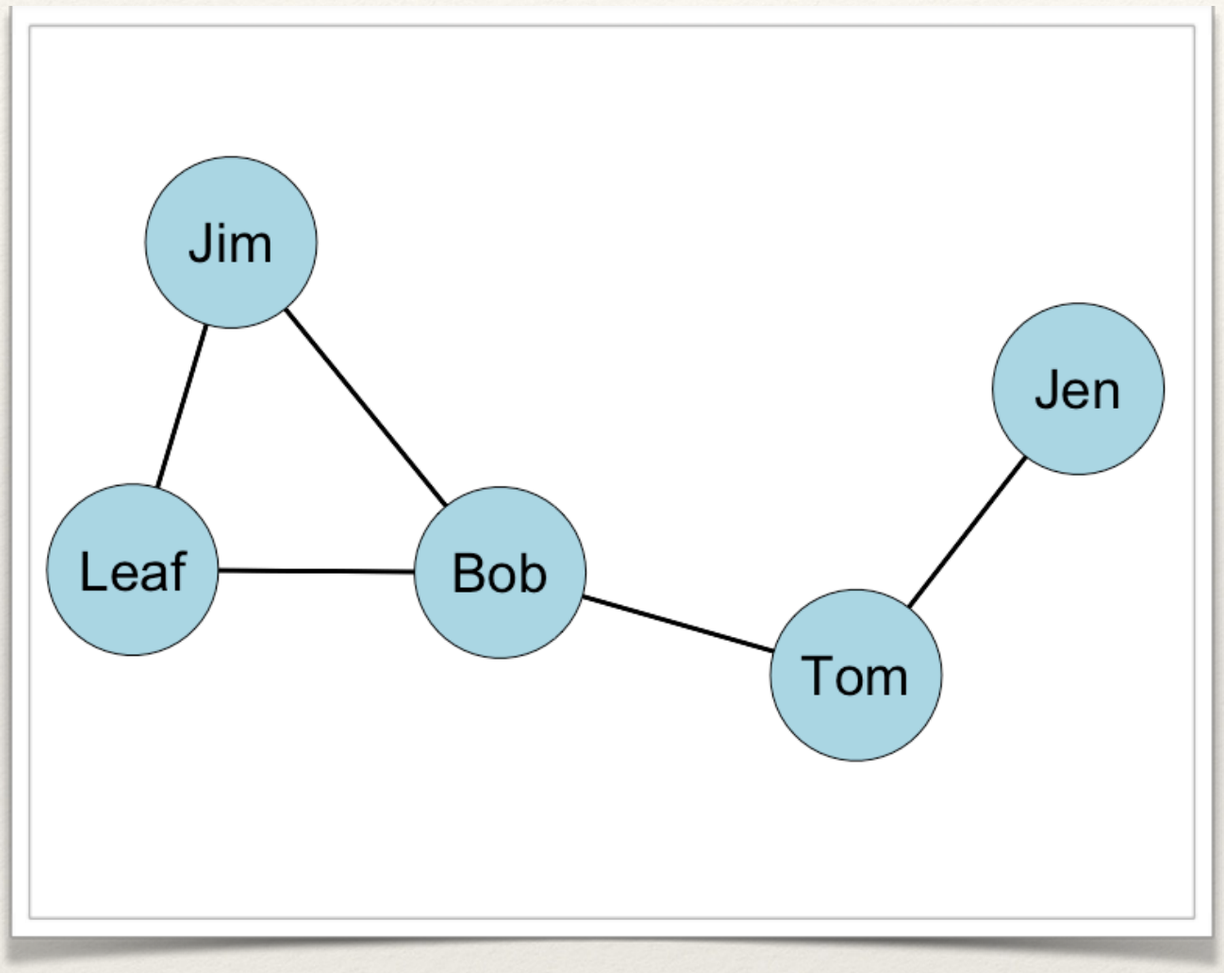


Distance Matrix

	Jen	Tom	Bob	Leaf	Jim
Jen		1	2		
Tom					
Bob					
Leaf					
Jim					

How far is Jen from Tom? From Bob?

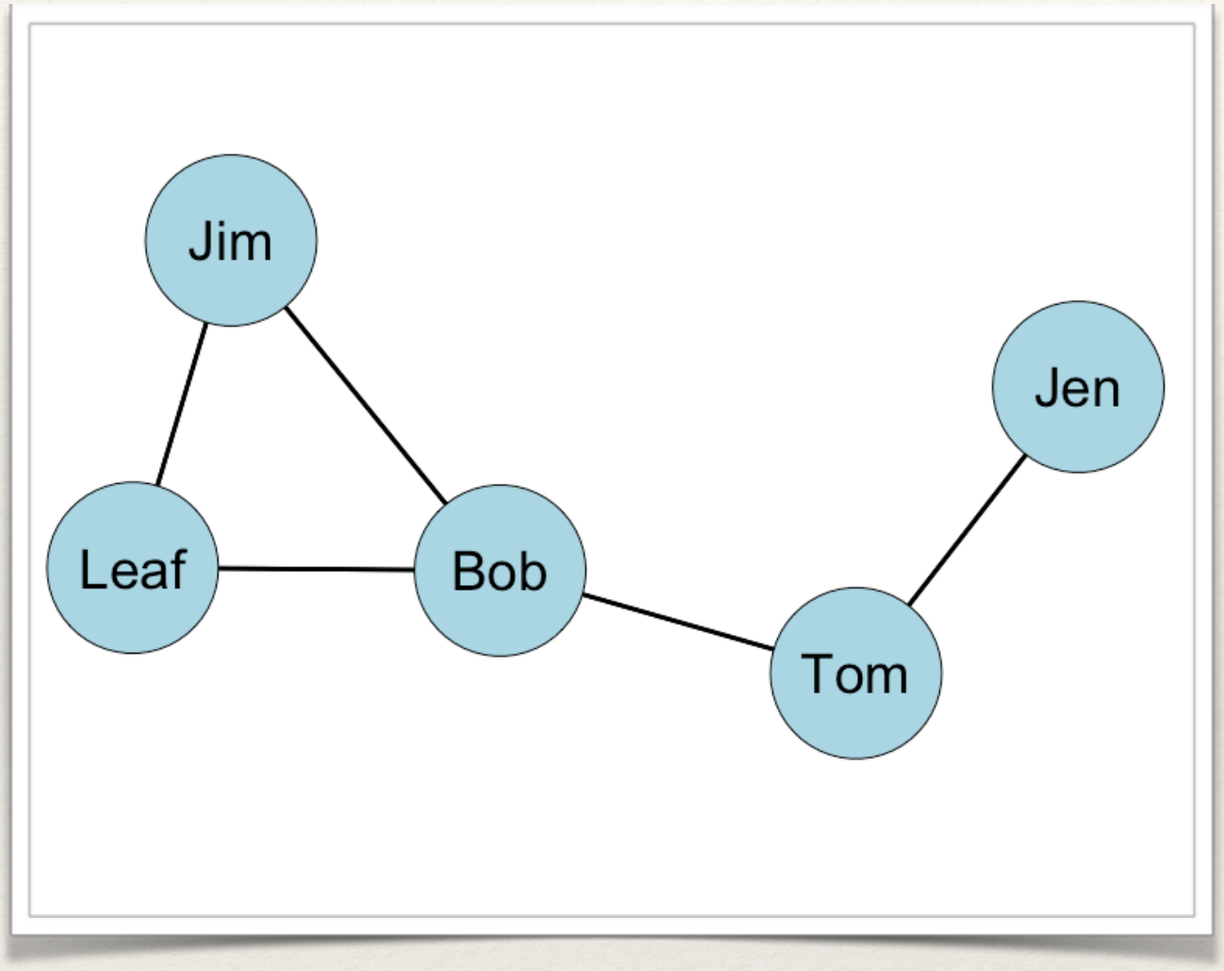
Example: Undirected, Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		1	2	?	?
Tom			?	?	?
Bob				?	?
Leaf					?
Jim					

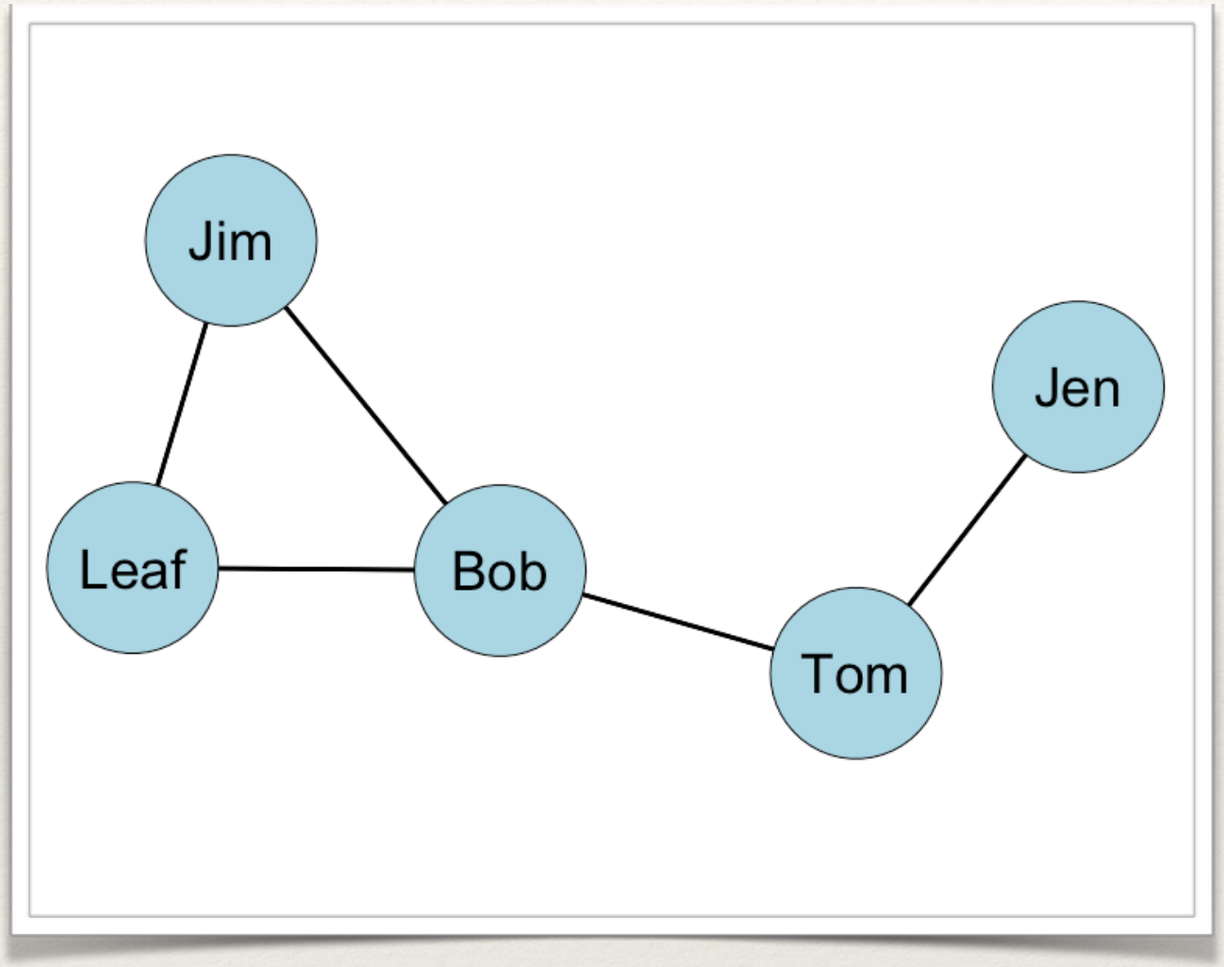
Now, fill in the rest…

Example: Undirected, Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3
Tom			1	2	2
Bob				1	1
Leaf					1
Jim					

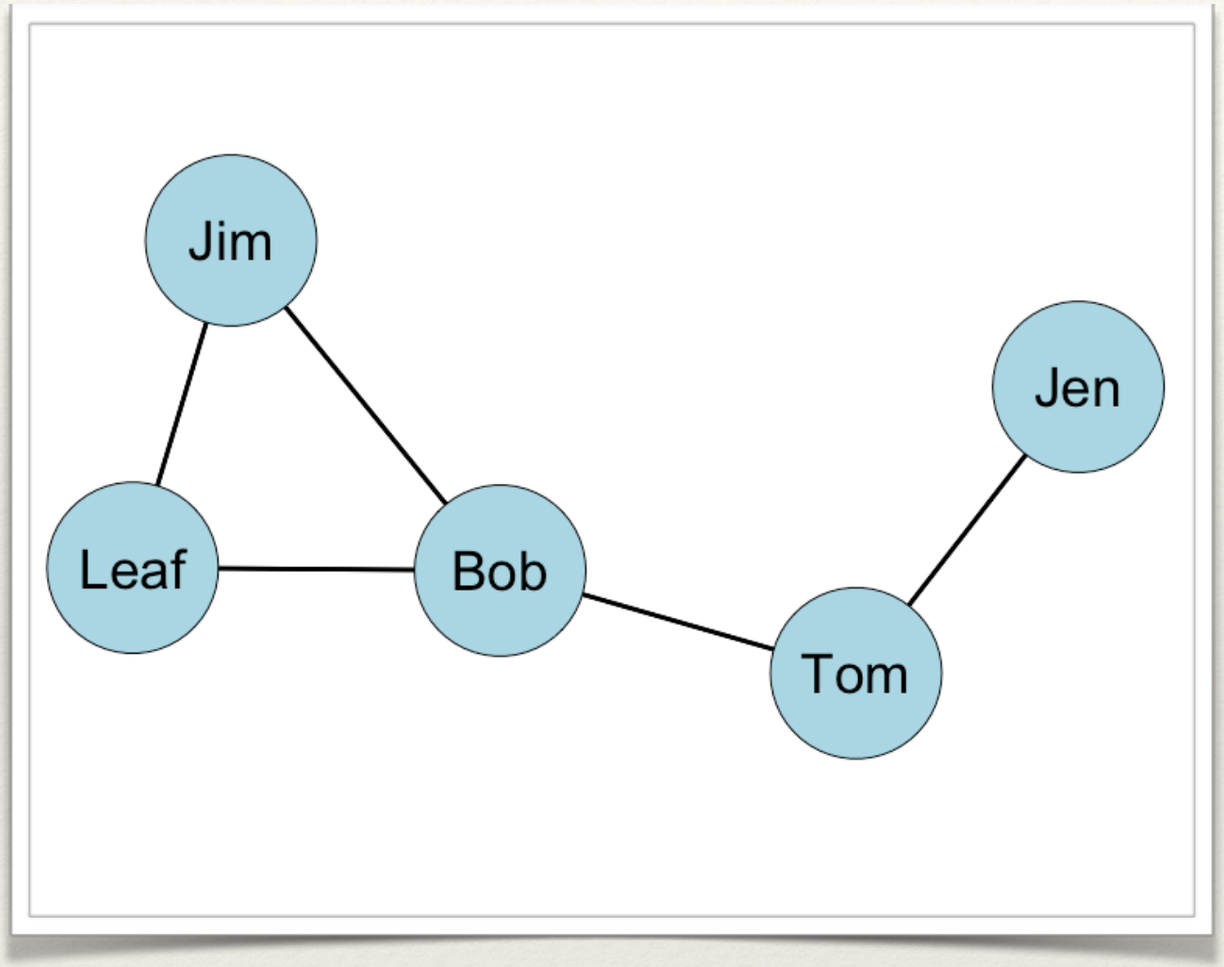
Example: Undirected, Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3
Tom	1		1	2	2
Bob	2	1		1	1
Leaf	3	2	1		1
Jim	3	2	1	1	

Since the graph is undirected, the distance matrix is symmetric about the diagonal.

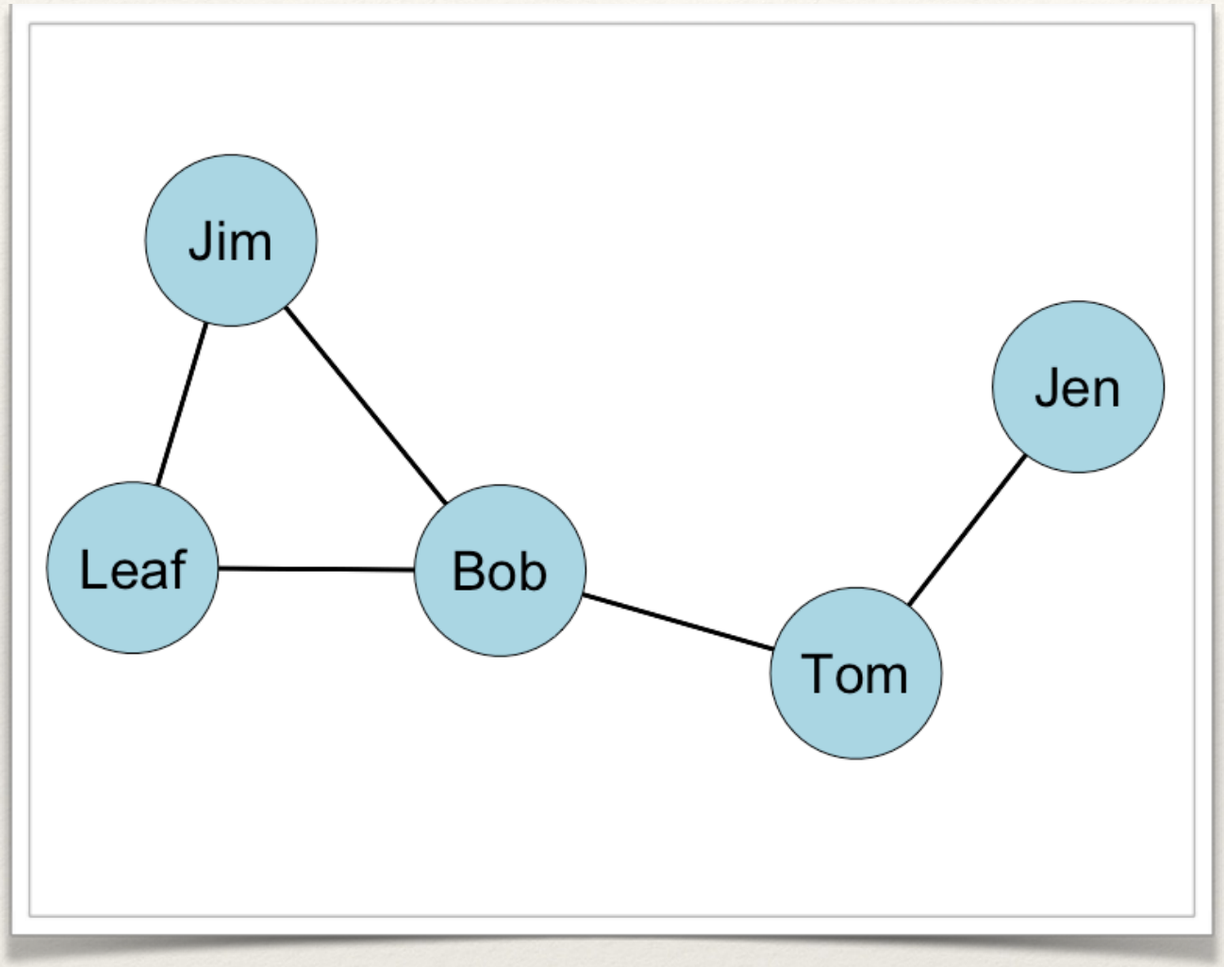
Example: Undirected, Binary Network



Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom	1		1	2	2	6
Bob	2	1		1	1	5
Leaf	3	2	1		1	7
Jim	3	2	1	1		7

The row sum of these is the distances.

Example: Undirected, Binary Network



Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
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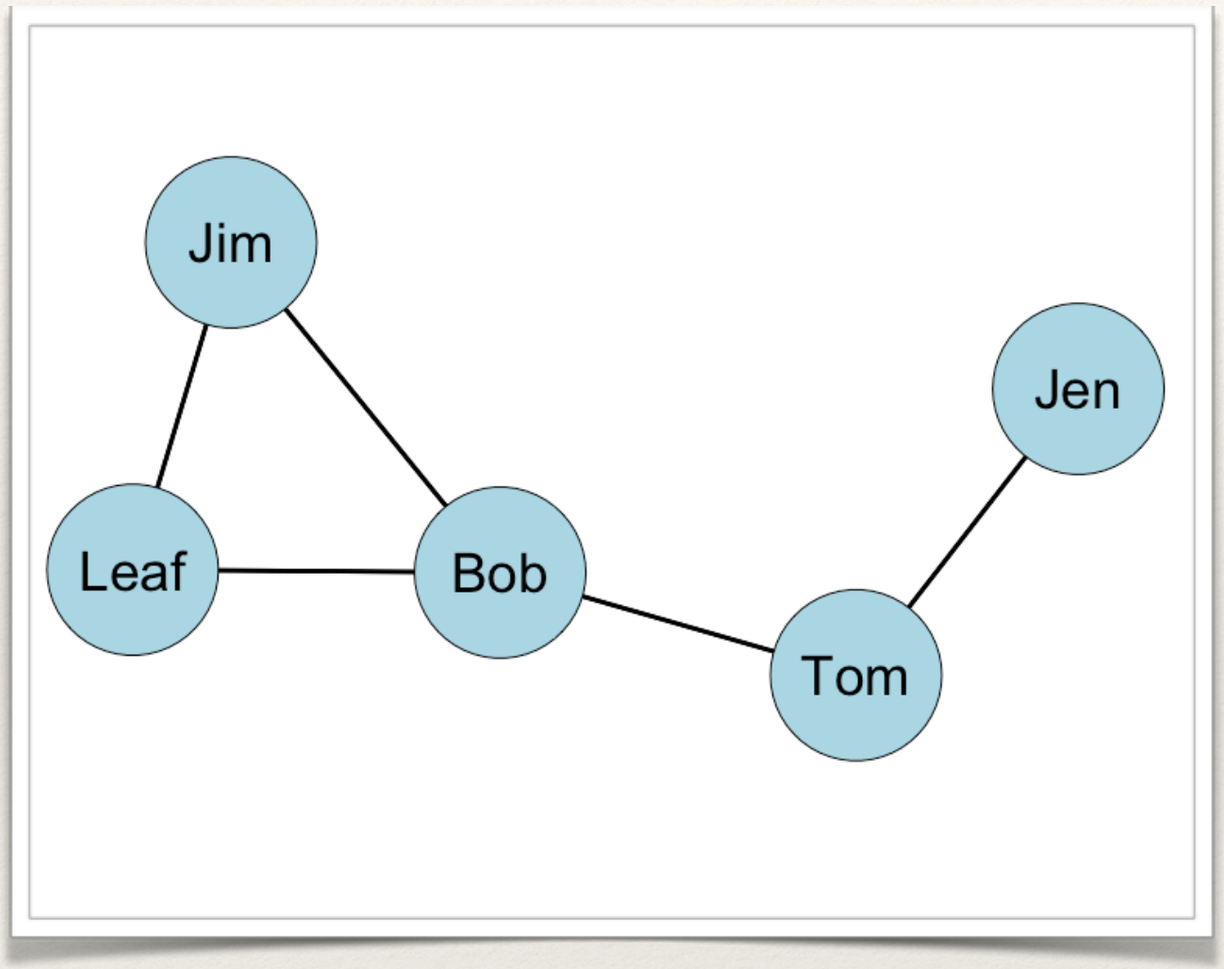
Tom	1		1	2	2	6
Bob	2	1		1	1	5
Leaf	3	2	1		1	7
Jim	3	2	1	1		7
Sum	9	6	5	7	7	

Since the graph is undirected, the column sum reports the same information.

Example: Undirected, Binary Network



By taking the reciprocal, we get the closeness.

Closeness Centrality

Jen = 1/9 = 0.111

Tom = 1/6 = 0.167

Bob = 1/5 = 0.200

Leaf = 1/7 = 0.143

Jim = 1/7 = 0.143

$$C_C(n_i) = \frac{1}{[\sum_{j=1}^{g} d(n_i, n_j)]}$$

Closeness Centrality: Undirected Binary Graphs

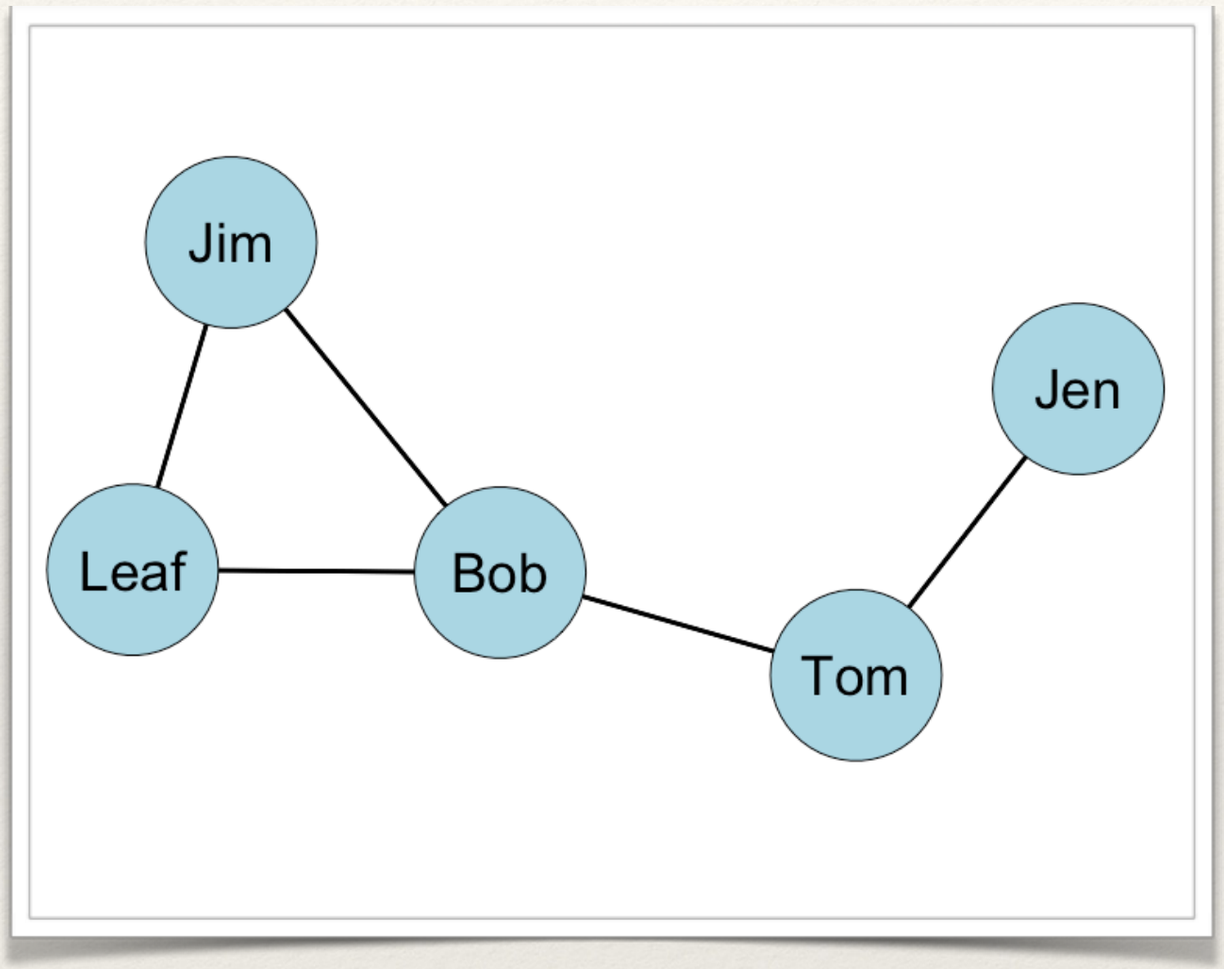
- Actor closeness centrality not only reflects each node's connectivity to other nodes but also depends on the size of the network, g.
- Having to sum over more nodes will push scores closer to zero.
  - *Solution?*

Standardized Closeness Centrality: Undirected Binary Graphs

$$C'_C(n_i) = [\sum_{j=1} d(n_i, n_j)]^{-1} \times [g - 1]$$

Multiply by g -1!

Example: Undirected, Binary Network



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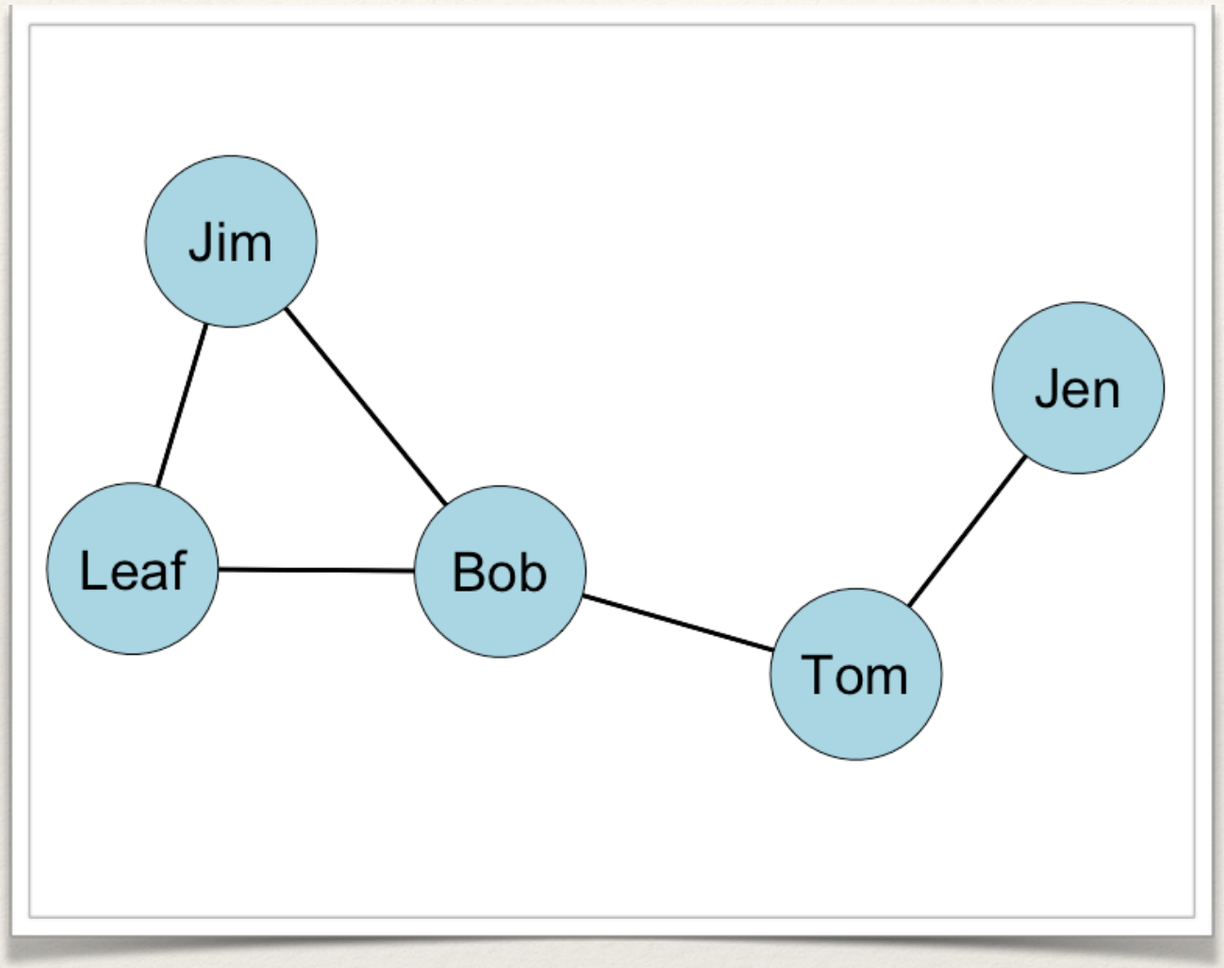
Multiplying by g-1 gives the standardized value

Example: Undirected, Binary Network



Standardized Closeness Centrality

Jen = 0.111*4 = 0.444

Tom = 0.167*4 = 0.668

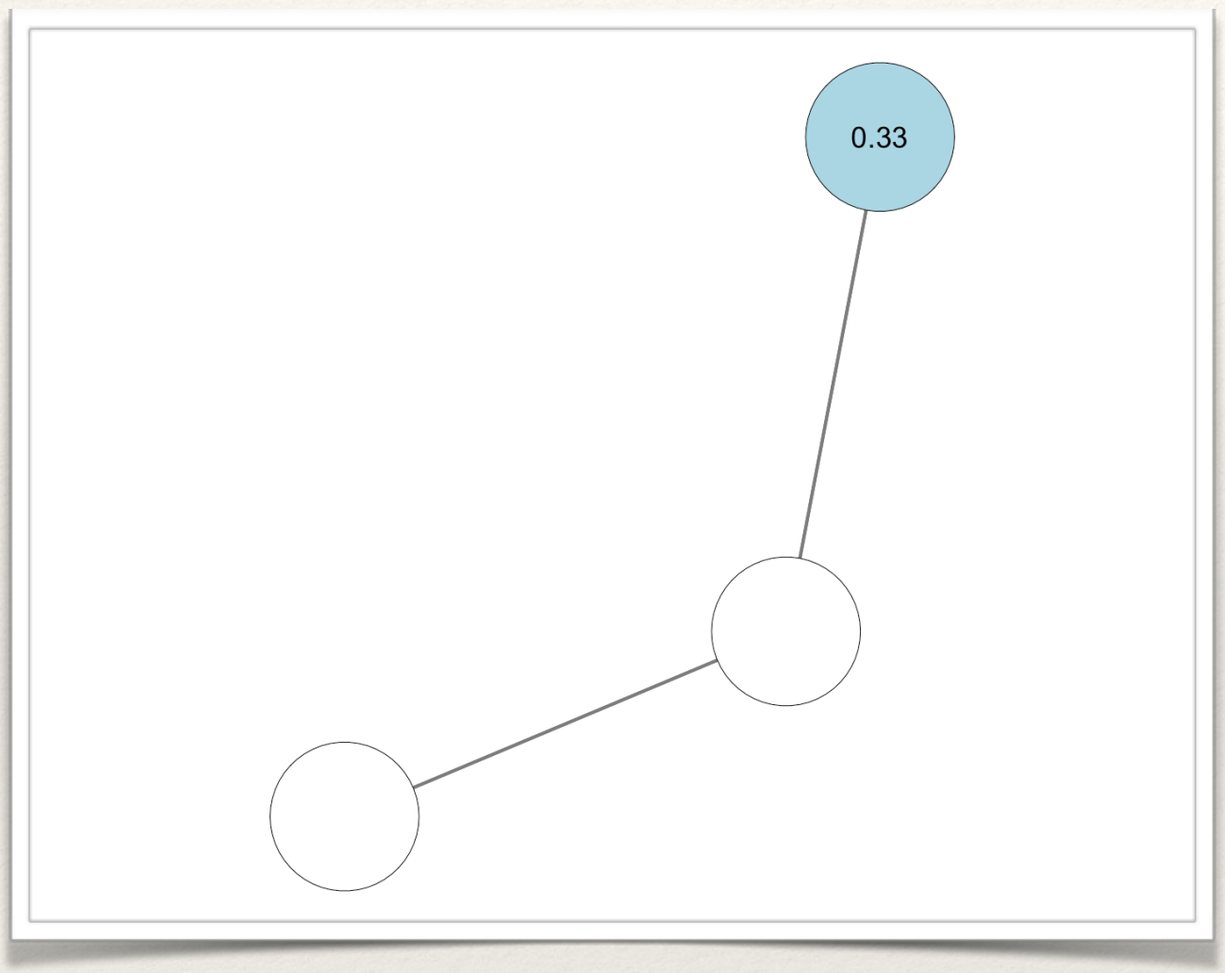
Bob = 0.200*4 = 0.800

Leaf = 0.143*4 = 0.572

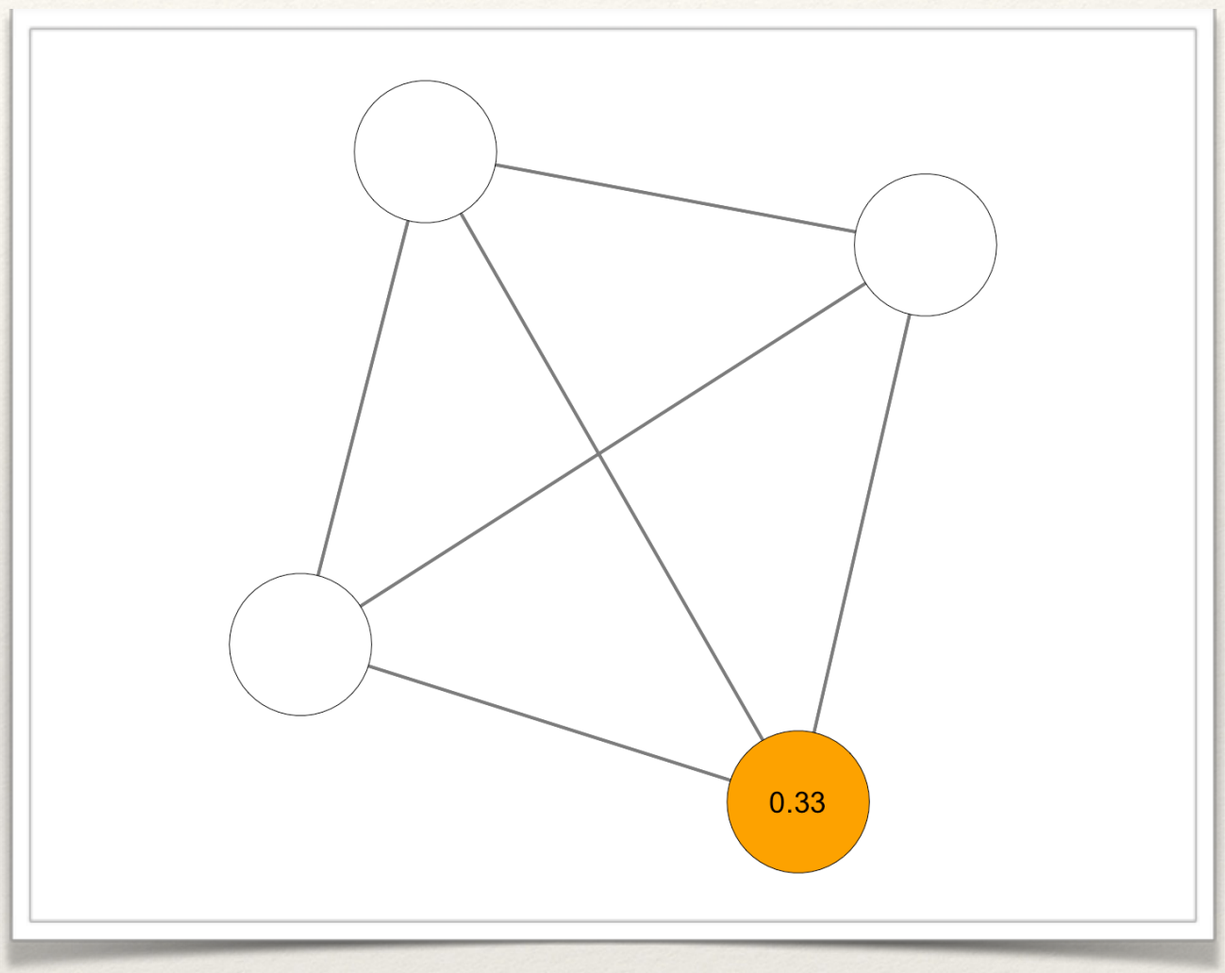
Jim = 0.143*4 = 0.572

Multiplying by g-1 gives the standardized value

Why standardization matters

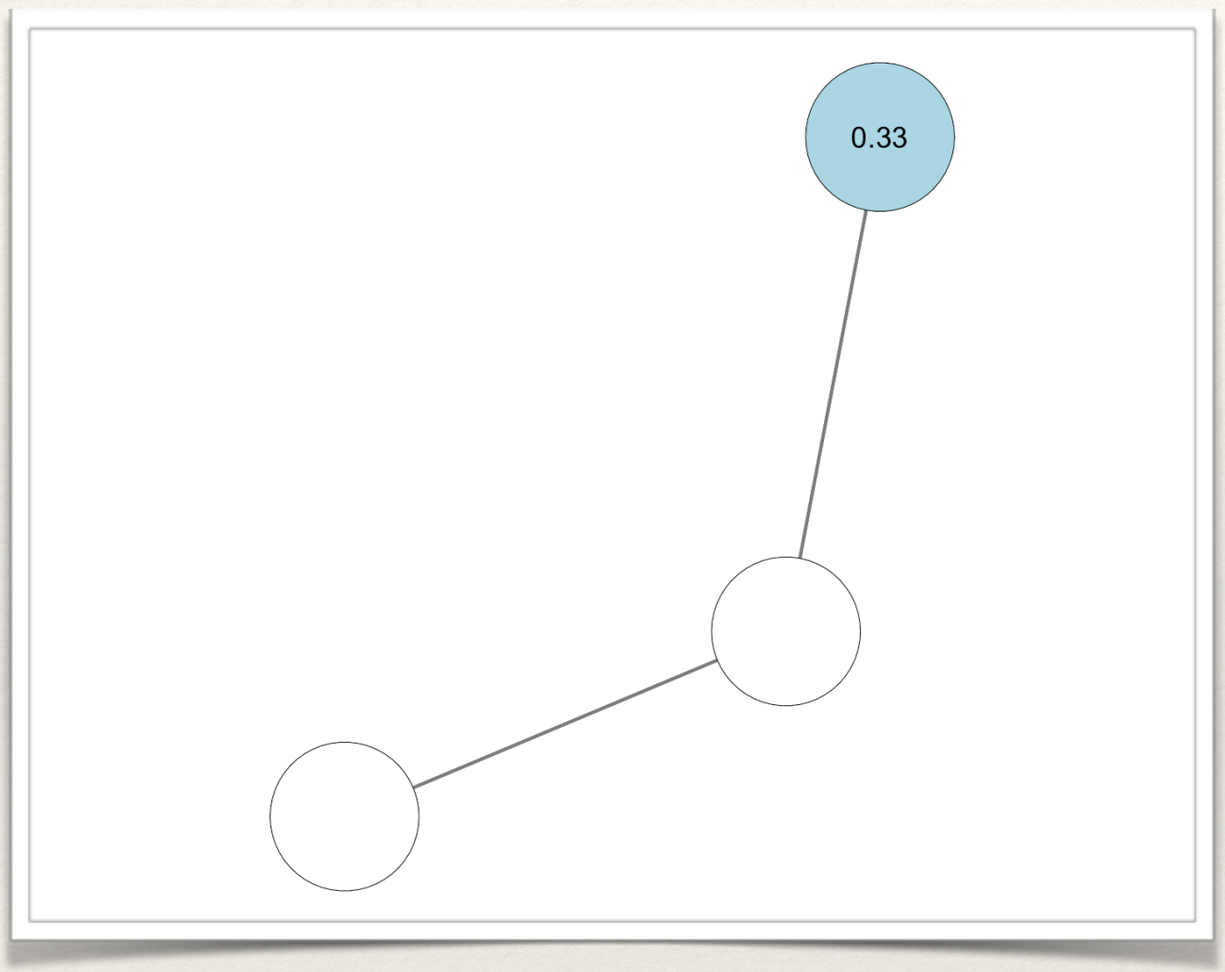


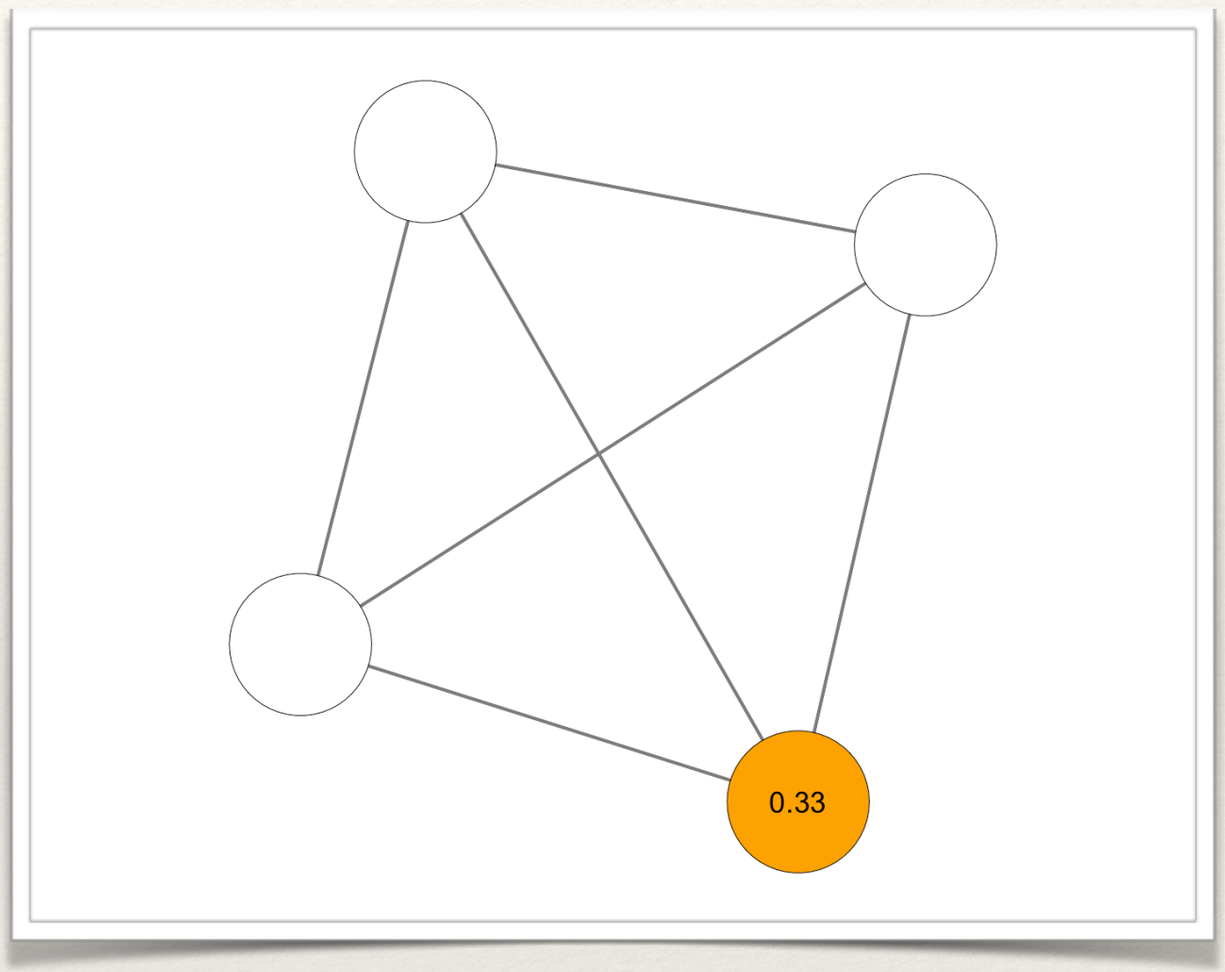
Network A: Raw



Network B: Raw

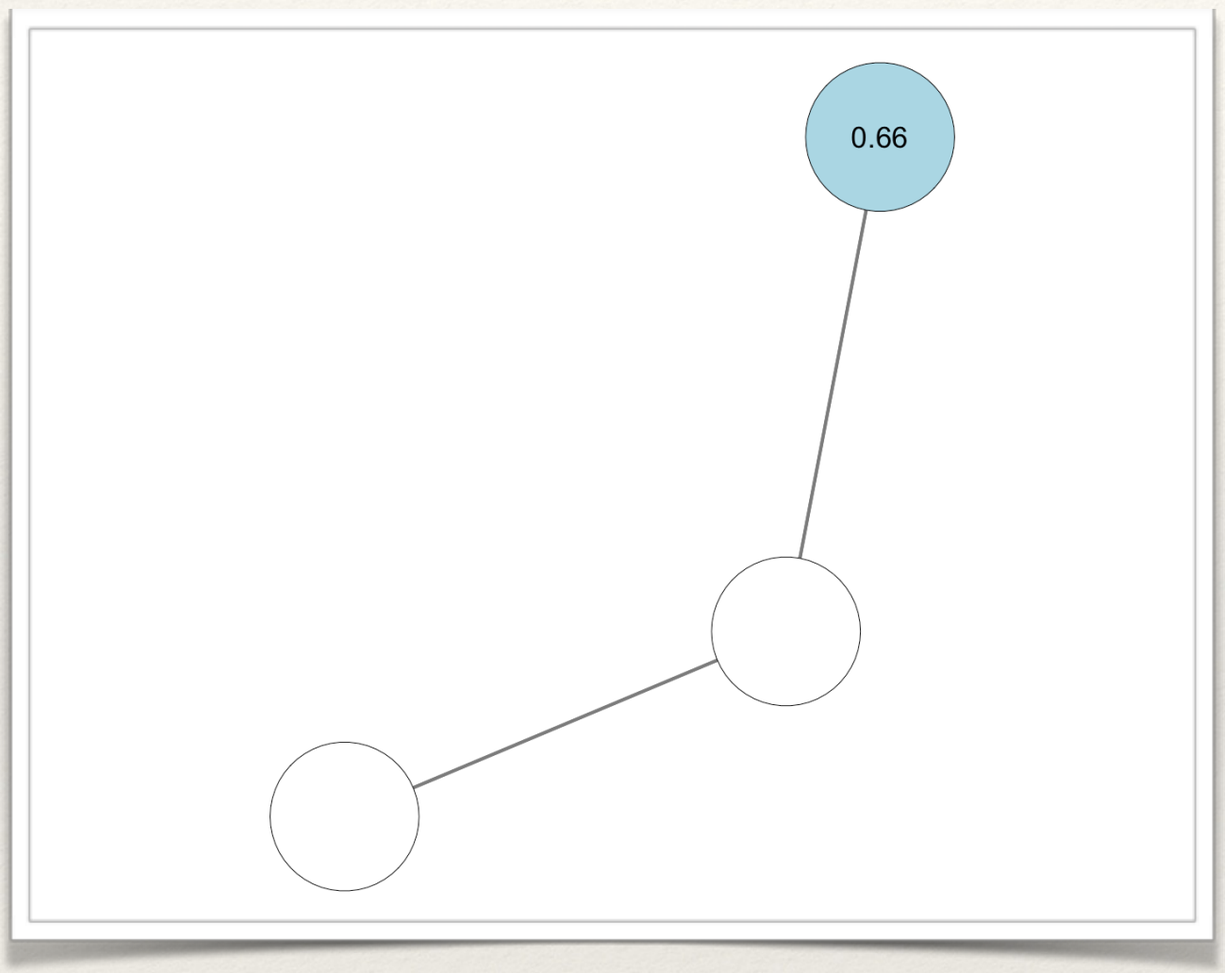
Why standardization matters



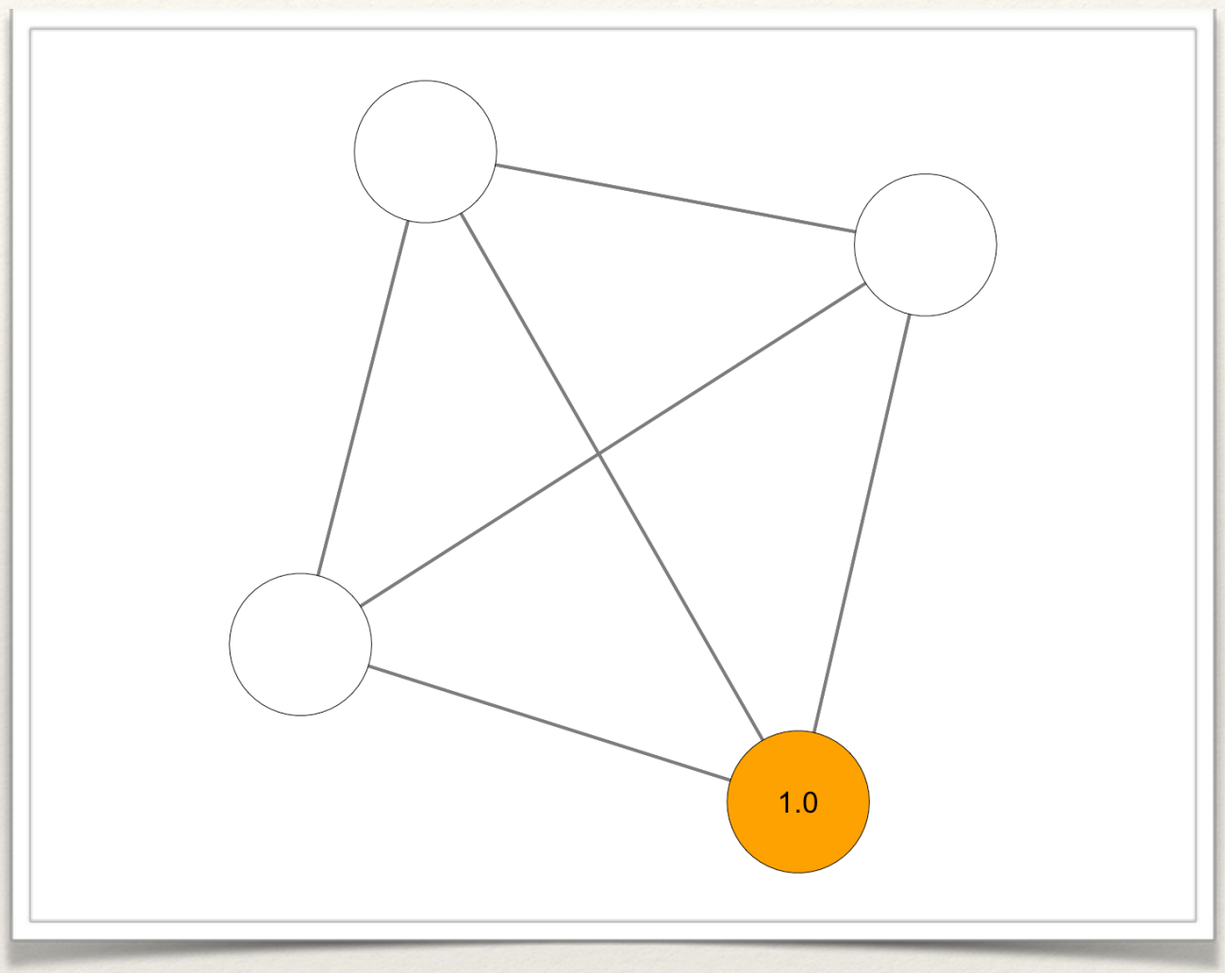


Are the focal nodes equally close to the other nodes?

Why standardization matters

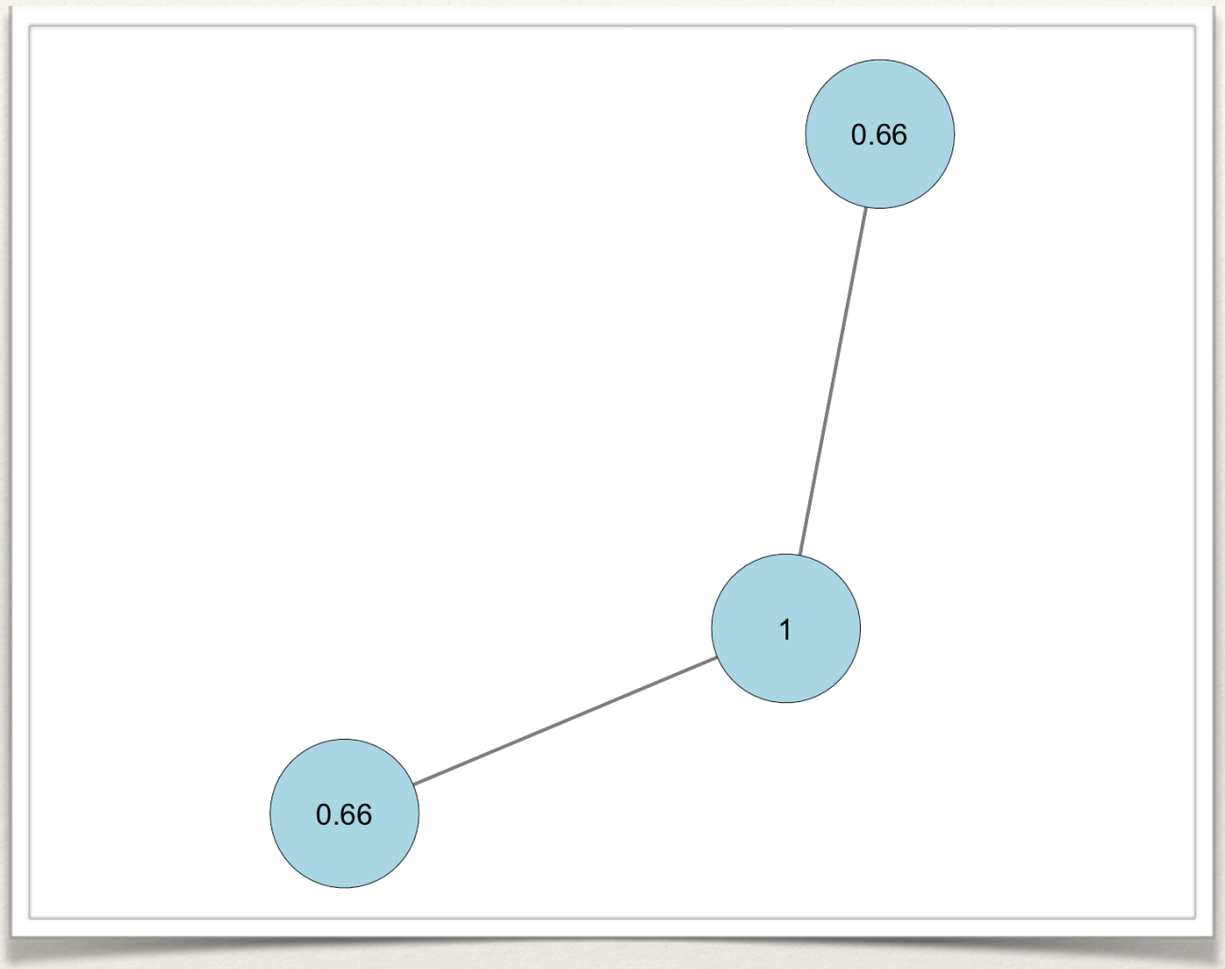


Network A: Standardized

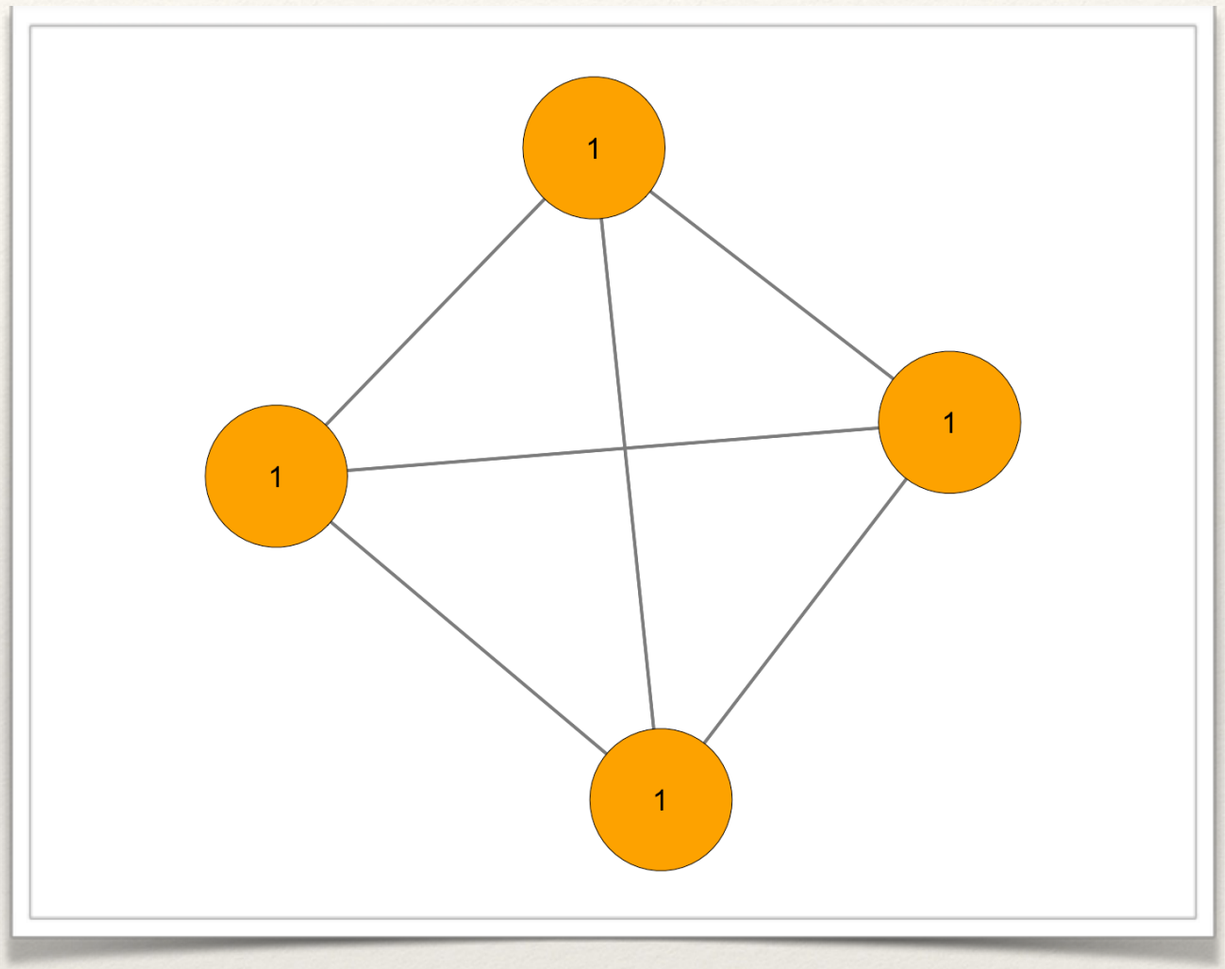


Network B: Standardized

Why standardization matters



Network A: Standardized



Network B: Standardized

Group Closeness Centrality: Undirected Binary Graphs

- We can also summarize the entire network, in terms of how close nodes are to each other.
- *Group closeness centrality* tells us how much variation there is in the closeness scores.

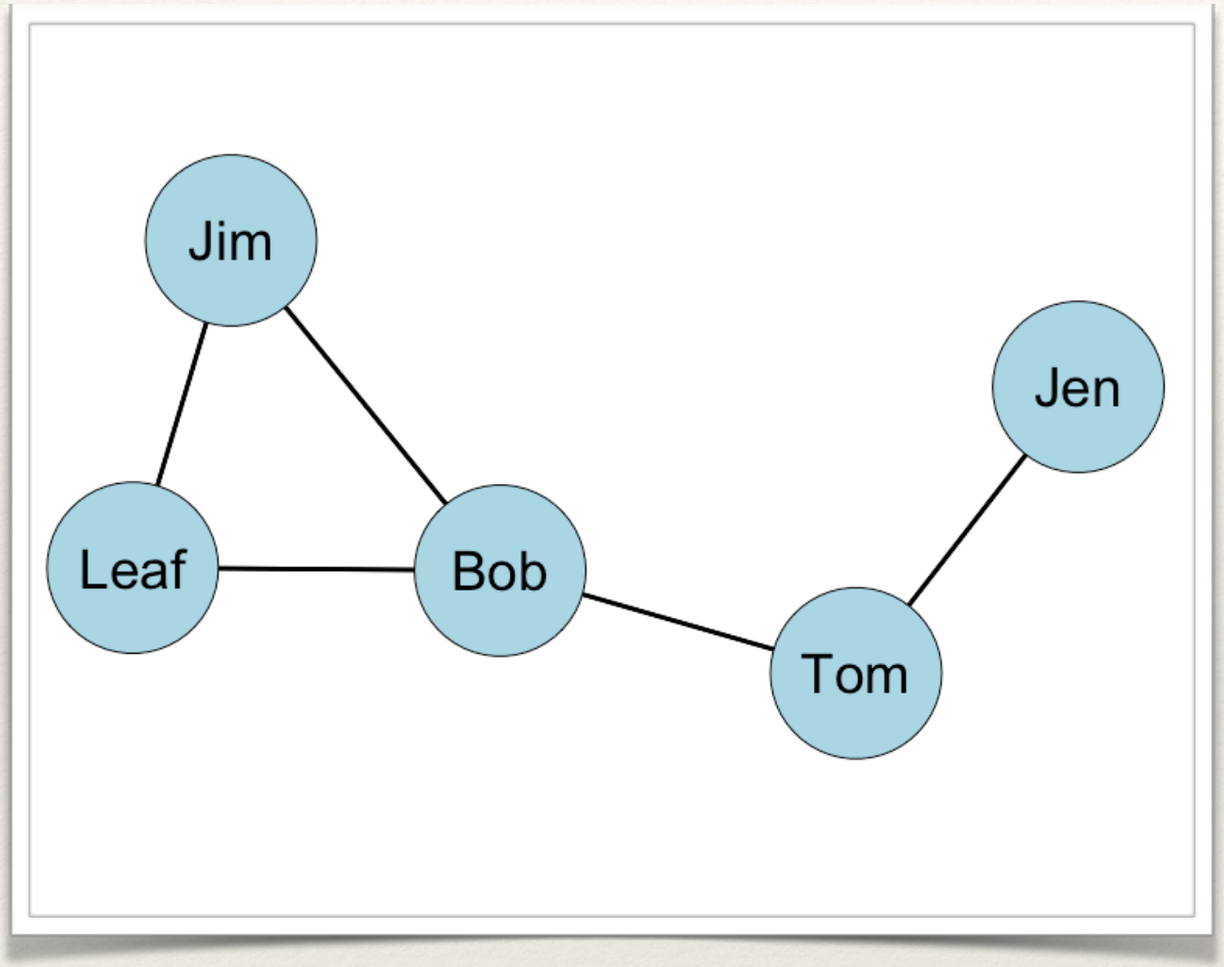
Group Closeness Centrality: Undirected Binary Graphs

Largest standardized closeness score

Standardized closeness score for actor i

$$C_C = \frac{\sum_{i=1}^{g} [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

Example: Undirected, Binary Network



What is the group closeness centralization score for this graph?

Standardized Closeness Centrality

Jen = 0.111*4 = 0.444

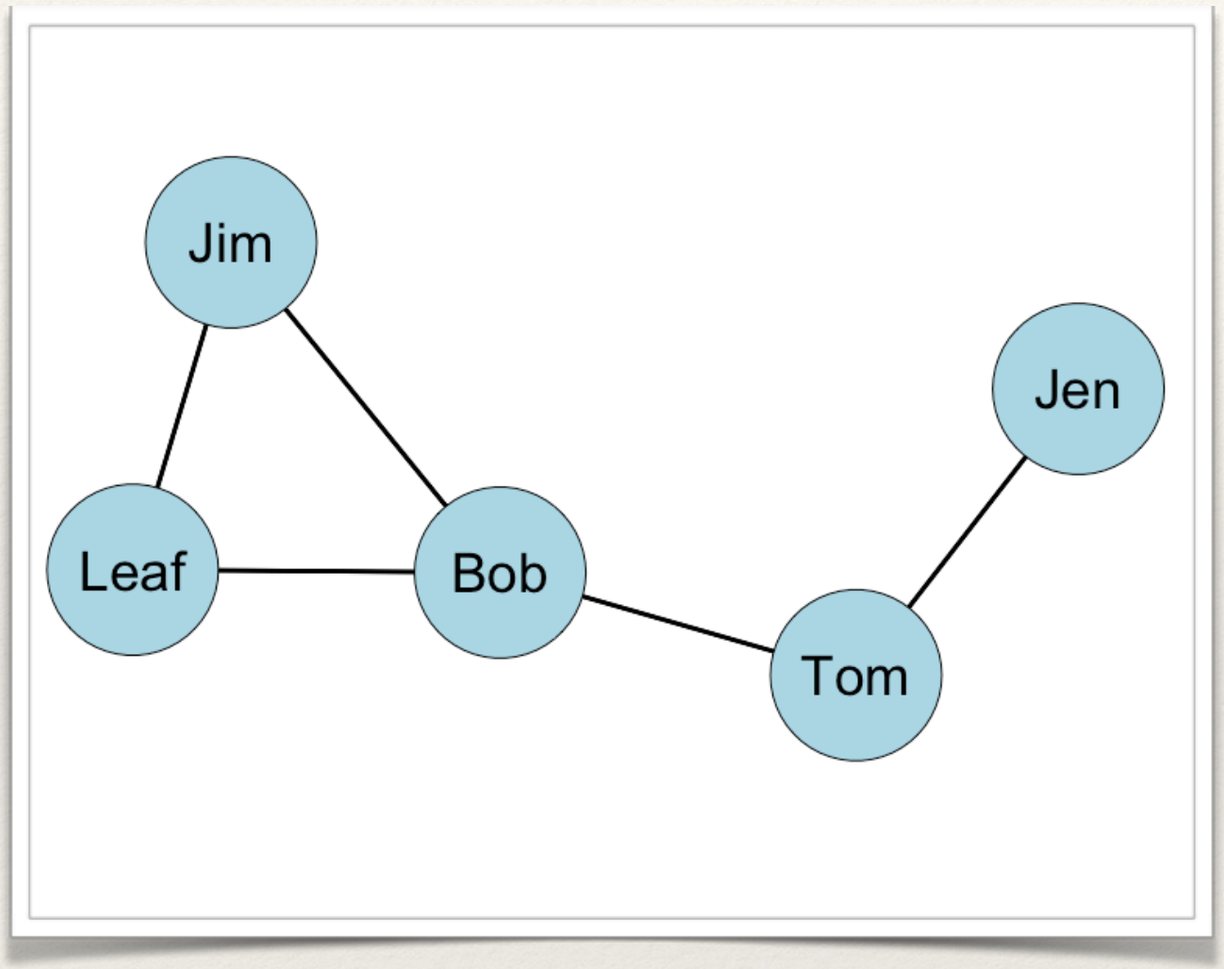
Tom = 0.167*4 = 0.668

Bob = 0.200*4 = 0.800

Leaf = 0.143*4 = 0.572

Jim = 0.143*4 = 0.572

Example: Undirected, Binary Network



Deviations of Standardized Closeness Centrality Scores

Jen = 0.800 - 0.444 = 0.356

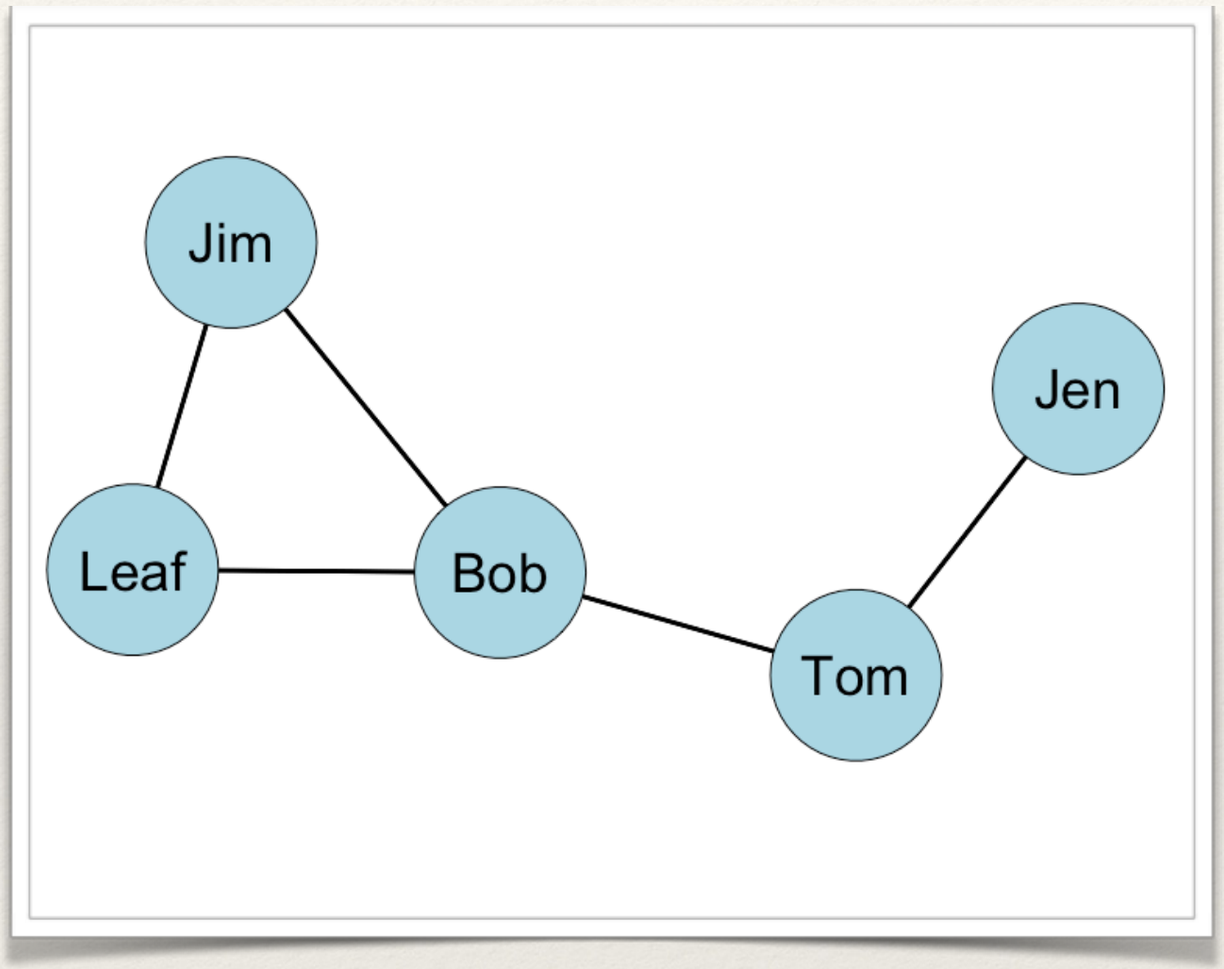
Tom = 0.800 - 0.668 = 0.132

Bob = 0.800 - 0.800 = 0.000

Leaf = 0.800 - 0.572 = 0.228

Jim = 0.800 - 0.572 = 0.228

Example: Undirected, Binary Network



Deviations of Standardized Closeness Centrality Scores

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Tom = 0.800 - 0.668 = 0.132

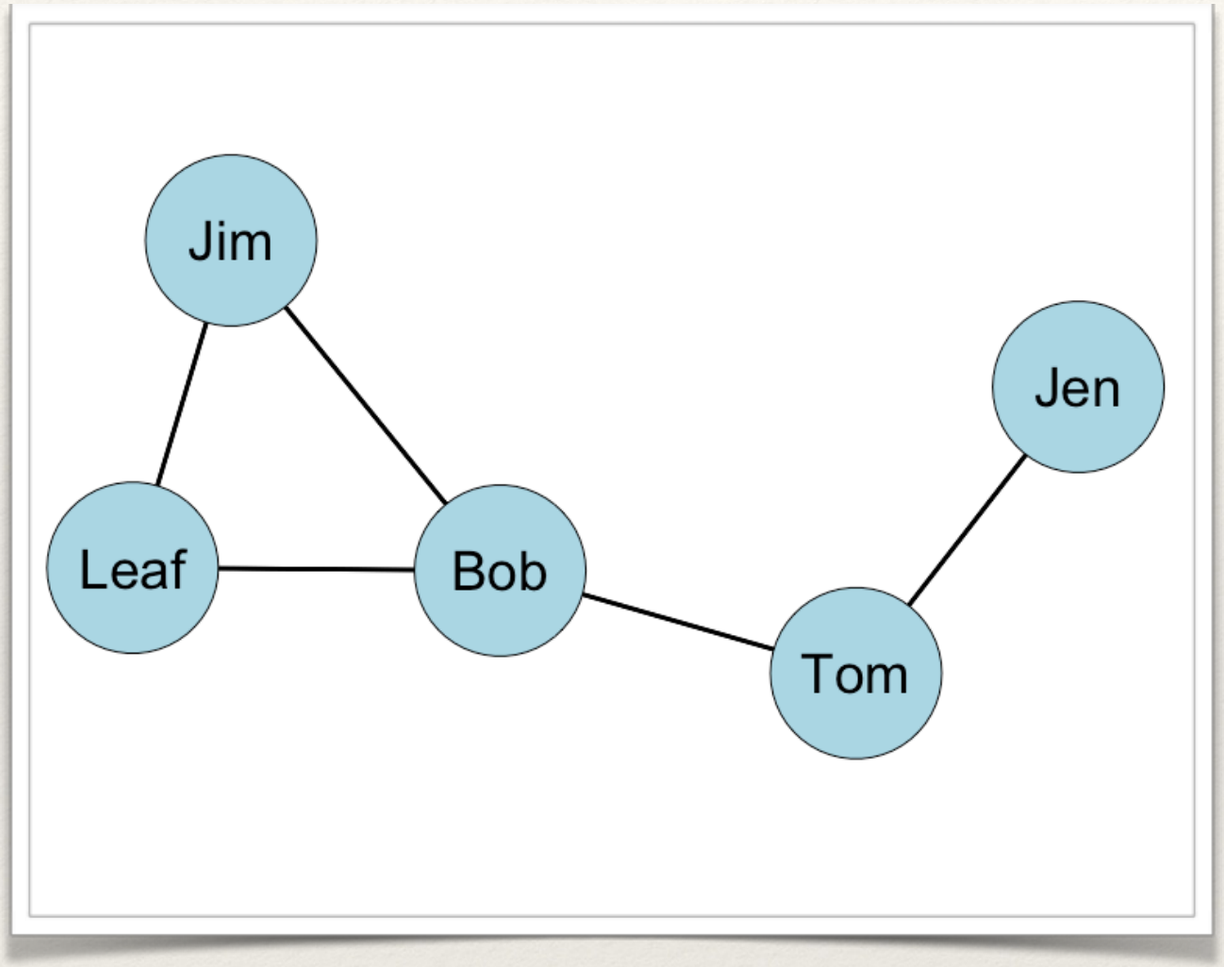
Bob = 0.800 - 0.800 = 0.000

Leaf = 0.800 - 0.572 = 0.228

Jim = 0.800 - 0.572 = 0.228

Sum = 0.944

Example: Undirected, Binary Network



$$C_C = \frac{\sum_{i=1}^{g} [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

Deviations of Standardized Closeness Centrality Scores

Jen = 0.800 - 0.444 = 0.356

Tom = 0.800 - 0.668 = 0.132

Bob = 0.800 - 0.800 = 0.000

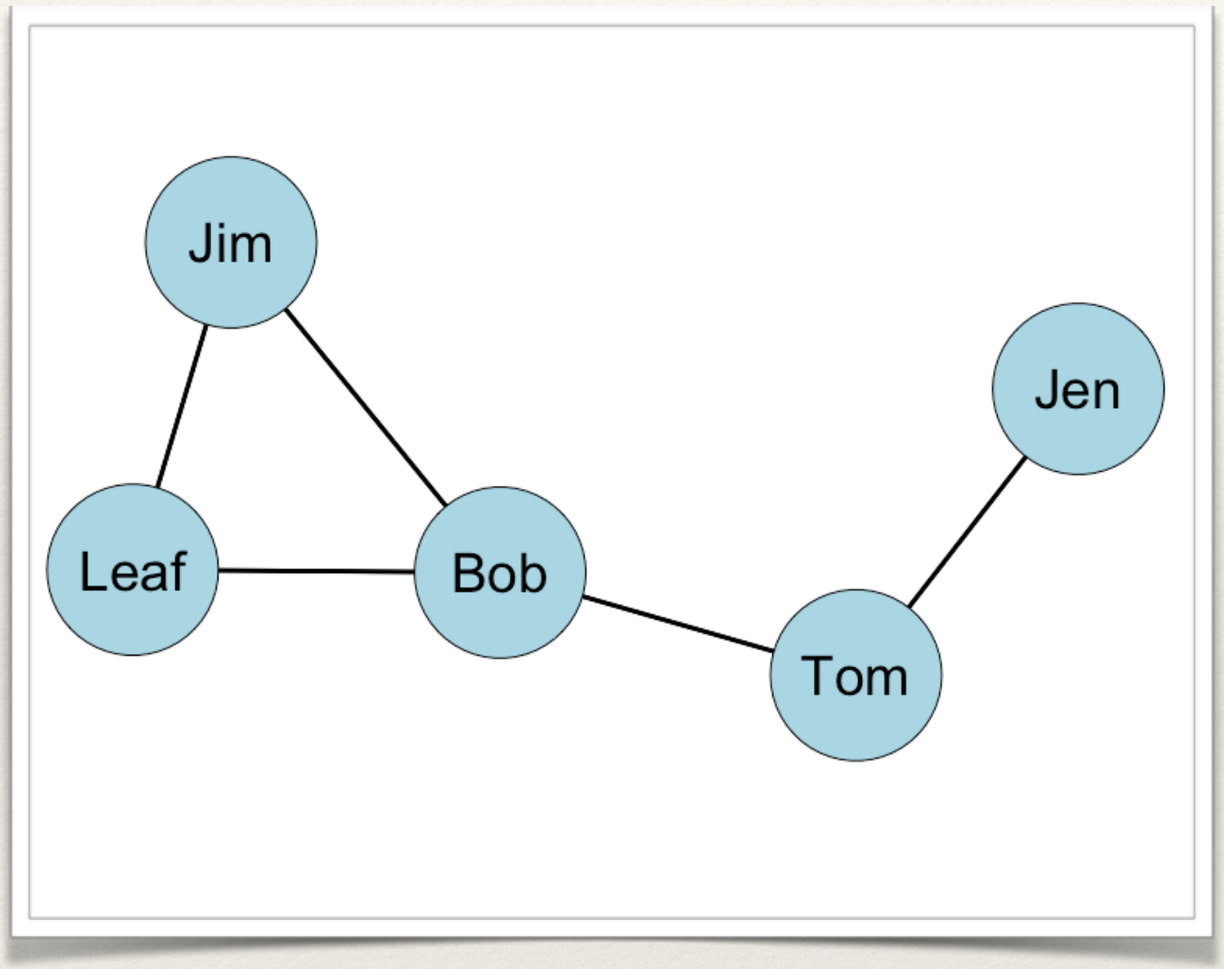
Leaf = 0.800 - 0.572 = 0.228

Jim = 0.800 - 0.572 = 0.228

Sum = 0.944

Denominator = [(5-2)(5-1)/(2*5-3)] = 1.714

Example: Undirected, Binary Network



Approximately 0.551

Deviations of Standardized Closeness Centrality Scores

Jen = 0.800 - 0.444 = 0.356

Tom = 0.800 - 0.668 = 0.132

Bob = 0.800 - 0.800 = 0.000

Leaf = 0.800 - 0.572 = 0.228

Jim = 0.800 - 0.572 = 0.228

Sum = 0.944

Denominator = [(5-2)(5-1)/(2*5-3)] = 1.714

Example: Undirected, Binary Network

$$C_C = \frac{\sum_{i=1}^{g} [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

$$= \frac{0.356 + 0.132 + 0.000 + 0.228 + 0.228}{[(5-2)(5-1)]/(2*5-3)} = \frac{0.944}{1.714} = 0.551$$

Betweenness Centrality

- We have seen how centrality can be conceptualized as:
  - Having a high number of ties
  - Being close to others in the network
  - We can also conceptualize centrality as a node that lies on a particular path between other nodes.
    - *Betweenness centrality* is based on the number of **shortest** paths between *j* and *k* that actor *i* resides on.

Undirected Networks

Betweenness Centrality: Undirected Binary Graphs

$$C_B(n_i) = \sum_{j<k} g_{jk}(n_i)/g_{jk}$$

Betweenness Centrality: Undirected Binary Graphs

The number of geodesics linking j to k.

$$C_B(n_i) = \sum_{j<k} g_{jk}(n_i)/g_{jk}$$

Betweenness Centrality: Undirected Binary Graphs

$$C_B(n_i) = \sum_{j<k} g_{jk}(n_i)/g_{jk}$$

The number of geodesics linking j to k.

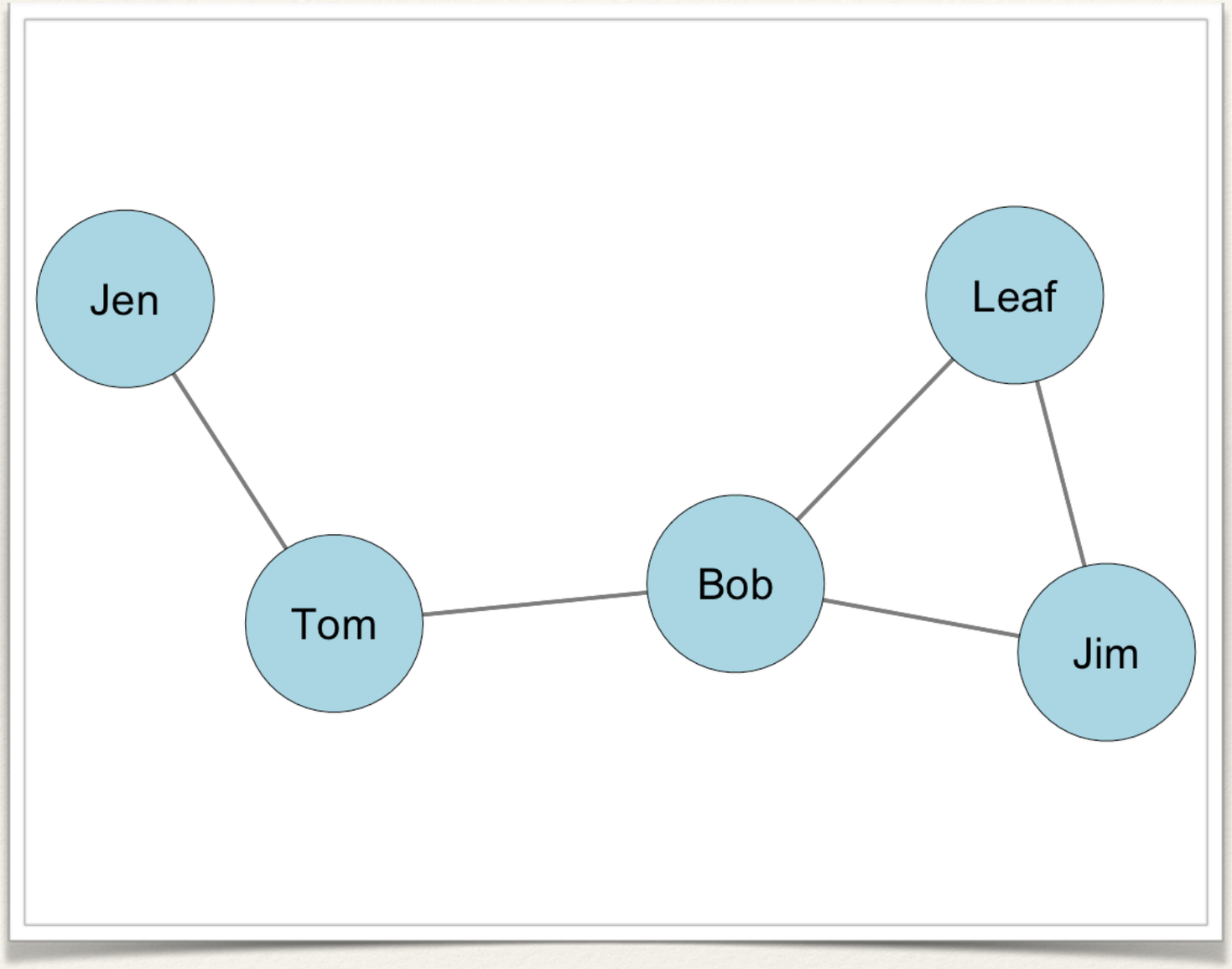
The number of geodesics linking j and k that contain i.

Betweenness Centrality: Undirected Binary Graphs

$$C_B(n_i) = \sum_{j<k} g_{jk}(n_i)/g_{jk}$$

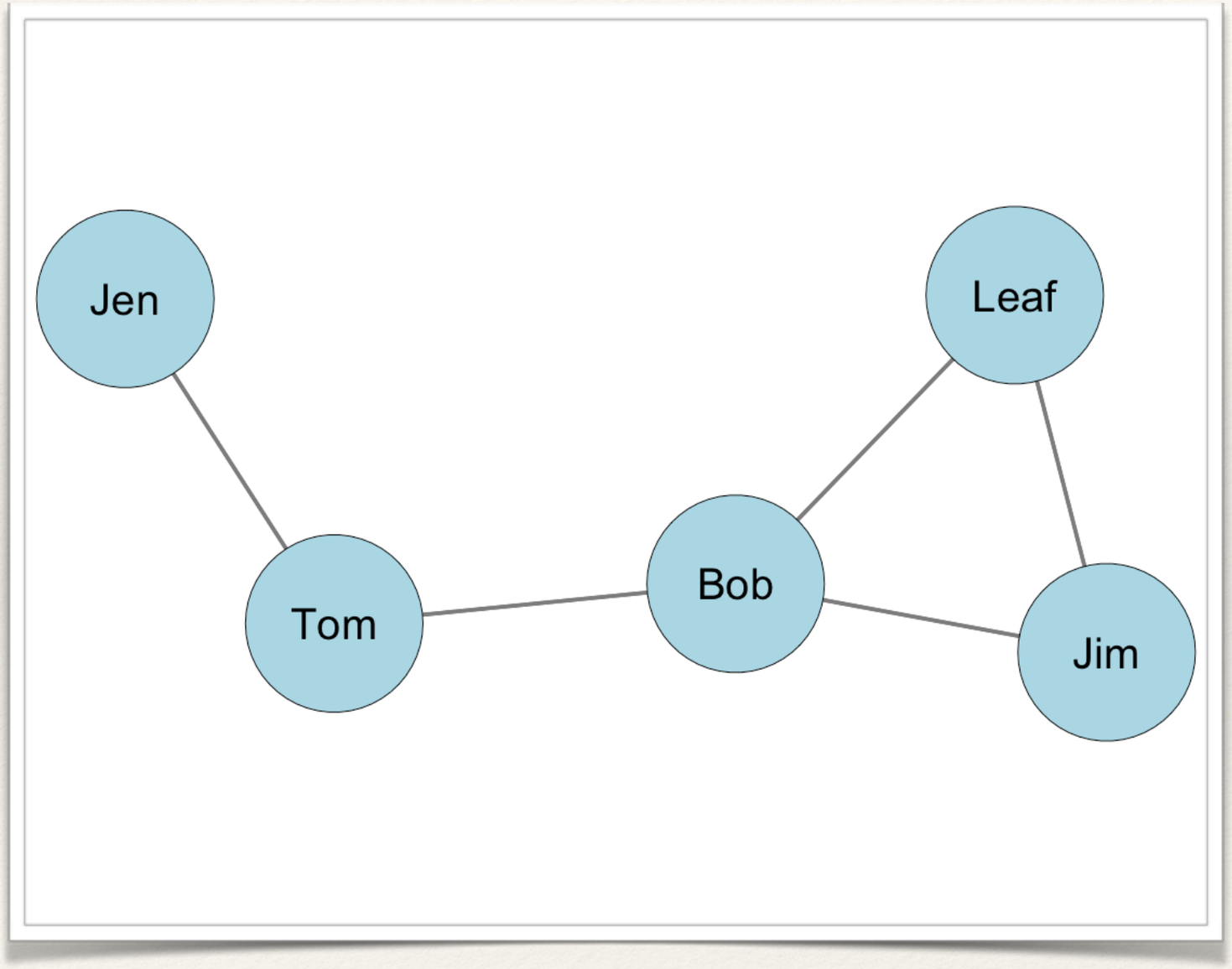
So, betweenness centrality is the ratio of the geodesics between *j* and *k* that contain *i*.

Example: Undirected, Binary Network



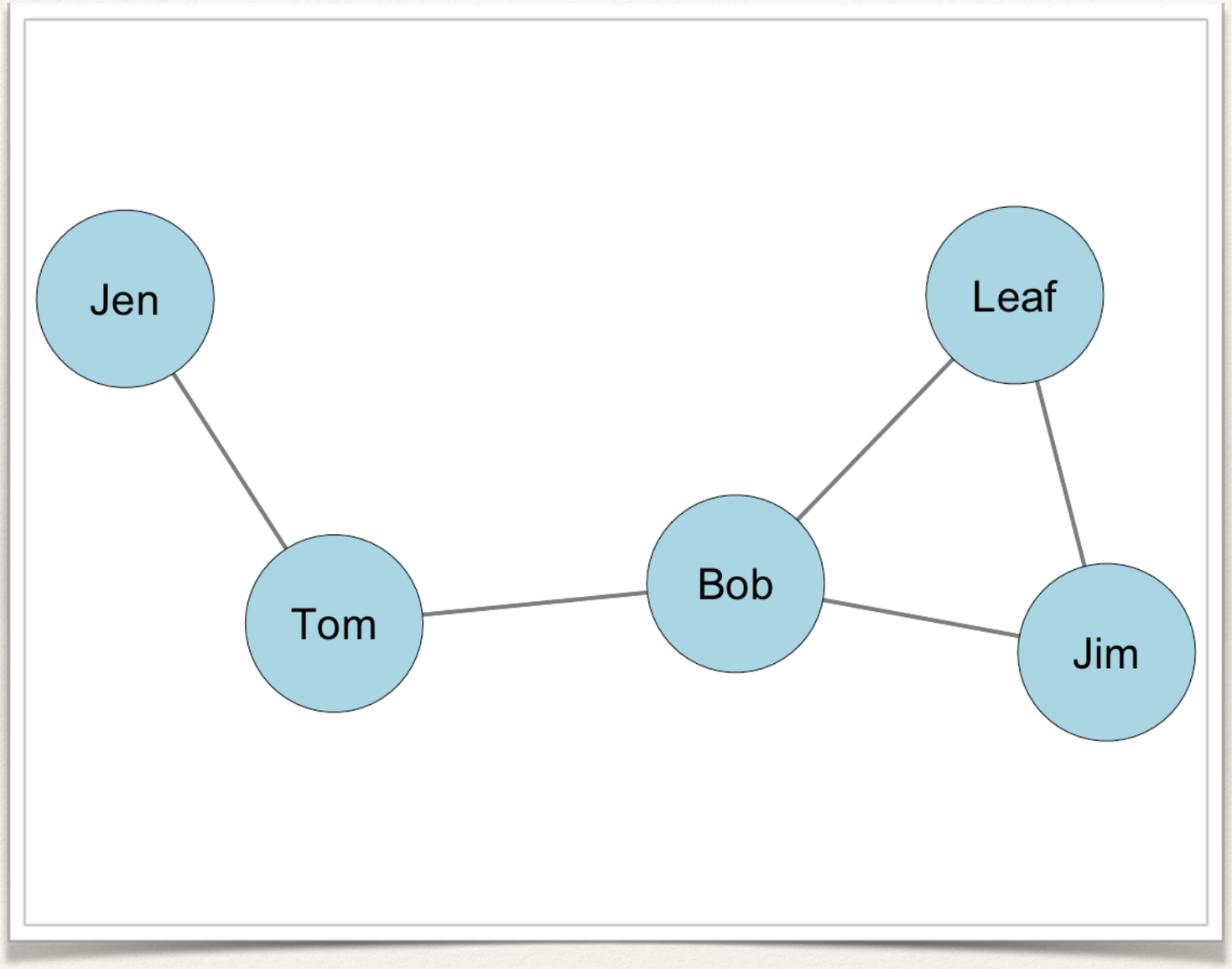
What are the paths between **Jen** and **Jim**?

Example: Undirected, Binary Network



Let's calculate the betweenness centrality for **Bob**.

Example: Undirected, Binary Network



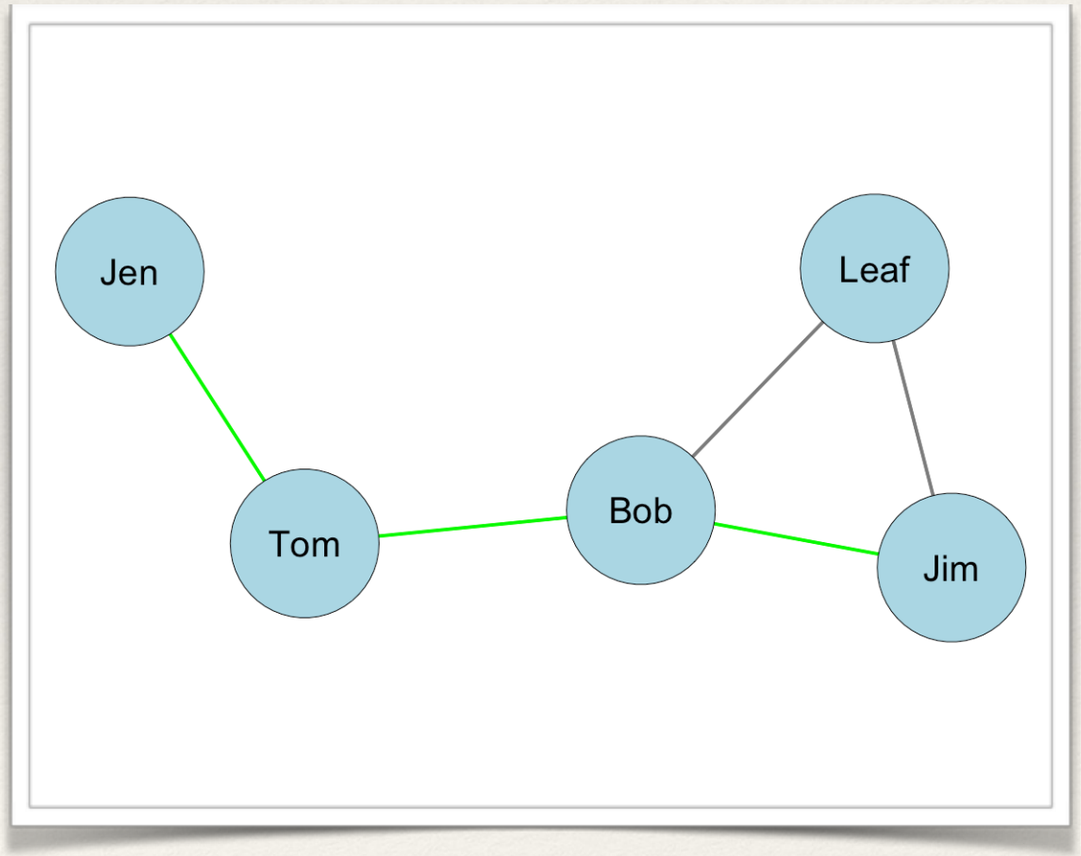
Let's calculate the betweenness centrality for **Bob**.

First, we need to find the geodesics for other nodes.

Then, figure out how many **Bob** occupies.

Example: Undirected, Binary Network

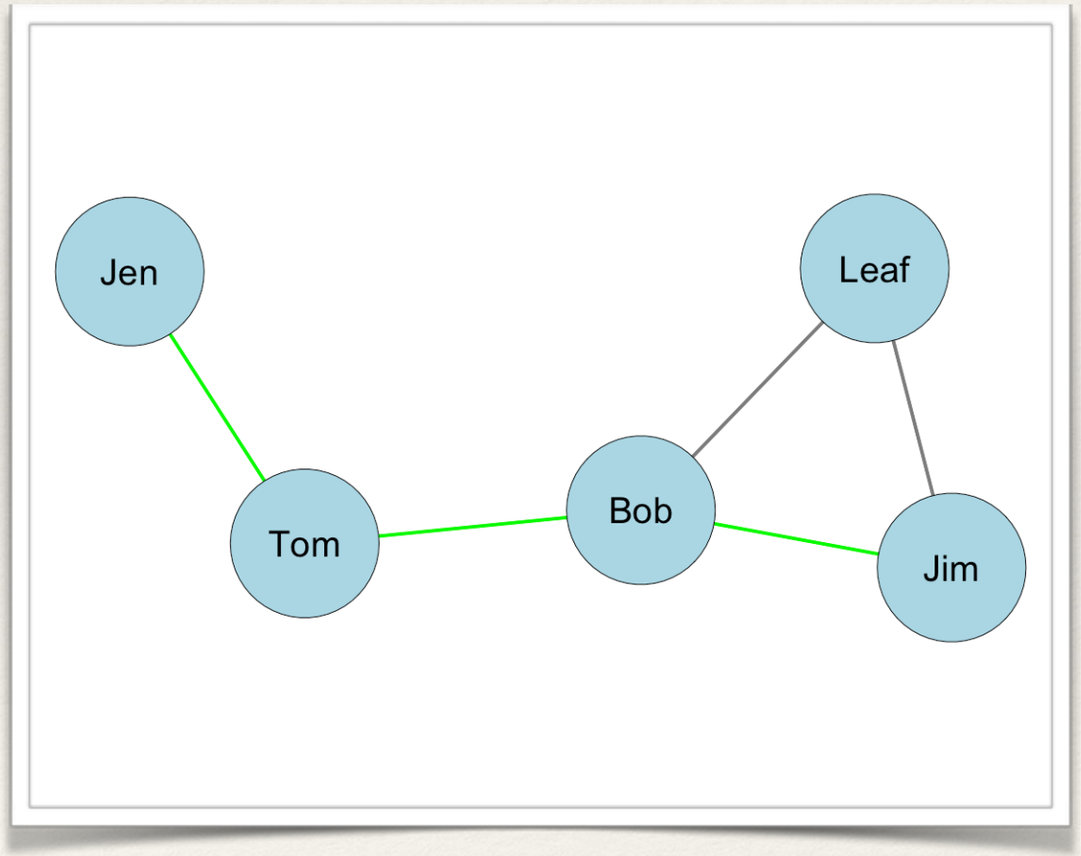
What are the paths between **Jen** and **Jim**?



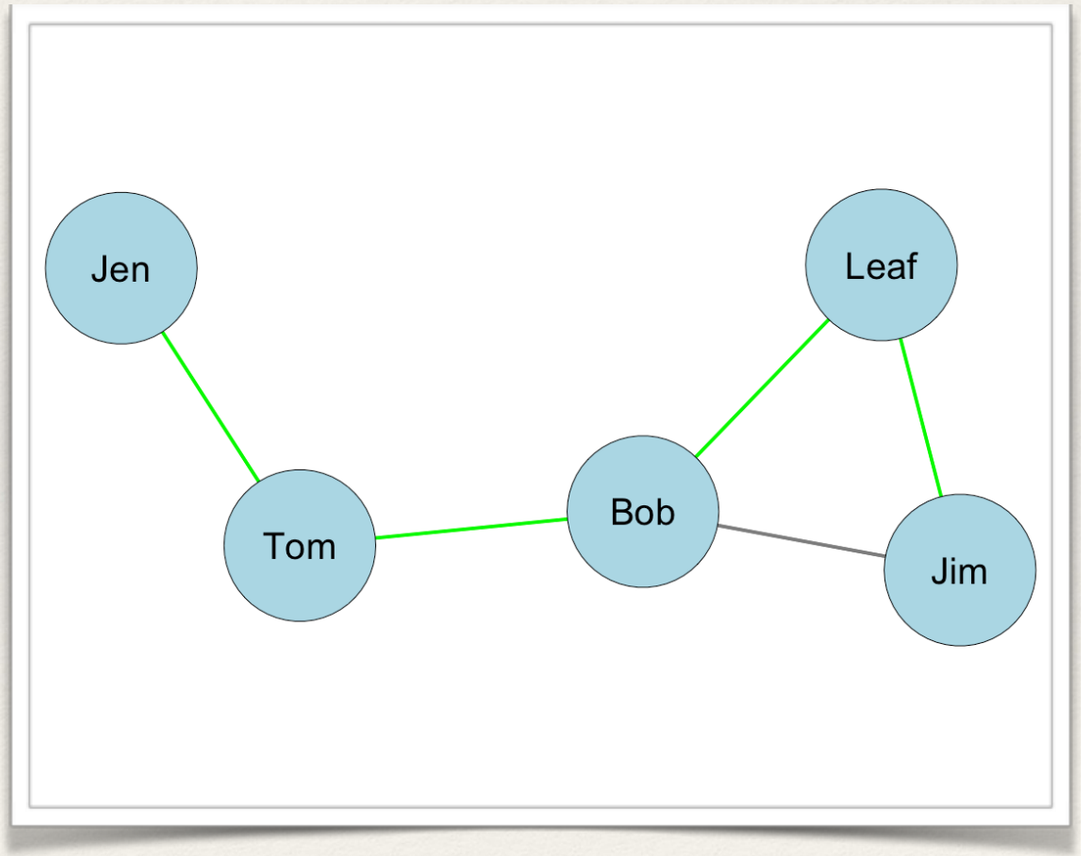
Jen-Tom-Bob-Jim

Example: Undirected, Binary Network

What are the paths between **Jen** and **Jim**?

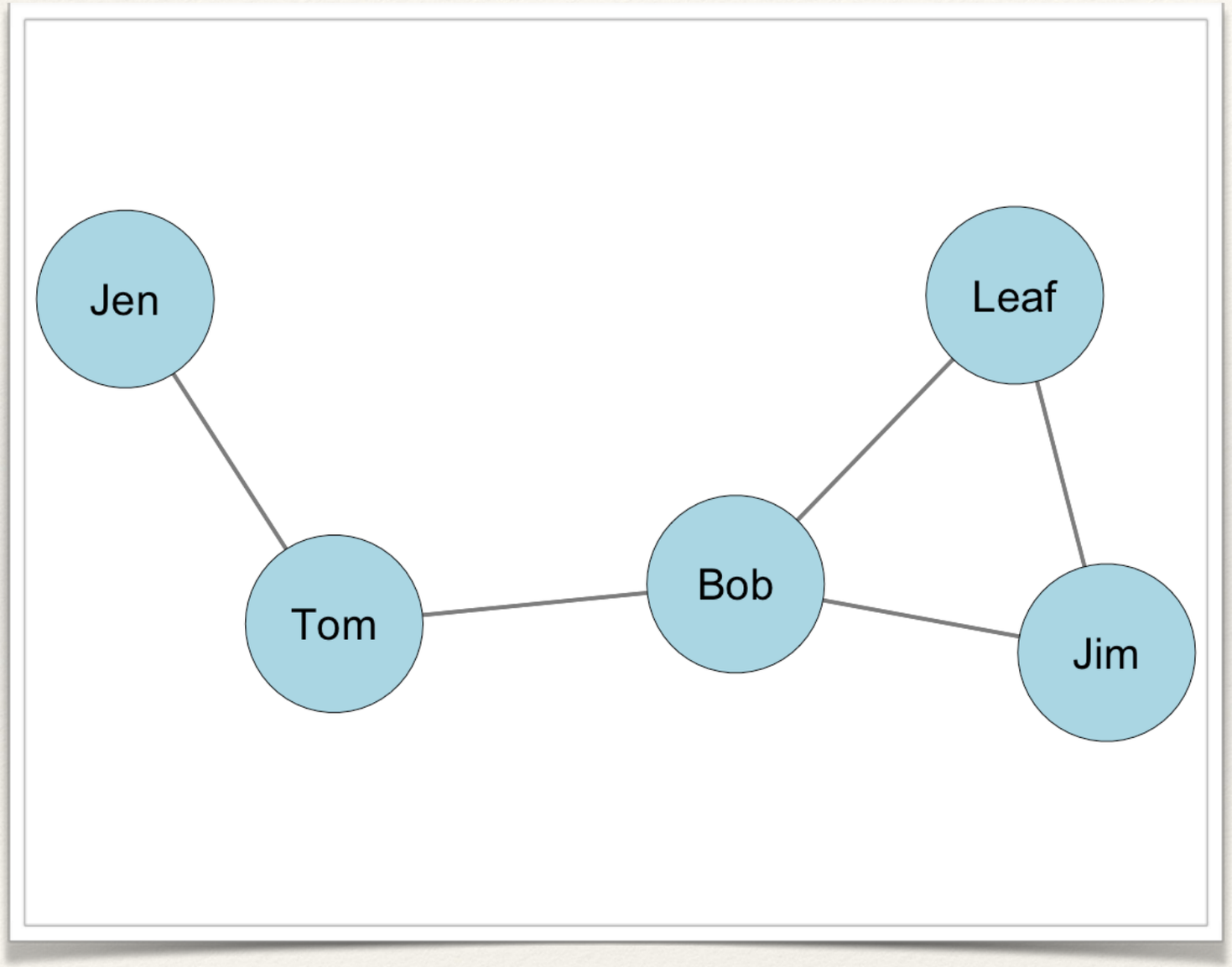


Jen-Tom-Bob-Jim



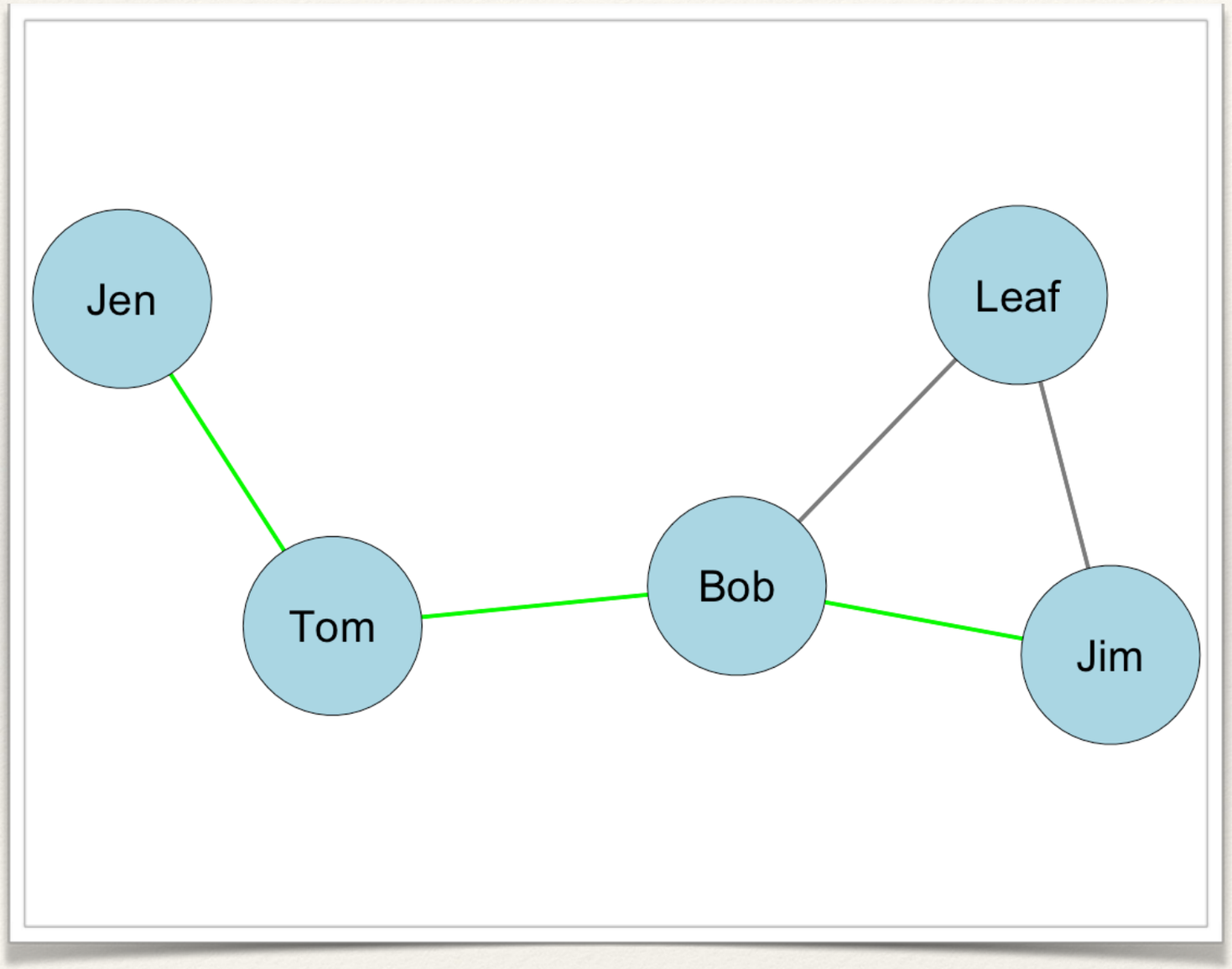
Jen-Tom-Bob-Leaf-Jim

Example: Undirected, Binary Network



What are the ***geodesic*** paths between **Jen** and **Jim**?

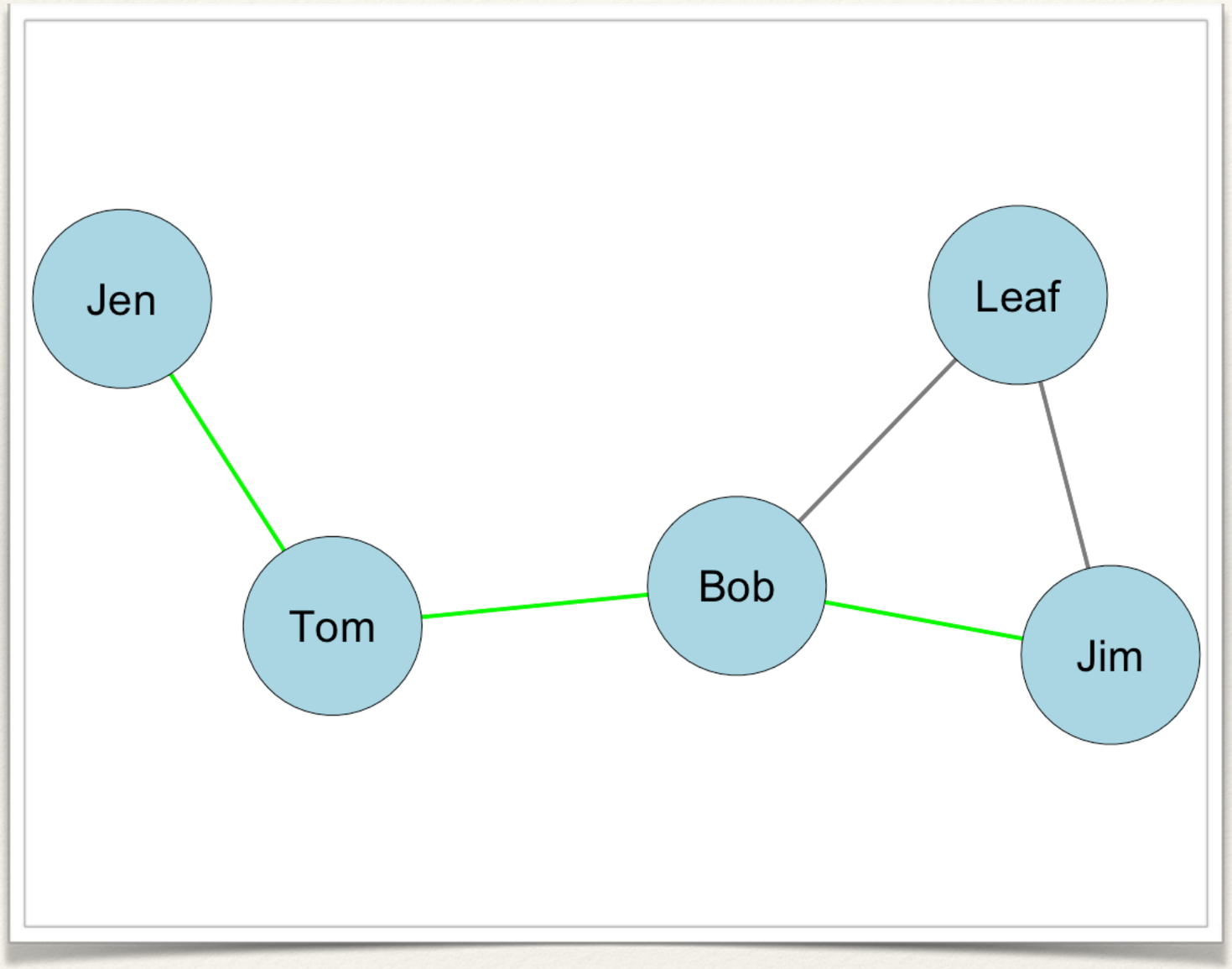
Example: Undirected, Binary Network



What are the ***geodesic*** paths between **Jen** and **Jim**?

Jen-Tom-Bob-Jim

Example: Undirected, Binary Network

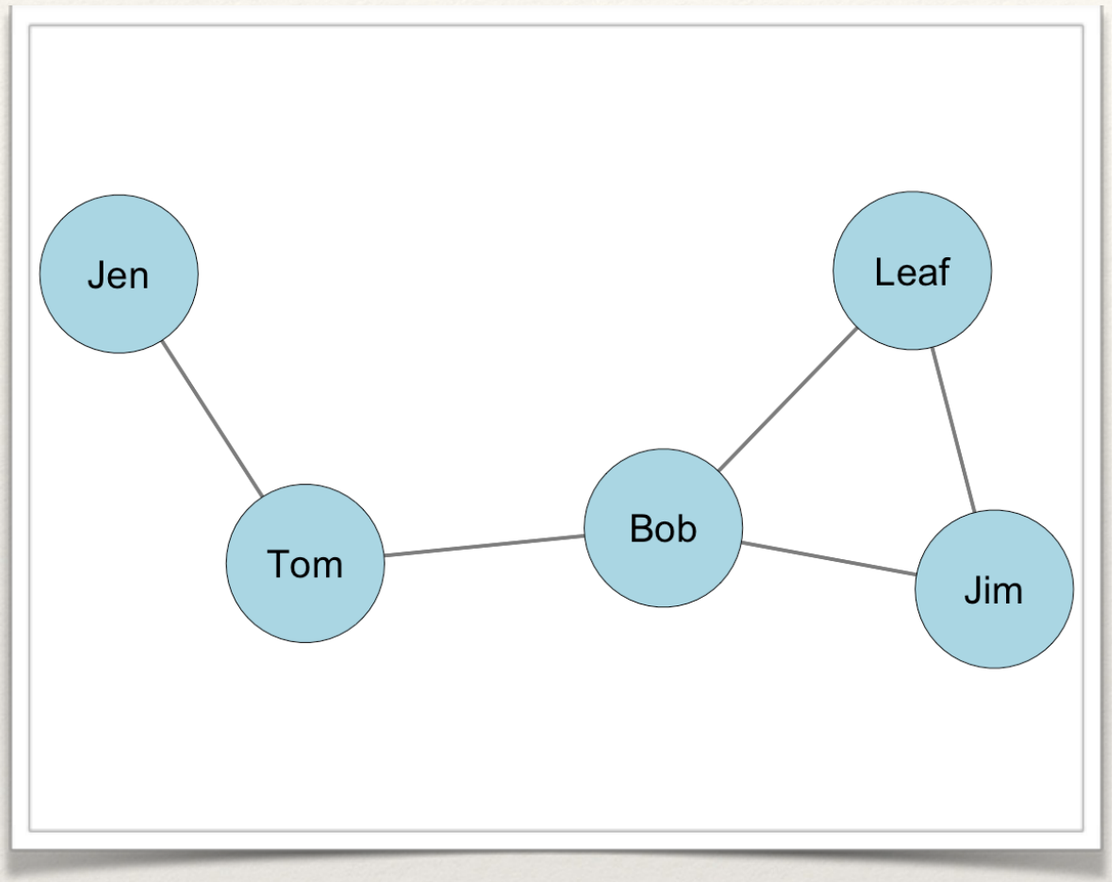


What are the ***geodesic*** paths between **Jen** and **Jim**?

Jen-Tom-Bob-Jim

Is there anyone on the geodesic between Jen and Jim?

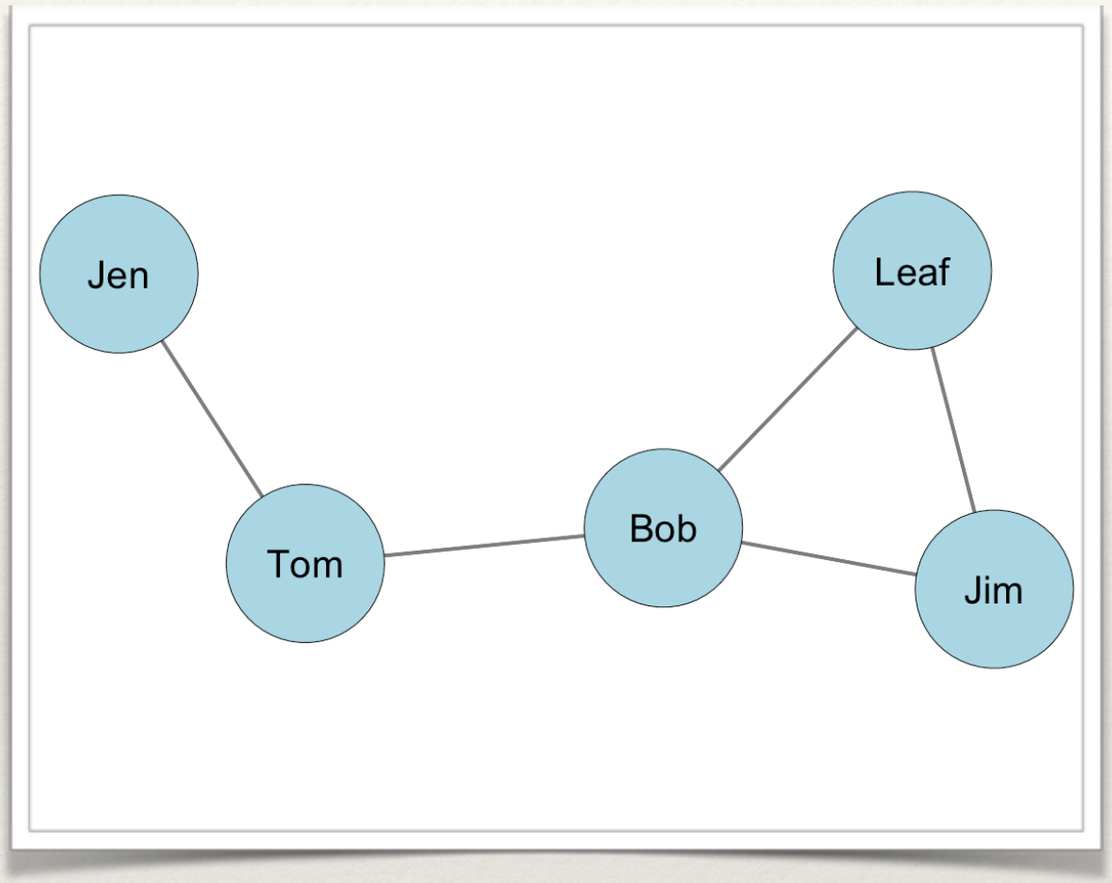
Example: Undirected, Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen				/?
Tom				
Leaf				
Jim				

How many geodesics from **Jen** to **Jim**?

Example: Undirected, Binary Network

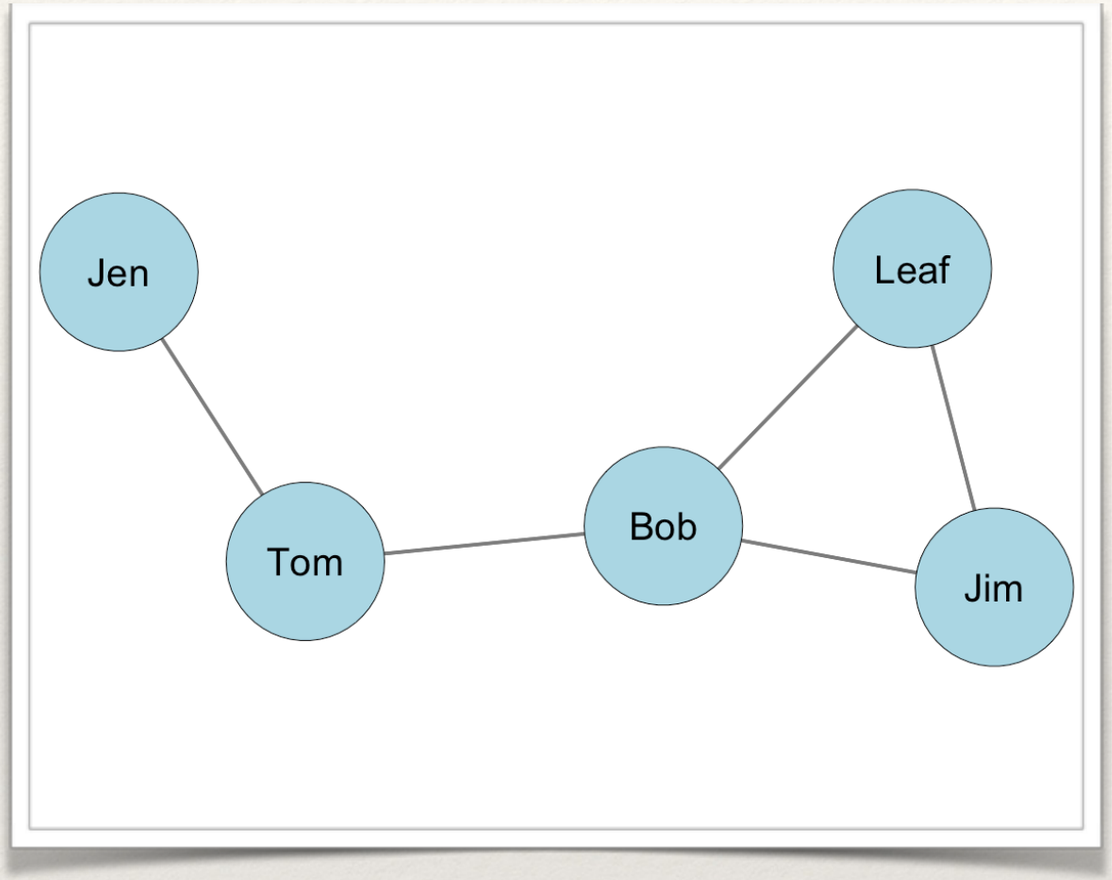


Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen				/1
Tom				
Leaf				
Jim				

There is 1 geodesic from **Jen** to **Jim**

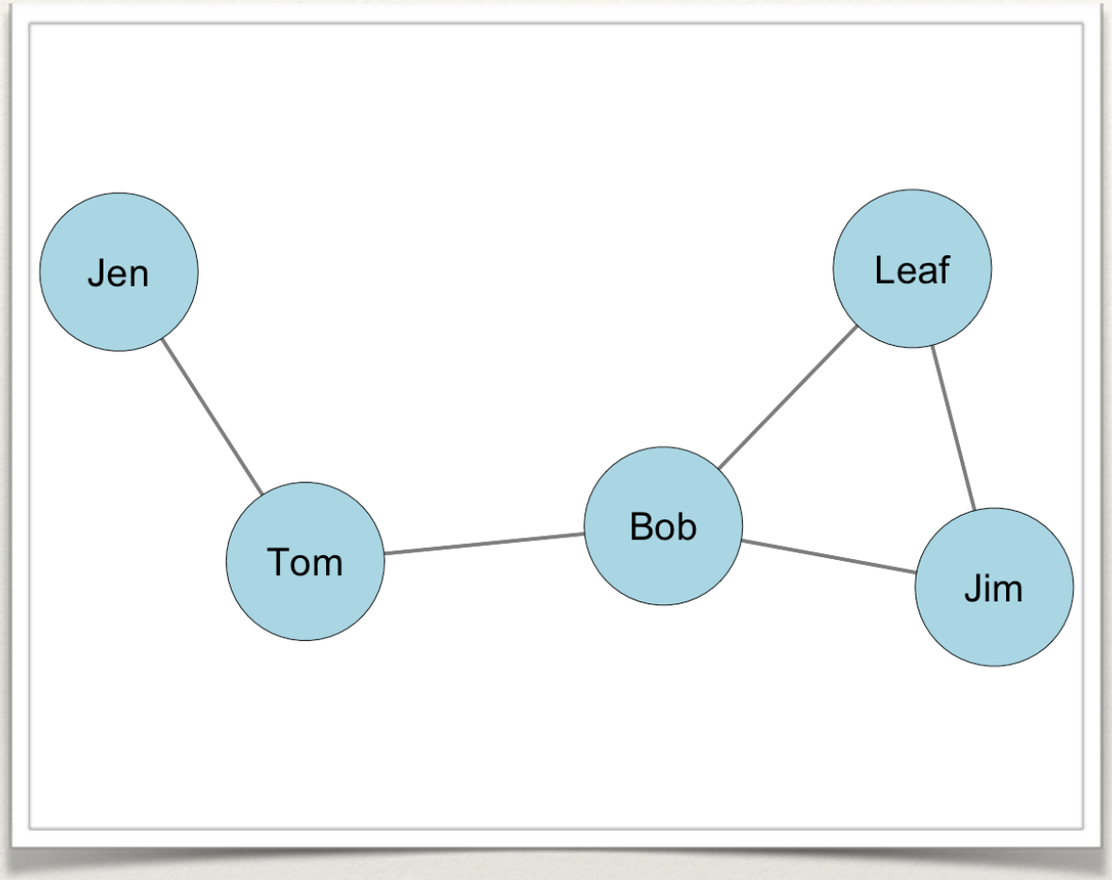
Example: Undirected, Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen				?/1
Tom				
Leaf				
Jim				

How many geodesics from **Jen** to **Jim** include **Bob**?

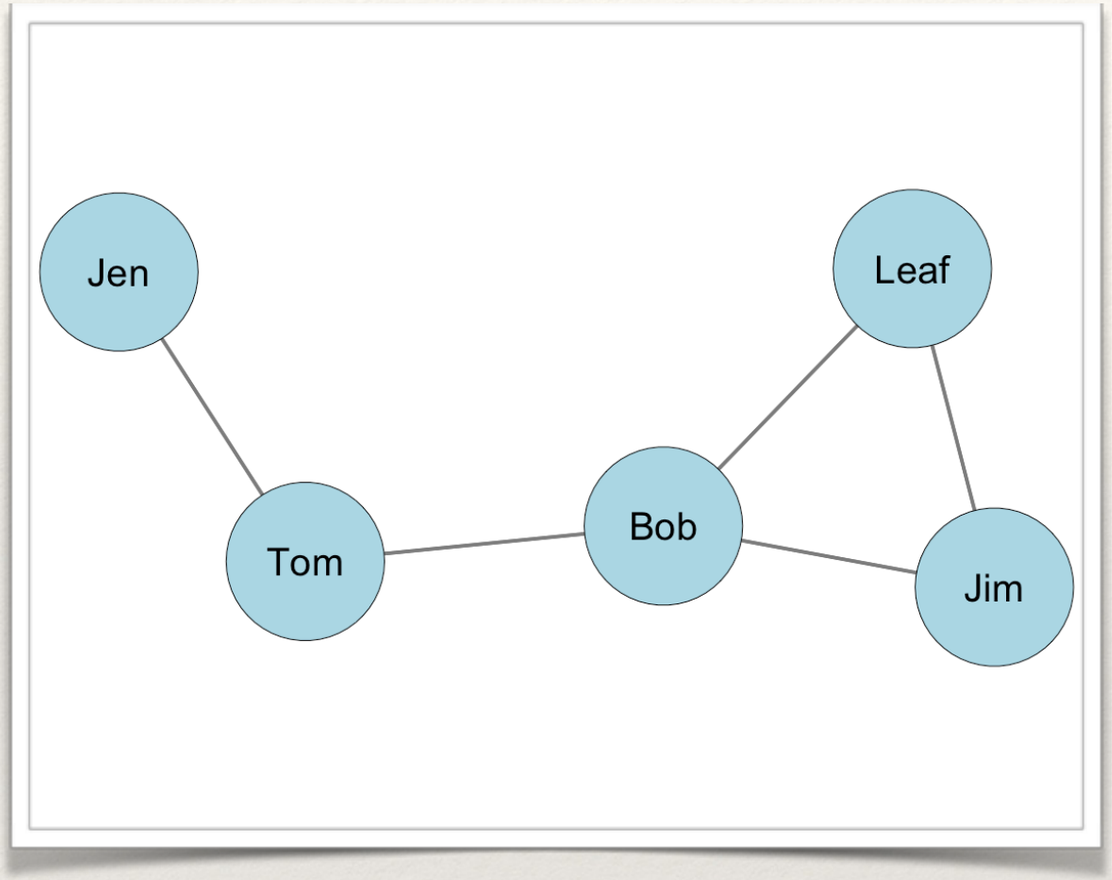
Example: Undirected, Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen				1/1
Tom				
Leaf				
Jim				

Bob is on the only geodesic from **Jen** to **Jim**

Example: Undirected, Binary Network

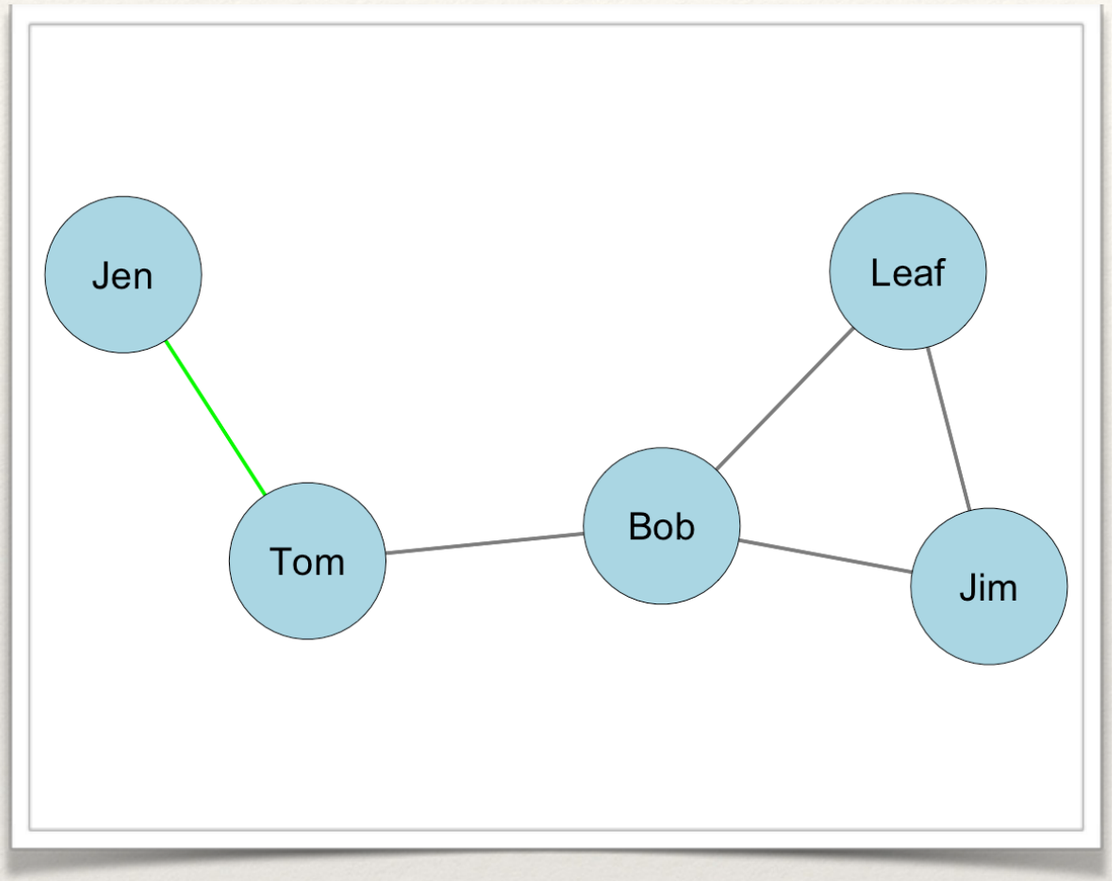


Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen		?/?		1/1
Tom				
Leaf				
Jim				

What about **Jen** to **Tom**?

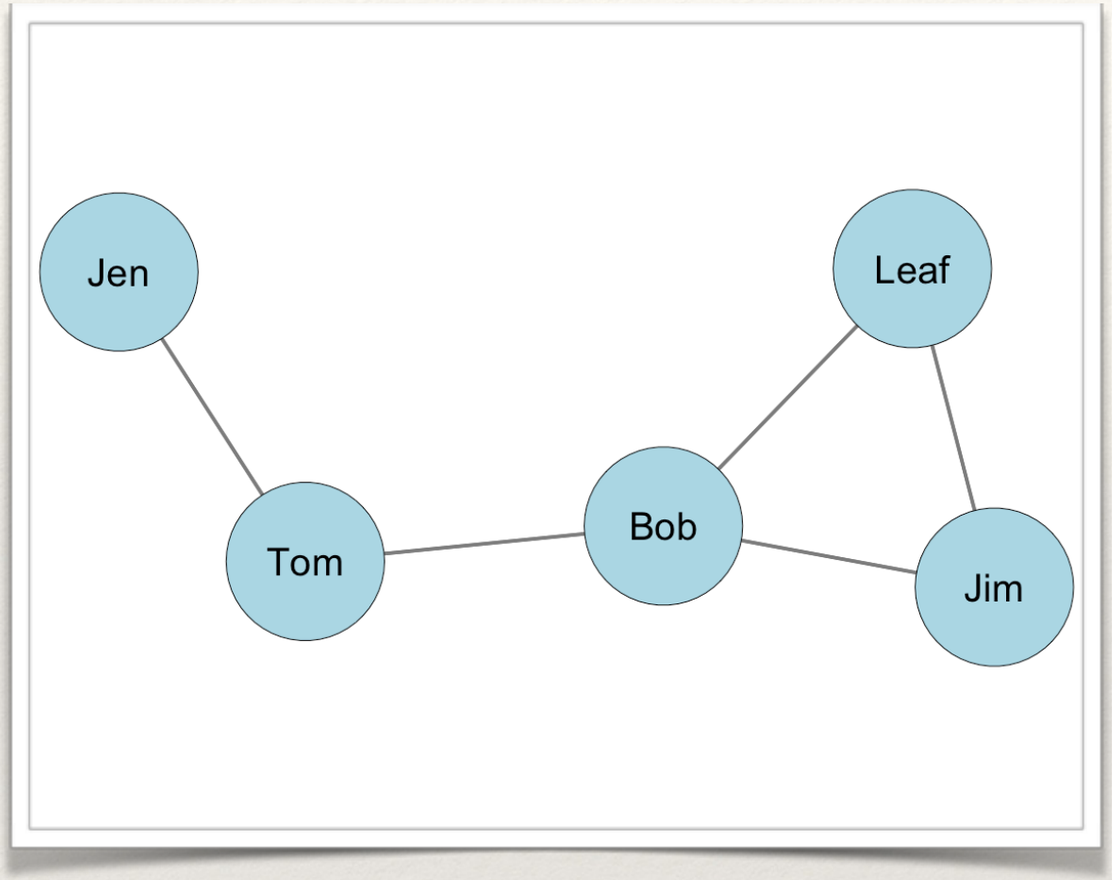
Example: Undirected, Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1		1/1
Tom				
Leaf				
Jim				

Bob is not on the only geodesic from **Jen** to **Tom**

Example: Undirected, Binary Network

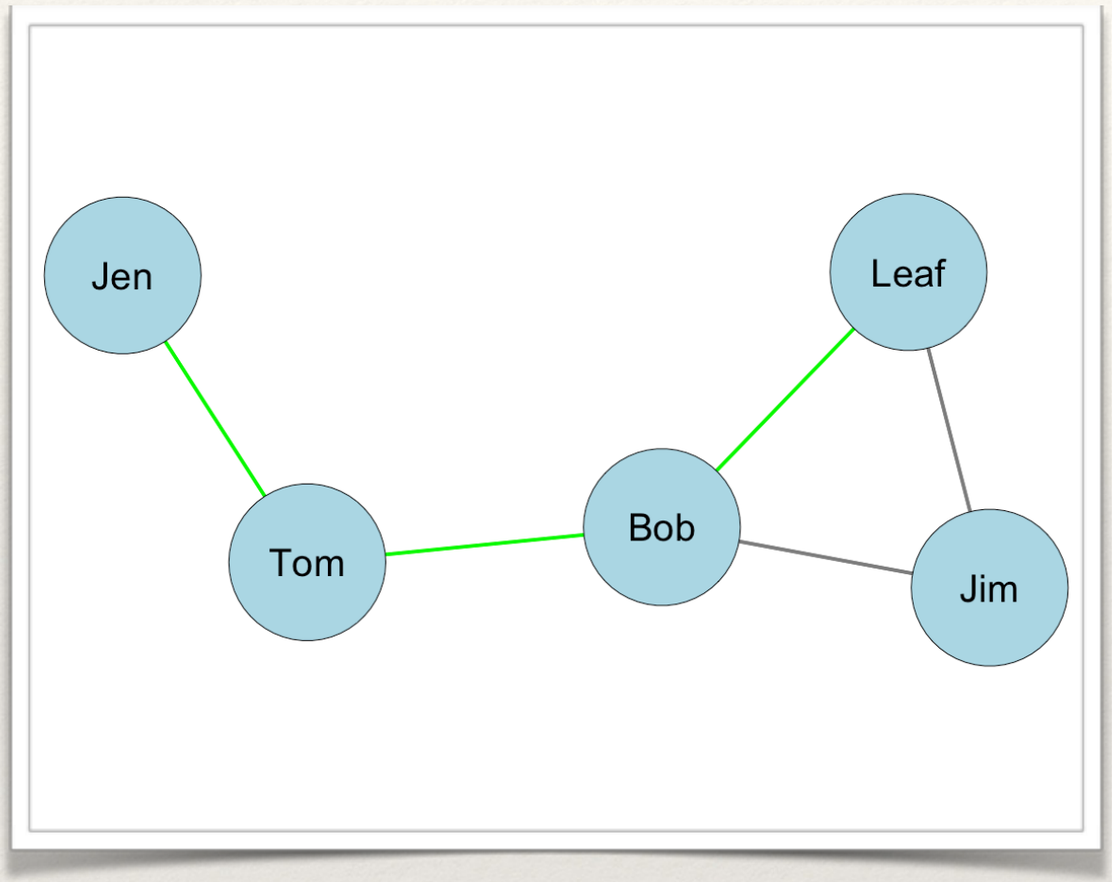


Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	?/?	1/1
Tom				
Leaf				
Jim				

What about **Jen** to **Leaf**?

Example: Undirected, Binary Network

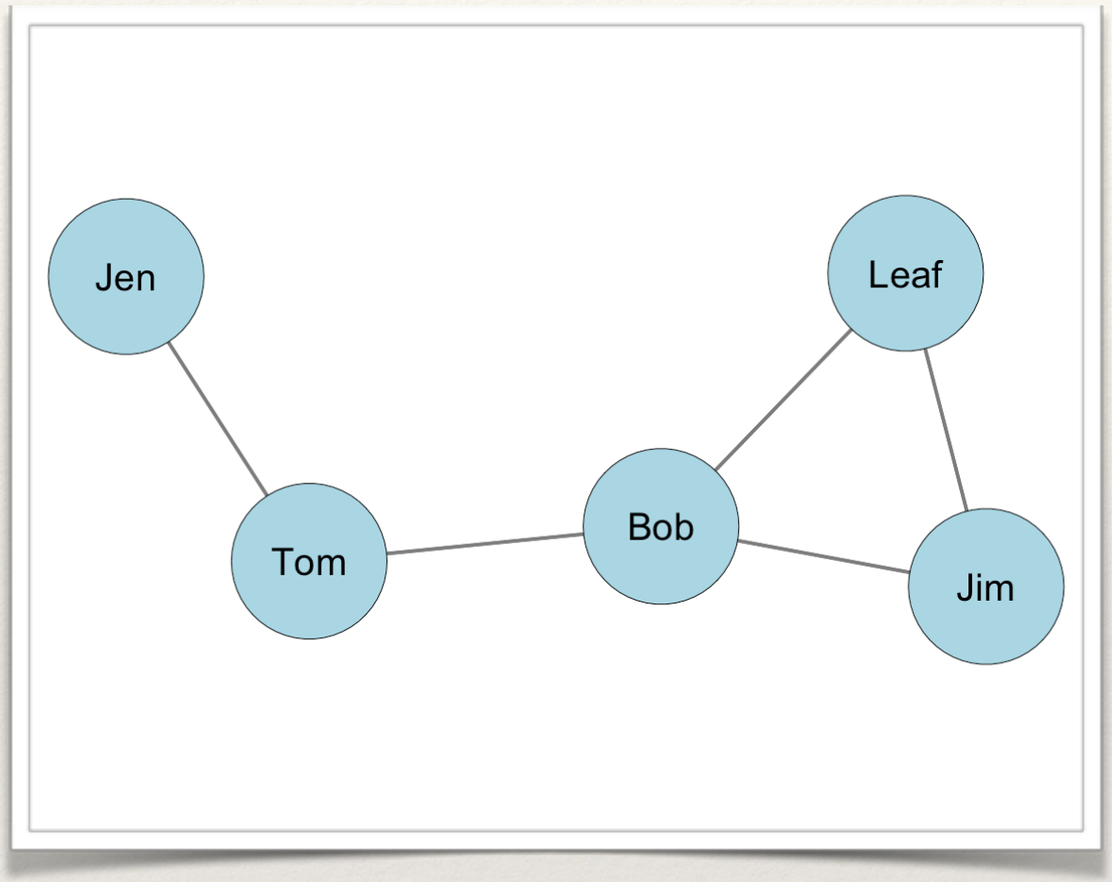


Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom				
Leaf				
Jim				

Bob is on the geodesic from **Jen** to **Leaf**

Example: Undirected, Binary Network



Of the geodesics between

Jen,Tom

Jen,Leaf

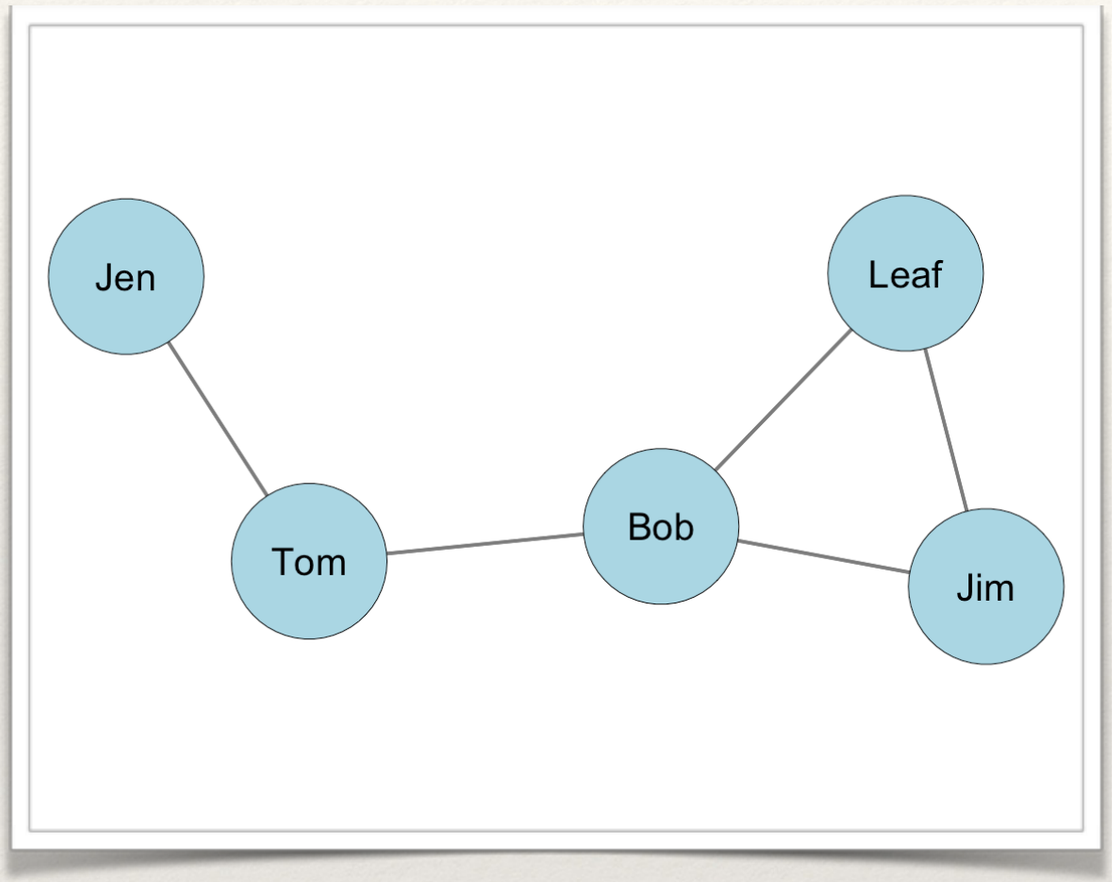
Jen,Tom

how many include **Bob**?

Geodesic Proportions for *Bob*

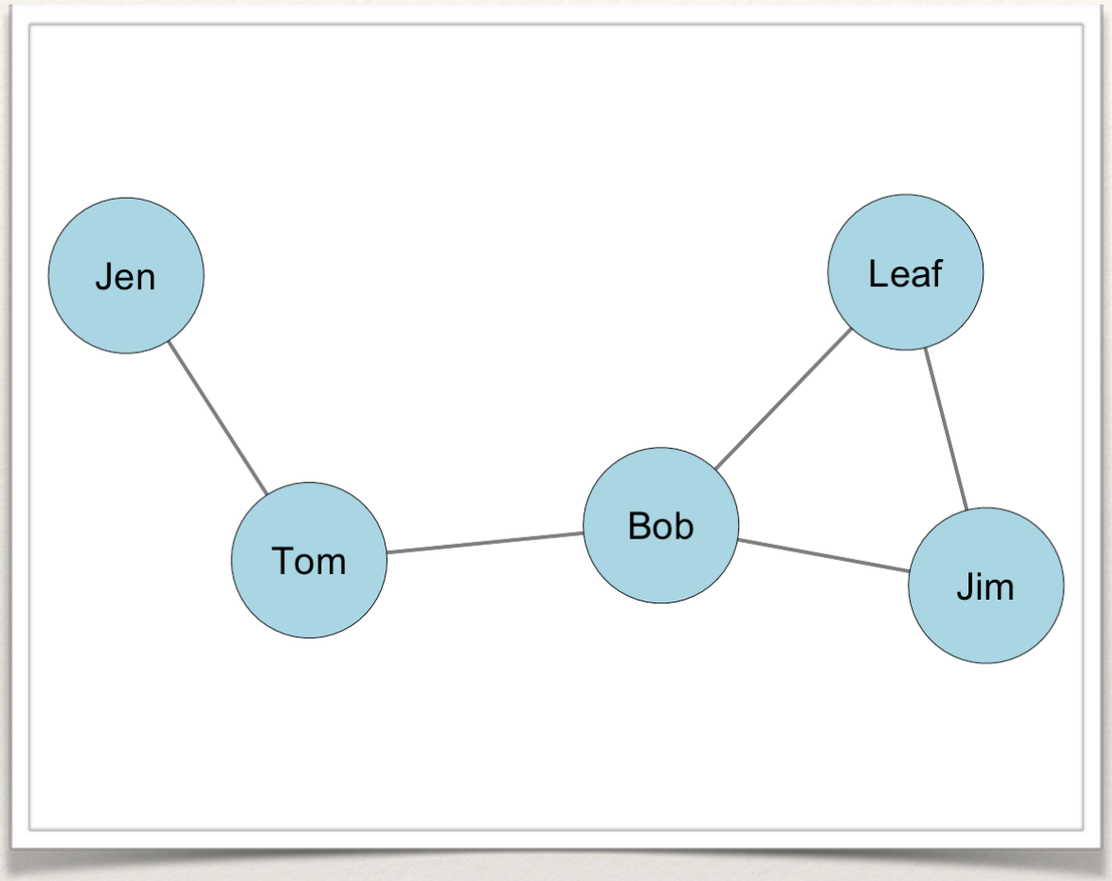
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom				
Leaf				
Jim				

Example: Undirected, Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom				
Leaf				
Jim				

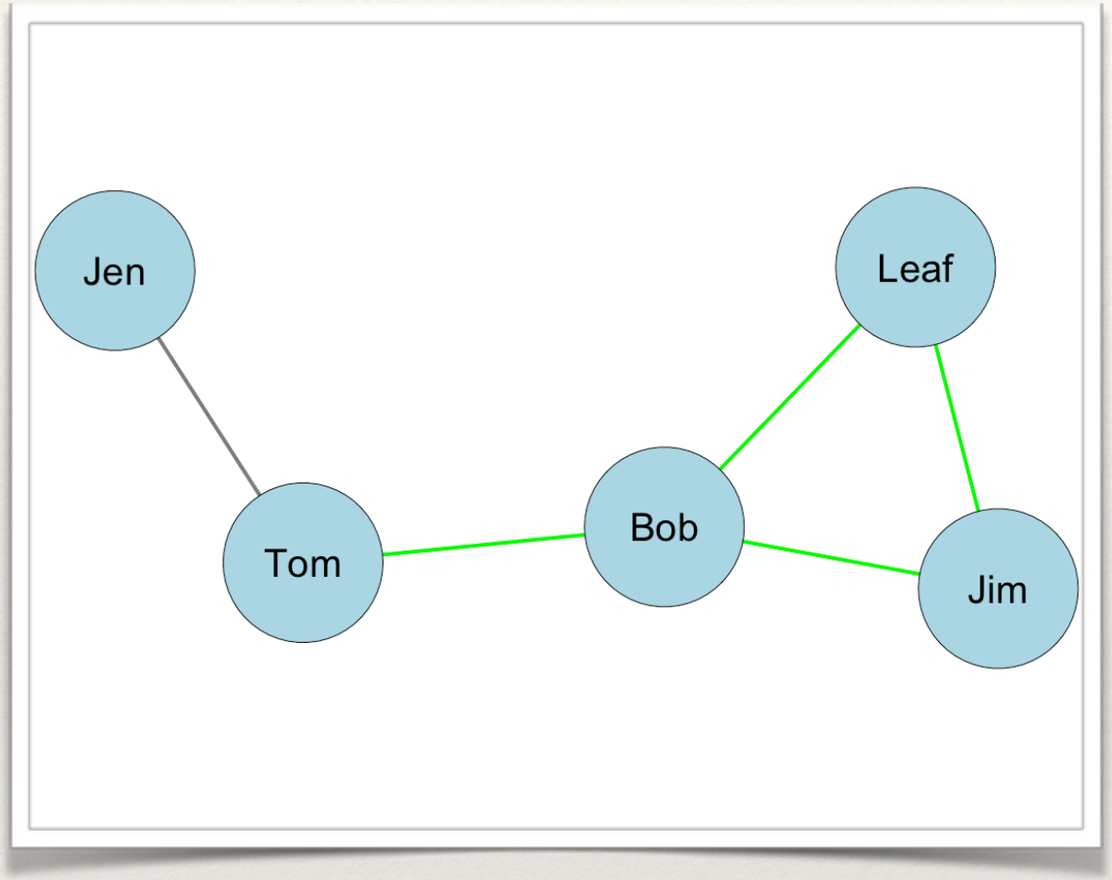
Example: Undirected, Binary Network



To finish, we need to calculate the geodesics for the rest of the matrix for **Bob**

Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom			?/?	?/?
Leaf				?/?
Jim				

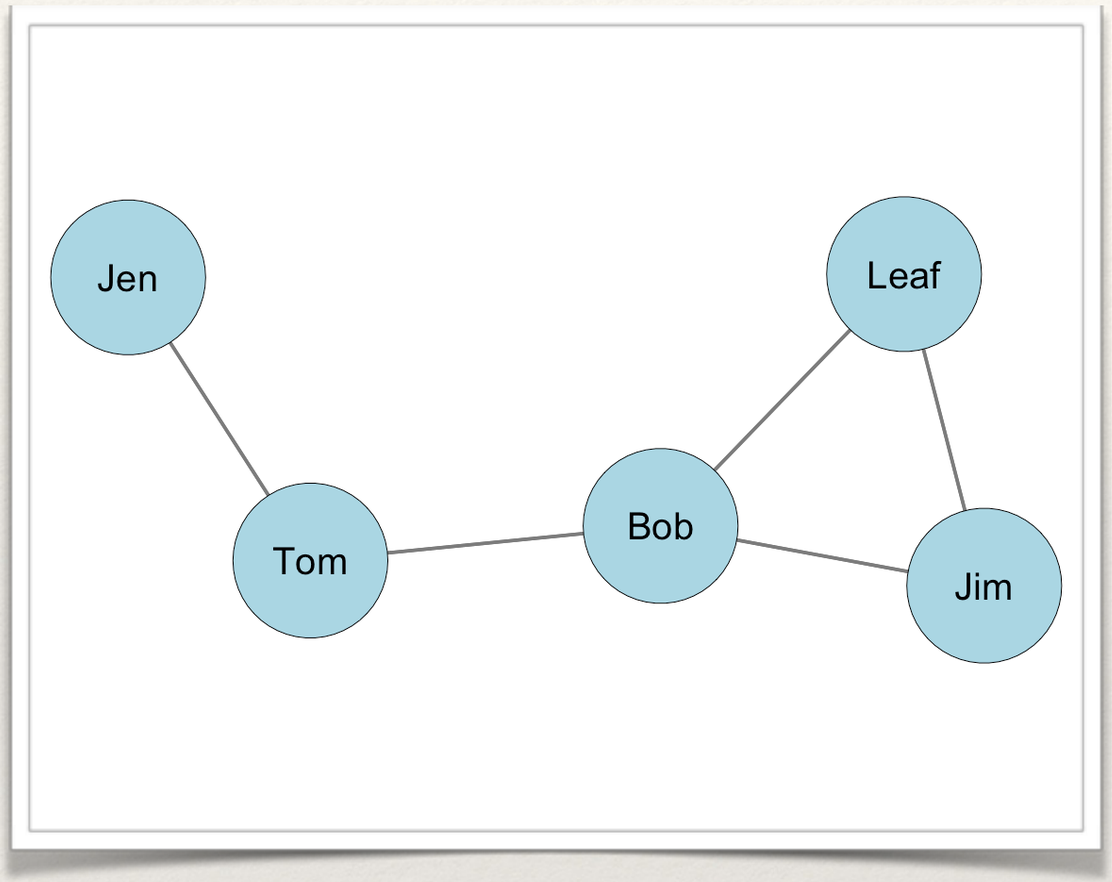
Example: Undirected, Binary Network



To finish, we need to calculate the geodesics for the rest of the matrix for **Bob**

Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom			1/1	1/1
Leaf				0/1
Jim				

Example: Undirected, Binary Network

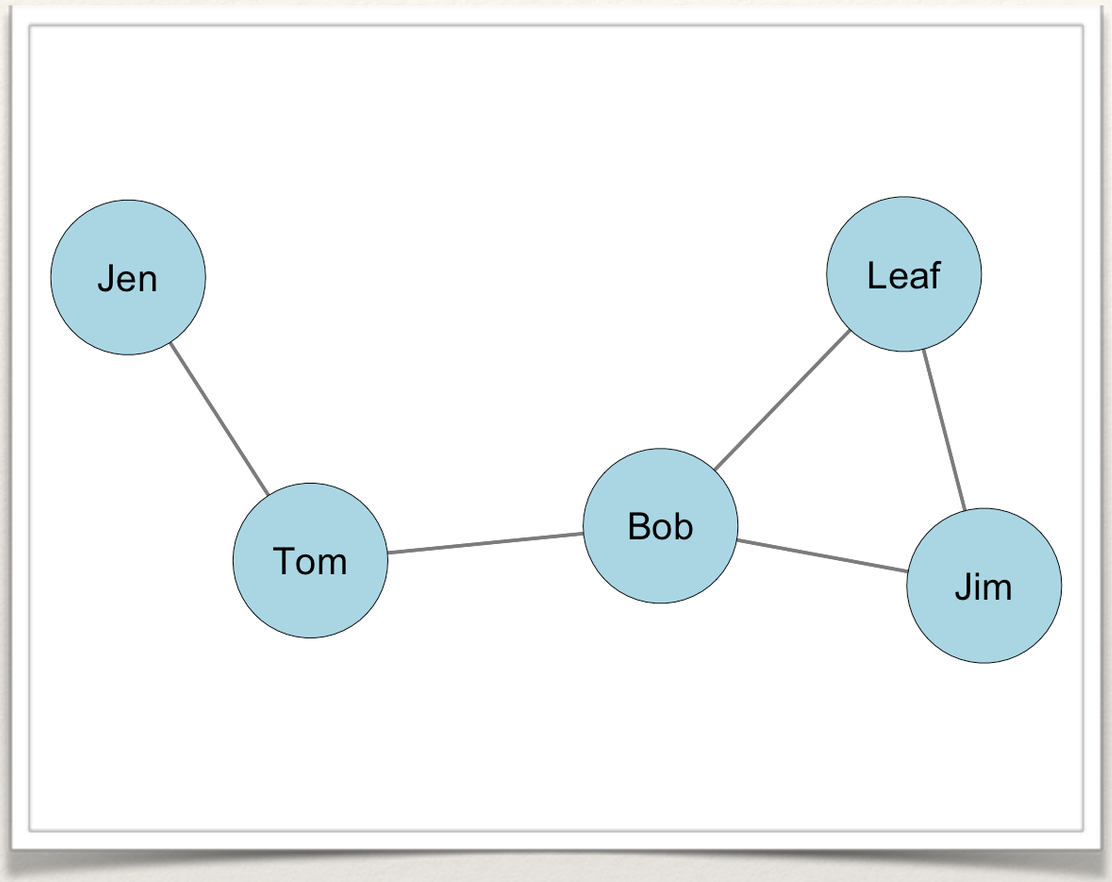


Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom			1/1	1/1
Leaf				0/1
Jim				

The sum of all these ratios is **Bob's** betweenness centrality score.

*What is **Bob's** score?*

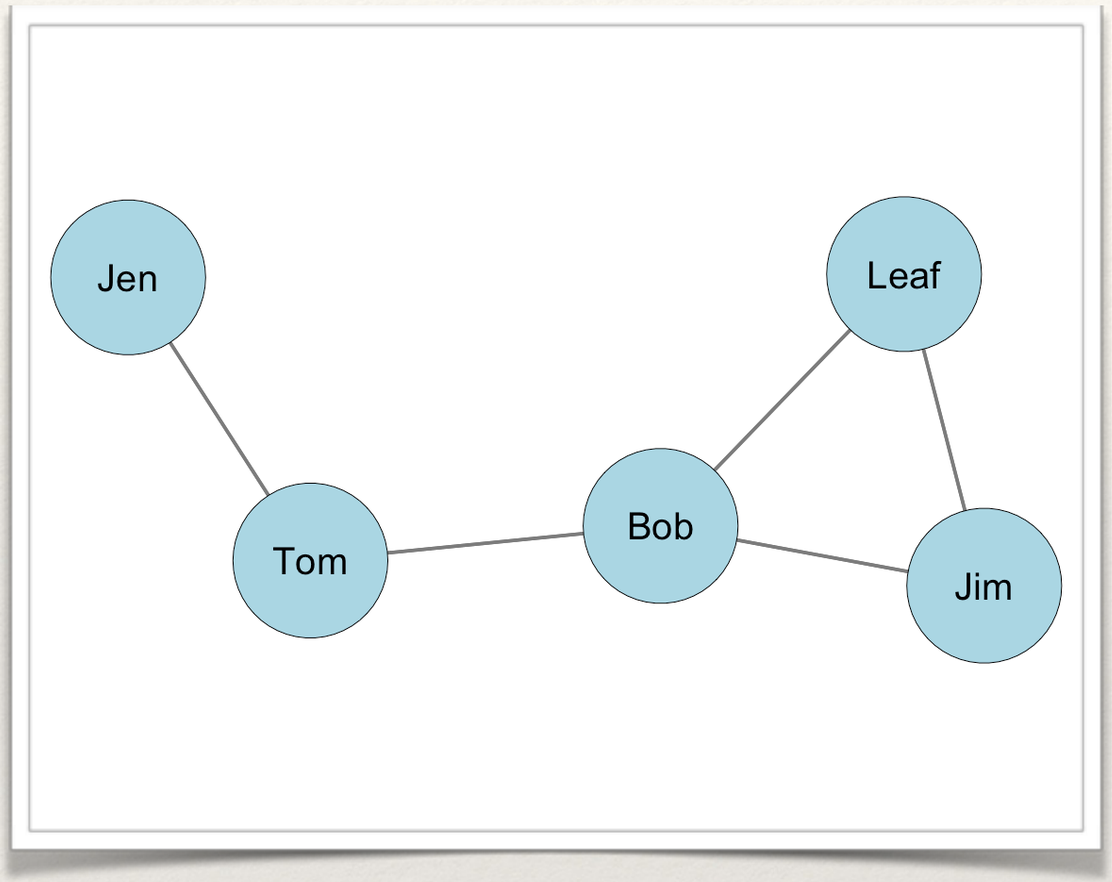
Example: Undirected, Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom			1/1	1/1
Leaf				0/1
Jim				

$$C_B(n_i) = \sum_{j<k} g_{jk}(n_i)/g_{jk}$$

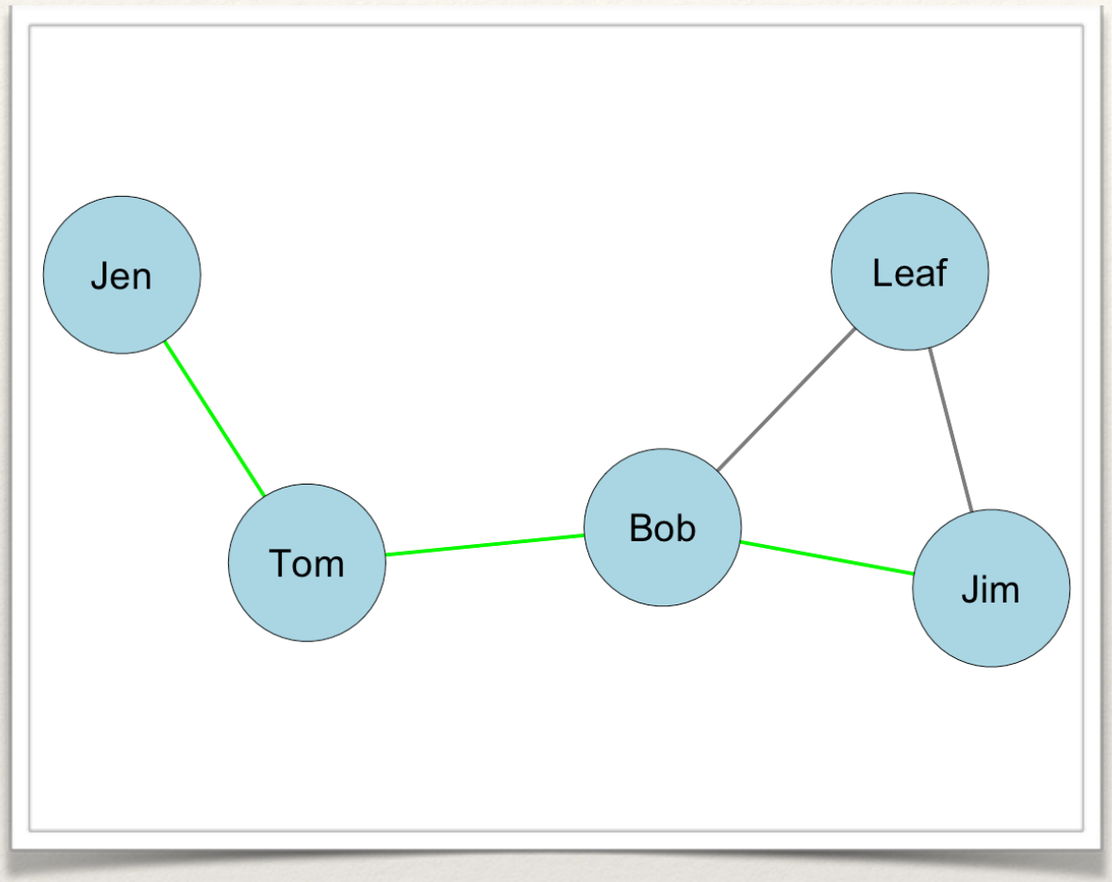
Example: Undirected, Binary Network



Let's do this now for **Leaf**. *What is* ***Leaf's*** *betweenness centrality?*

Geodesic Proportions for *Leaf*				
	Jen	*Tom*	*Bob*	*Jim*
Jen		?/?	?/?	?/?
Tom			?/?	?/?
Bob				?/?
Jim				

Example: Undirected, Binary Network



Geodesic Proportions for *Leaf*				
	Jen	*Tom*	*Bob*	*Jim*
Jen		0/1	0/1	0/1
Tom			0/1	0/1
Bob				0/1
Jim				

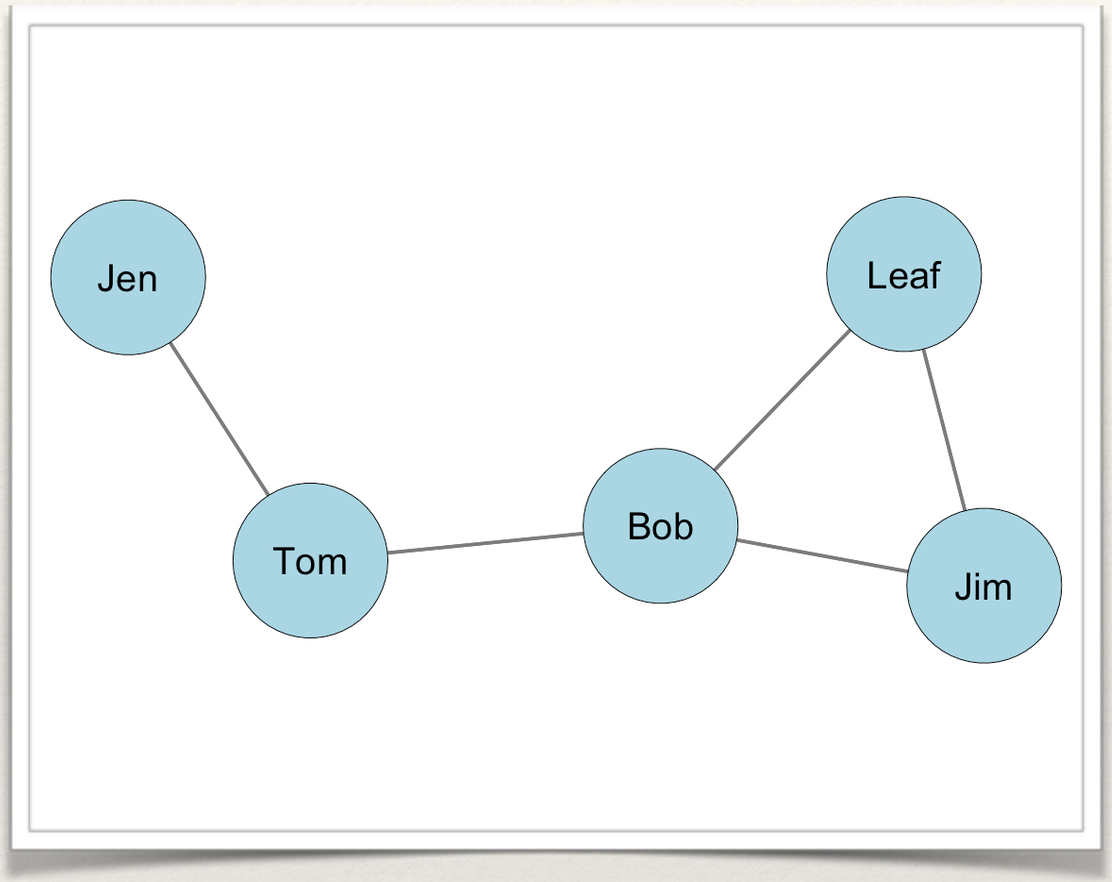
Leaf's betweenness centrality is 0

Betweenness Centrality: Undirected Binary Graphs

$$C'_B(n_i) = \frac{\sum_{j<k} g_{jk}(n_i)/g_{jk}}{[(g-1)(g-2)/2]} = \frac{C_B(n_i)}{[(g-1)(g-2)/2]}$$

The maximum number of pairs of actors not including n_i

Example: Undirected, Binary Network



Unstandardized (raw) for **Bob**: 4

Standardized for **Bob**:
4 / [(5-1)(5-2)/2] = 0.667

Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom			1/1	1/1
Leaf				0/1
Jim				

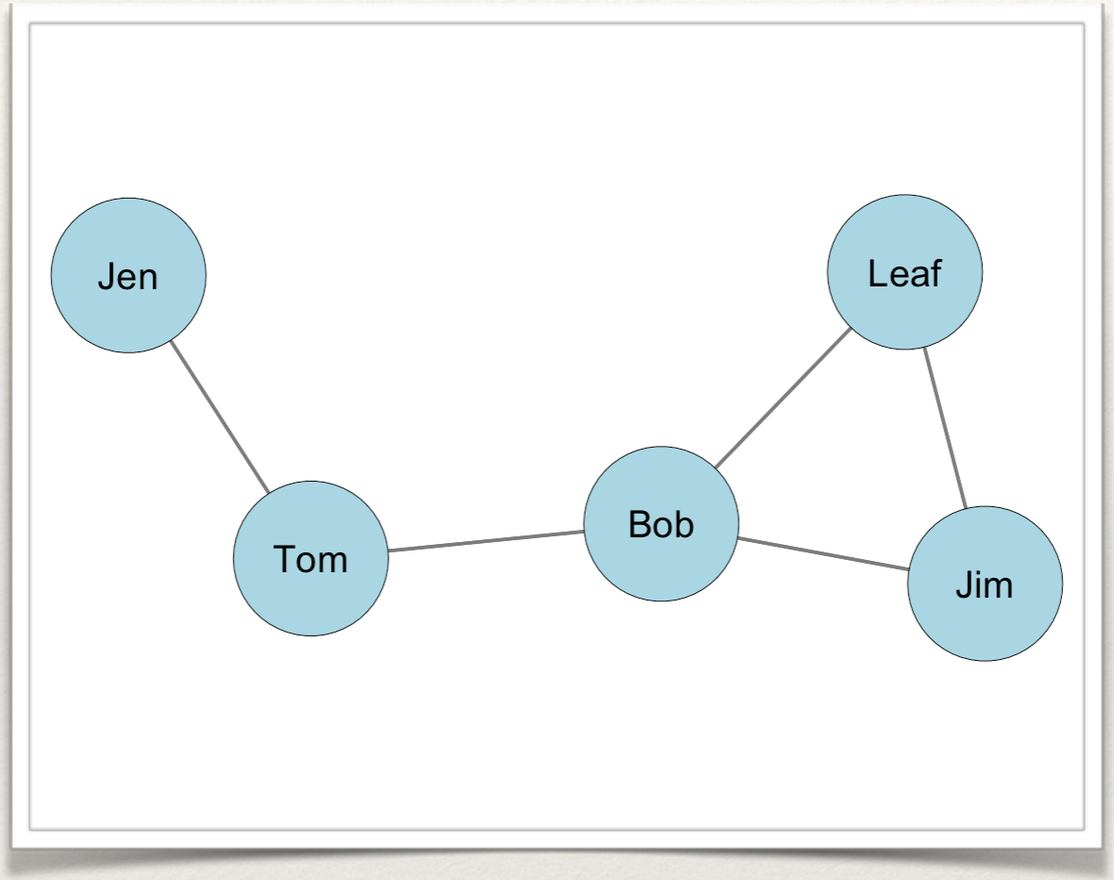
Group Betweenness Centrality: Undirected Binary Graphs

Largest value of the standardized betweenness for the network

Standardized betweenness score for actor i

$$C_B = \frac{\sum_{i=1}^{g} [C'_B(n^*) - C'_B(n_i)]}{(g-1)}$$

Example: Undirected, Binary Network



Standardized Betweenness Centrality Scores

Jen = 0.000

Tom = 0.500

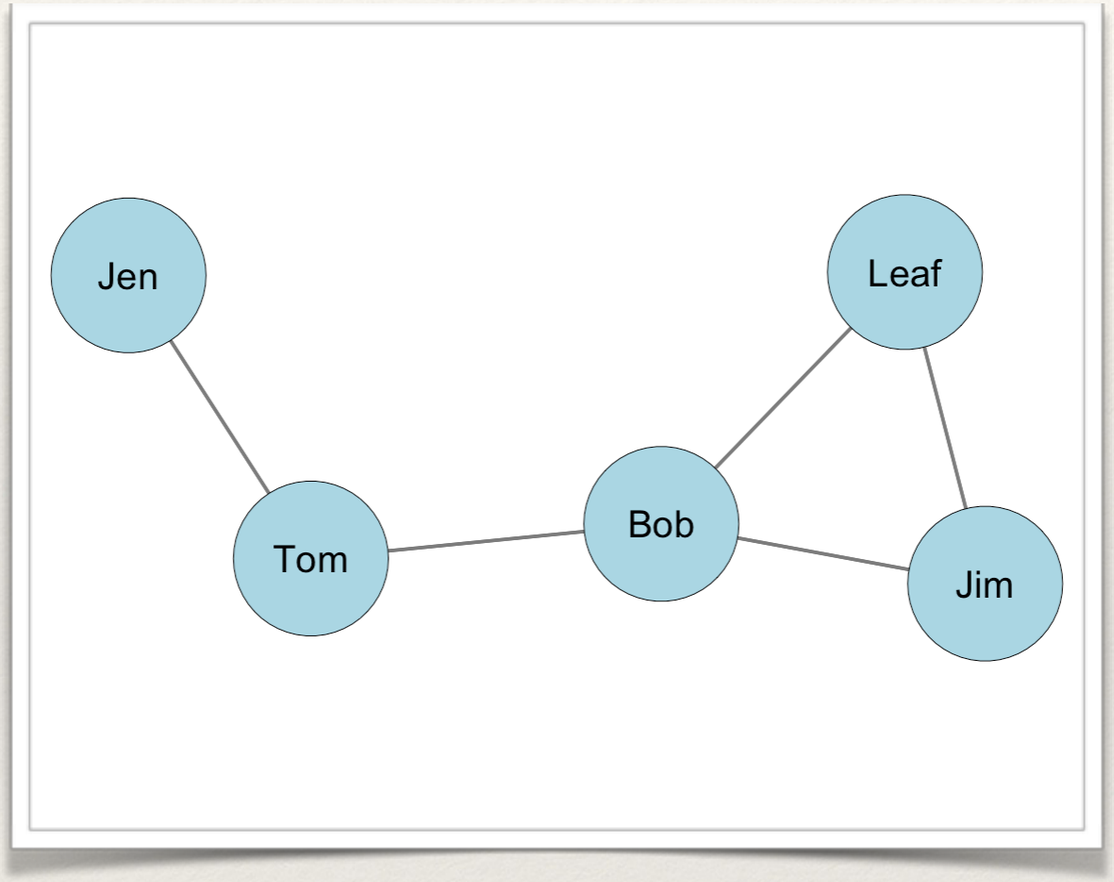
Bob = 0.667

Leaf = 0.000

Jim = 0.000

What is the group betweenness centralization score for this graph?

Example: Undirected, Binary Network



Deviated Standardized Betweenness Centrality Scores

Jen = 0.667 - 0.000 = 0.667

Tom = 0.667 - 0.500 = 0.167

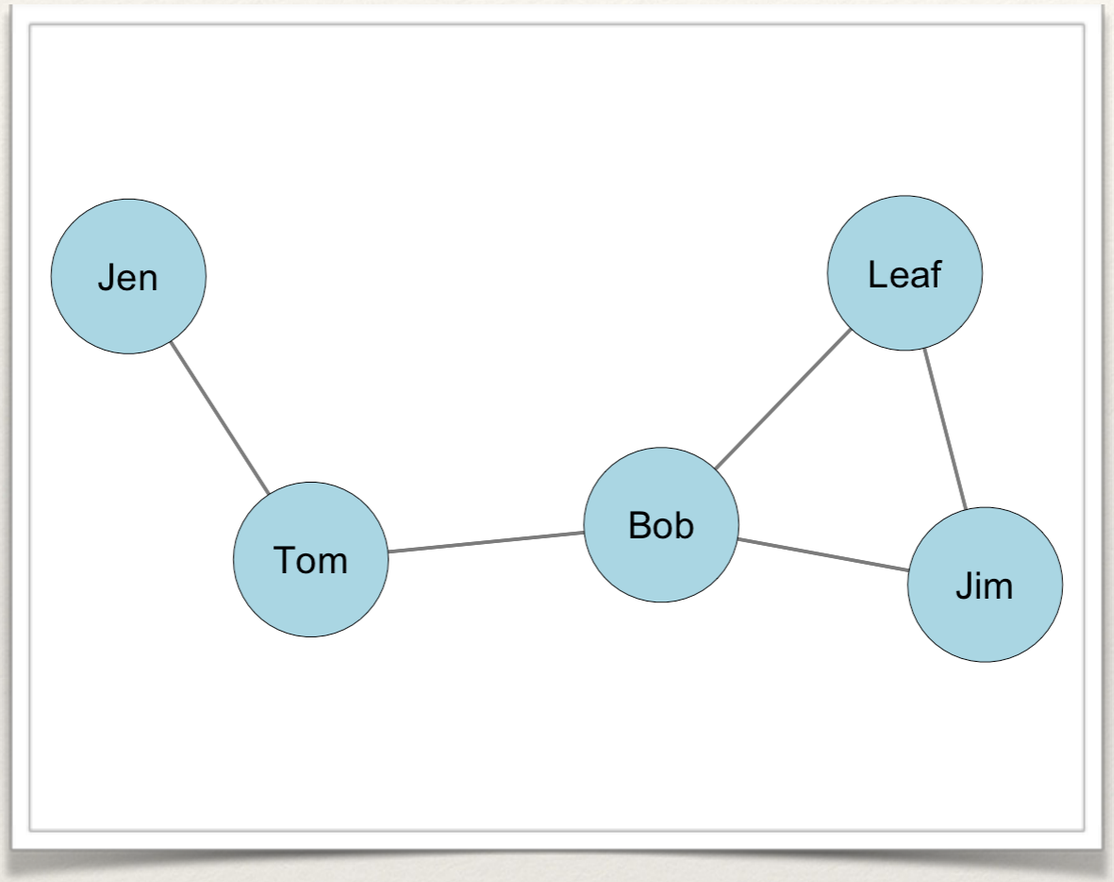
Bob = 0.667 - 0.667 = 0.000

Leaf = 0.667 - 0.000 = 0.667

Jim = 0.667 - 0.000 = 0.667

What is the group betweenness centralization score for this graph?

Example: Undirected, Binary Network



What is the group betweenness centralization score for this graph?

Deviated Standardized Betweenness Centrality Scores

Jen = 0.667 - 0.000 = 0.667

Tom = 0.667 - 0.500 = 0.167

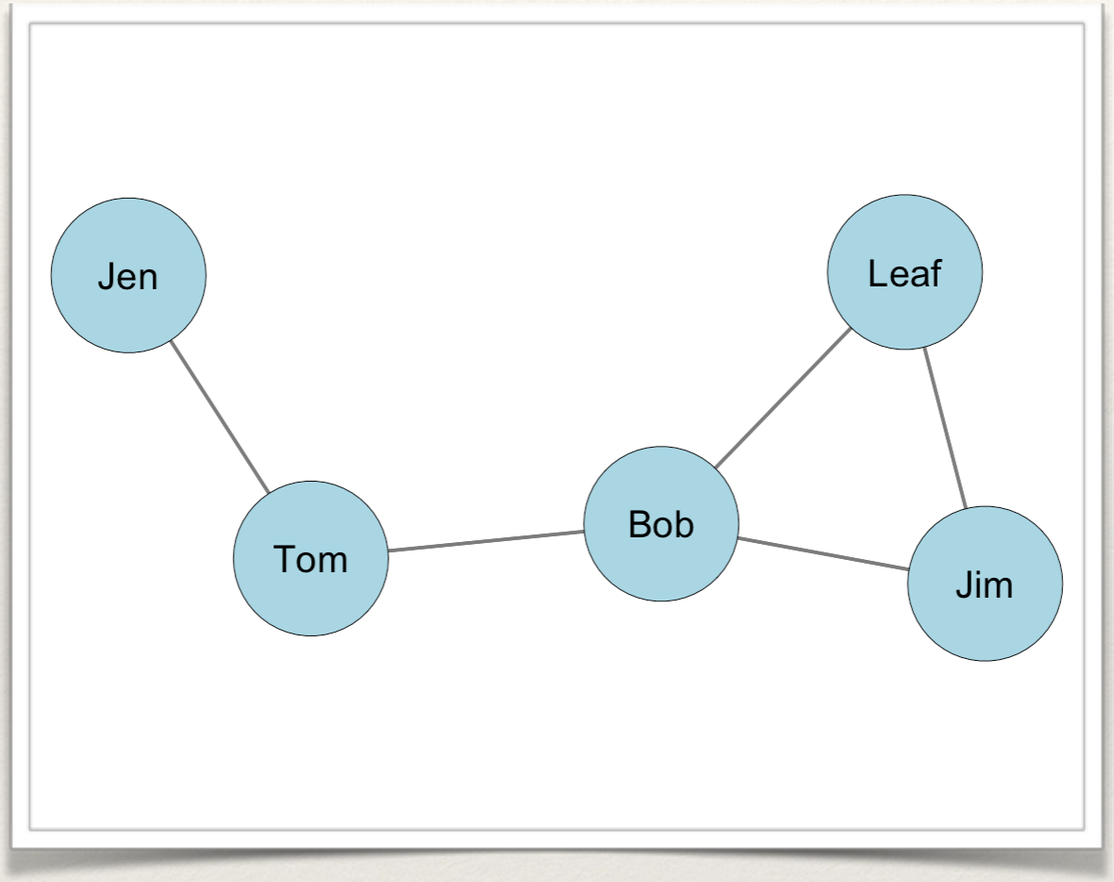
Bob = 0.667 - 0.667 = 0.000

Leaf = 0.667 - 0.000 = 0.667

Jim = 0.667 - 0.000 = 0.667

Sum = 2.168

Example: Undirected, Binary Network



$$C_B = \frac{\sum_{i=1}^{g} [C'_B(n^*) - C'_B(n_i)]}{(g-1)}$$

Deviated Standardized Betweenness Centrality Scores

Jen = 0.667 - 0.000 = 0.667

Tom = 0.667 - 0.500 = 0.167

Bob = 0.667 - 0.667 = 0.000

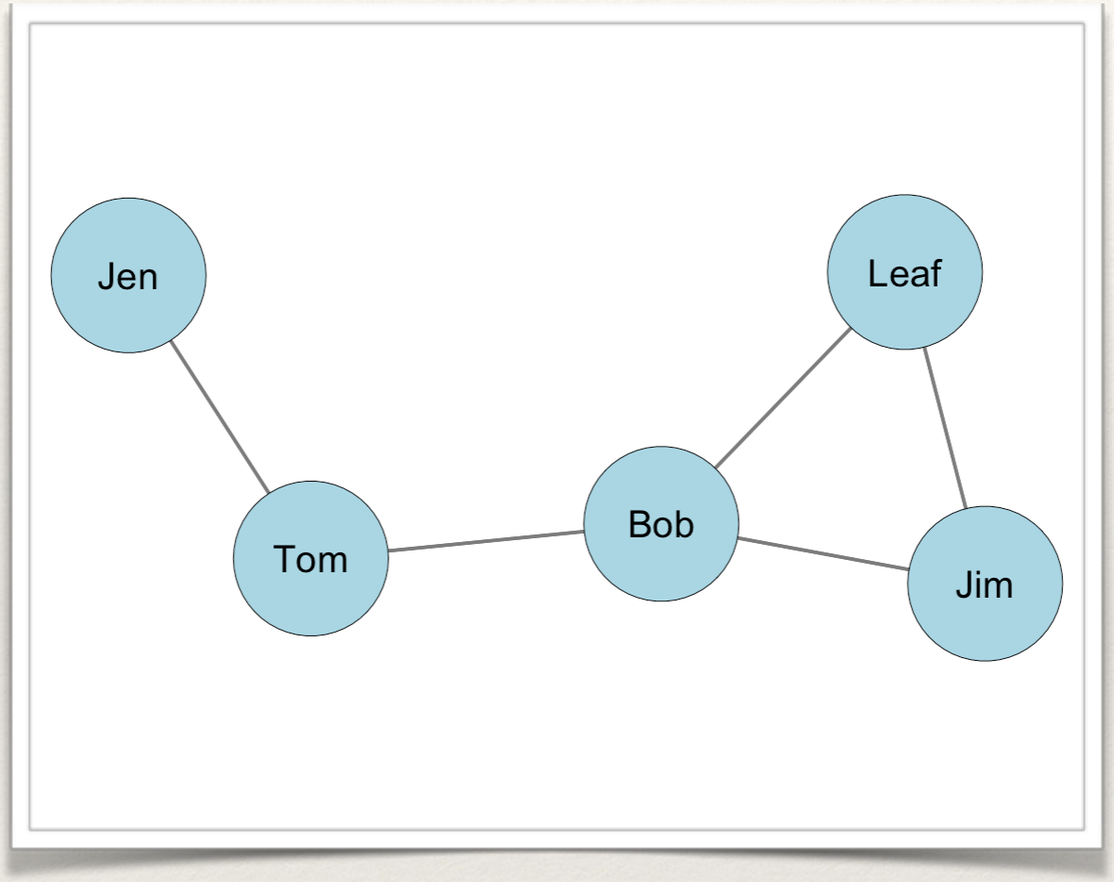
Leaf = 0.667 - 0.000 = 0.667

Jim = 0.667 - 0.000 = 0.667

Sum = 2.168

Denominator = 4

Example: Undirected, Binary Network



2.168 / 4 = **0.542**

Deviated Standardized Betweenness Centrality Scores

Jen = 0.667 - 0.000 = 0.667

Tom = 0.667 - 0.500 = 0.167

Bob = 0.667 - 0.667 = 0.000

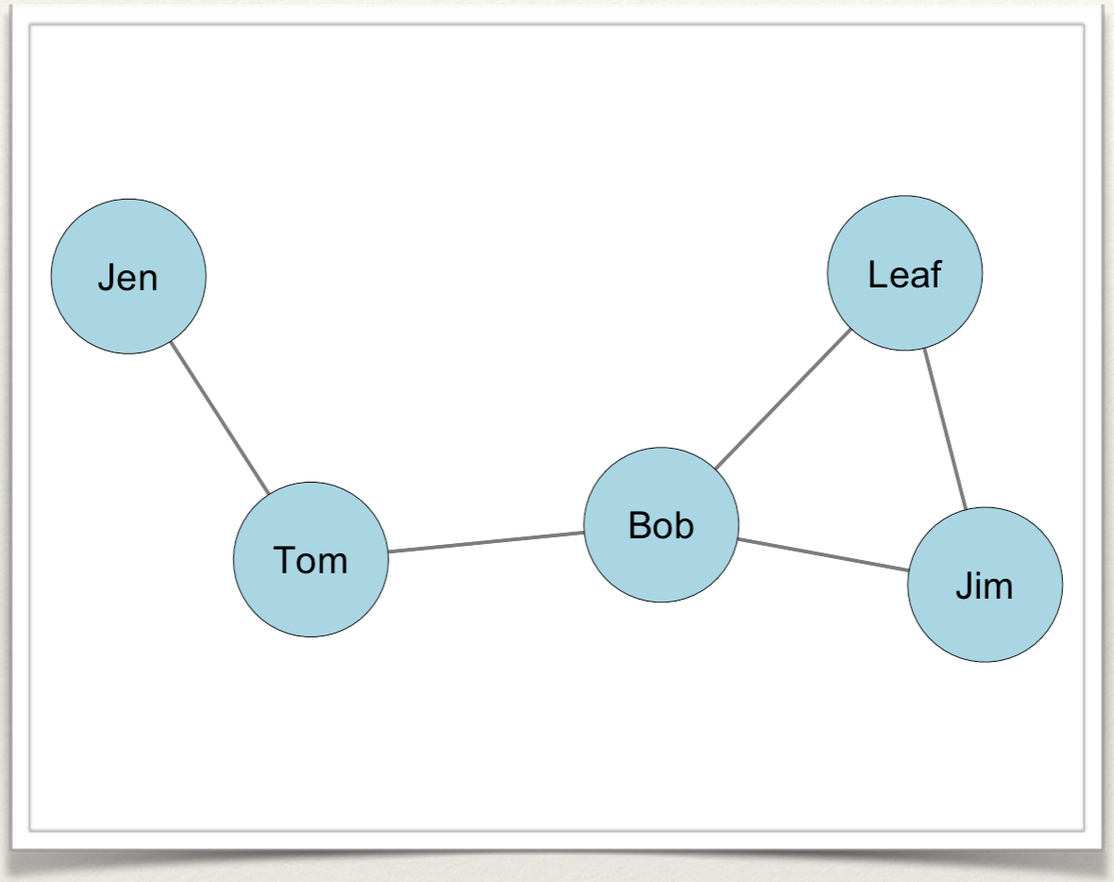
Leaf = 0.667 - 0.000 = 0.667

Jim = 0.667 - 0.000 = 0.667

Sum = 2.168

Denominator = 4

Example: Undirected, Binary Network



Compare the centralization scores:

Degree = 0.416
Closeness = 0.552
Betweenness = 0.542

What can we say about the differences in the centralization scores for each type of centrality?

Empirical Example

THE SOCIAL ORGANIZATION OF CONSPIRACY: ILLEGAL NETWORKS IN THE HEAVY ELECTRICAL EQUIPMENT INDUSTRY*

WAYNE E. BAKER
University of Chicago

ROBERT R. FAULKNER
University of Massachusetts

American Sociological Review, 1993, Vol. 58 (December:837–860)

- https://www.jstor.org/stable/2095954

Empirical Example

- If you are going to conspire, you face a dilemma:
  - You have to balance **secrecy** and **task performance**.

Empirical Example

- If you are going to conspire, you face a dilemma:
  - As a result, your network should *look* a certain way.

Organization Objective	Information-Processing Requirement	
	High	Low
Concealment	Decentralized networks	Decentralized networks
Coordination	Decentralized networks	Centralized networks

Figure 1. Concealment Versus Coordination: Theoretical Expectations

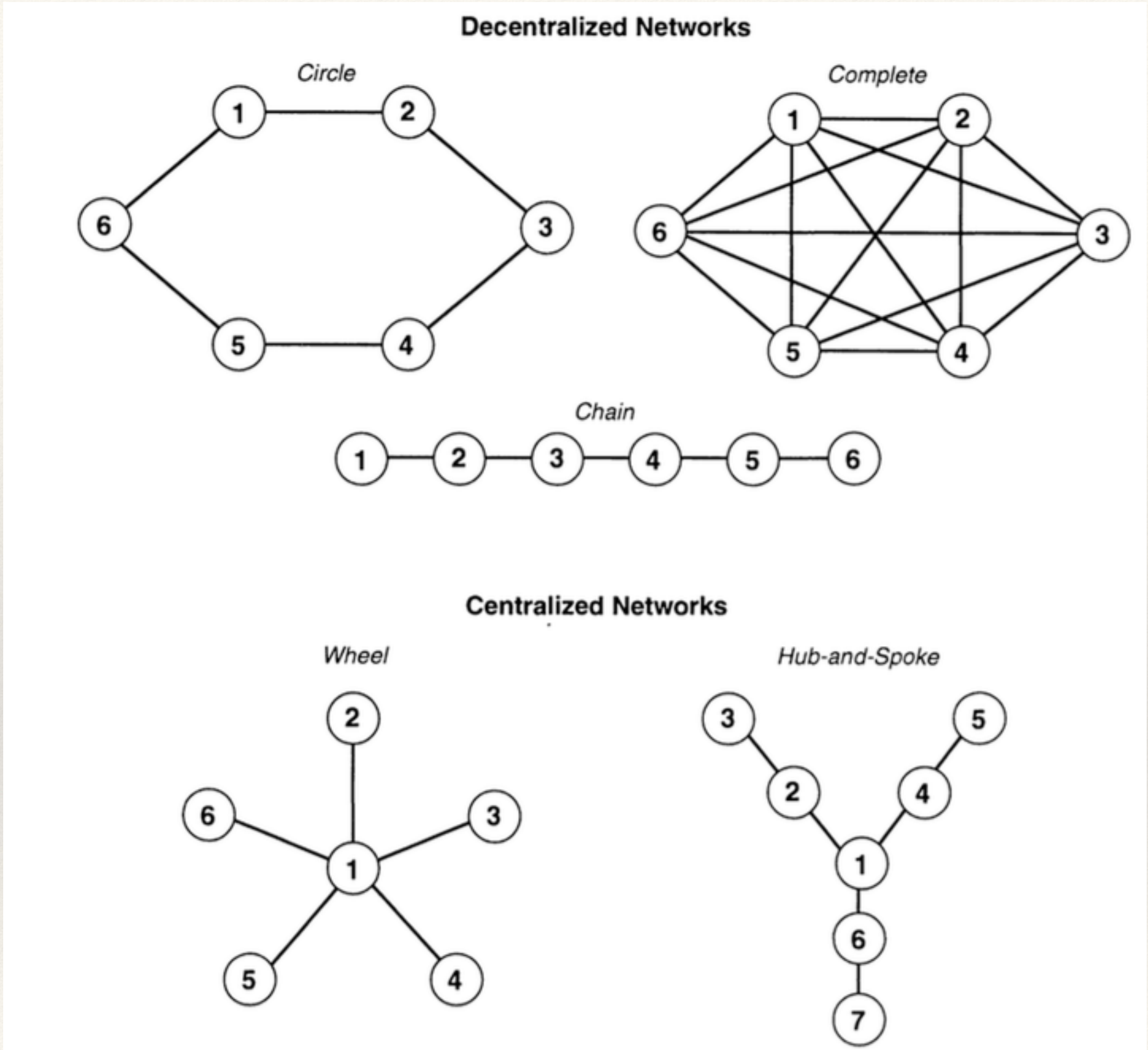


Figure 2. Five Examples of Communication Networks

Empirical Example

- Findings:
  - The low information processing settings had the *least* centralization.
  - Supporting the **secrecy** hypothesis.

Table 1. Network Characteristics and Outcomes for Three Price-Fixing Conspiracies

Network Characteristic and Outcome	Conspiracy		
	Switchgear	Transformers	Turbines
Network Characteristic			
Size (number of participants)	33	21	24
Density	23.3	32.4	35.5
Nieminen graph centralization (degree)	41.7	36.1	51.4
Freeman graph centralization (betweenness)	21.3	17.6	24.2
Sabidussi graph centralization (farness)	39.0	37.4	60.8

Empirical Example

- Findings:
  - The high information processing settings had the *most* centralization.
  - Counter to expectation.

Table 1. Network Characteristics and Outcomes for Three Price-Fixing Conspiracies

Network Characteristic and Outcome	Conspiracy		
	Switchgear	Transformers	Turbines
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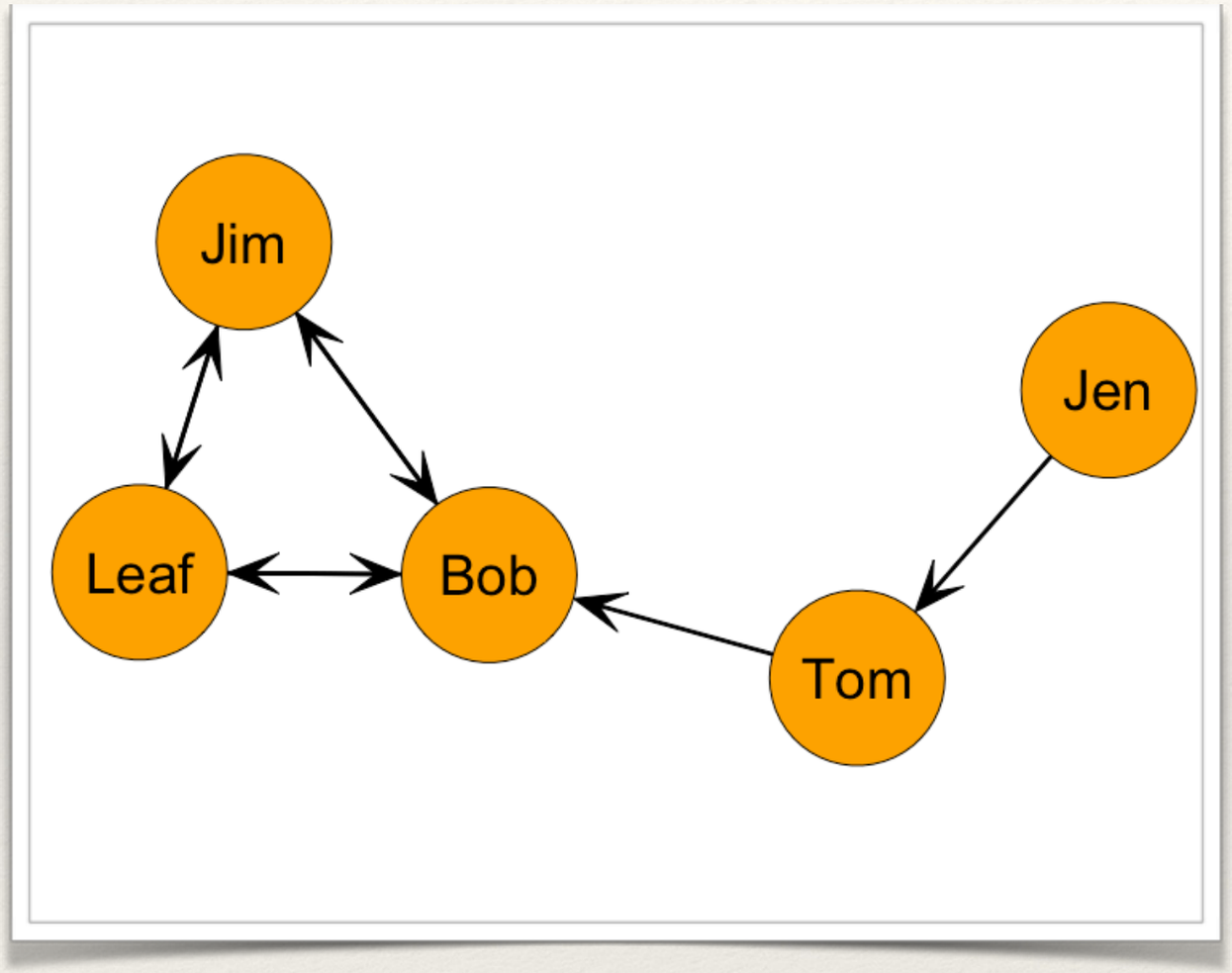
Sabidussi graph centralization (farness)	39.0	37.4	60.8

Directed Networks

Closeness and Betweenness Centrality: Directed Binary Graphs

- Recall that in a directed network, the directionality matters.
- As a result, we have to consider how this might influence our measures.

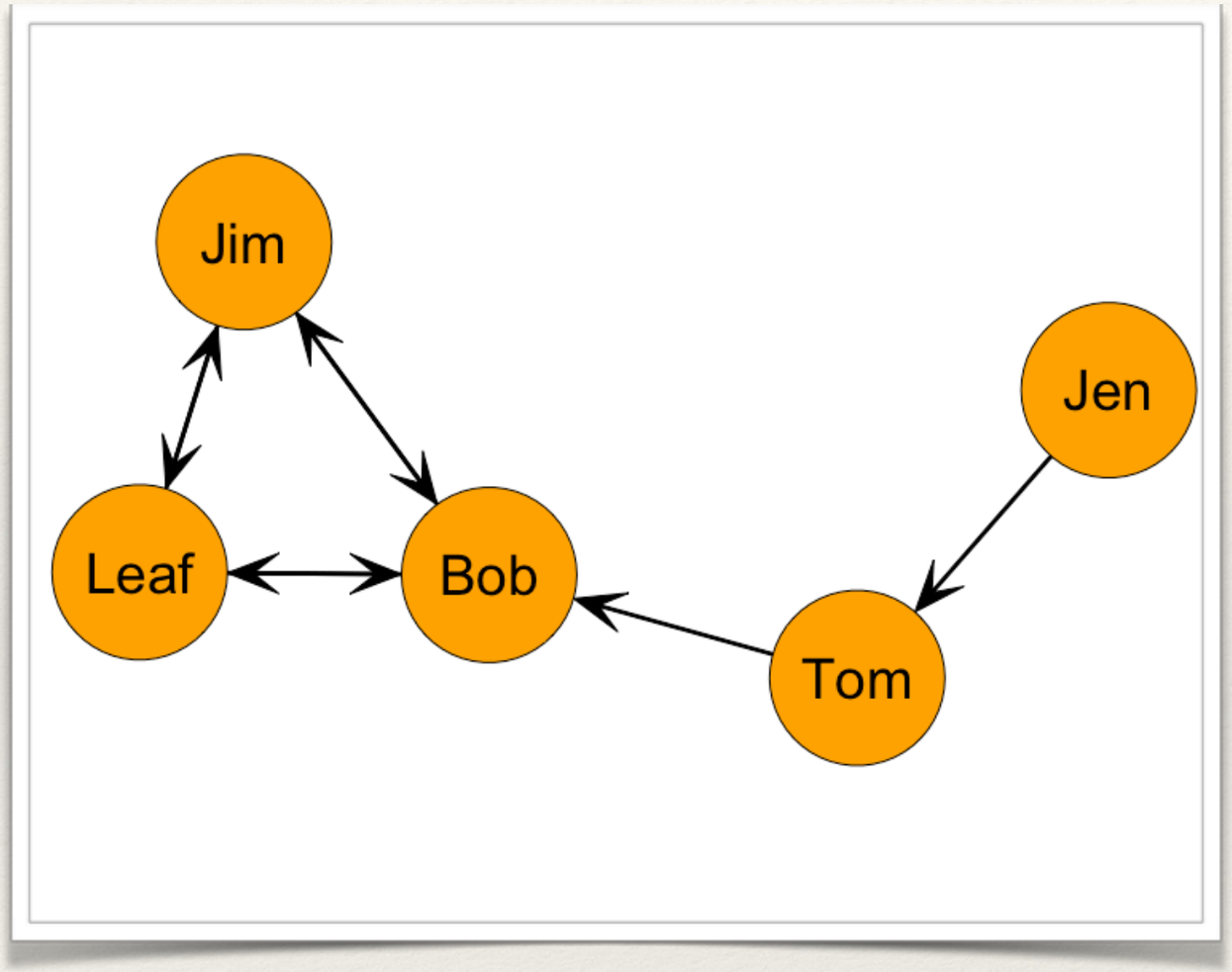
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen					
Tom					
Bob					
Leaf					
Jim					

*How far is **Jen** from Tom? From Bob?*

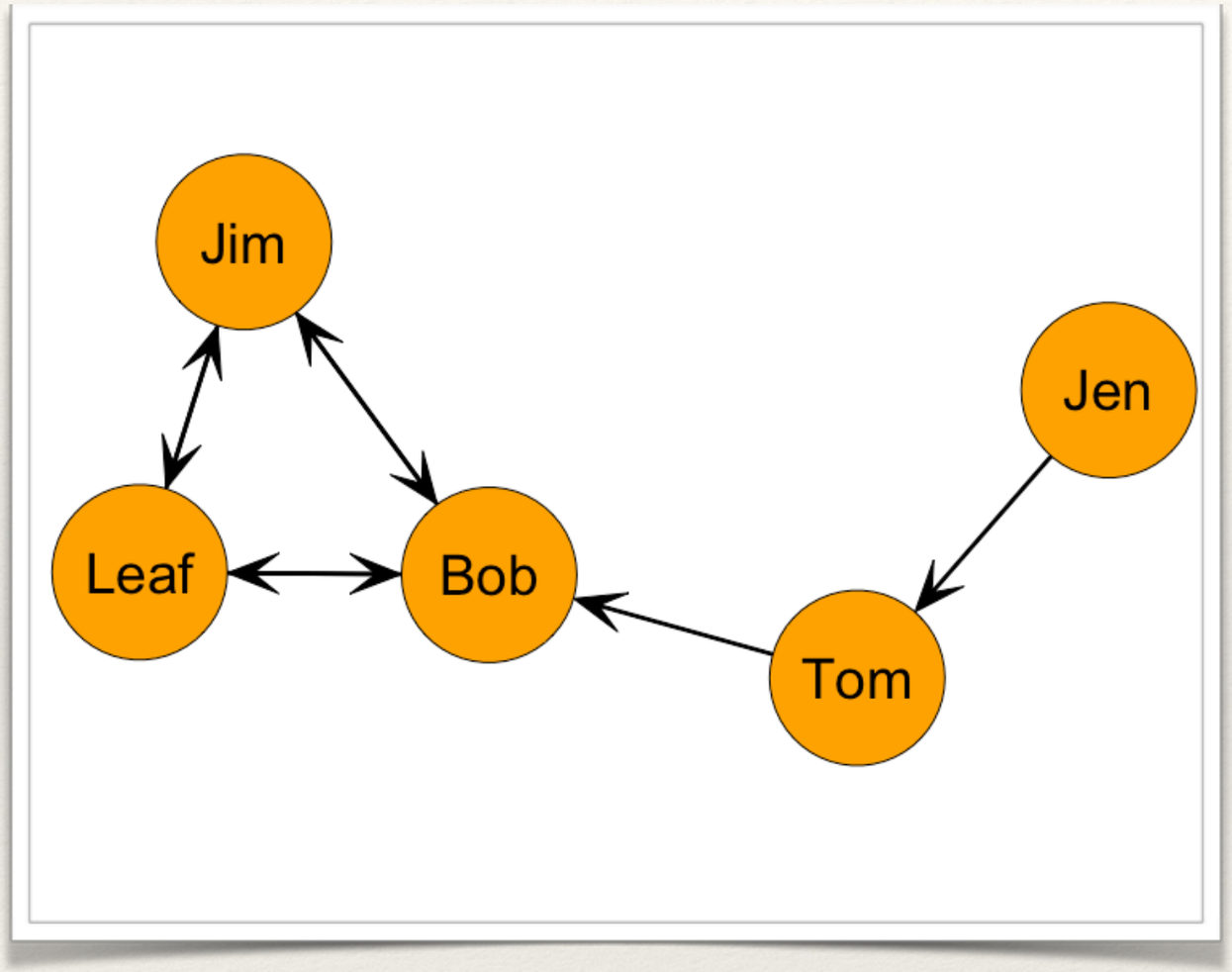
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen					
Tom					
Bob					
Leaf					
Jim					

*How far is **Jen** from Tom? From Bob? Depends on direction.*

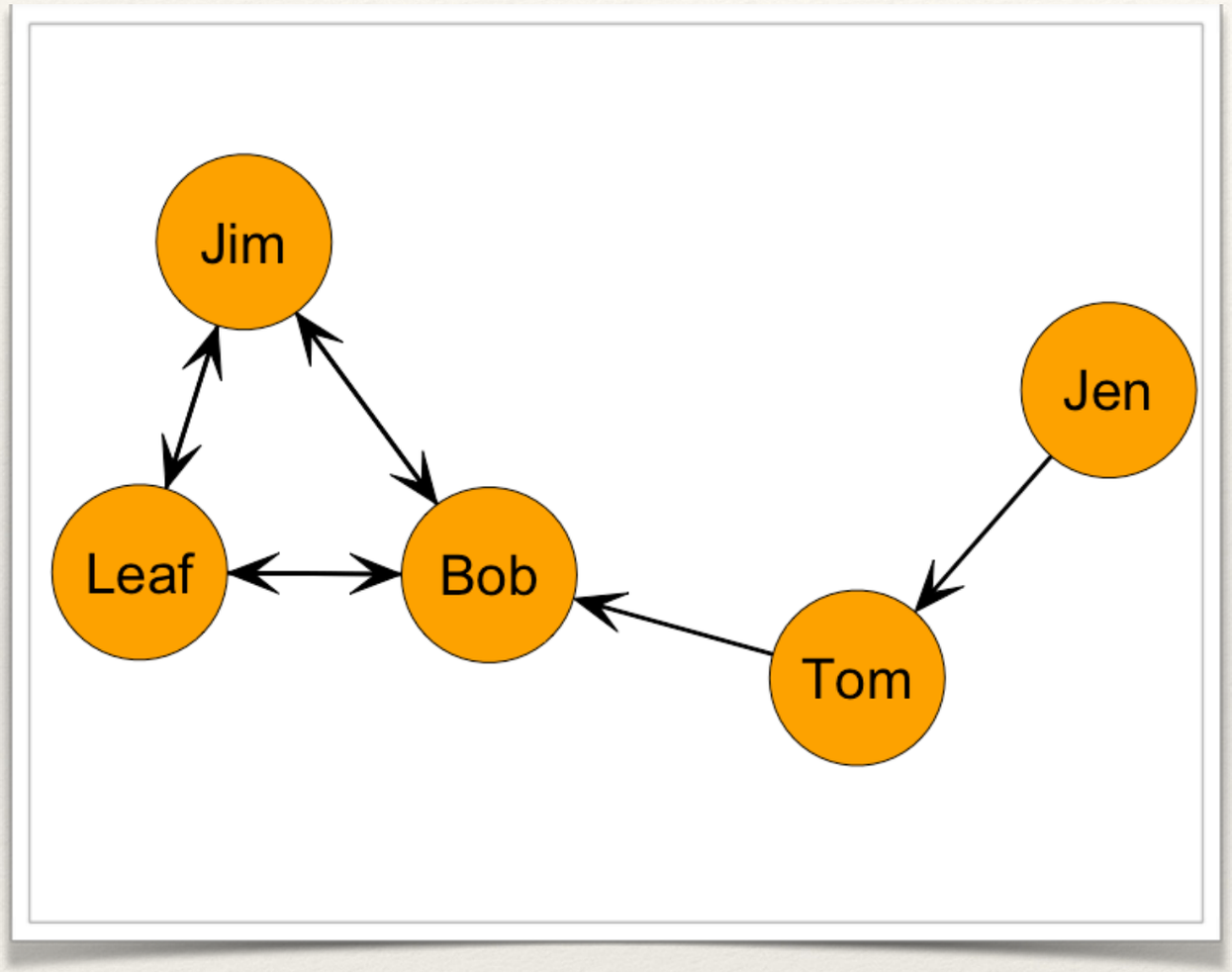
Example: Closeness Centrality for Directed Binary Network



Closeness centrality in directed graphs usually focuses on the send network (i.e. outgoing ties).

Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen					
Tom					
Bob					
Leaf					
Jim					

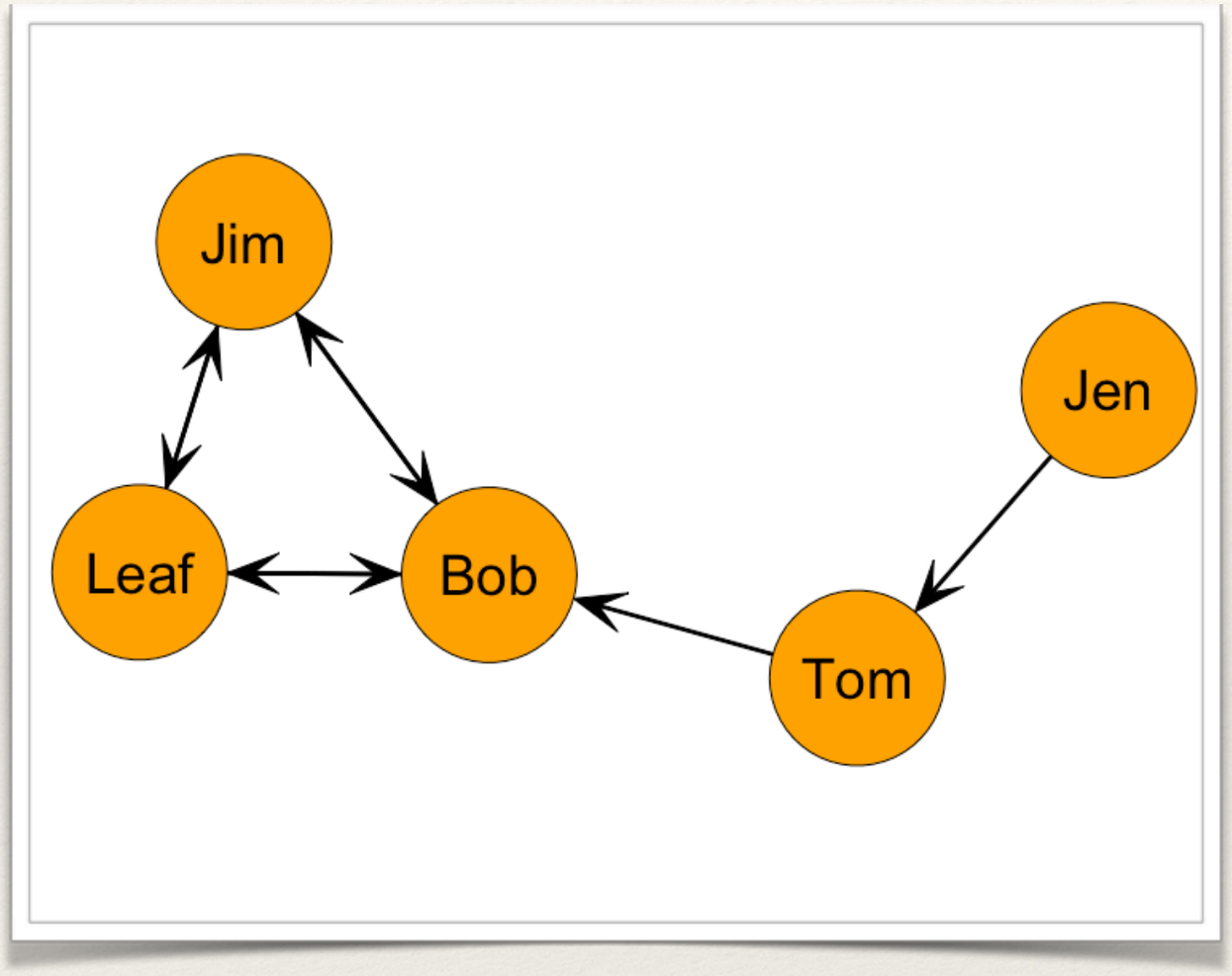
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		?	?		
Tom					
Bob					
Leaf					
Jim					

*How far is **Jen** from Tom? From Bob?*

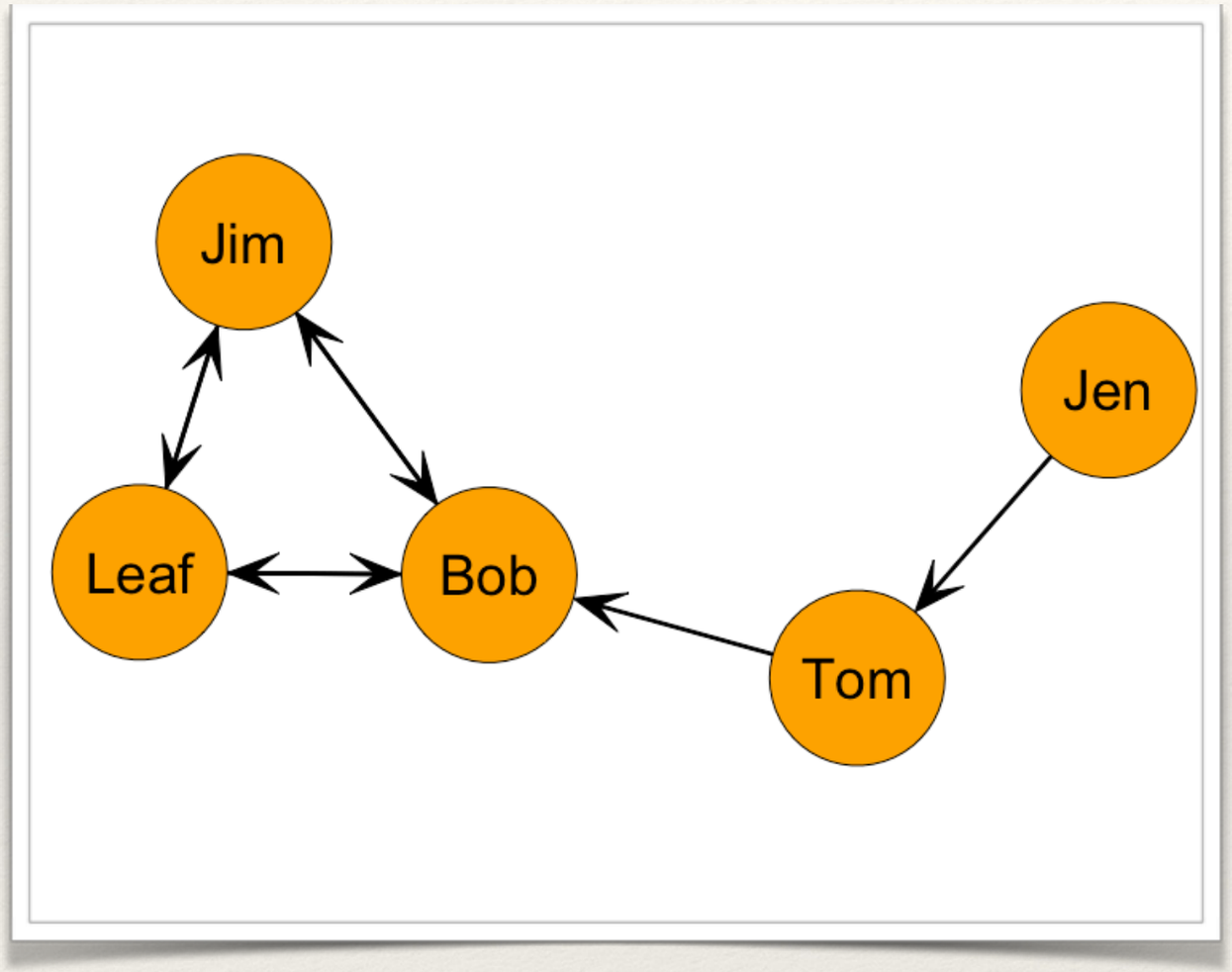
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		1	2		
Tom					
Bob					
Leaf					
Jim					

*How far is **Jen** from Tom? From Bob?*

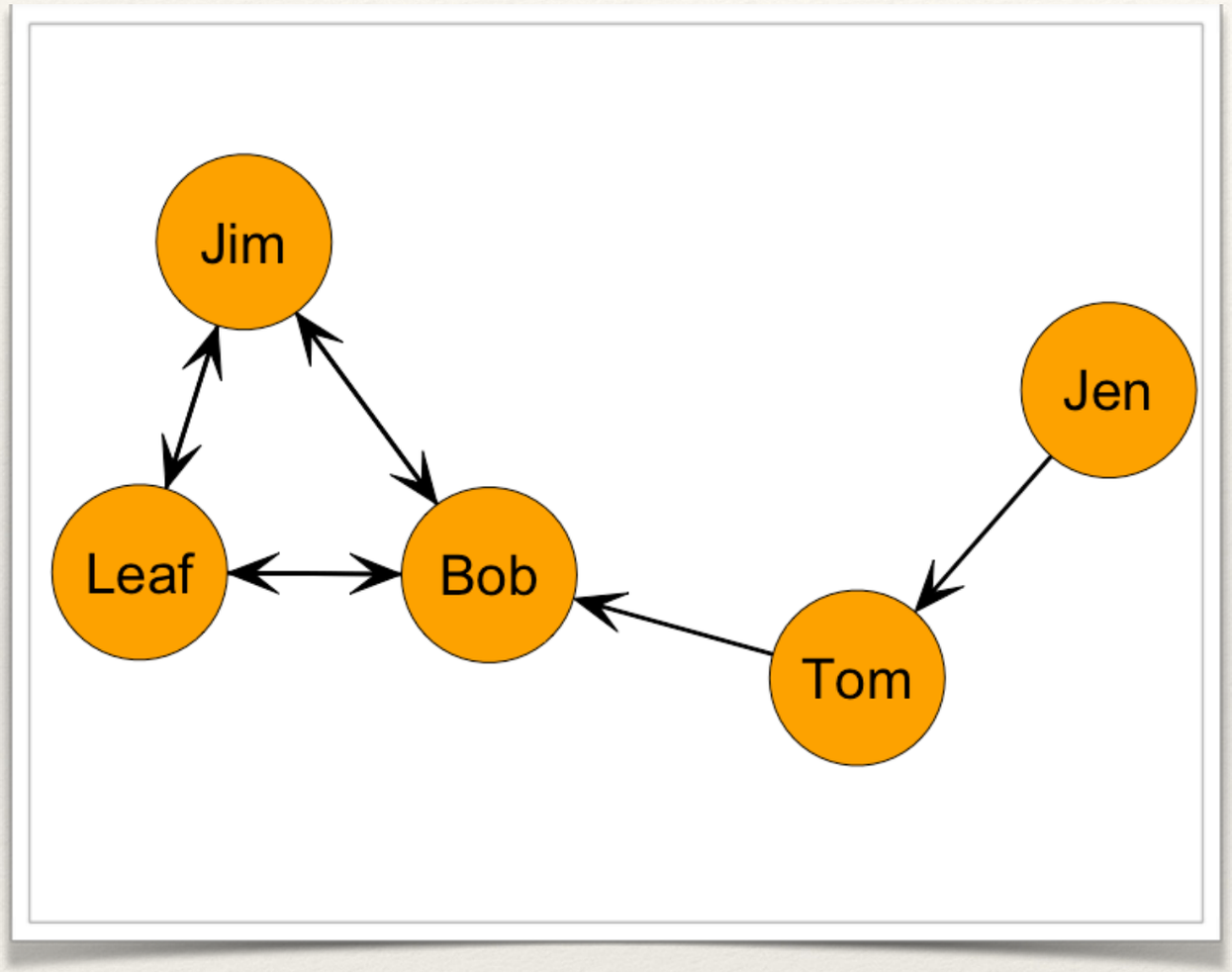
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		1	2	?	?
Tom					
Bob					
Leaf					
Jim					

What about the rest?

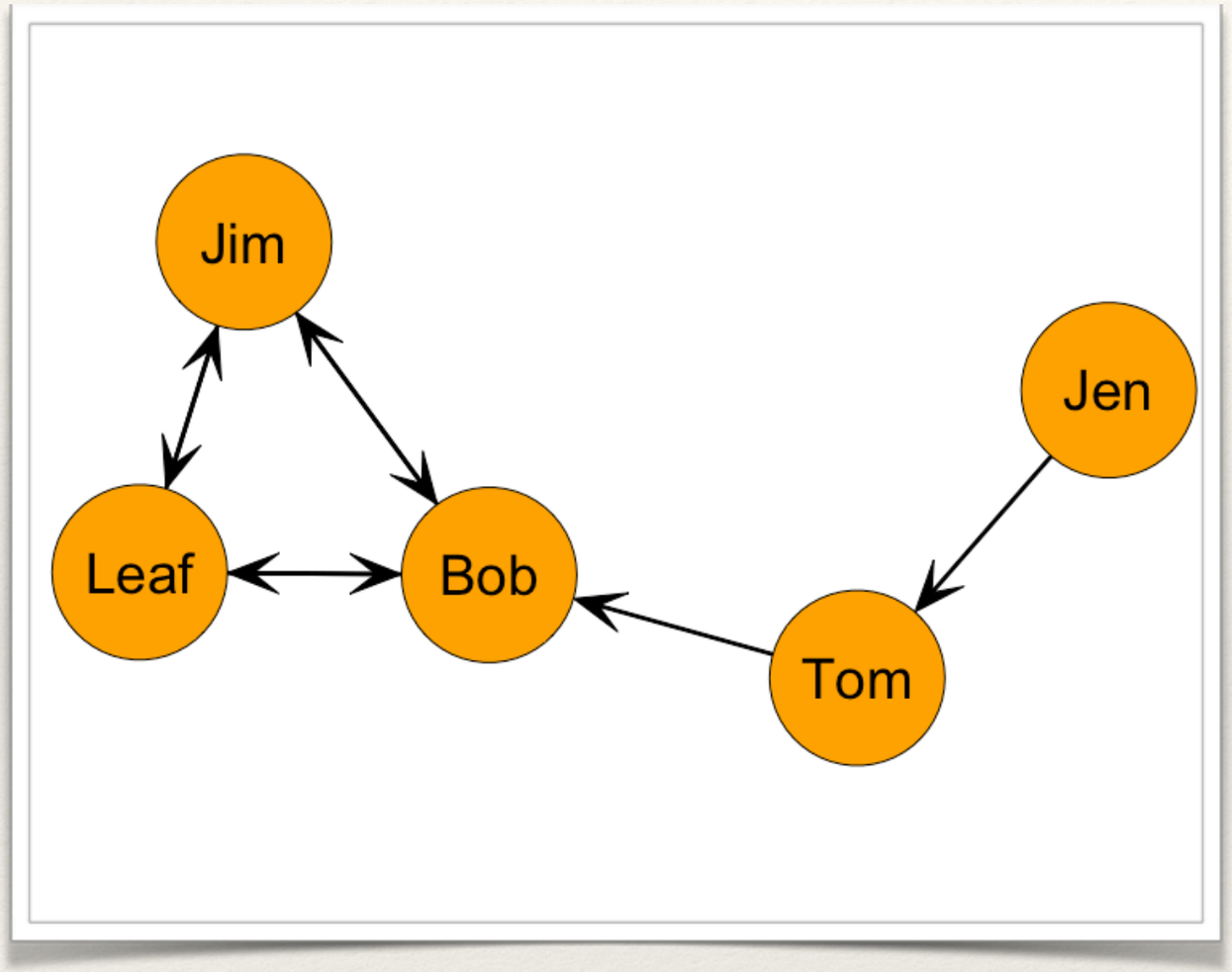
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3
Tom					
Bob					
Leaf					
Jim					

*What is **Jen's** closeness in the graph?*

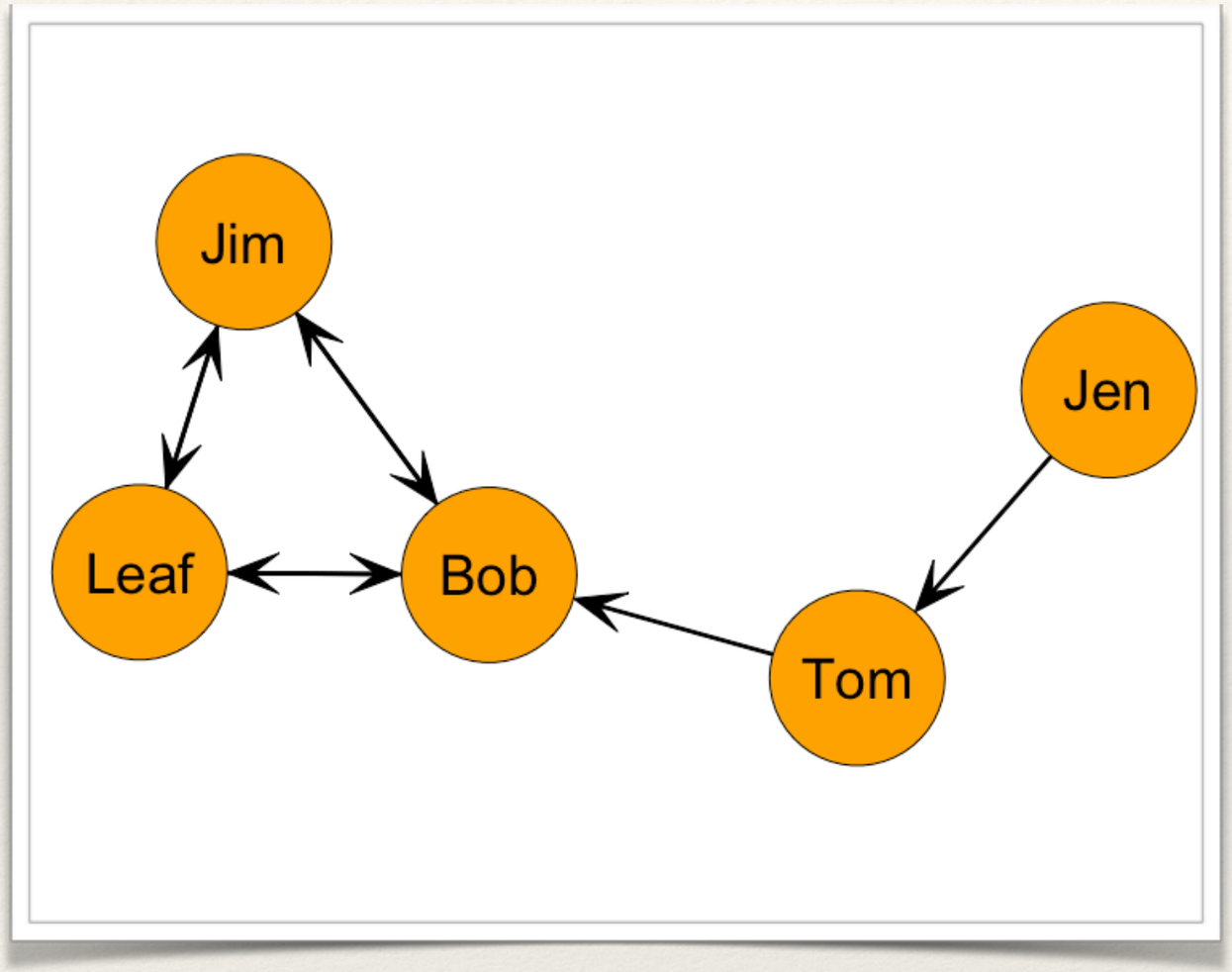
Example: Closeness Centrality for Directed Binary Network



Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom						
Bob						
Leaf						
Jim						

*What is **Jen's** closeness in the graph?*

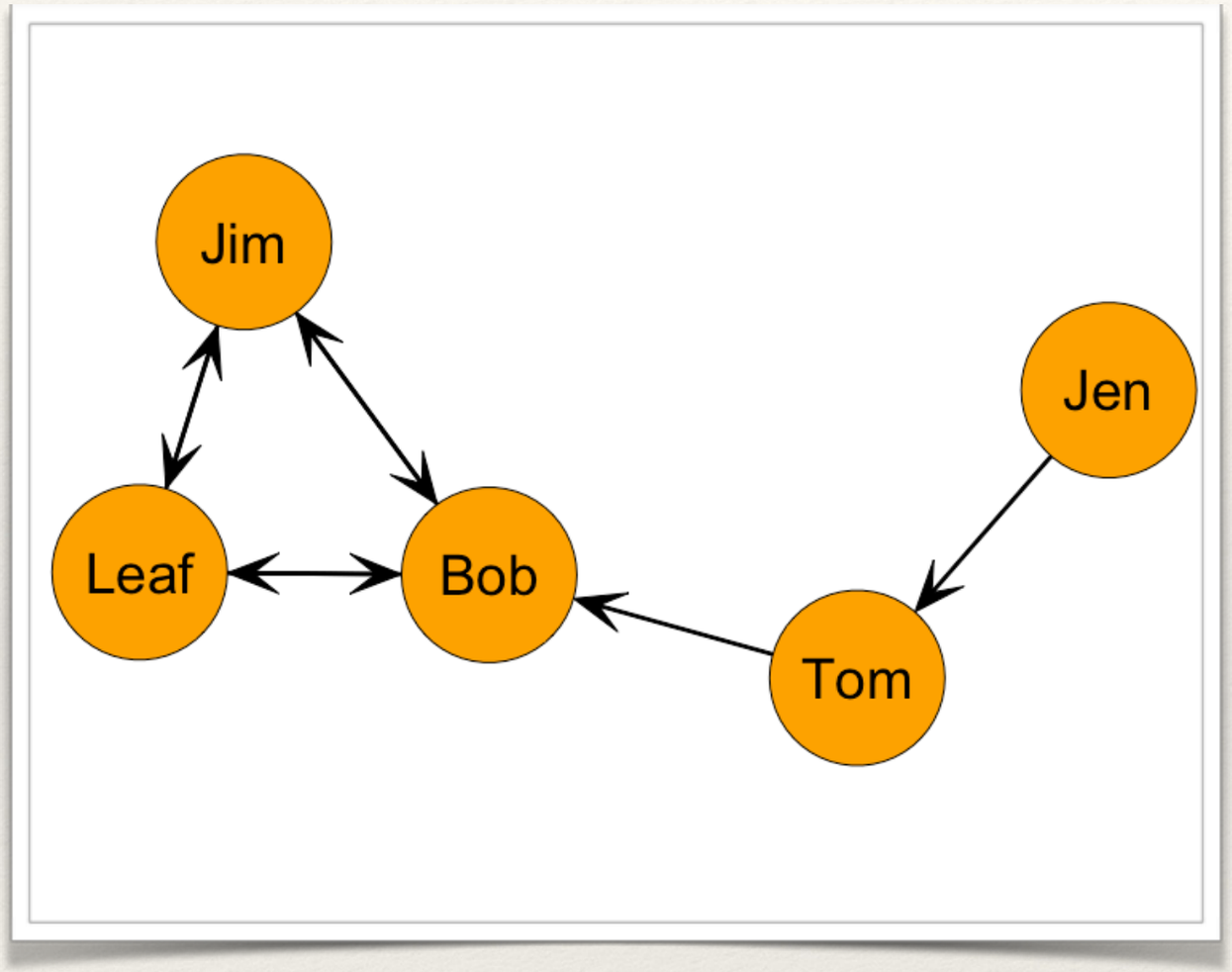
Example: Closeness Centrality for Directed Binary Network



Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom						
Bob						
Leaf						
Jim						

Just take the inverse, which is 1/9 = 0.111.

Example: Closeness Centrality for Directed Binary Network

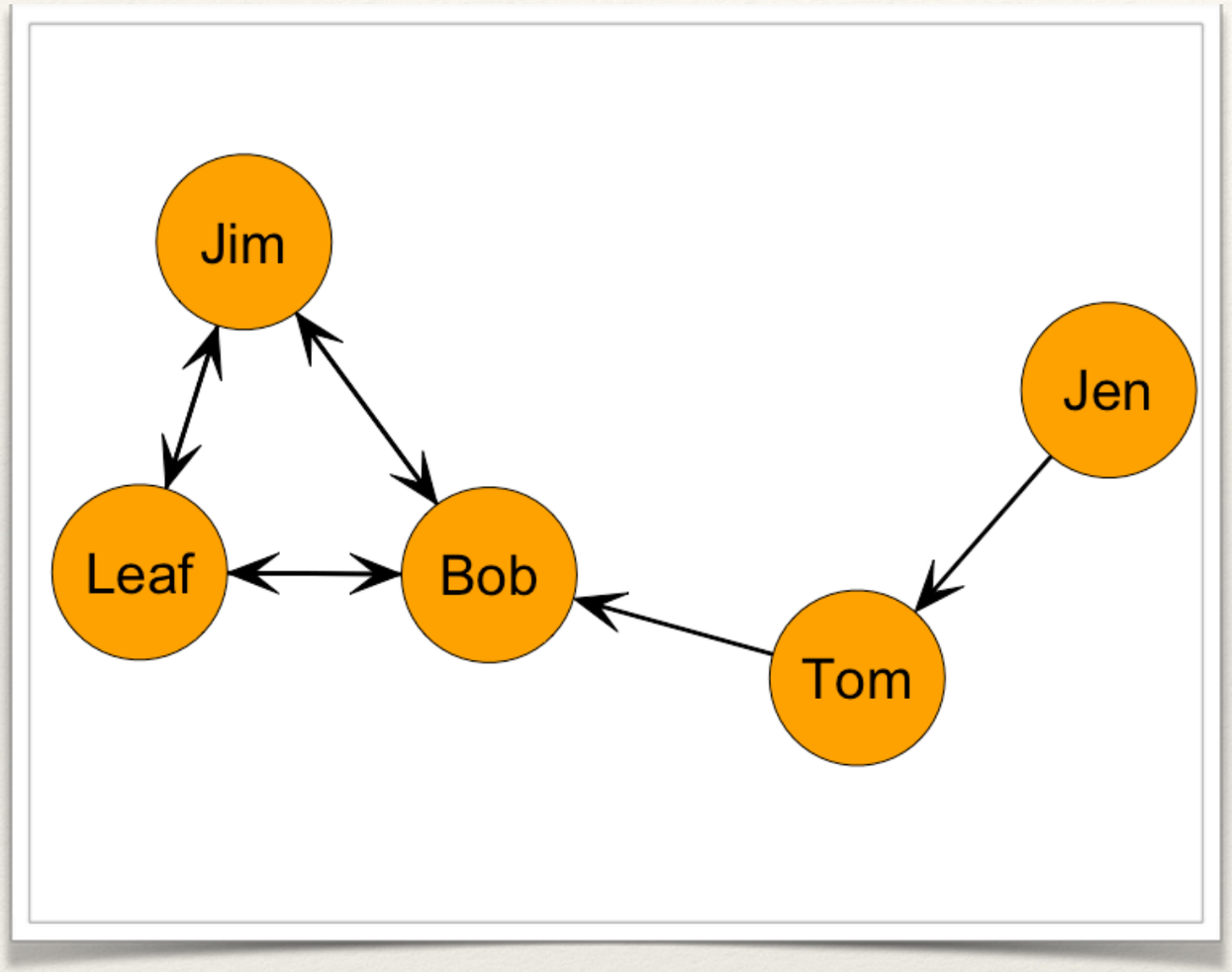


Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom						
Bob						
Leaf						
Jim						

*Then, **Jen's** standardized score is*

(1/9)(g-1) = 0.111*4 = 0.444*

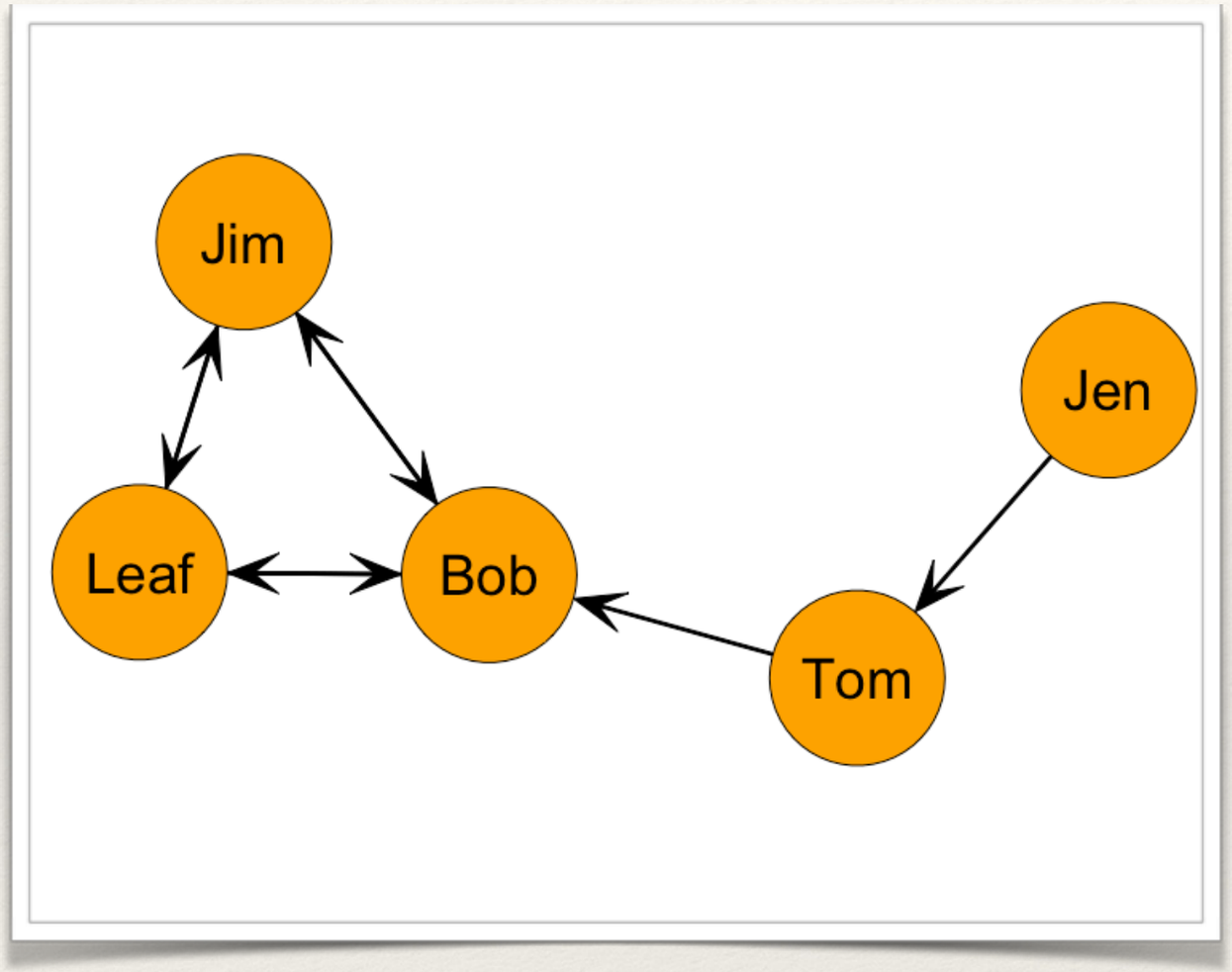
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen					
Tom					
Bob	?	?		?	?
Leaf					
Jim					

*What about **Bob**? How far is **Bob** from everyone?*

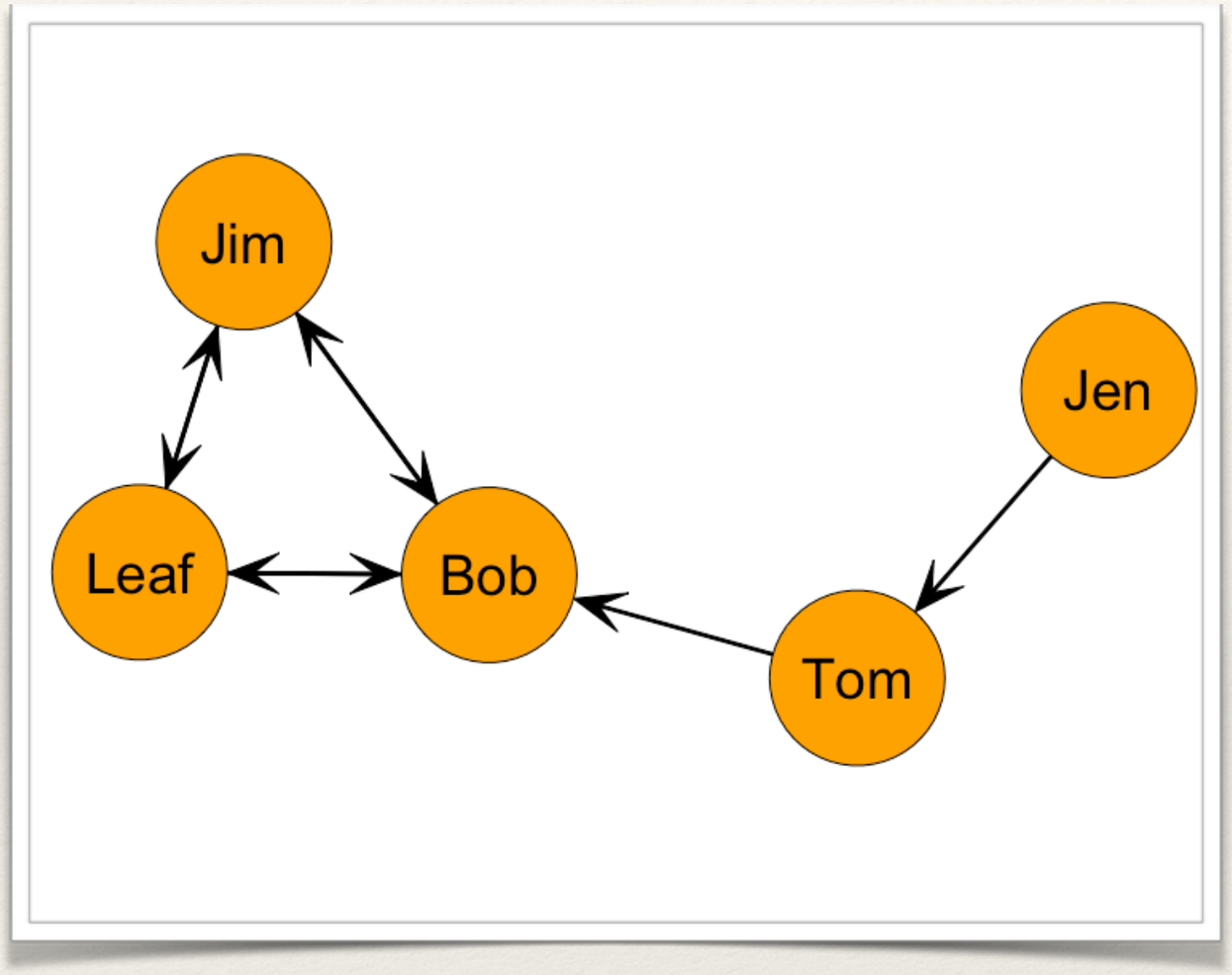
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen					
Tom					
Bob	Inf	Inf		?	?
Leaf					
Jim					

*Why is **Bob's** distance to **Tom** infinite?*

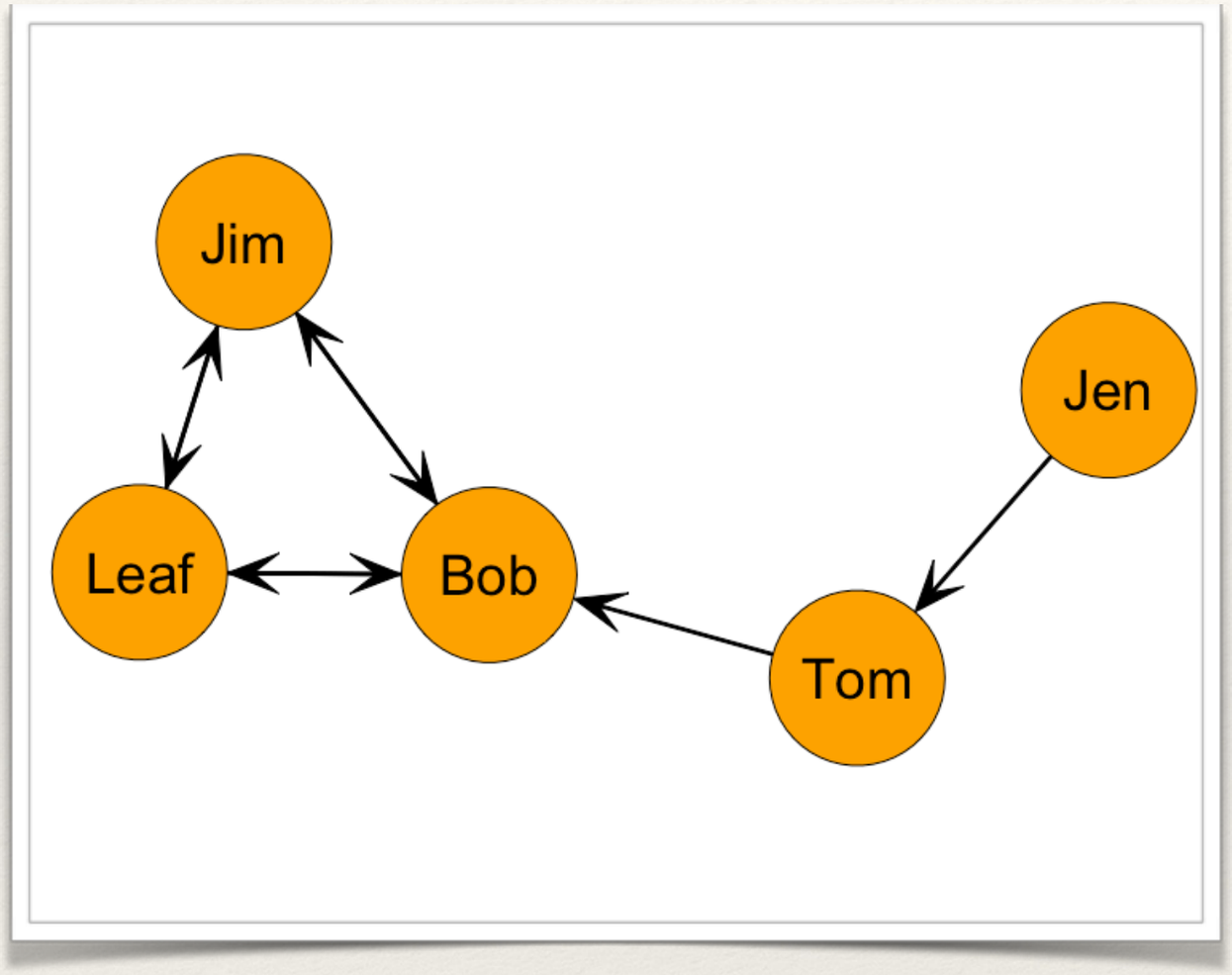
Example: Closeness Centrality for Directed Binary Network



Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen						
Tom						
Bob	Inf	Inf		1	1	2
Leaf						
Jim						

*What is **Bob's** closeness?*

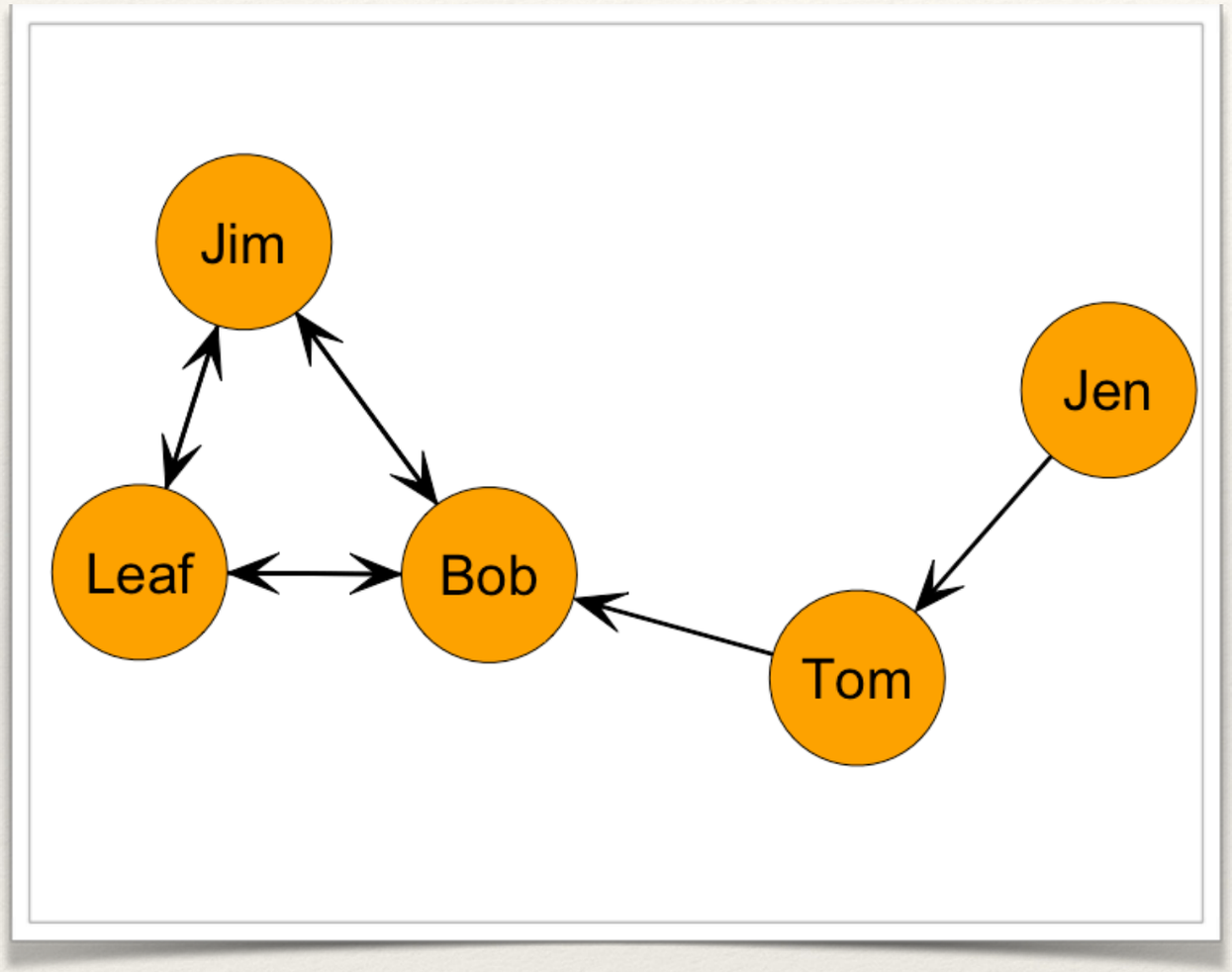
Example: Closeness Centrality for Directed Binary Network



Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen						
Tom						
Bob	Inf	Inf		1	1	2
Leaf						
Jim						

Take the inverse and ***Bob's*** *score is:* $1/2 = 0.5$

Example: Closeness Centrality for Directed Binary Network

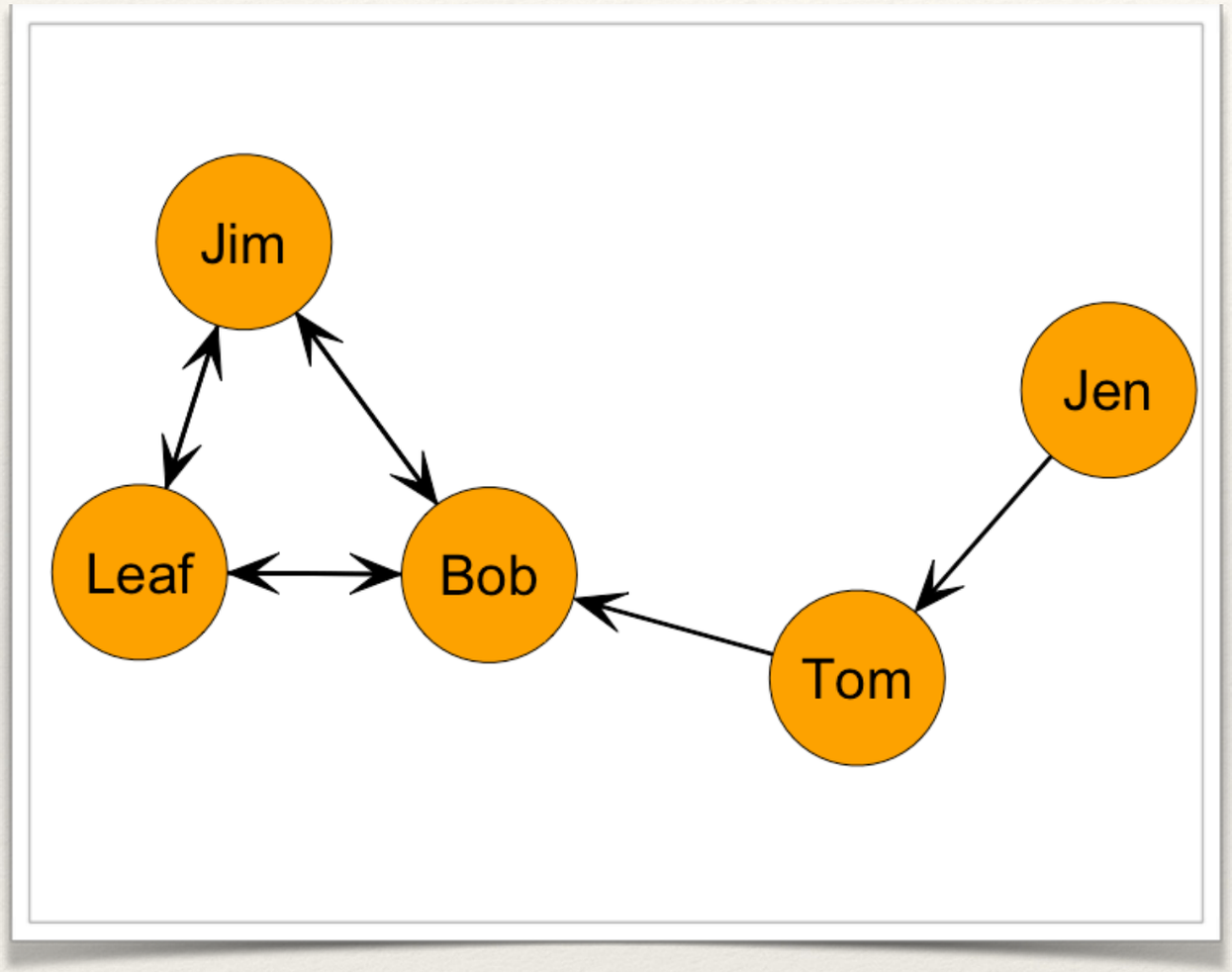


Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen						
Tom						
Bob	Inf	Inf		1	1	2
Leaf						
Jim						

*Then, **Bob's** standardized score is*

$(1/2)*(g-1) = 0.5*4 = 0.2$

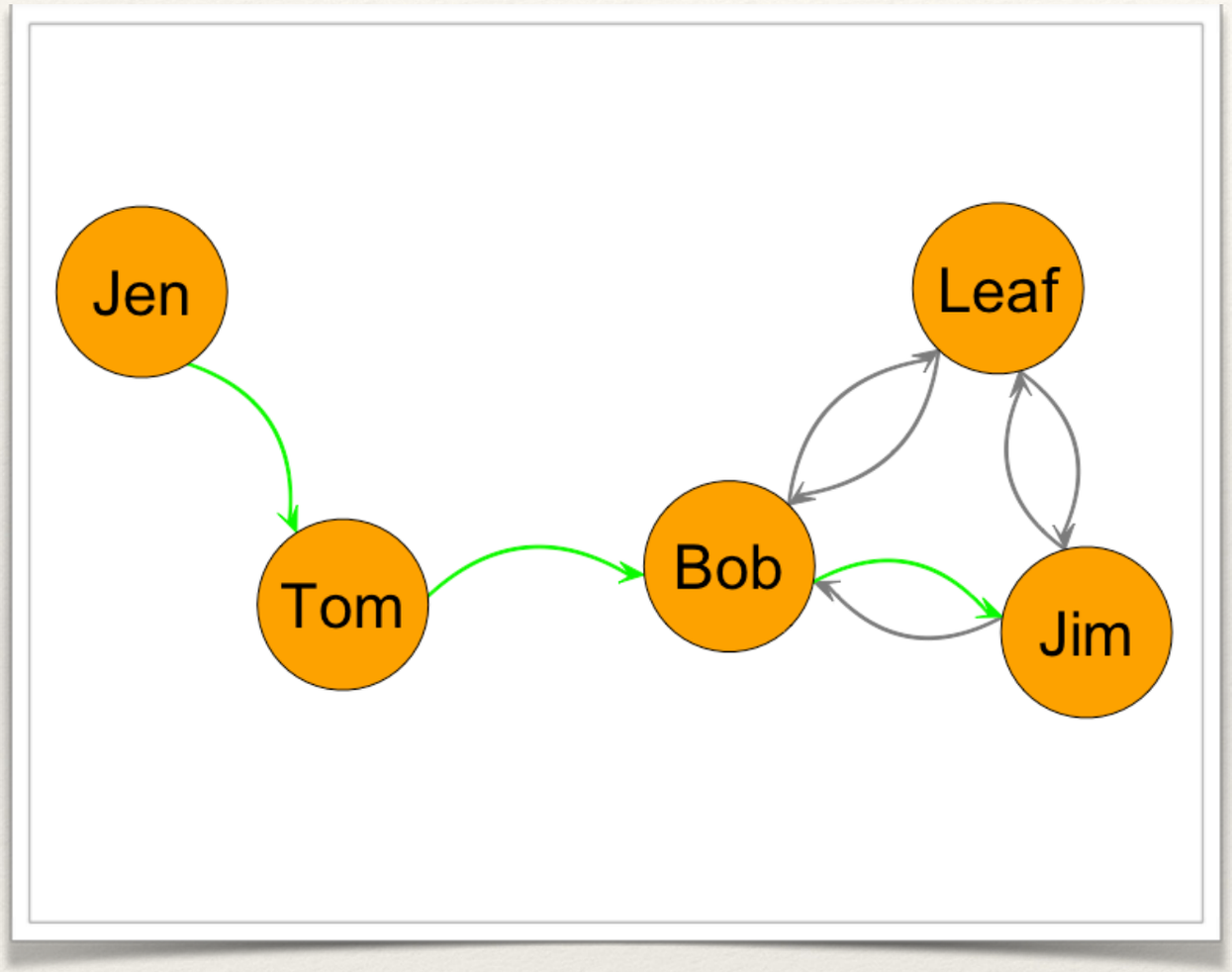
Example: Closeness Centrality for Directed Binary Network



Distance Matrix					
	Jen	Tom	Bob	Leaf	Jim
Jen					
Tom					
Bob					
Leaf					
Jim					

*The **column** sums will reflect the **outdegree,** and would tell you have close <u>others</u> are to you.*

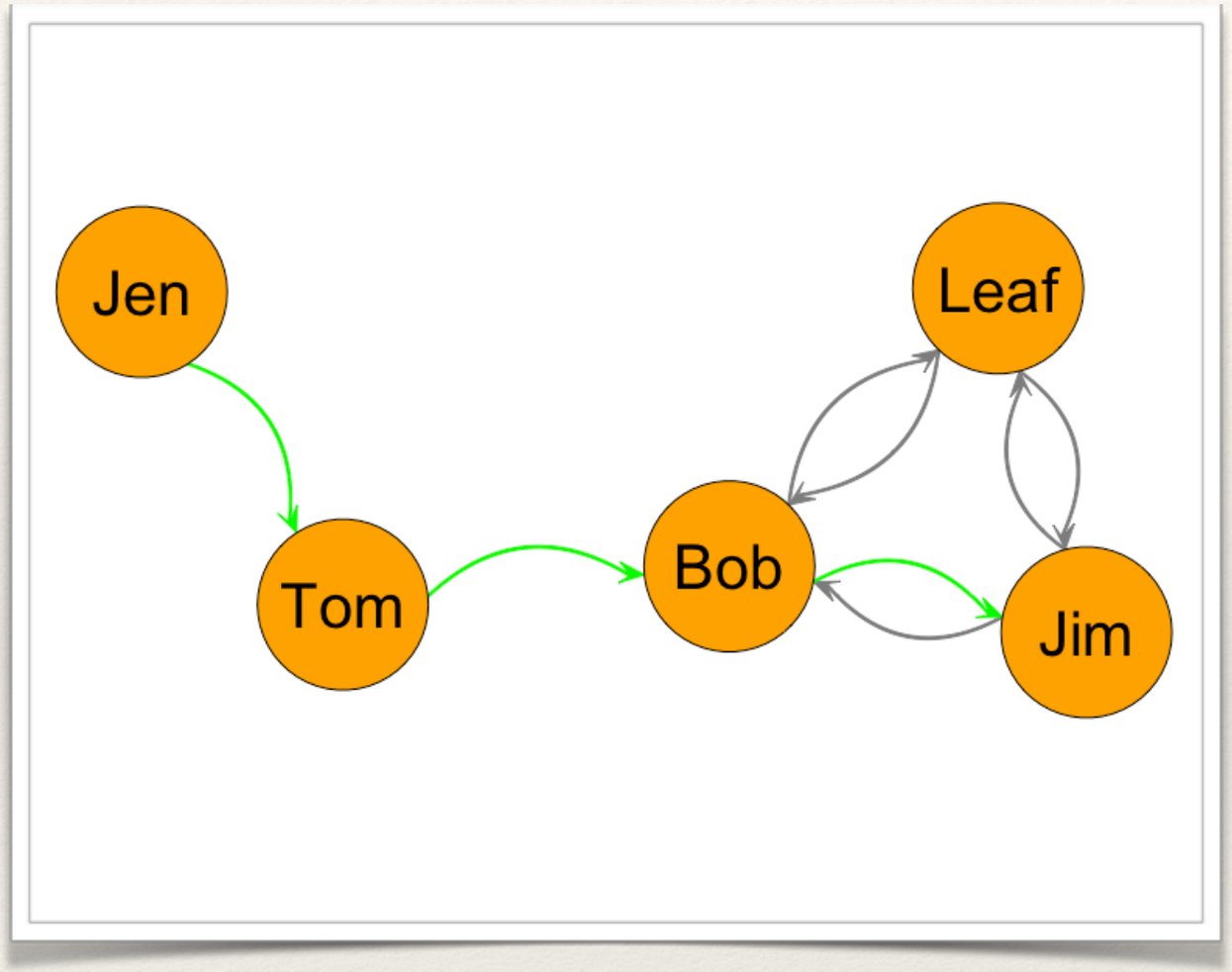
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen				/?
Tom				
Leaf				
Jim				

Betweenness would be the same: look at the the sending behavior.

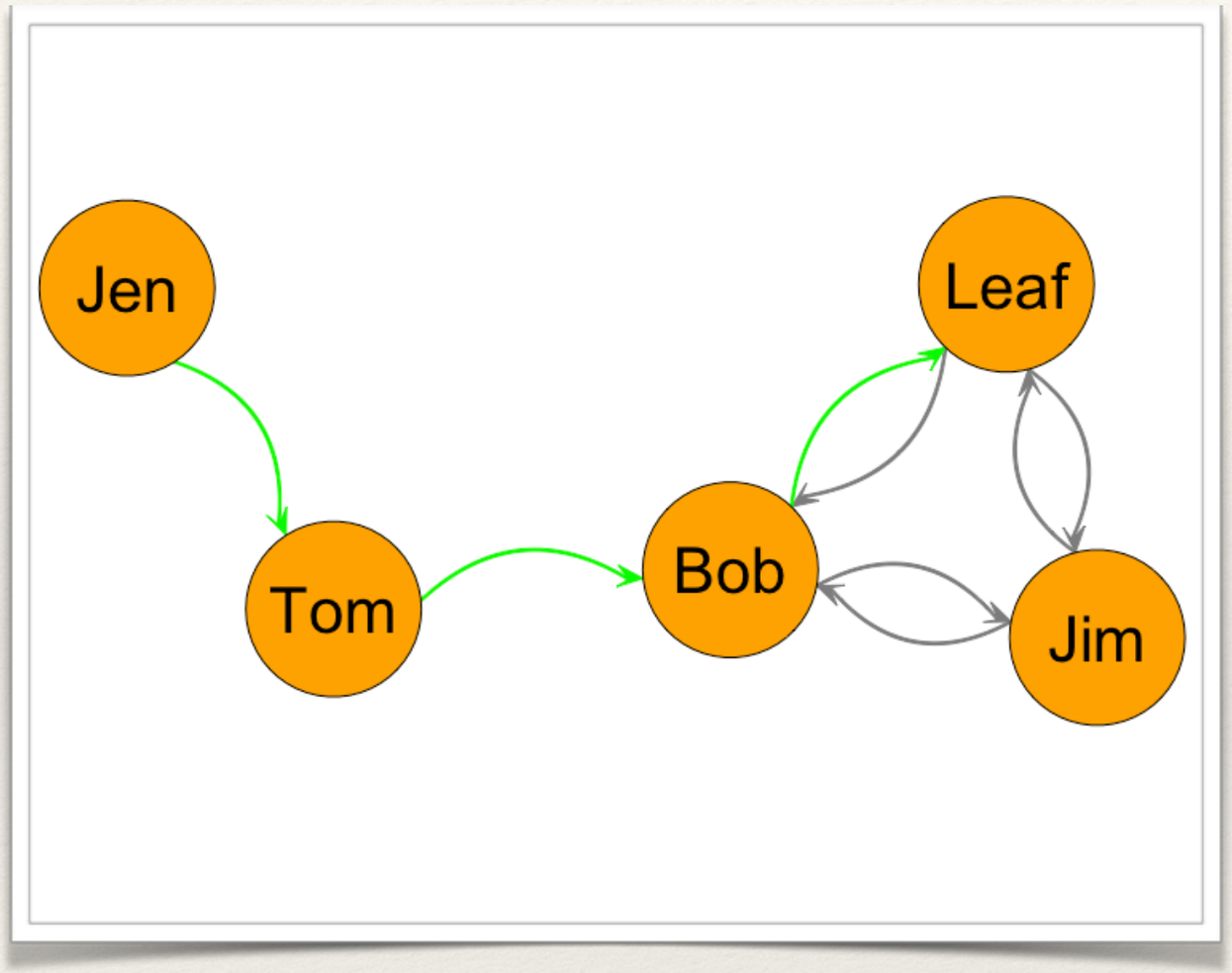
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen				1/1
Tom				
Leaf				
Jim				

Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen			?	1/1
Tom				
Leaf				
Jim				

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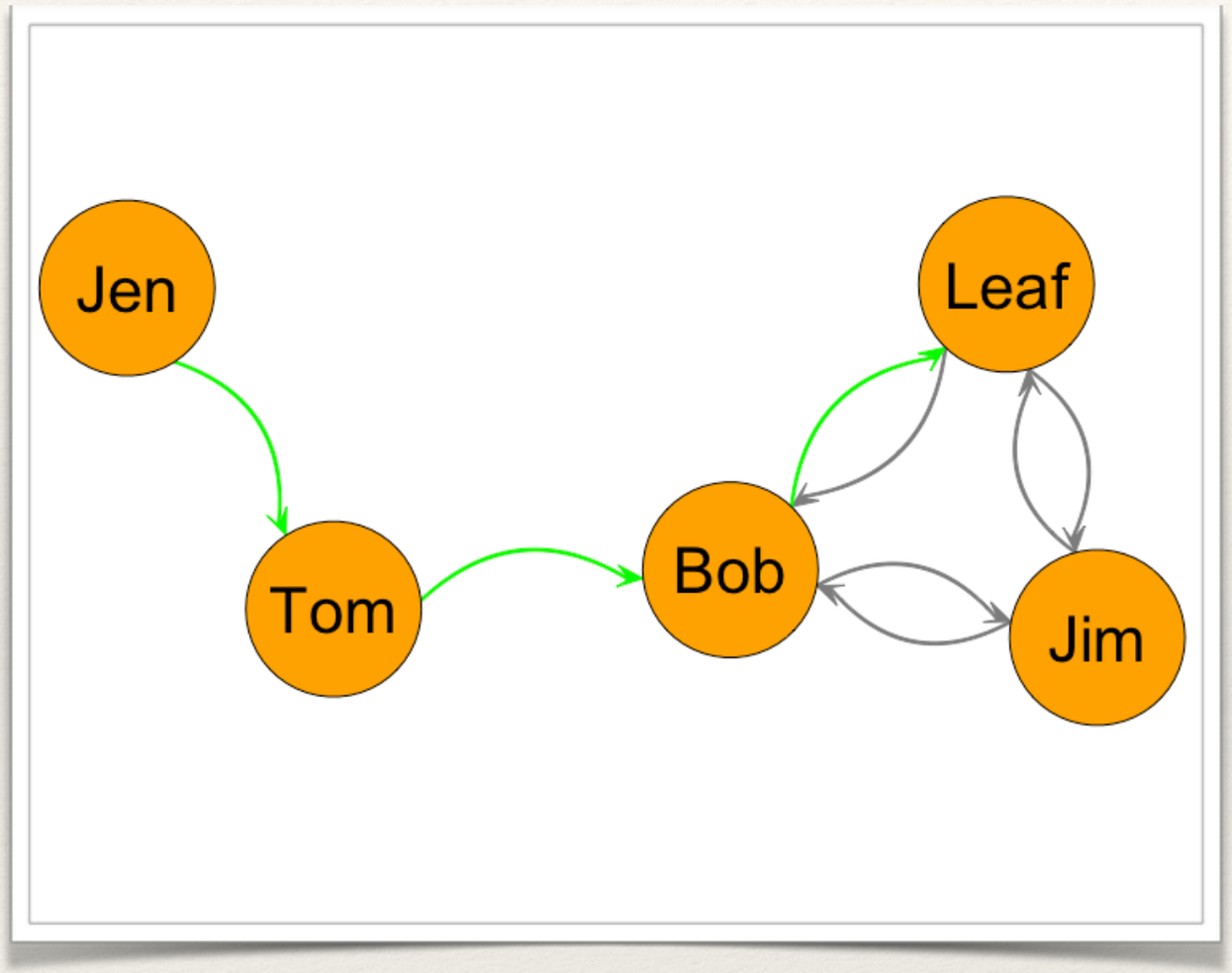

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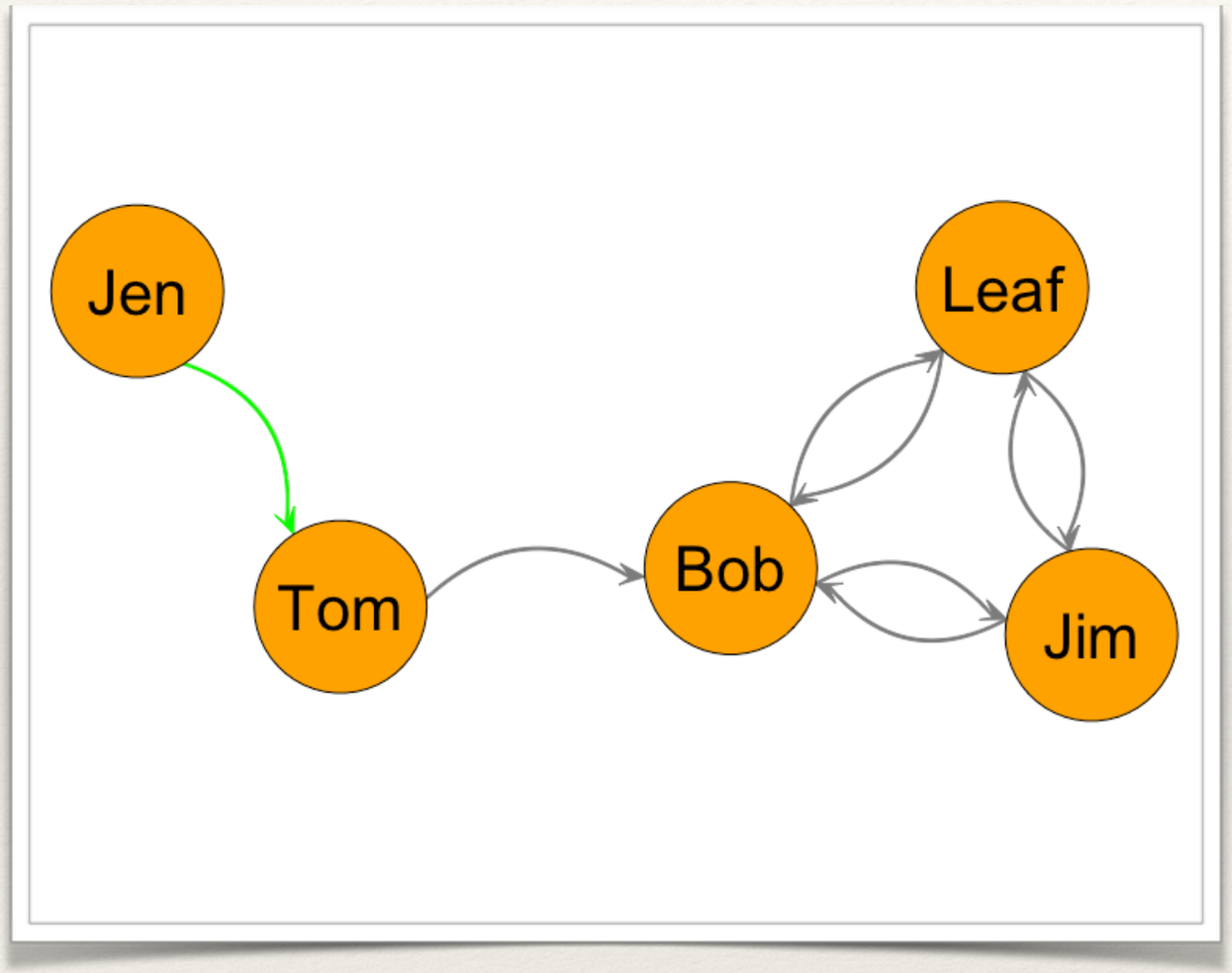
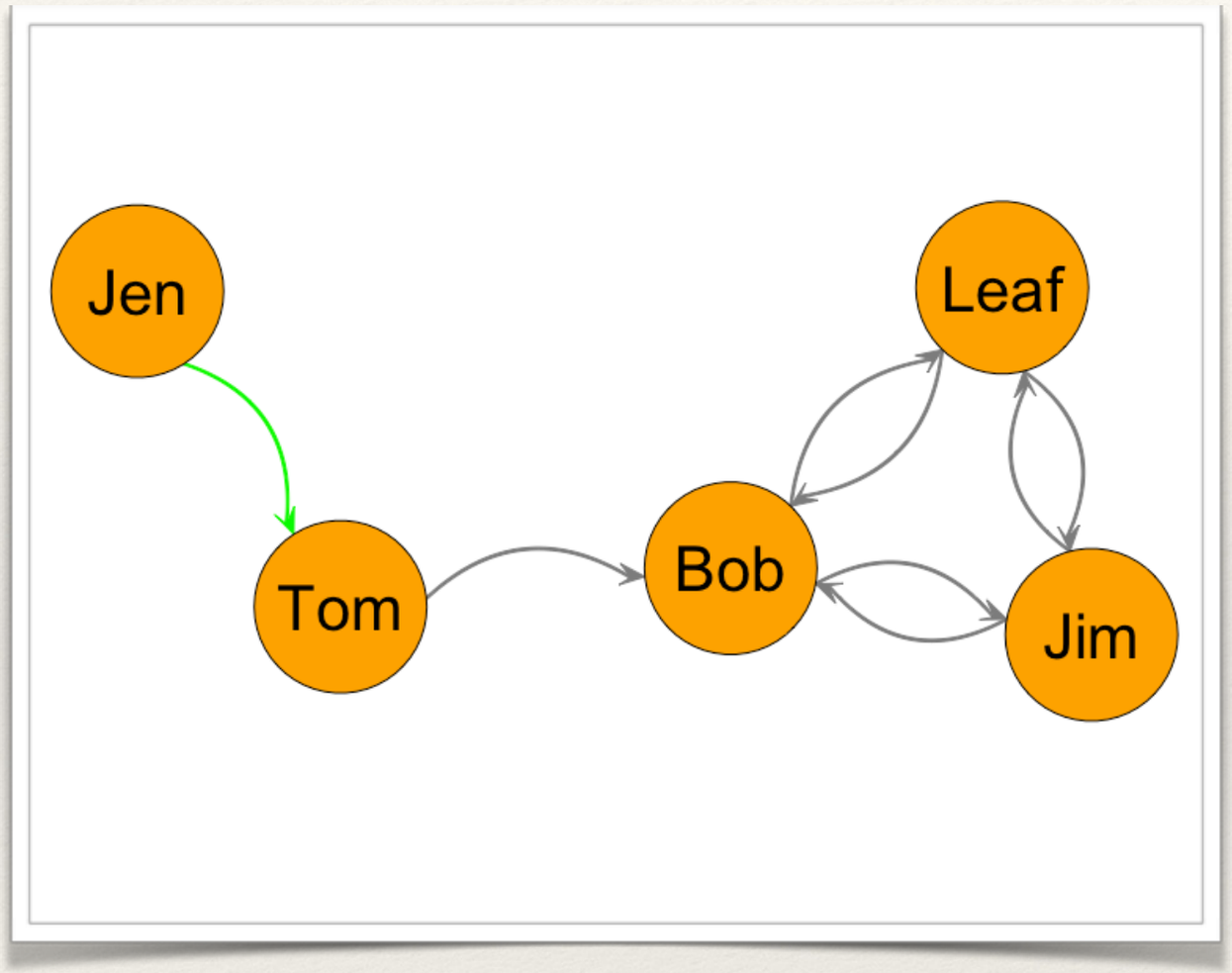


Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen		?	1/1	1/1
Tom				
Leaf				
Jim				

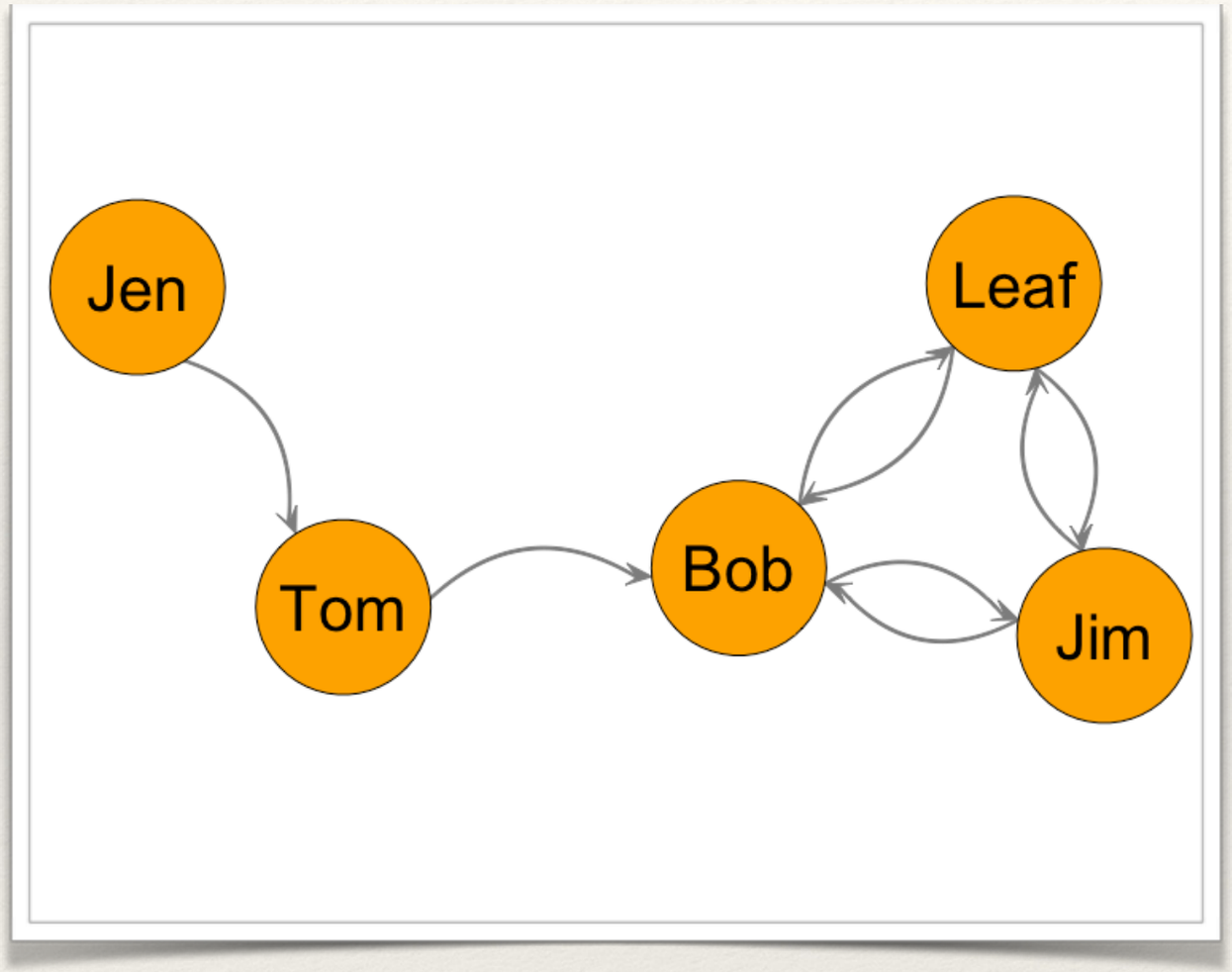
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom				
Leaf				
Jim				

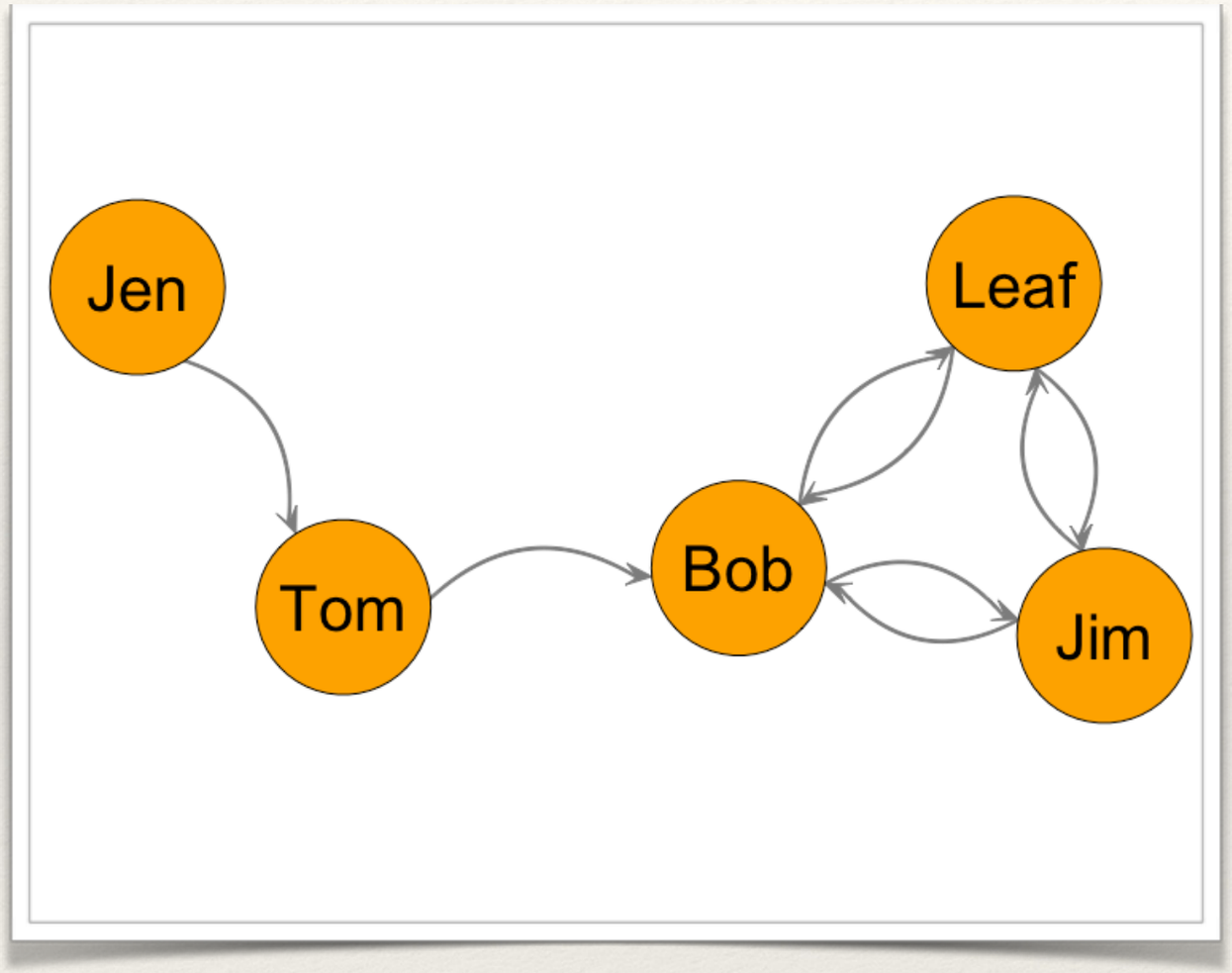
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

Then, we just complete the matrix.

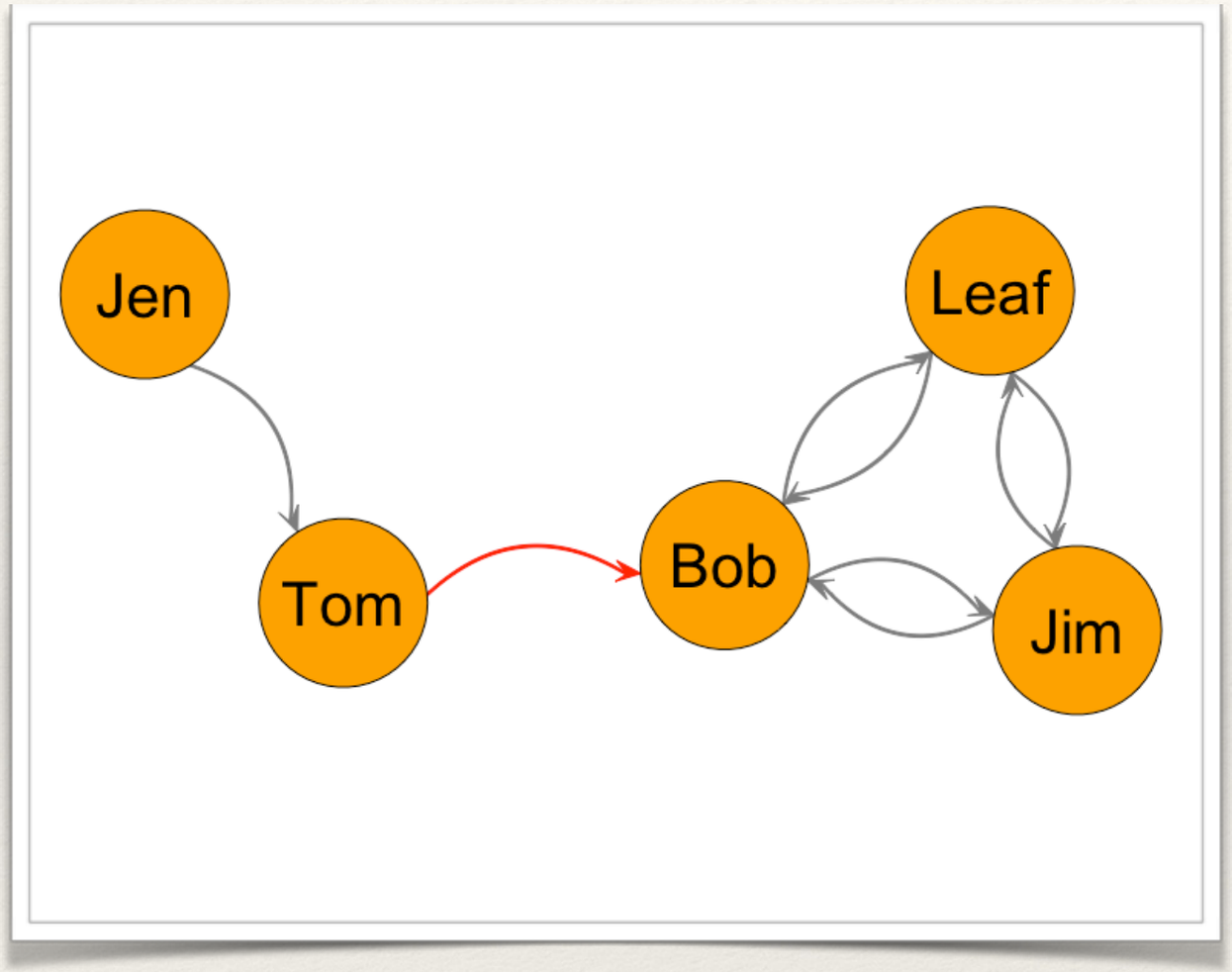
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*				
	Jen	*Tom*	*Leaf*	*Jim*
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

Why no geodesics?

Example: Betweenness Centrality for Directed Binary Network



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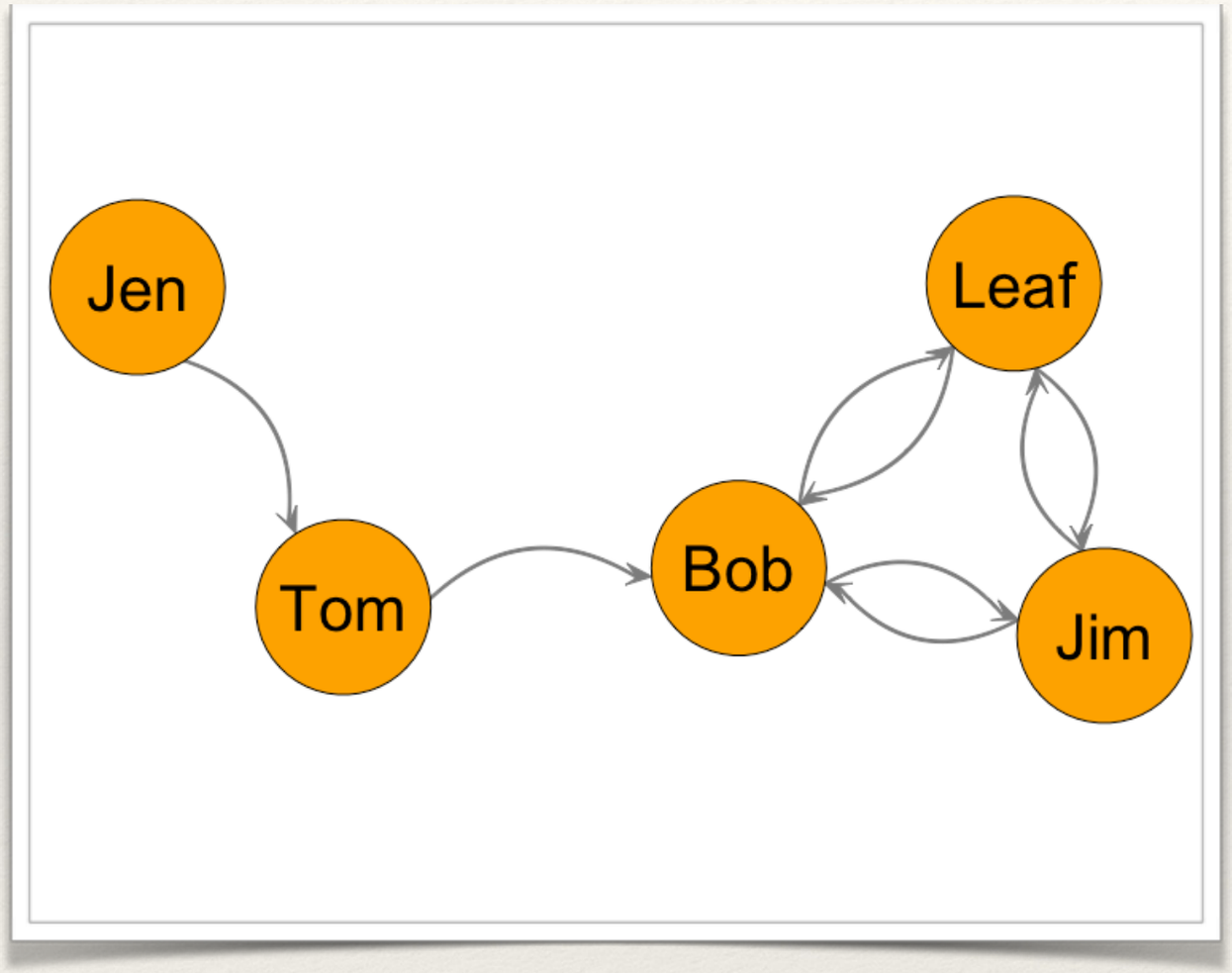
Jim	0/0	0/0	0/1	

Tom and **Jen** cannot be reached past **Bob**, because there are no outgoing ties from **Bob**.

Example: Betweenness Centrality for Directed Binary Network



Unstandardized (raw) for **Bob**: 4

Standardized for **Bob**:
4 / [(5-1)(5-2)/2] = 0.667

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	Jen	*Tom*	*Leaf*	*Jim*
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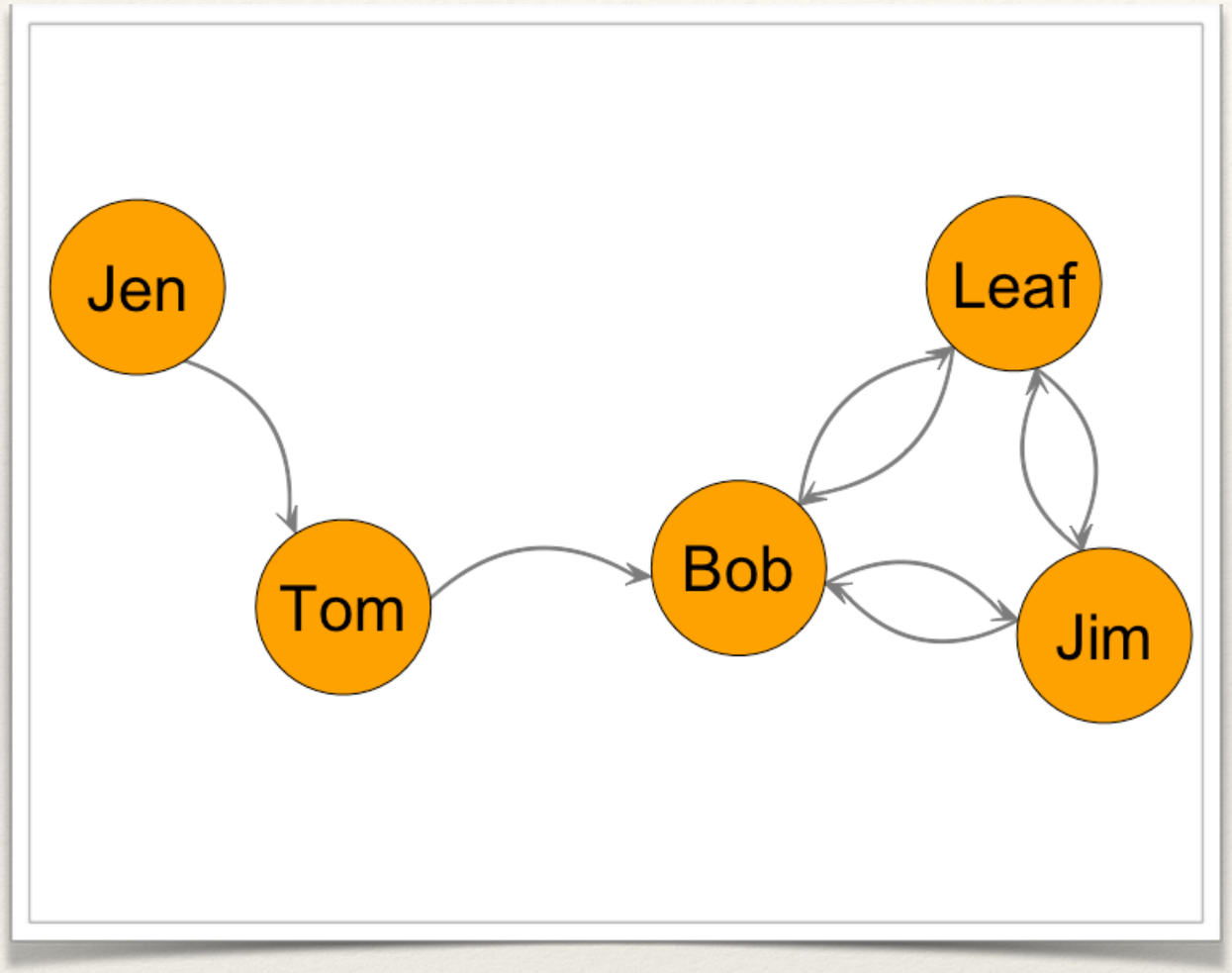
Example: Betweenness Centrality for Directed Binary Network



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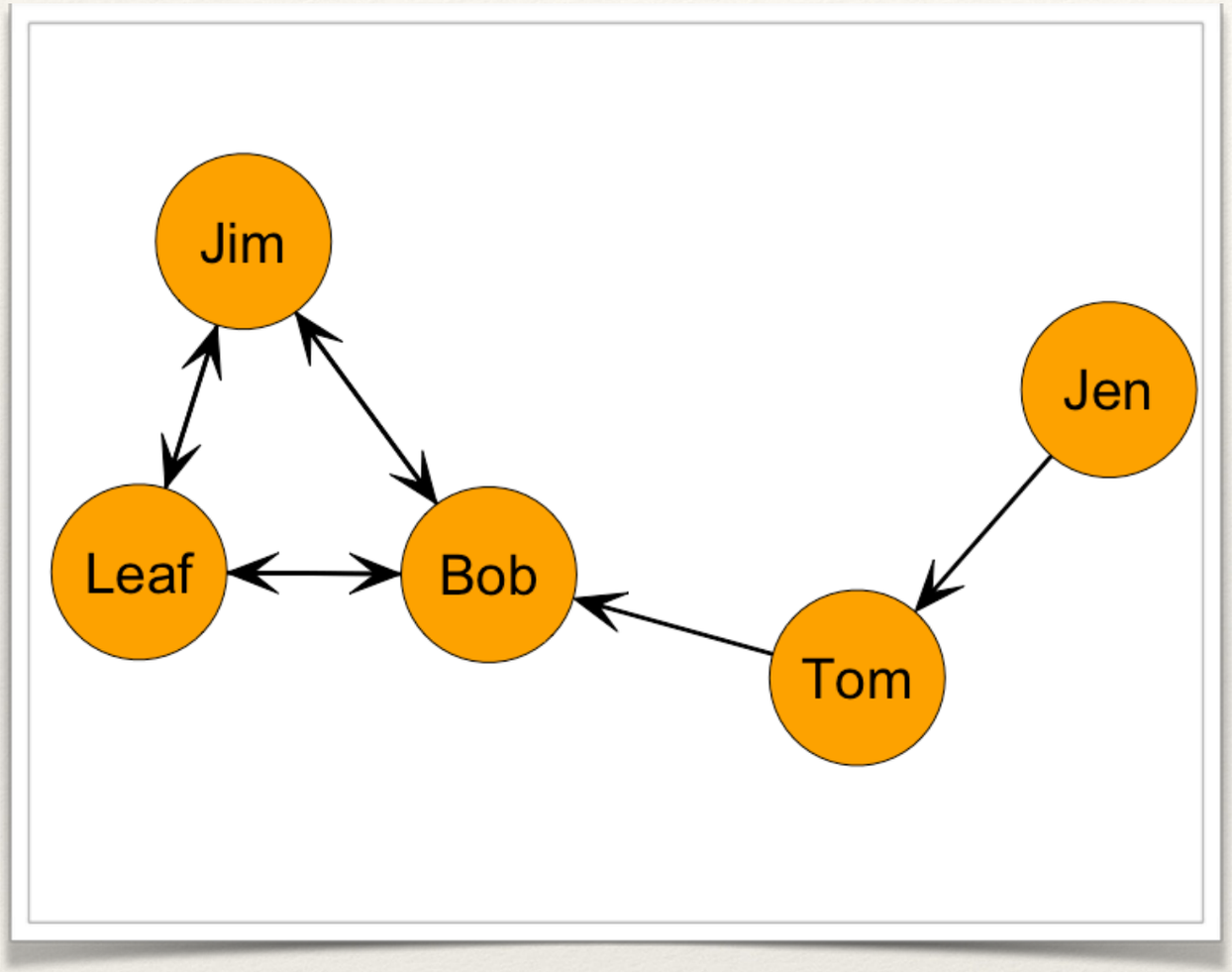
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

Example: Undirected, Binary Network



Compare the centralization scores:

Degree = 0.375

Closeness = 0.555

Betweenness = 0.270

What can we say about the differences in the centralization scores for each type of centrality?

Comparing Measures of Centrality

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important/active alters		Probably multiple paths in the network, ego is near many people, but so are others
High Betweenness	Ego's few ties are crucial for network flow	Very rare. Would mean that ego monopolizes the ties from a small number of people to many others	

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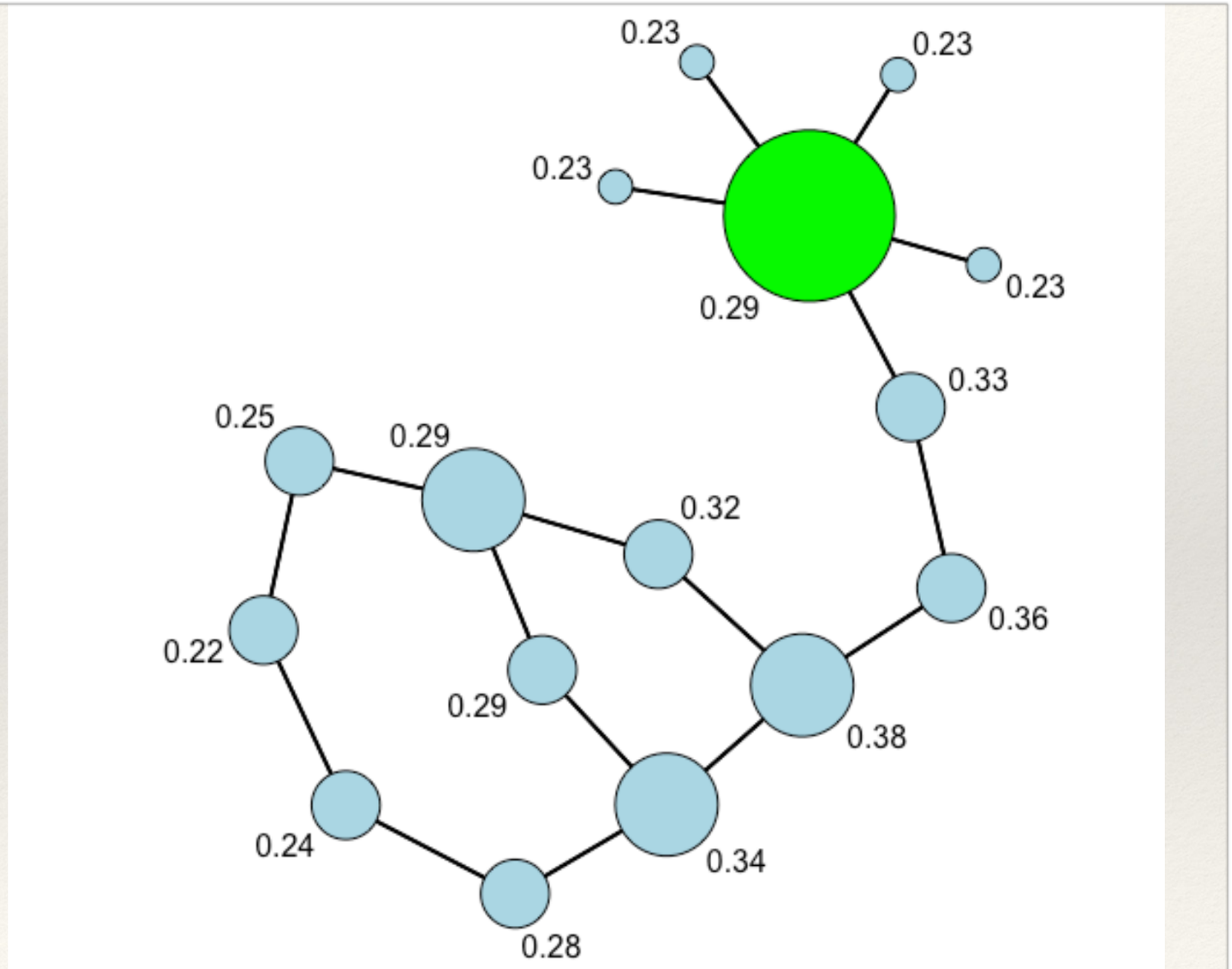
High Degree, Low Closeness



Nodes sized by degree

Nodes labeled by closeness centrality

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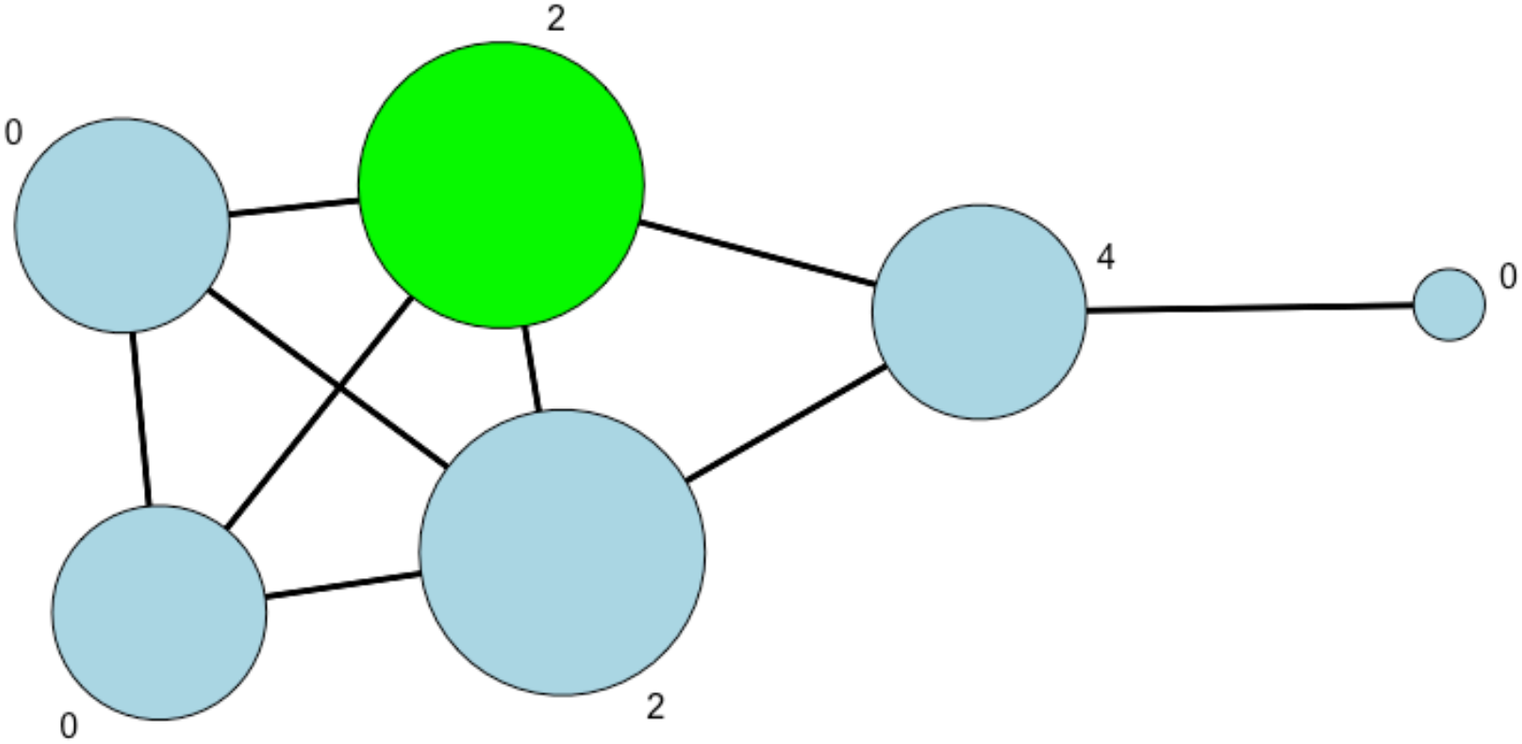

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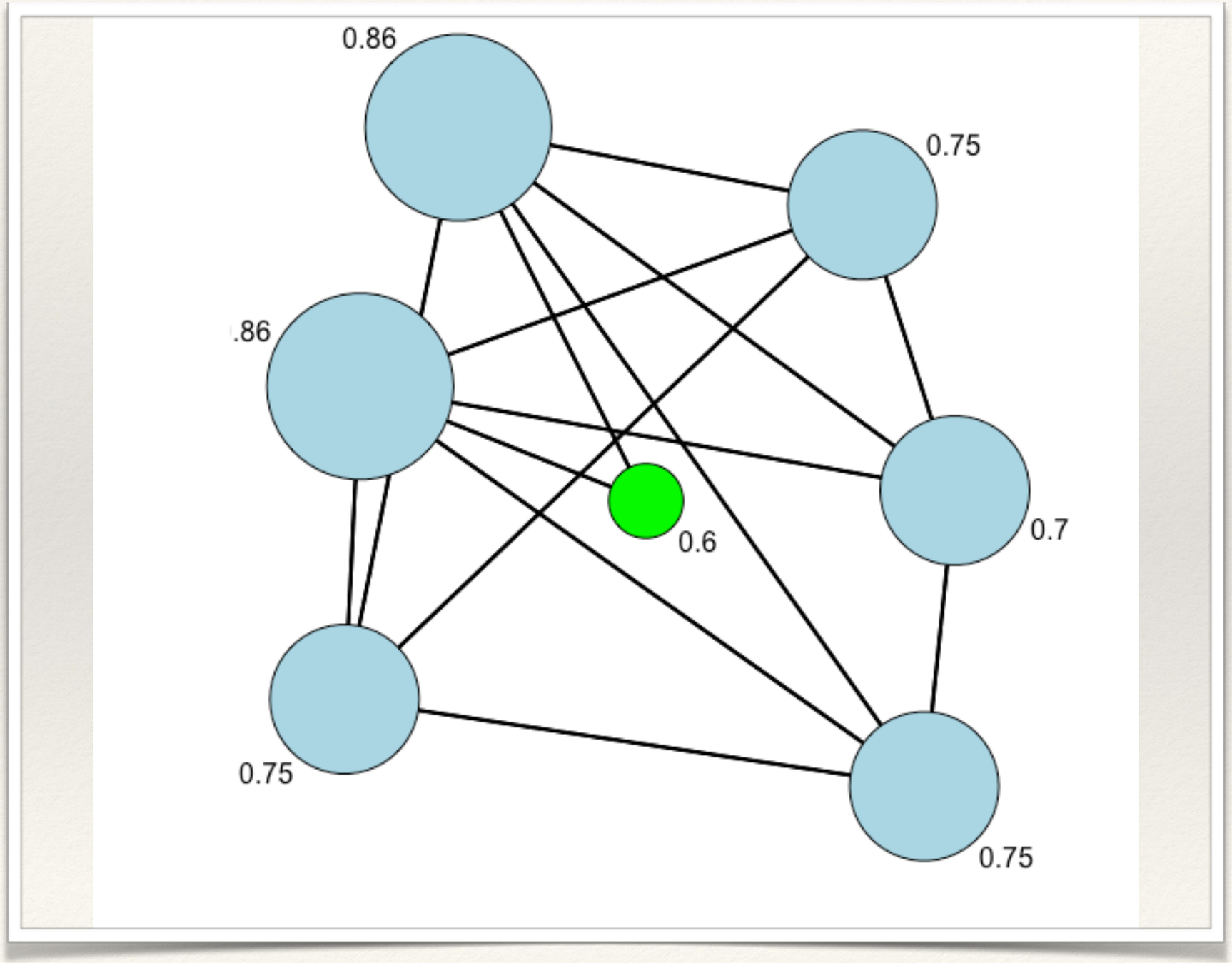
High Closeness, Low Degree



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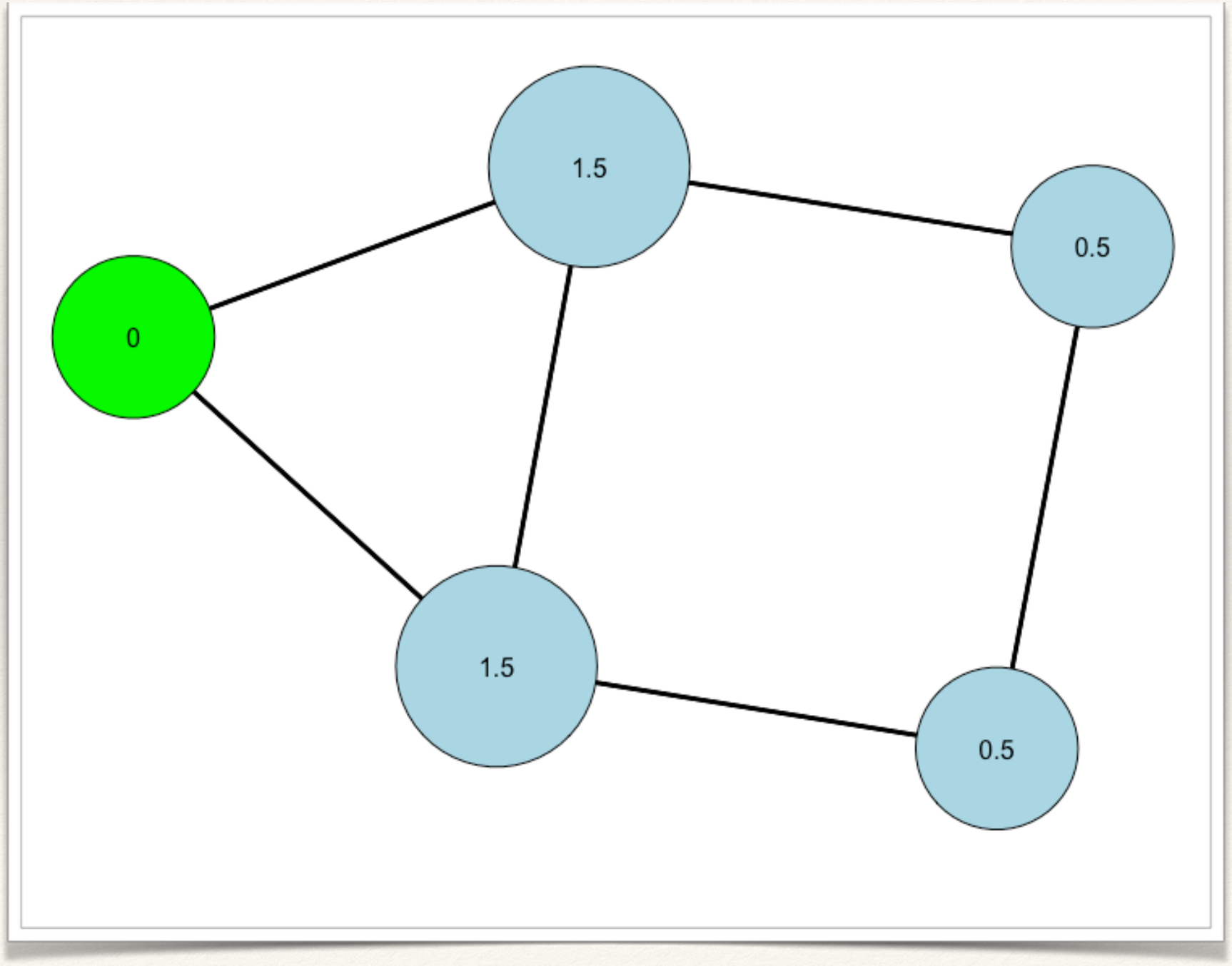
High Closeness, Low Betweenness



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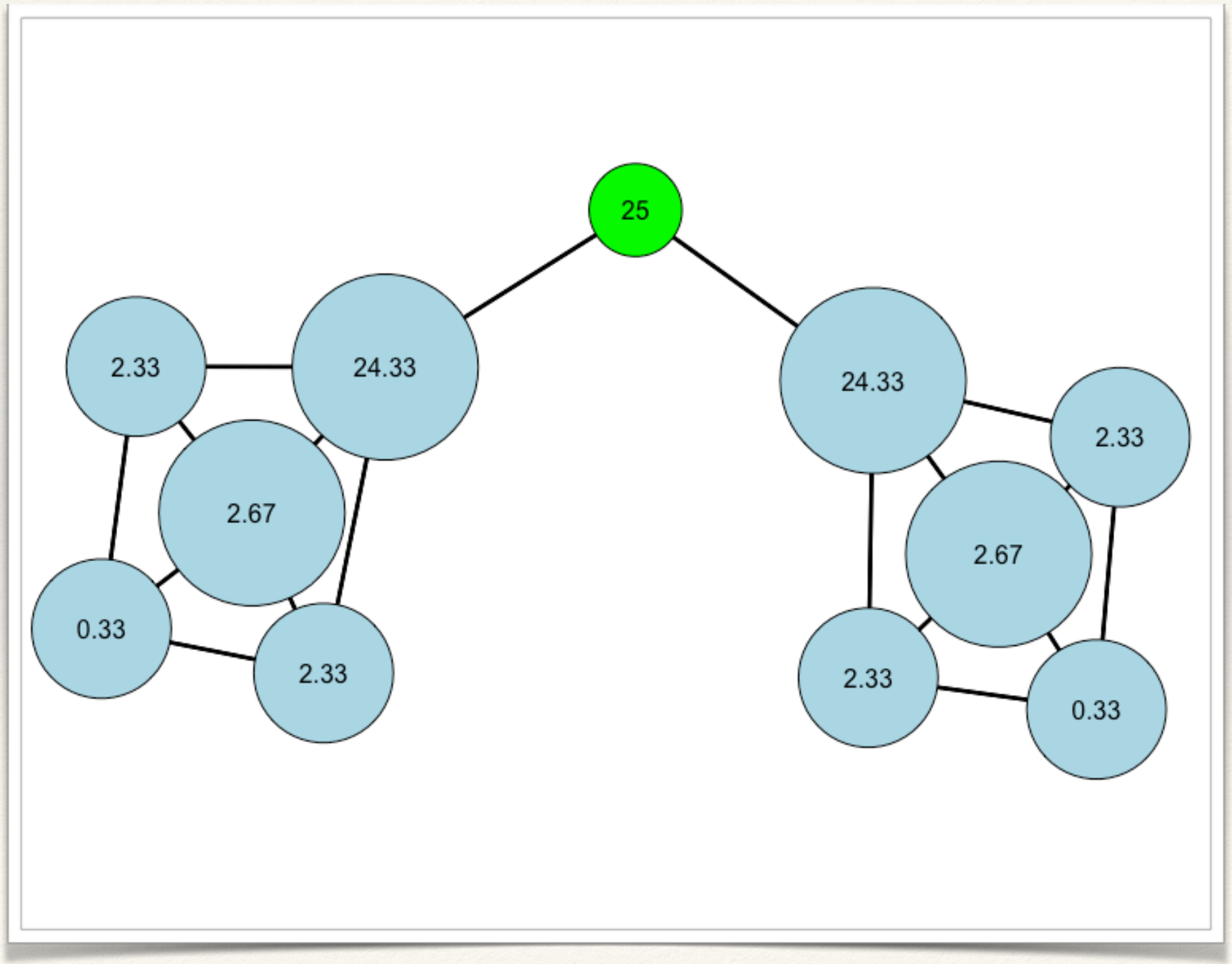
High Betweenness, Low Degree



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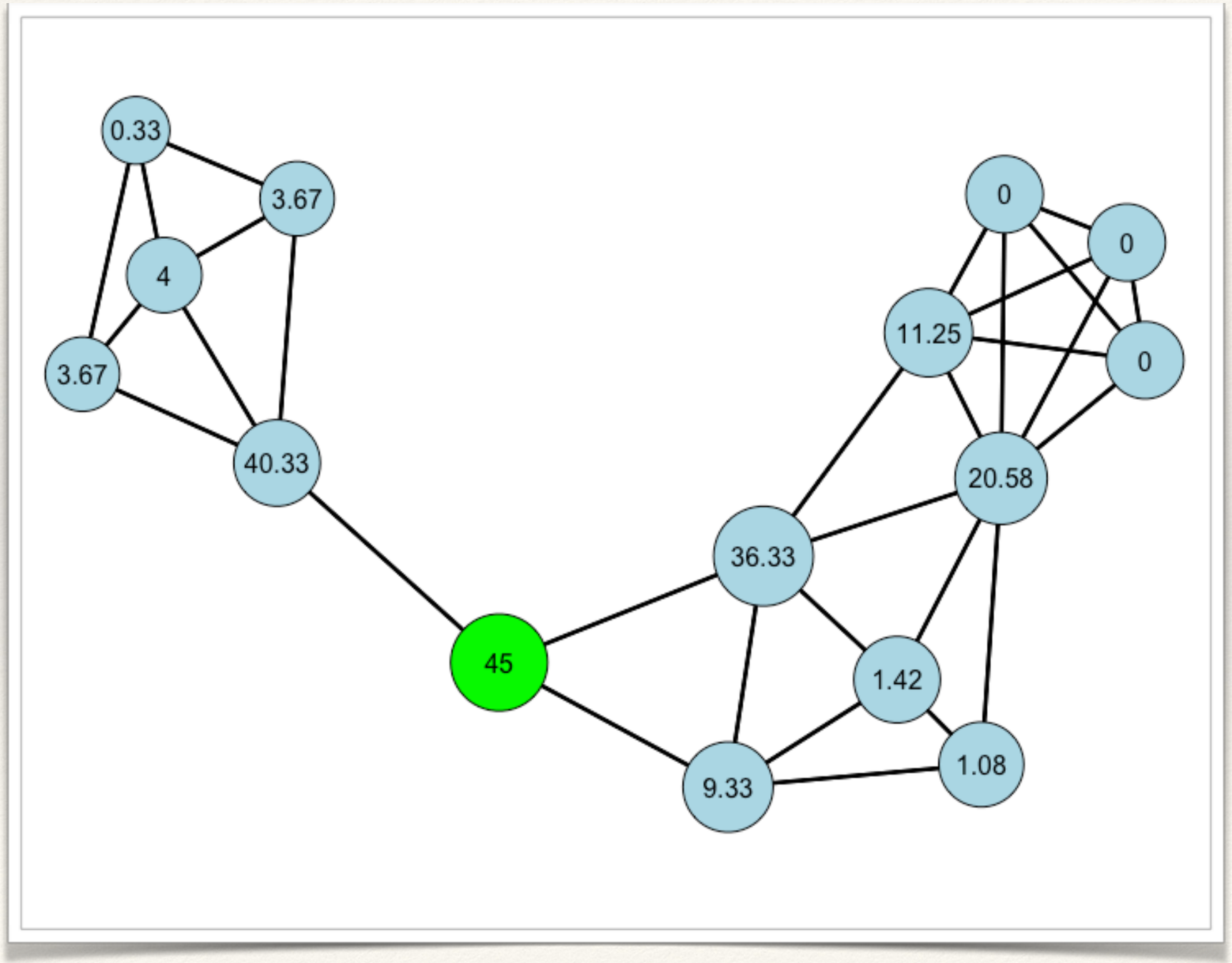

Nodes sized by closeness centrality

Nodes labeled by betweenness centrality

Learning Goals

- Revisit the conceptualization of "centrality".
- Understand calculation of *closeness* centrality and *betweenness* centrality.
- Compare different measures of centrality.

Questions?