

Statistical Analysis of Networks

Introduction to Exponential Random Graph Models

Motivating Example

JUSTICE QUARTERLY
2022, VOL. 39, NO. 3, 553–584
<https://doi.org/10.1080/07418825.2020.1807588>



Trusting the Untrustworthy: The Social Organization of Trust Among Incarcerated Women

Jacob T.N. Young^a and Dana L. Haynie^b

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ABSTRACT

Although the benefits of trust are well documented across a variety of settings, little empirical attention has been dedicated to trust in carceral settings, particularly among incarcerated women. Knowing how individuals in prison establish relationships of trust with one another is crucial for understanding how individuals adjust to conditions of confinement. Using data from 133 incarcerated women in a Pennsylvania prison unit, this study adopts a network approach to examine the role of individual and structural determinants of trust using exponential random graph models. Findings provide weak support for the claim that individual determinants (e.g. age, religious affiliation) shape whether women are more likely to trust someone to support them during an argument or a dispute. Instead, our findings show that structural determinants are the primary drivers of trust relationships. Trust is deeply entwined with friendship relations among women who get along with each other. Our approach paves a new path for the examination of trust in correctional settings and other criminological contexts.

ARTICLE HISTORY

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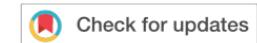
KEYWORDS

Trust; women; incarceration; network; ergm

- ❖ What does it *mean* to trust someone?

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- ❖ What does it *mean* to trust someone?
- ❖ Precision with conceptualization and operationalization

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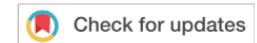
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- ❖ Trust people like you?
- ❖ Or, is it structural?

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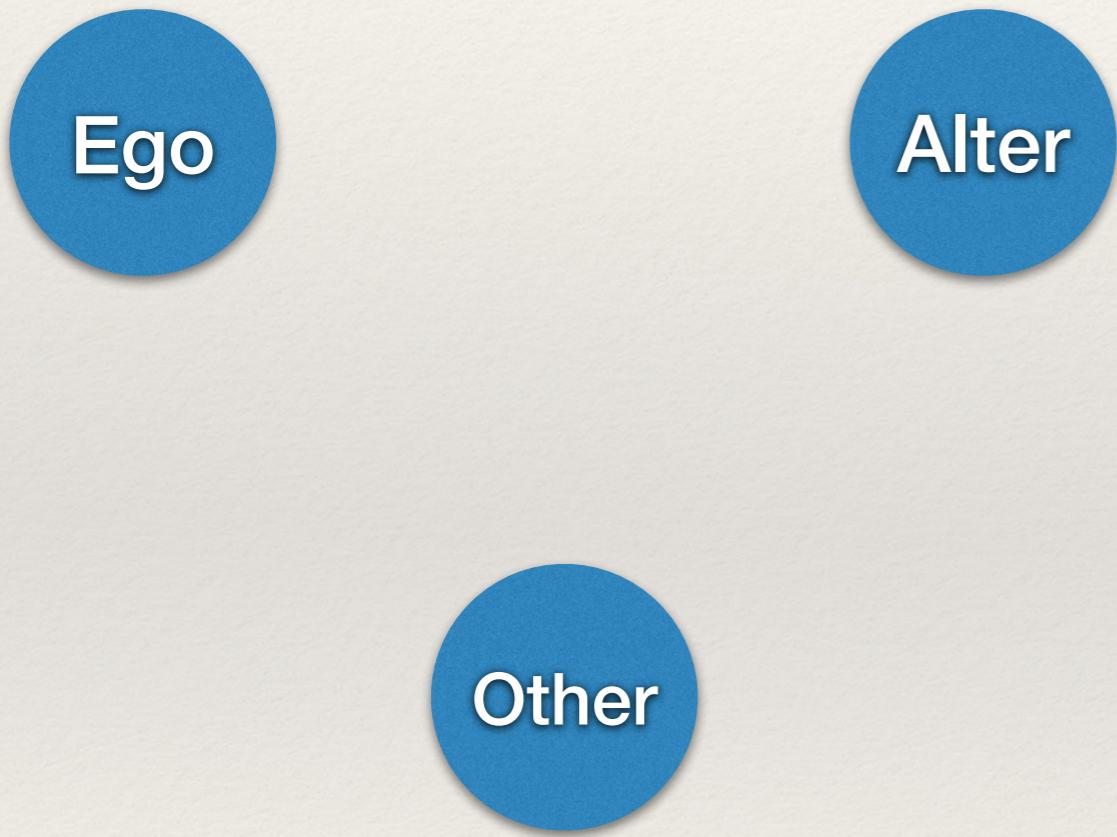
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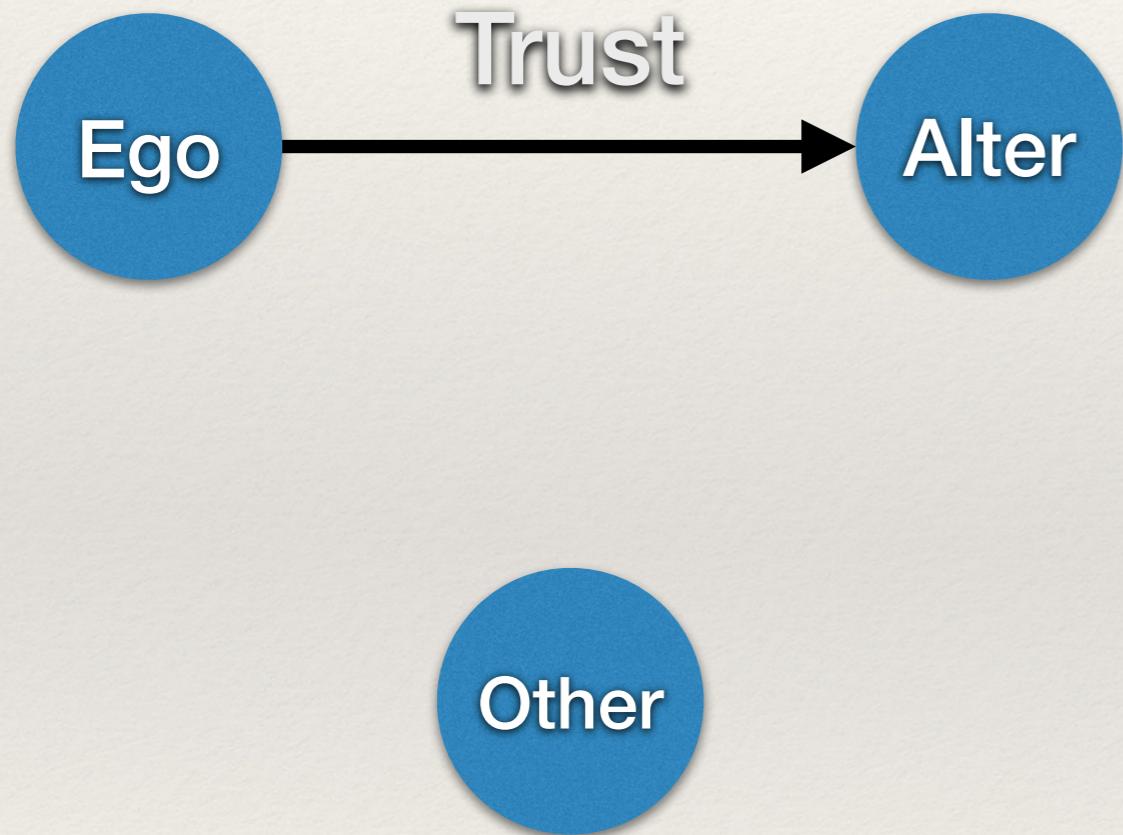
Motivating Example

- ❖ Is trust “embedded”? (Buskins 2002)
 - ❖ Dyadic Embeddedness
 - ❖ Network Embeddedness

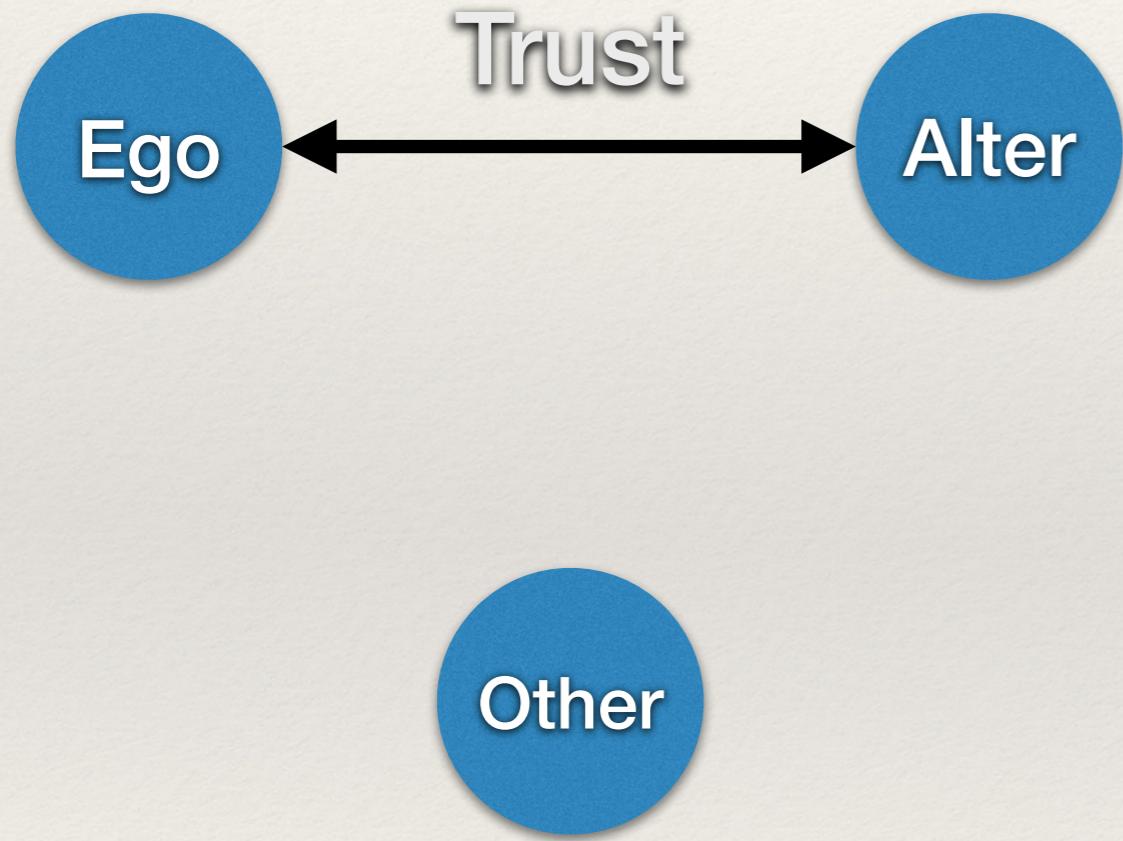
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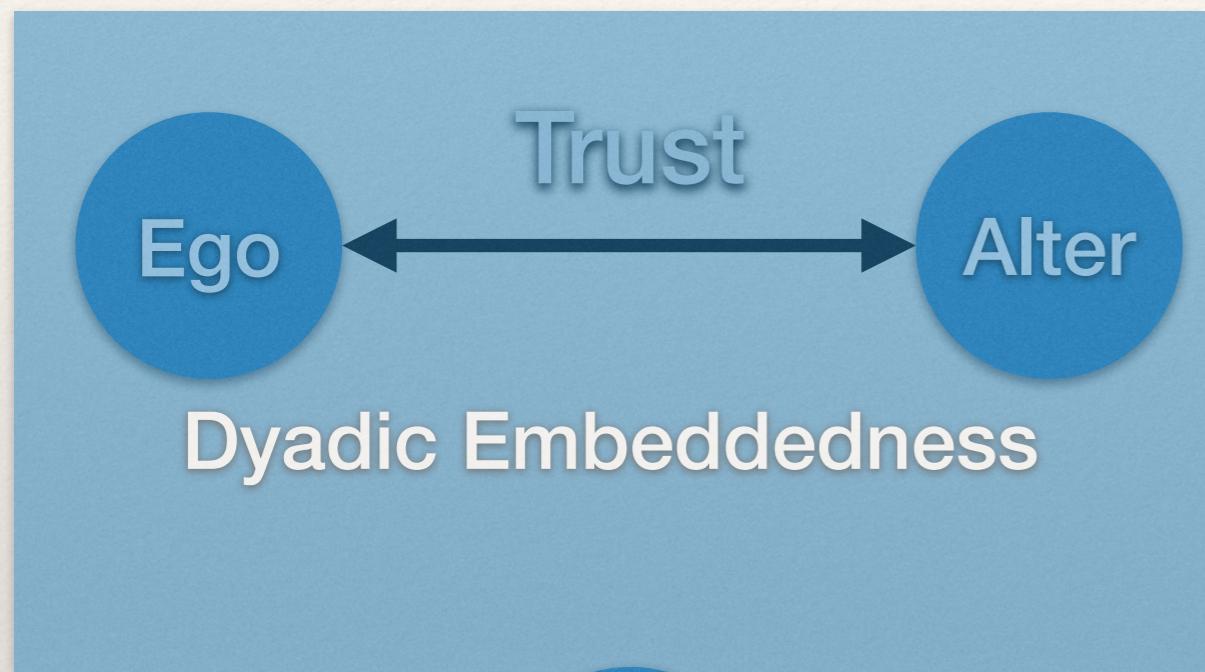
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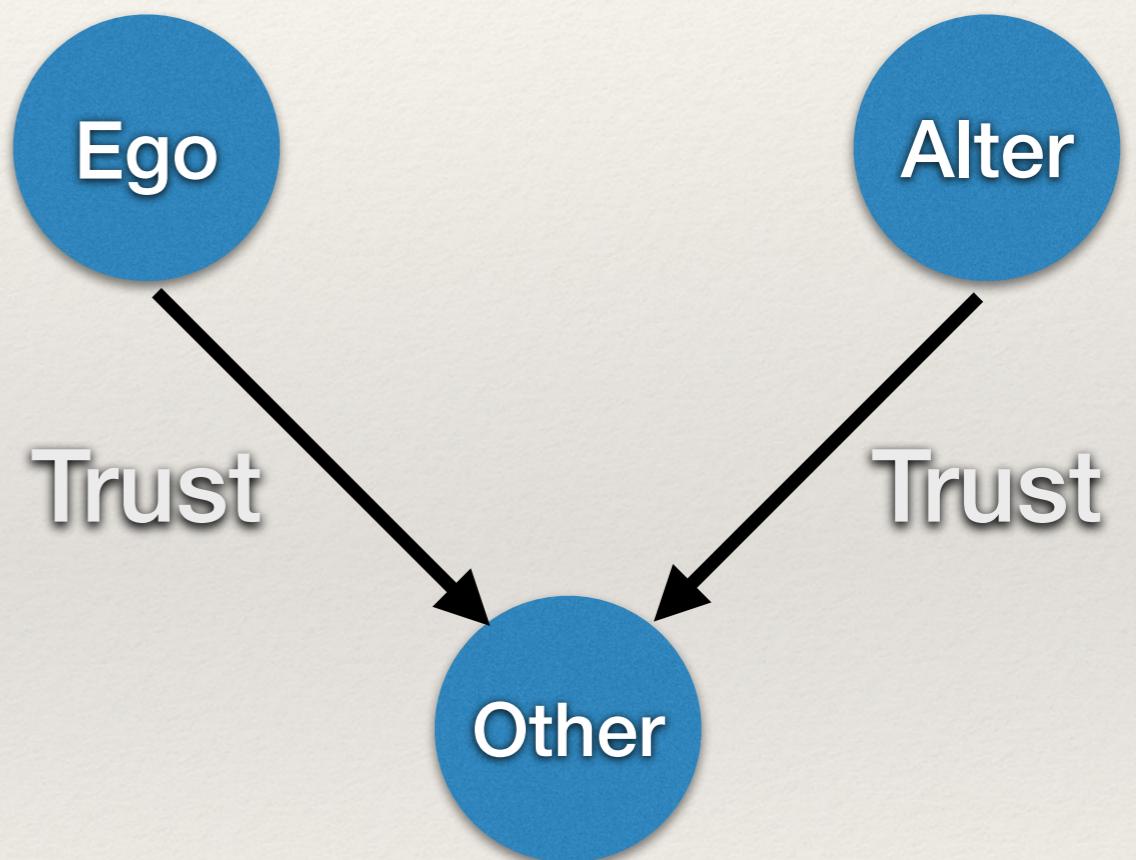
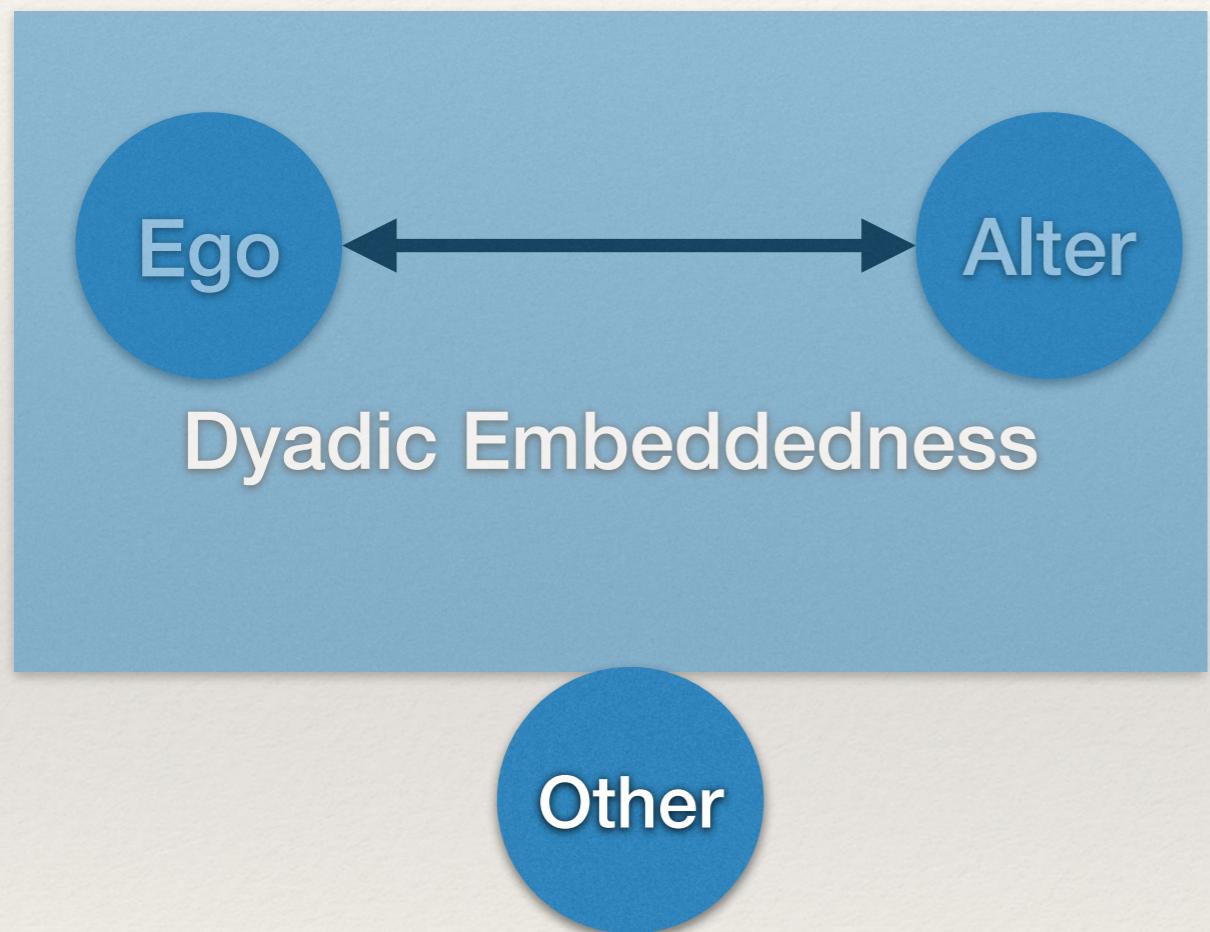
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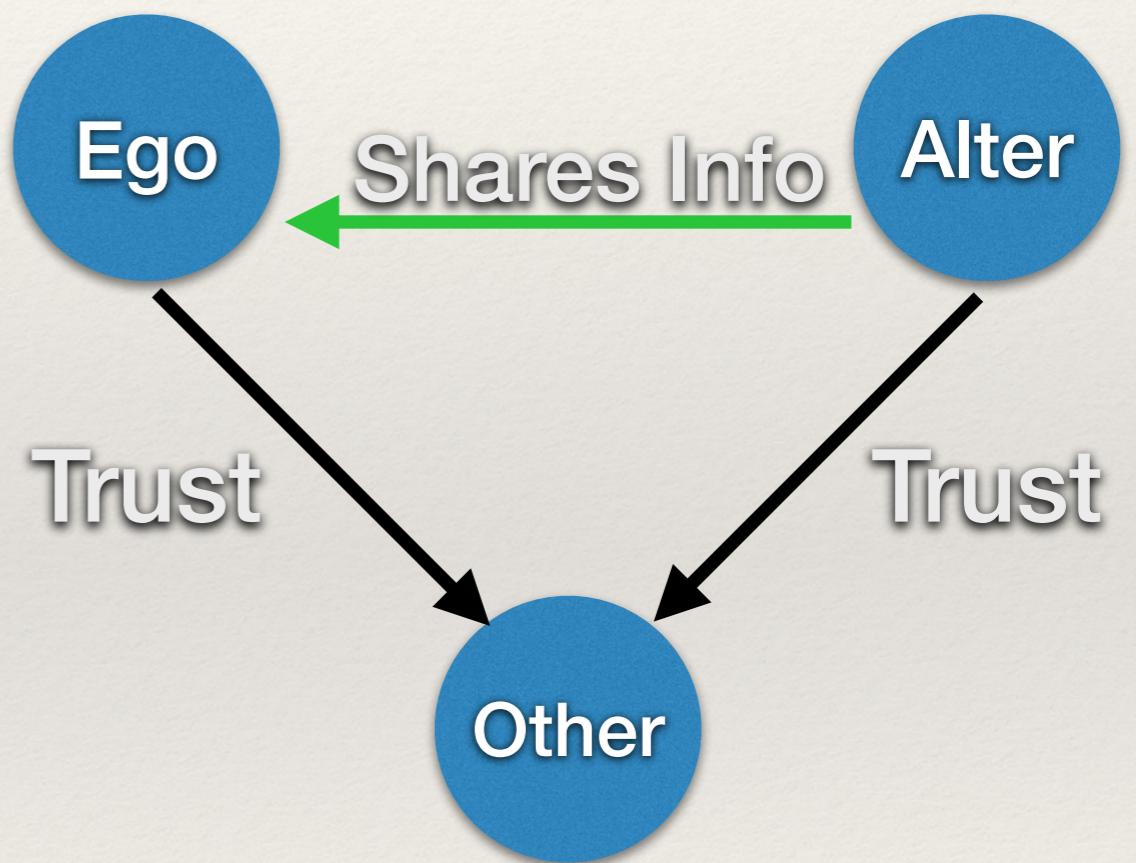
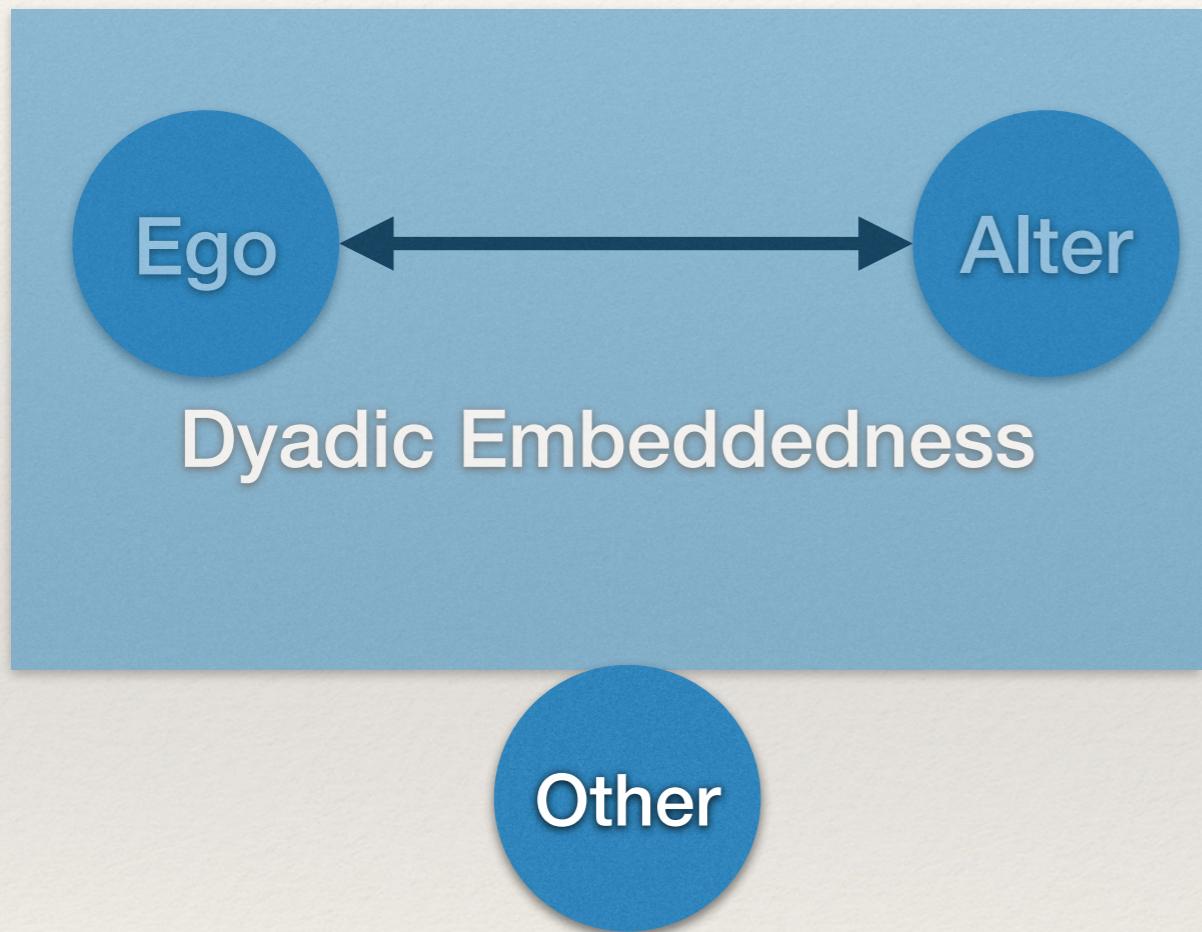
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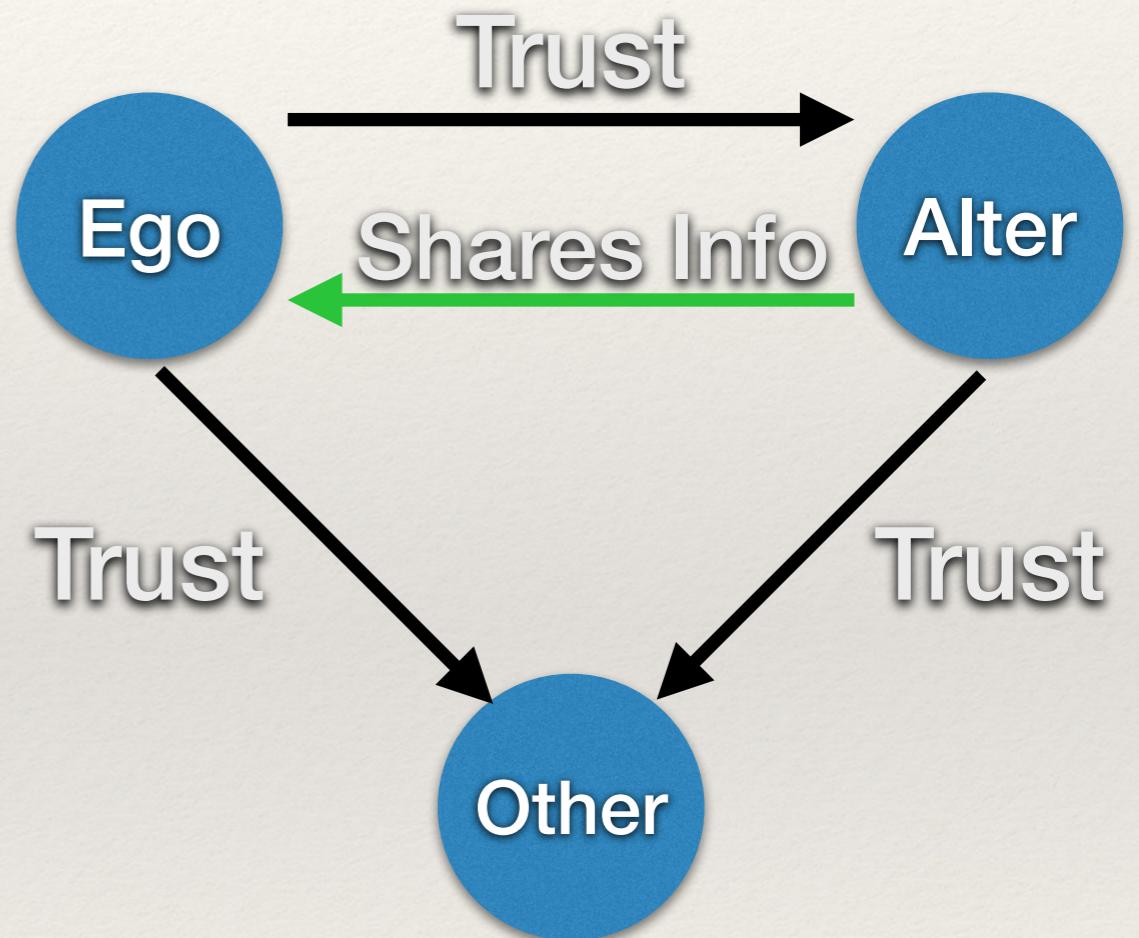
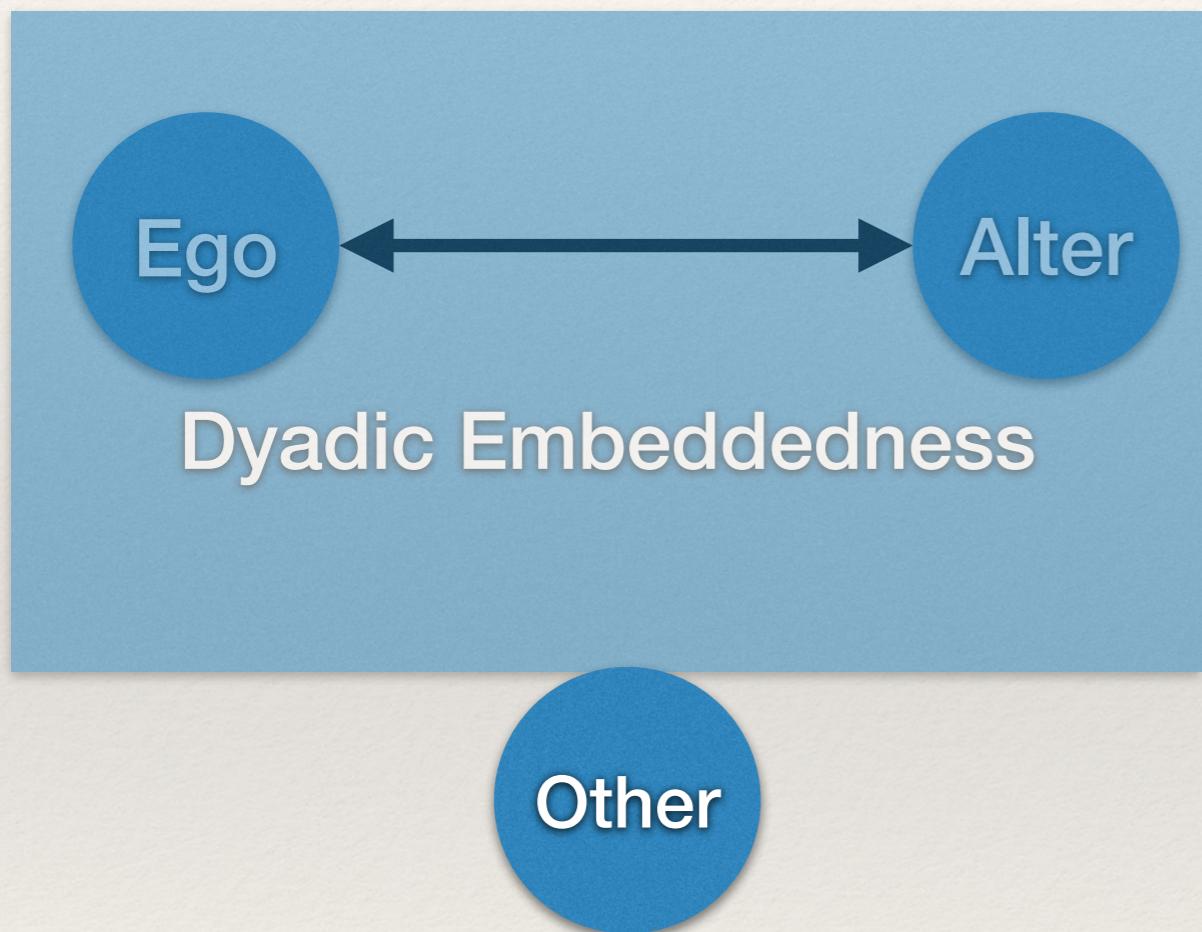
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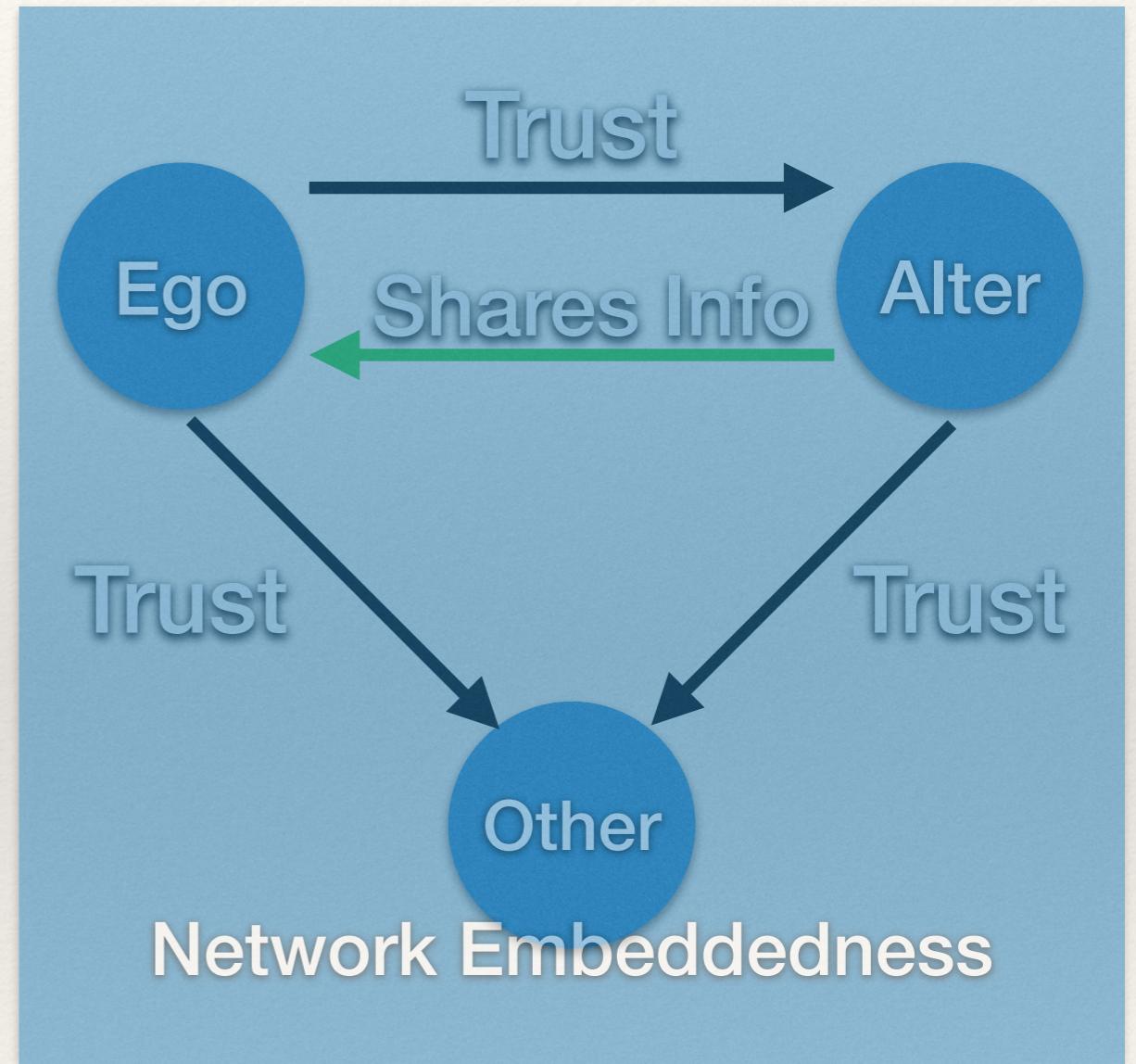
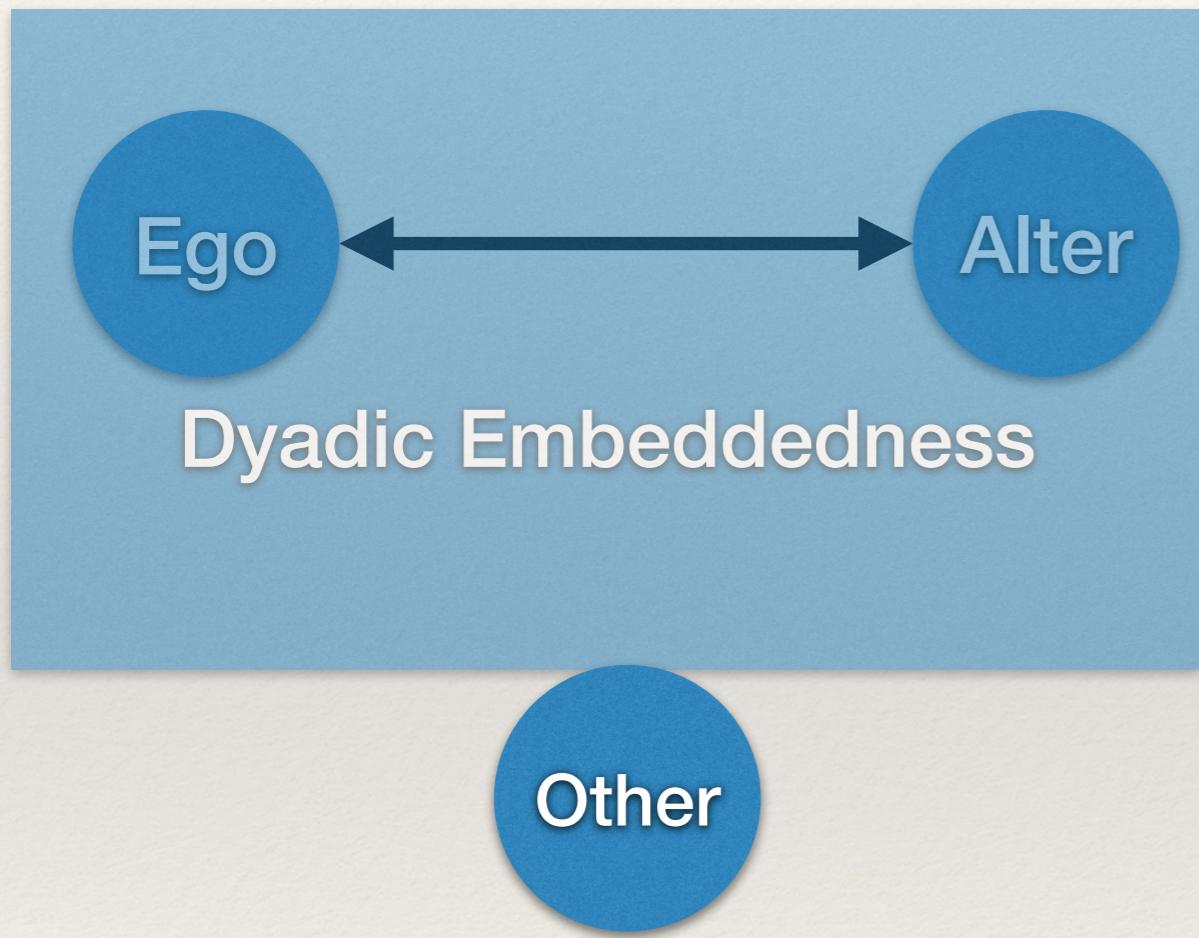
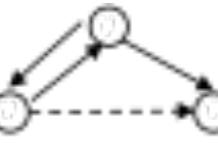


Table 1: Hypotheses and Network Configurations

Hypothesis	Network Configuration ^{a,b}
1. i is more likely to trust j if j is older, has served more time in prison, and has spent more time on the unit.	
2. i is more likely to trust j if j is the same religious affiliation as i	
3. i is no more likely to trust j if j is the same race/ethnicity as i	
4. i is more likely to trust j if i gets along with j	
5. i is more likely to trust j if j trusts i	
6. i is more likely to trust j if a) i trusts k and k trusts j and b) k gets along with i	
7. i is more likely to trust j as i 's brokerage in the get along with network increases	
8. i is less likely to trust j as j 's brokerage in the get along with network increases	
<i>Notes :</i>	
^a Dashed lines indicate the hypothesized trust tie.	
^b Black/White nodes indicate the presence/absence or higher/lower value of an attribute, respectively.	

- ❖ How to test these hypotheses?

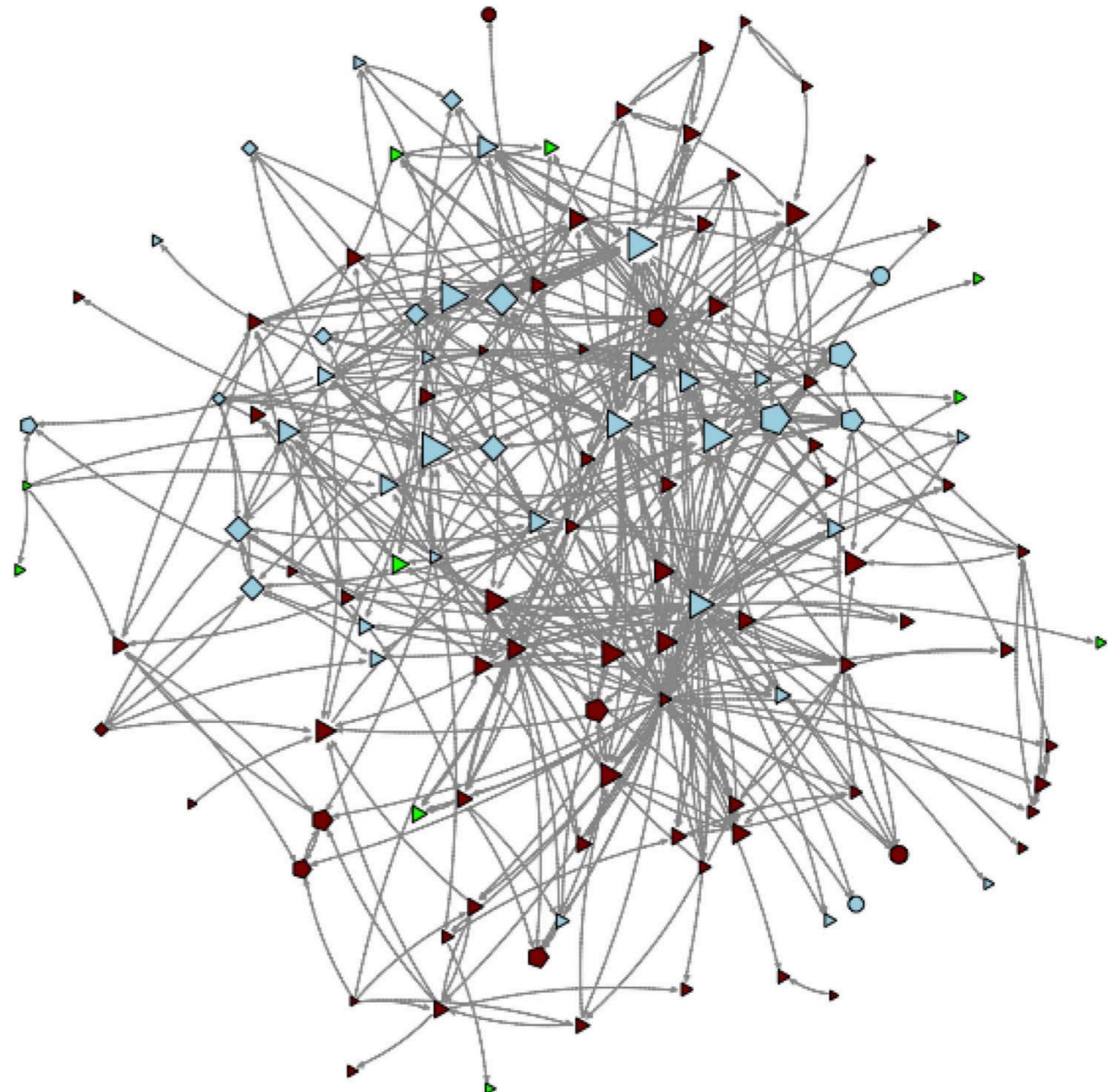


Figure 1. Trust Network for 131 Women.

Notes:

Nodes colored by Race/Ethnicity (White = Red; African American = Blue; Hispanic = Green).

Nodes shaped by Religious Affiliation (Triangle = Christian; Muslim = Square; None = Pentagon; Circle = Other).

Nodes sized proportional to indegree centrality (i.e. larger nodes received more trust nominations).

6 isolates excluded from plot.

❖ ERGMS!

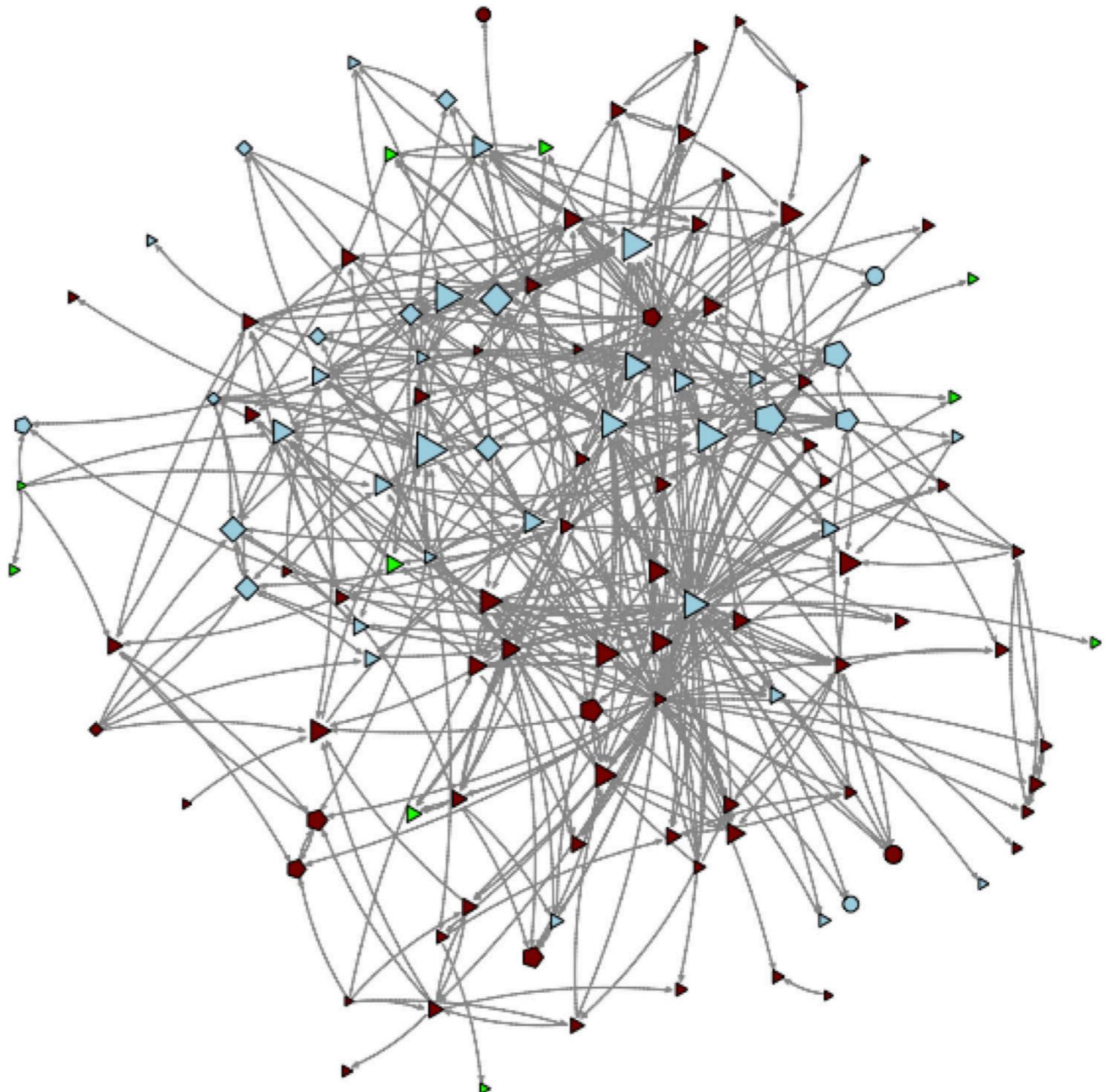


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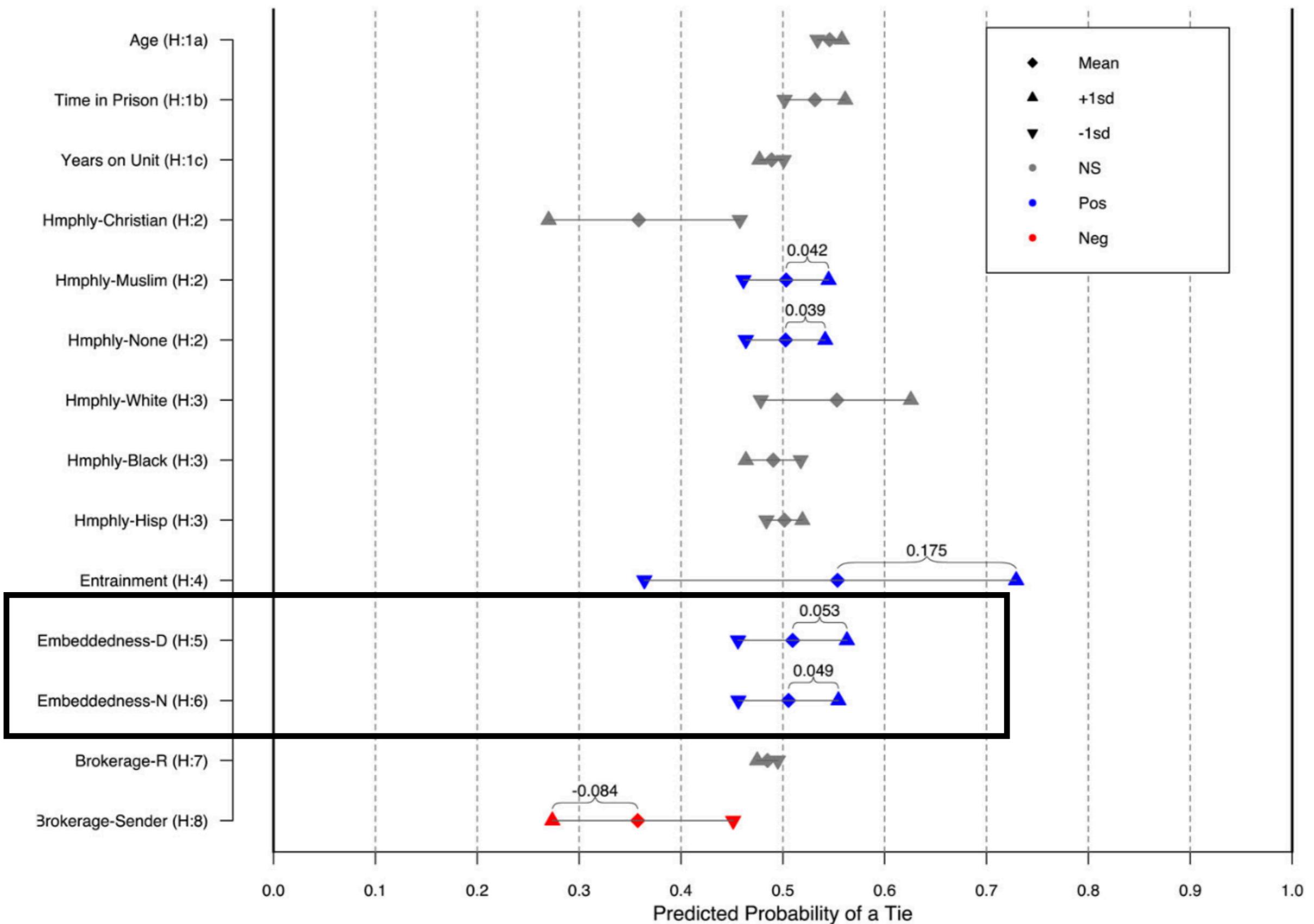


Figure 2. Predicted Probabilities for Selected Estimates (Estimates shown for Mean, $+1\text{sd}$, -1sd).

Statistical Analysis of Networks

Introduction to Exponential Random Graph Models

Learning Goals

- ❖ By the end of this lecture, you should be able to answer these questions:
 - ❖ What is the underlying logic of exponential random graph models (ERGMs)?
 - ❖ What is the historical development of ERGMs and what are the various properties of the models that were developed over time?
 - ❖ What are *network configurations* and how do they help us operationalize theoretical concepts?

Introduction

- ❖ **So far:**
 - ❖ We have been doing “descriptive statistics” with network data.
 - ❖ Example: how are the degrees distributed in this network?
- ❖ **Now:**
 - ❖ We want to shift toward “inferential statistics” with network data.
 - ❖ Example: is the distribution of degrees different from a network where ties form at random?

Introduction

- ❖ **New questions:**
 - ❖ *How do networks form?*
 - ❖ *What are the micro patterns that generate global structure?*
 - ❖ *How likely is it that we would observe these configurations if ties formed at random?*
- ❖ Exponential Random Graph Models or ERGMs, provide a **model** (an account of what **governs the formation of a network**) for examining such questions.

Modeling Networks

- ❖ Why should we care? Why not be satisfied with descriptive statistics?
 - ❖ Complexity and randomness
 - ❖ Statistical inference and hypothesis testing
 - ❖ Global structure from local structure (micro-macro problem)

Logic of Random Graph Modeling

- ❖ We have an **observed network**, and want to know about the **stochastic** process by which it came about.
- ❖ Conceptual Analog:
 - ❖ Sampling from a normal distribution.
 - ❖ We don't get the **exact** same data (i.e. sample *statistics* differ from population *parameters*).
 - ❖ But, there is some **process** generating our sample statistics (i.e. central tendency and dispersion).

Logic of Random Graph Modeling

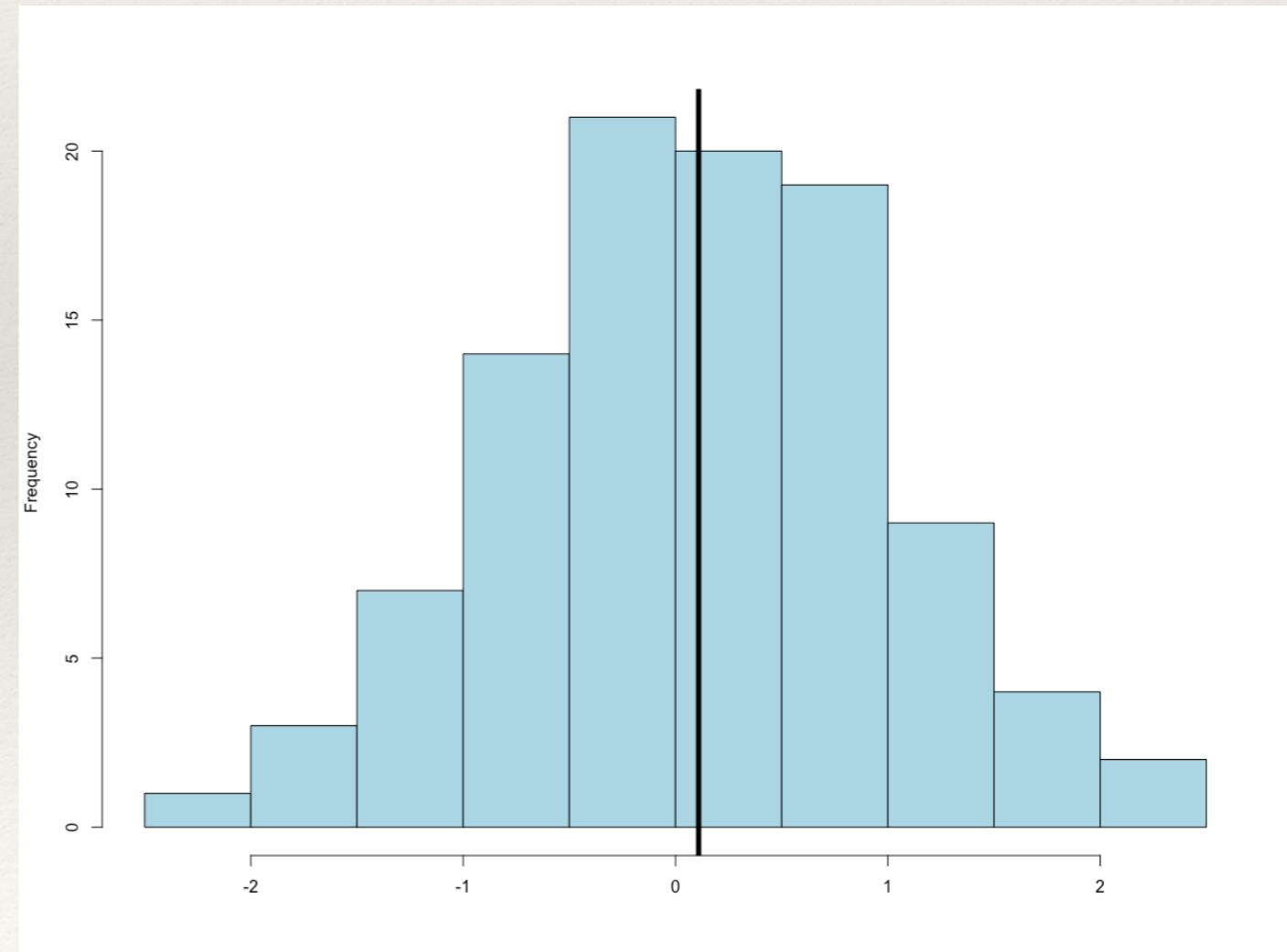
- ❖ Simulate some data from a standard normal distribution and calculate the mean.

Logic of Random Graph Modeling

- ❖ Simulate some data from a standard normal distribution and calculate the mean.

100 random
draws from a
distribution with
a mean = 0

Sample mean = 0.108

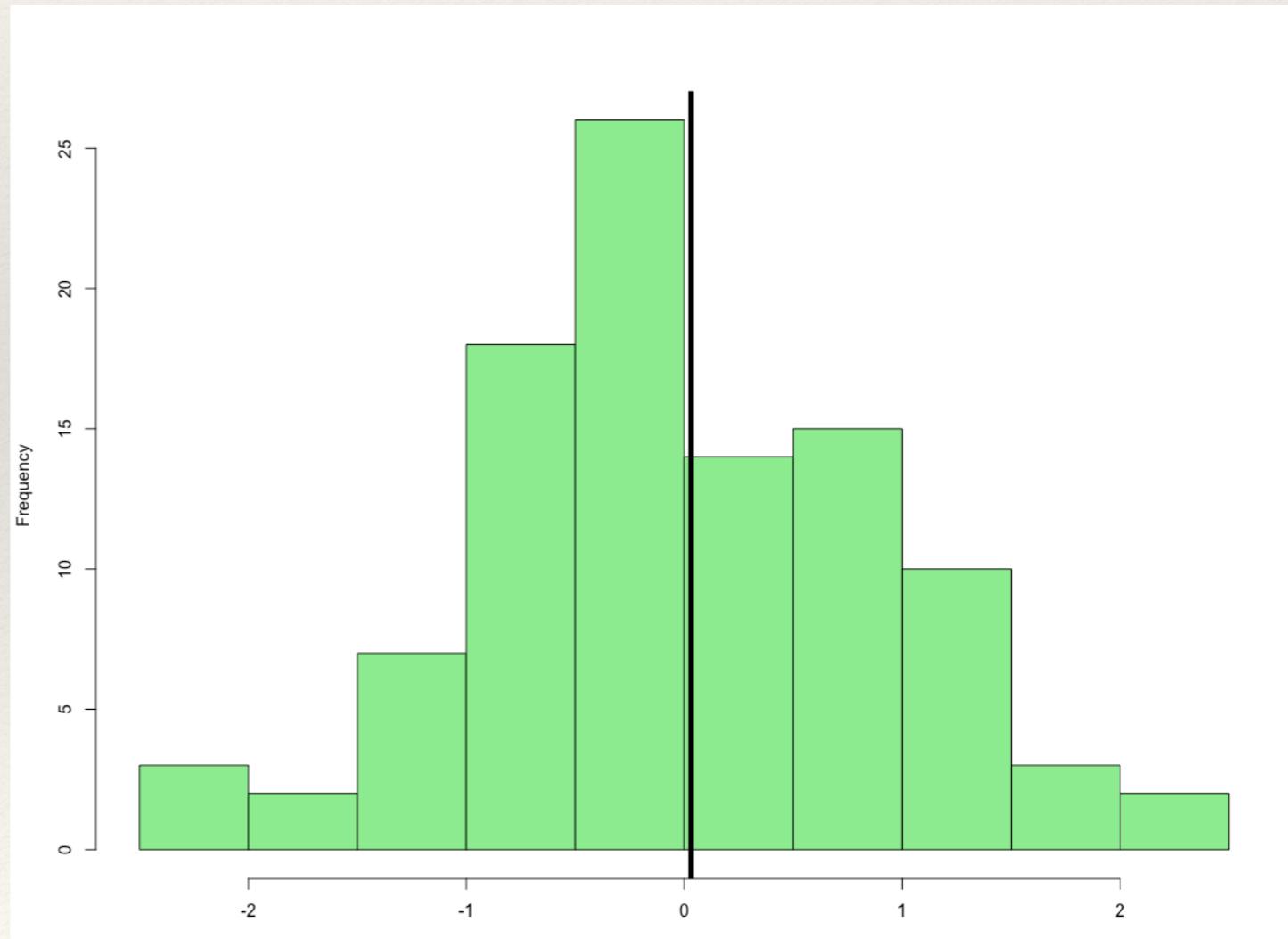


Logic of Random Graph Modeling

- ❖ Simulate some data from a standard normal distribution and calculate the mean.

100 random
draws from a
distribution with
a mean = 0

Sample mean = 0.031

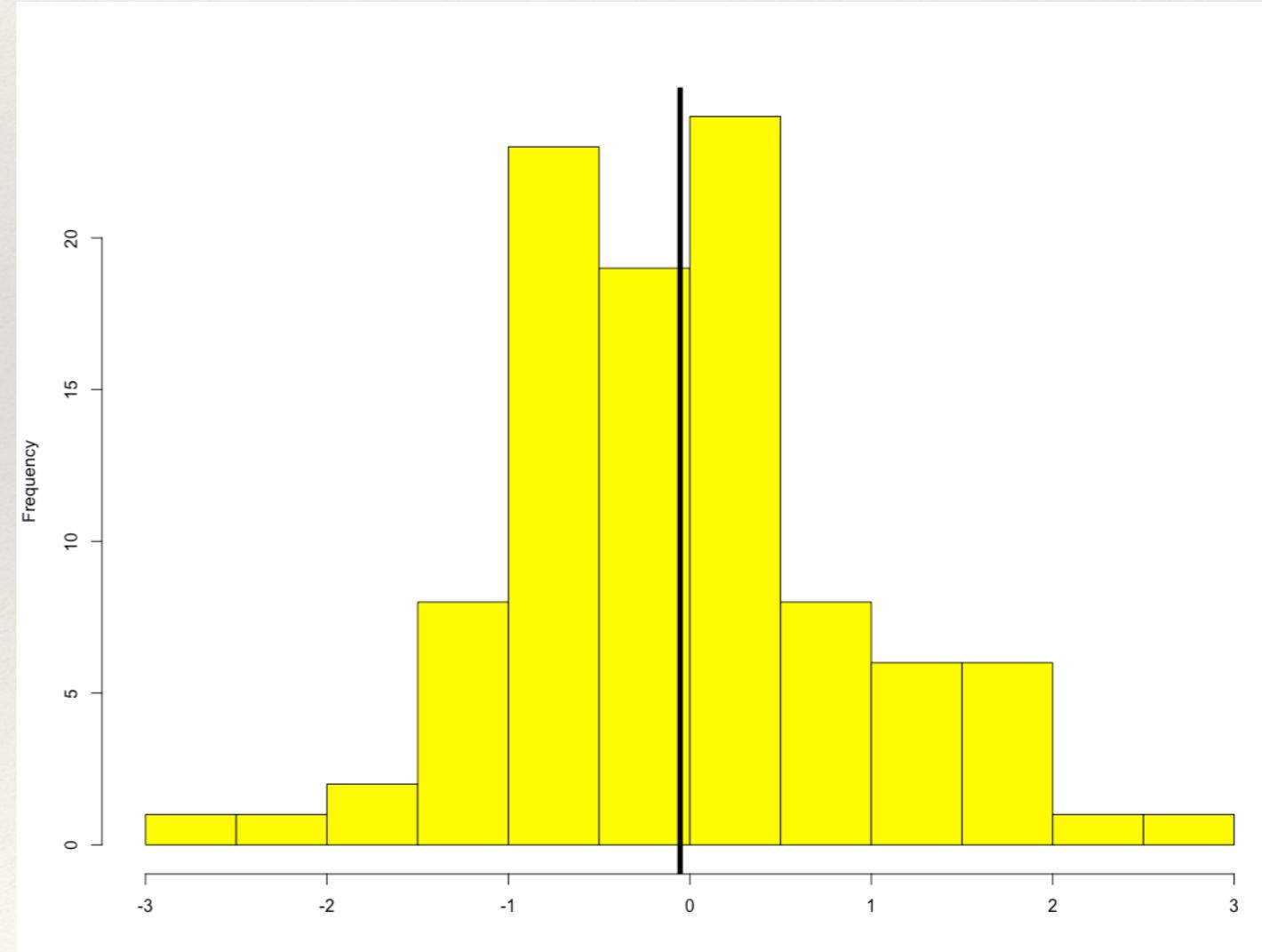


Logic of Random Graph Modeling

- ❖ Simulate some data from a standard normal distribution and calculate the mean.

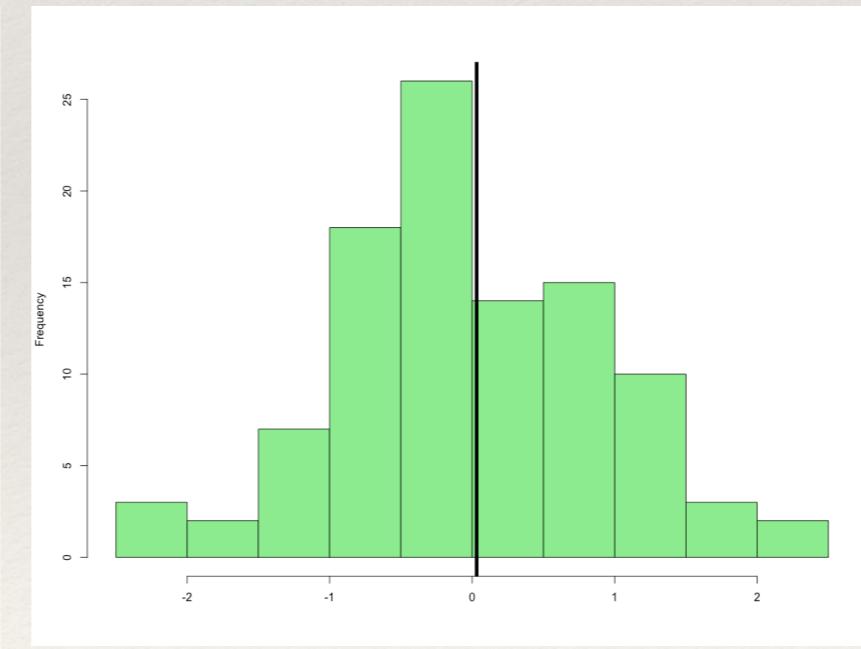
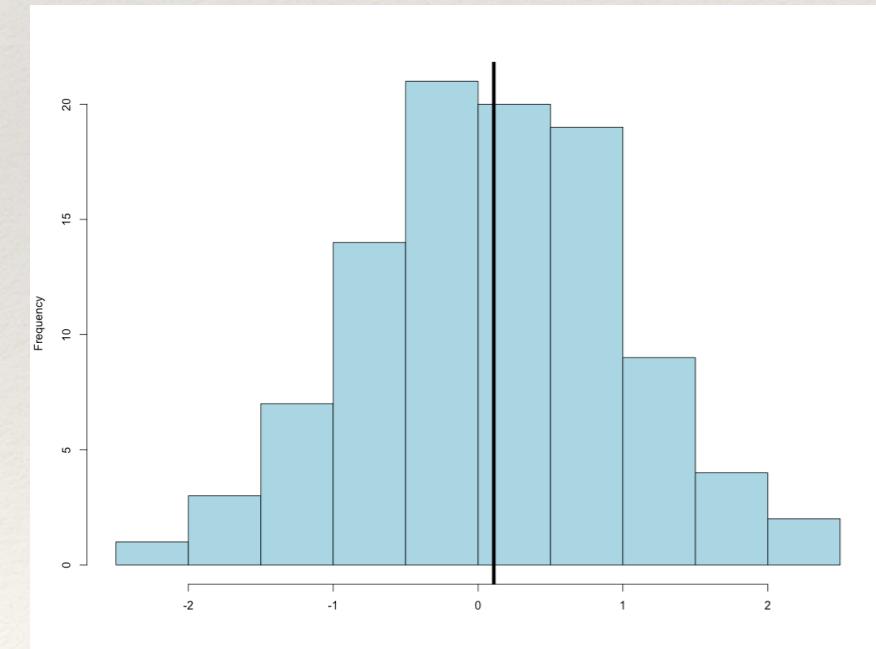
100 random
draws from a
distribution with
a mean = 0

Sample mean = -0.053



Logic of Random Graph Modeling

- ❖ There are data generated from the same distribution, but have different means.

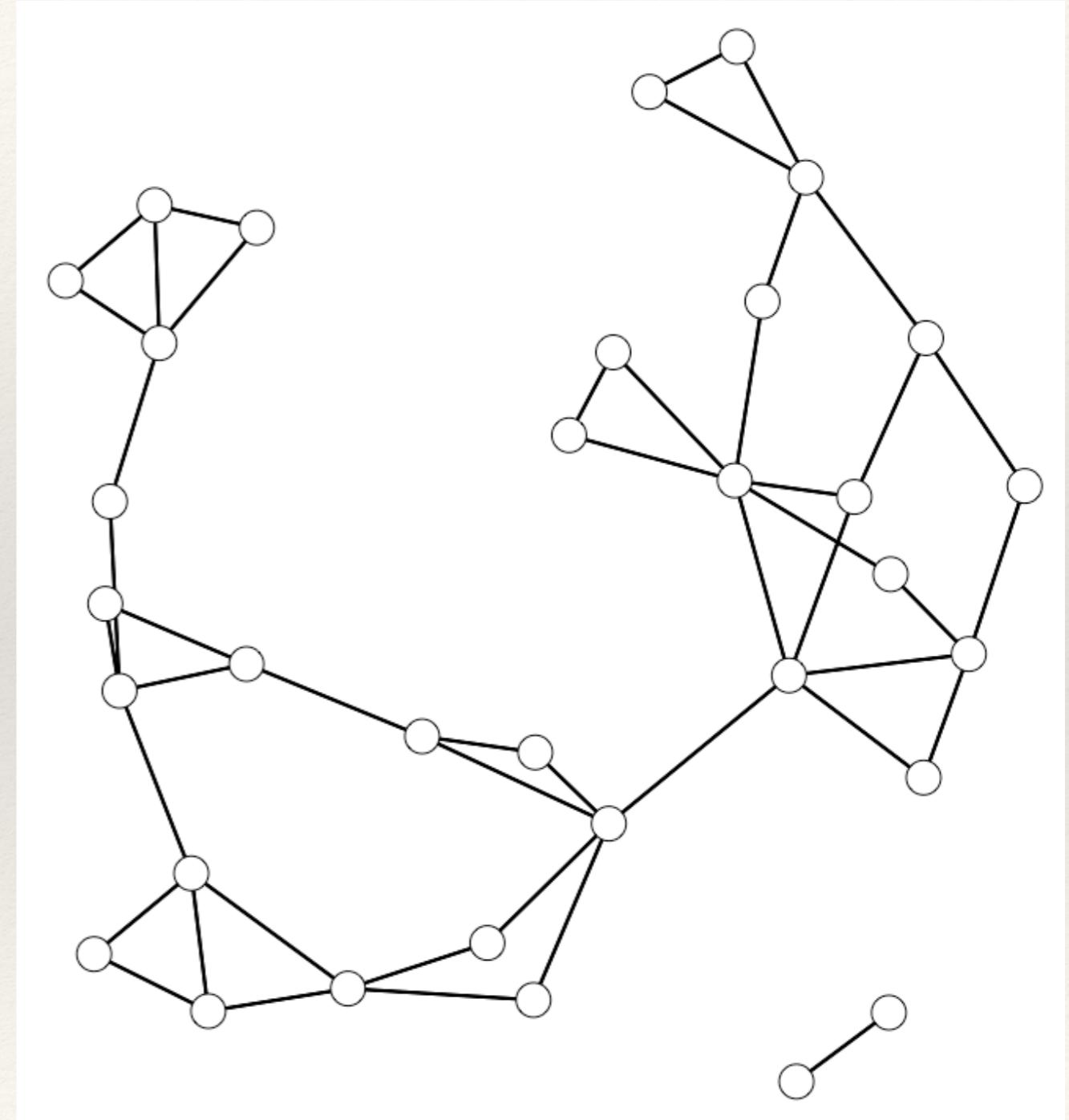


Logic of Random Graph Modeling

- ❖ In a similar way, we want to examine the *parameters* which generate the network we have observed.
 - ❖ Did it come about because people:
 - ❖ Reciprocate relationships?
 - ❖ Nominate popular others?
 - ❖ Close triads?

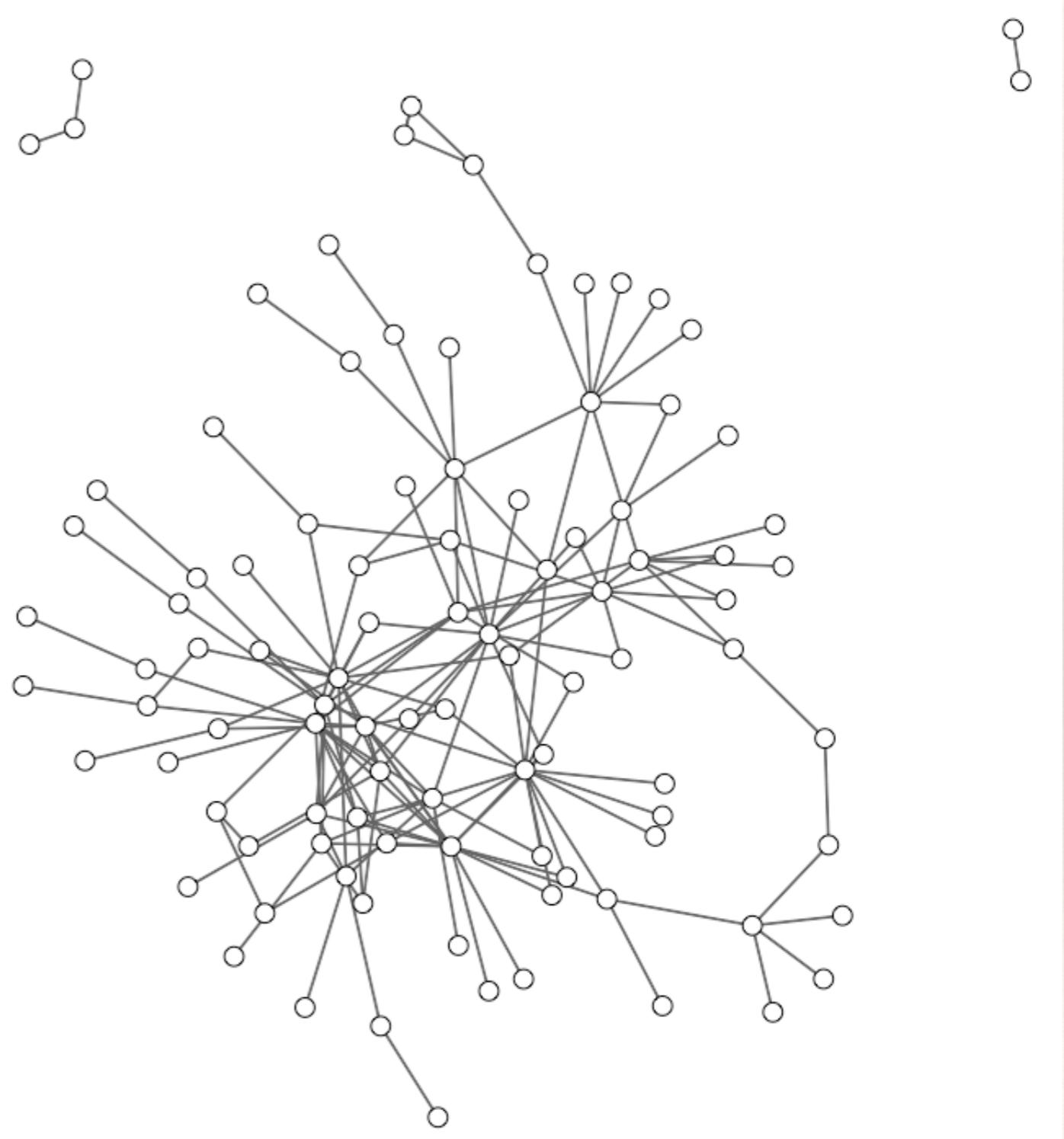
Example

What pattern
do you see in
these data?



Example

What pattern
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Emergent Structure

- ❖ Different processes can lead to similar outcomes.

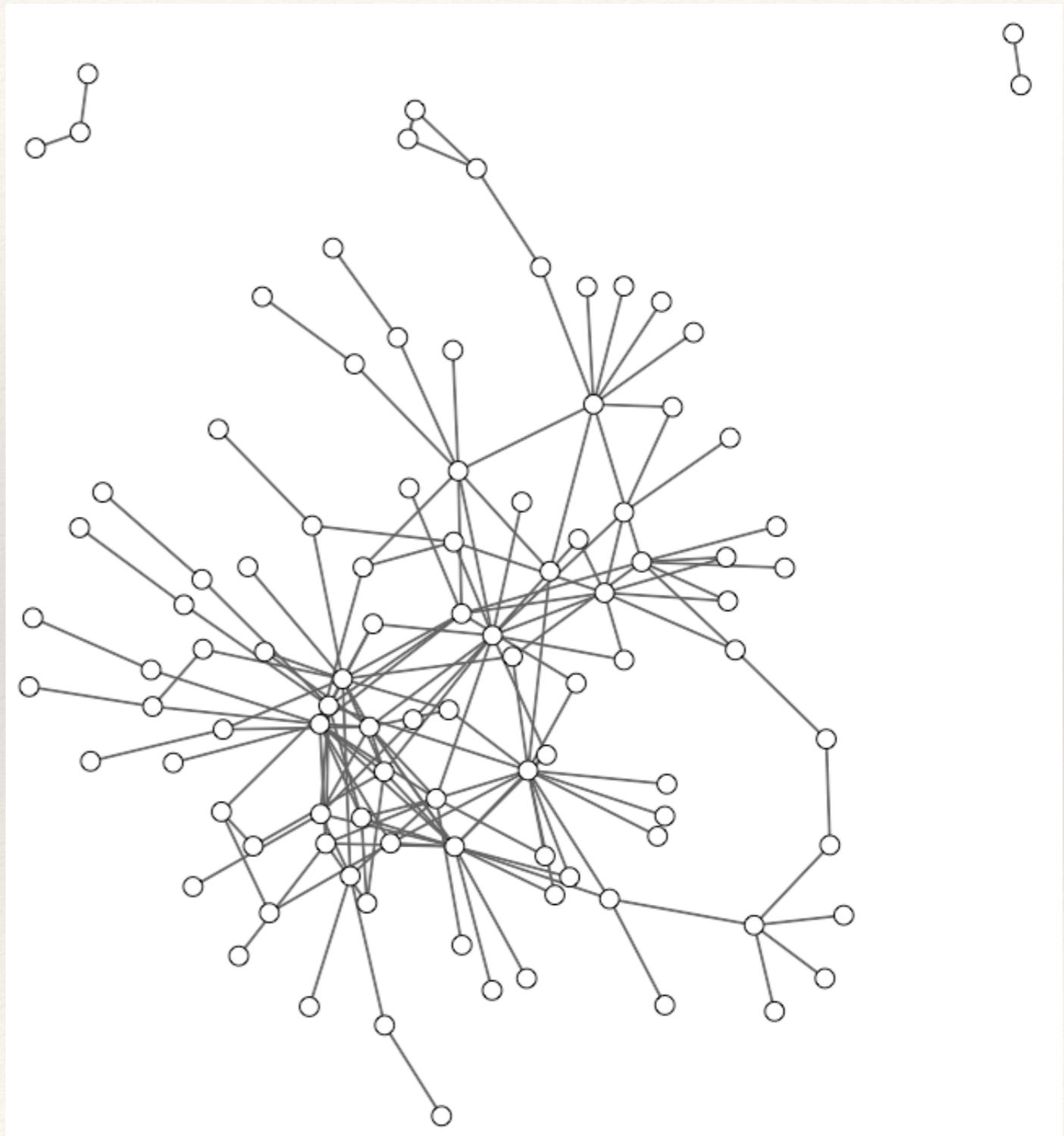
Example

Different processes can lead to similar outcomes:

Sociality-highly active persons create clusters.

Homophily-assortative mixing by attribute creates clusters.

Transitivity-triangles create clusters.



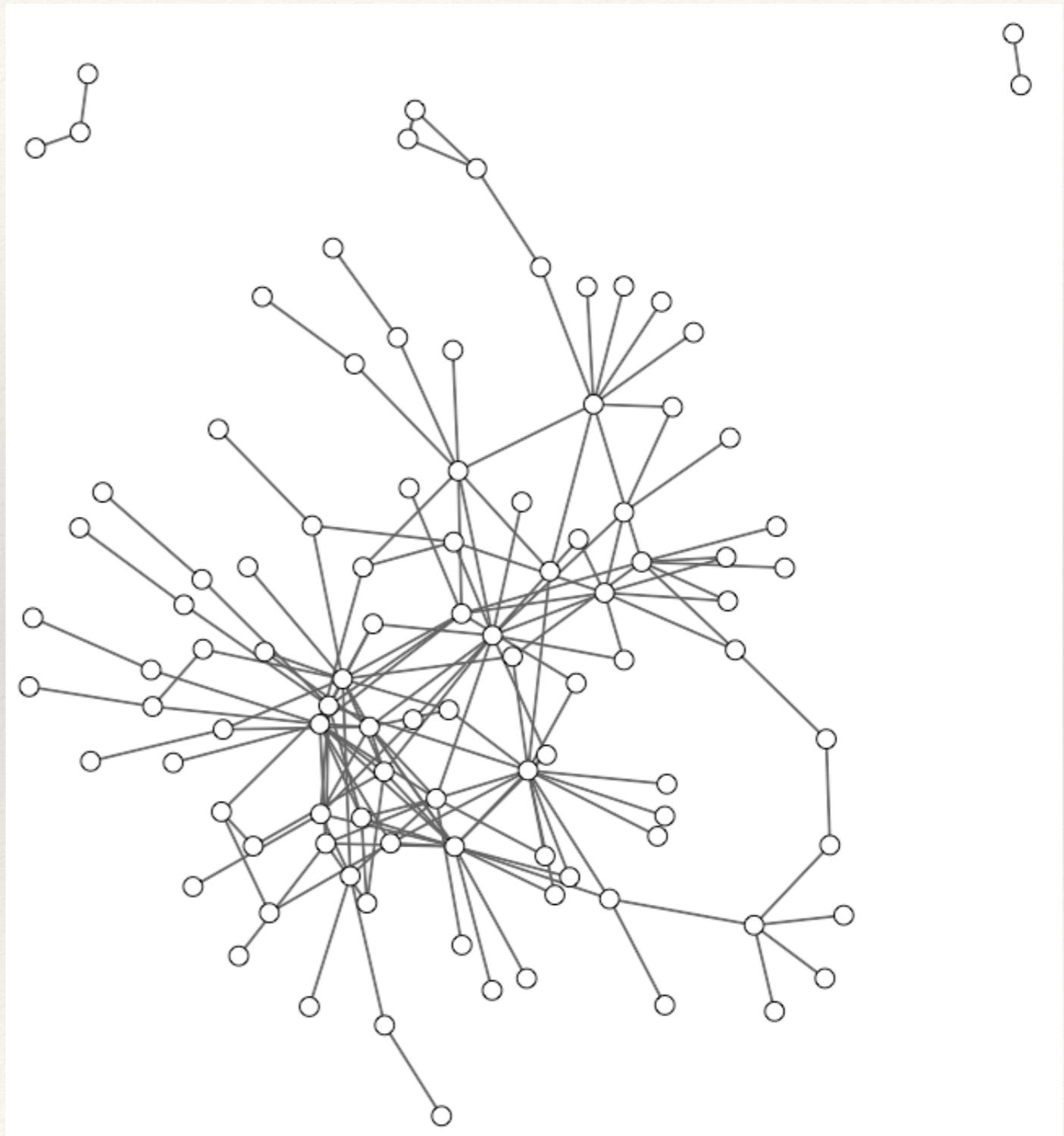
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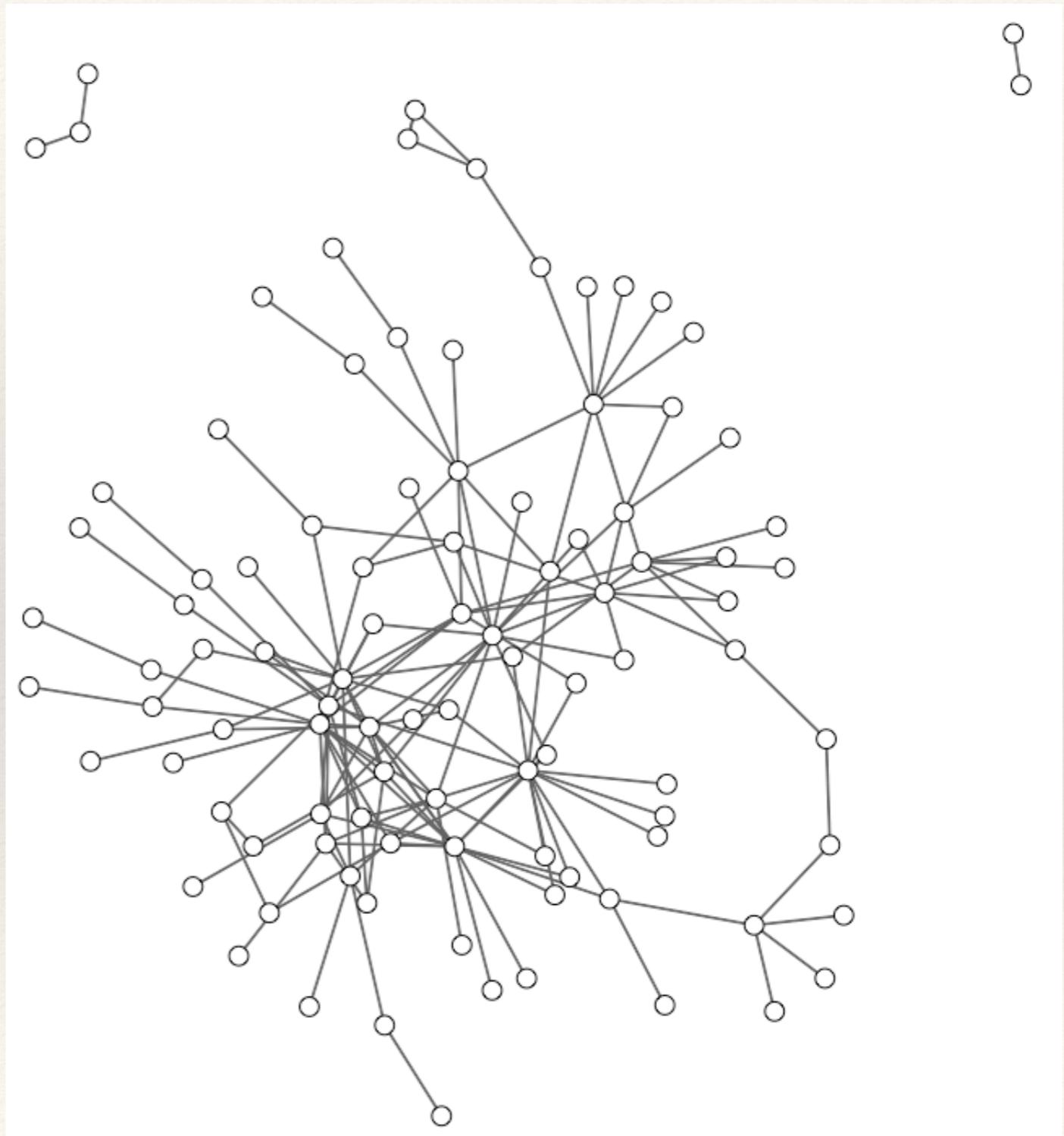
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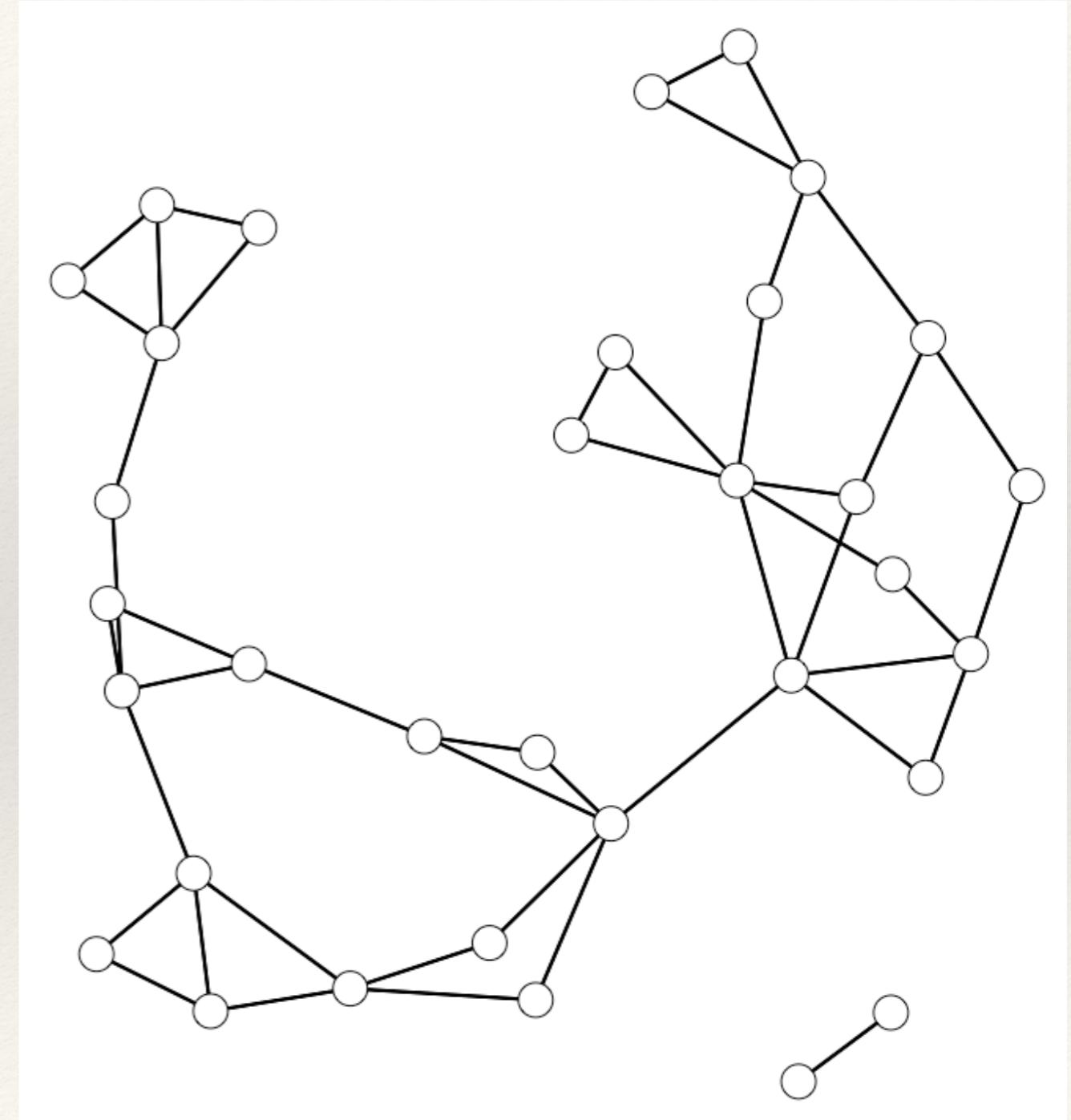
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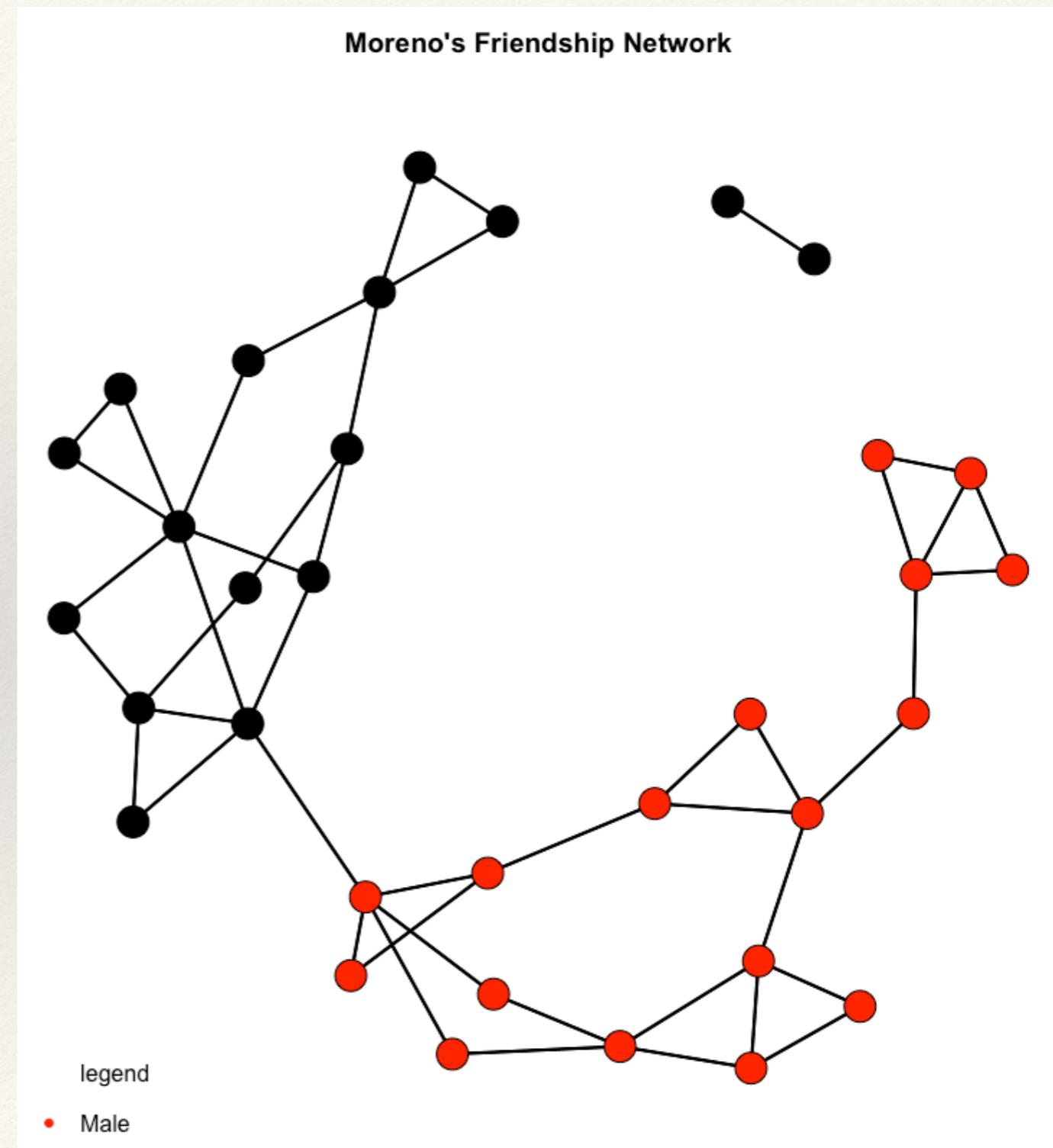
Example

What pattern
do you see in
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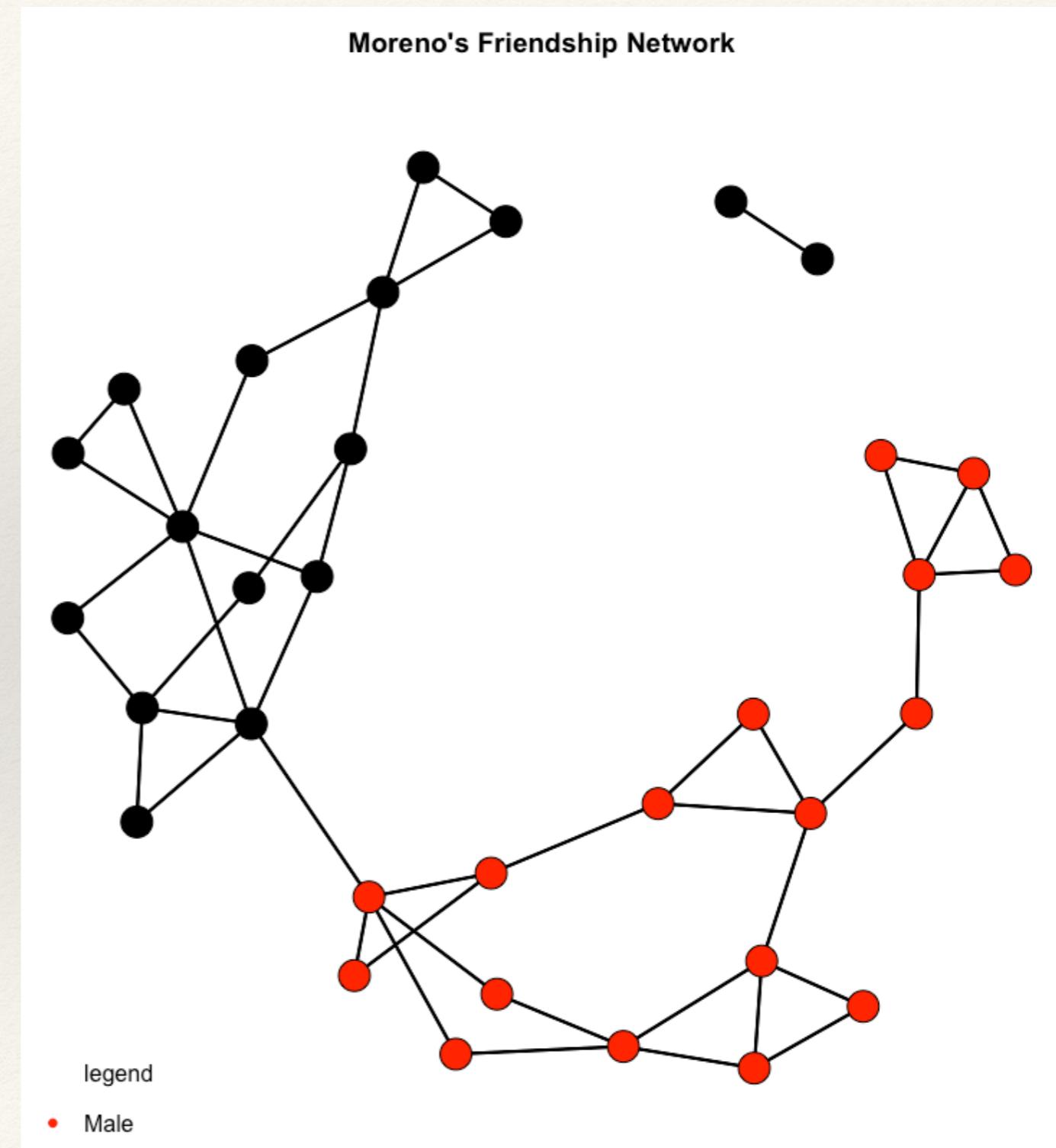
Example

Here, it is probably matching based on attributes.



Example

But, we can
test that
hypothesis!

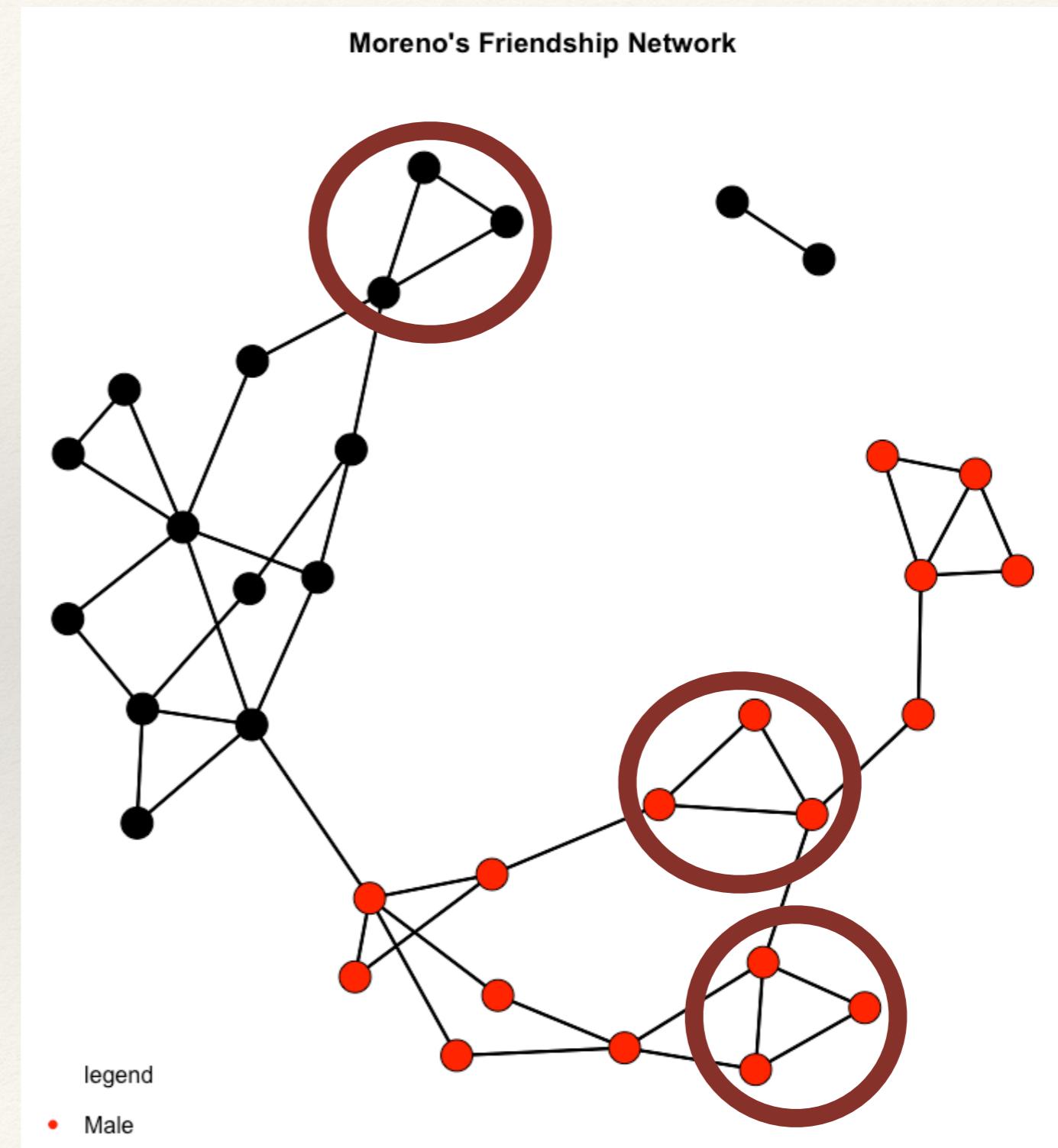


Example

- ❖ Think about a triad of individuals with the same attribute...

Example

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Example

- ❖ Think about a triad of individuals with the same attribute...
 - ❖ Did this configuration occur because:
 - ❖ People tend to choose friends who are like them? (“birds of a feather”)
 - ❖ People who have friends in common tend to become friends? (“friend of a friend”)

Emergent Structure

- ❖ Different processes can lead to similar outcomes.
- ❖ We want to be able to fit these terms simultaneously and identify the independent effects of each process on the overall outcome.

Logic of Random Graph Modeling

- ❖ We can then evaluate how probable a particular network is among all possible networks that could exist on a set of actors.
 - ❖ The observed network is only one realization from a set of possible networks with similar characteristics (think back to the sampling example).
- ❖ Robins et al. (2007: 176)
 - ❖ “The range of possible networks, and their probability of occurrence under the model, is represented by a *probability distribution* on the set of all possible graphs”

Logic of Random Graph Modeling

- ❖ Once we have estimated the parameters of the probability distribution, we can sample graphs at random and compare their characteristics with those of the observed network.
 - ❖ If the model is good, then sampled graphs will resemble the observed network (visually and descriptively)
 - ❖ If this is the case, we can conjecture that the modeled structural effects could explain the emergence of the network.

Exponential Random Graph Formulation

- ❖ Express the probability of observing a tie between nodes i and j given some terms (i.e. network configurations).
- ❖ A general framework for expressing different types of models.
 - ❖ Think of each model as “theory of network dependence”.
 - ❖ We will look at four model types.

Dependence Assumptions

- ❖ Edge independence (Bernoulli/Simple Random graphs)
 - ❖ *How likely is a tie between i and j?*
 - ❖ Erdos and Renyi (1959)
 - ❖ The probability of a tie is the number of edges.

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c} \right) \exp\{\theta L(y)\}$$

Exponential Random Graph Formulation

Probability of a tie
being observed

$$P(Y = y) = \left(\frac{1}{c} \right) \exp\{\theta L(y)\}$$

Exponential Random Graph Formulation

Probability of a tie
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Normalizing
constant

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c} \right) \exp\{\theta L(y)\}$$

Probability of a tie
being observed

Normalizing constant

coefficient for the
edge term

The diagram illustrates the components of the Exponential Random Graph formulation. It shows the probability mass function $P(Y = y)$ as a product of a normalizing constant $\left(\frac{1}{c}\right)$ and a weight function $\exp\{\theta L(y)\}$. Annotations with arrows identify these parts: a downward arrow points to the weight function from the text 'Probability of a tie being observed'; an upward arrow points to the normalizing constant from the text 'Normalizing constant'; and a vertical arrow points to the coefficient θ from the text 'coefficient for the edge term'.

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c} \right) \exp\{\theta L(y)\}$$

Probability of a tie
being observed

Normalizing
constant

Number of
edges in y

coefficient for the
edge term

The diagram illustrates the components of the Exponential Random Graph Formulation. The equation is $P(Y = y) = \left(\frac{1}{c} \right) \exp\{\theta L(y)\}$. Four annotations provide context for each part:

- "Probability of a tie being observed" points to the term $P(Y = y)$.
- "Normalizing constant" points to the fraction $\left(\frac{1}{c} \right)$.
- "Number of edges in y " points to the term $L(y)$.
- "coefficient for the edge term" points to the coefficient θ .

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c} \right) \exp\{\theta L(y)\}$$



Note the absence of
any dependence
with other edges

Dependence Assumptions

- ❖ Edge independence (Bernoulli/Simple Random graphs)
 - ❖ Assumes all ties are independent of one another.
 - ❖ Assumes nodes do not vary in their tie propensity.
 - ❖ Does a poor job capturing: clustering, the degree distribution(s).
 - ❖ But, a **baseline** model: a good reference for comparing more complex models.

Dependence Assumptions

- ❖ Dyadic independence (p1 models)
 - ❖ *How likely is a tie between i and j?*
 - ❖ Holland and Leinhardt (1981)
 - ❖ Depends on the attractiveness of the node, as nodes differ in their indegree.
 - ❖ Depends on whether the tie is reciprocal, Did j send a tie to i ?

Exponential Random Graph Formulation

$$P(Y = y) \propto \exp \left(\mu L(y) + \sum_i^N \alpha_i y_{i+} + \sum_j^N \beta_j y_{+j} + \rho M(y) \right)$$


Number of edges in
the network

Exponential Random Graph Formulation

$$P(Y = y) \propto \exp \left(\mu L(y) + \sum_i^N \alpha_i y_{i+} + \sum_j^N \beta_j y_{+j} + \rho M(y) \right)$$

Number of outgoing ties

Number of incoming ties

Number of edges in the network

The diagram illustrates the components of the Exponential Random Graph Model (ERGM) formula. The formula is:

$$P(Y = y) \propto \exp \left(\mu L(y) + \sum_i^N \alpha_i y_{i+} + \sum_j^N \beta_j y_{+j} + \rho M(y) \right)$$

The terms are annotated as follows:

- $\mu L(y)$ is labeled "Number of edges in the network".
- $\sum_i^N \alpha_i y_{i+}$ is labeled "Number of outgoing ties".
- $\sum_j^N \beta_j y_{+j}$ is labeled "Number of incoming ties".
- $\rho M(y)$ is unlabeled.

Exponential Random Graph Formulation

$$P(Y = y) \propto \exp \left(\mu L(y) + \sum_i^N \alpha_i y_{i+} + \sum_j^N \beta_j y_{+j} + \rho M(y) \right)$$

Number of outgoing ties

Number of mutual ties

Number of incoming ties

Number of edges in the network

↑

↑

↑

Exponential Random Graph Formulation

$$P(Y = y) \propto \exp\left(\mu L(y) + \sum_i^N \alpha_i y_{i+} + \sum_j^N \beta_j y_{+j} + \rho M(y)\right)$$

Note the absence of
terms for **other**
dyads.

Dependence Assumptions

- ❖ Dyadic independence (p1 models)
 - ❖ Assumes that two dyads are conditionally independent.
 - ❖ A tie between i and j does not depend on a tie with k .
 - ❖ Does a poor job capturing transitivity in networks.

Dependence Assumptions

- ❖ Dyadic dependence (p^* models/Markov graphs)
 - ❖ *How likely is a tie between i and j?*
 - ❖ **Frank and Strauss (1986)**
 - ❖ Tie probability between i and j depends on ties that i and j have with others.
 - ❖ Example: Tie between Chris and Lisa is dependent on Lisa's relationship with Ewan.
 - ❖ Edges that do not have a node in common are conditionally independent (Markov assumption).

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c}\right) \exp\{\theta L(y) + \sigma_k S_k(y) + \dots + \tau T(y)\}$$



Number of edges in
the network

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c}\right) \exp\left\{\theta L(y) + \sigma_k S_k(y) + \dots + \tau T(y)\right\}$$

Number of edges in
the network

Number of k -star
configurations in
the network

```
graph TD; A["P(Y = y) = (1/c) exp{θL(y) + σkSk(y) + ... + τT(y)}"] -- "↑" --> B["Number of edges in the network"]; A -- "↓" --> C["Number of k-star configurations in the network"]
```

Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c}\right) \exp\left\{\theta L(y) + \sigma_k S_k(y) + \dots + \tau T(y)\right\}$$

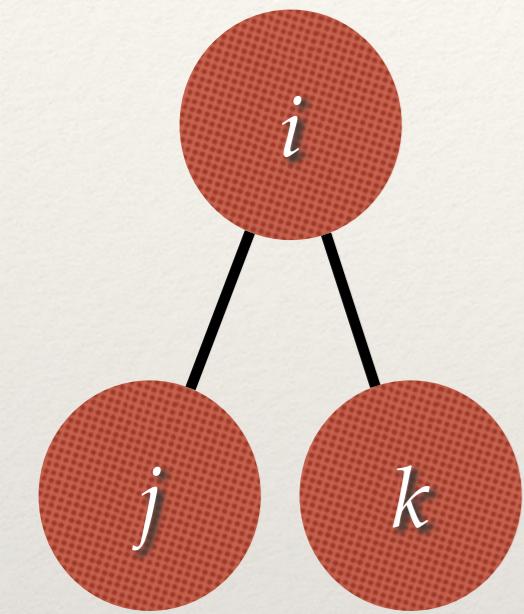
Number of edges in the network

Number of triangles in the network

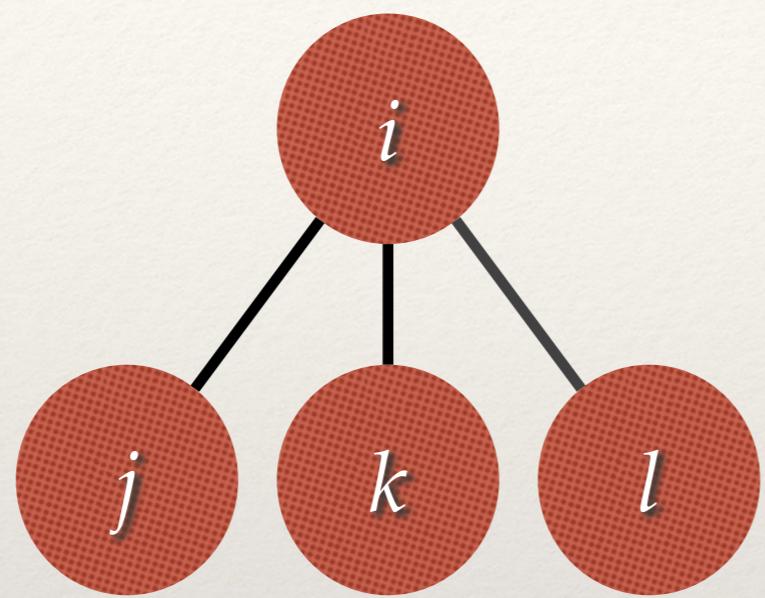
Number of k -star configurations in the network

The diagram illustrates the components of the exponential random graph formulation. The formula is $P(Y = y) = \left(\frac{1}{c}\right) \exp\left\{\theta L(y) + \sigma_k S_k(y) + \dots + \tau T(y)\right\}$. Arrows point from the terms $L(y)$, $S_k(y)$, \dots , and $T(y)$ to their respective labels: 'Number of edges in the network', 'Number of triangles in the network', and 'Number of k -star configurations in the network'.

Configurations

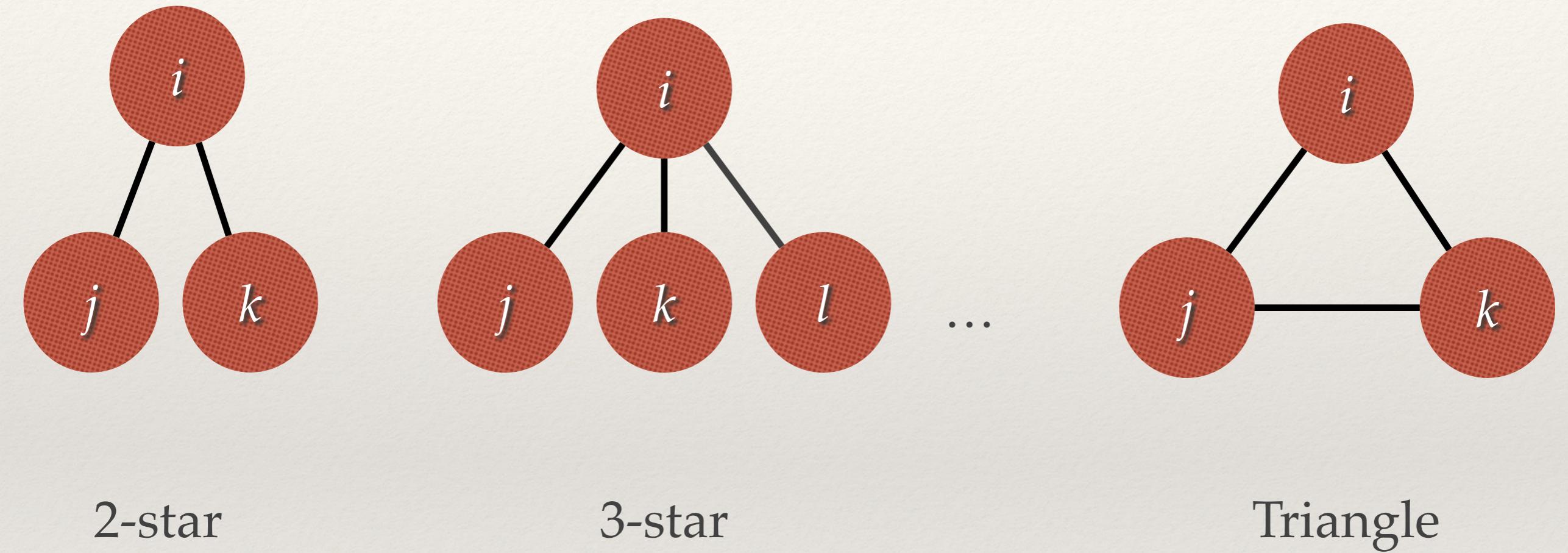


2-star



3-star

Configurations



Exponential Random Graph Formulation

$$P(Y = y) = \left(\frac{1}{c}\right) \exp\{\theta L(y) + \sigma_k S_k(y) + \dots + \tau T(y)\}$$

Counting these up,
allows us to
represent the
dependence
between the dyads.

Dependence Assumptions

- ❖ Dyadic dependence (p^* models/Markov graphs)
 - ❖ The original formulation could not handle actor covariates.
 - ❖ Example: Do males send more ties?
 - ❖ The model was later extended to include actor covariates.

Exponential Random Graph Formulation

All dyads except i
and j .

$$\text{logit} \left(P \left(Y_{ij} = 1 \mid n \text{ actors}, Y_{ij}^C \right) \right) = \sum_{k=1}^{\kappa} \theta_k \delta_{z_k(y)}$$

Exponential Random Graph Formulation

$$\text{logit} \left(P \left(Y_{ij} = 1 \mid n \text{ actors}, Y_{ij}^C \right) \right) = \sum_{k=1}^{\kappa} \theta_k \delta_{z_k(y)}$$

All dyads except i and j .

Coefficients for network statistics

Exponential Random Graph Formulation

$$\text{logit} \left(P \left(Y_{ij} = 1 \mid n \text{ actors}, Y_{ij}^C \right) \right) = \sum_{k=1}^{\kappa} \theta_k \delta_{z_k(y)}$$

All dyads except i and j .

Change statistic

Coefficients for network statistics

```
graph TD; A["All dyads except i and j."] --> B["P(Y_ij = 1 | n actors, Y_ij^C)"]; C["Coefficients for network statistics"] --> D["theta_k delta_z_k(y)"]; E["Change statistic"] --> F["sum_{k=1}^{\kappa} theta_k delta_z_k(y)"]
```

Exponential Random Graph Formulation

$$\text{logit} \left(P \left(Y_{ij} = 1 \mid n \text{ actors}, Y_{ij}^C \right) \right) = \sum_{k=1}^{\kappa} \theta_k \delta_{z_k(y)}$$

All dyads except i and j .

Change statistic

Looks like a **logistic regression**, right?!?

Coefficients for network statistics

```
graph TD; A[All dyads except i and j.] --> C[Y_{ij}^C]; B[Change statistic] --> D[delta_{z_k(y)}]; E[Coefficients for network statistics] --> F[theta_k]
```

Dependence Assumptions

- ❖ Dyadic dependence (p^* models/Markov graphs)
 - ❖ It is like a logistic regression, except:
 - ❖ Change statistic is not just the value of the independent variable (it is the change in the variable).
 - ❖ Conditional statement of left-hand side of equation (logistic regression assumes independence across units).

Dependence Assumptions

- ❖ Higher-Order Dependence Models
 - ❖ p^* models struggle with *degeneracy*, meaning that the networks simulated from the model do not match well with the observed network.
 - ❖ Recent work has addressed this problem by defining more complex dependencies among dyads.
 - ❖ Examples:
 - ❖ Social circuit models (i.e. $i-j$, $j-k$, $k-l$).
 - ❖ Geometrically weighted terms.

ERGM Theory

- ❖ *So what do we do with this?*
- ❖ Robins and Lusher (2013: 11)
 - ❖ “It is a theoretical and empirical task to delineate the various forms of dependence that are exhibited in actual social structures. We regard this as social network theory at a fundamental level...”
 - ❖ “The process of theory translation requires the alignment of theoretical concepts with network configurations.”

Network Configurations and Processes

- ❖ Conceptualization and Operationalization!
 - ❖ What is happening? (Conceptualization)
 - ❖ What would that “look like”? (Operationalization)

Network Configurations and Processes

Configuration

Process

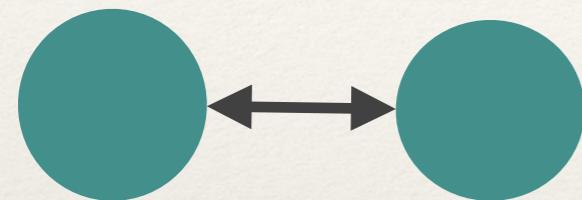
Network Configurations and Processes

Configuration

Process

Friendship is
reciprocated

Network Configurations and Processes



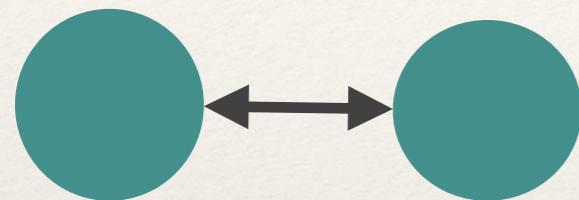
Configuration

Mutuality

Process

Friendship is
reciprocated

Network Configurations and Processes



Configuration

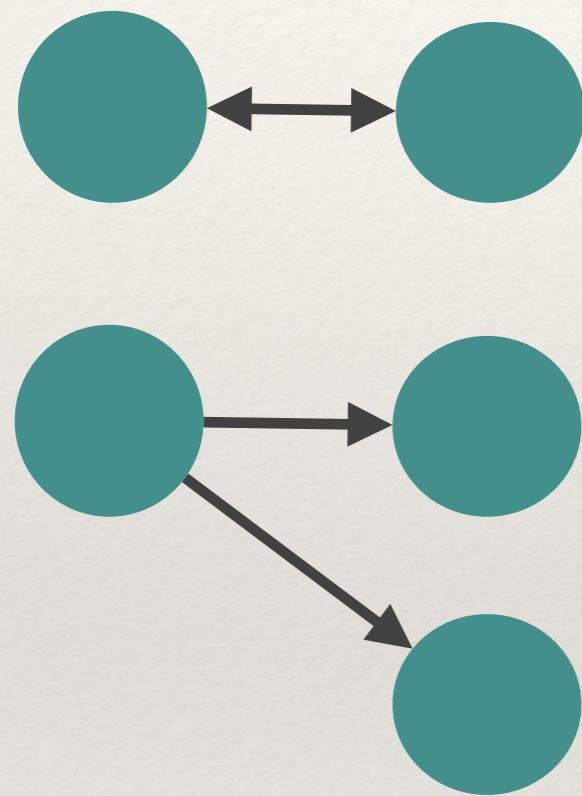
Mutuality

Process

Friendship is reciprocated

Some people are very social

Network Configurations and Processes



Configuration

Mutuality

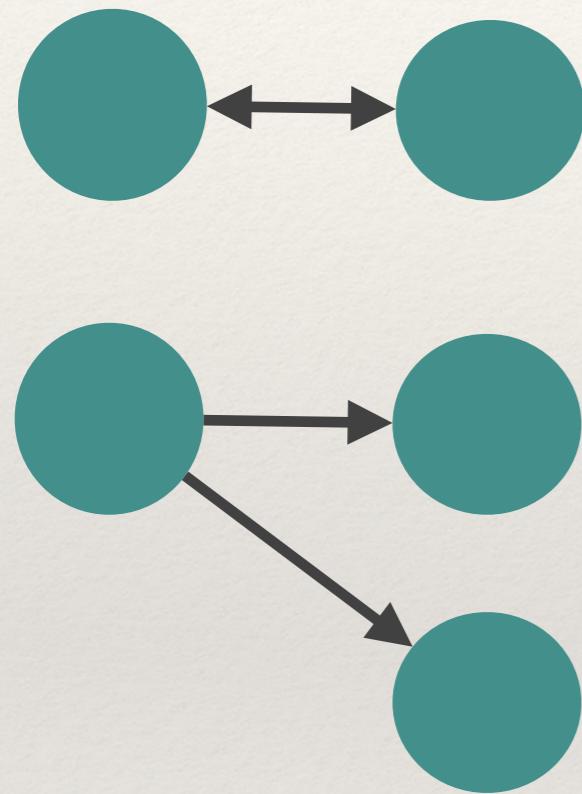
Process

Friendship is reciprocated

Outdegree

Some people
are very social

Network Configurations and Processes



Configuration

Mutuality

Process

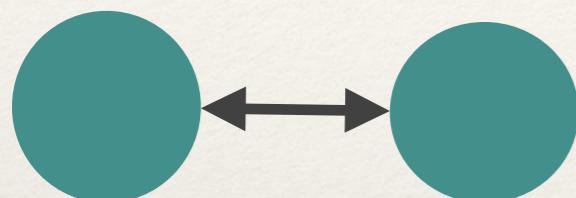
Friendship is reciprocated

Outdegree

Some people are very social

Some people are very popular

Network Configurations and Processes

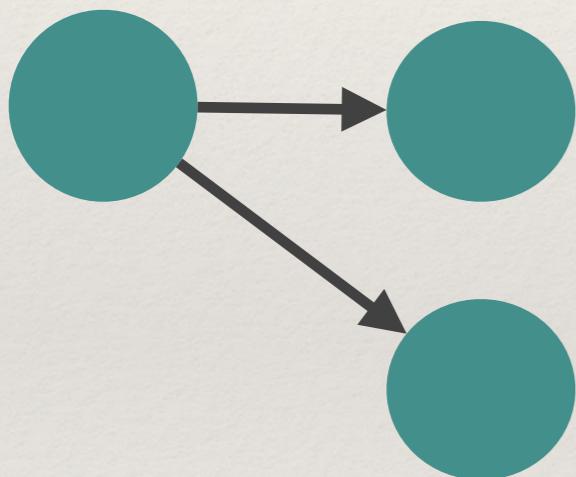


Configuration

Mutuality

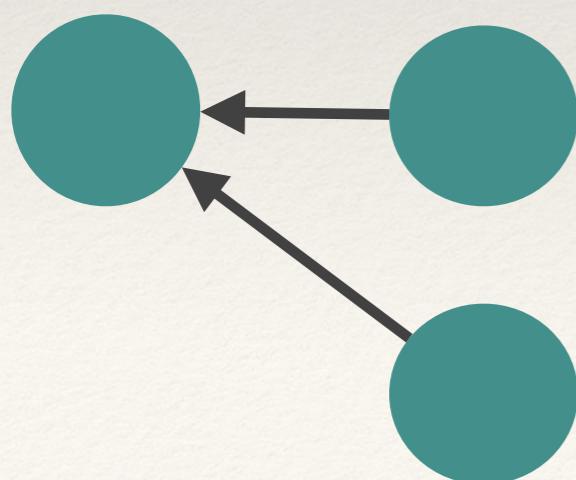
Process

Friendship is reciprocated



Outdegree

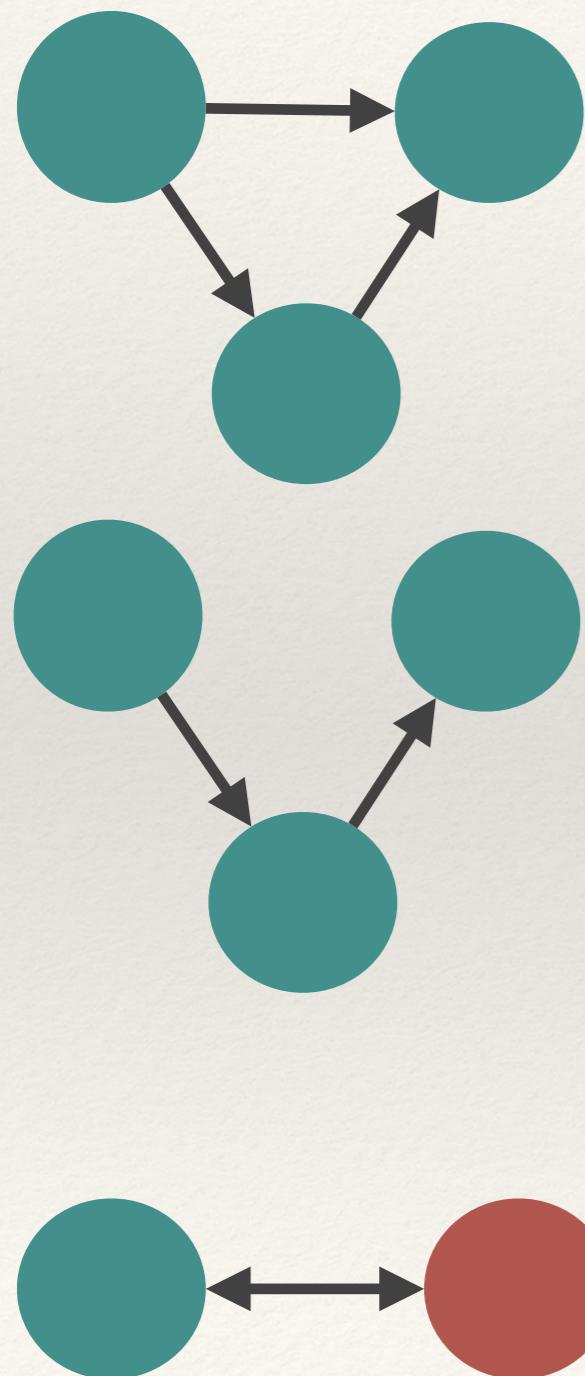
Some people are very social



Indegree

Some people are very popular

Network Configurations and Processes



Configuration

Transitivity

2-path

Heterophily

Process

Friends of friends
become friends

Betweenness
positions are
preferred

Opposites attract

Network Configurations and Processes

- ❖ Conceptualization and Operationalization!
 - ❖ What is happening? (Conceptualization)
 - ❖ What would that “look like”? (Operationalization)

Network Configurations and Processes

How & why social ties form?

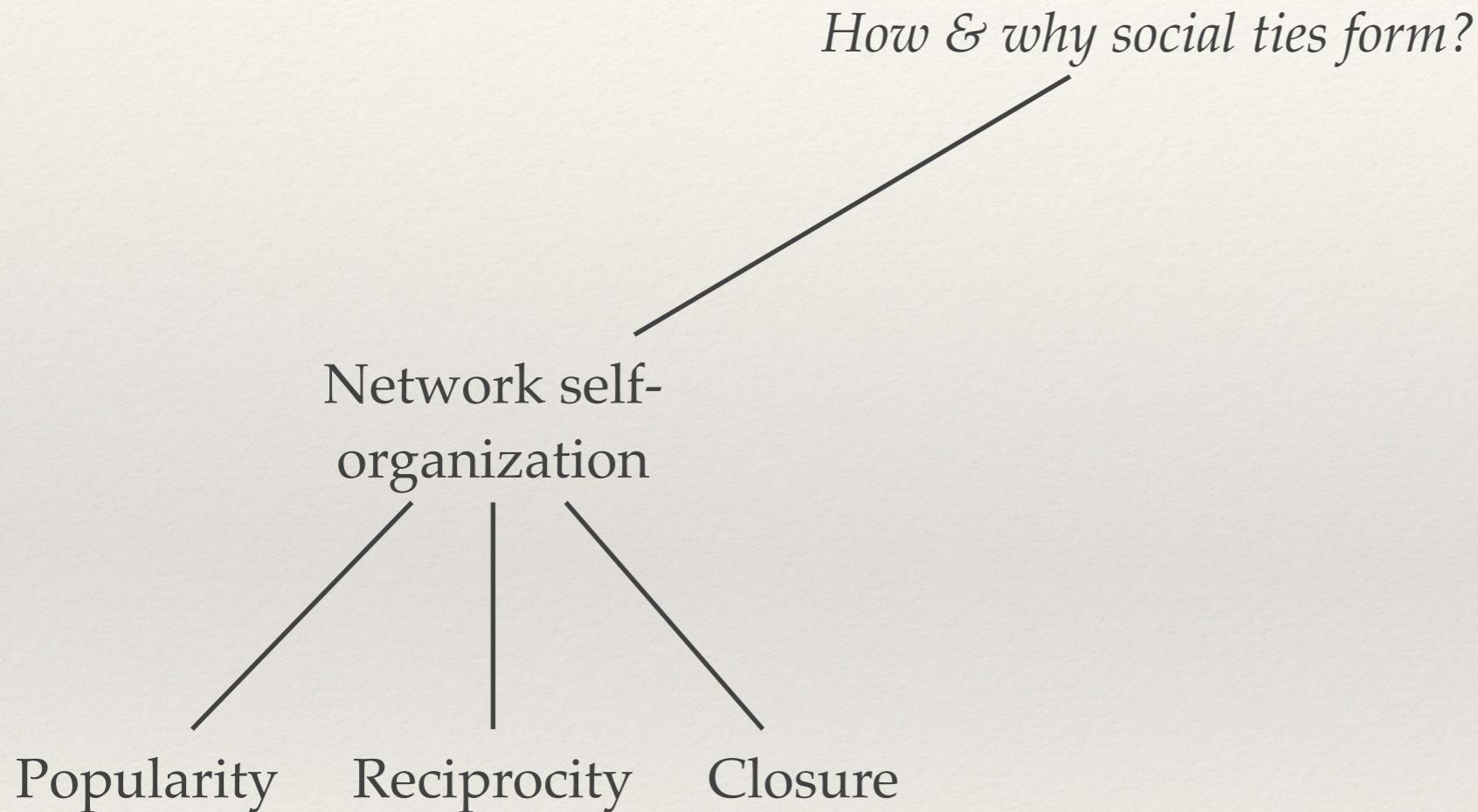
Network Configurations and Processes

How & why social ties form?

Network self-organization

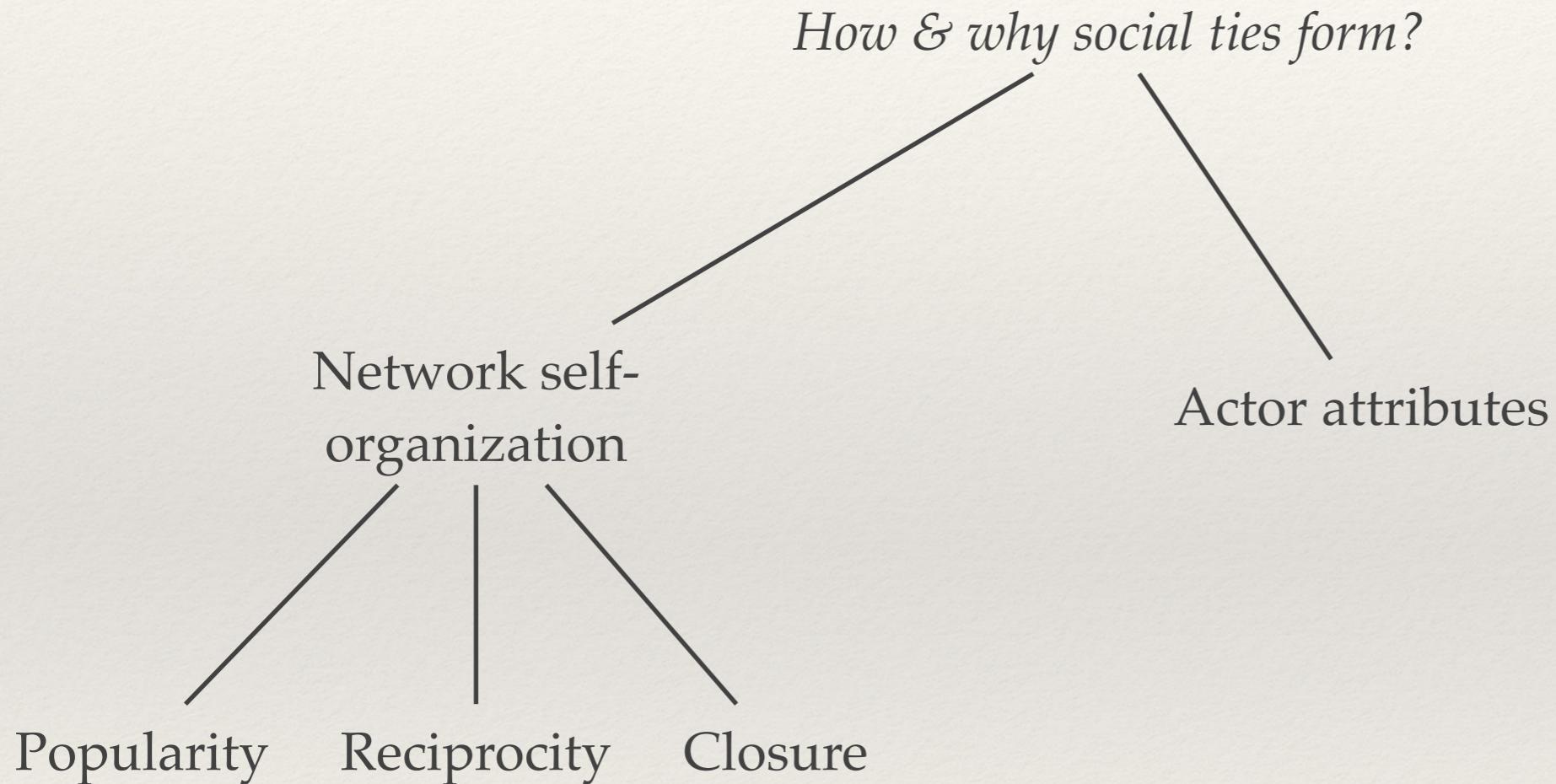


Network Configurations and Processes



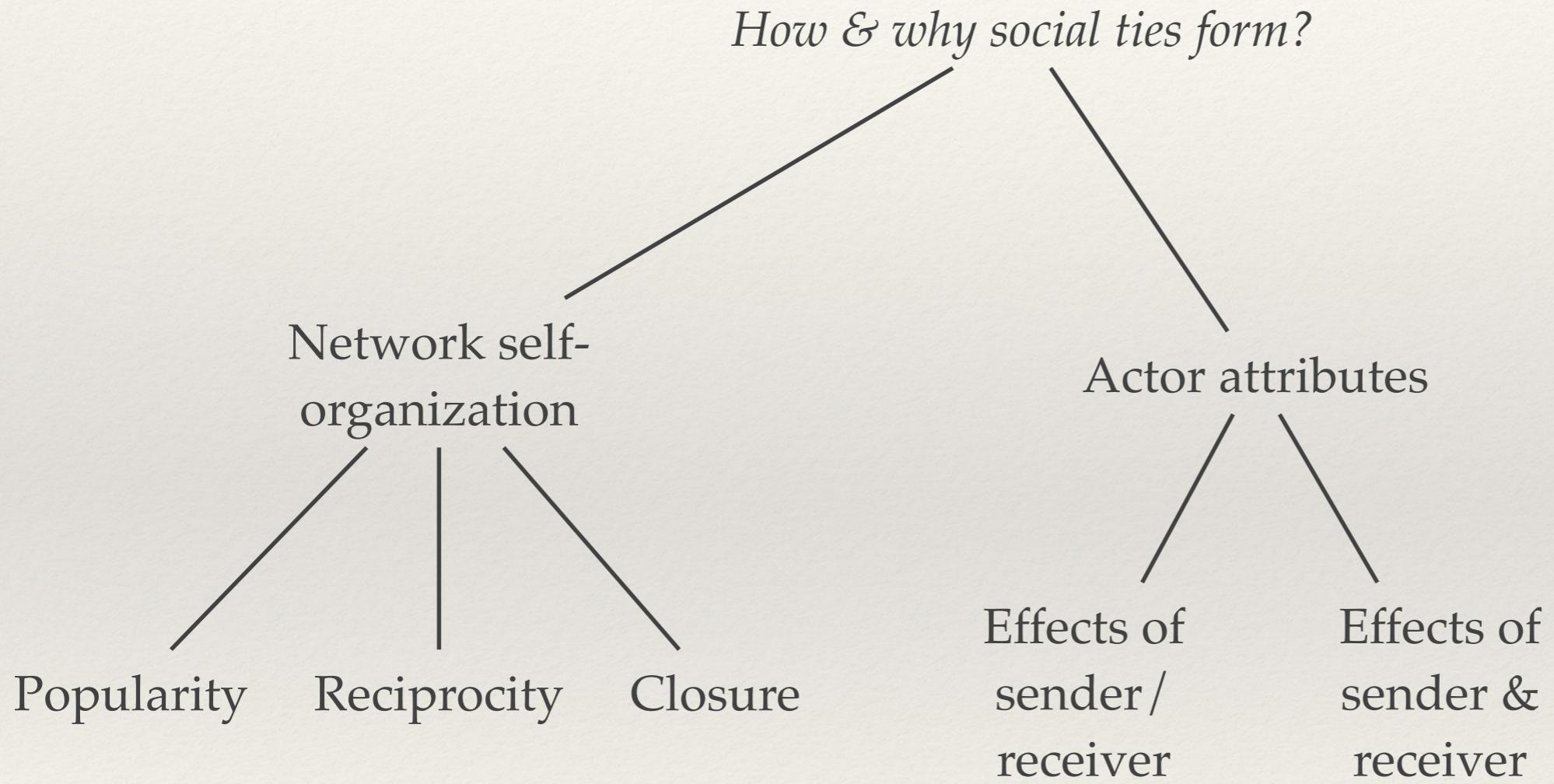
Adapted from Lusher, Koskinen, and Robins (2013) "Exponential Random Graph Models of Social Networks..."

Network Configurations and Processes



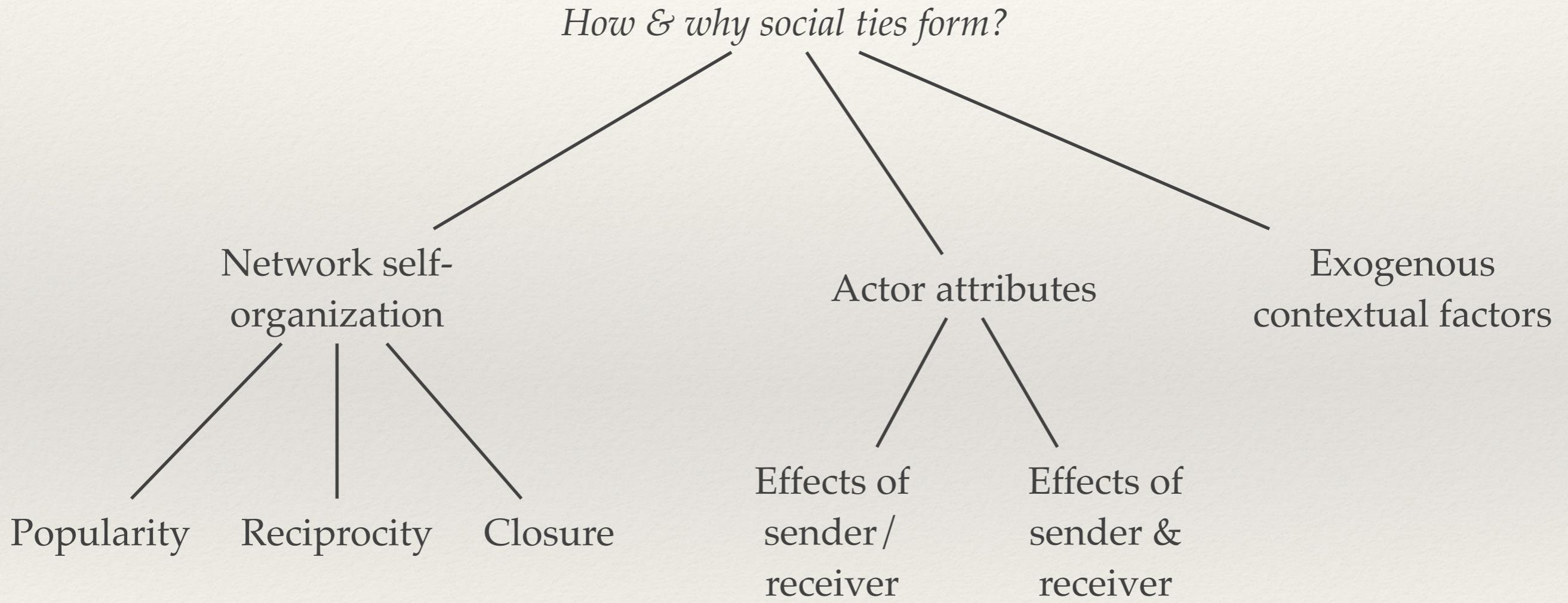
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Network Configurations and Processes



Adapted from Lusher, Koskinen, and Robins (2013) "Exponential Random Graph Models of Social Networks..."

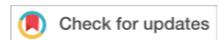
Network Configurations and Processes



Adapted from Lusher, Koskinen, and Robins (2013) "Exponential Random Graph Models of Social Networks..."

Network Configurations and Processes

- ❖ As we work through developing a model, we want to identify the **network configurations** that capture the theoretical process we are interested in testing.



Trusting the Untrustworthy: The Social Organization of Trust Among Incarcerated Women

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ABSTRACT

Although the benefits of trust are well documented across a variety of settings, little empirical attention has been dedicated to trust in carceral settings, particularly among incarcerated women. Knowing how individuals in prison establish relationships of trust with one another is crucial for understanding how individuals adjust to conditions of confinement. Using data from 133 incarcerated women in a Pennsylvania prison unit, this study adopts a network approach to examine the role of individual and structural determinants of trust using exponential random graph models. Findings provide weak support for the claim that individual determinants (e.g. age, religious affiliation) shape whether women are more likely to trust someone to support them during an argument or a dispute. Instead, our findings show that structural determinants are the primary drivers of trust relationships. Trust is deeply entwined with friendship relations among women who get along with each other. Our approach paves a new path for the examination of trust in correctional settings and other criminological contexts.

ARTICLE HISTORY

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KEYWORDS

Trust; women; incarceration; network; ergm

Table 1: Hypotheses and Network Configurations

Hypothesis	Network Configuration ^{a,b}
1. i is more likely to trust j if j is older, has served more time in prison, and has spent more time on the unit.	
2. i is more likely to trust j if j is the same religious affiliation as i	
3. i is no more likely to trust j if j is the same race/ethnicity as i	
4. i is more likely to trust j if i gets along with j	
5. i is more likely to trust j if j trusts i	
6. i is more likely to trust j if a) i trusts k and k trusts j and b) k gets along with i	
7. i is more likely to trust j as i 's brokerage in the get along with network increases	
8. i is less likely to trust j as j 's brokerage in the get along with network increases	

Notes :

^aDashed lines indicate the hypothesized trust tie.

^bBlack/White nodes indicate the presence/absence or higher/lower value of an attribute, respectively.

Learning Goals

- ❖ By the end of this lecture, you should be able to answer these questions:
 - ❖ What is the underlying logic of exponential random graph models (ERGMs)?
 - ❖ What is the historical development of ERGMs and what are the various properties of the models that were developed over time?
 - ❖ What are *network configurations* and how do they help us operationalize theoretical concepts?

Questions?