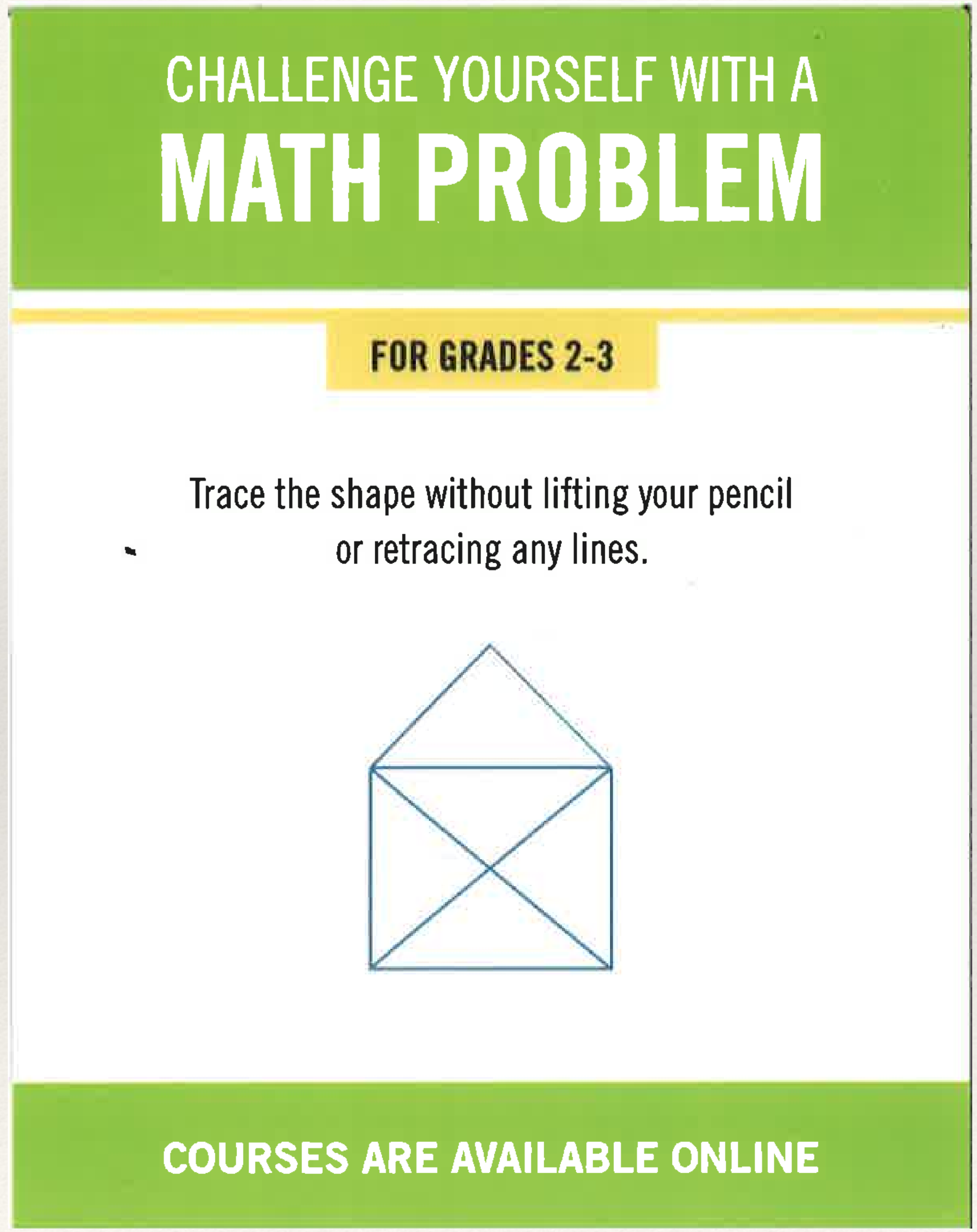
*Statistical Analysis of Networks*

# Projection & Weighted Graphs

# Challenge yourself with a problem



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# Motivating Example *Revisited*

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**Diffusion of Ideas and Technology: The Role of Networks in Influencing the Endorsement and Use of On-Officer Video Cameras**

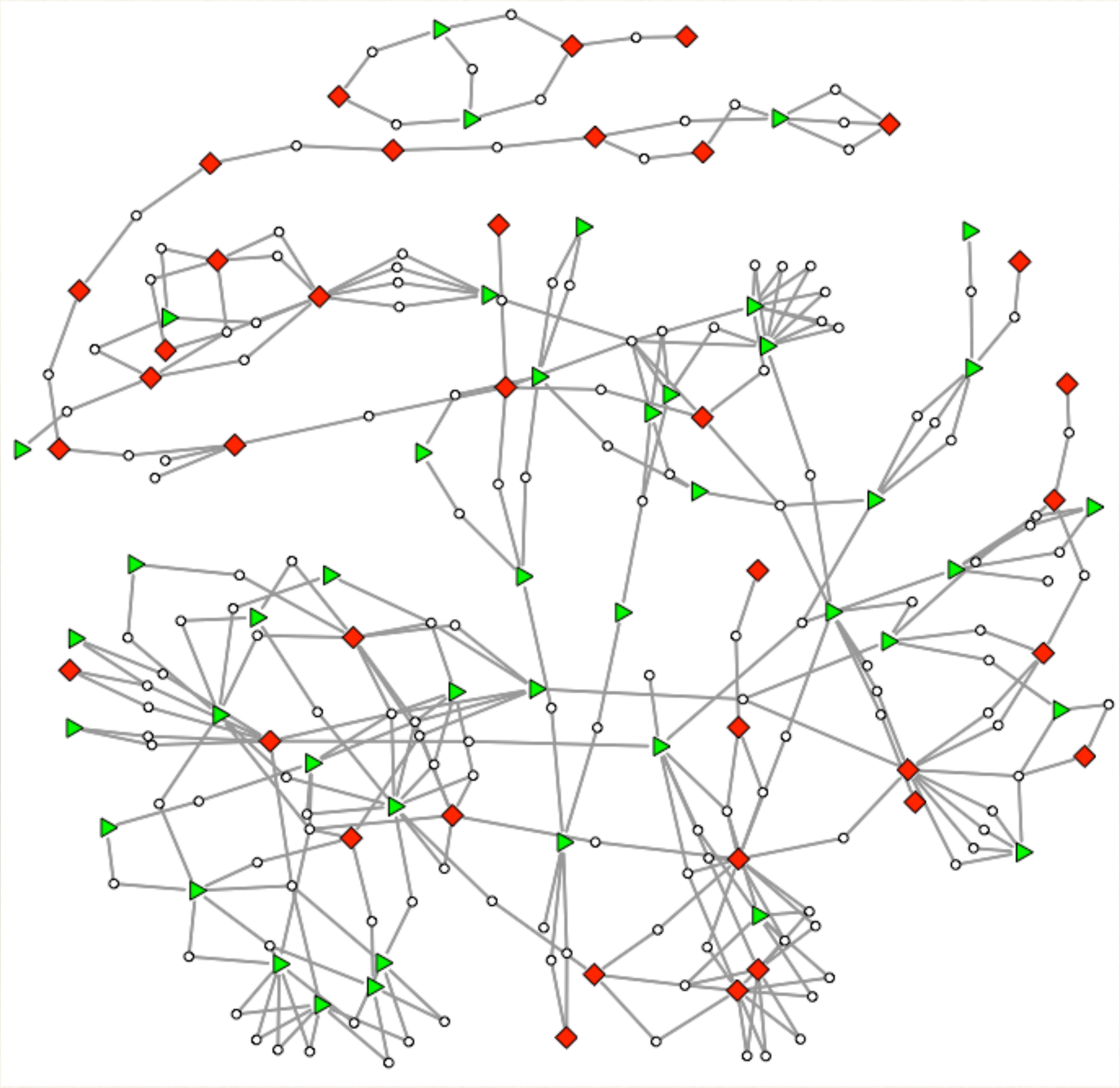
Jacob T. N. Young[1] and Justin T. Ready[1]

- Questions:
  - How do police officers "frame" body-worn cameras?
  - Is the meaning officers attribute to cameras created and transmitted in groups?

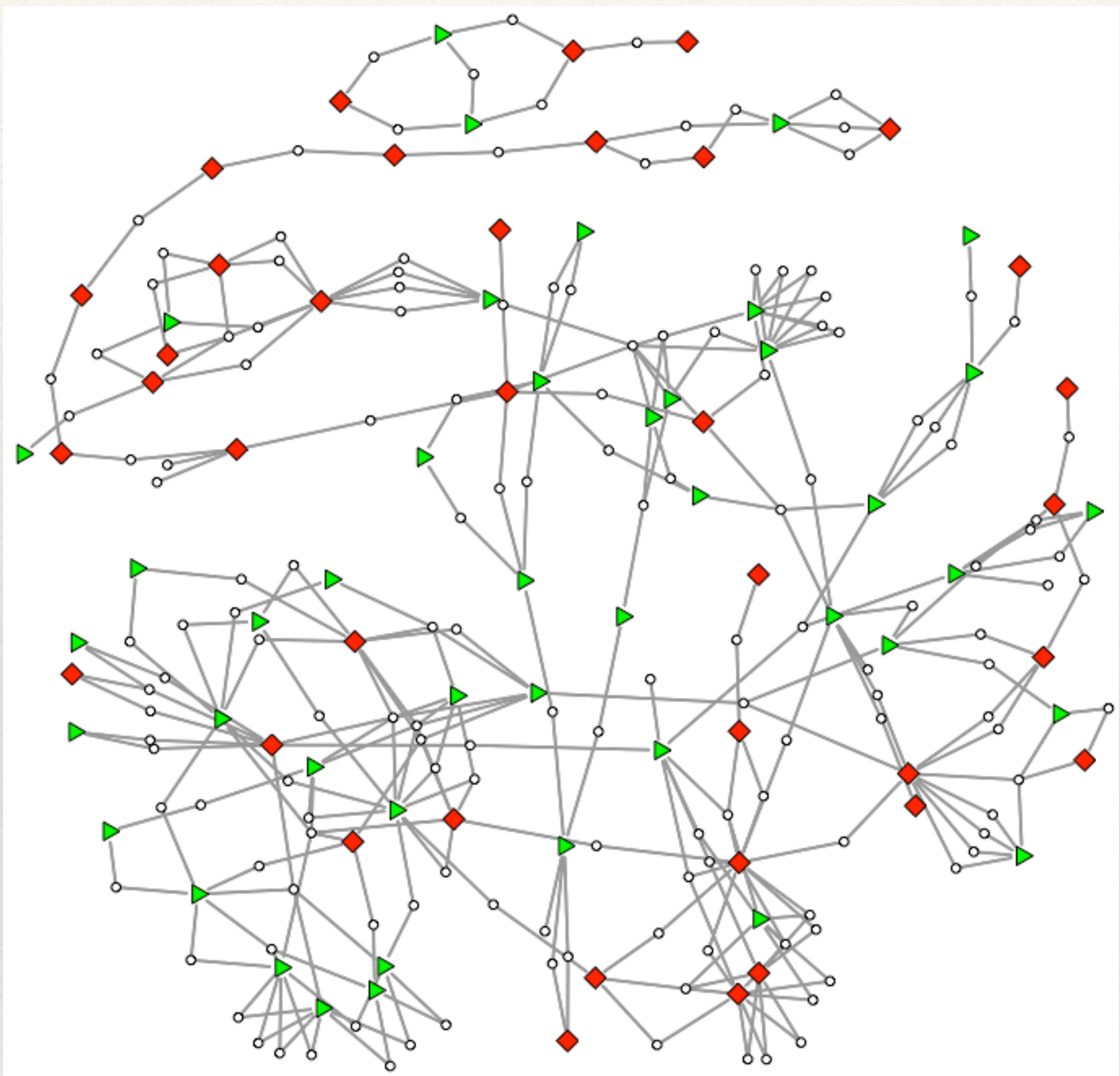
What is the **concept** of interest?

How is it **conceptualized**?

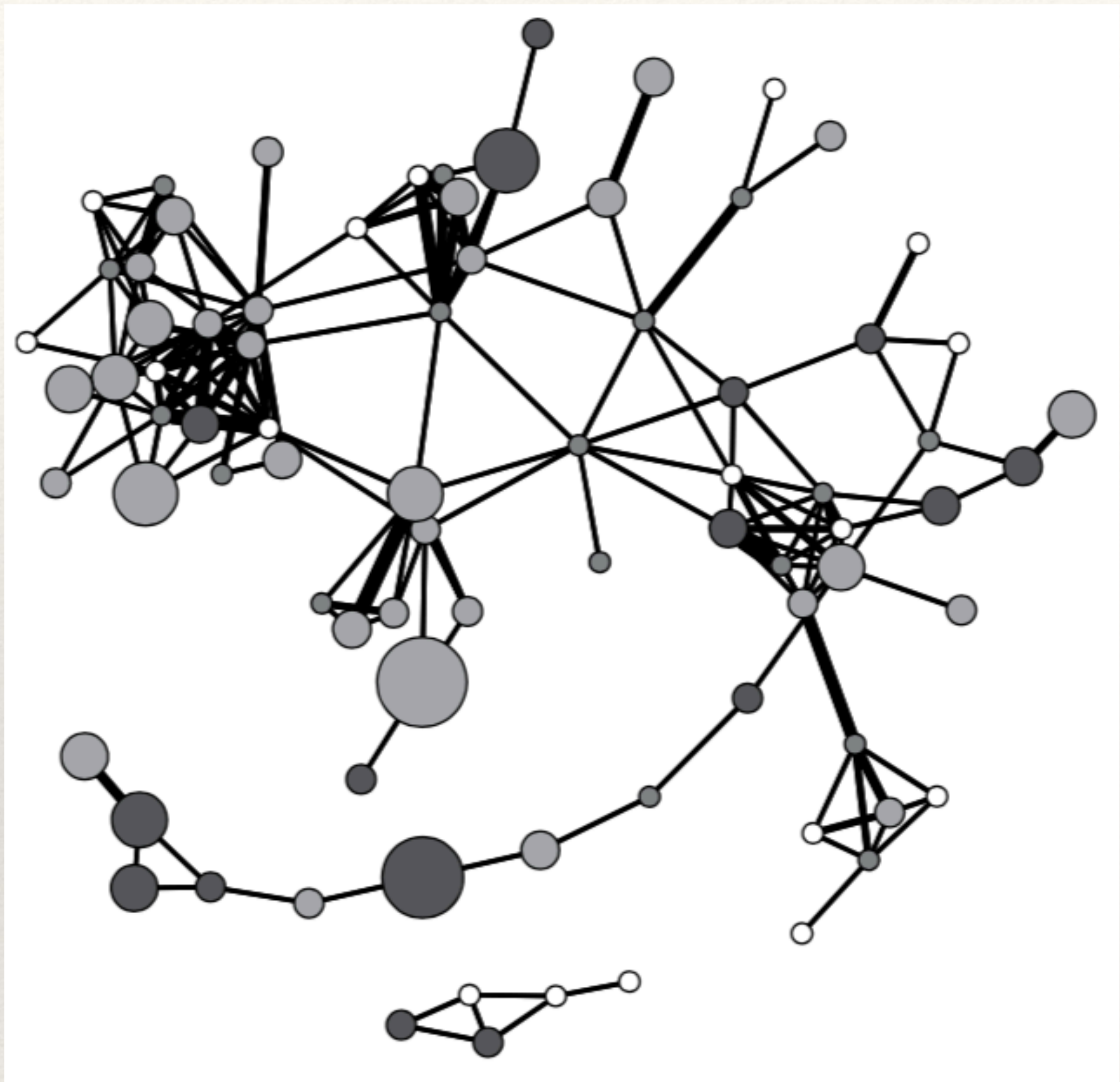
How is it **operationalized**?



Two-mode network of officers connected by incidents



One-mode network of officers connected to officers



*What do the connections represent in this network?*

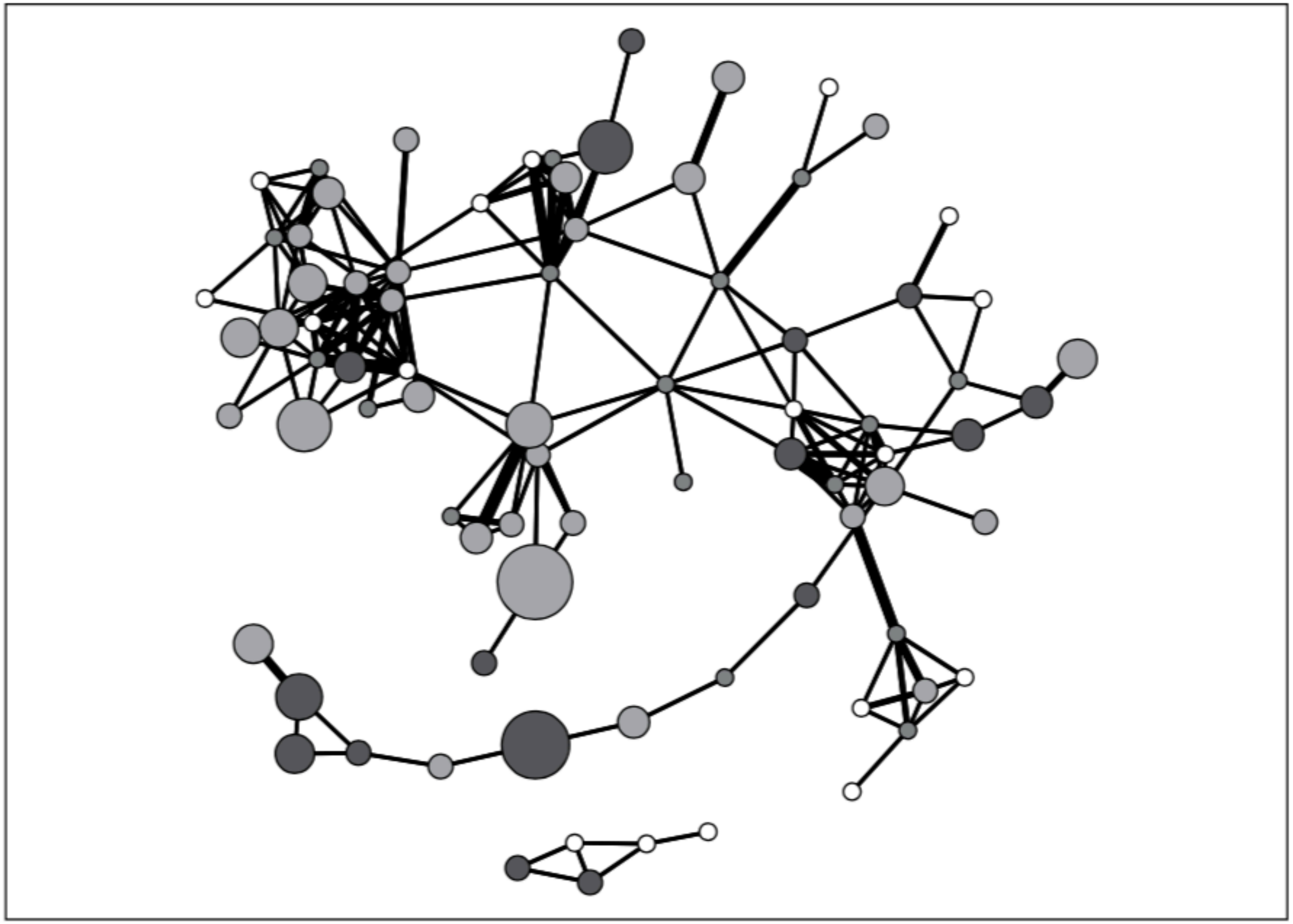


**Figure 5.** One-mode network of officers.

*Note.* Node size is proportional to change in legitimacy: darker = more negative; white = no change. Line size is proportional to number of shared incidents.

Findings: Officers views of cameras changed based on who they interacted with through the network

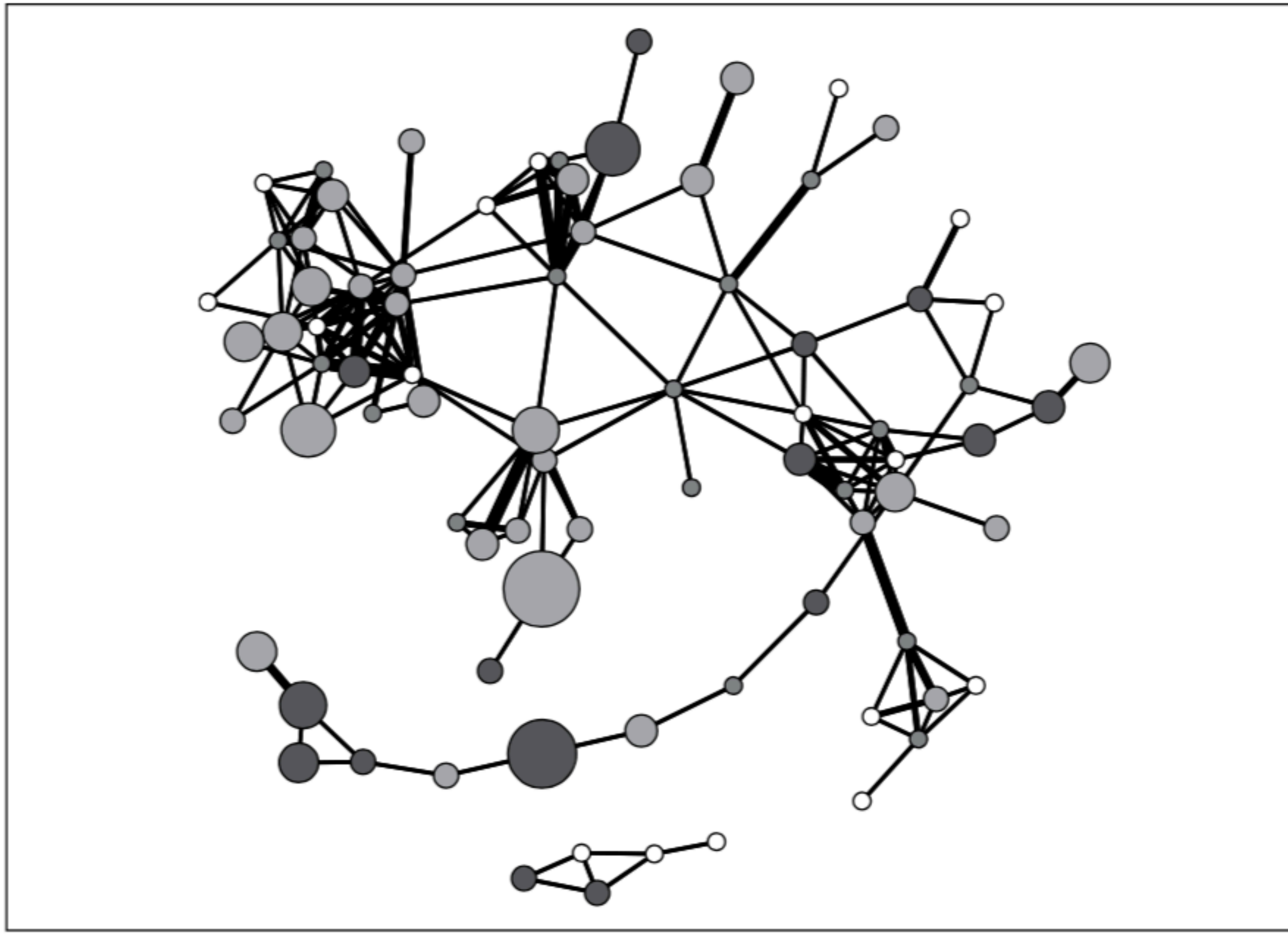


**Figure 5.** One-mode network of officers.

*Note.* Node size is proportional to change in legitimacy: darker = more negative; white = no change. Line size is proportional to number of shared incidents.

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# Projection & Weighted Graphs

# Learning Goals

- At the end of the lecture, you should be able to answer these questions:
  - How can we create **unipartite** graphs from **bipartite** graphs?
  - What is the difference between **dichotomized** projections and **summation** projections?

# Projection

- The process by which we map the connectivity between modes to a single mode.
  - Example
    - Two-mode network is people in groups.
      - By projecting, we get:
        - One-mode network of people connected to people *by* groups.
        - One-mode network of groups connected *by* people.

# Projection

- Breiger (1974)
  - We can build the adjacency matrix for each projected network through matrix algebra.
    - Specifically, multiplying an adjacency matrix by it’s **transpose**.
      - The transpose of a matrix simply reverses the columns and rows:
        - $A^T_{ij} = A_{ji}$

# Projection

- Breiger (1974)
  - The two-mode, *NxM*, adjacency matrix, when multiplied by it's **transpose**, produces either:
    - An *MxM* matrix (ties among *M* nodes via *N*).
    - An *NxN* matrix (ties among *N* nodes via *M*).

# Transposition

Matrix **A**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | 1 | 1 | 0 | 0 | 0 |
| B | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 0 | 0 |
| D | 0 | 1 | 1 | 1 | 1 |
| E | 0 | 0 | 1 | 0 | 0 |
| F | 0 | 0 | 1 | 0 | 1 |

order is 6x5

Matrix $\mathbf{A}^{\mathbf{T}}$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 |

order is 5x6

# Projection

- Matrix Multiplication Rules
  - To multiply two matrices, the number of columns in the first matrix must match the number of rows in the second matrix.
    - Example: 5x6 X 6x5 works, but not 5x6 X 5x6
  - The **product** matrix has the number of rows equal to the first matrix and the number of columns equal to the second matrix.
    - Example: 5x6 X 6x5 = 5x5

# Projection

- Product Matrix
  - The product matrix is the projected graph.
    - Recall that there are two:
      - **A** X **A**$^t$ (the "people" matrix **P**)
      - And the **A**$^t$ X **A** (the "group" matrix **G**)
        - *What does each one mean?*

# Matrix Multiplication

Matrix **A**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | 1 | 1 | 0 | 0 | 0 |
| B | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 0 | 0 |
| D | 0 | 1 | 1 | 1 | 1 |
| E | 0 | 0 | 1 | 0 | 0 |
| F | 0 | 0 | 1 | 0 | 1 |

order is 6x5

# Matrix Multiplication

Matrix **A**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
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| B | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 0 | 0 |
| D | 0 | 1 | 1 | 1 | 1 |
| E | 0 | 0 | 1 | 0 | 0 |
| F | 0 | 0 | 1 | 0 | 1 |

order is 6x5

X

Matrix $\mathbf{A}^{\mathbf{T}}$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 |

order is 5x6

# Matrix Multiplication

Matrix **A**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| A | 1 | 1 | 0 | 0 | 0 |
| B | 1 | 0 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 0 | 0 |
| D | 0 | 1 | 1 | 1 | 1 |
| E | 0 | 0 | 1 | 0 | 0 |
| F | 0 | 0 | 1 | 0 | 1 |

order is 6x5

X

Matrix $\mathbf{A}^{\mathbf{T}}$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 0 | 0 |
| 3 | 0 | 0 | 0 | 1 | 1 | 1 |
| 4 | 0 | 0 | 0 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 1 | 0 | 1 |

order is 5x6

The product matrix is
6x6

# Projection by Multiplication

$$\mathbf{A} \times \mathbf{A}^\mathbf{T} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

6x5 X 5x6 = 6x6

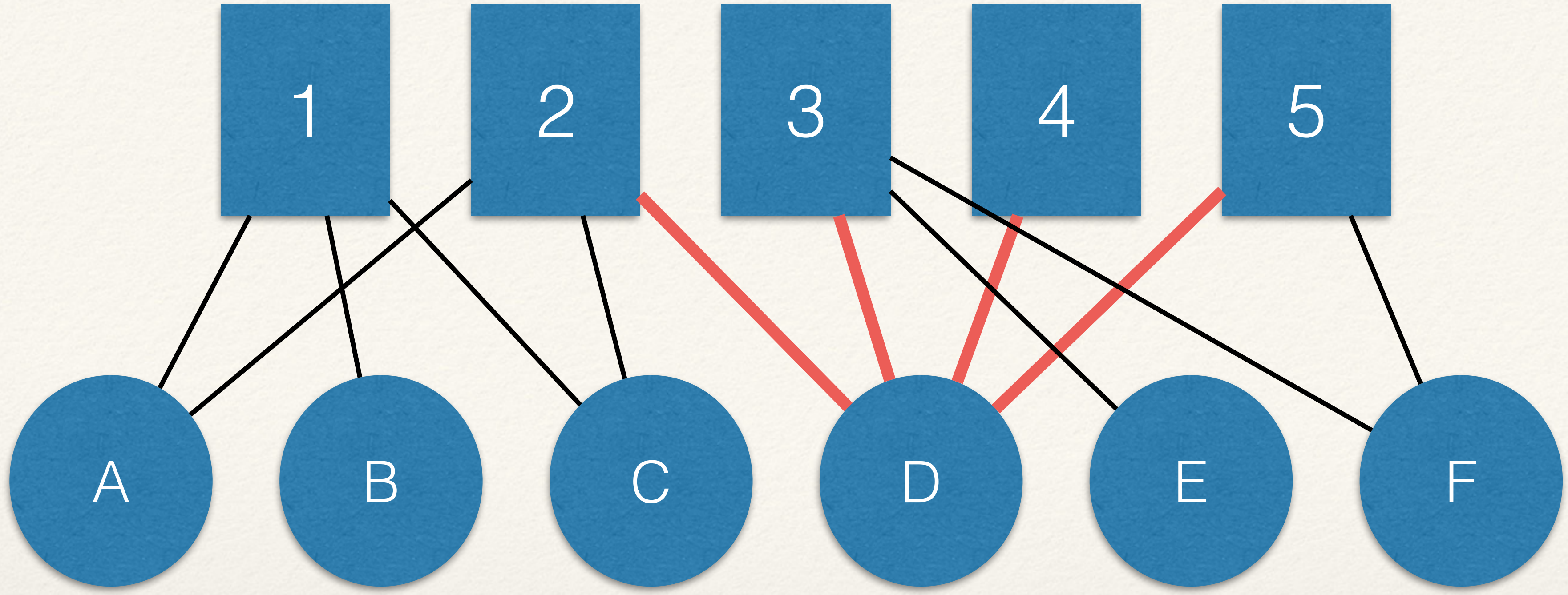
# Projection by Multiplication

We want to know how people are connected by groups (i.e. the rows of our two-mode adjacency matrix)

$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

6x5 X 5x6 = 6x6



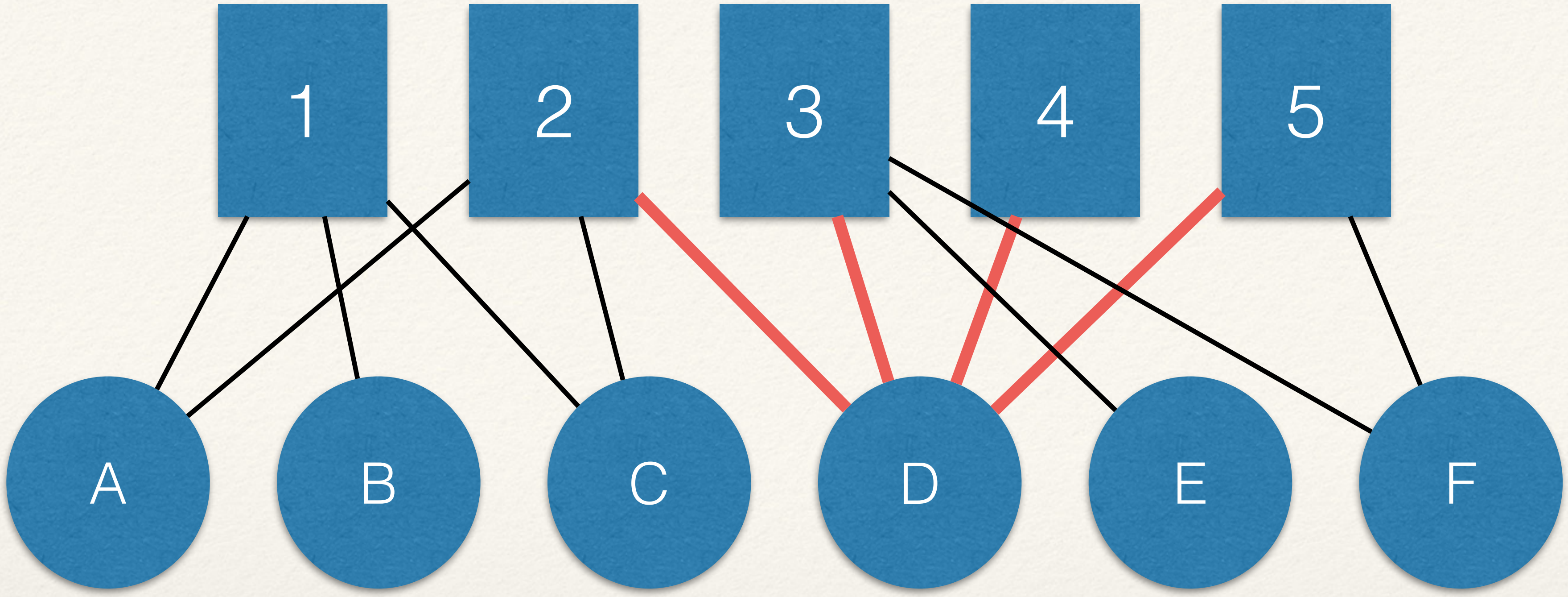

$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

The diagonal is the count of ties **the person** has with two-mode vertices

For example, D is in 4 groups

6x5 X 5x6 = 6x6

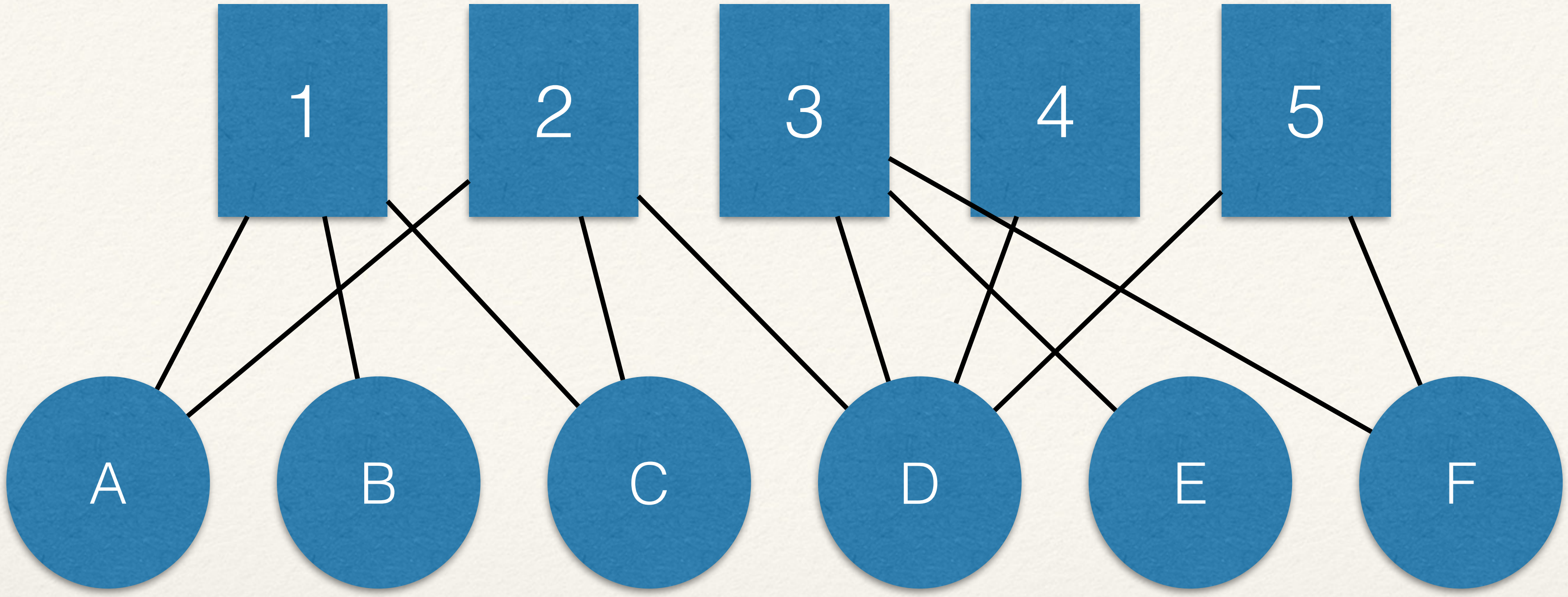



$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

*What statistic does the diagonal give us?*

6x5 X 5x6 = 6x6

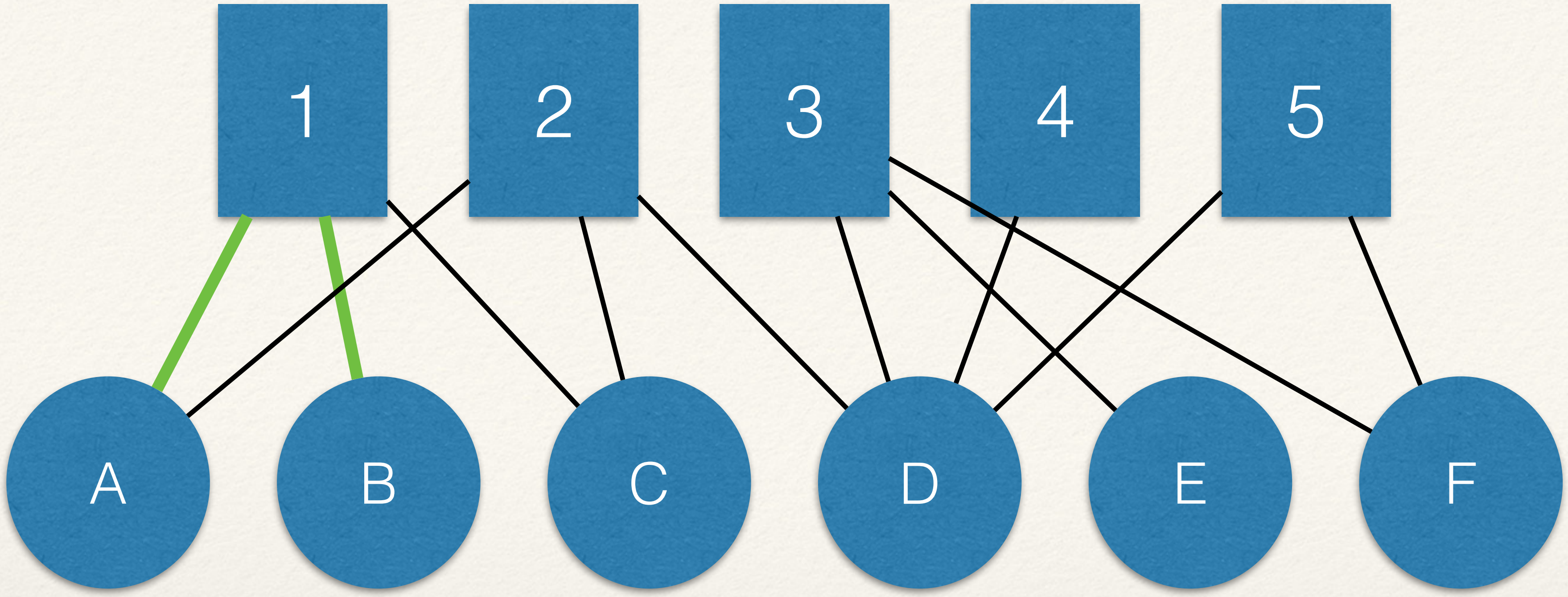



$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

Note, that the projection forces the product matrix to be symmetric (i.e. undirected graph)

6x5 X 5x6 = 6x6



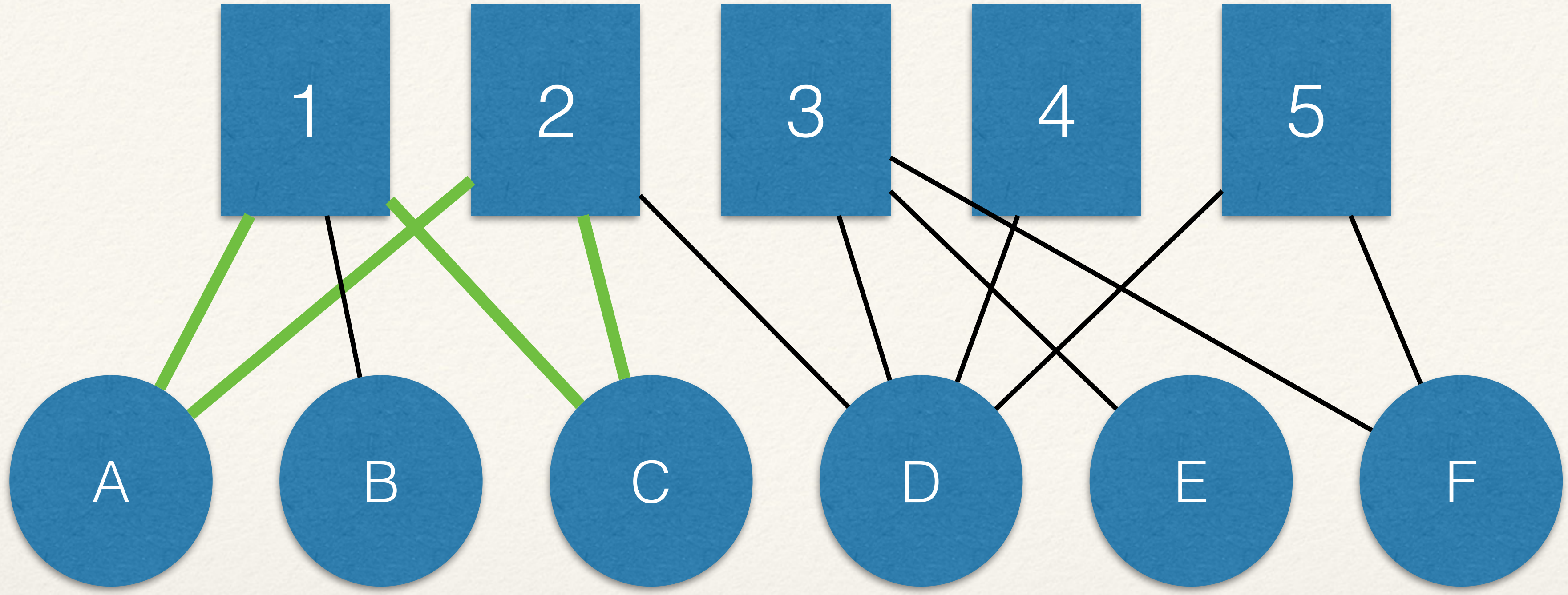

$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

$6 \times 5 \ X \ 5 \times 6 = 6 \times 6$

The off-diagonal entries indicate the number of ways that vertices in the first mode are connected by vertices in the second mode

A and B are linked through a single vertex, 1



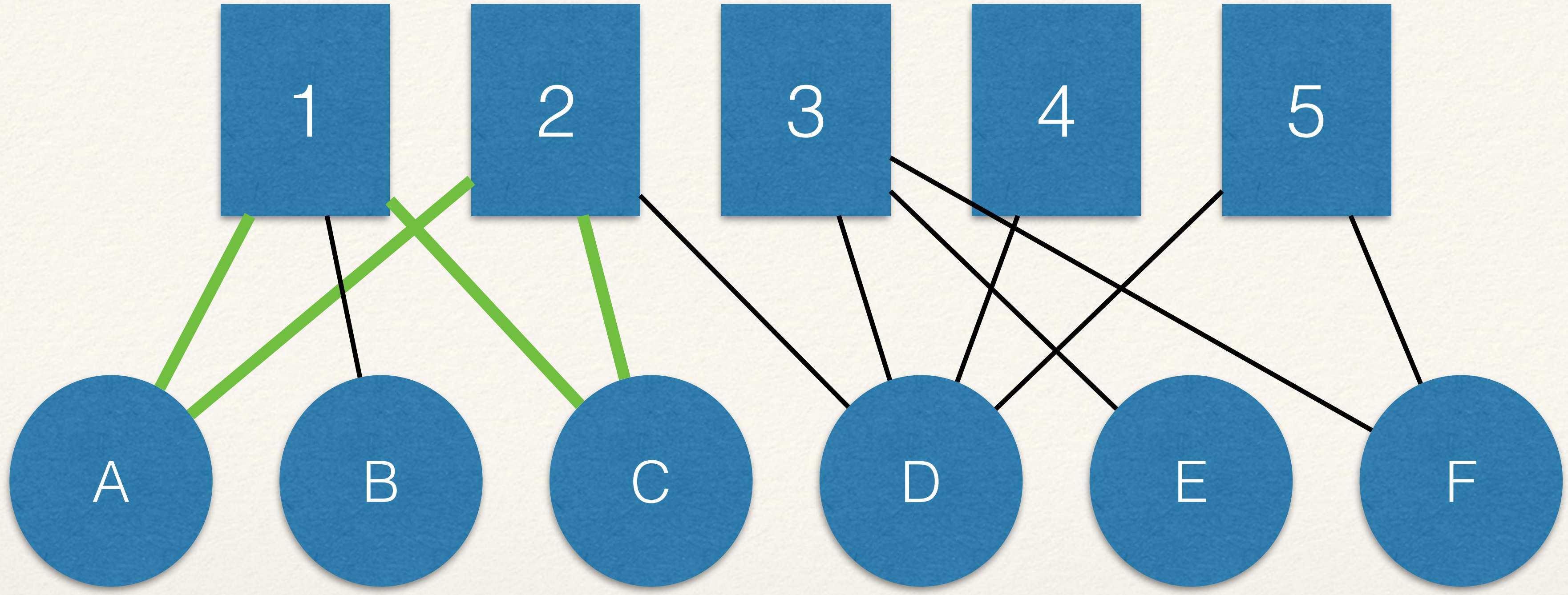

$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

A and C are linked through two vertices,1 and 2

So, if these are groups, A and C are members of 2 of the same groups

6x5 X 5x6 = 6x6




$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

**NOTE: these are counts of shared vertices, not edge counts**

6x5 X 5x6 = 6x6

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 | 0 | 0 |
| B | 1 | 0 | 1 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 0 | 1 | 1 |
| E | 0 | 0 | 0 | 1 | 0 | 1 |
| F | 0 | 0 | 0 | 1 | 1 | 0 |

If we treat any tie greater than 0 as binary, *called* ***dichotomizing***, and recode the diagonal as 0, we get an undirected, one-mode network

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 | 0 | 0 |
| B | 1 | 0 | 1 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 0 | 1 | 1 |
| E | 0 | 0 | 0 | 1 | 0 | 1 |
| F | 0 | 0 | 0 | 1 | 1 | 0 |

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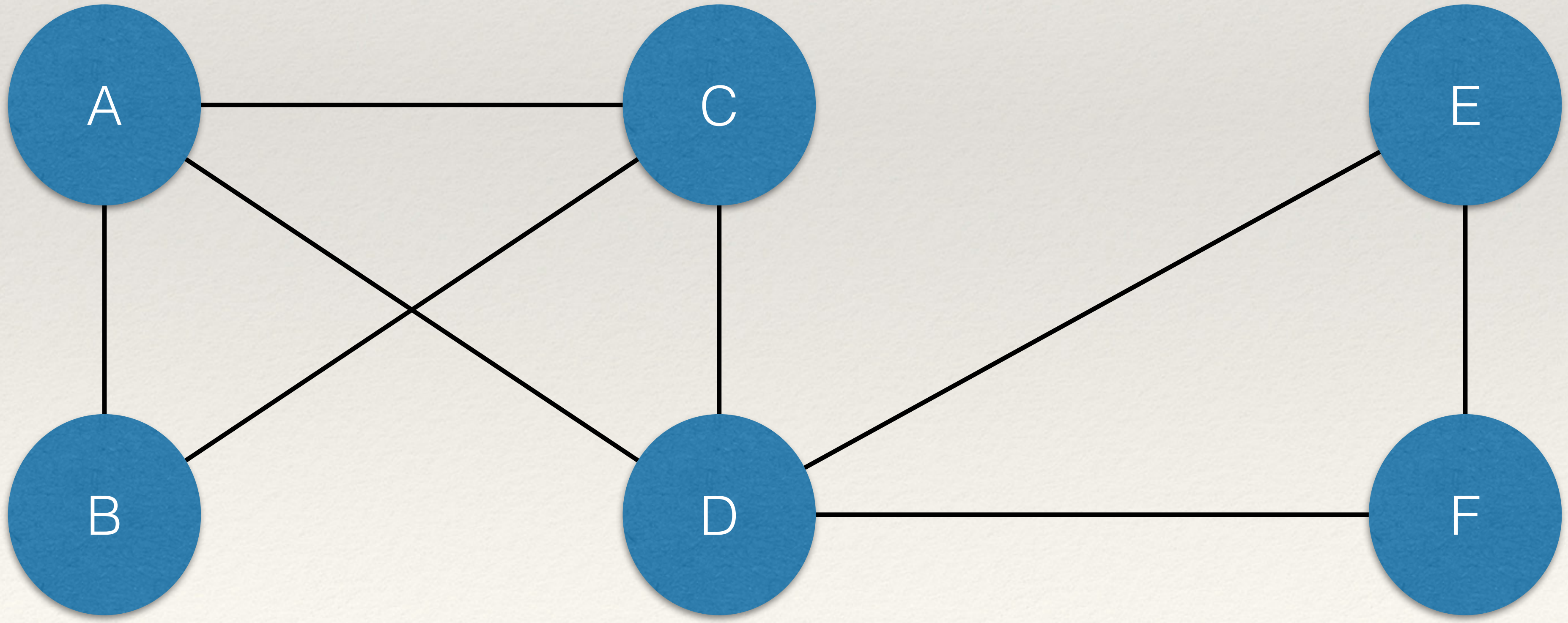


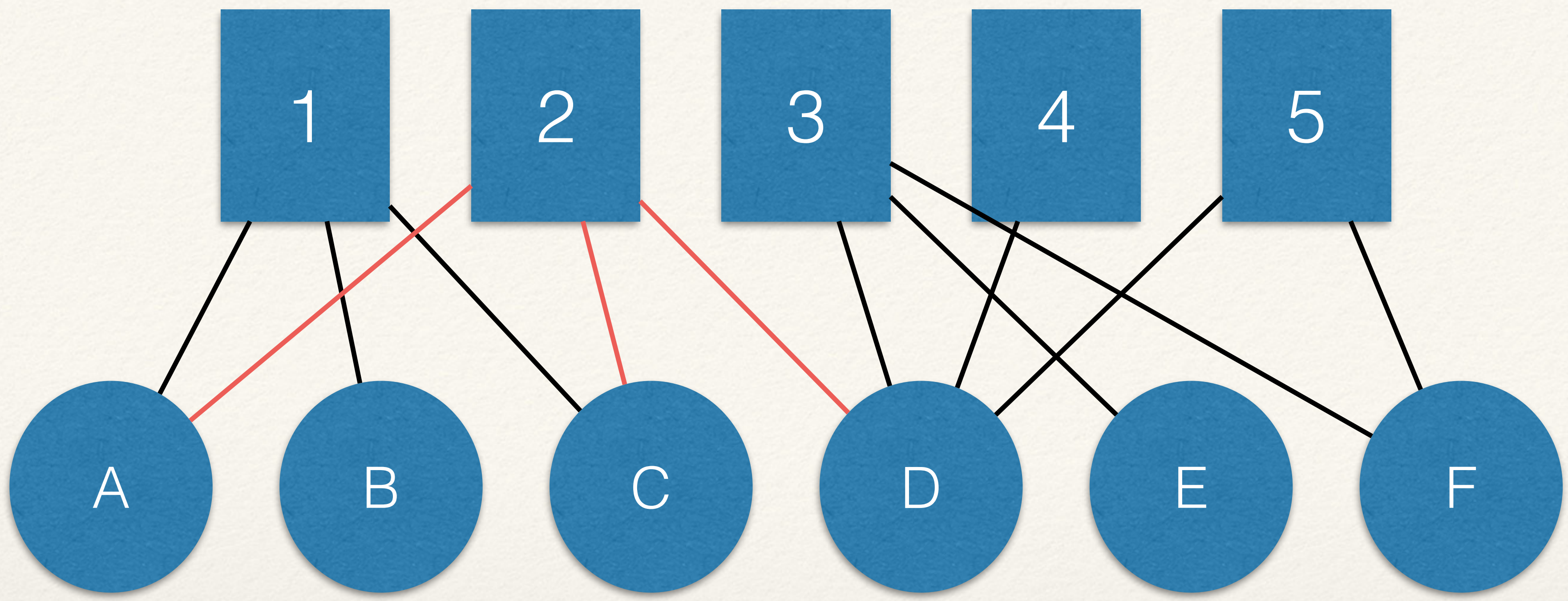
# Projection by Multiplication

We want to know how groups are connected by people (i.e. the columns of our two-mode adjacency matrix)

$$\mathbf{A}^{\mathbf{T}} \times \mathbf{A} = \mathbf{G}$$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

5x6 X 6x5 = 5x5



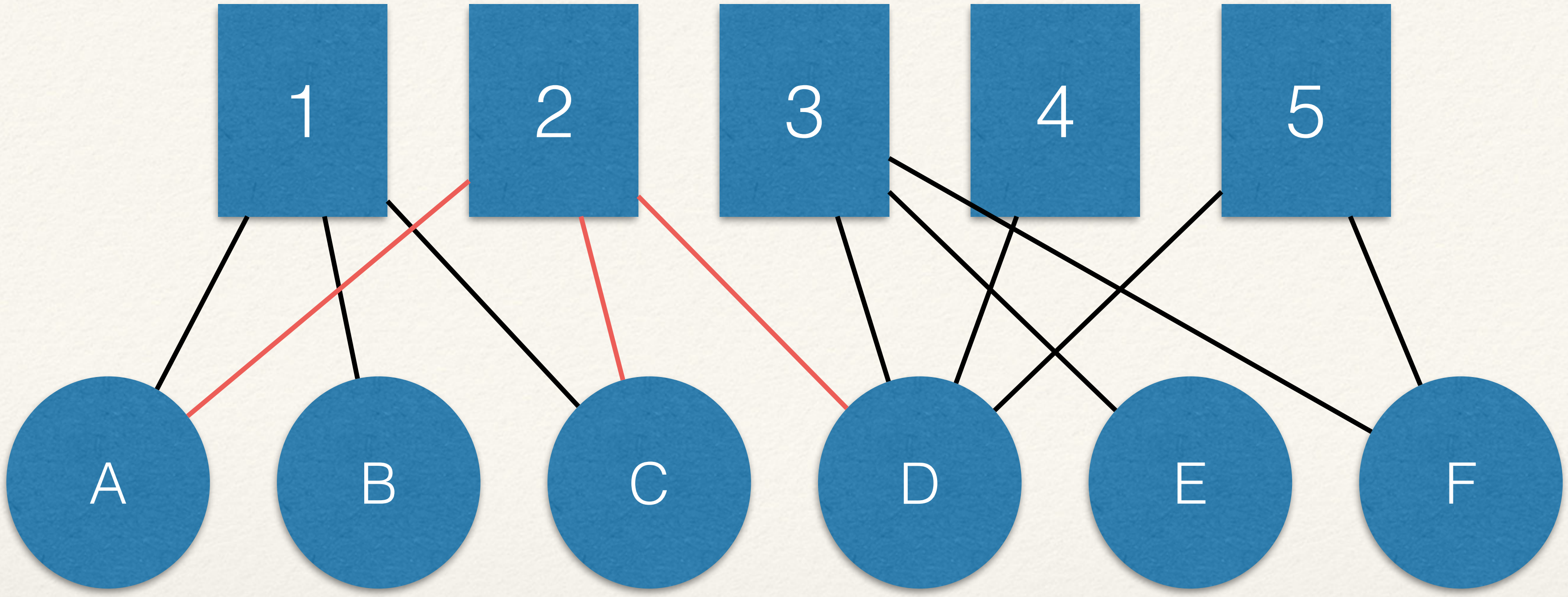

$$\mathbf{A}^{\mathbf{T}} \times \mathbf{A} = \mathbf{G}$$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

The diagonal is the count of ties the **group** has with two-mode vertices

For example, 2 has 3 people

5x6 X 6x5 = 5x5

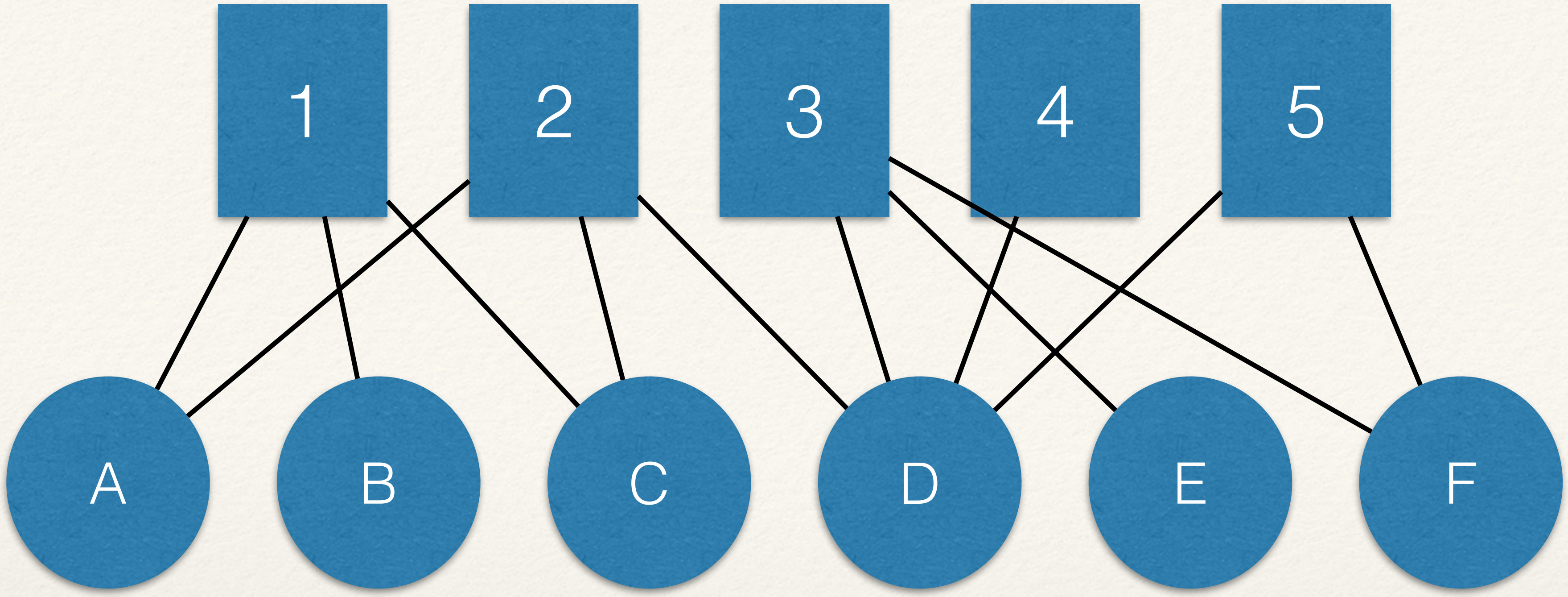



$\mathbf{A}^{\mathbf{T}} \times \mathbf{A} = \mathbf{G}$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

5x6 X 6x5 = 5x5

*What statistic does the diagonal give us?*

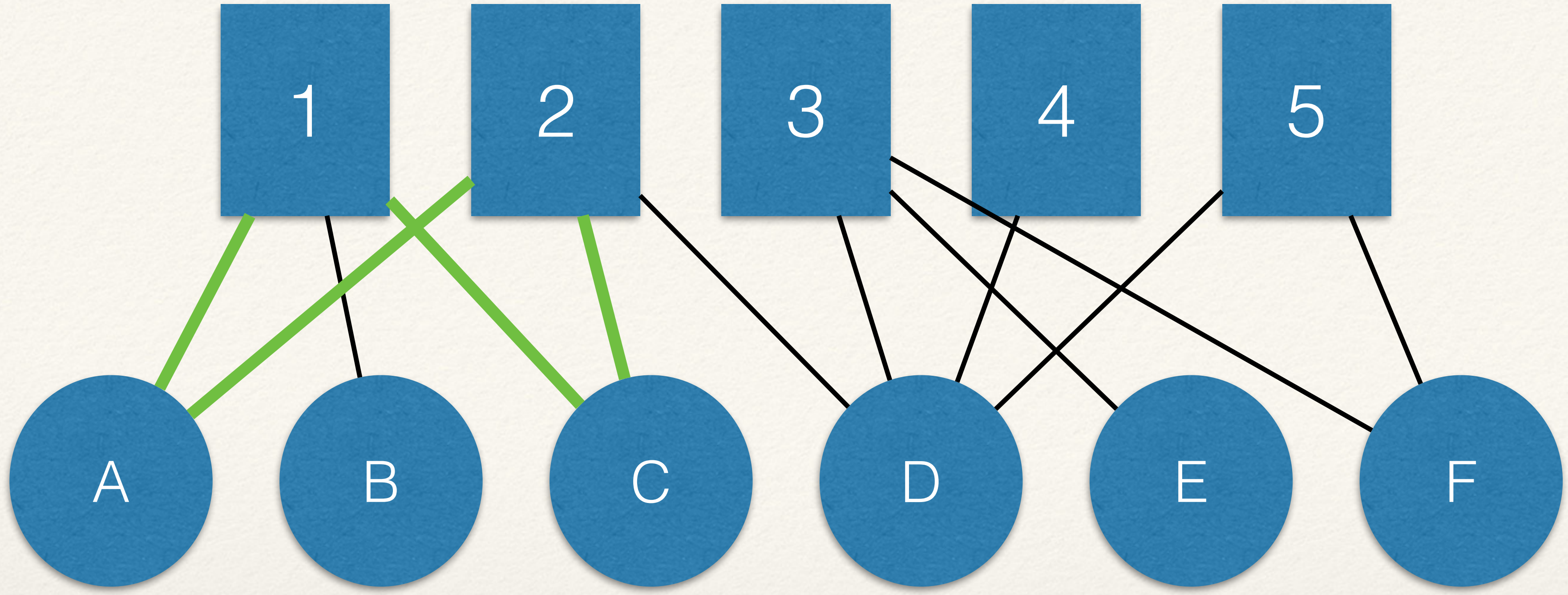



$\mathbf{A}^\mathbf{T} \times \mathbf{A} = \mathbf{G}$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

5x6 X 6x5 = 5x5

Note, that the projection forces the product matrix to be symmetric (i.e. undirected graph)



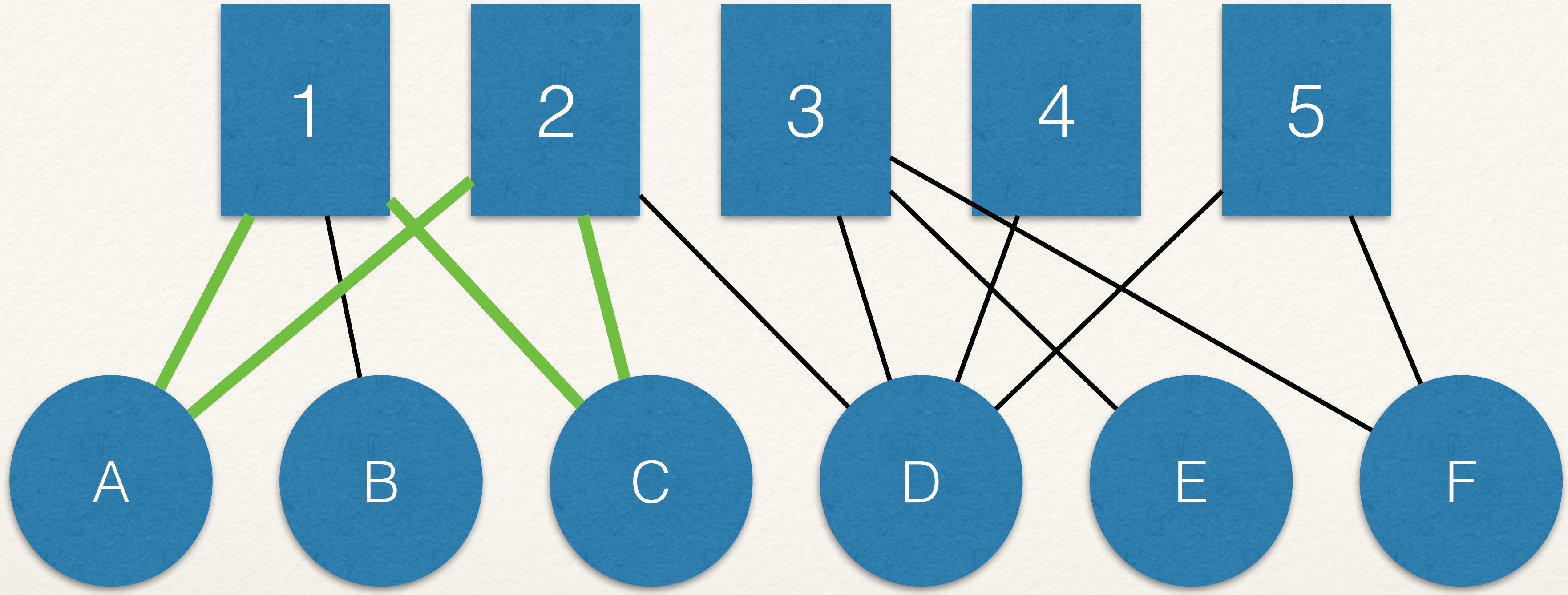

$$\mathbf{A}^{\mathbf{T}} \times \mathbf{A} = \mathbf{G}$$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

5x6 X 6x5 = 5x5

The off-diagonal entries indicate the number of ways that vertices in the second mode are connected by vertices in the first mode

1 and 2 are connected by 2 vertices, A and C




$$\mathbf{A}^{\mathbf{T}} \times \mathbf{A} = \mathbf{G}$$

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

$5x6 \times 6x5 = 5x5$

**NOTE: these are counts of shared vertices, not edge counts**

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

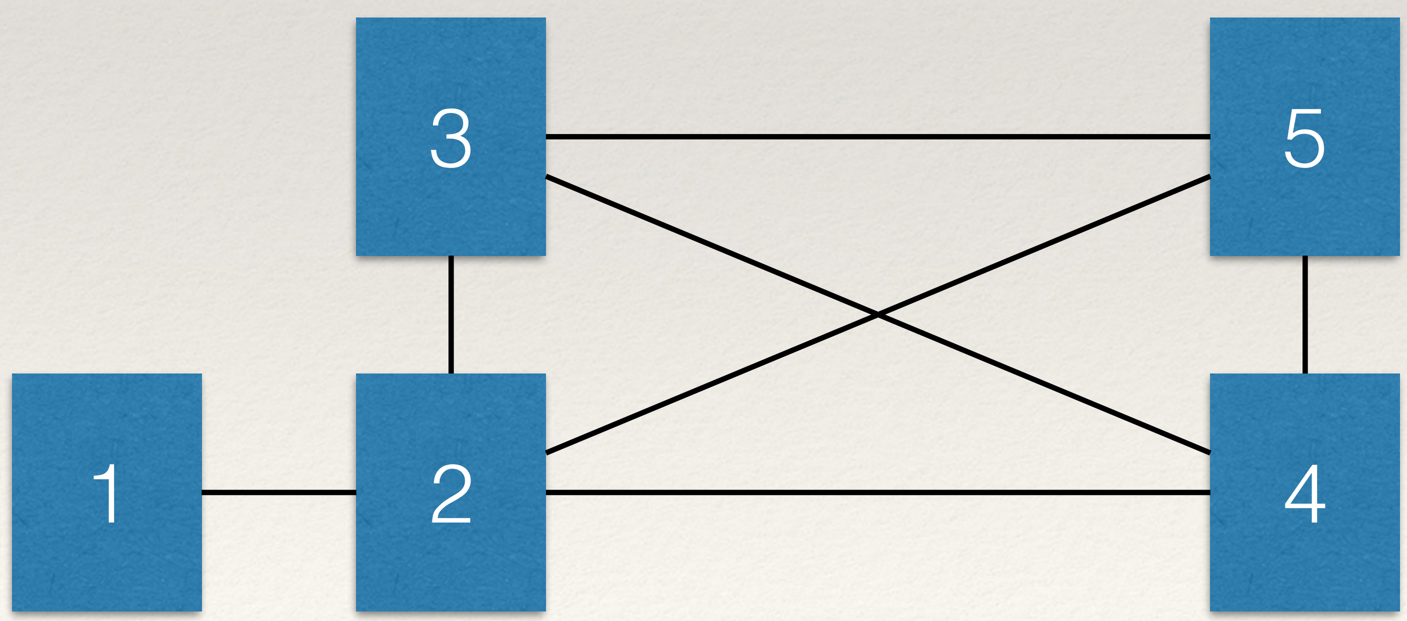
| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 1 |
| 3 | 0 | 1 | 0 | 1 | 1 |
| 4 | 0 | 1 | 1 | 0 | 1 |
| 5 | 0 | 1 | 1 | 1 | 0 |

If we treat any tie greater than 0 as binary, *called* ***dichotomizing***, and recode the diagonal as 0, we get an undirected, one-mode network

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 3 | 2 | 0 | 0 | 0 |
| 2 | 2 | 3 | 1 | 1 | 1 |
| 3 | 0 | 1 | 3 | 1 | 2 |
| 4 | 0 | 1 | 1 | 1 | 1 |
| 5 | 0 | 1 | 2 | 1 | 2 |

| | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 2 | 1 | 0 | 1 | 1 | 1 |
| 3 | 0 | 1 | 0 | 1 | 1 |
| 4 | 0 | 1 | 1 | 0 | 1 |
| 5 | 0 | 1 | 1 | 1 | 0 |

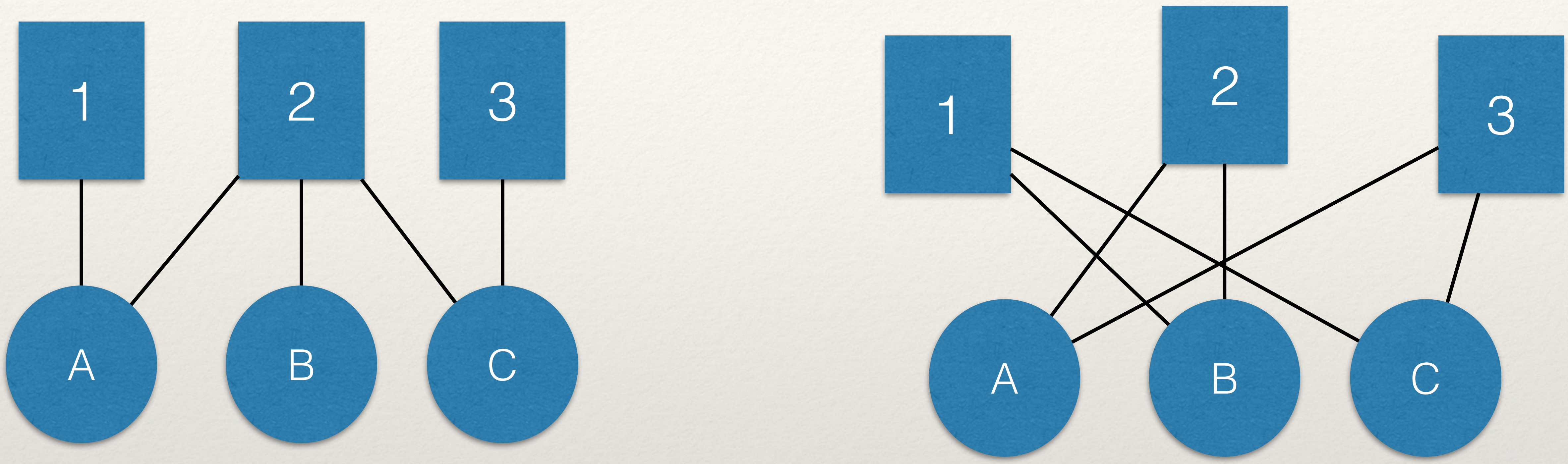
If we treat any tie greater than 0 as binary, *called* ***dichotomizing***, and recode the diagonal as 0, we get an undirected, one-mode network



# Projection

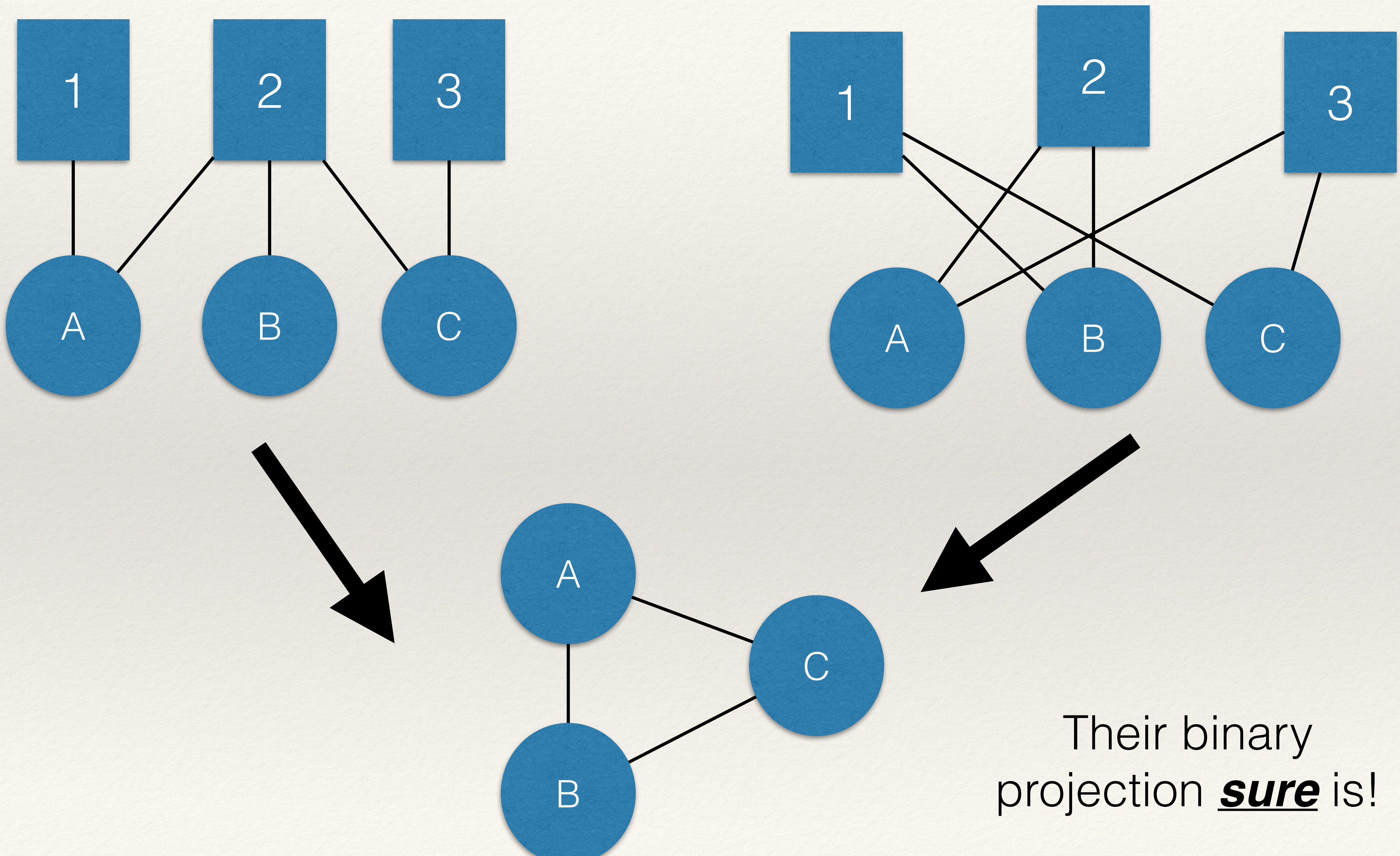
- To project, or not to project?
    - As noted by many scholars, there is data loss when we project and binarize the data.
        - Sometimes, this can be misleading.

# Projection and Data Loss



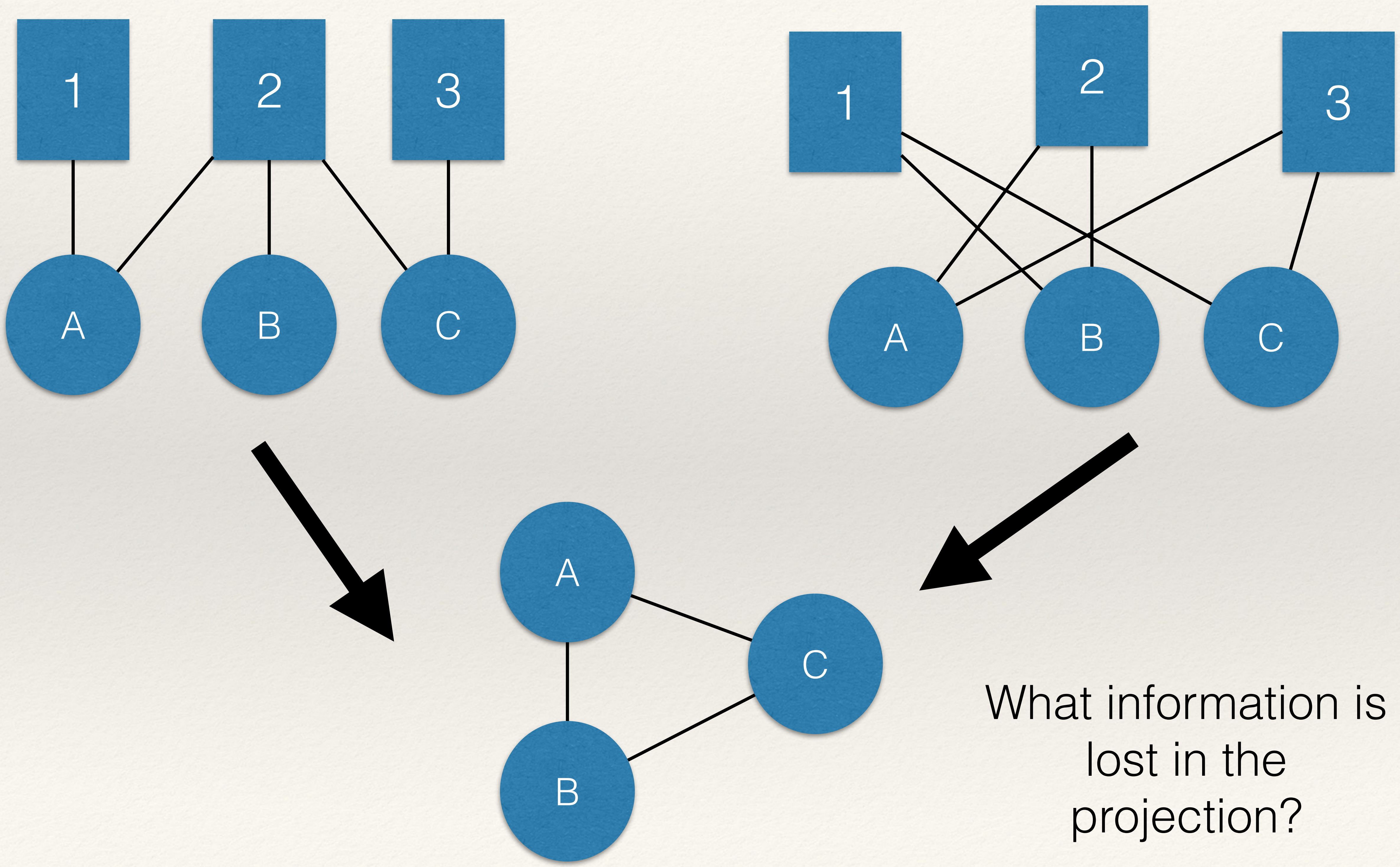

Are these bipartite graphs the same?

# Projection and Data Loss




Their binary
projection ***sure*** is!

# Projection and Data Loss




What information is lost in the projection?

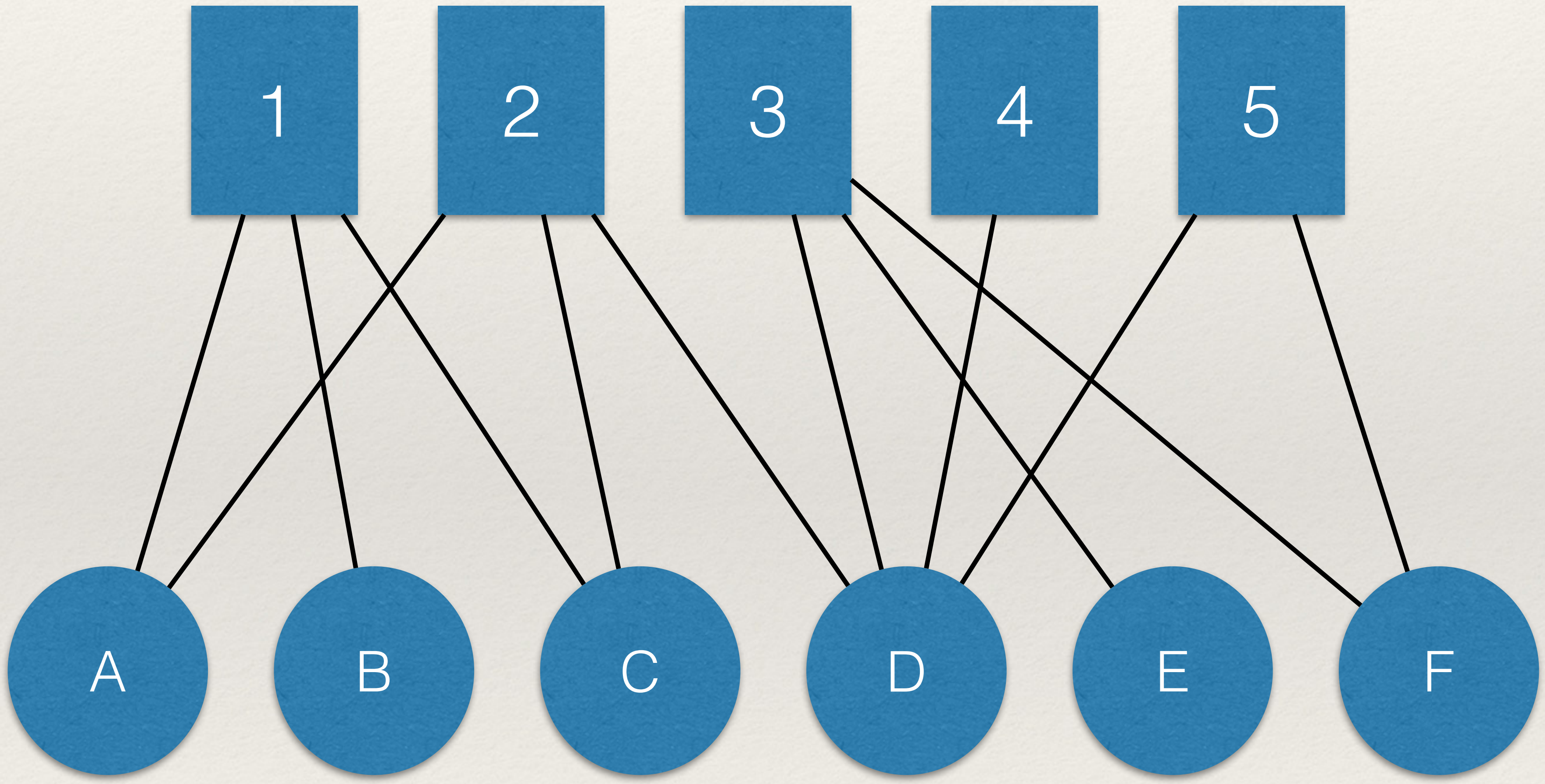
# Projection

- So what do we do?
  - When you can, “keep it real” by keeping it two-mode.
  - If you must project, minimize data loss by weighting edges.

# Weighted Edges

- We can use the information from the bipartite graph to weight the edges in the network.
- These weights can be used in a plot and/or in the analysis.
  - The most common method is to sum the ties between two actors (i.e. *summation method*).

# Projection



| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

→

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 0 | 1 | 1 | 1 | 0 | 0 |
| B | 1 | 0 | 1 | 0 | 0 | 0 |
| C | 1 | 1 | 0 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 0 | 1 | 1 |
| E | 0 | 0 | 0 | 1 | 0 | 1 |
| F | 0 | 0 | 0 | 1 | 1 | 0 |

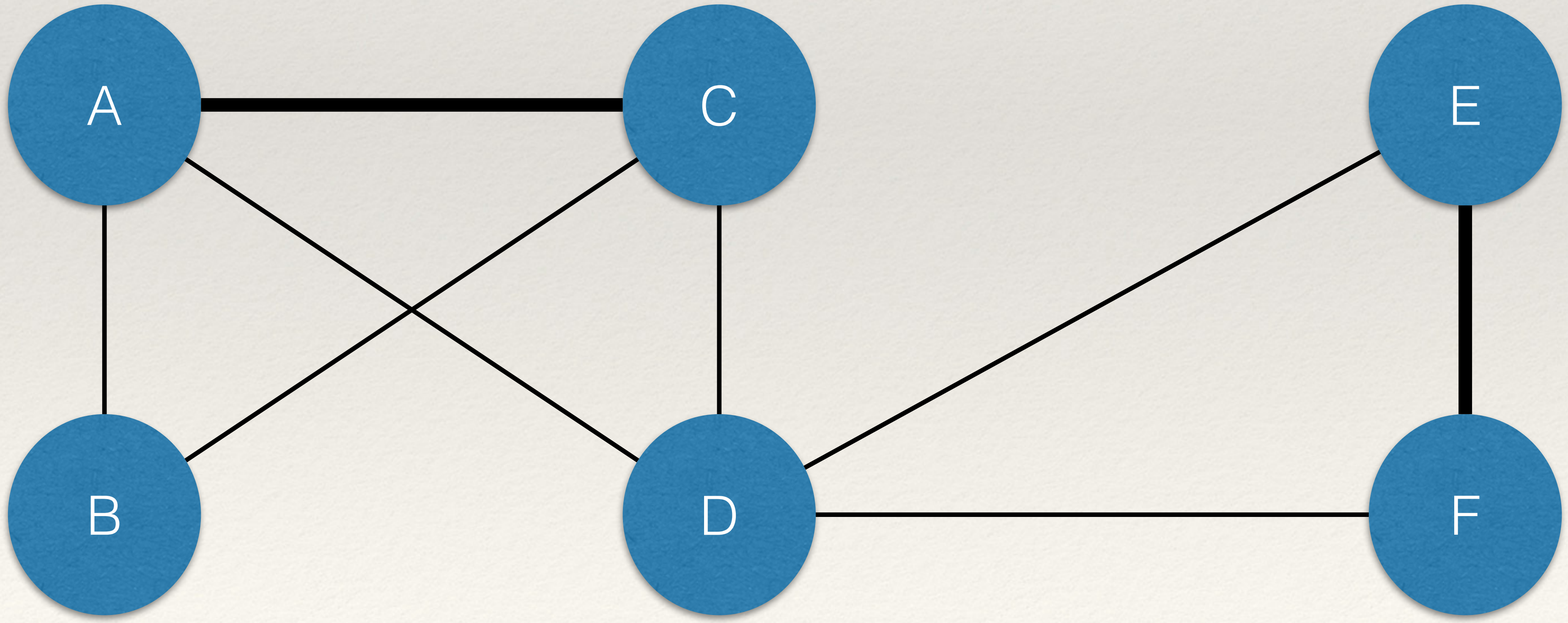
If we treat any tie greater than 0 as binary, *called **dichotomizing***, and recode the diagonal as 0, we get an undirected, one-mode network

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

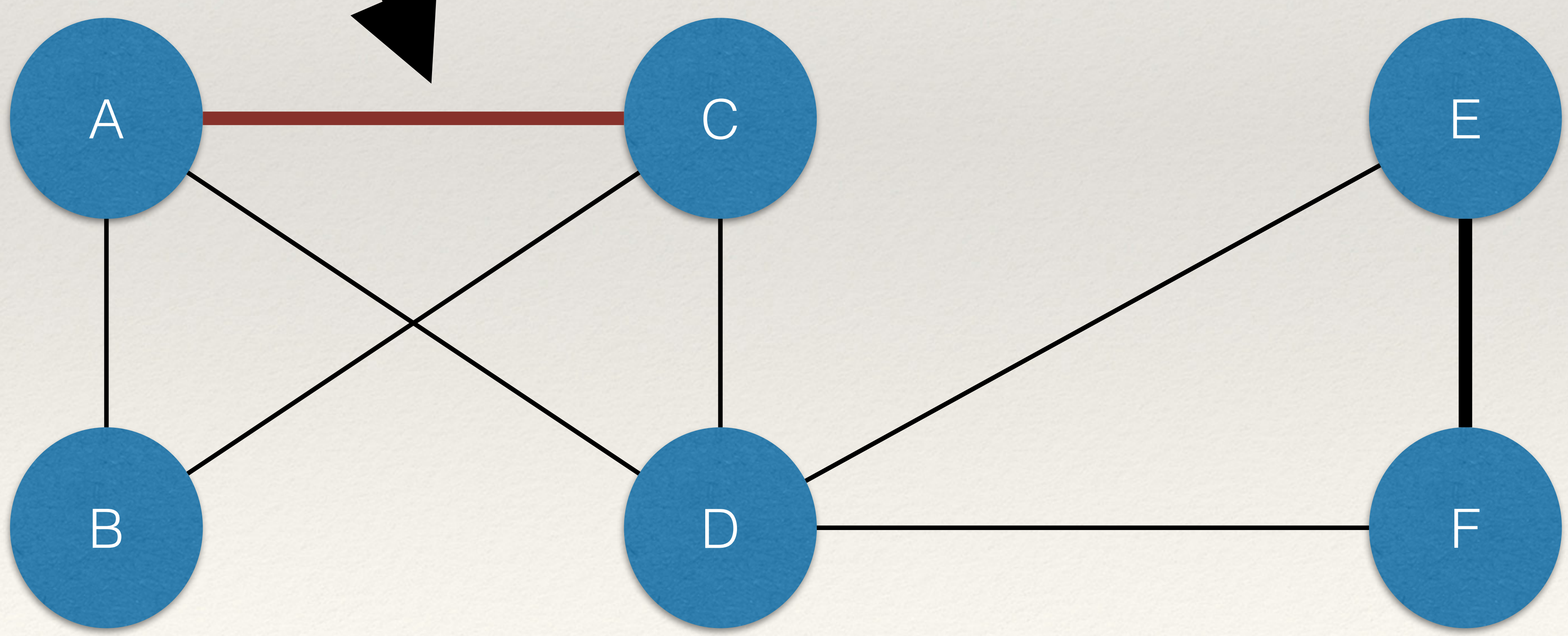
The off-diagonal entries are the tie weights



## $A \times A^T = P$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

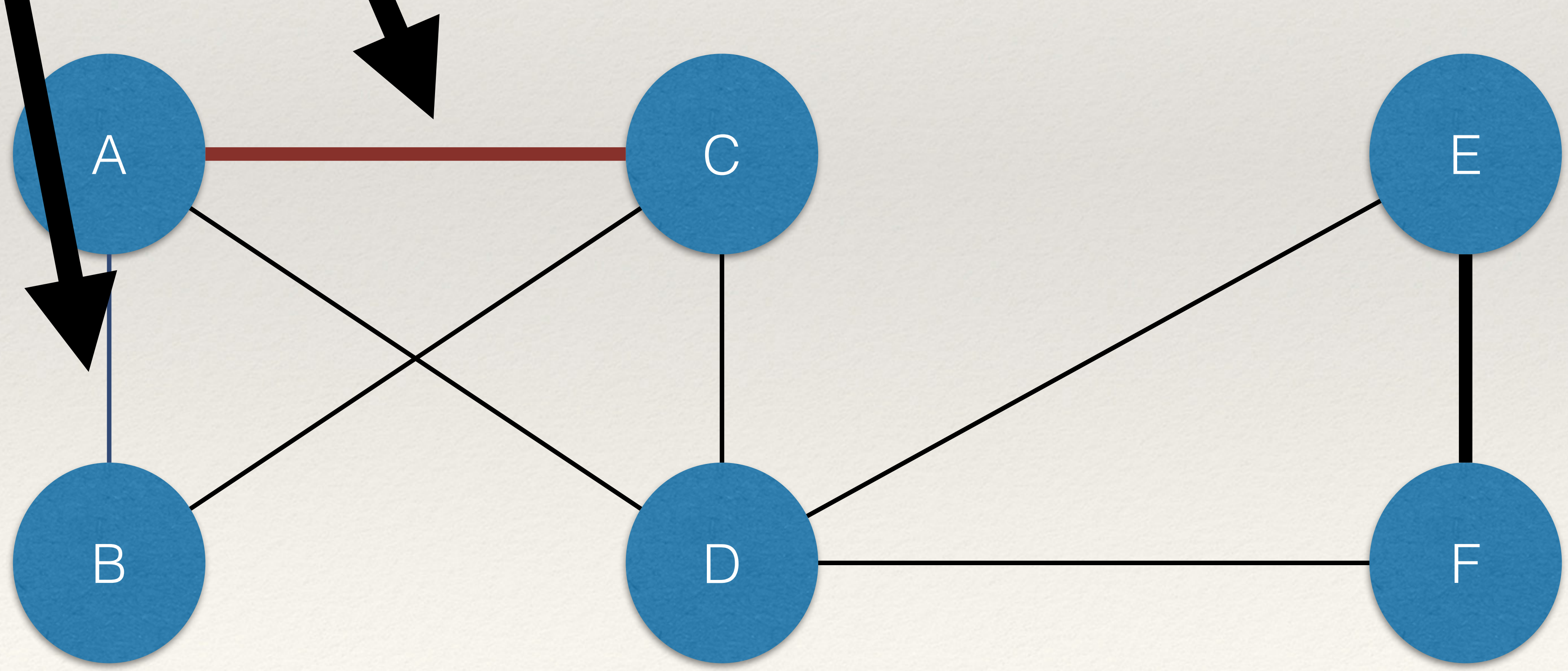
The weight corresponds to the number of nodes in the second mode that the two actors share in the bipartite graph.



$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | [illegible] | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

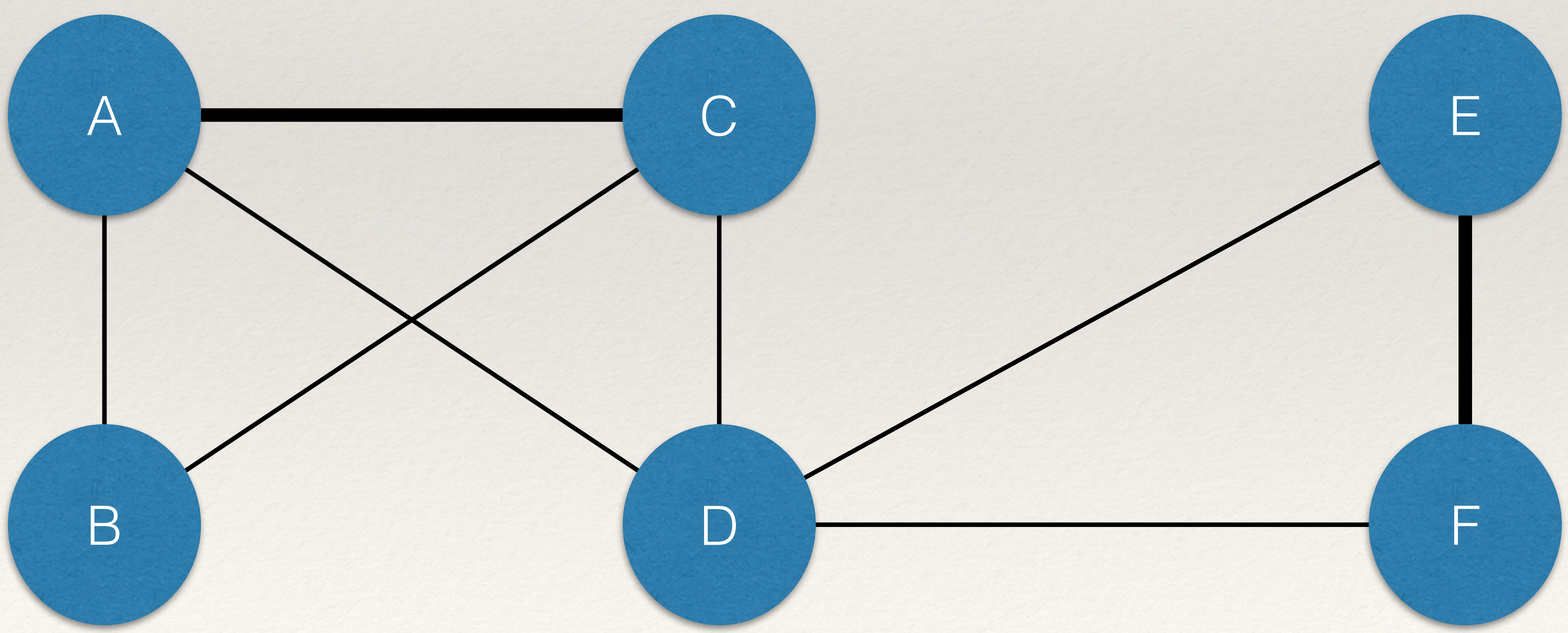
The weight corresponds to the number of nodes in the second mode that the two actors share in the bipartite graph.

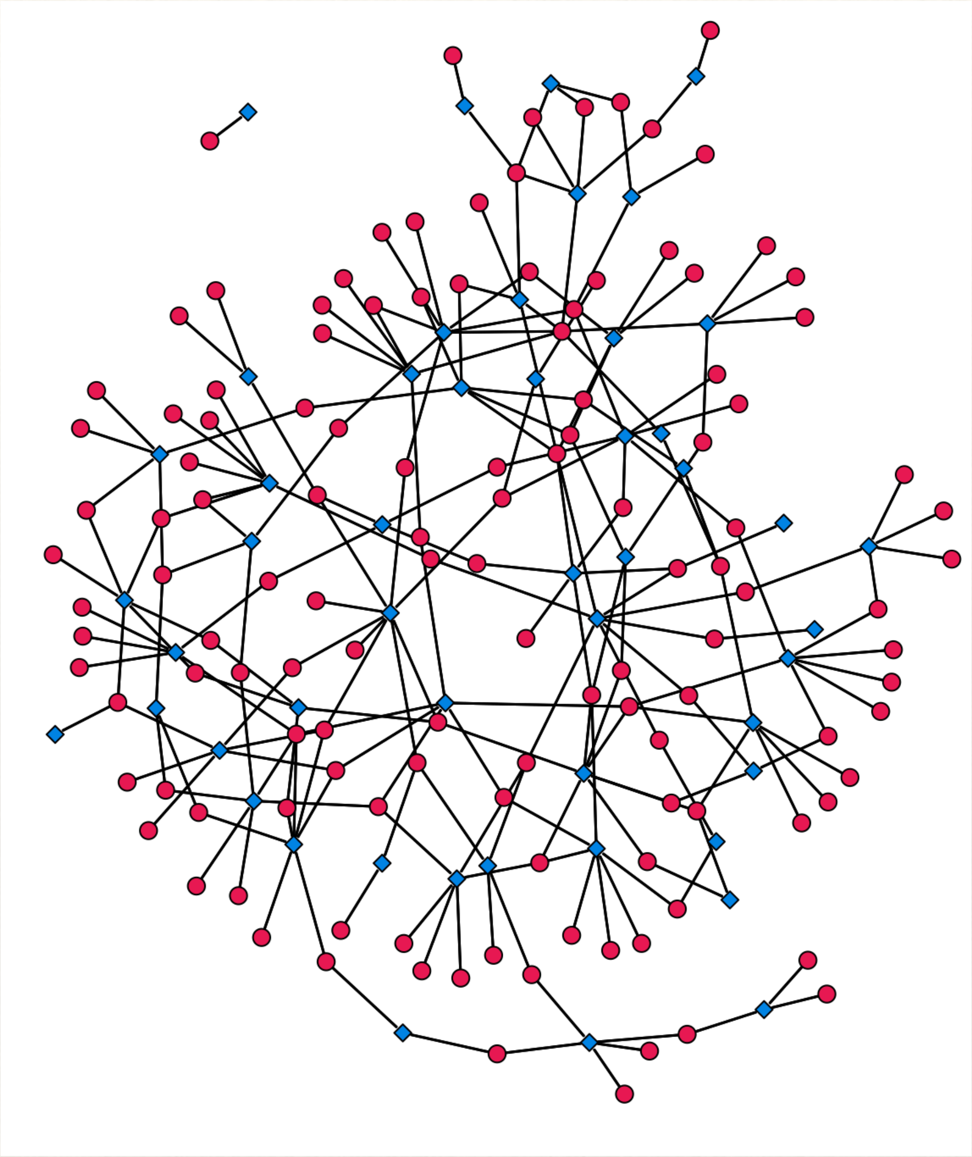


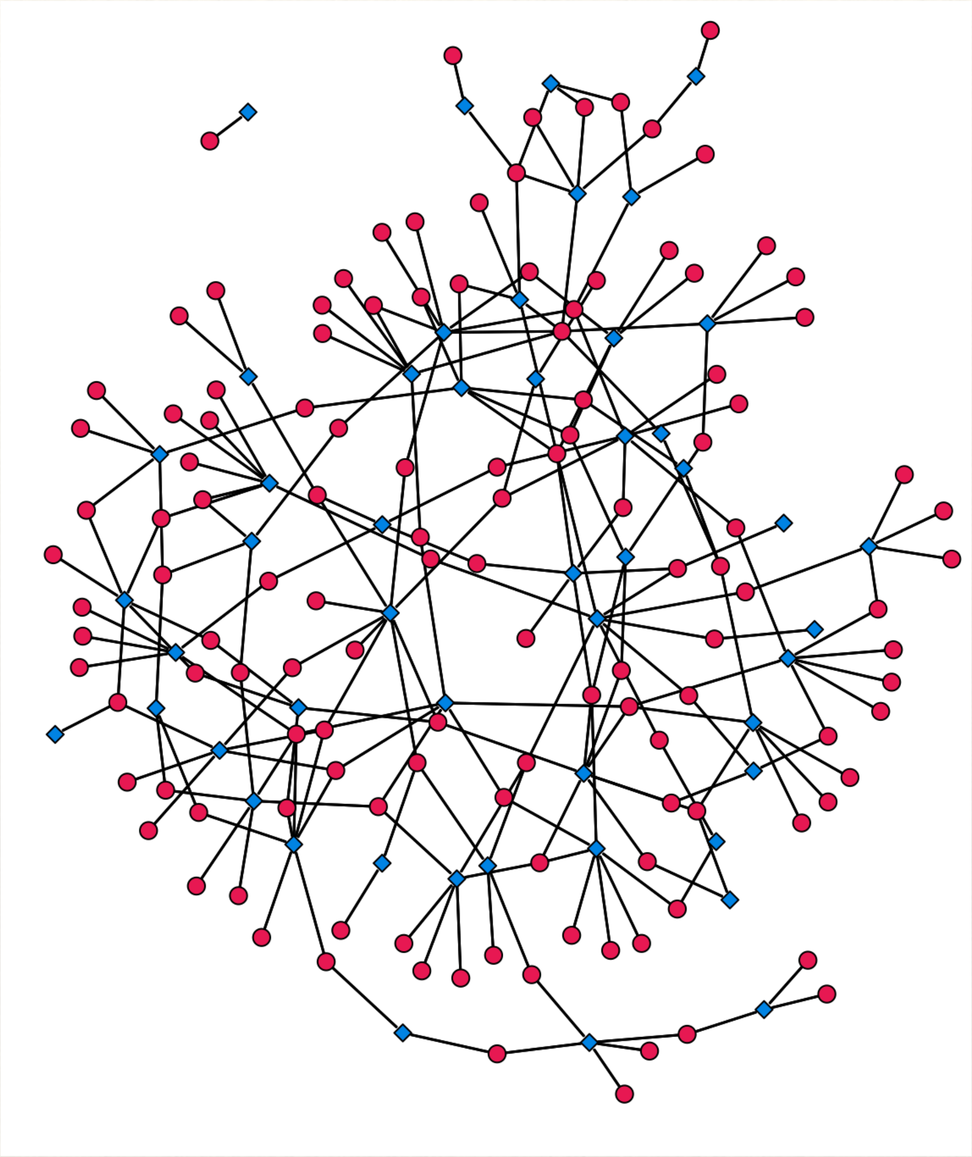
$$\mathbf{A} \times \mathbf{A}^{\mathbf{T}} = \mathbf{P}$$

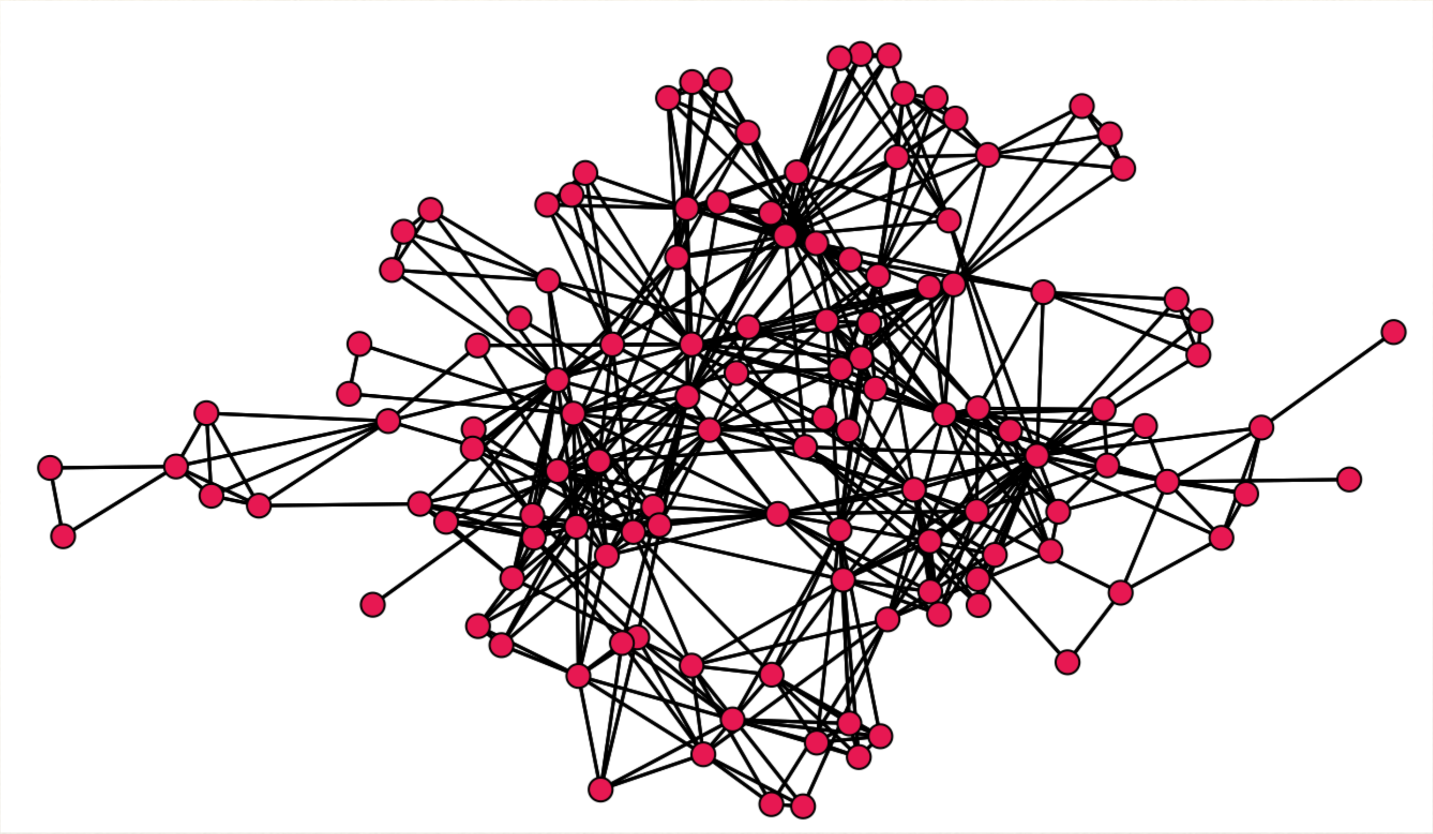
| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| A | 2 | 1 | 2 | 1 | 0 | 0 |
| B | 1 | 1 | 1 | 0 | 0 | 0 |
| C | 2 | 1 | 2 | 1 | 0 | 0 |
| D | 1 | 0 | 1 | 4 | 1 | 2 |
| E | 0 | 0 | 0 | 1 | 1 | 1 |
| F | 0 | 0 | 0 | 2 | 1 | 2 |

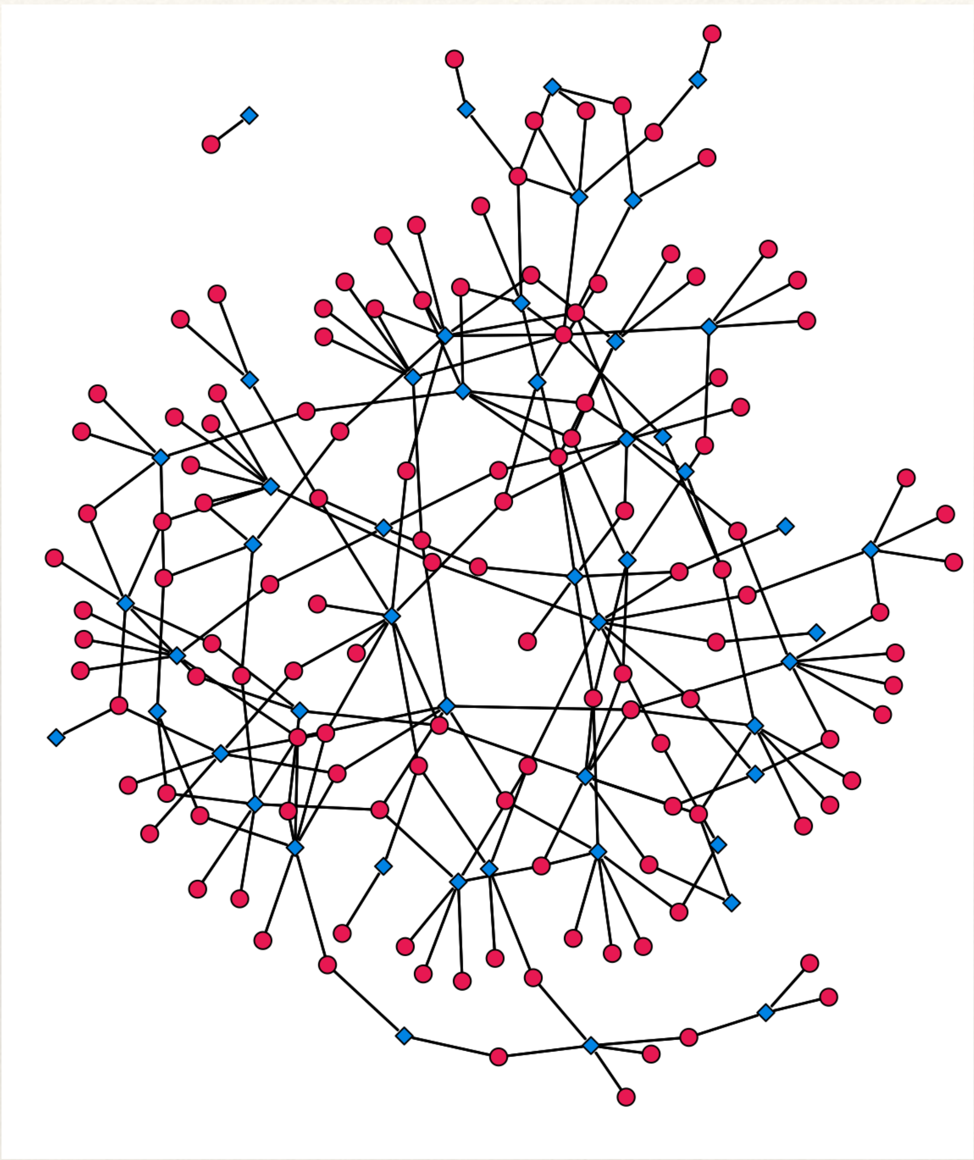
These weights are returned as the product matrix.

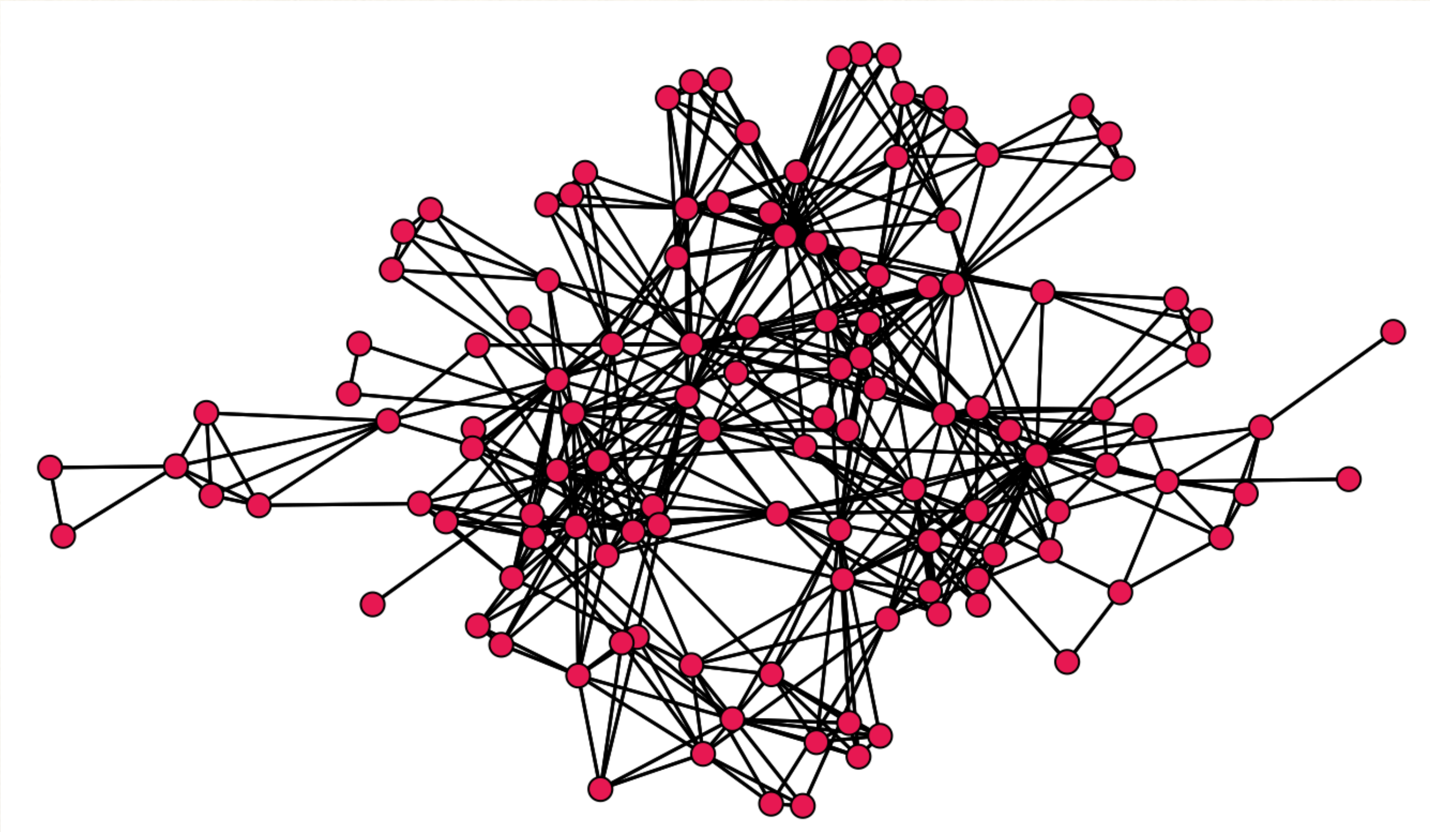


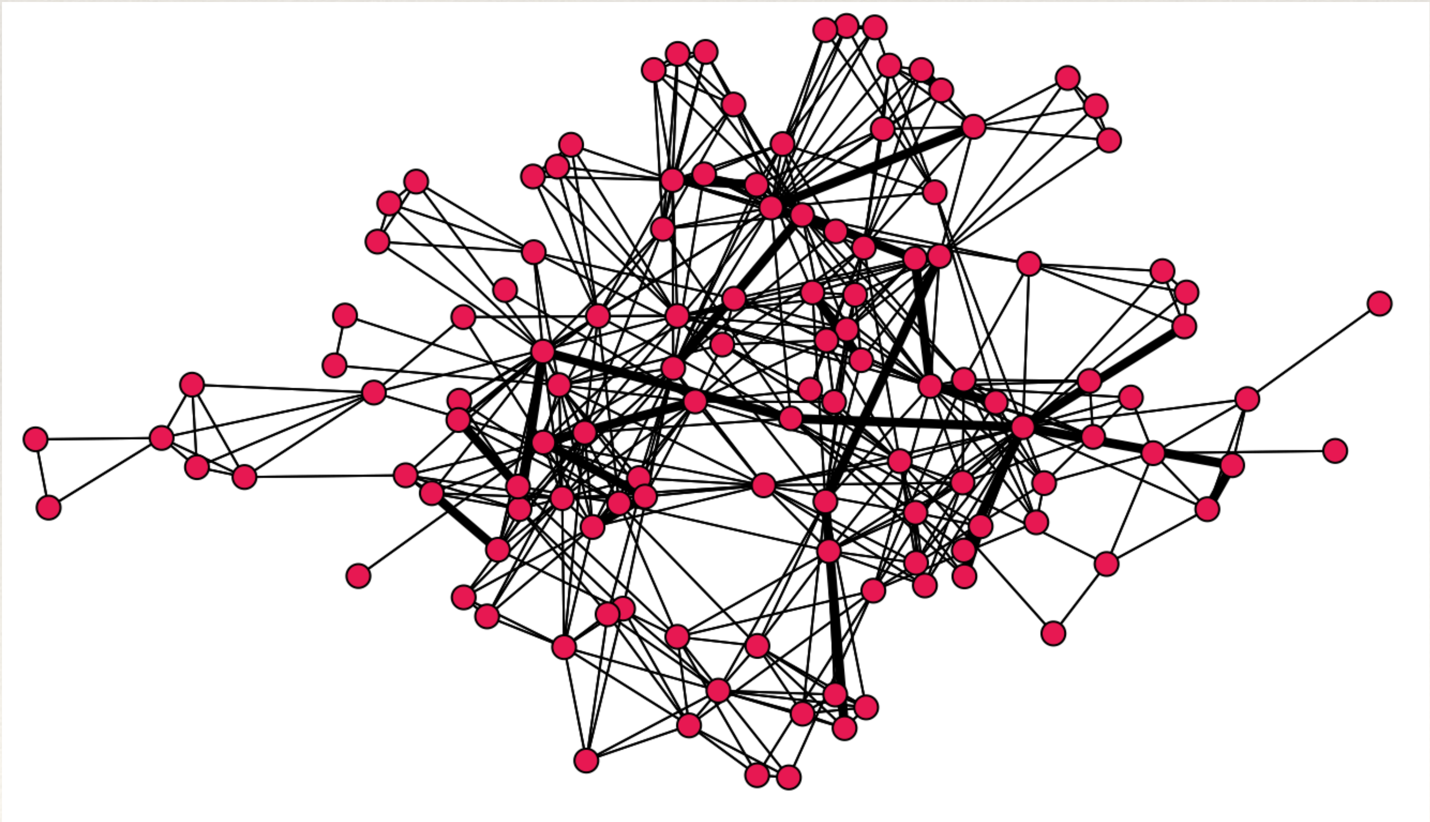


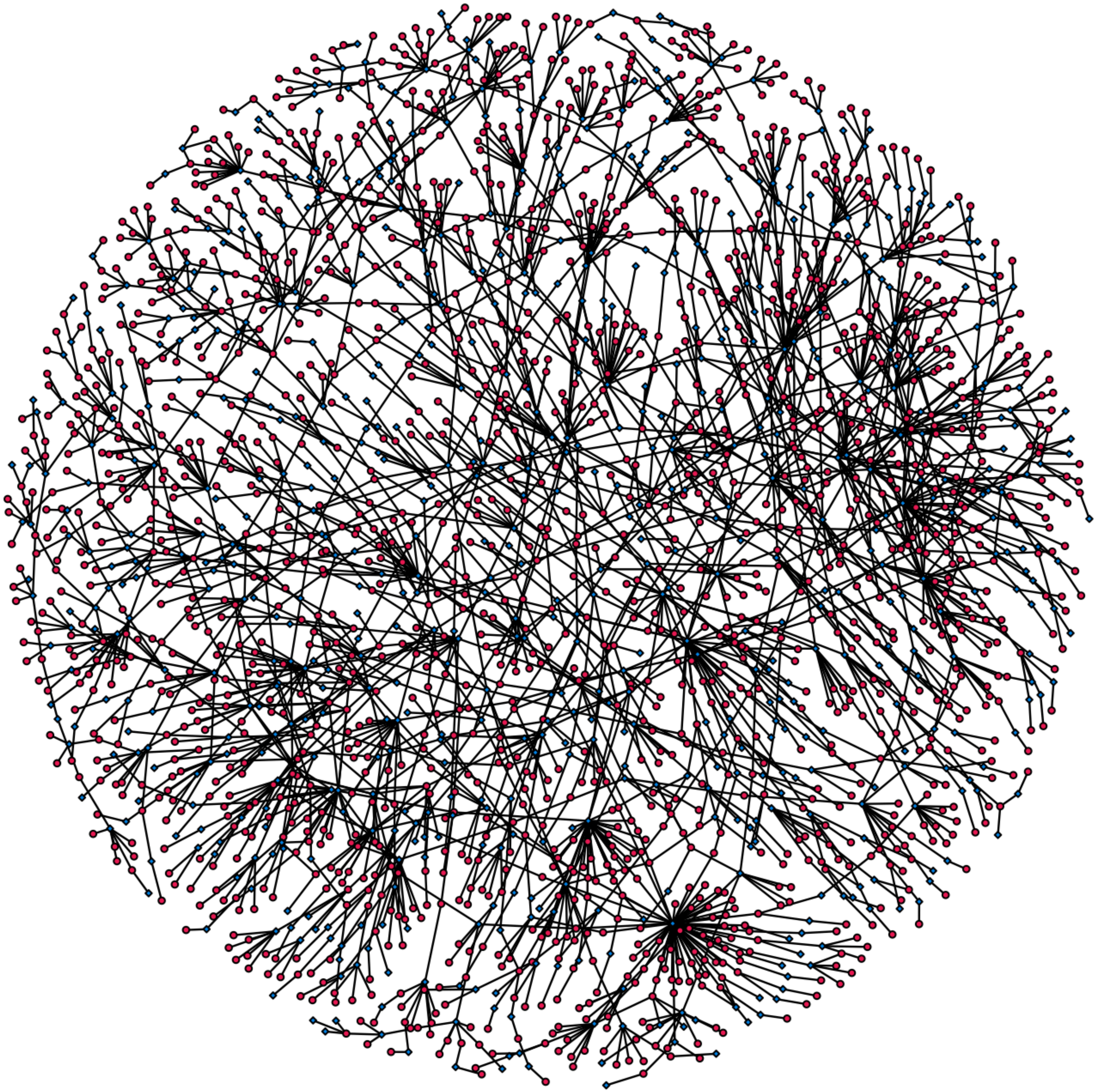


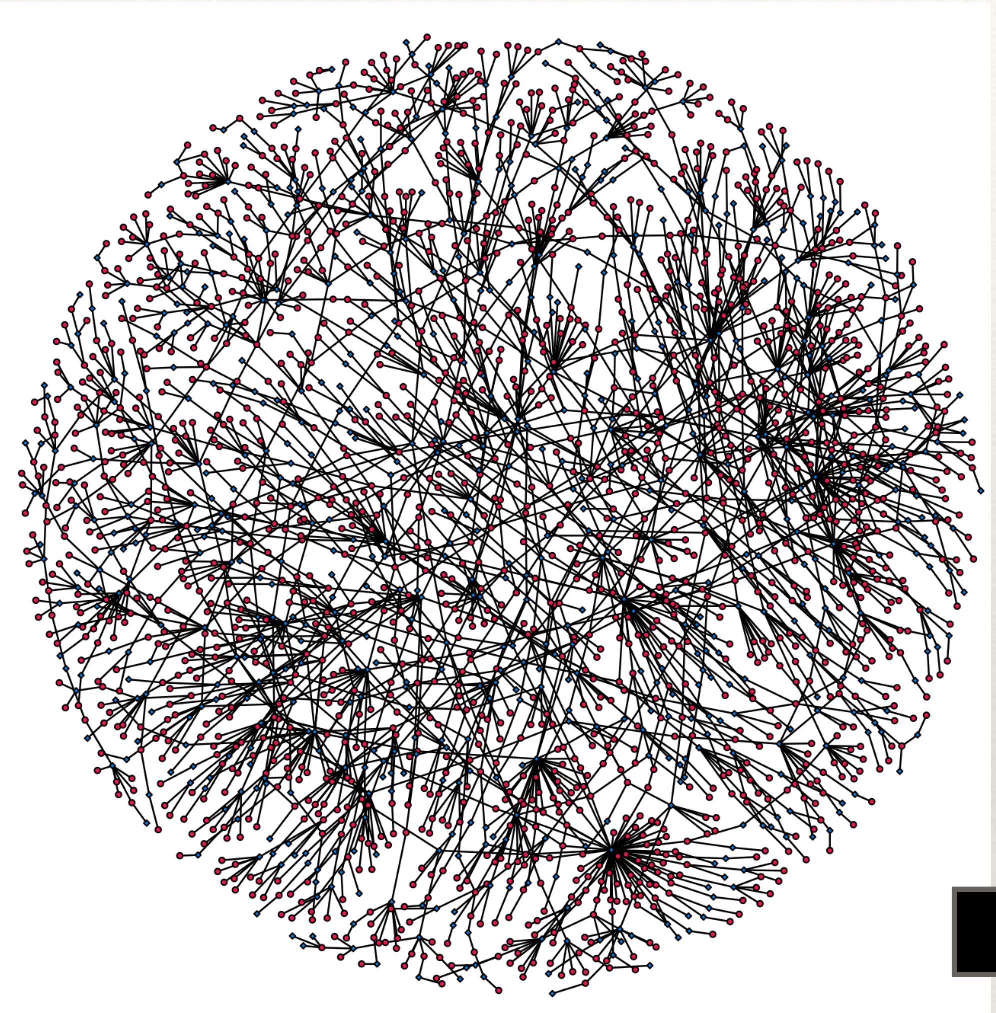


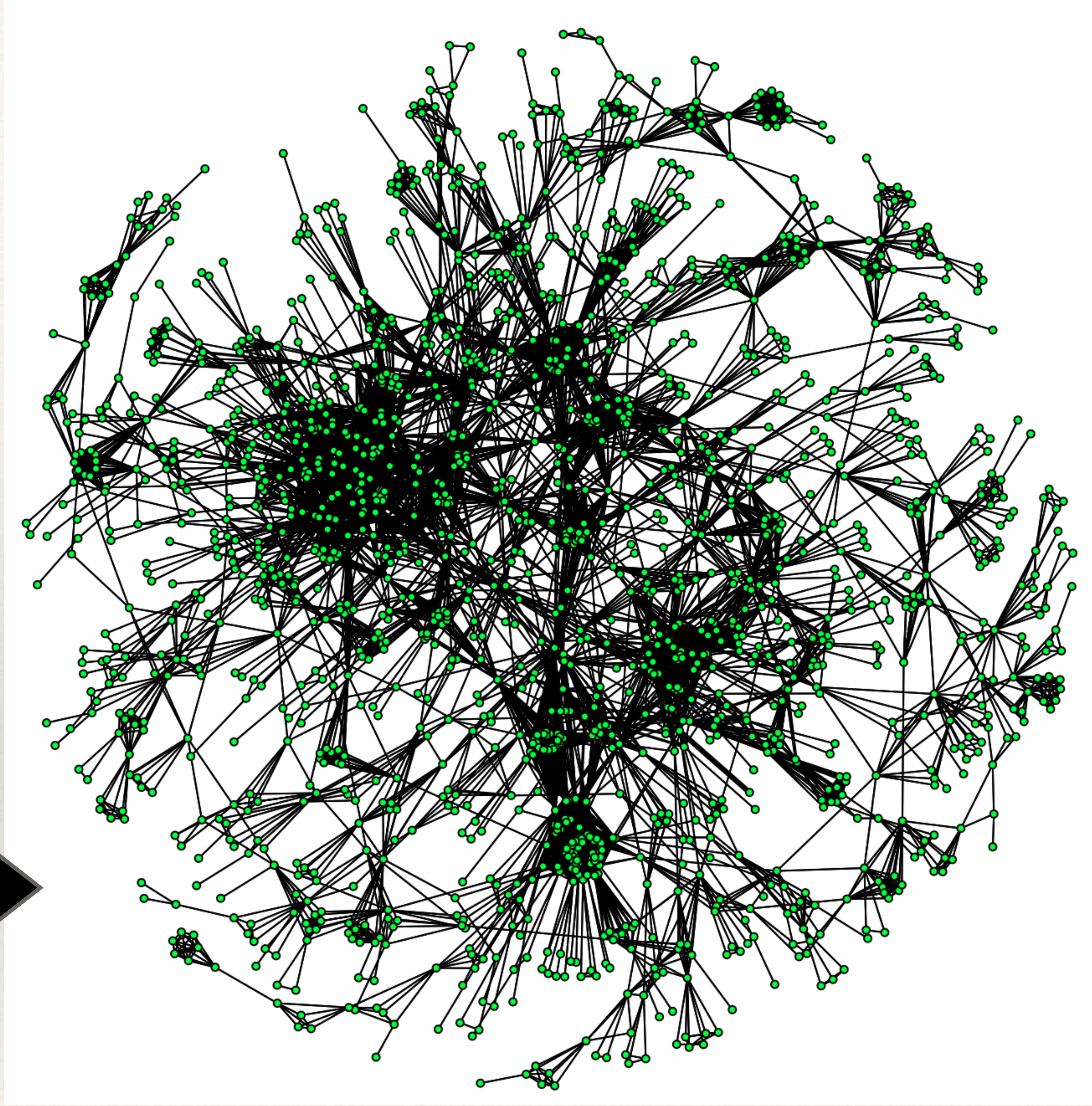


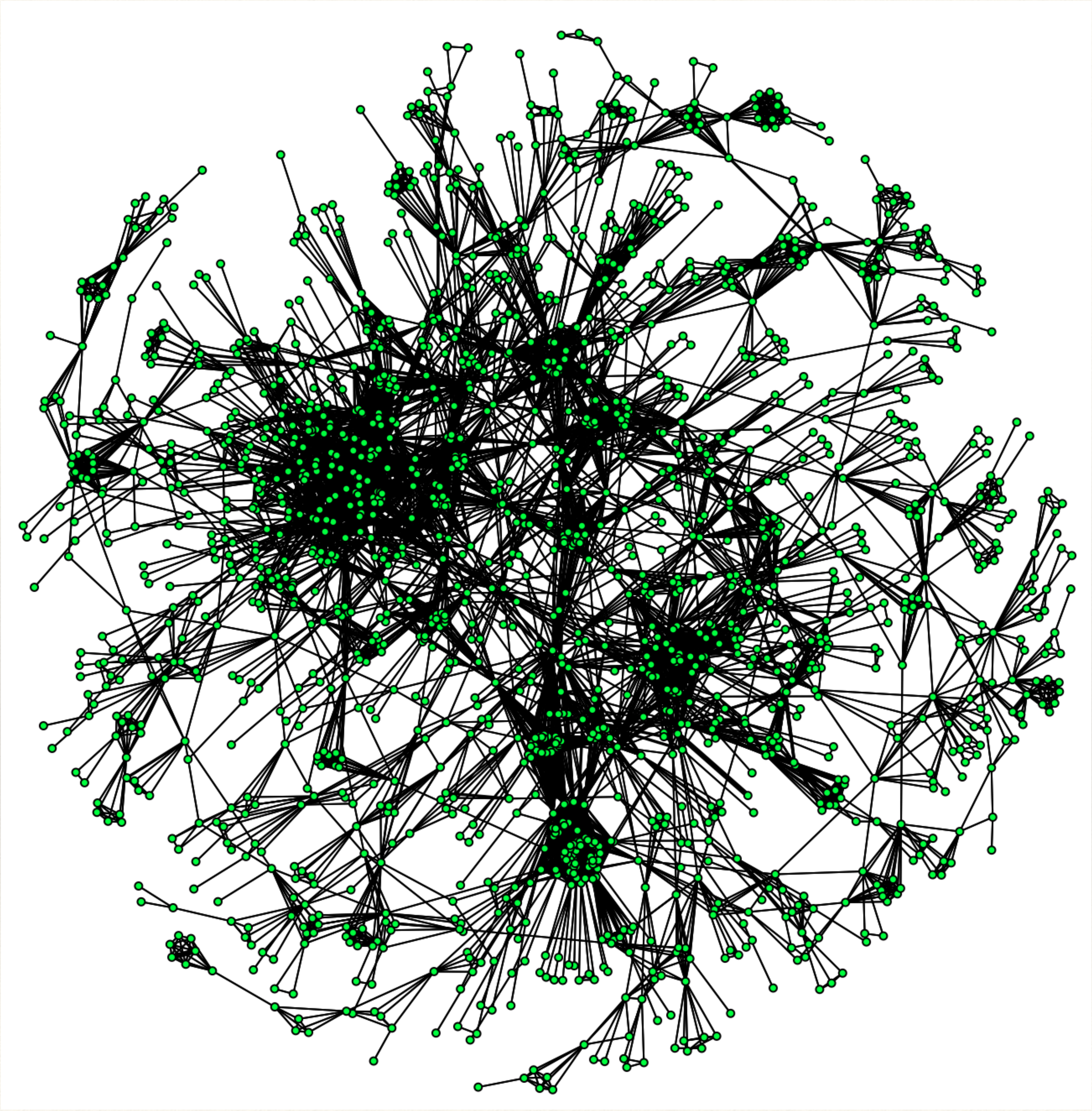












# Learning Goals

- At the end of the lecture, you should be able to answer these questions:
  - How can we create **unipartite** graphs from **bipartite** graphs?
  - What is the difference between **dichotomized** projections and **summation** projections?

# Questions?