

*Statistical Analysis of Networks*

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# Closeness & Betweenness Centrality

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# Learning Goals

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- ❖ At the end of the lecture, you should be able to answer these questions:
  - ❖ What are some different ways we can conceptualize “centrality”?
  - ❖ What is *closeness* and *betweenness* centrality?
  - ❖ How do we calculate these measures for undirected and directed graphs?
  - ❖ What do comparing these measures tell us about the structure of a network?

When we say a *node* is “central,”  
what do we mean conceptually?

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# Degree Centrality

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- ❖ Nodes that have many edges are “central”.
  - ❖ We can operationalize this using the degree (for undirected graphs) or the indegree / outdegree (for directed graphs).

# Motivating Example

J Youth Adolescence (2014) 43:104–115  
DOI 10.1007/s10964-013-9946-0

EMPIRICAL RESEARCH

## “Role Magnets”? An Empirical Investigation of Popularity Trajectories for Life-Course Persistent Individuals During Adolescence

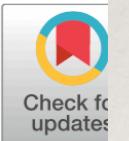
Jacob T. N. Young

**PNAS**

RESEARCH ARTICLE

POLITICAL SCIENCES

OPEN ACCESS



## Disrupting hate: The effect of deplatforming hate organizations on their online audience

Daniel Robert Thomas<sup>a,1</sup> and Laila A. Wahedi<sup>a</sup>

Edited by Timothy Wilson, University of Virginia, Charlottesville, VA; received August 17, 2022; accepted January 20, 2023

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# Conceptualization

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- ❖ “Everyone agrees, it seems, that centrality is an important structural attribute of networks. All concede that it is related to a high degree to other important group properties and processes. But there consensus ends.” (Freeman, 1978/1979: 217)
- ❖ The type of measure we use depends on the substantive question of interest.
  - ❖ Various measures of centrality are correlated, but they operationalize different concepts.

What are some other ways a node  
can be “central”?

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# Closeness Centrality

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- ❖ How “close” is a node to other nodes?
  - ❖ In a graph, *closeness centrality* measures how near a node is to the other nodes in the network.
  - ❖ Nodes that are central are those that can reach other nodes in short distances.

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# Closeness Centrality

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- ❖ Closeness is based on the inverse of the distance of each actor to every other actor.
- ❖ Terminology:
  - ❖ A **geodesic** is the shortest path between two nodes.
  - ❖ The **distance**,  $d(n_i, n_j)$ , is the length of the path between  $i$  and  $j$ .

# Undirected Networks

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# Closeness Centrality: Undirected Binary Graphs

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$$C_C(n_i) = \left[ \sum_{j=1}^g d(n_i, n_j) \right]^{-1}$$

# Closeness Centrality: Undirected Binary Graphs

$$C_C(n_i) = \left[ \sum_{j=1}^g d(n_i, n_j) \right]^{-1}$$

↑  
How far is  $i$  from  
every node?

# Closeness Centrality: Undirected Binary Graphs

$$C_C(n_i) = \left[ \sum_{j=1}^g d(n_i, n_j) \right]^{-1}$$

How far is  $i$  from  
every node?

Then take the  
inverse of the sum.

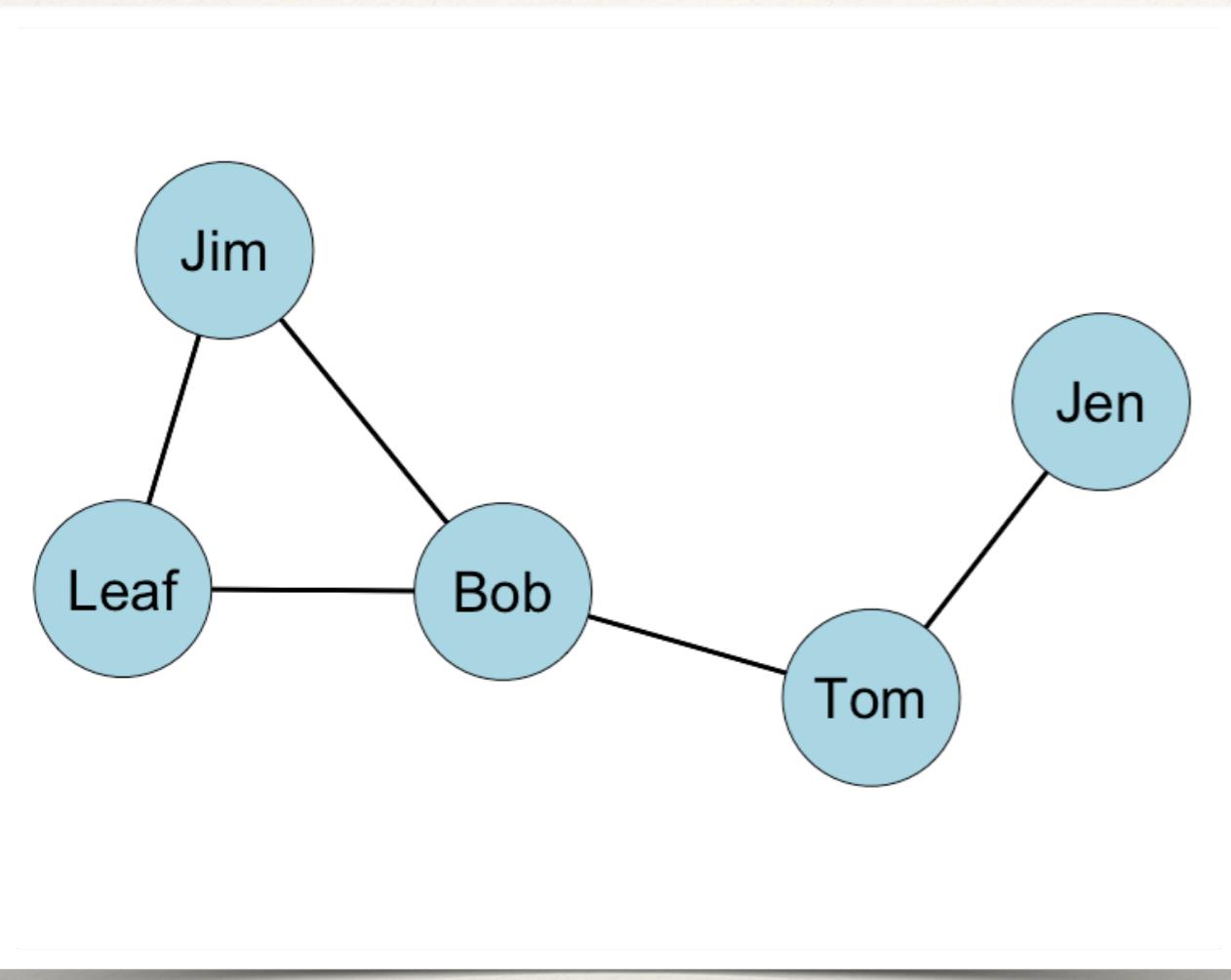
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# Closeness Centrality: Undirected Binary Graphs

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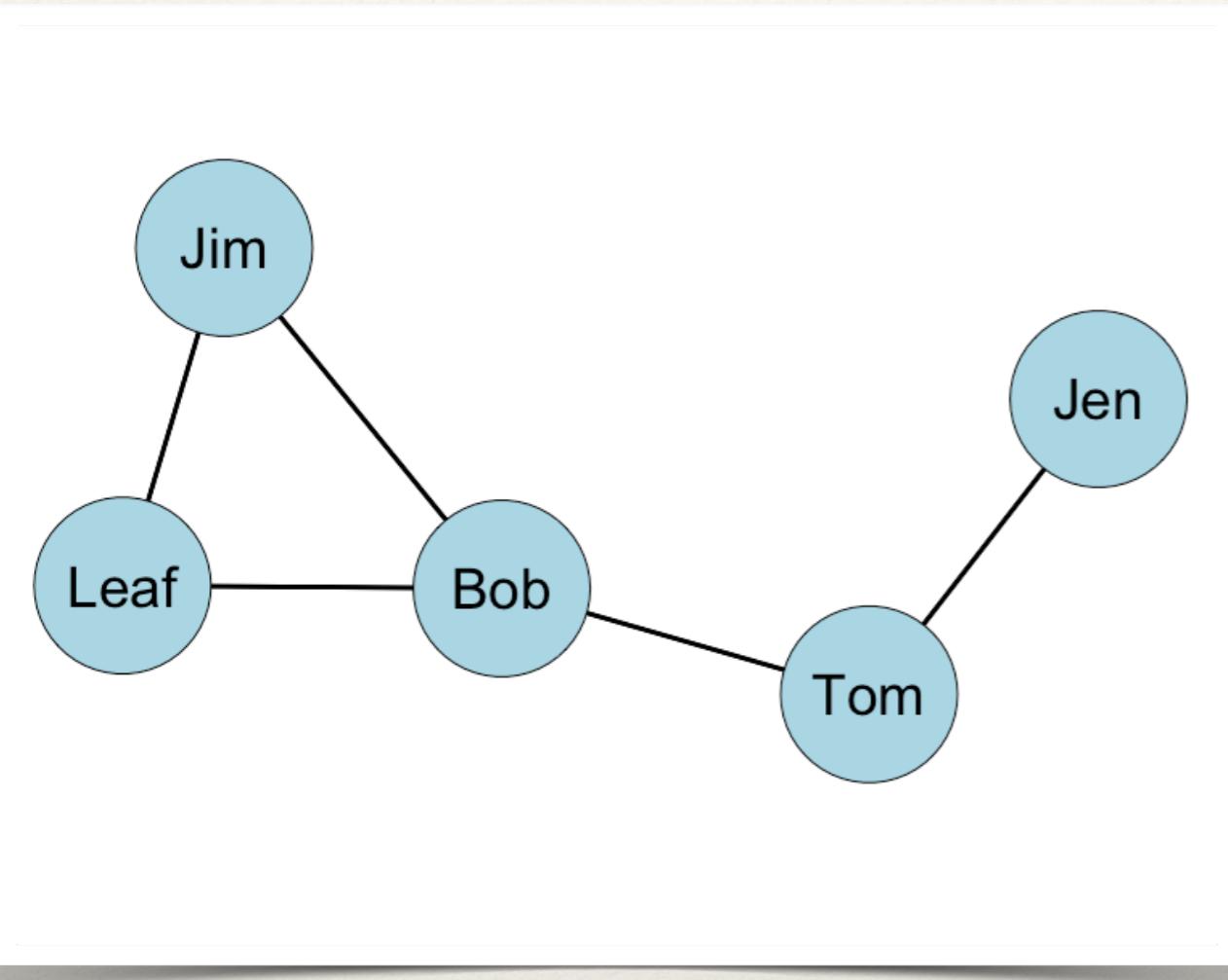
$$C_C(n_i) = \left[ \sum_{j=1}^g d(n_i, n_j) \right]^{-1} = \frac{1}{\left[ \sum_{j=1}^g d(n_i, n_j) \right]}$$

# Example: Undirected, Binary Network



We want a matrix of the distances between each node.

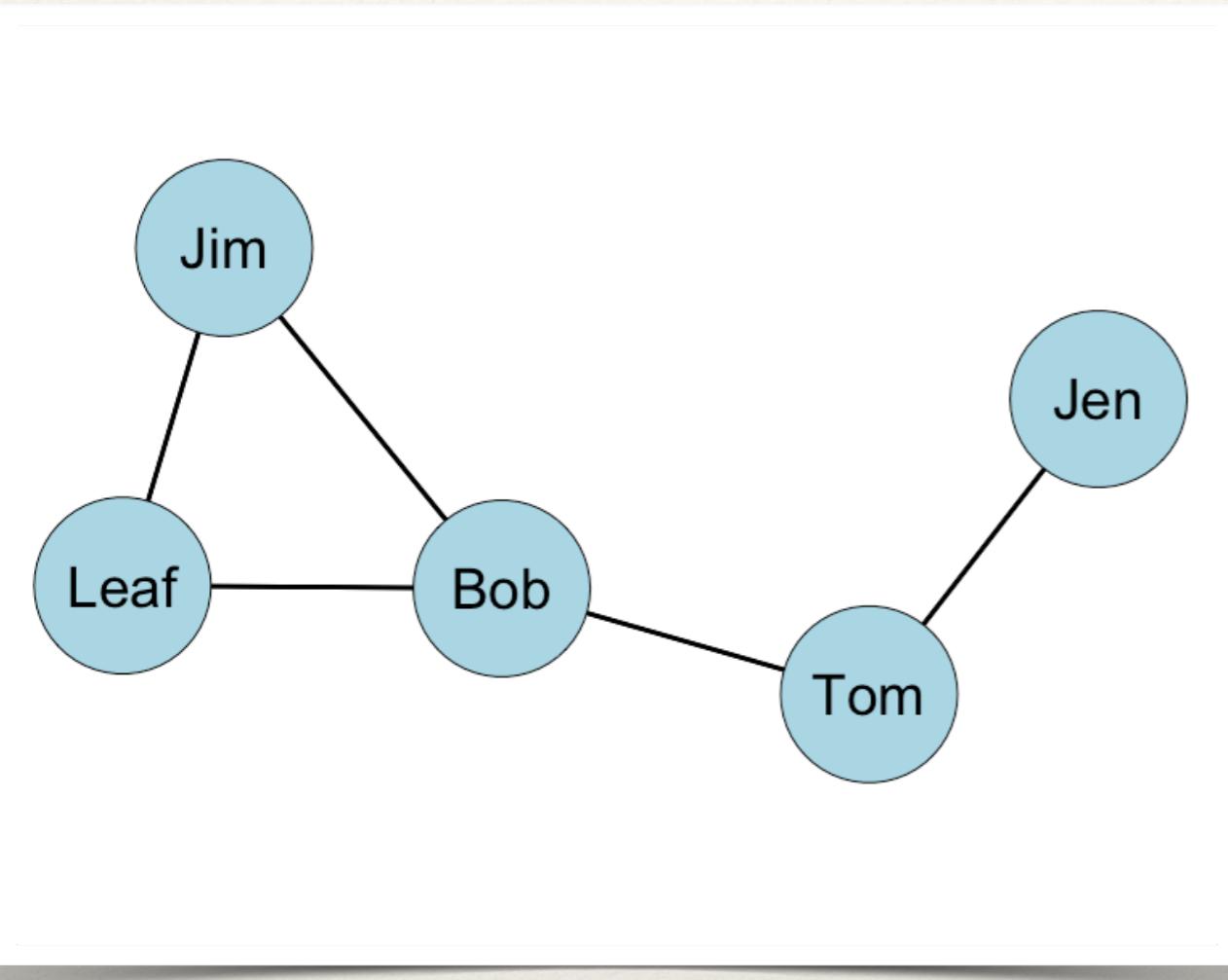
# Example: Undirected, Binary Network



*How far is Jen from Tom? From Bob?*

		Jen	Tom	Bob	Leaf	Jim
Jen		?	?			
Tom						
Bob						
Leaf						
Jim						

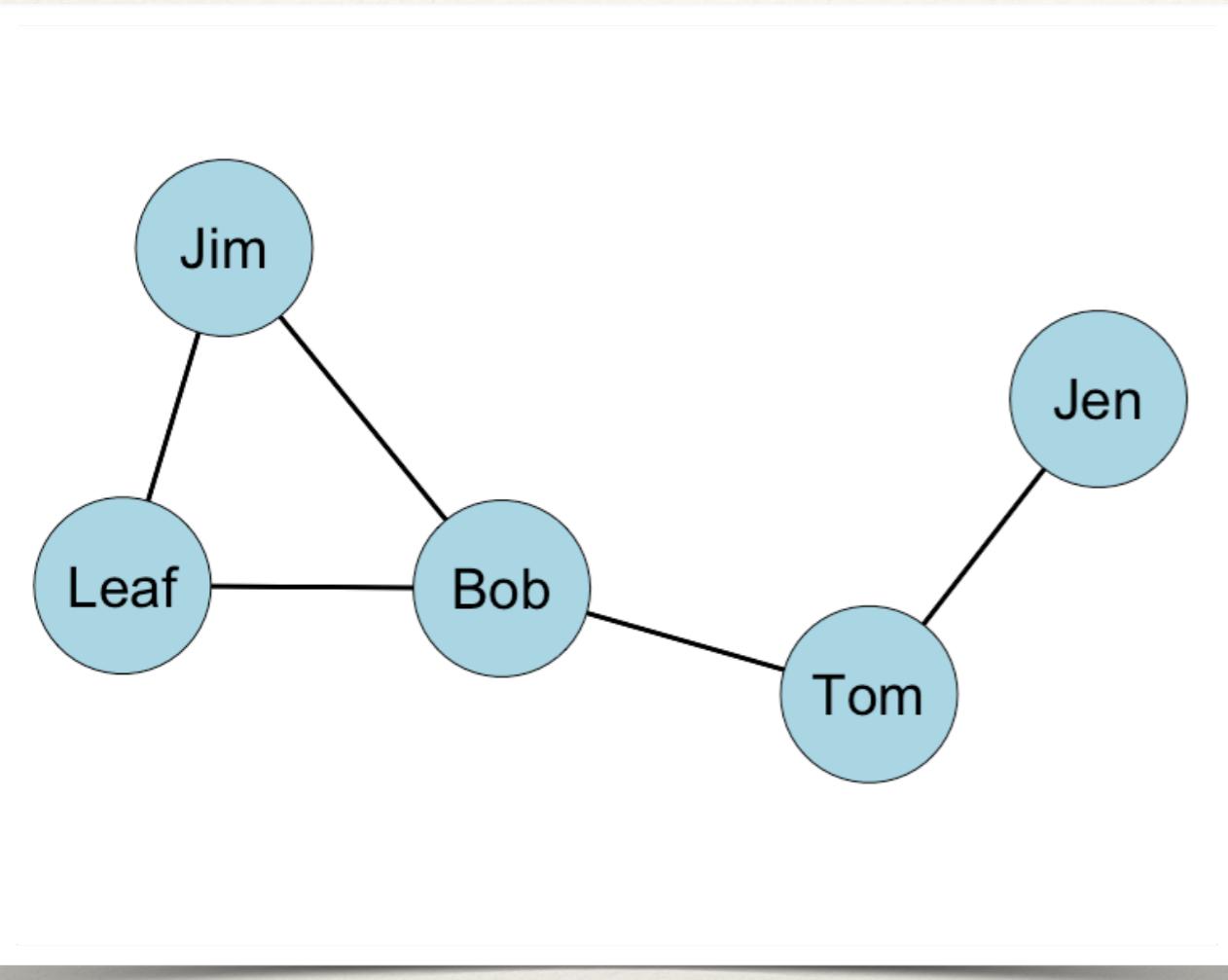
# Example: Undirected, Binary Network



*How far is Jen from Tom? From Bob?*

		Jen	Tom	Bob	Leaf	Jim
Jen			1			
Tom						
Bob						
Leaf						
Jim						

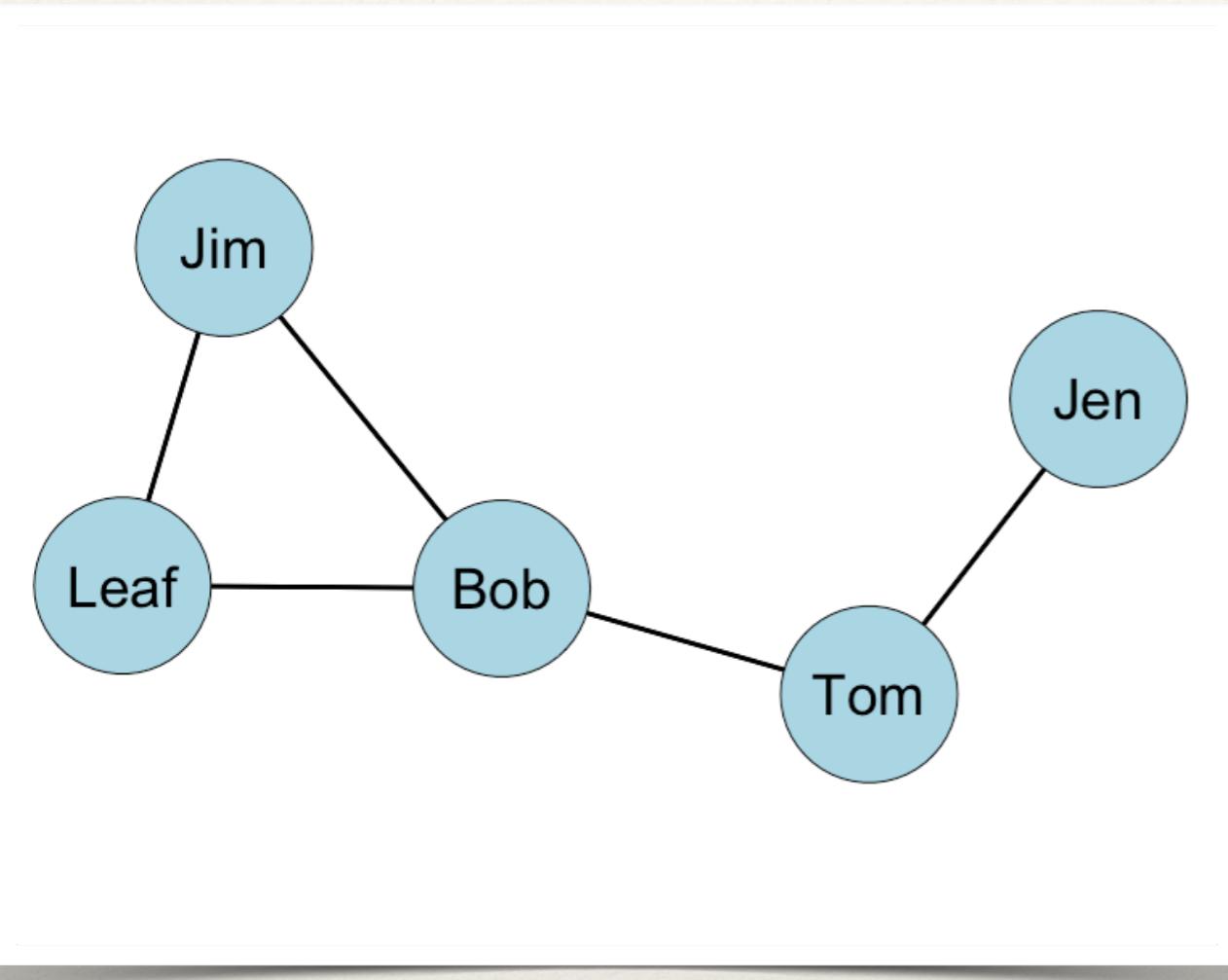
# Example: Undirected, Binary Network



*How far is Jen from Tom? From Bob?*

		Jen	Tom	Bob	Leaf	Jim
Jen			1	2		
Tom						
Bob						
Leaf						
Jim						

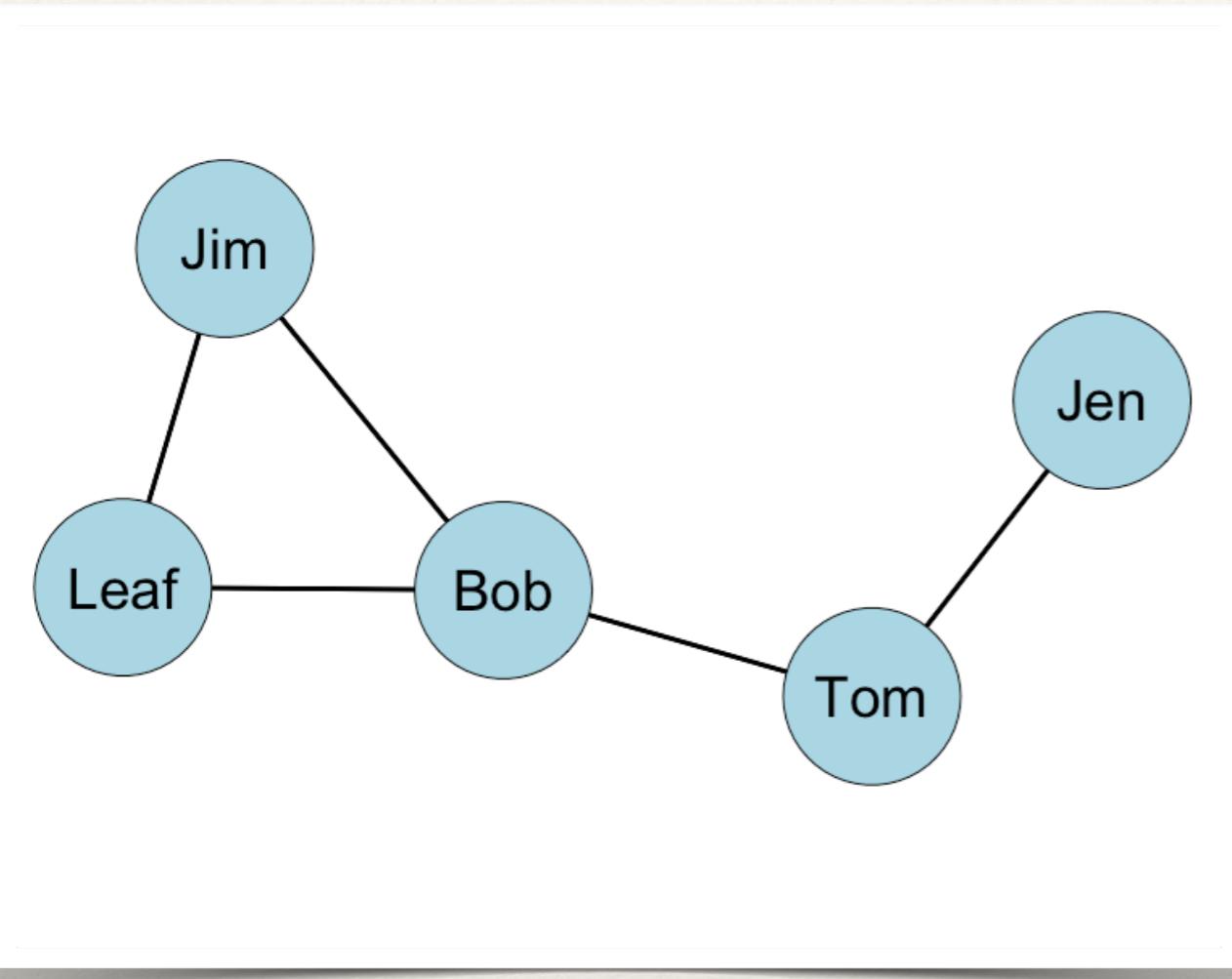
# Example: Undirected, Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	?	?	?
Tom			?	?	?	?
Bob				?	?	?
Leaf					?	?
Jim						?

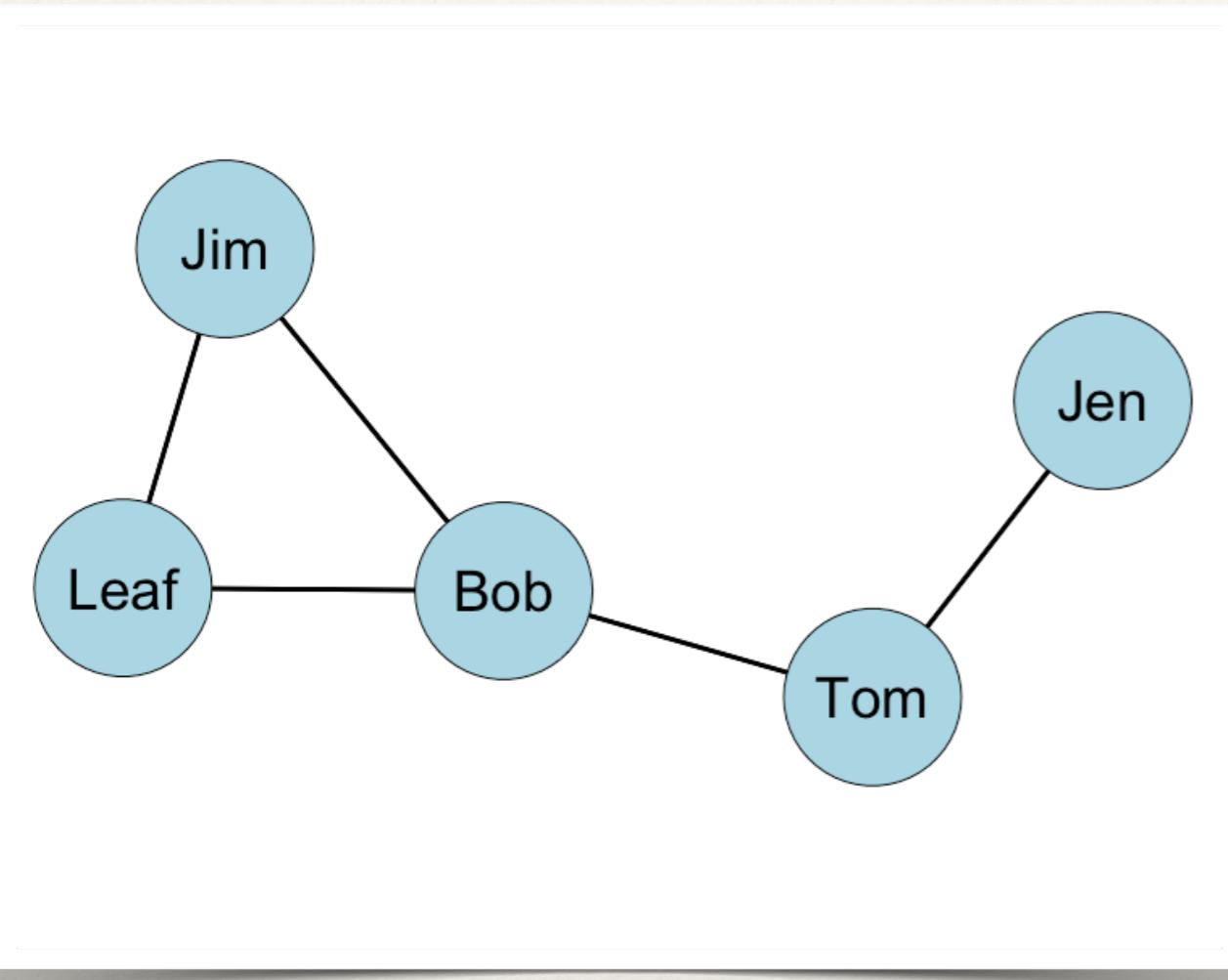
*Now, fill in the rest...*

# Example: Undirected, Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3	
Tom			1	2	2	
Bob				1	1	
Leaf					1	
Jim						1

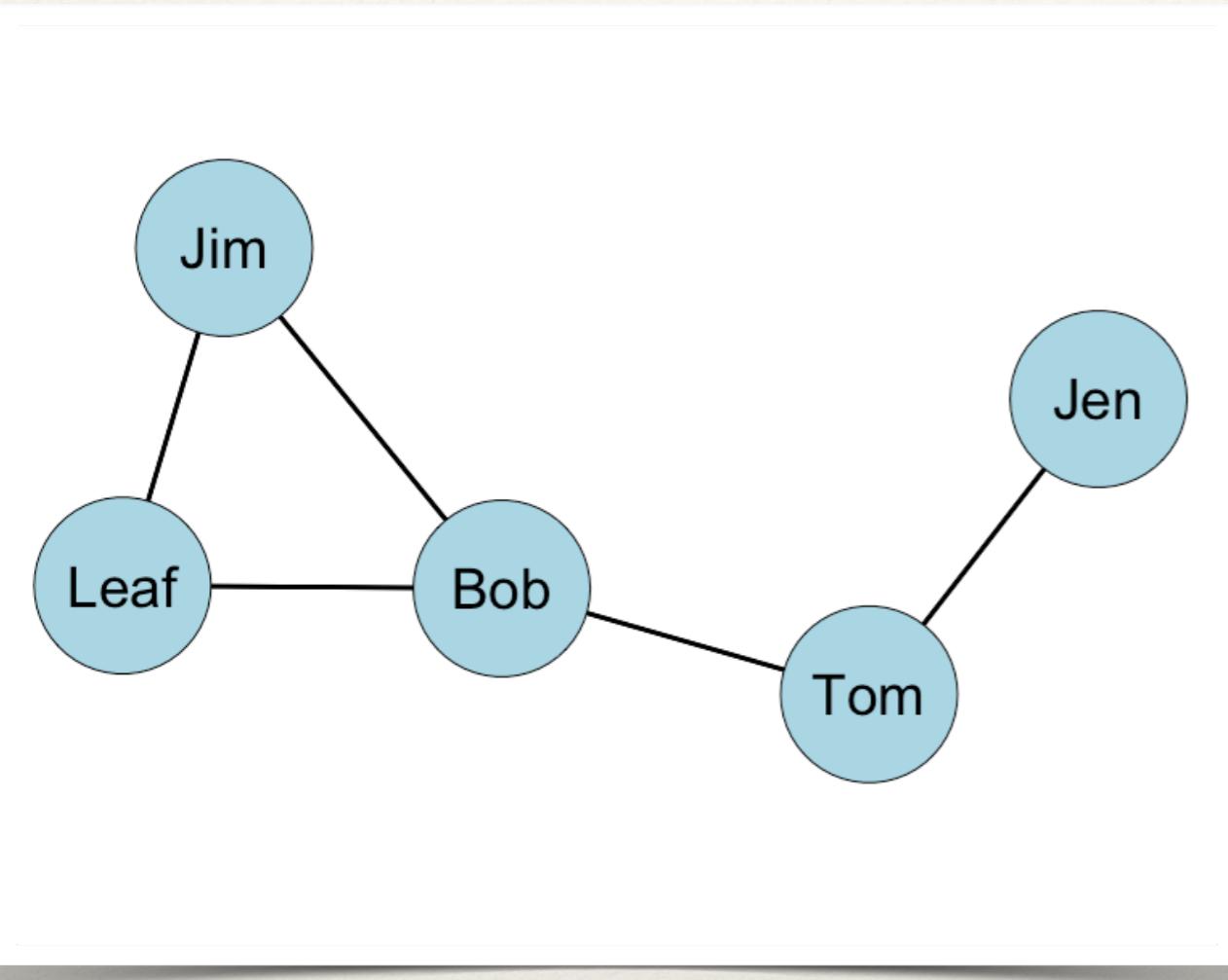
# Example: Undirected, Binary Network



*Since the graph is undirected, the distance matrix is symmetric about the diagonal.*

		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3	3
Tom	1		1	2	2	2
Bob	2	1		1	1	1
Leaf	3	2	1		1	1
Jim	3	2	1	1		1

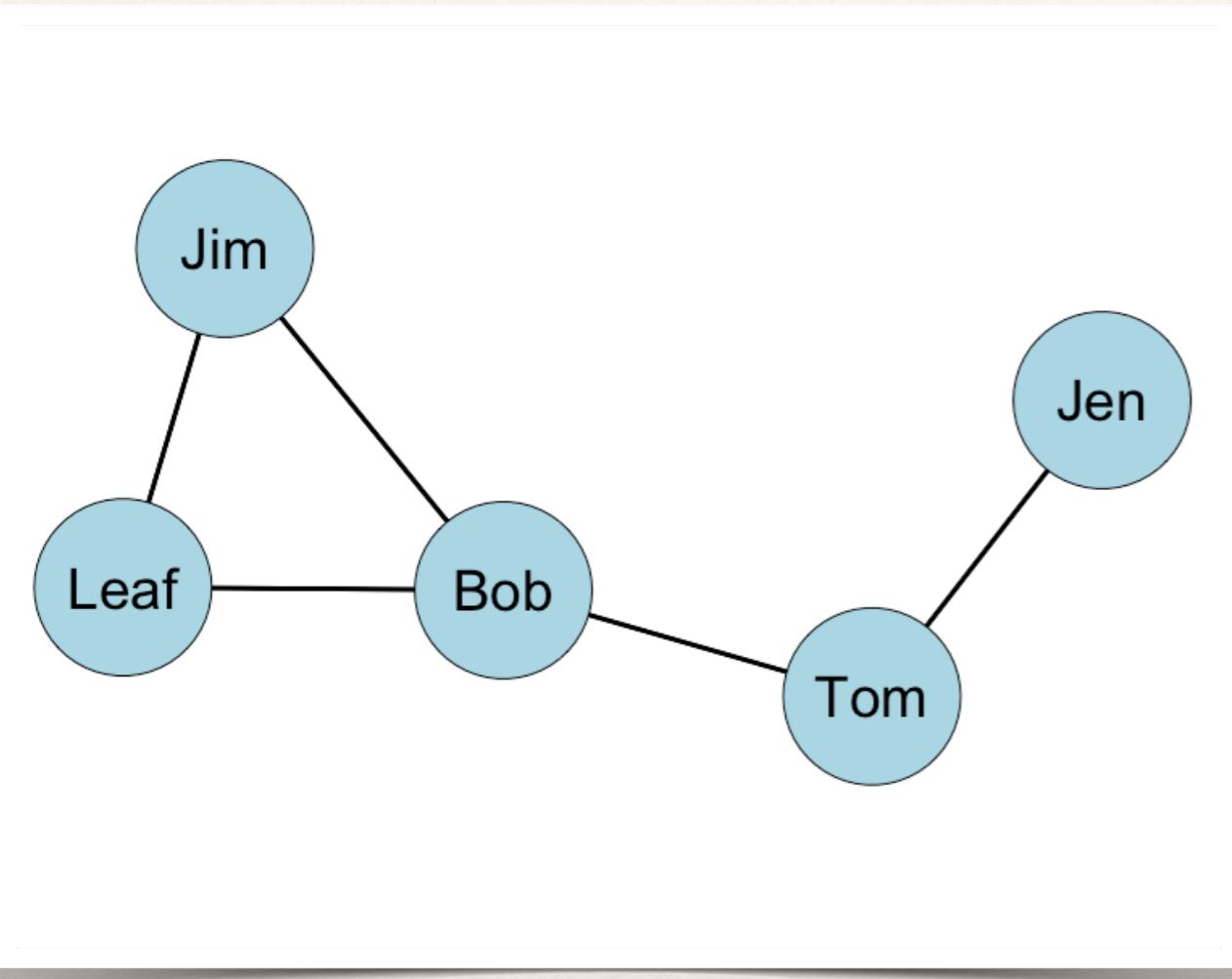
# Example: Undirected, Binary Network



*The row sum of these is the distances.*

Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen	1	2	3	3	3	9
Tom	1		1	2	2	6
Bob	2	1		1	1	5
Leaf	3	2	1		1	7
Jim	3	2	1	1		7

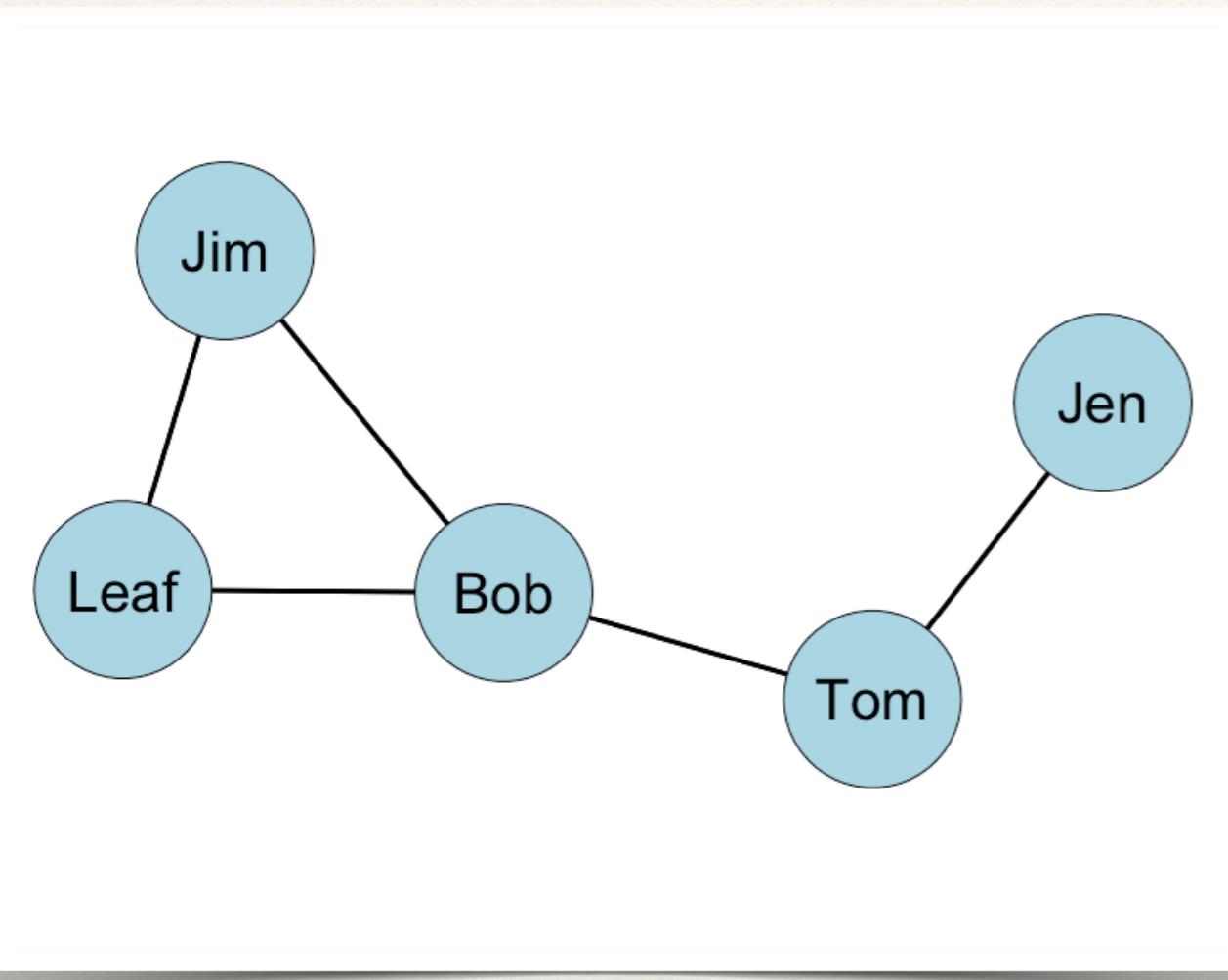
# Example: Undirected, Binary Network



*Since the graph is undirected, the column sum reports the same information.*

Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom	1		1	2	2	6
Bob	2	1		1	1	5
Leaf	3	2	1		1	7
Jim	3	2	1	1		7
Sum	9	6	5	7	7	

# Example: Undirected, Binary Network



By taking the reciprocal, we get the closeness.

## Closeness Centrality

$$\text{Jen} = 1/9 = 0.111$$

$$\text{Tom} = 1/6 = 0.167$$

$$\text{Bob} = 1/5 = 0.200$$

$$\text{Leaf} = 1/7 = 0.143$$

$$\text{Jim} = 1/7 = 0.143$$

$$C_C(n_i) = \frac{1}{\left[ \sum_{j=1}^g d(n_i, n_j) \right]}$$

# Closeness Centrality: Undirected Binary Graphs

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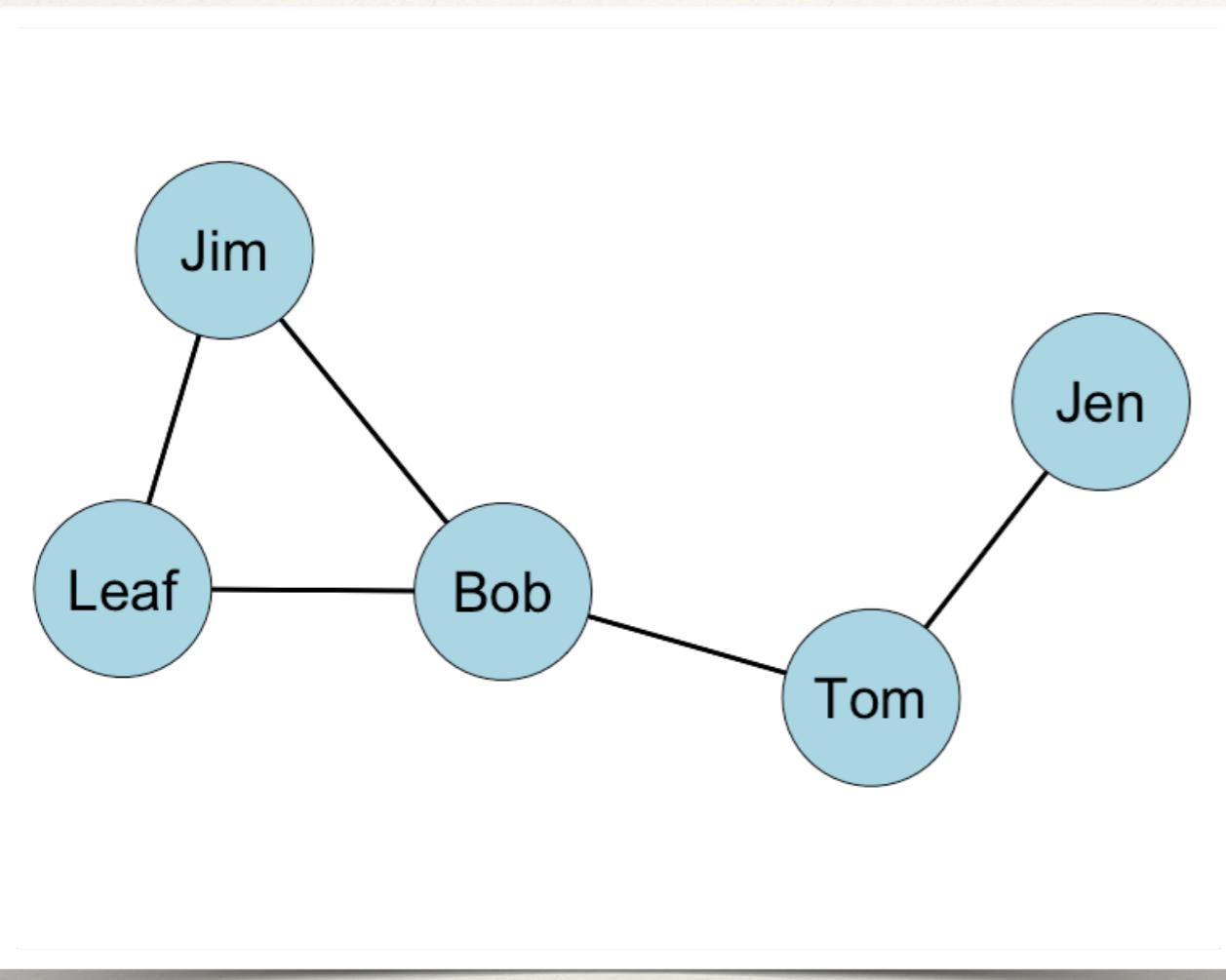
- ❖ Actor closeness centrality not only reflects each node's connectivity to other nodes but also depends on the size of the network,  $g$ .
- ❖ Having to sum over more nodes will push scores closer to zero.
  - ❖ *Solution?*

## Standardized Closeness Centrality: Undirected Binary Graphs

$$C'_C(n_i) = \left[ \sum_{j=1}^g d(n_i, n_j) \right]^{-1} \times [g - 1]$$

Multiply by  $g - 1!$

# Example: Undirected, Binary Network



## Closeness Centrality

$$\text{Jen} = 1/9 = 0.111$$

$$\text{Tom} = 1/6 = 0.167$$

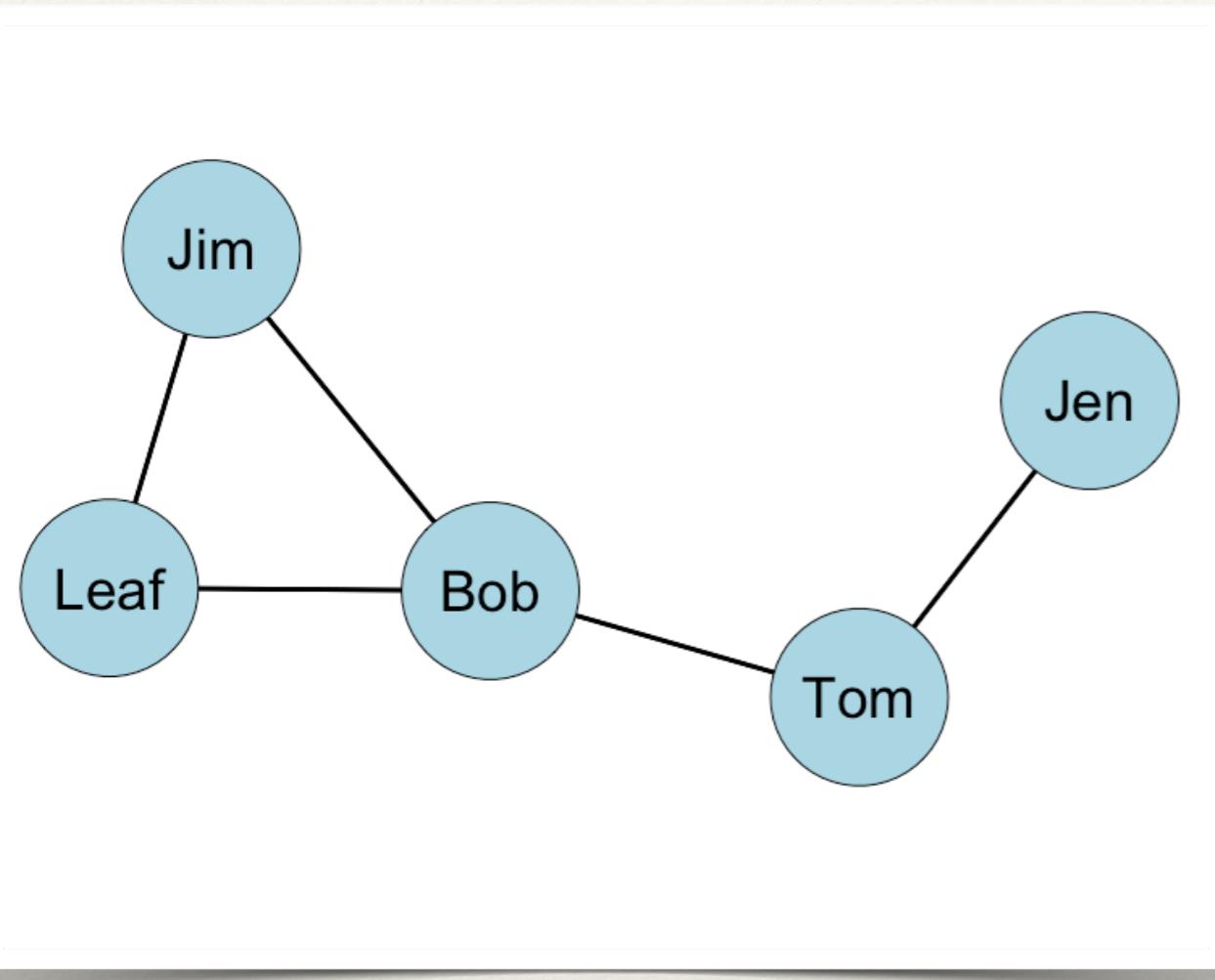
$$\text{Bob} = 1/5 = 0.200$$

$$\text{Leaf} = 1/7 = 0.143$$

$$\text{Jim} = 1/7 = 0.143$$

*Multiplying by  $g-1$  gives the  
standardized value*

# Example: Undirected, Binary Network



Standardized Closeness

Centrality

$$\text{Jen} = 0.111^*4 = 0.444$$

$$\text{Tom} = 0.167^*4 = 0.668$$

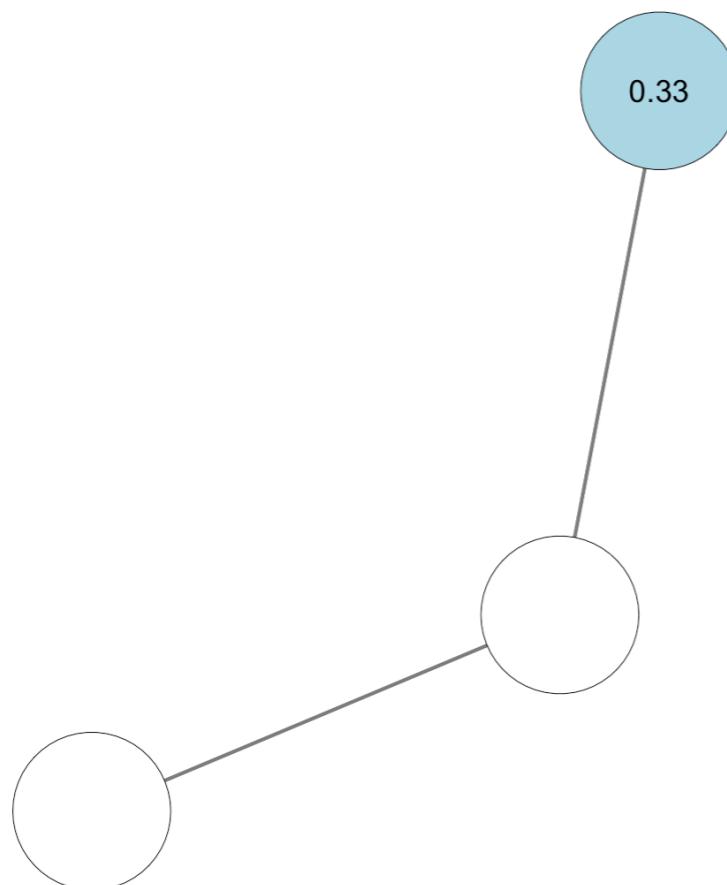
$$\text{Bob} = 0.200^*4 = 0.800$$

$$\text{Leaf} = 0.143^*4 = 0.572$$

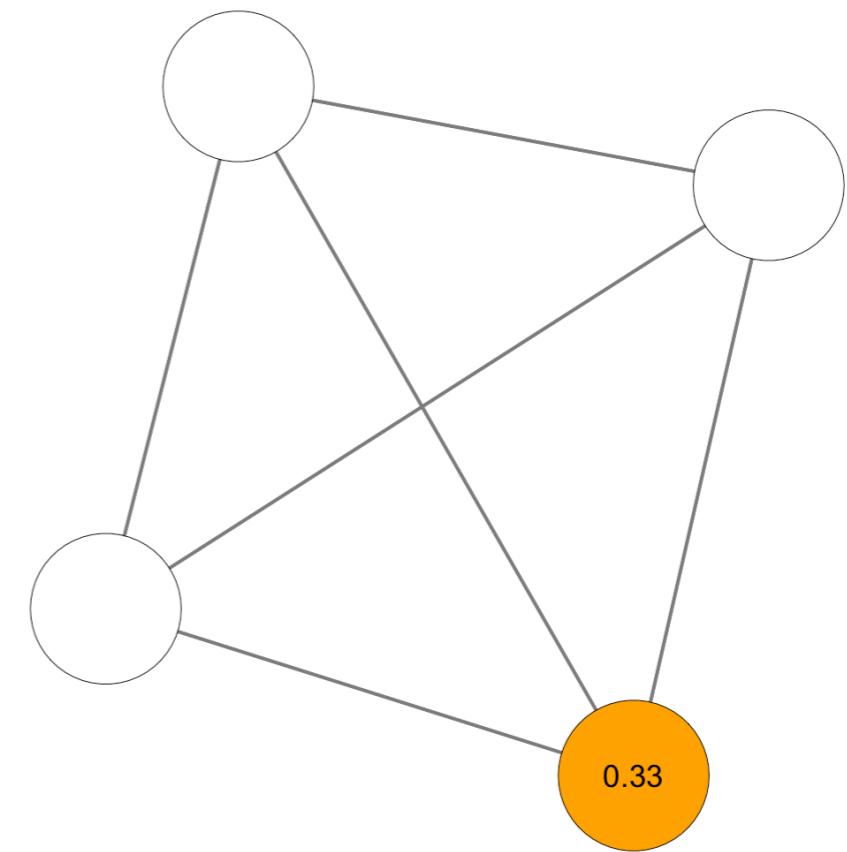
$$\text{Jim} = 0.143^*4 = 0.572$$

*Multiplying by g-1 gives the  
standardized value*

# Why standardization matters

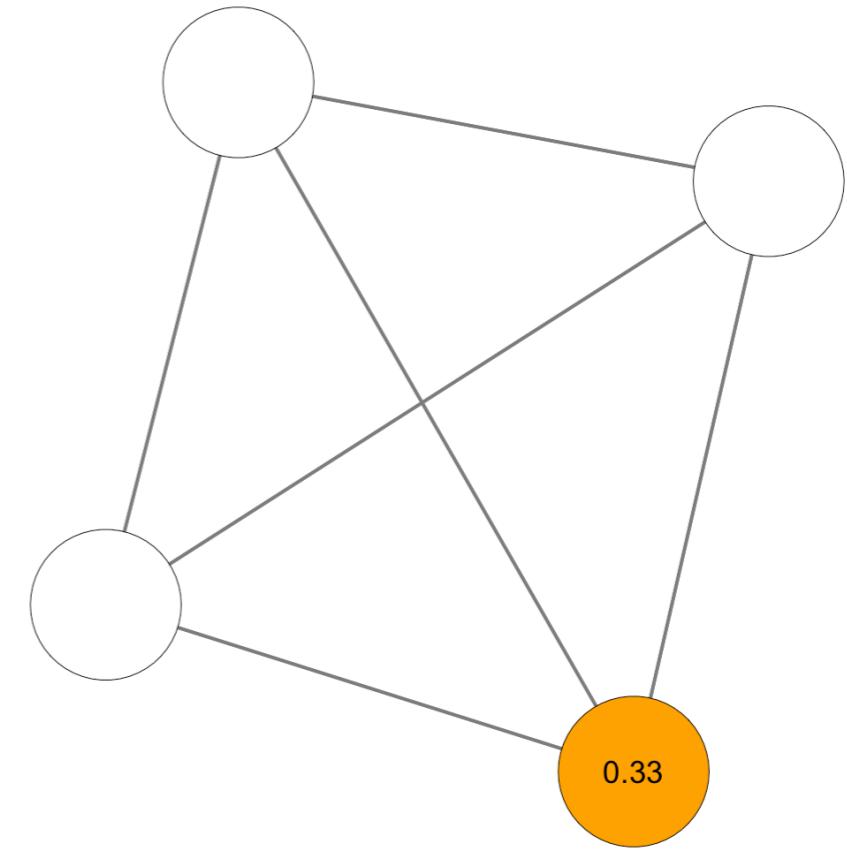
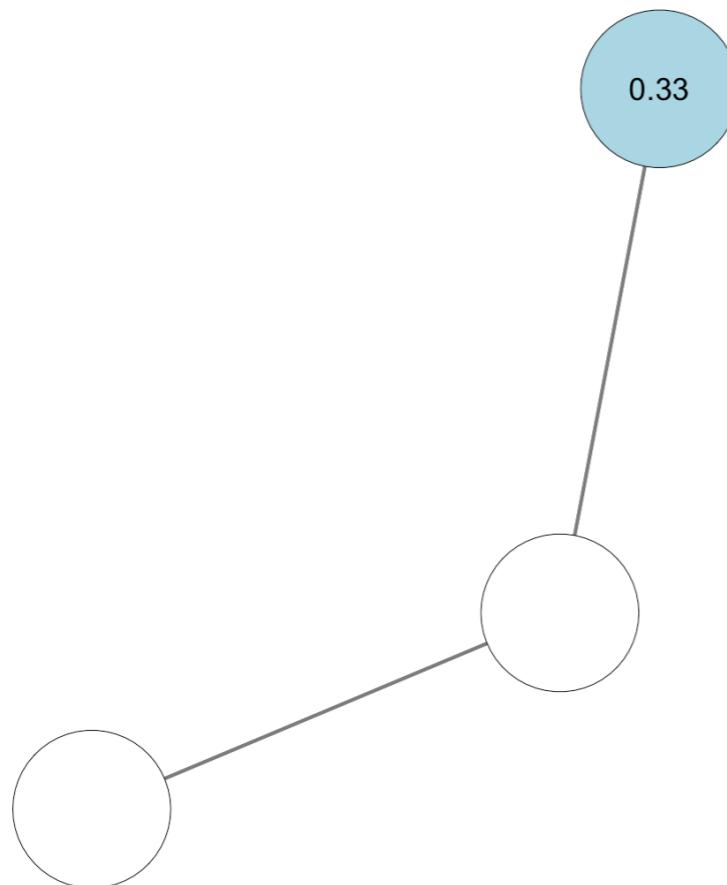


*Network A: Raw*



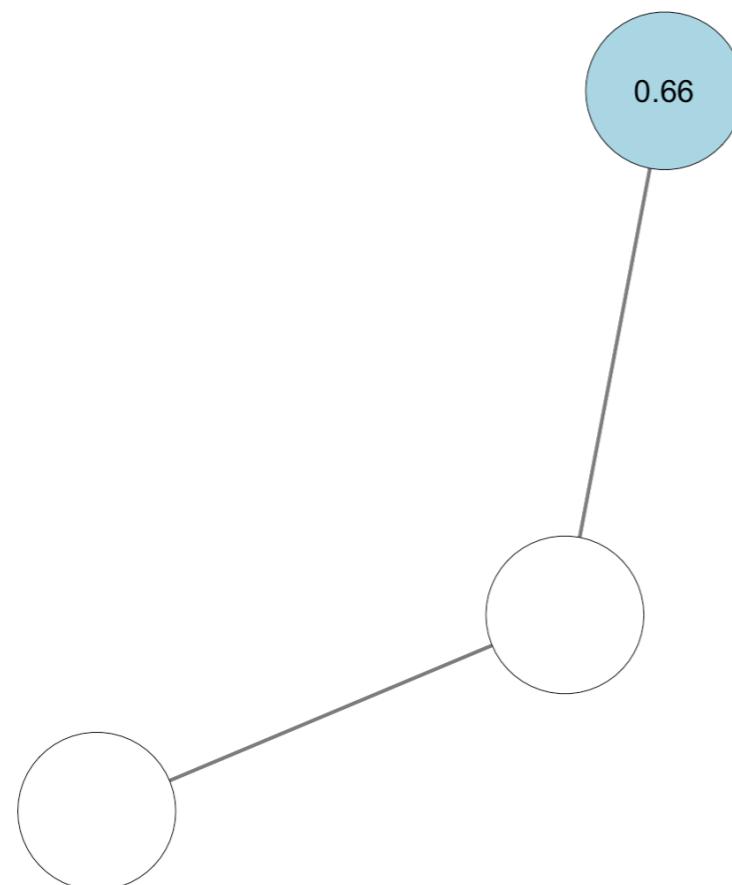
*Network B: Raw*

# Why standardization matters

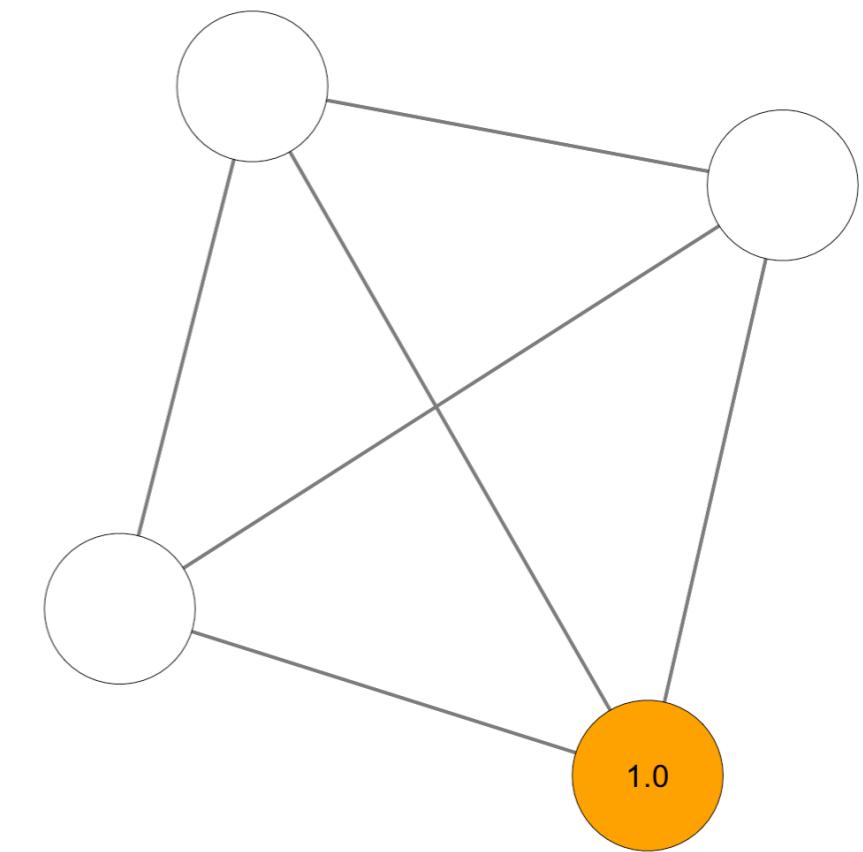


*Are the focal nodes equally close to  
the other nodes?*

# Why standardization matters

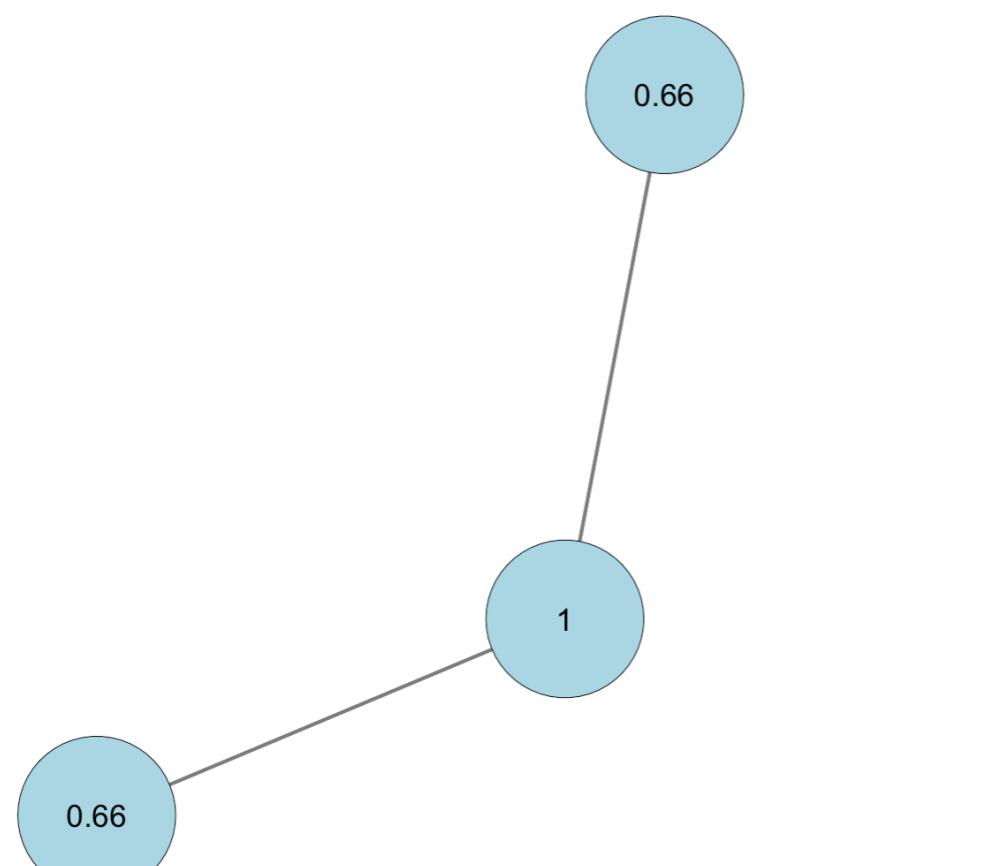


*Network A: Standardized*

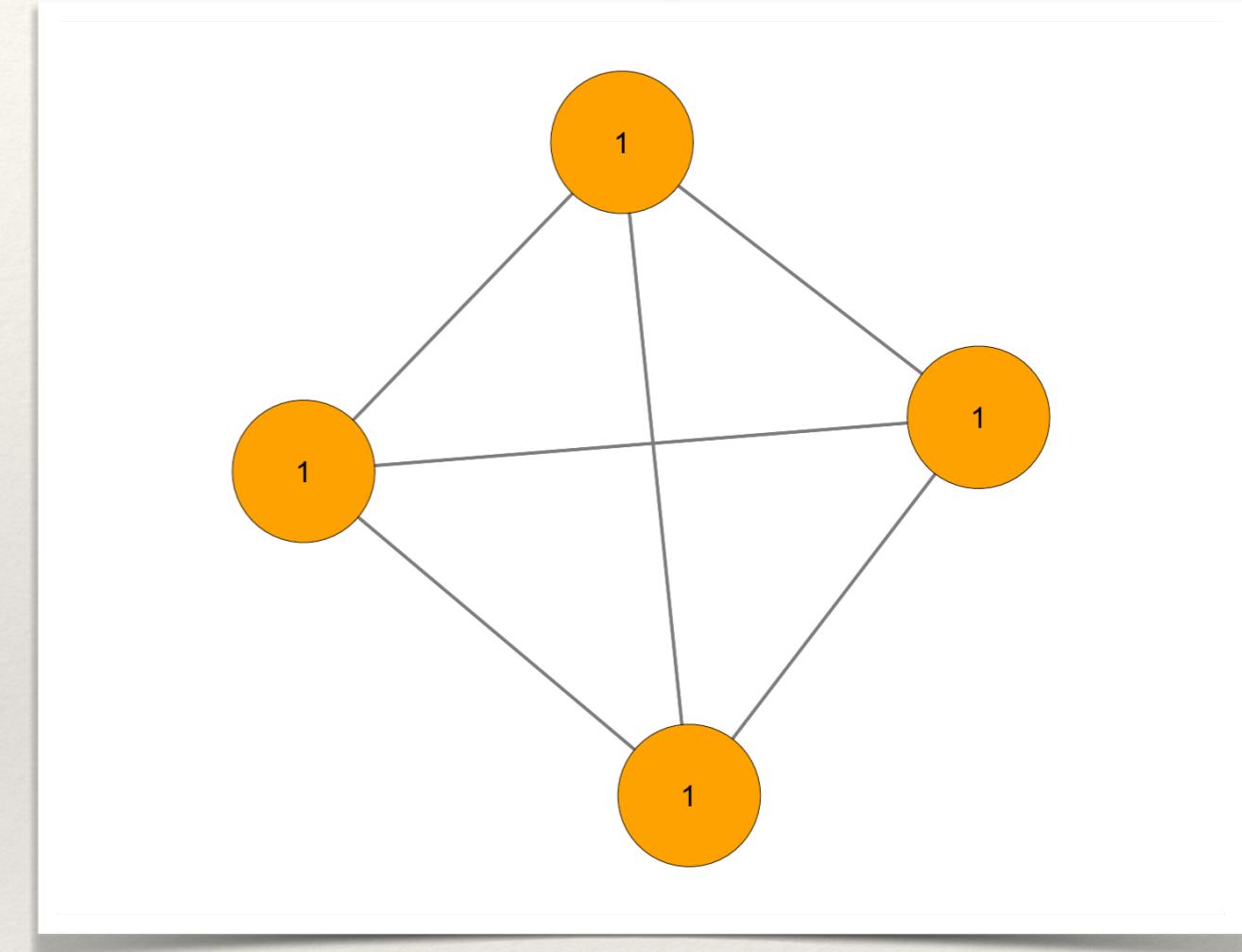


*Network B: Standardized*

# Why standardization matters



*Network A: Standardized*



*Network B: Standardized*

# Centralization

- ❖ We can describe the dispersion in the centrality scores.
  - ❖ In sna, this is referred to as *centralization*.



# Group Closeness Centralization: Undirected Binary Graphs

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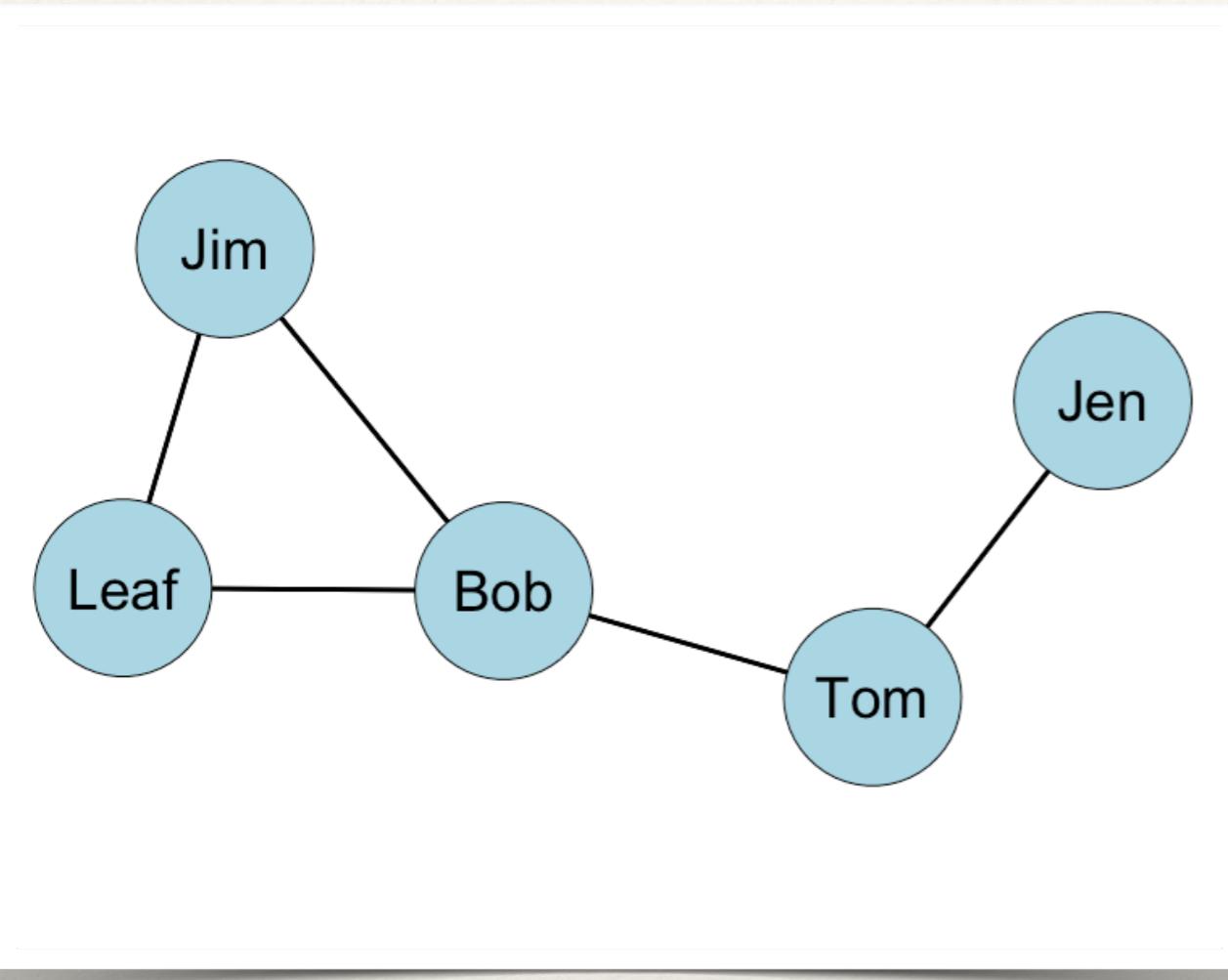
- ❖ Group *closeness* centralization describes the dispersion in the closeness scores between the nodes.
- ❖ The values are between 0 and 1 (as before).
  - ❖ Where scores closer to 0 indicate a decentralized structure and scores closer to 1 indicate a hierarchical structure.

# Group Closeness Centralization: Undirected Binary Graphs

$$C_C = \frac{\sum_{i=1}^g [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

Largest  
standardized  
closeness score      Standardized  
closeness score for  
actor  $i$

# Example: Undirected, Binary Network



Standardized Closeness

Centrality

$$\text{Jen} = 0.111^*4 = 0.444$$

$$\text{Tom} = 0.167^*4 = 0.668$$

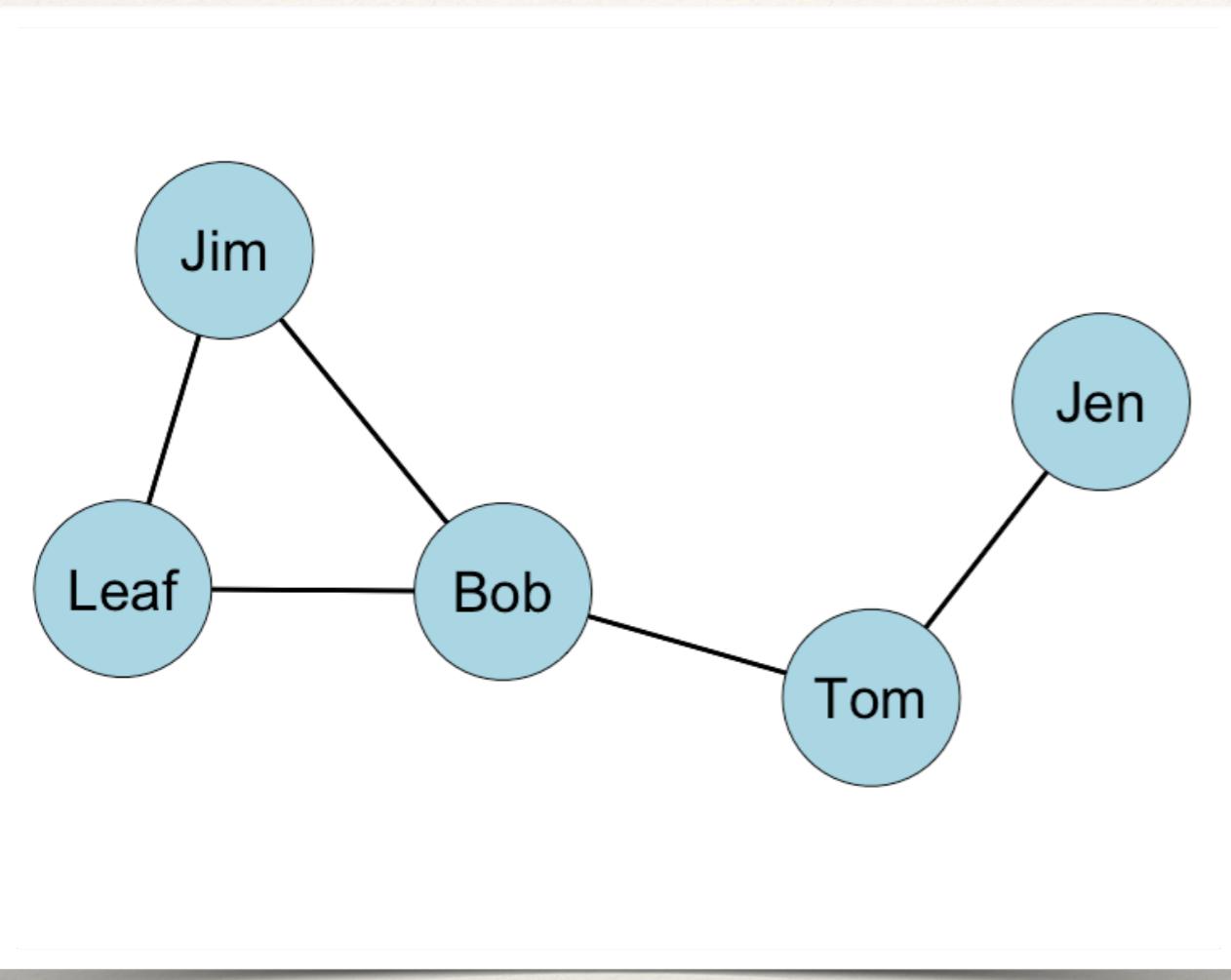
$$\text{Bob} = 0.200^*4 = 0.800$$

$$\text{Leaf} = 0.143^*4 = 0.572$$

$$\text{Jim} = 0.143^*4 = 0.572$$

*What is the group closeness  
centralization score for this graph?*

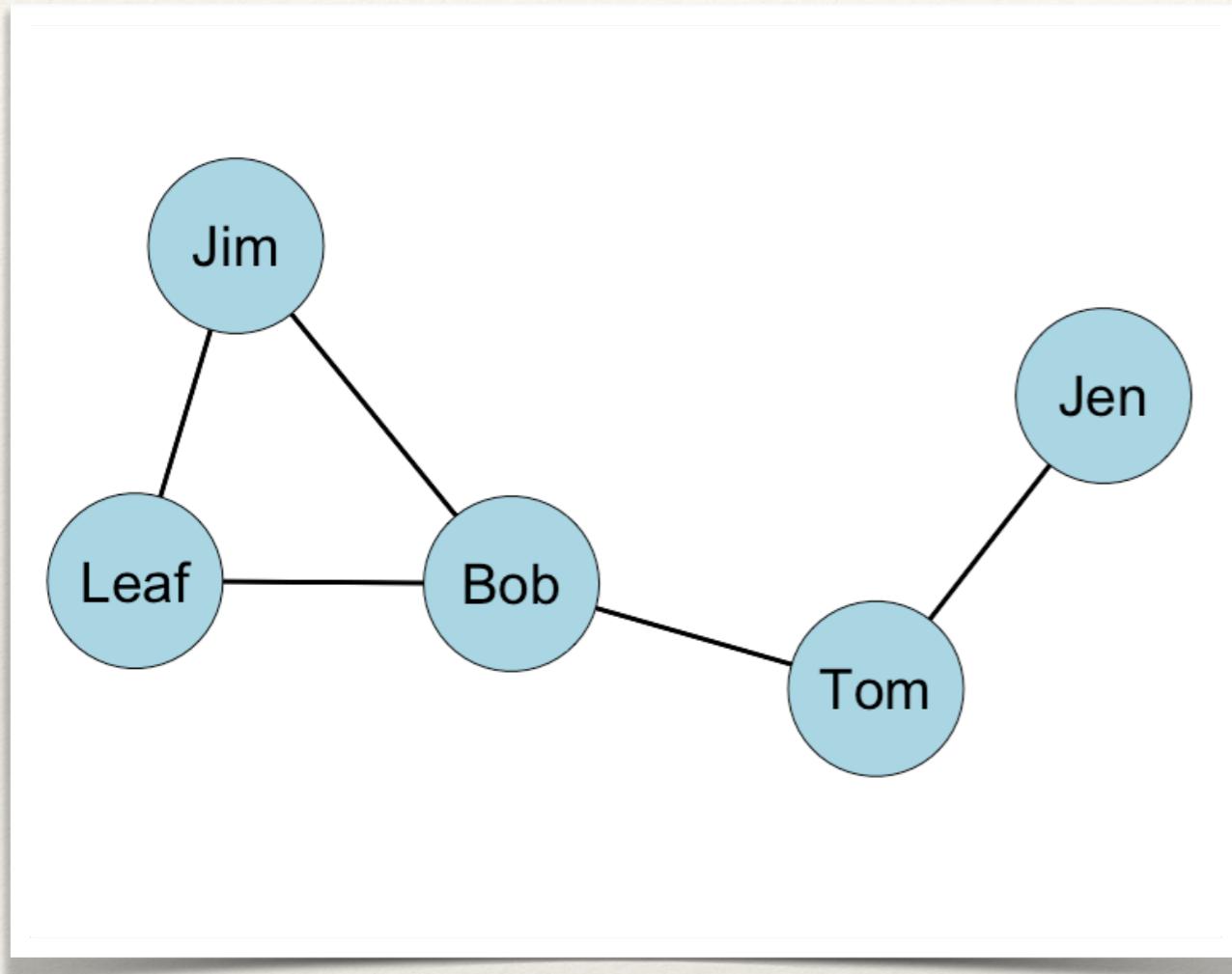
# Example: Undirected, Binary Network



Deviations of Standardized Closeness Centrality Scores

Jen =  $0.800 - 0.444 = 0.356$   
Tom =  $0.800 - 0.668 = 0.132$   
Bob =  $0.800 - 0.800 = 0.000$   
Leaf =  $0.800 - 0.572 = 0.228$   
Jim =  $0.800 - 0.572 = 0.228$

# Example: Undirected, Binary Network



Deviations of  
Standardized Closeness

Centrality Scores

$$\text{Jen} = 0.800 - 0.444 = 0.356$$

$$\text{Tom} = 0.800 - 0.668 = 0.132$$

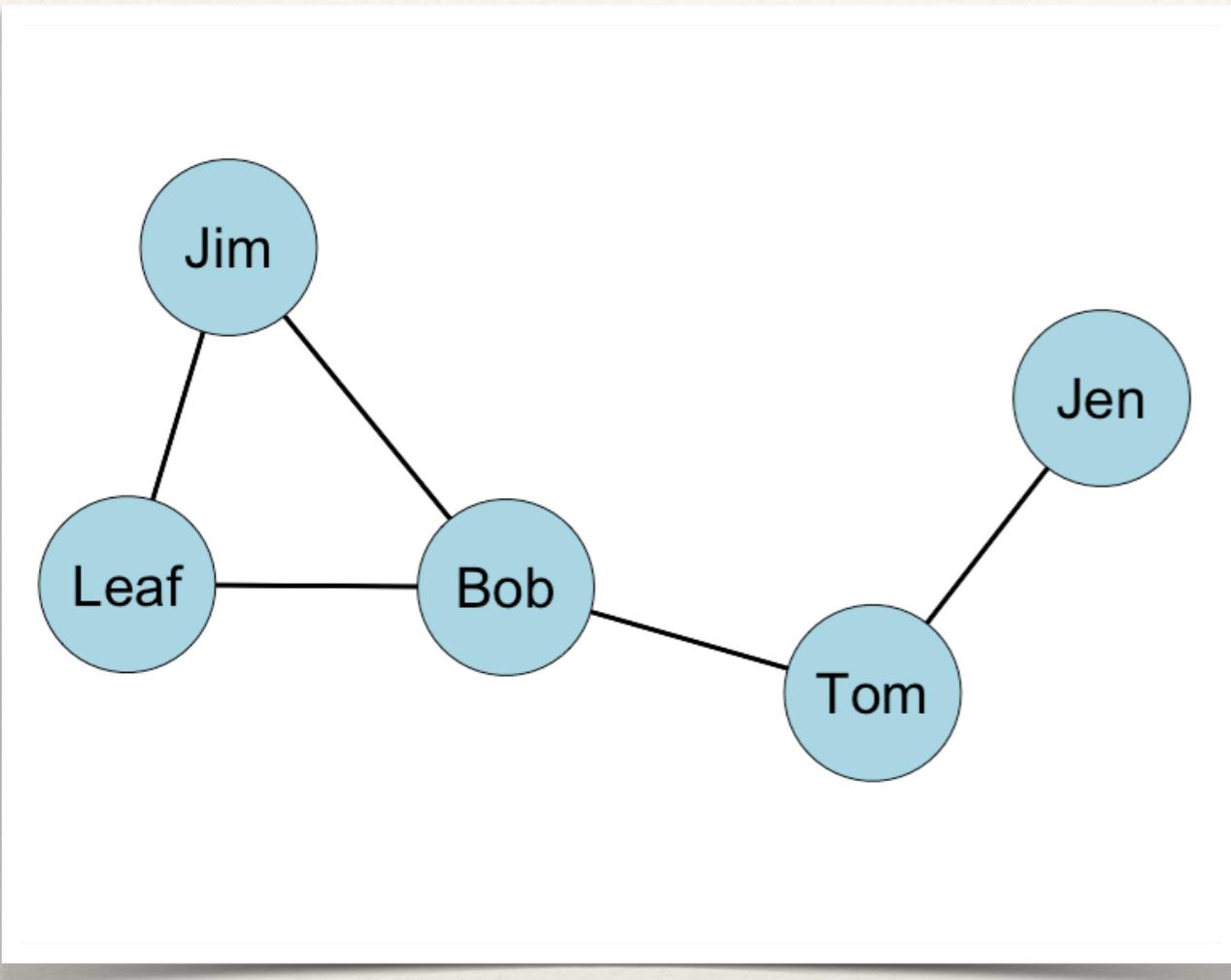
$$\text{Bob} = 0.800 - 0.800 = 0.000$$

$$\text{Leaf} = 0.800 - 0.572 = 0.228$$

$$\text{Jim} = 0.800 - 0.572 = 0.228$$

$$\text{Sum} = 0.944$$

# Example: Undirected, Binary Network



Deviations of Standardized Closeness Centrality Scores

$$\text{Jen} = 0.800 - 0.444 = 0.356$$

$$\text{Tom} = 0.800 - 0.668 = 0.132$$

$$\text{Bob} = 0.800 - 0.800 = 0.000$$

$$\text{Leaf} = 0.800 - 0.572 = 0.228$$

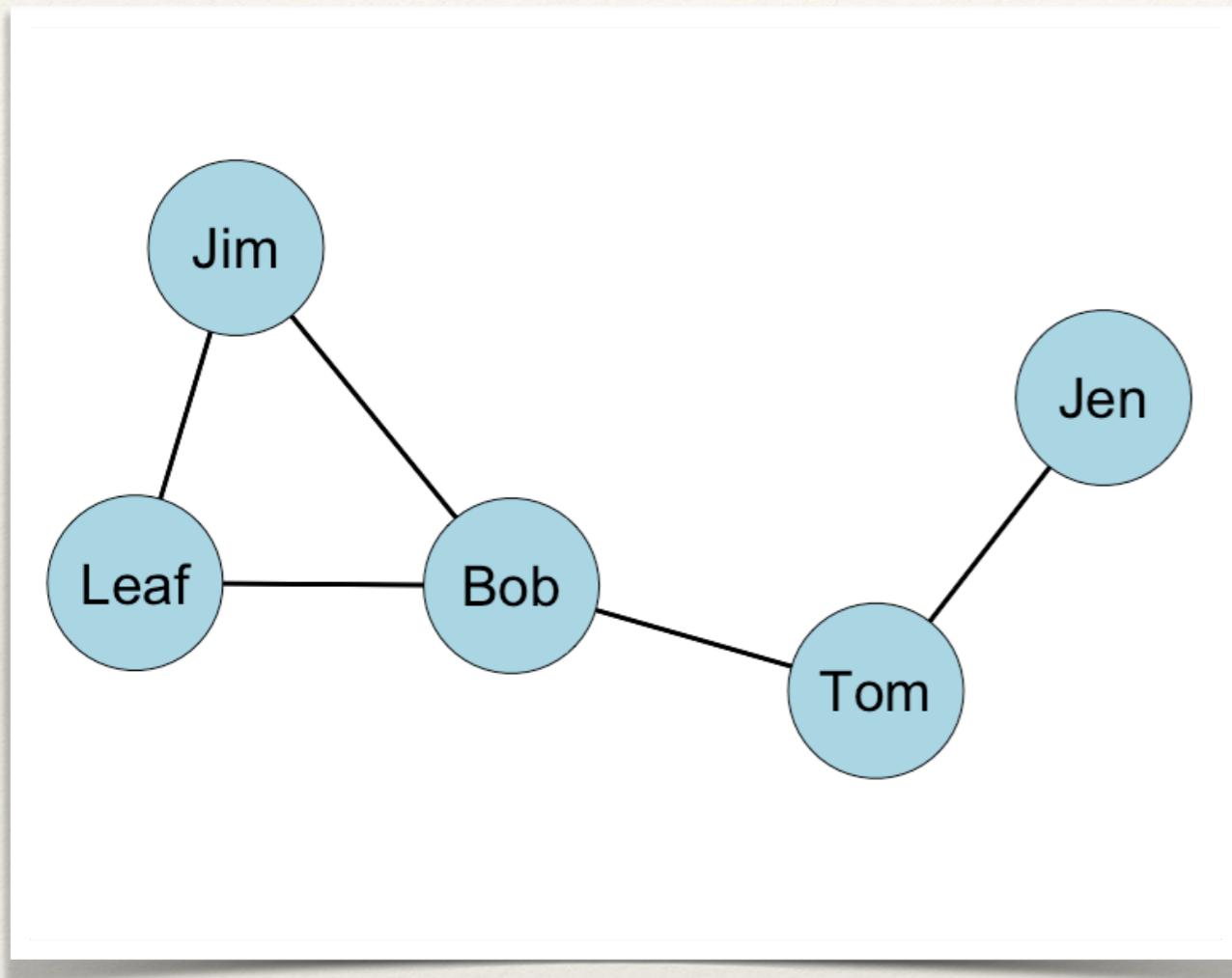
$$\text{Jim} = 0.800 - 0.572 = 0.228$$

$$\text{Sum} = 0.944$$

$$\begin{aligned} \text{Denominator} &= \\ &[(5-2)(5-1)/(2*5-3)] = 1.714 \end{aligned}$$

$$C_C = \frac{\sum_{i=1}^g [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

# Example: Undirected, Binary Network



*Approximately 0.551*

Deviations of  
Standardized Closeness  
Centrality Scores

$$\text{Jen} = 0.800 - 0.444 = 0.356$$

$$\text{Tom} = 0.800 - 0.668 = 0.132$$

$$\text{Bob} = 0.800 - 0.800 = 0.000$$

$$\text{Leaf} = 0.800 - 0.572 = 0.228$$

$$\text{Jim} = 0.800 - 0.572 = 0.228$$

$$\text{Sum} = 0.944$$

$$\begin{aligned}\text{Denominator} &= \\ &[(5-2)(5-1) / (2 \cdot 5 - 3)] = 1.714\end{aligned}$$

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# Example: Undirected, Binary Network

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$$C_C = \frac{\sum_{i=1}^g [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

$$= \frac{0.356 + 0.132 + 0.000 + 0.228 + 0.228}{[(5-2)(5-1)]/(2*5-3)} = \frac{0.944}{1.714} = 0.551$$

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# Betweenness Centrality

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- ❖ We have seen how centrality can be conceptualized as:
  - ❖ Having a high number of ties
  - ❖ Being close to others in the network
    - ❖ We can also conceptualize centrality as a node that lies on a particular path between other nodes.
    - ❖ *Betweenness centrality* is based on the number of **shortest** paths between  $j$  and  $k$  that actor  $i$  resides on.

# Undirected Networks

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# Betweenness Centrality: Undirected Binary Graphs

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$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

# Betweenness Centrality: Undirected Binary Graphs

The number of  
geodesics linking  $j$   
to  $k$ .

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / \downarrow g_{jk}$$

# Betweenness Centrality: Undirected Binary Graphs

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

The number of geodesics linking  $j$  to  $k$ .

The number of geodesics linking  $j$  and  $k$  that contain  $i$ .

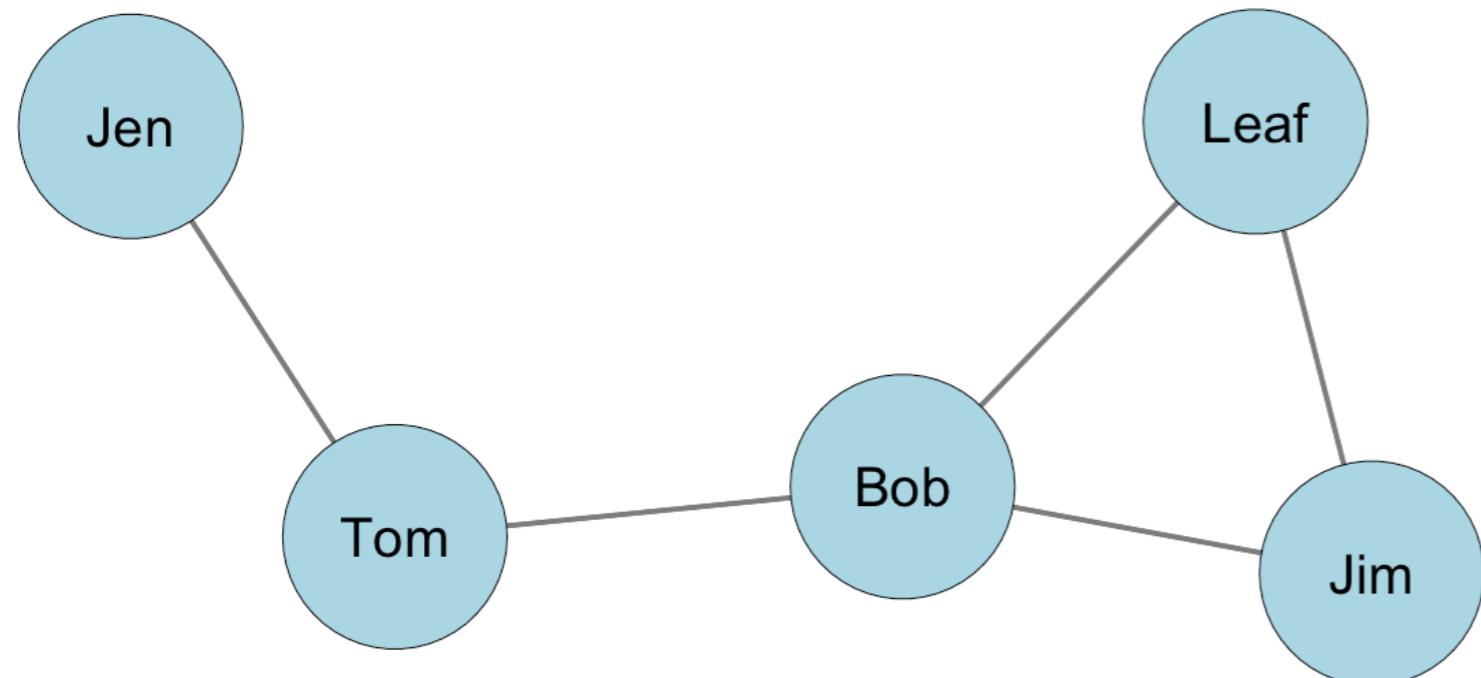
# Betweenness Centrality: Undirected Binary Graphs

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$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

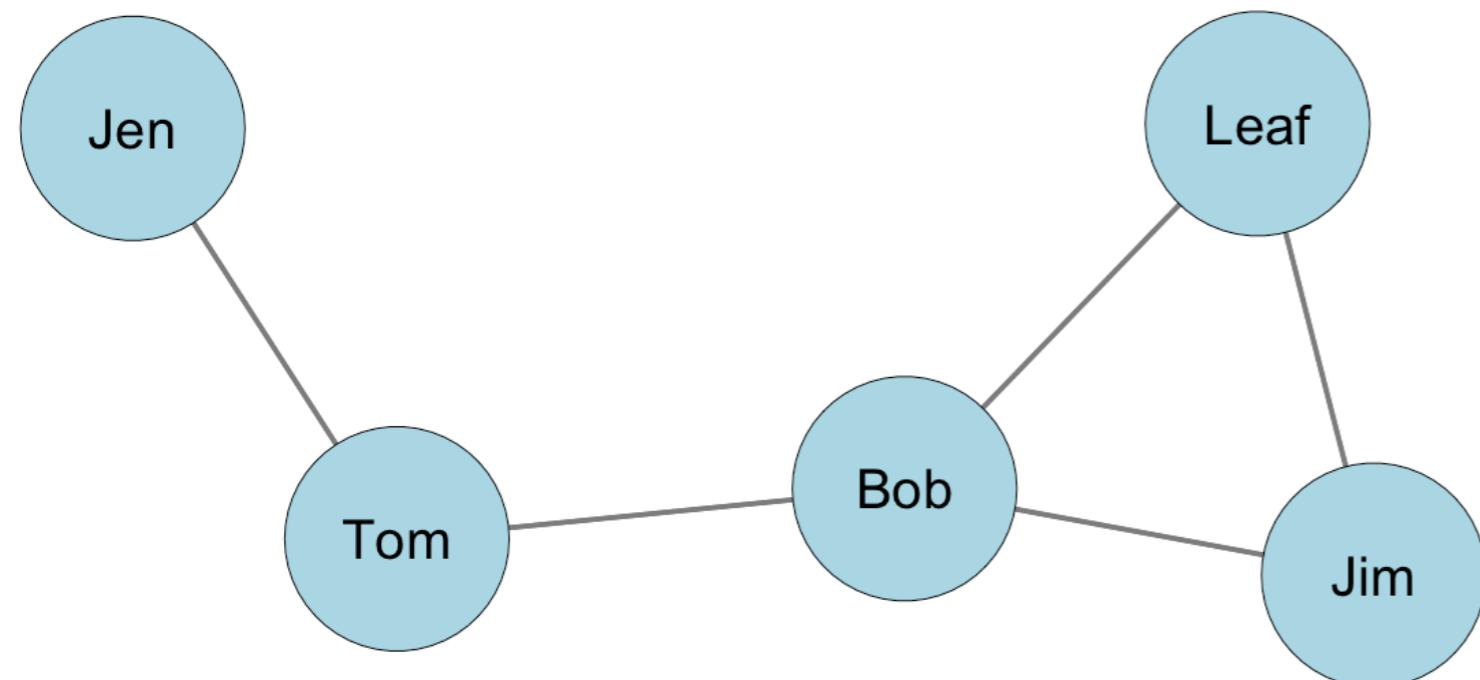
So, betweenness centrality is the ratio of the geodesics between  $j$  and  $k$  that contain  $i$ .

# Example: Undirected, Binary Network



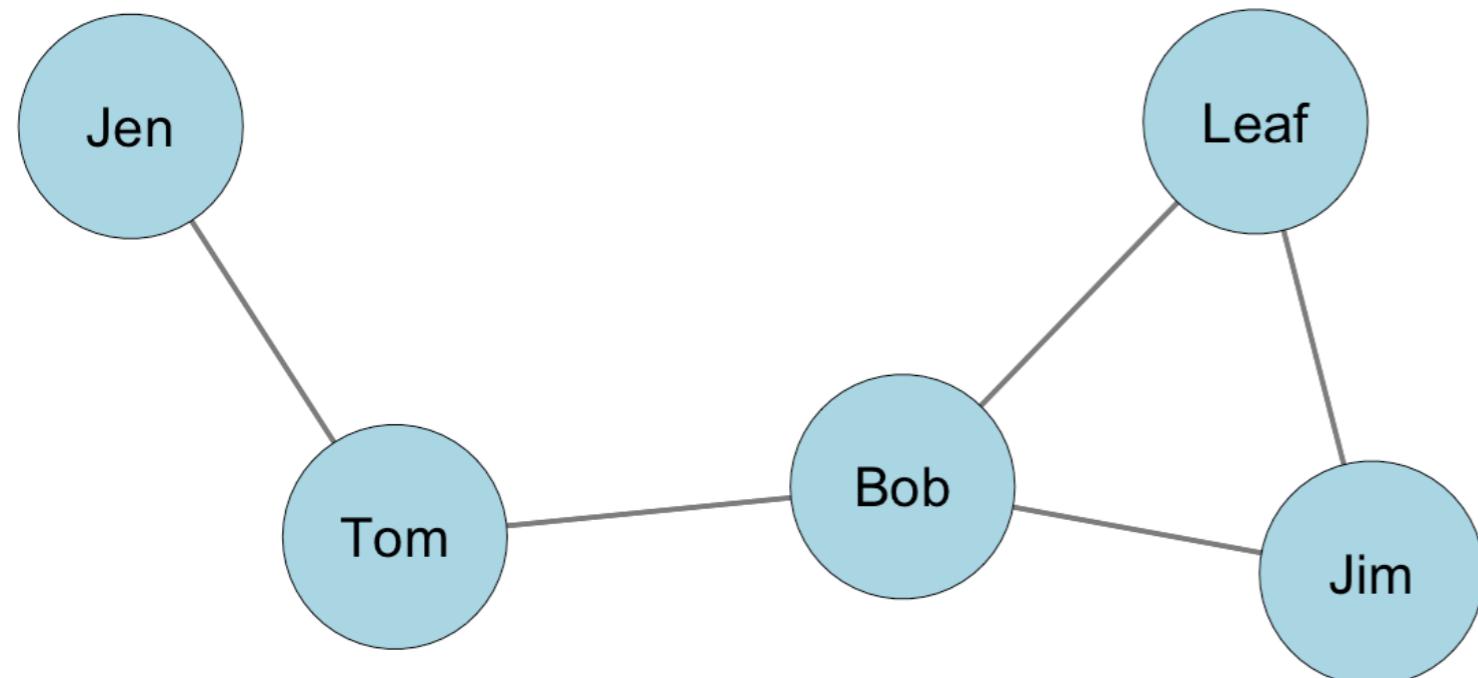
What are the paths  
between Jen and Jim?

# Example: Undirected, Binary Network



Let's calculate the betweenness centrality for **Bob**.

# Example: Undirected, Binary Network



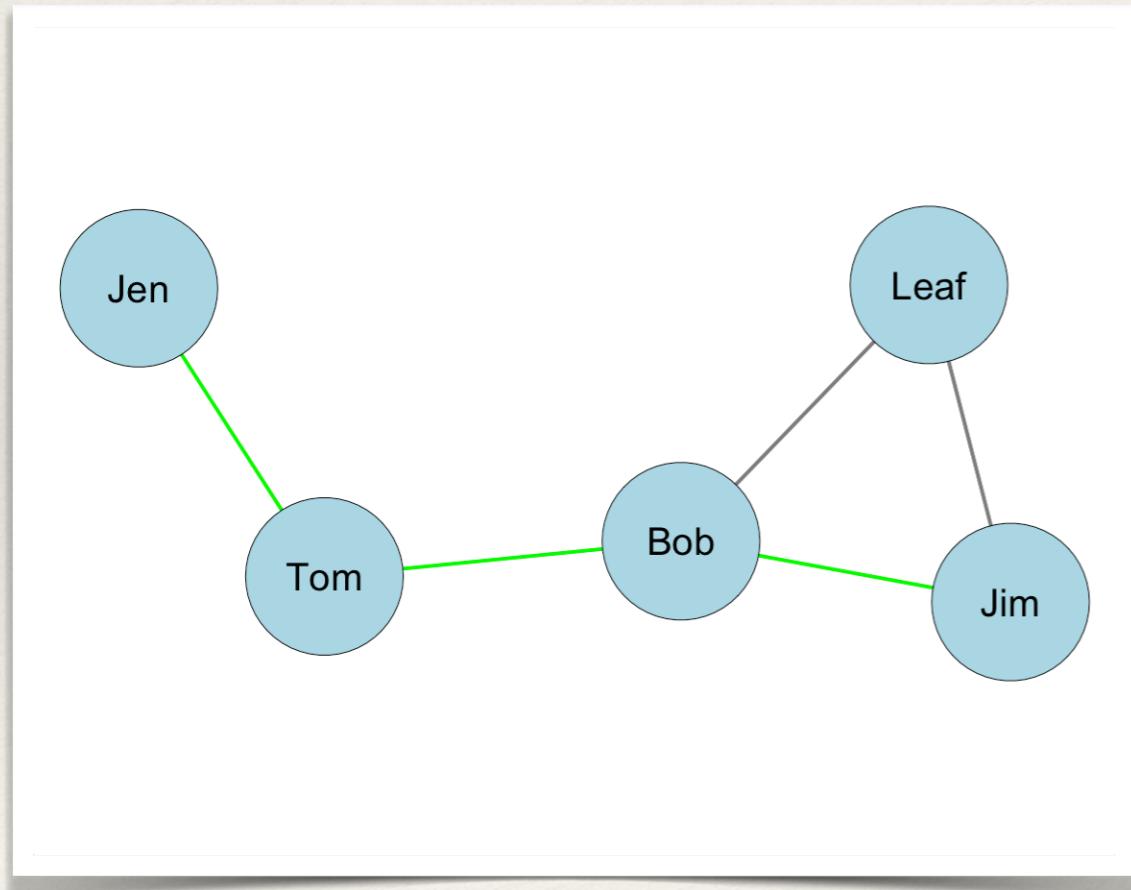
Let's calculate the betweenness centrality for **Bob**.

First, we need to find the geodesics for other nodes.

Then, figure out how many **Bob** occupies.

# Example: Undirected, Binary Network

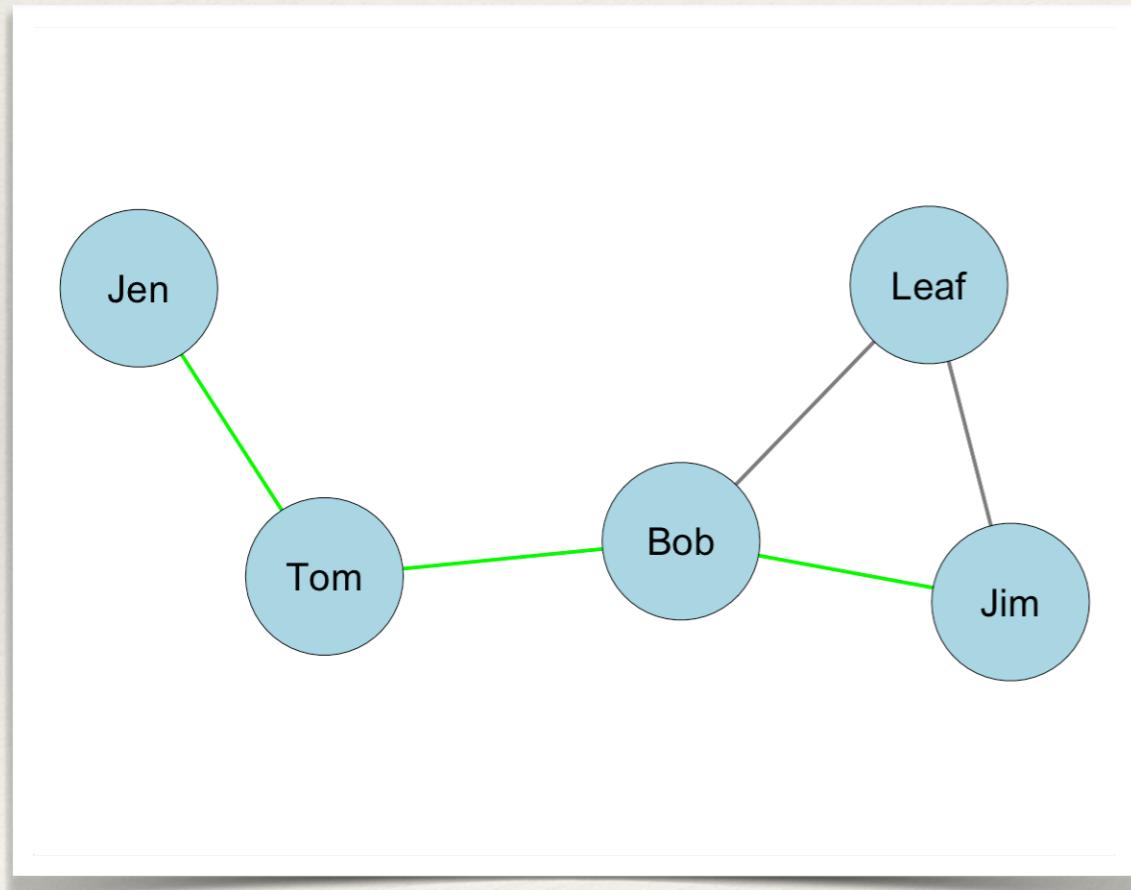
What are the paths between Jen and Jim?



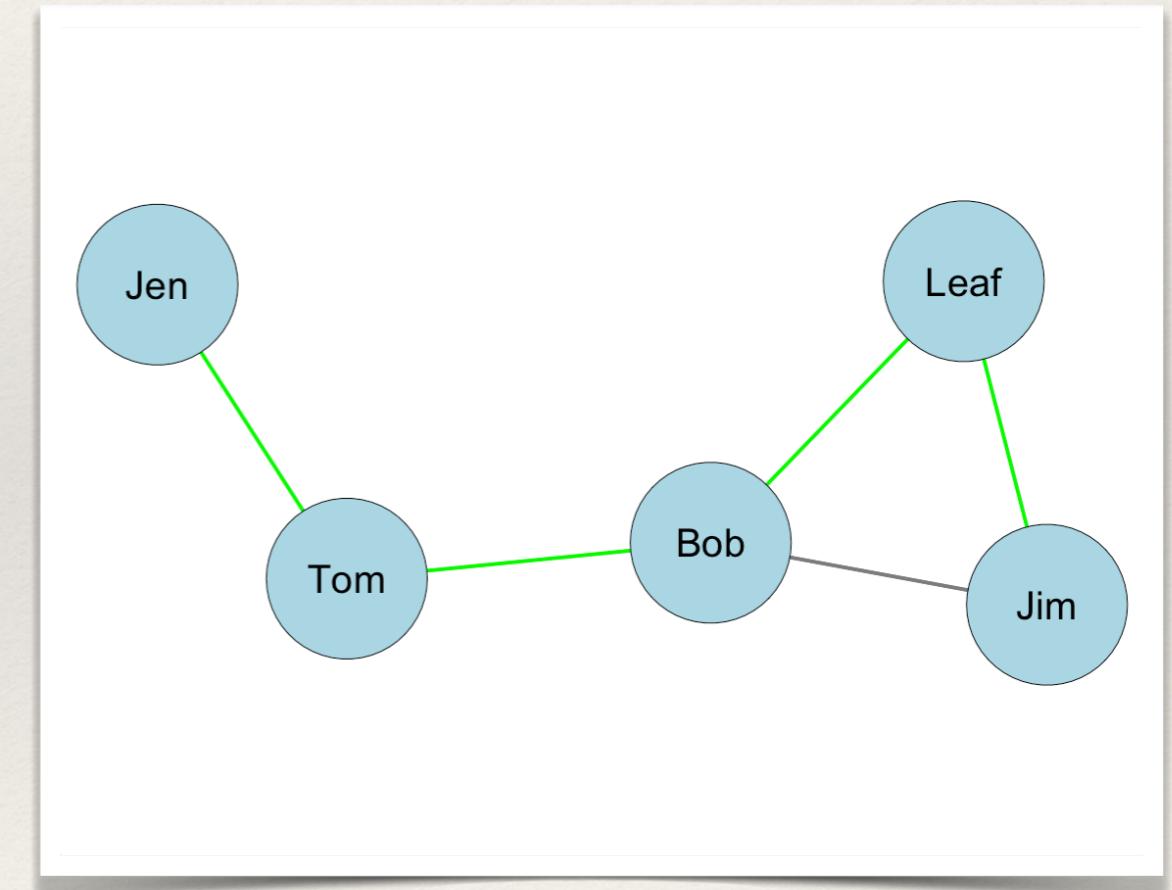
Jen-Tom-Bob-Jim

# Example: Undirected, Binary Network

What are the paths between Jen and Jim?

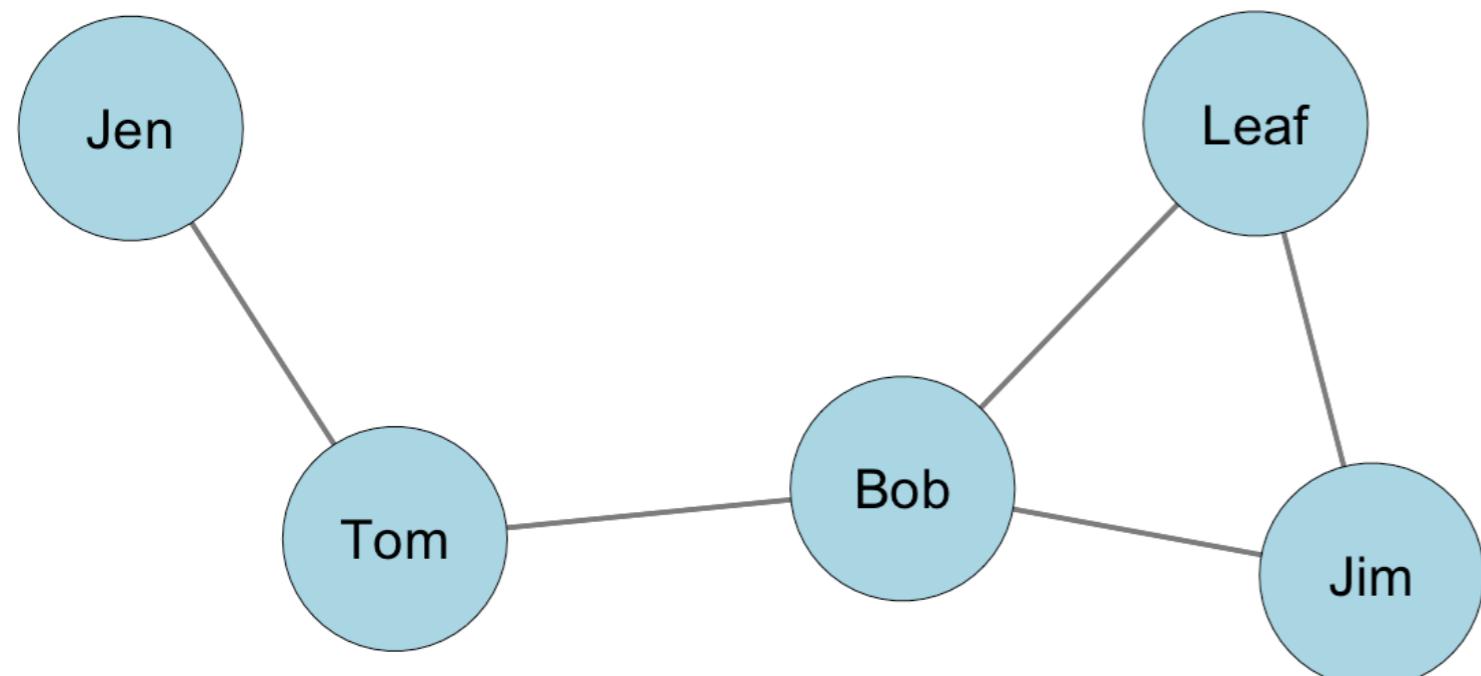


Jen-Tom-Bob-Jim



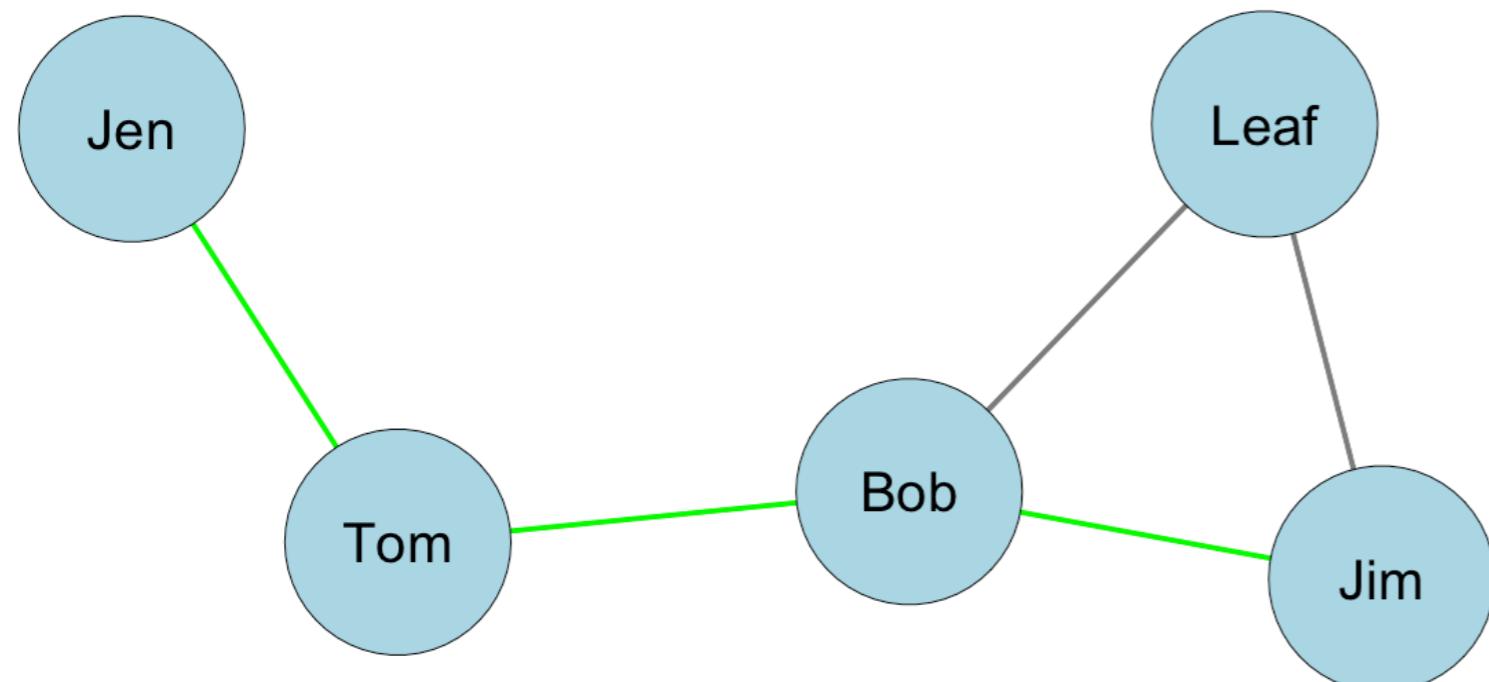
Jen-Tom-Bob-Leaf-Jim

# Example: Undirected, Binary Network



What are the geodesic paths between Jen and Jim?

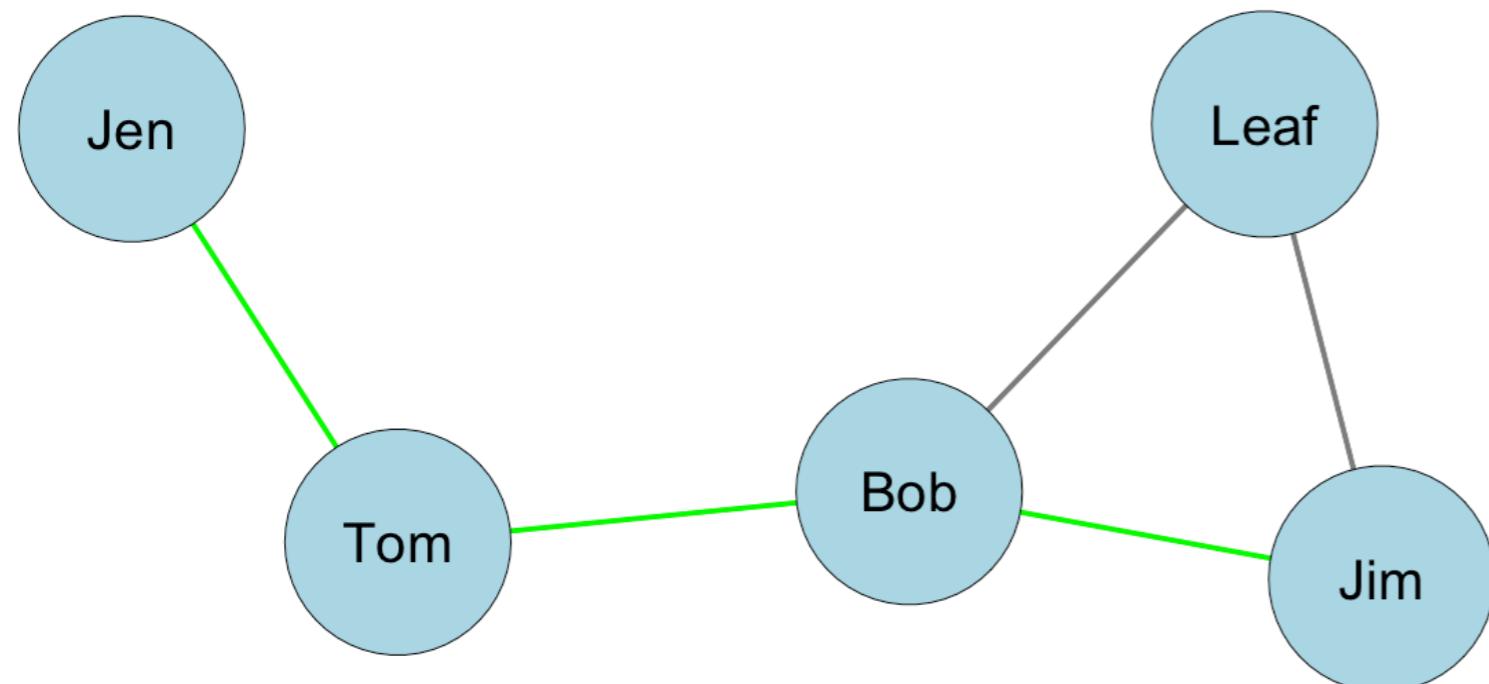
# Example: Undirected, Binary Network



What are the geodesic paths between Jen and Jim?

Jen-Tom-Bob-Jim

# Example: Undirected, Binary Network

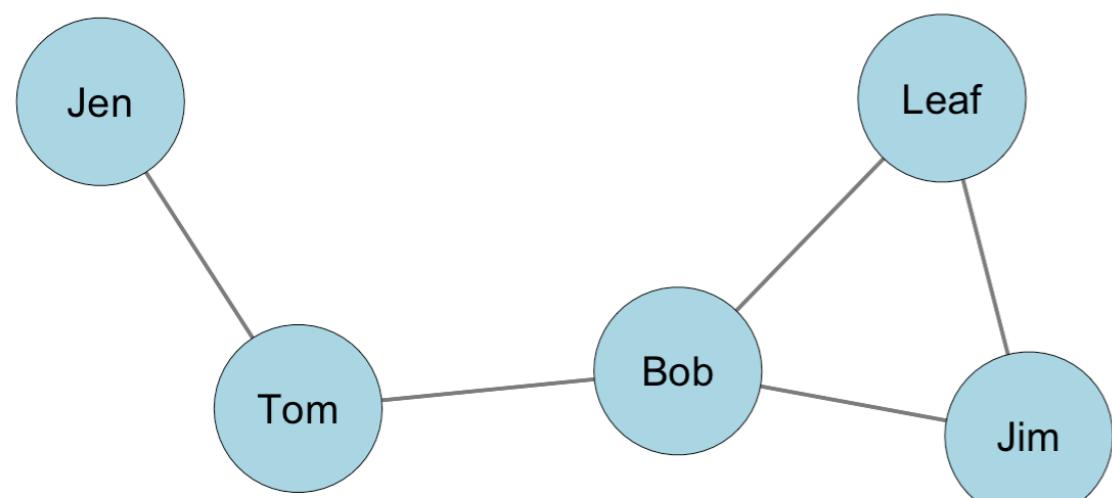


What are the geodesic paths between Jen and Jim?

Jen-Tom-Bob-Jim

*Is there anyone on the geodesic between Jen and Jim?*

# Example: Undirected, Binary Network

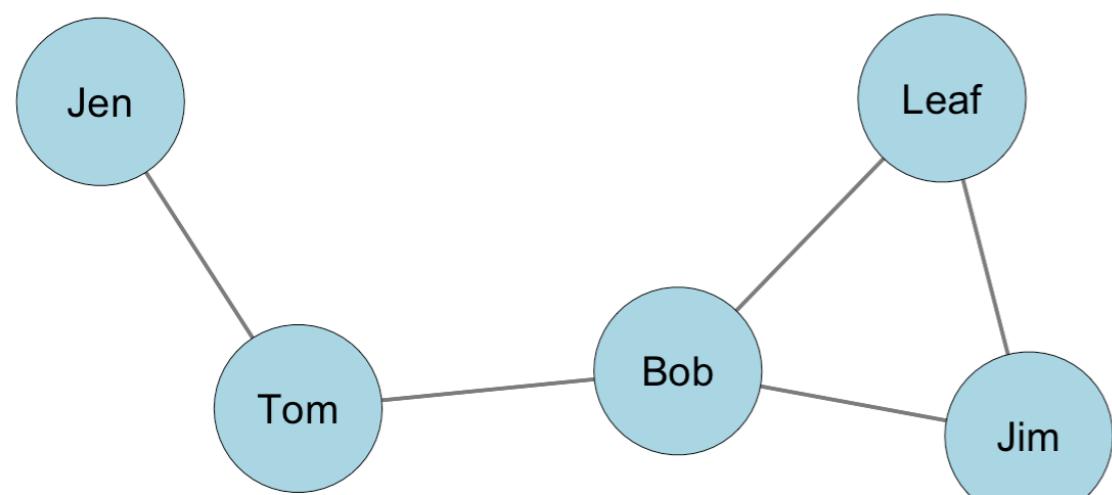


How many geodesics  
from Jen to Jim?

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen				
				/?

# Example: Undirected, Binary Network

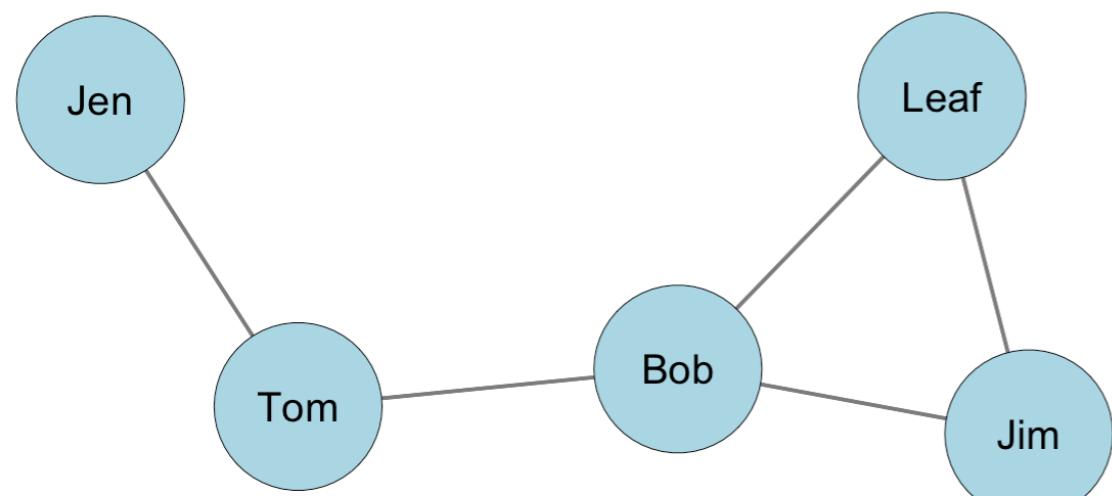


There is 1 geodesic  
from **Jen** to **Jim**

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

# Example: Undirected, Binary Network

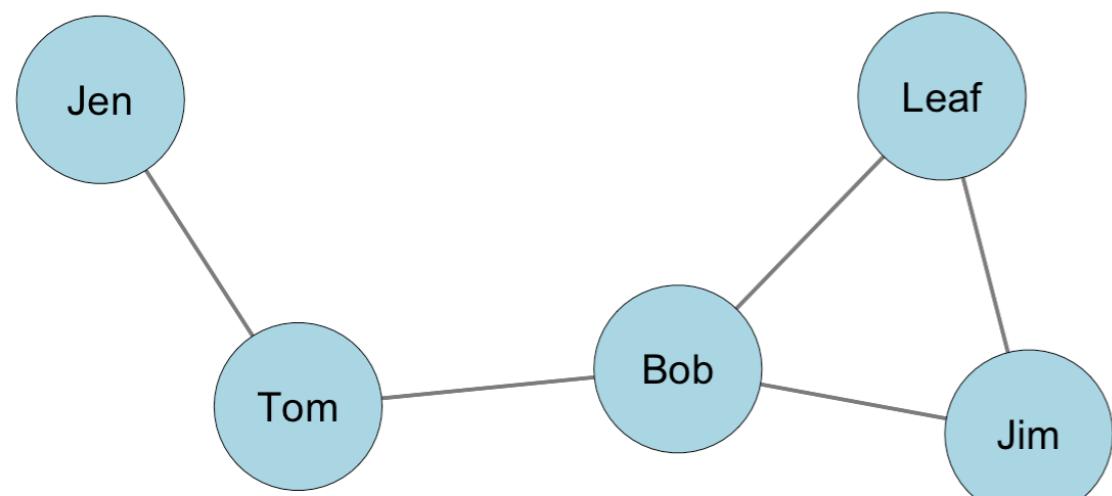


How many geodesics  
from **Jen** to **Jim**  
include **Bob**?

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				?
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

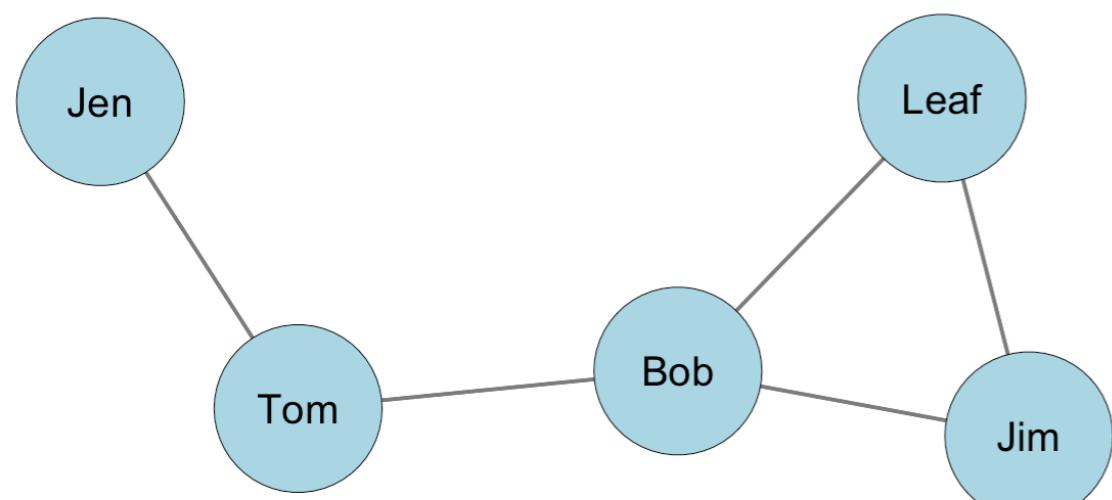
# Example: Undirected, Binary Network



**Bob** is on the  
only geodesic  
from **Jen** to **Jim**

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				1/1

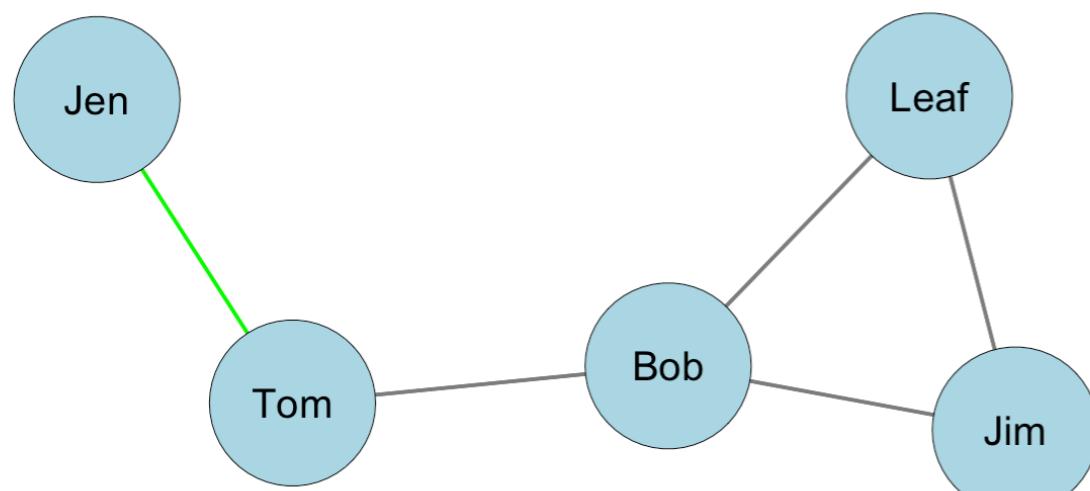
# Example: Undirected, Binary Network



What about Jen  
to Tom?

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

# Example: Undirected, Binary Network

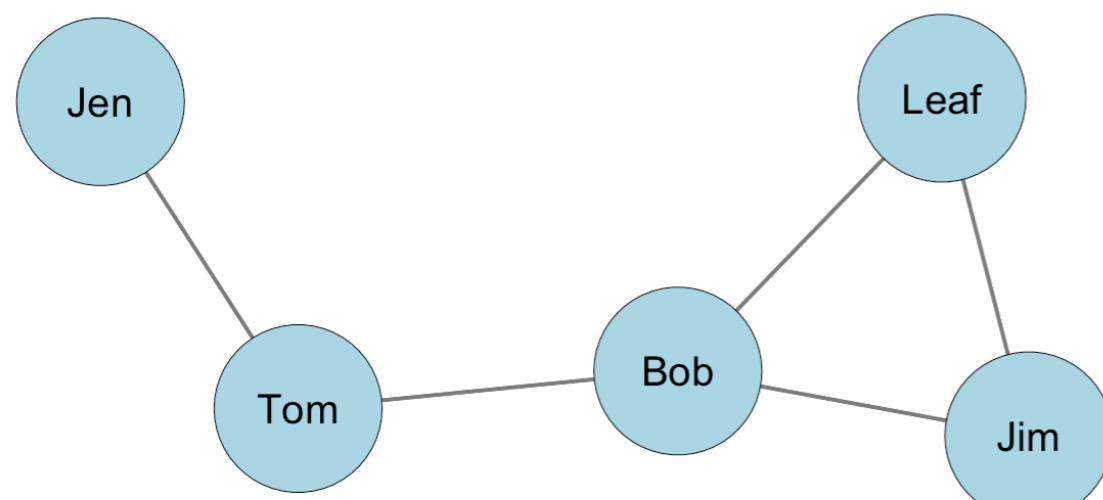


**Bob** is not on  
the only  
geodesic from  
Jen to Tom

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1		1/1
Tom				
Leaf				
Jim				

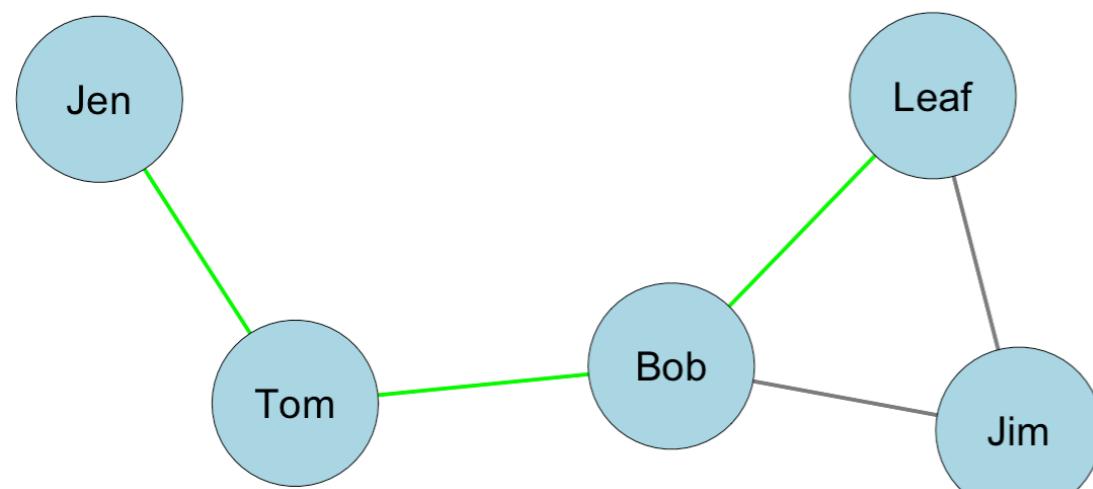
# Example: Undirected, Binary Network



What about Jen  
to Leaf?

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	/?	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

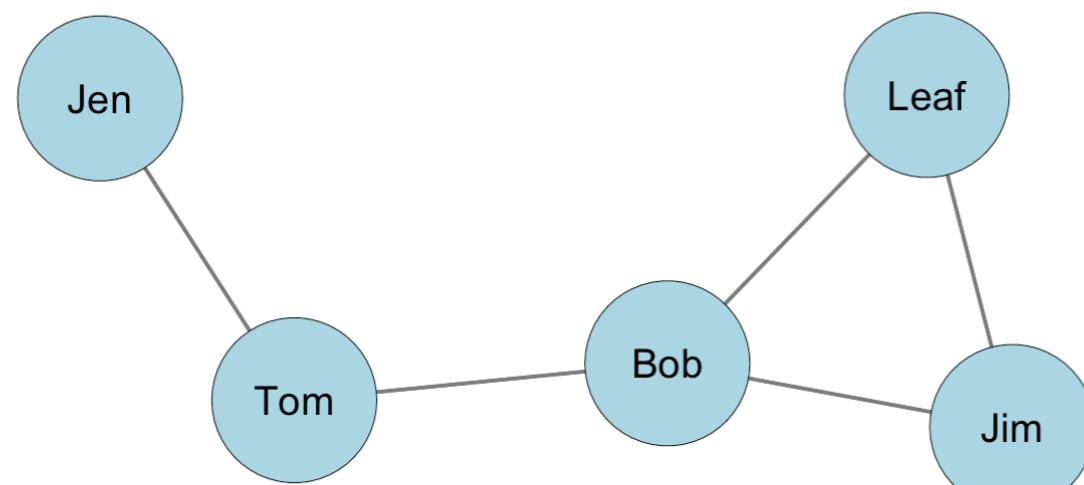
# Example: Undirected, Binary Network



**Bob** is on the  
geodesic from  
**Jen** to **Leaf**

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

# Example: Undirected, Binary Network

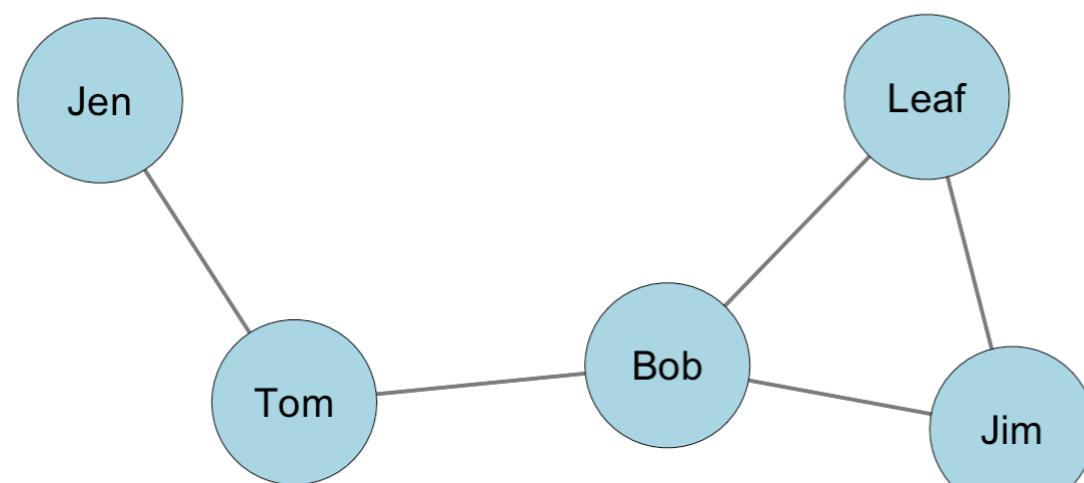


Of the geodesics between  
**Jen, Tom**  
**Jen, Leaf**  
**Jen, Tom**  
how many include Bob?

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

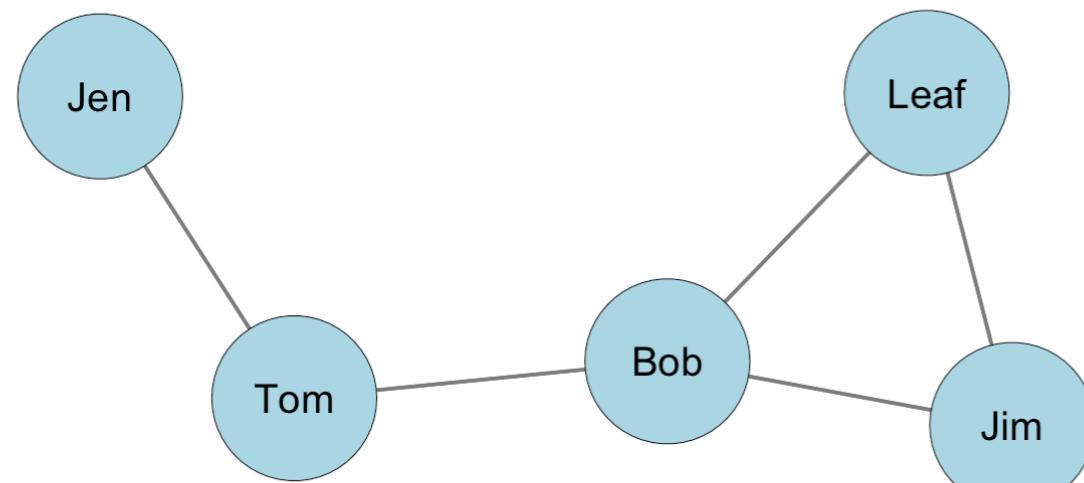
# Example: Undirected, Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1

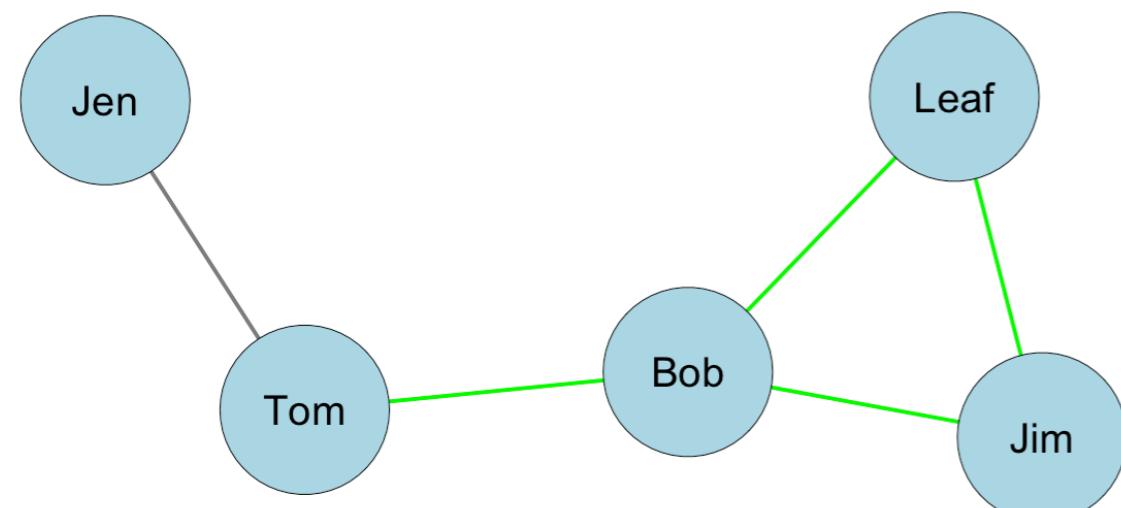
# Example: Undirected, Binary Network



To finish, we need to calculate the geodesics for the rest of the matrix for Bob

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			?/?	?/?
<i>Leaf</i>				?/?
<i>Jim</i>				

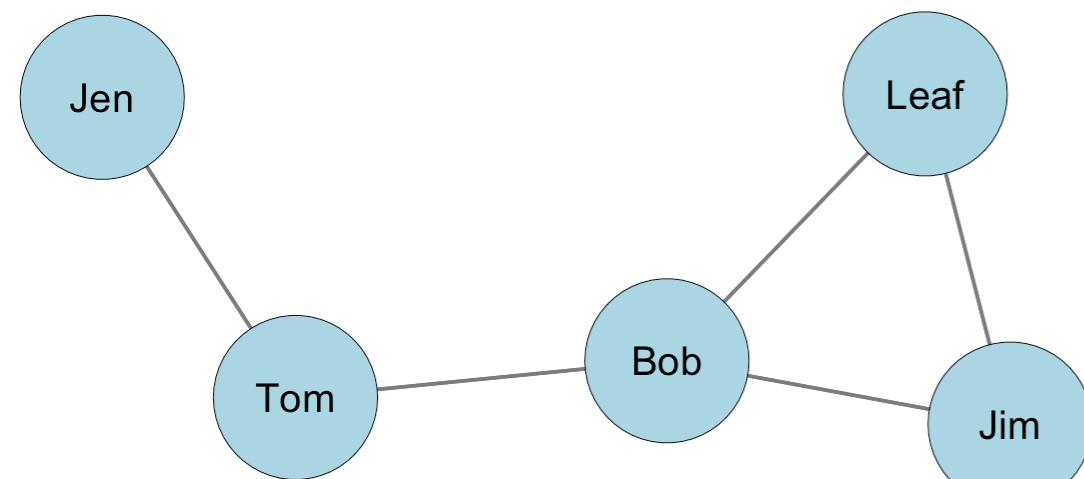
# Example: Undirected, Binary Network



To finish, we need to calculate the geodesics for the rest of the matrix for Bob

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

# Example: Undirected, Binary Network



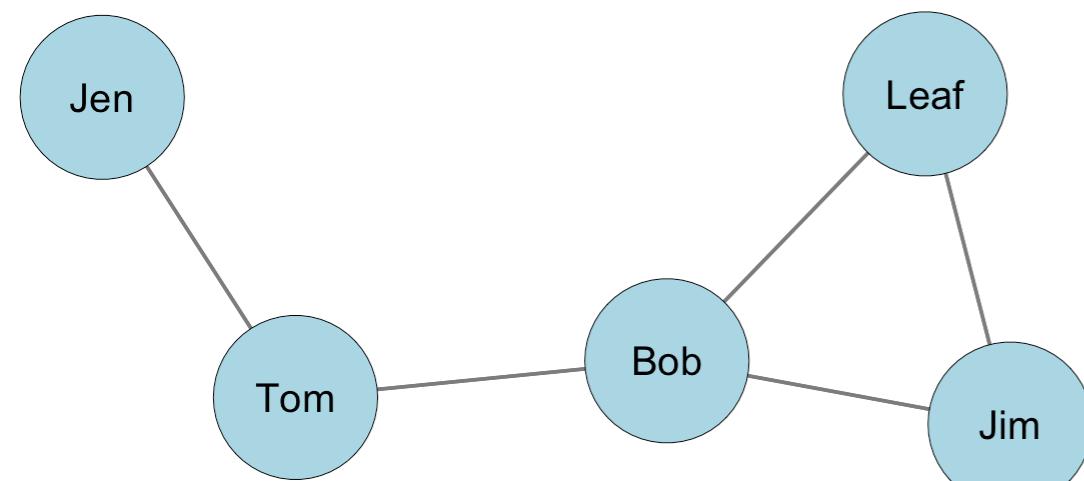
The sum of all these ratios is **Bob's** betweenness centrality score.

*What is Bob's score?*

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

# Example: Undirected, Binary Network

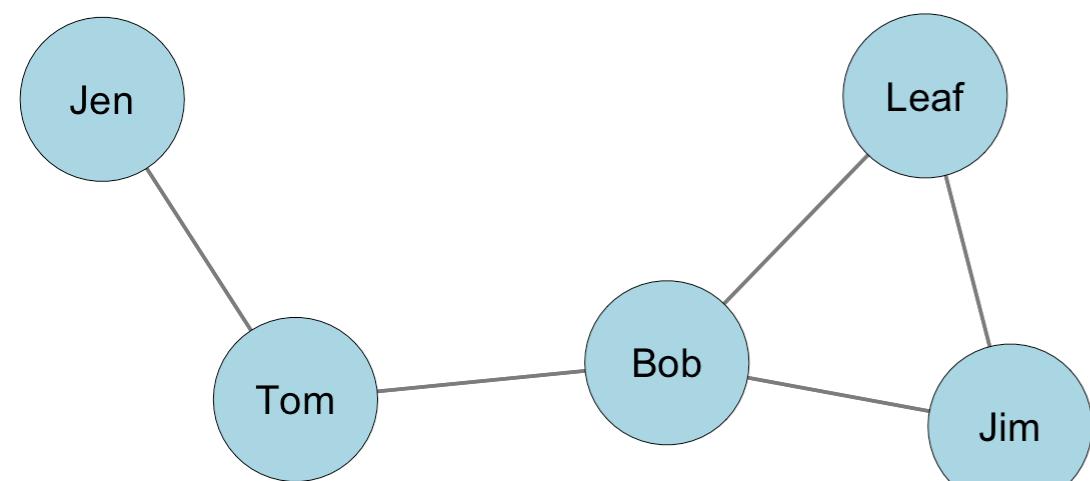


Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

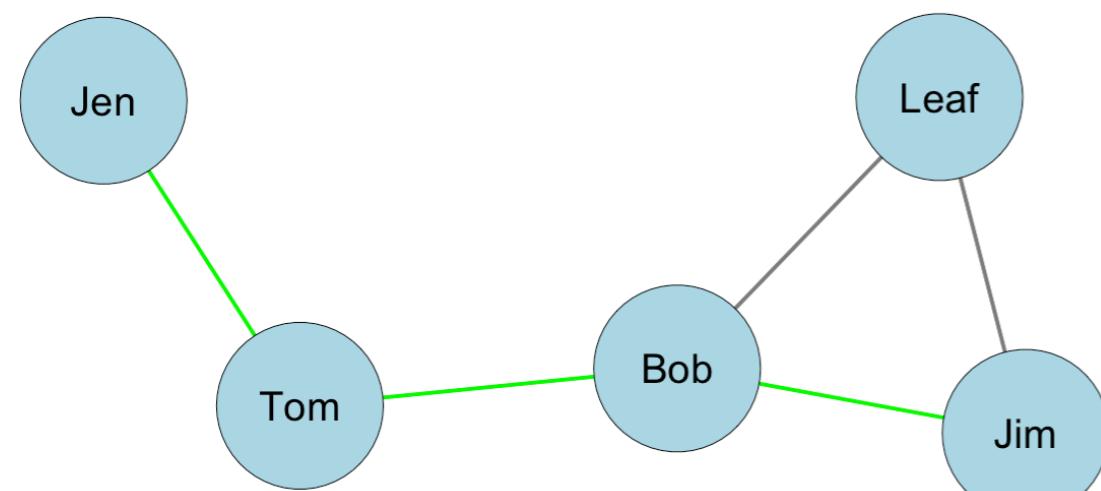
# Example: Undirected, Binary Network



Let's do this now for **Leaf**. *What is Leaf's betweenness centrality?*

Geodesic Proportions for Leaf				
	Jen	Tom	Bob	Jim
Jen		??/?	??/?	??/?
Tom			??/?	??/?
Bob				??/?
Jim				

# Example: Undirected, Binary Network



Leaf's betweenness centrality is 0



Geodesic Proportions for Leaf

	Jen	Tom	Bob	Jim
Jen		0/1	0/1	0/1
Tom			0/1	0/1
Bob				0/1
Jim				

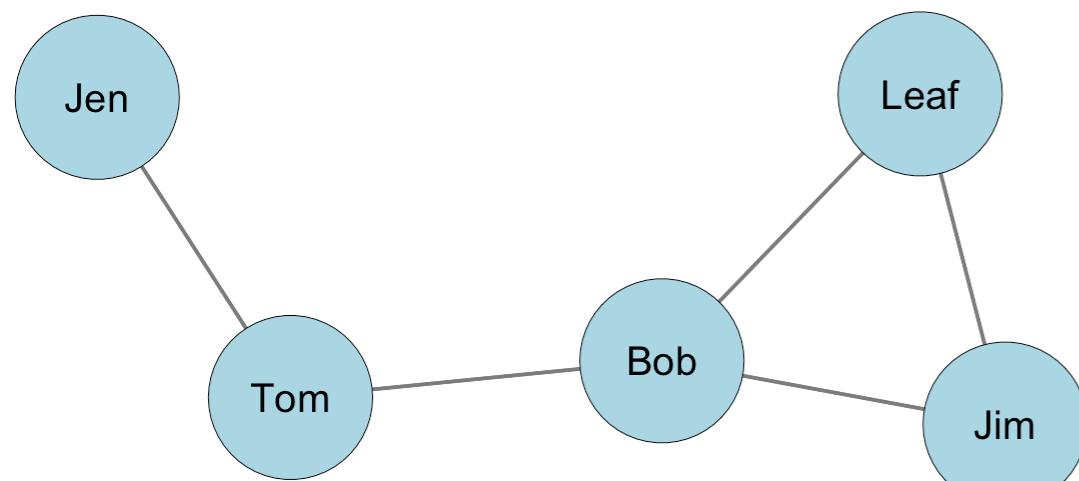
# Betweenness Centrality: Undirected Binary Graphs

$$C'_B(n_i) = \frac{\sum_{j < k} g_{jk}(n_i)/g_{jk}}{[(g-1)(g-2)/2]} = \frac{C_B(n_i)}{[(g-1)(g-2)/2]}$$



The maximum  
number of pairs of  
actors not including  $n_i$

# Example: Undirected, Binary Network



Unstandardized (raw) for Bob: 4

Standardized for Bob:

$$4 / [(5-1)(5-2)/2] = 0.667$$

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

---

# Group Closeness Centralization: Undirected Binary Graphs

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- ❖ We can also summarize the entire network, in terms of how close nodes are to each other.
  - ❖ *Group closeness centralization* tells us how much variation there is in the closeness scores.

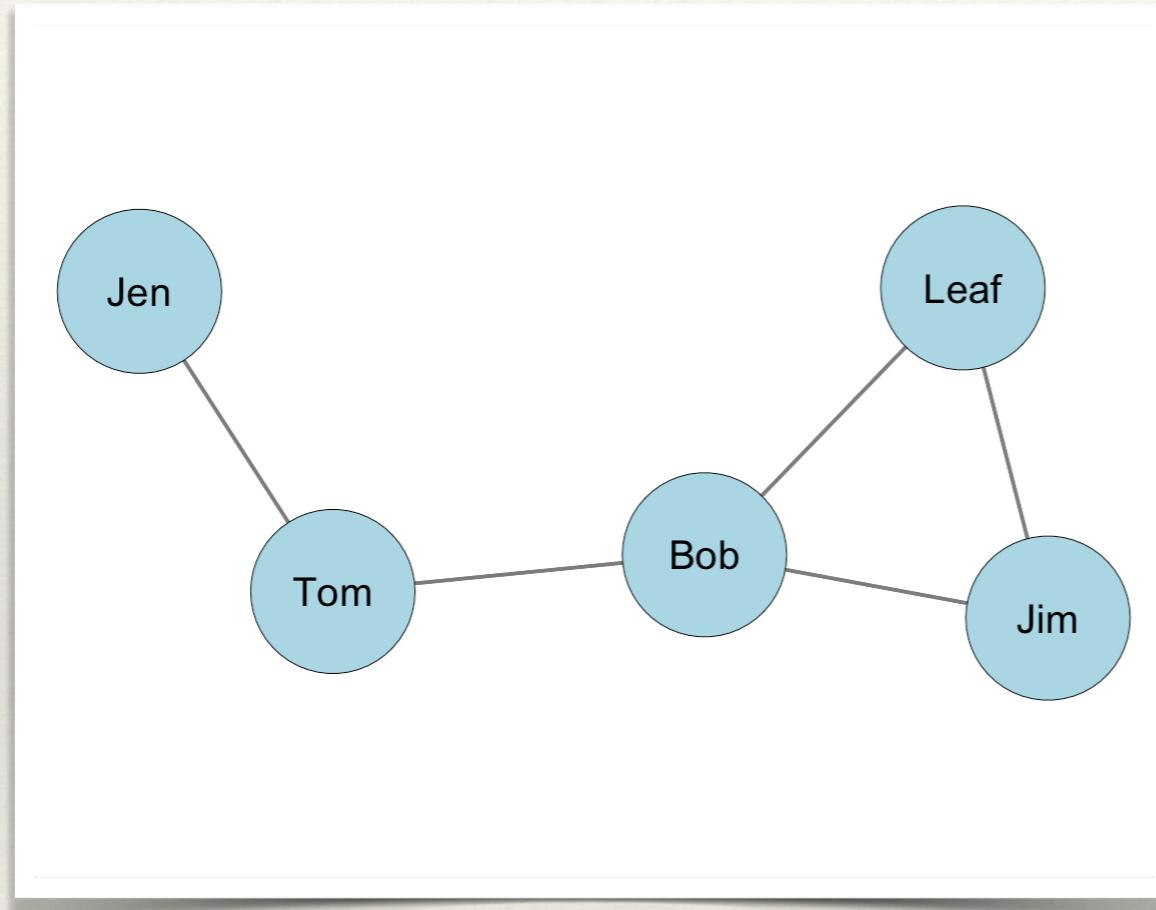
# Group Betweenness Centrality: Undirected Binary Graphs

$$C_B = \frac{\sum_{i=1}^g [C'_B(n^*) - C'_B(n_i)]}{(g - 1)}$$

Largest value of the standardized betweenness for the network

Standardized betweenness score for actor  $i$

# Example: Undirected, Binary Network



Standardized Betweenness  
Centrality Scores

Jen = 0.000

Tom = 0.500

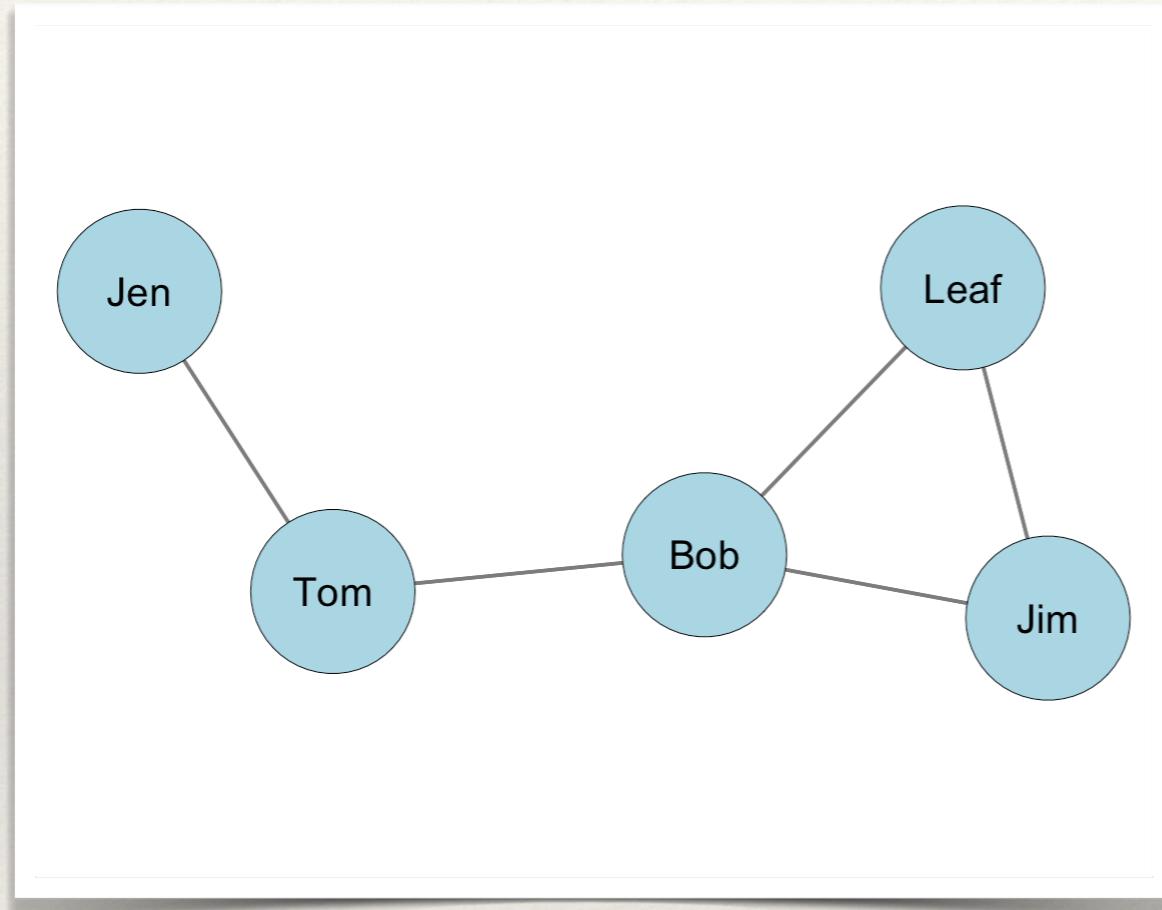
Bob = 0.667

Leaf = 0.000

Jim = 0.000

*What is the group betweenness centralization score for this graph?*

# Example: Undirected, Binary Network



*What is the group betweenness centralization score for this graph?*

## Deviated Standardized Betweenness Centrality

### Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

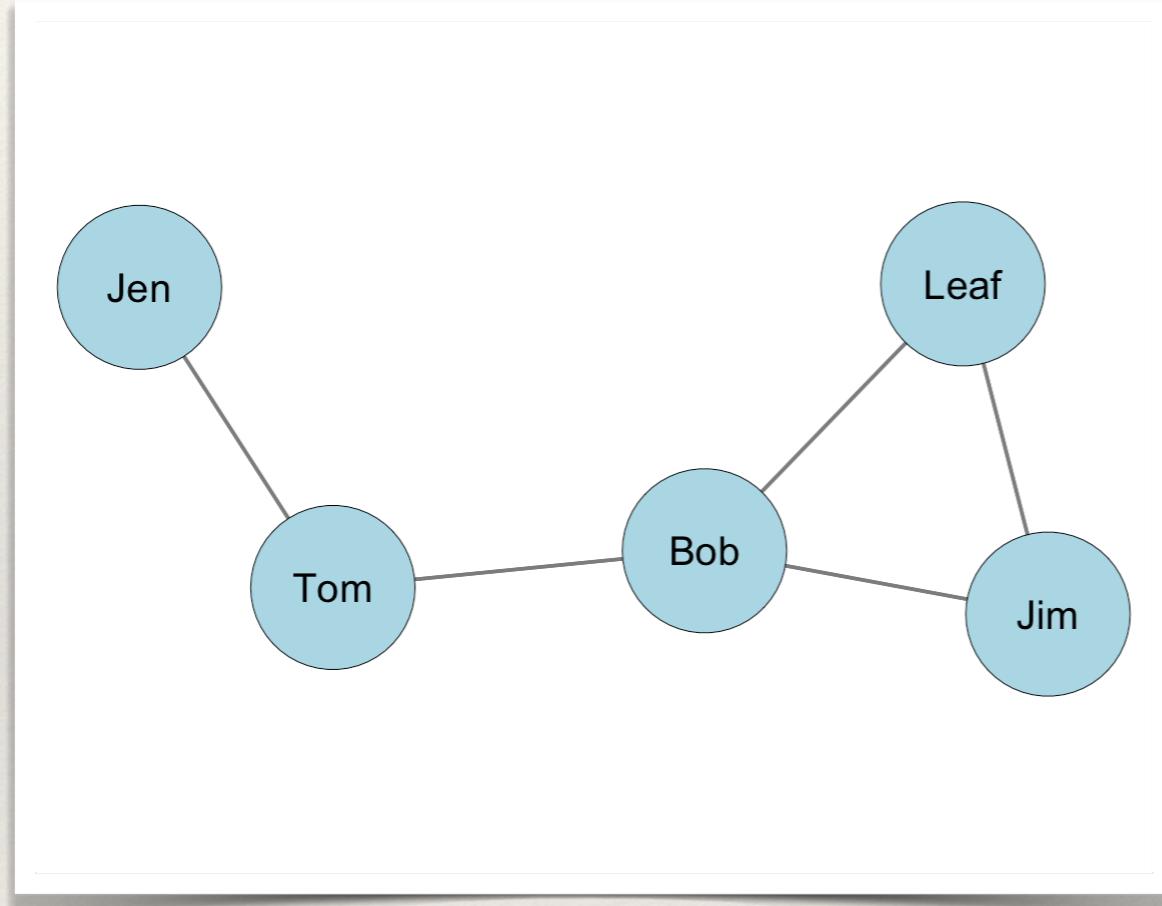
$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

# Example: Undirected, Binary Network



Deviated Standardized  
Betweenness Centrality  
Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

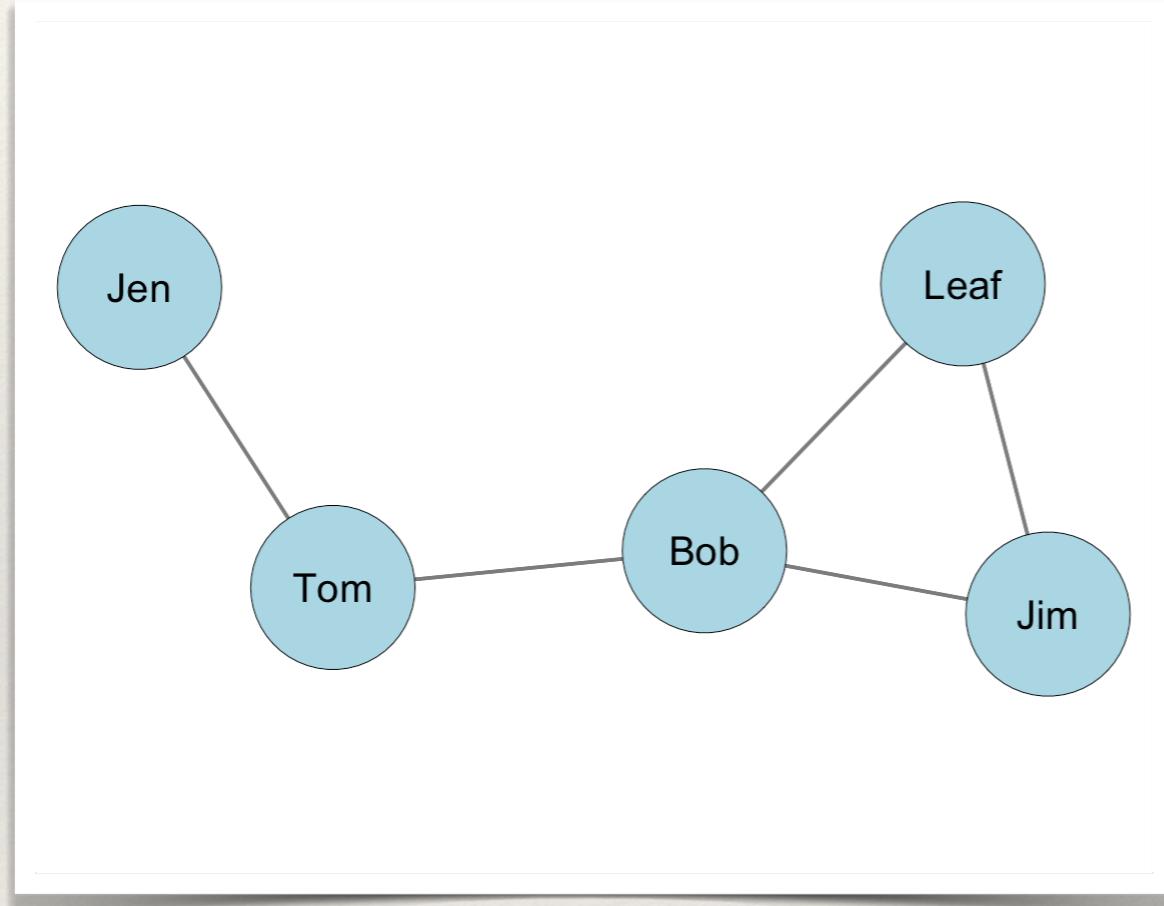
$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

$$\text{Sum} = 2.168$$

*What is the group betweenness centralization score for this graph?*

# Example: Undirected, Binary Network



$$C_B = \frac{\sum_{i=1}^g [C'_B(n^*) - C'_B(n_i)]}{(g - 1)}$$

Deviated Standardized  
Betweenness Centrality  
Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

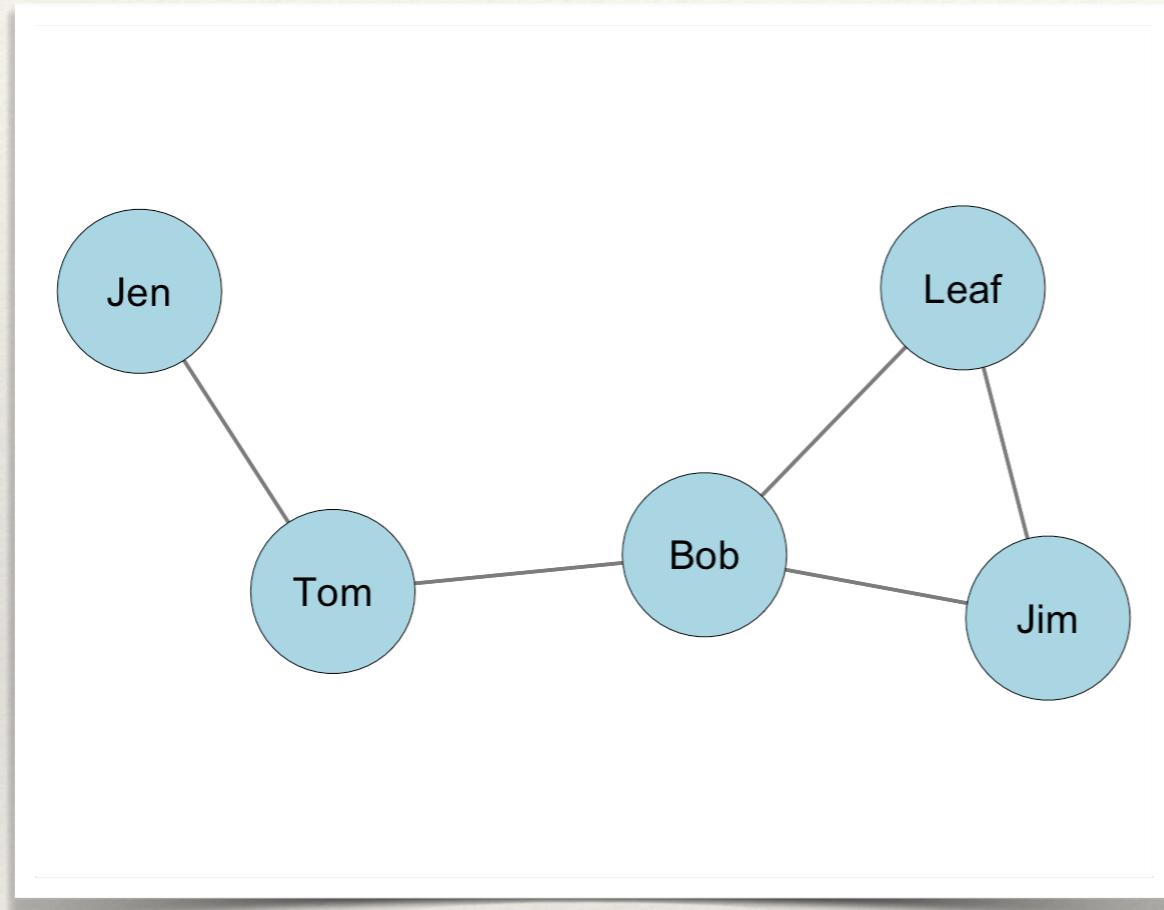
$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

$$\text{Sum} = 2.168$$

$$\text{Denominator} = 4$$

# Example: Undirected, Binary Network



$$2.168 / 4 = 0.542$$

Deviated Standardized  
Betweenness Centrality  
Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

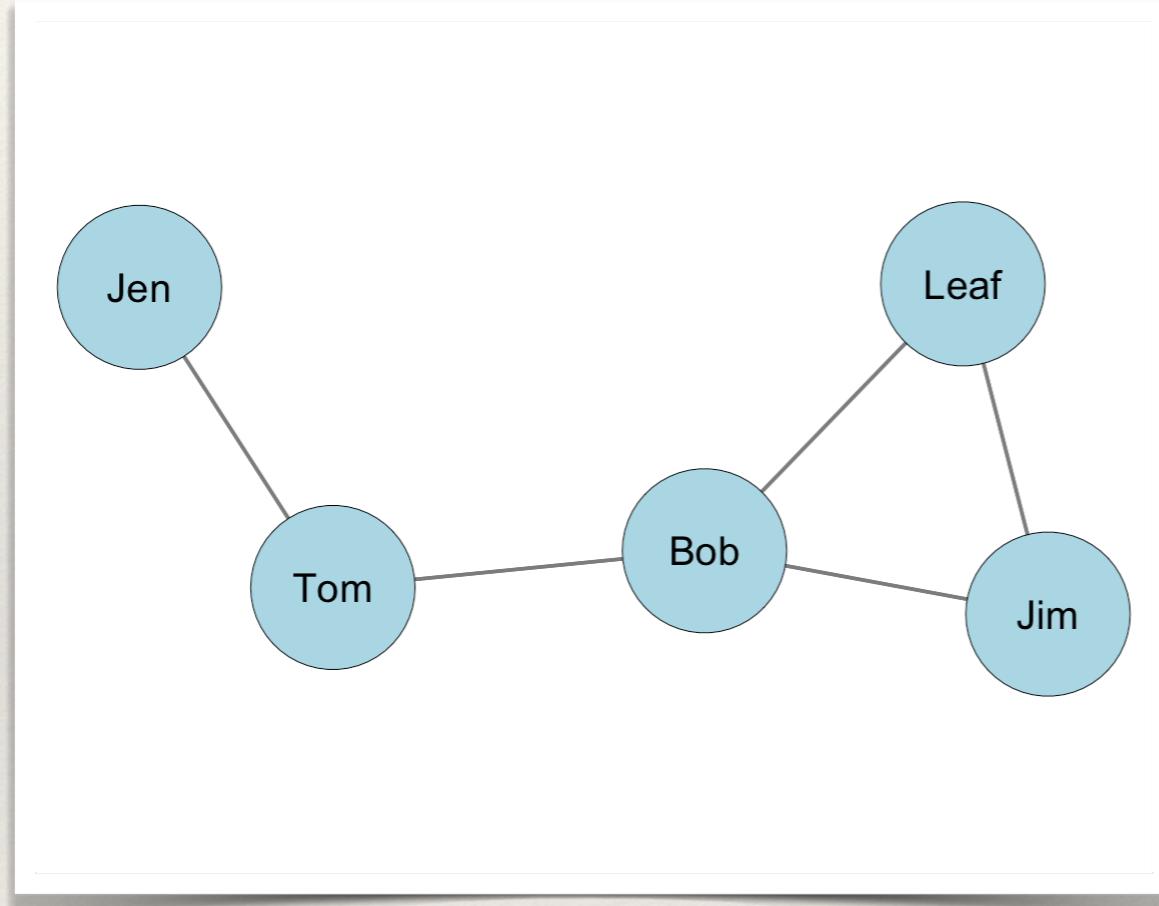
$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

$$\text{Sum} = 2.168$$

$$\text{Denominator} = 4$$

# Example: Undirected, Binary Network



Compare the centralization scores:

Degree = 0.416

Closeness = 0.552

Betweenness = 0.542

*What can we say about the differences in the centralization scores for each type of centrality?*

# Directed Networks

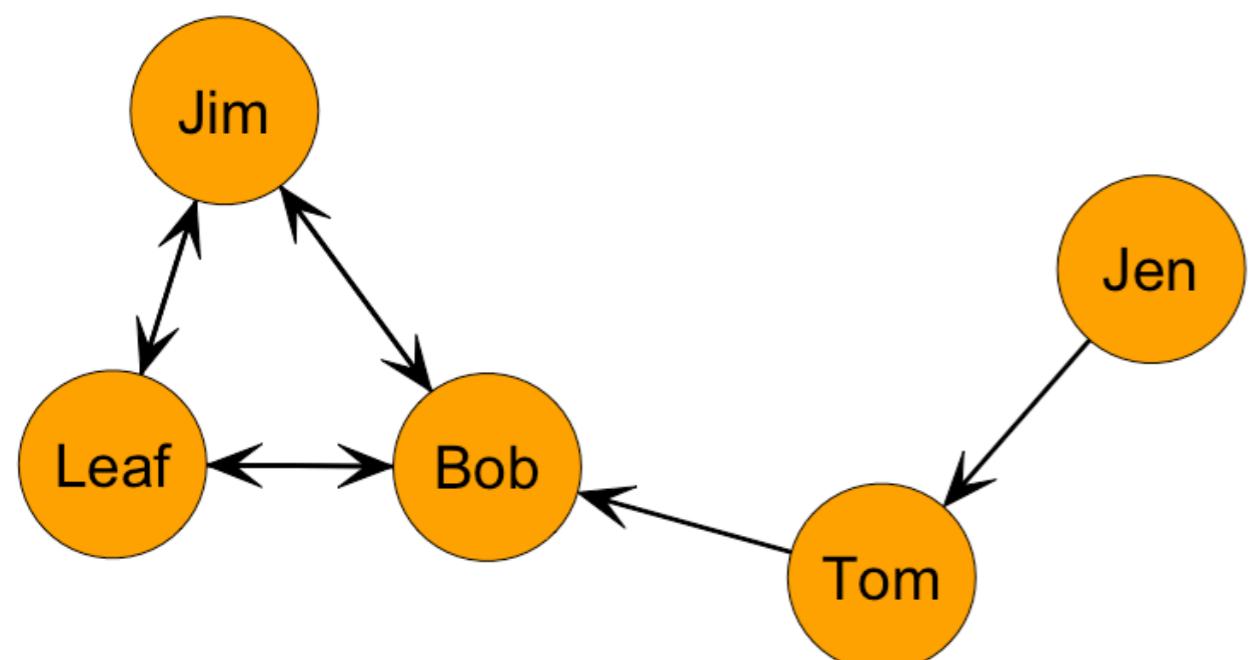
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## Closeness and Betweenness Centrality: Directed Binary Graphs

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- ❖ Recall that in a directed network, the directionality matters.
- ❖ As a result, we have to consider how this might influence our measures.

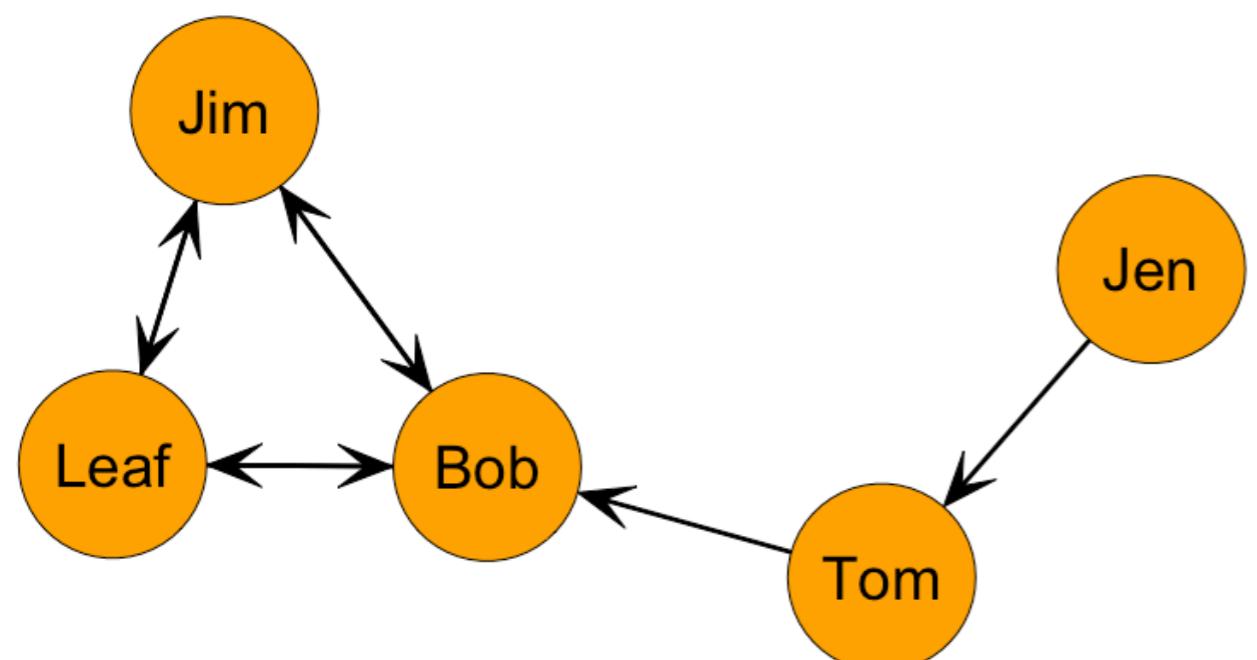
# Example: Closeness Centrality for Directed Binary Network



*How far is Jen from Tom? From Bob?*

		Jen	Tom	Bob	Leaf	Jim
Jen						

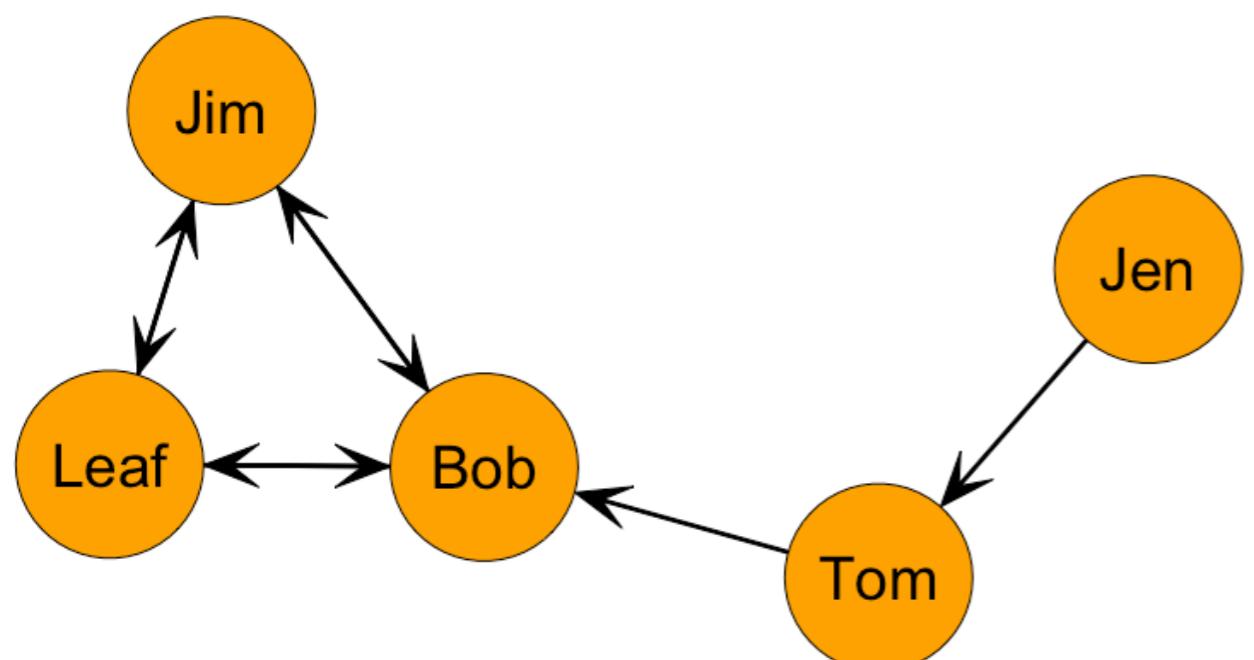
# Example: Closeness Centrality for Directed Binary Network



*How far is Jen from Tom? From Bob? Depends on direction.*

		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob						
Leaf						
Jim						

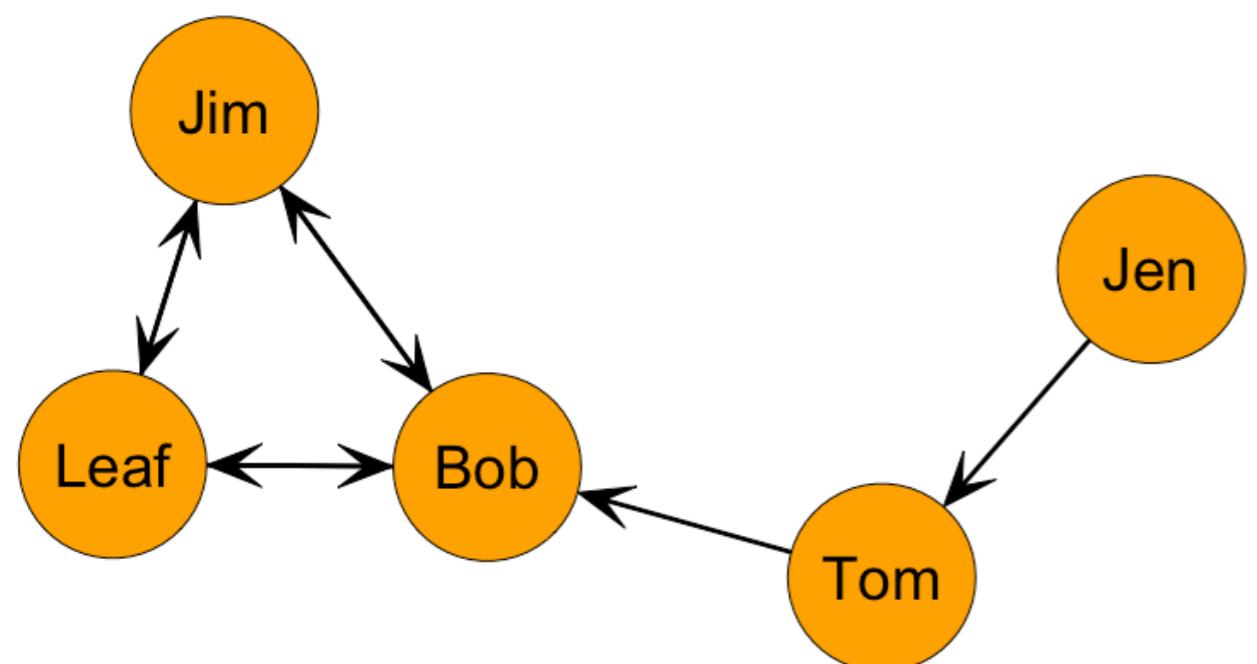
# Example: Closeness Centrality for Directed Binary Network



*Closeness centrality in directed graphs usually focuses on the send network (i.e. outgoing ties).*

		Distance Matrix				
		Jen	Tom	Bob	Leaf	Jim
Jen	Jen					
	Tom					
	Bob					
	Leaf					
	Jim					

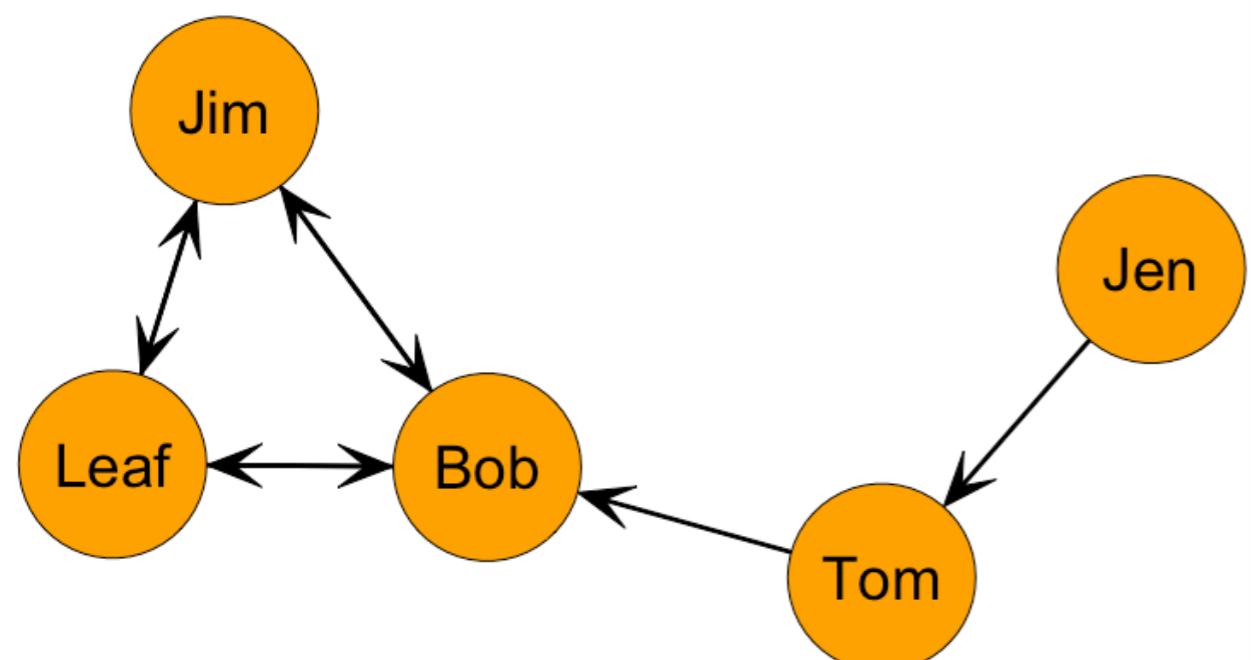
# Example: Closeness Centrality for Directed Binary Network



*How far is Jen from Tom? From Bob?*

		Jen	Tom	Bob	Leaf	Jim
Jen		?	?			
Tom						
Bob						
Leaf						
Jim						

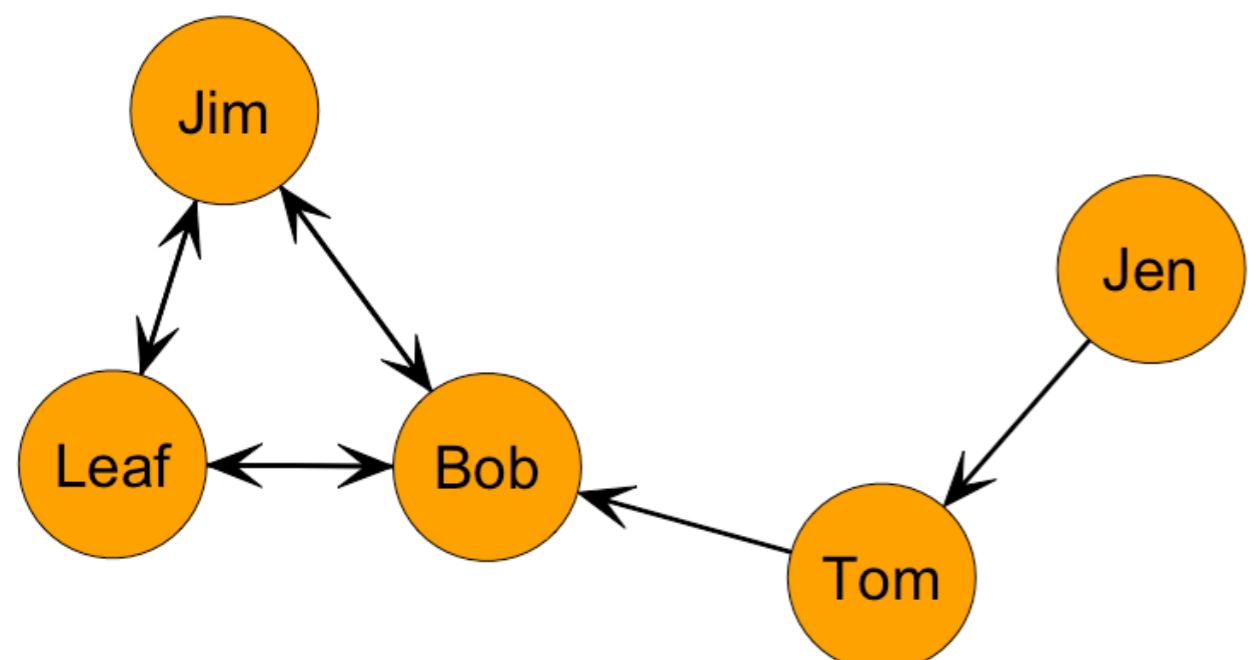
# Example: Closeness Centrality for Directed Binary Network



*How far is Jen from Tom? From Bob?*

		Jen	Tom	Bob	Leaf	Jim
Jen		1	2			
Tom						
Bob						
Leaf						
Jim						

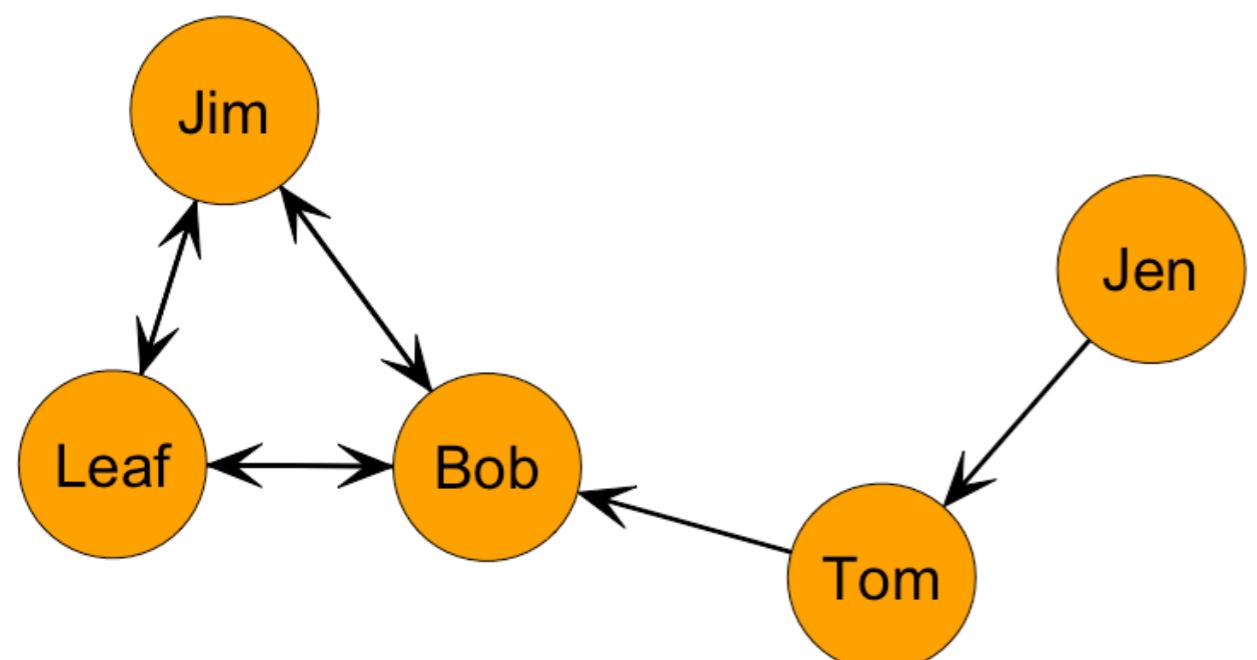
# Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	?	?	
Tom						
Bob						
Leaf						
Jim						

*What about the rest?*

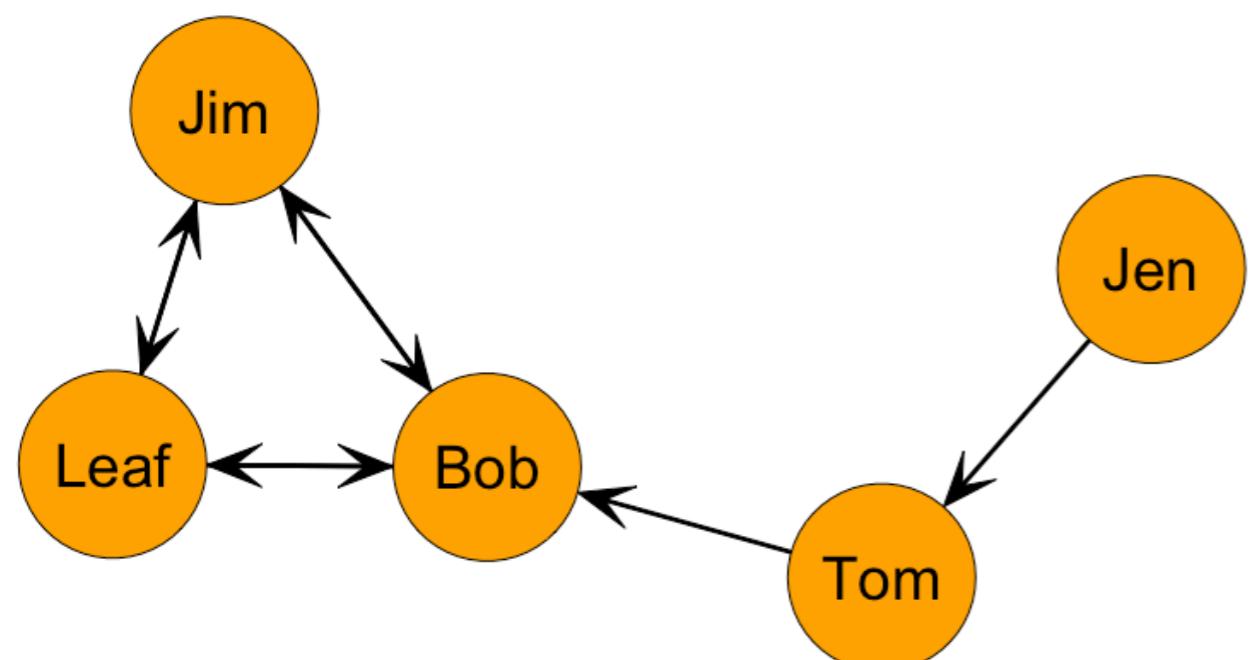
# Example: Closeness Centrality for Directed Binary Network



*What is Jen's closeness in the graph?*

		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3	
Tom						
Bob						
Leaf						
Jim						

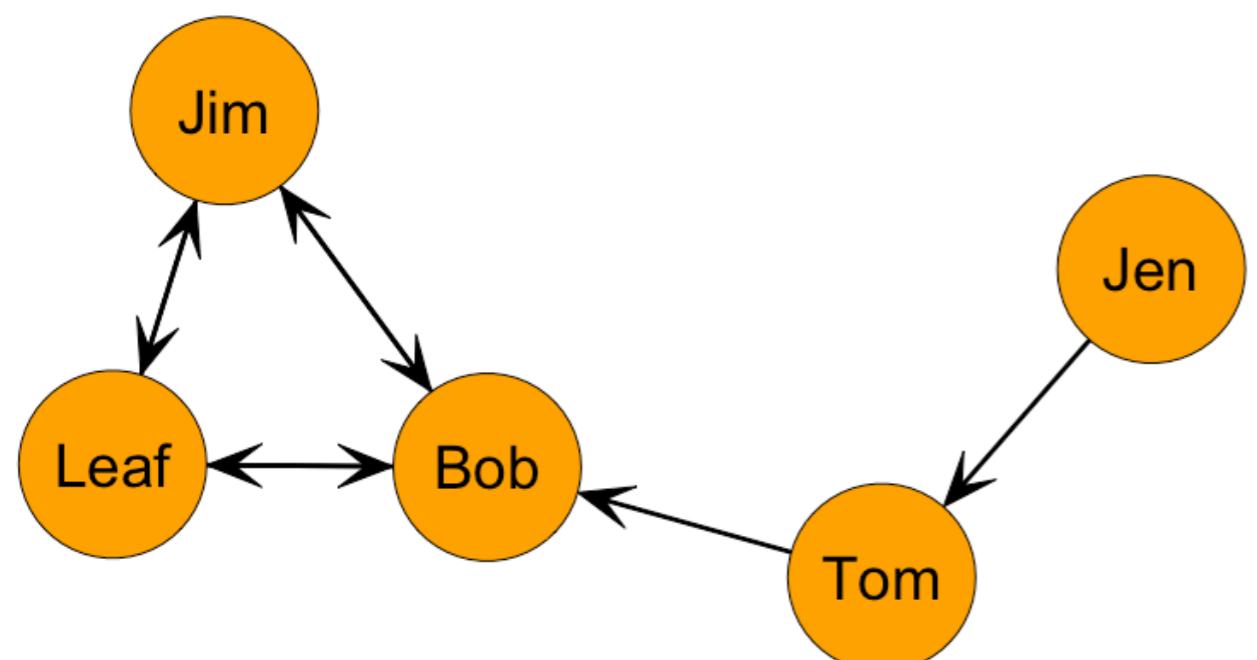
# Example: Closeness Centrality for Directed Binary Network



*What is Jen's closeness in the graph?*

		Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9	
Tom							
Bob							
Leaf							
Jim							

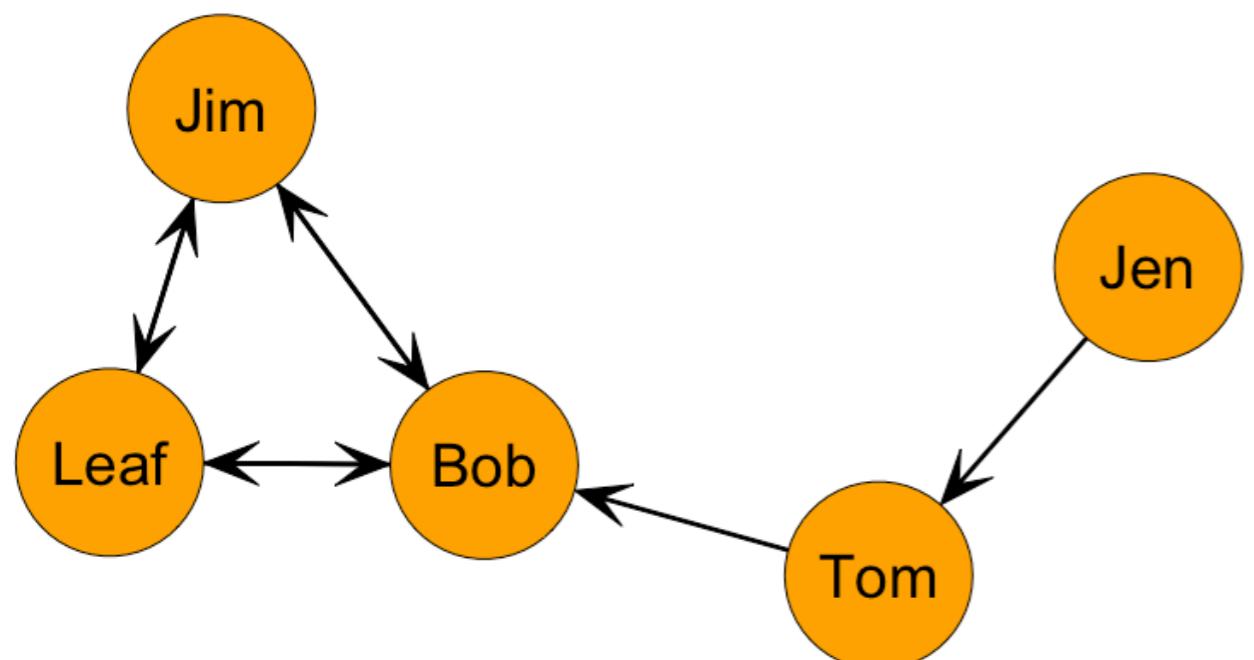
# Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9	
Tom							
Bob							
Leaf							
Jim							

*Just take the inverse, which is  $1/9$   
= 0.111.*

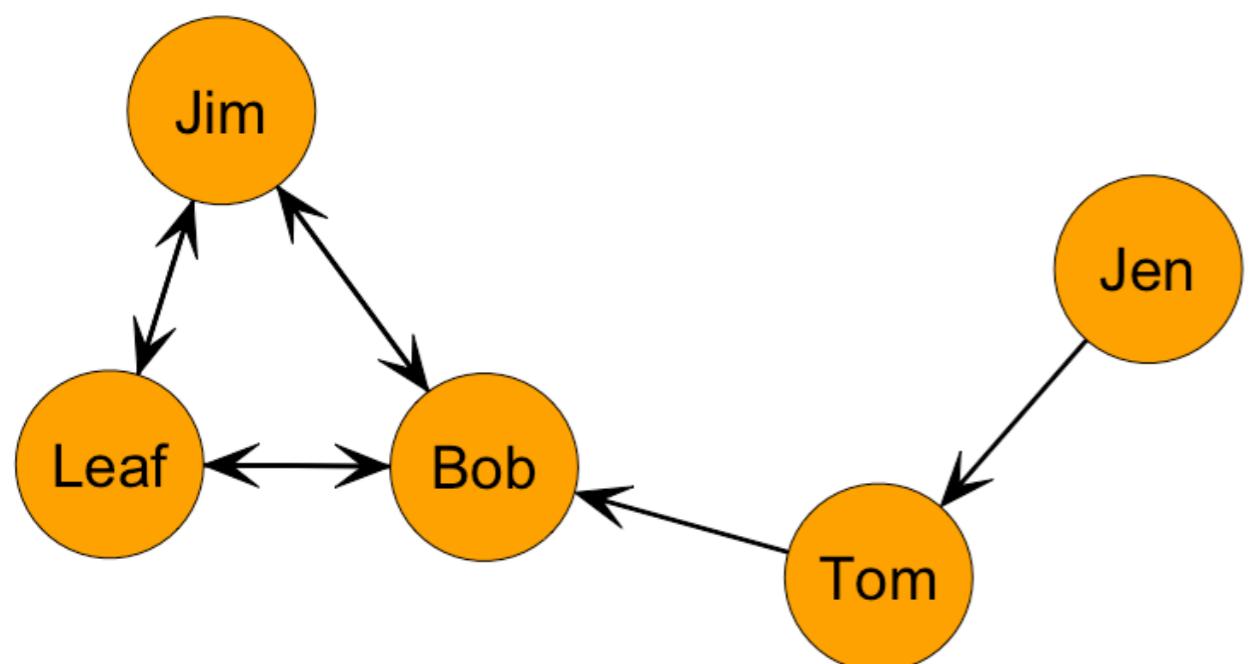
## Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9	
Tom							
Bob							
Leaf							
Jim							

Then, Jen's standardized score is  
 $(1/9) * (g-1) = 0.111 * 4 = 0.444$

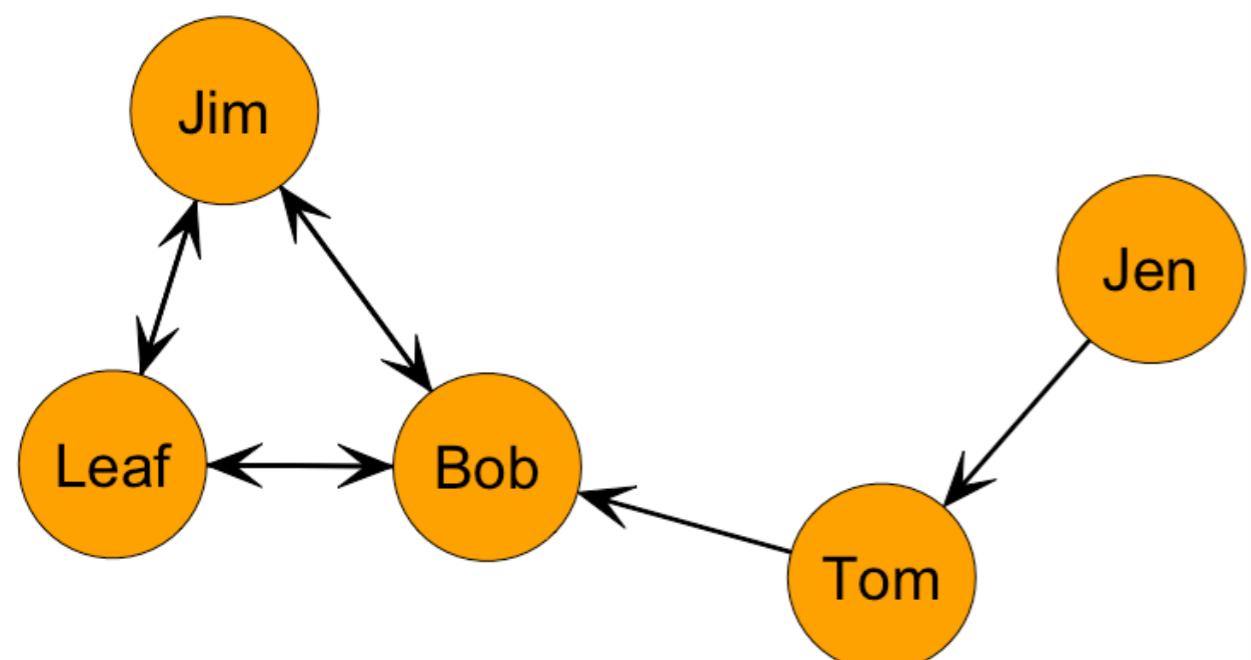
# Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob	?	?			?	?
Leaf						
Jim						

*What about Bob? How far is Bob from everyone?*

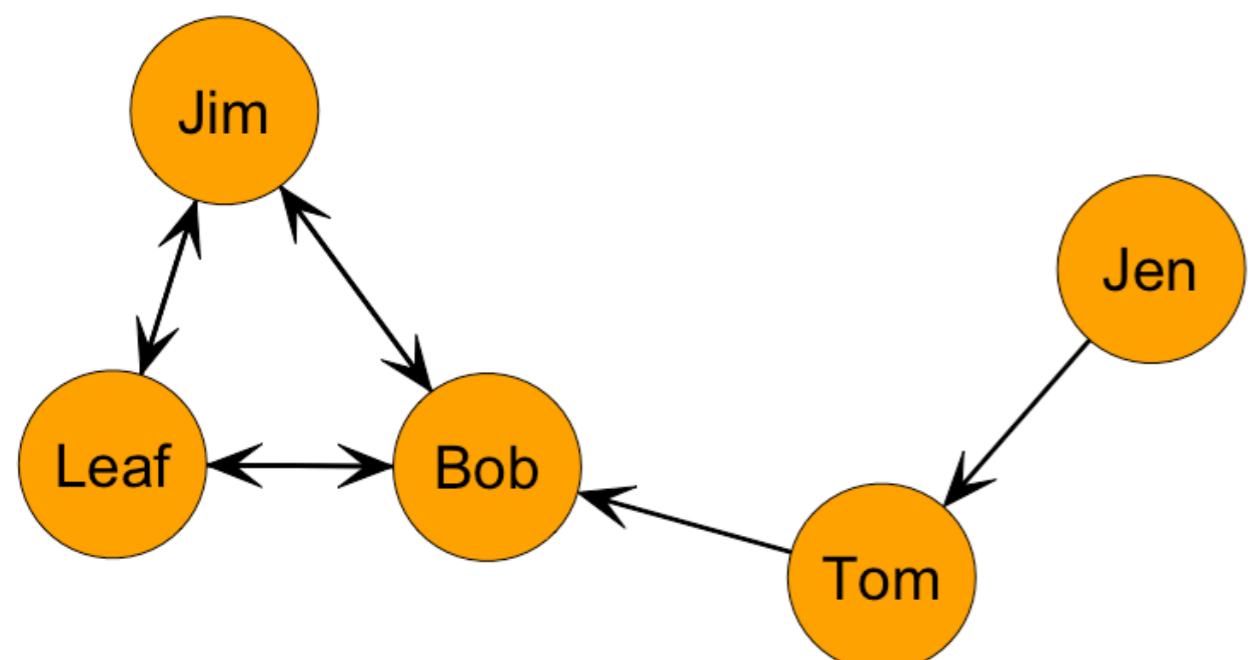
# Example: Closeness Centrality for Directed Binary Network



*Why is Bob's distance to Tom infinite?*

		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob	Inf	Inf			?	?
Leaf						
Jim						

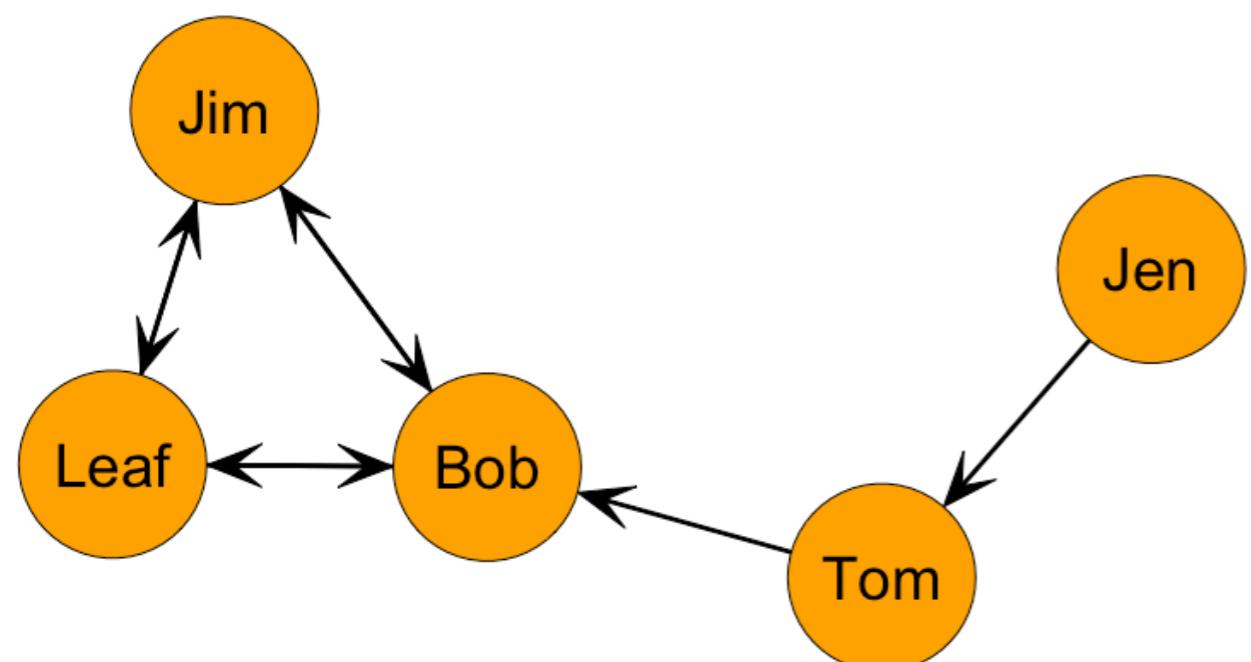
# Example: Closeness Centrality for Directed Binary Network



*What is Bob's closeness?*

		Jen	Tom	Bob	Leaf	Jim	Sum
Jen							
Tom							
Bob	Inf	Inf			1	1	2
Leaf							
Jim							

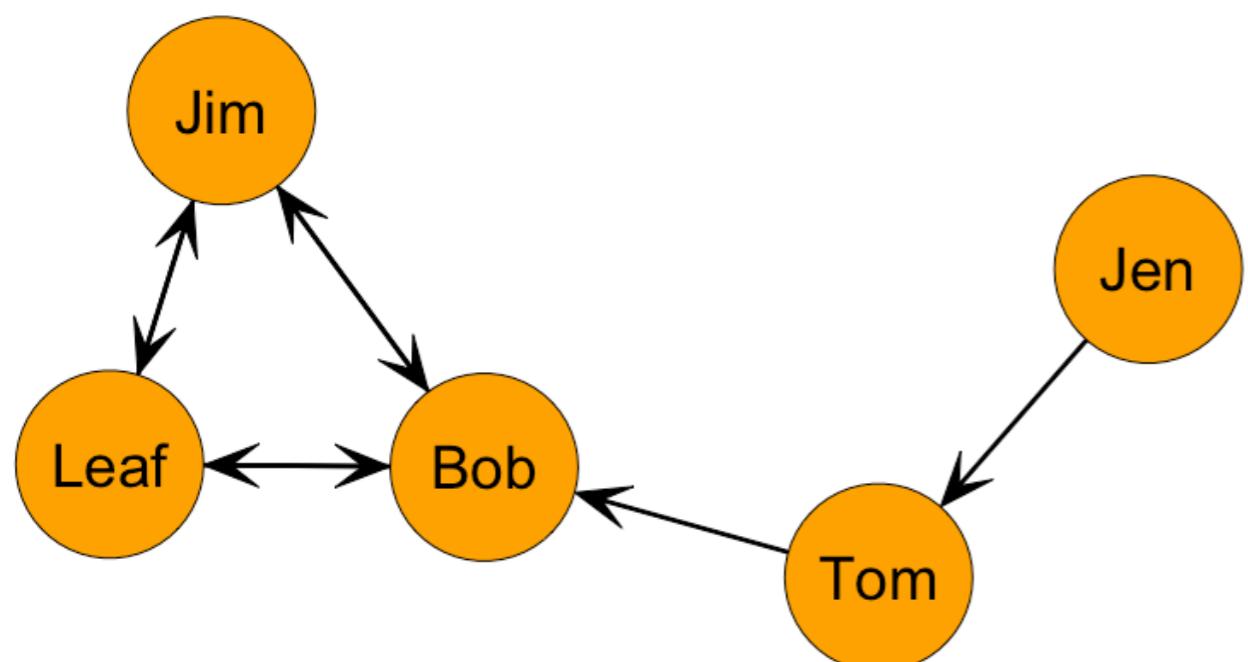
# Example: Closeness Centrality for Directed Binary Network



*Take the inverse and Bob's score  
is:  $1/2 = 0.5$*

		Jen	Tom	Bob	Leaf	Jim	Sum
Jen							
Tom							
Bob		Inf	Inf			1	1
Leaf							
Jim							

## Example: Closeness Centrality for Directed Binary Network

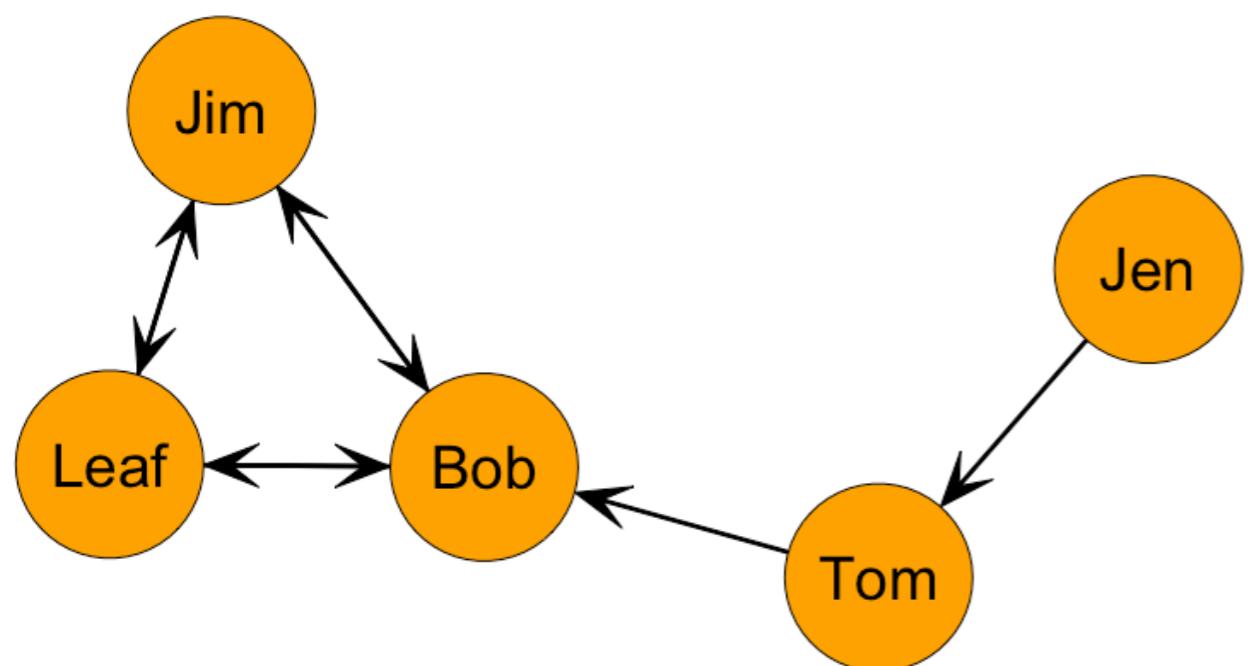


		Jen	Tom	Bob	Leaf	Jim	Sum
Jen							
Tom							
Bob		Inf	Inf		1	1	2
Leaf							
Jim							

Then, Bob's standardized score is

$$(1/2)^*(g-1) = 0.5^*4 = 0.2$$

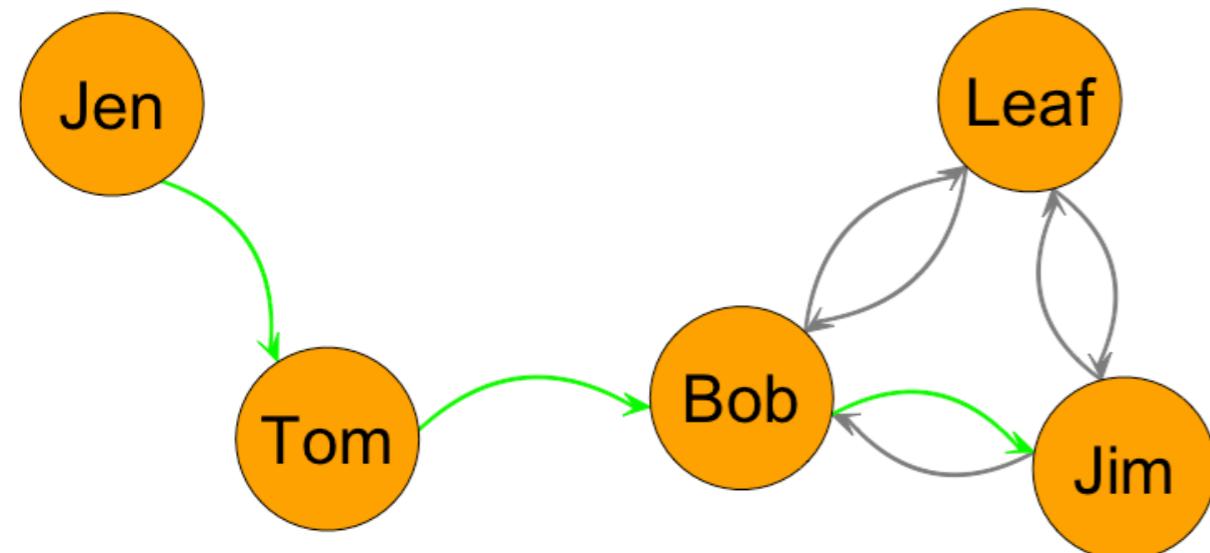
## Example: Closeness Centrality for Directed Binary Network



*The column sums will reflect the outdegree, and would tell you have close others are to you.*

		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob						
Leaf						
Jim						

## Example: Betweenness Centrality for Directed Binary Network

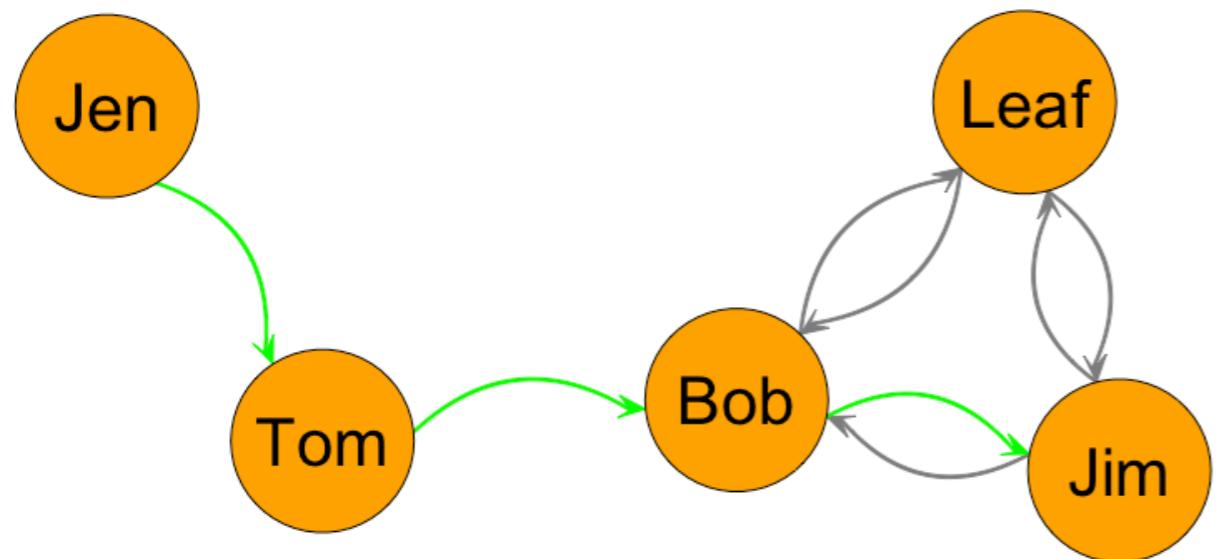


Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen				
Jim				/?

Betweenness would be  
the same: look at the  
the sending behavior.

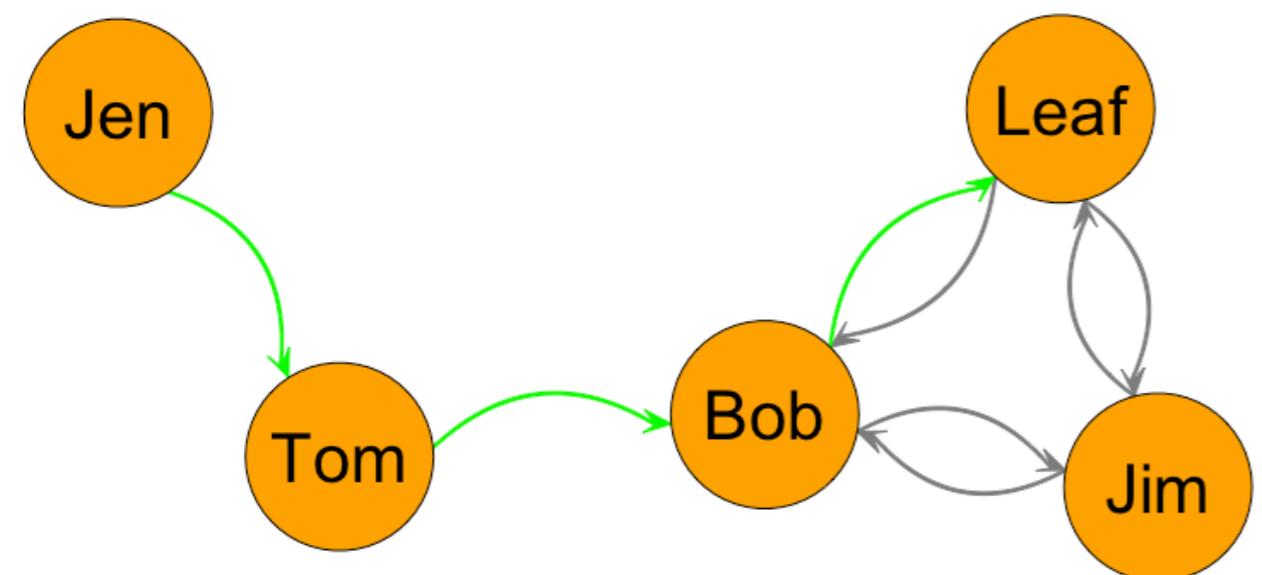
# Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

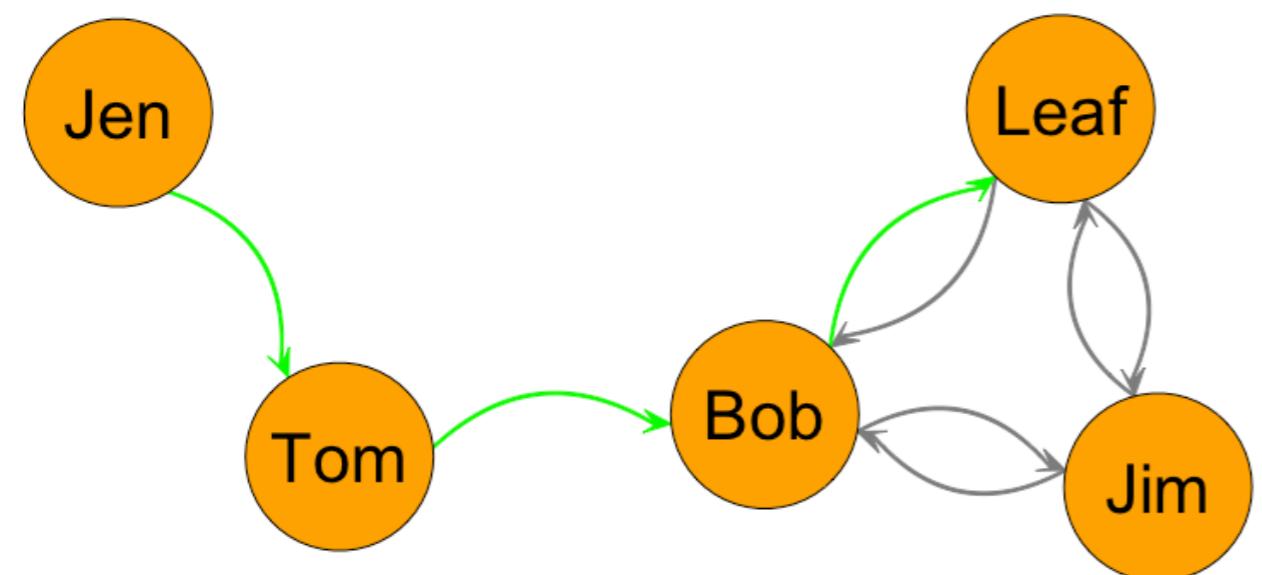
# Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen			?	1/1
Tom				
Leaf				
Jim				

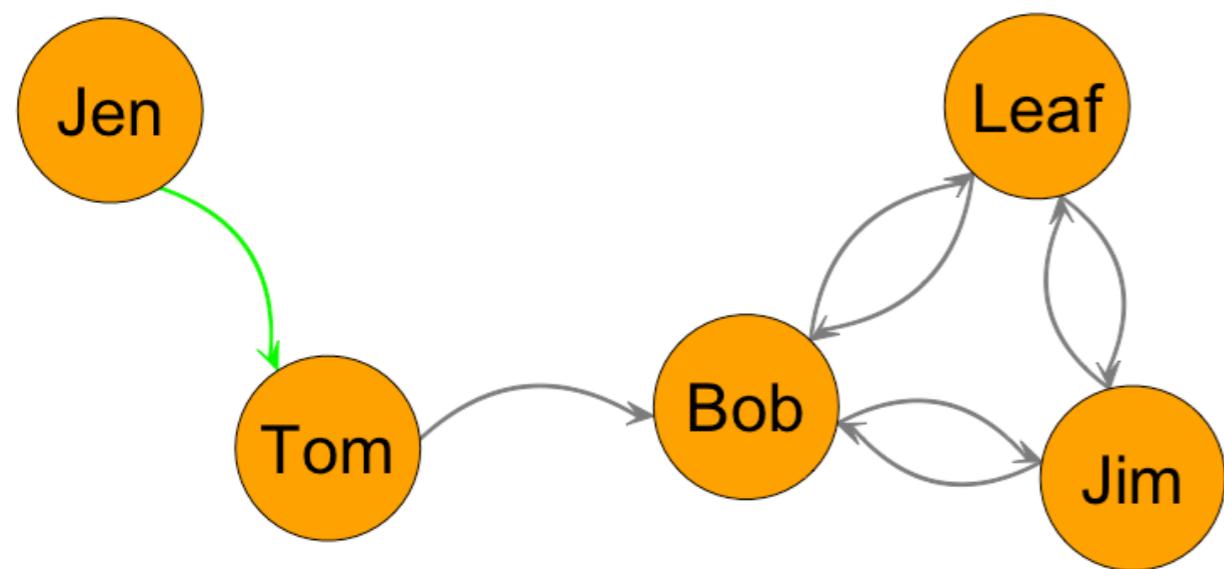
# Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen			1/1	1/1
Tom				
Leaf				
Jim				

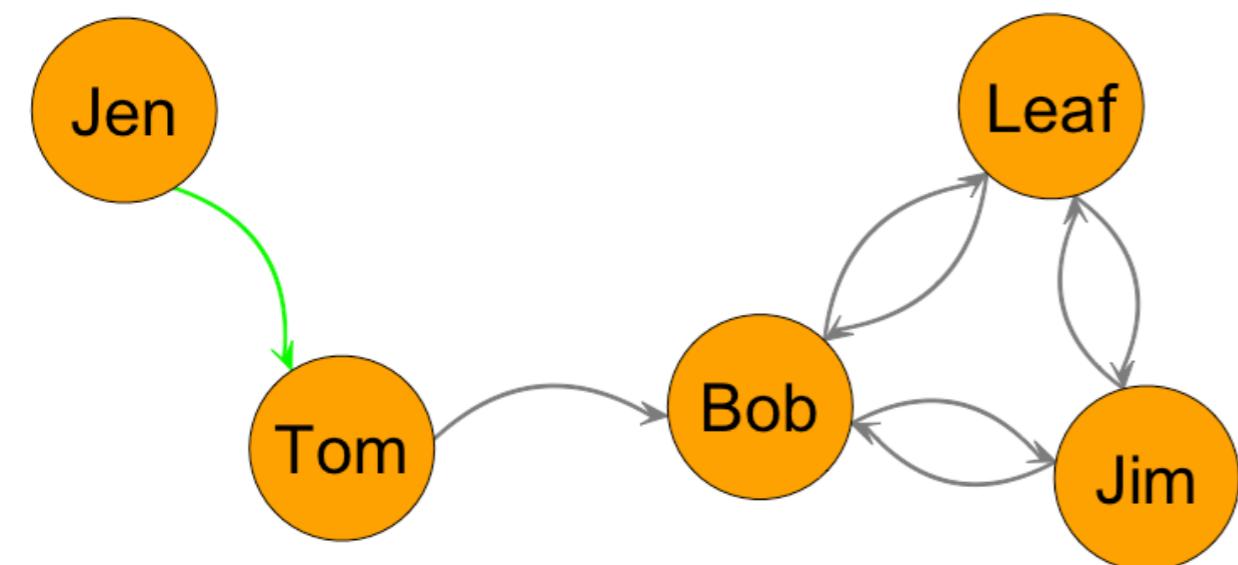
# Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		?	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

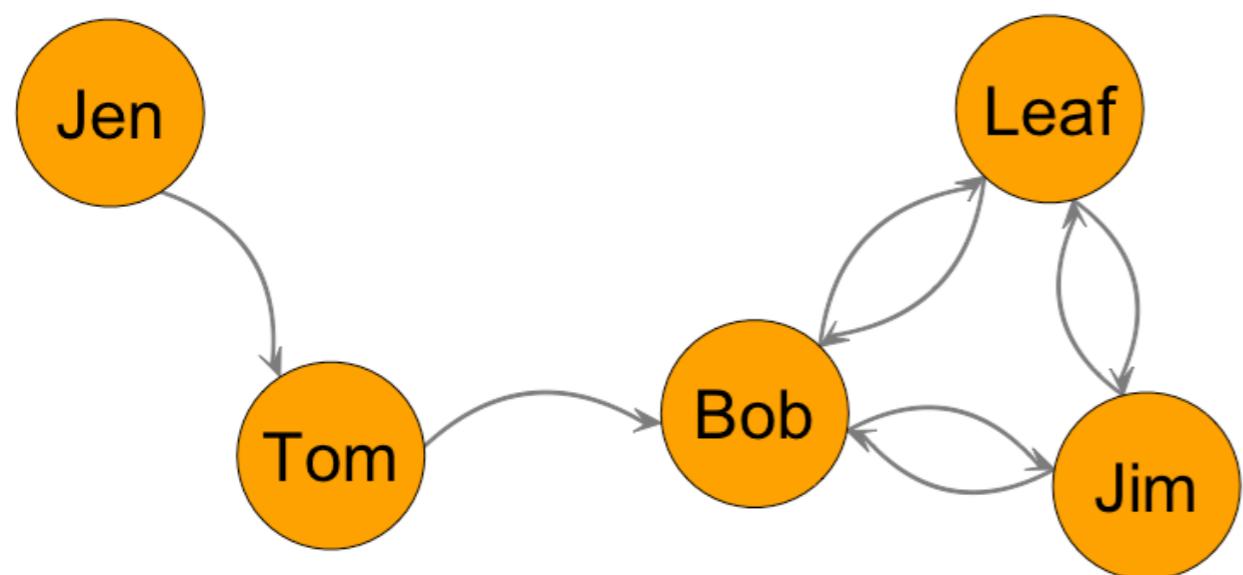
# Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

# Example: Betweenness Centrality for Directed Binary Network

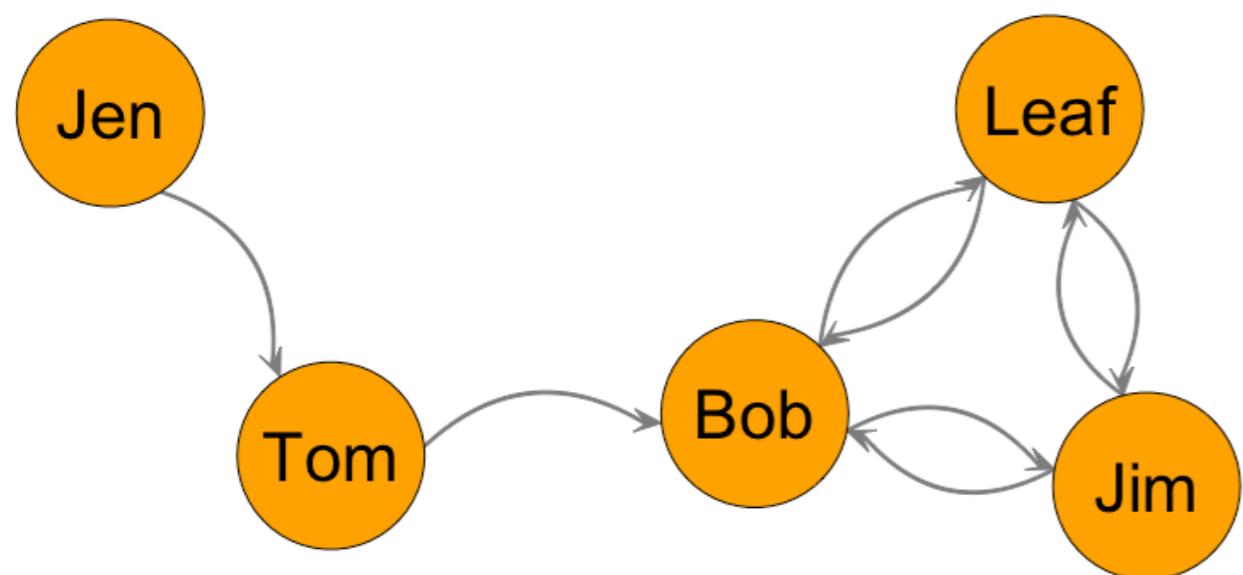


Then, we just complete the matrix.

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen	0/1	1/1	1/1	
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

# Example: Betweenness Centrality for Directed Binary Network

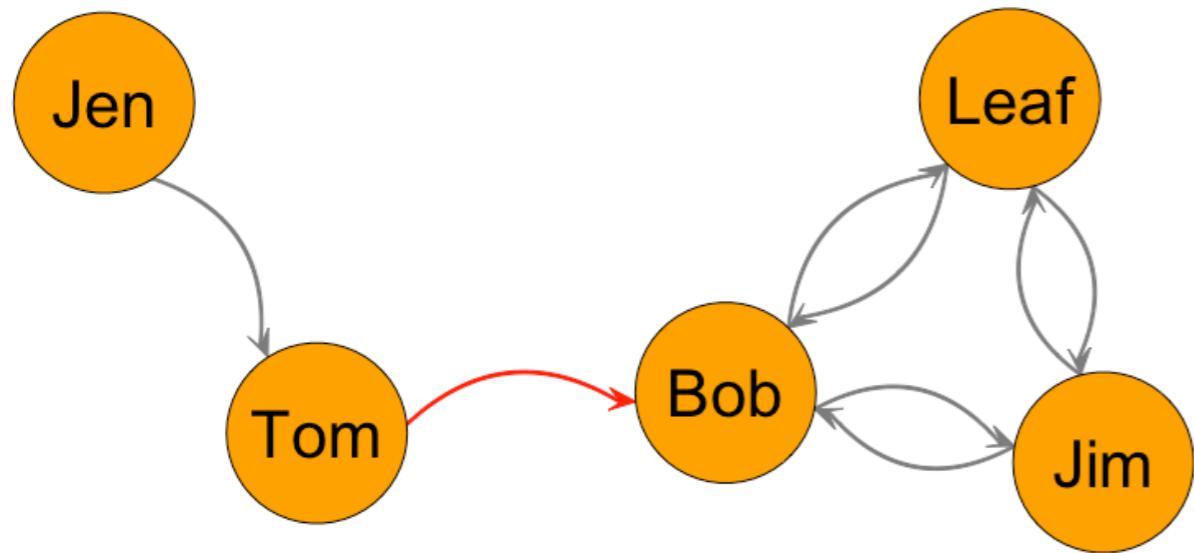


Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

Why no geodesics?

## Example: Betweenness Centrality for Directed Binary Network

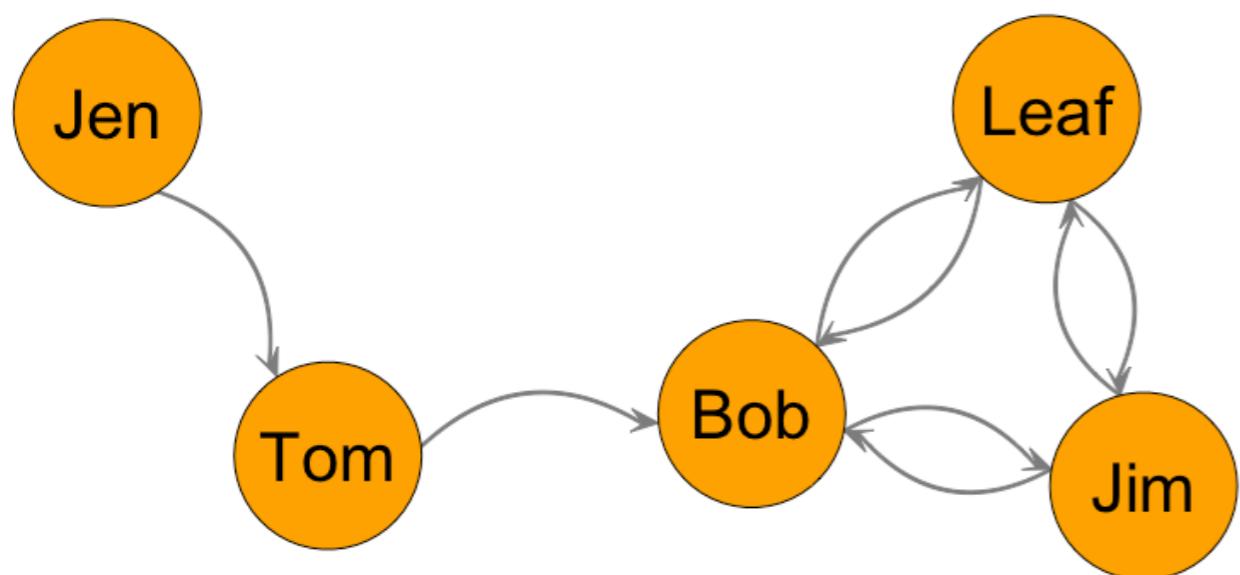


**Tom** and **Jen** cannot be reached past **Bob**, because there are no outgoing ties from **Bob**.

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>	0/0		1/1	1/1
<i>Leaf</i>	0/0	0/0		0/1
<i>Jim</i>	0/0	0/0	0/1	

# Example: Betweenness Centrality for Directed Binary Network



Unstandardized (raw) for Bob: 4

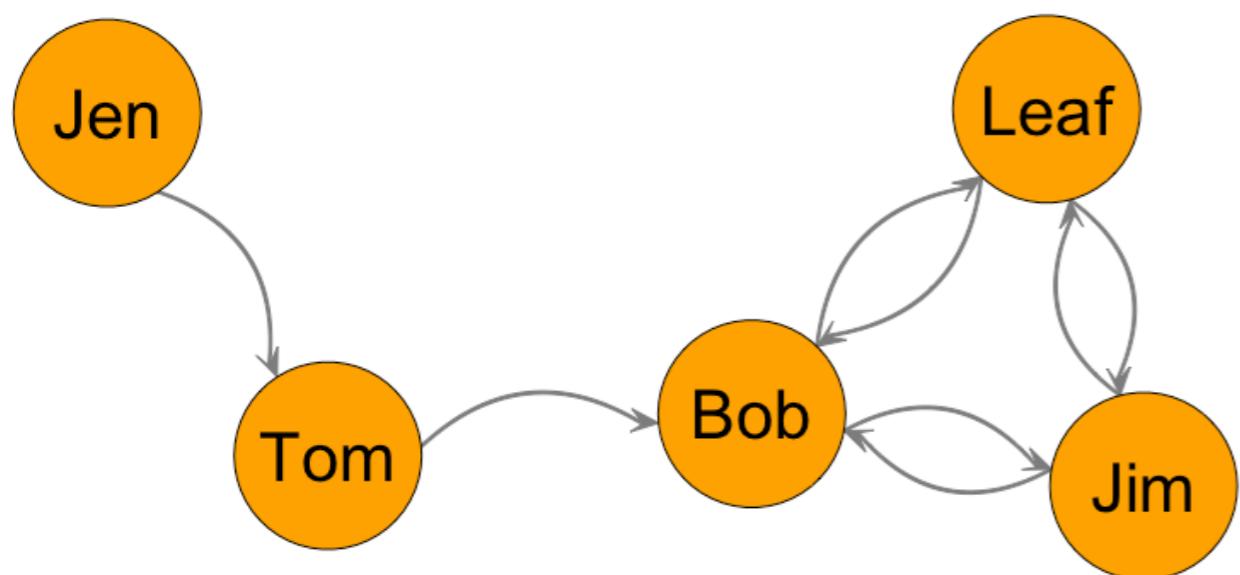
Standardized for Bob:

$$4 / [(5-1)(5-2)/2] = 0.667$$

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

# Example: Betweenness Centrality for Directed Binary Network



Unstandardized (raw) for Bob: 4

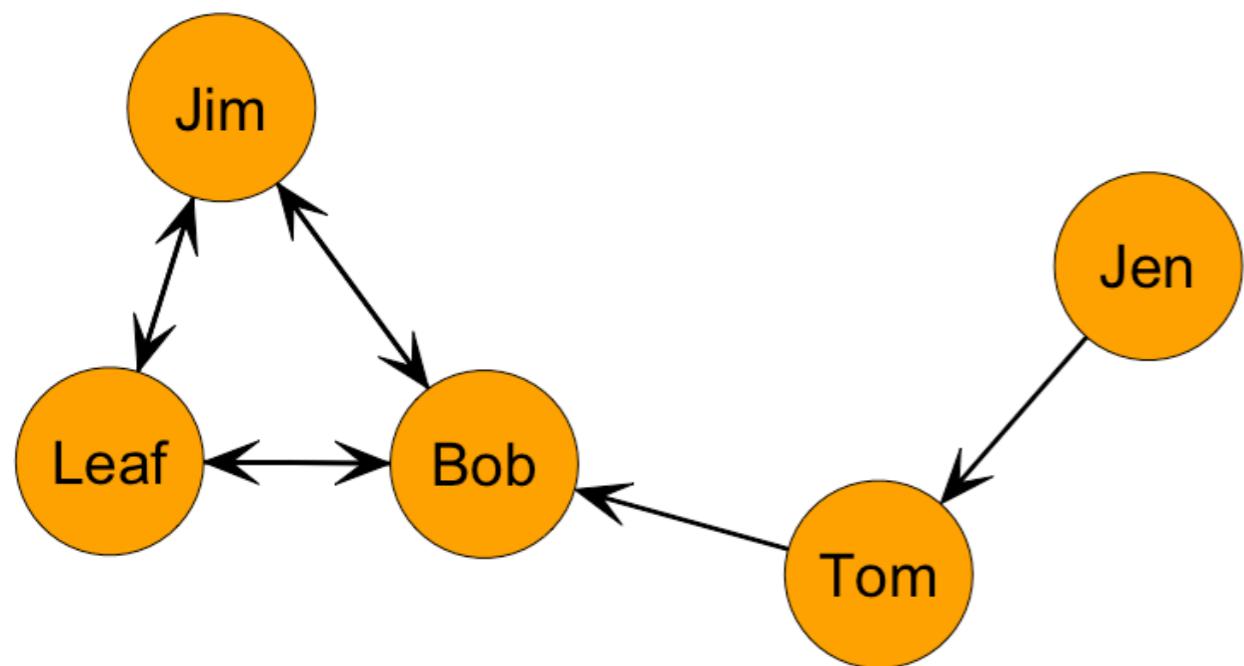
Standardized for Bob:

$$4 / [(5-1)(5-2)/2] = 0.667$$

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

# Example: Undirected, Binary Network



Compare the centralization scores:

Degree = 0.375

Closeness = 0.555

Betweenness = 0.270

*What can we say about the differences in the centralization scores for each type of centrality?*

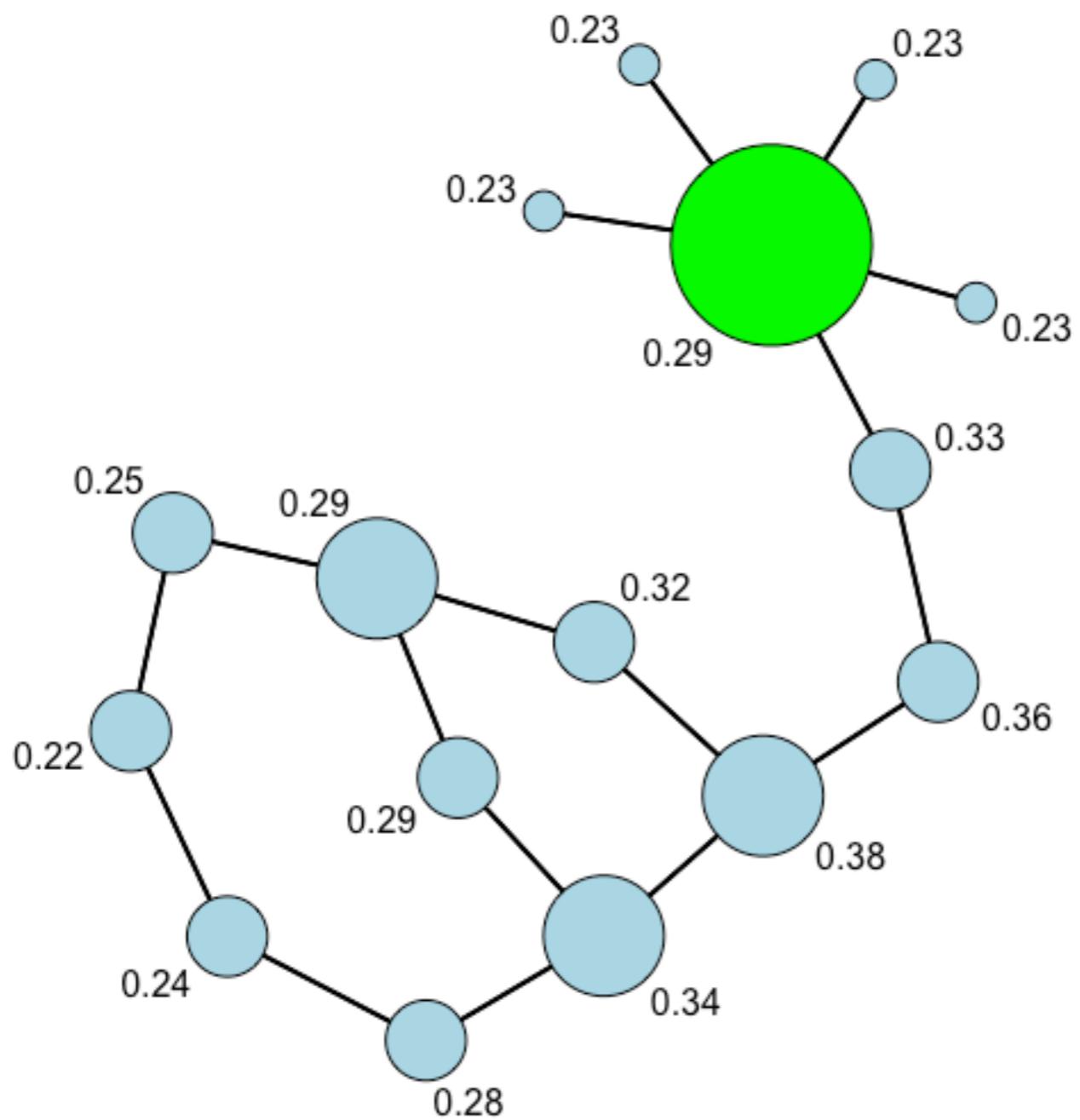
# Comparing Measures of Centrality

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important/active alters		Probably multiple paths in the network, ego is near many people, but so are others
High Betweenness	Ego's few ties are crucial for network flow	Very rare. Would mean that ego monopolizes the ties from a small number of people to many others	

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# High Degree, Low Closeness



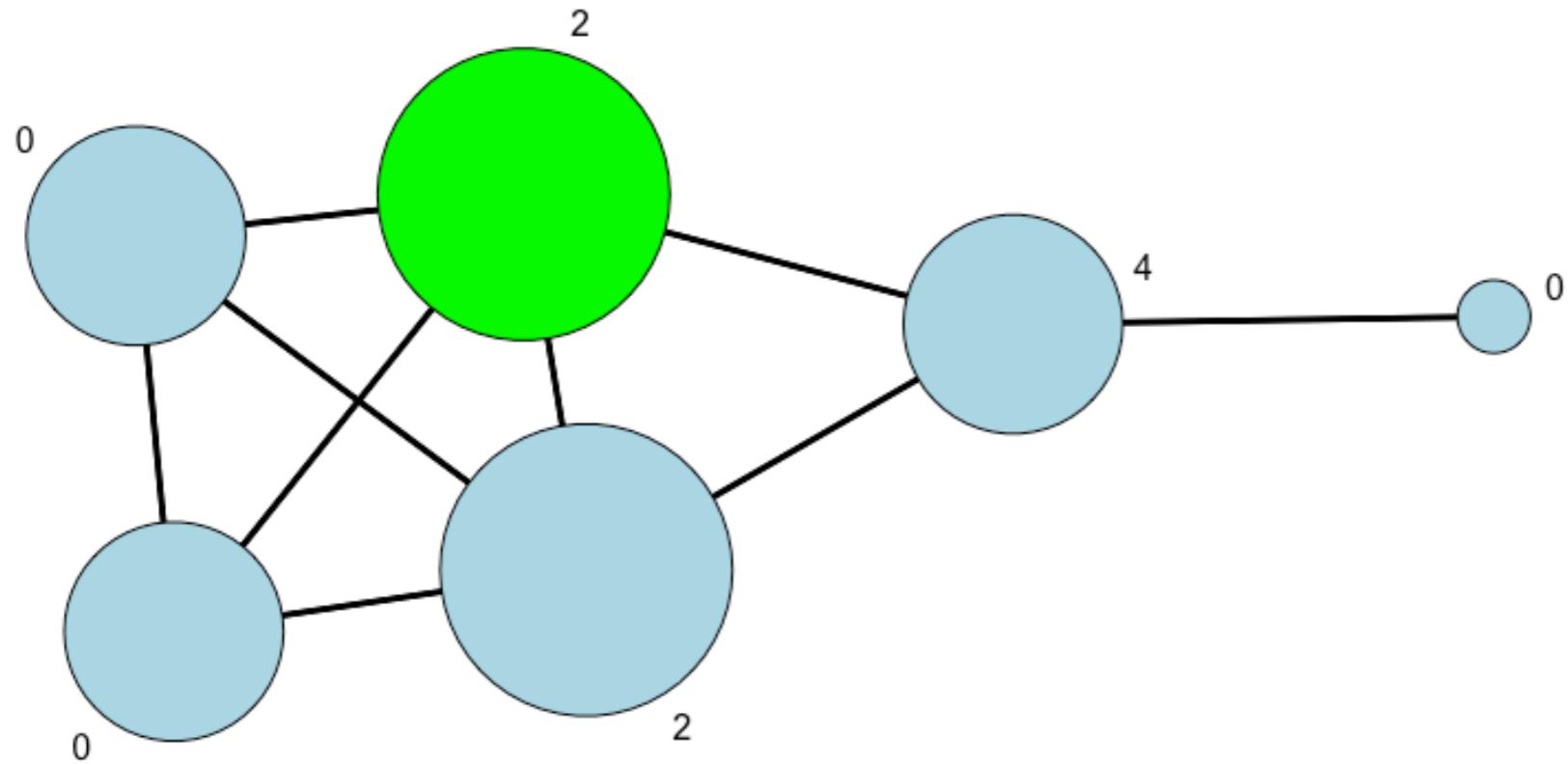
*Nodes sized by  
degree*

*Nodes labeled by  
closeness  
centrality*

# Comparing Measures of Centrality

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# High Degree, Low Betweenness



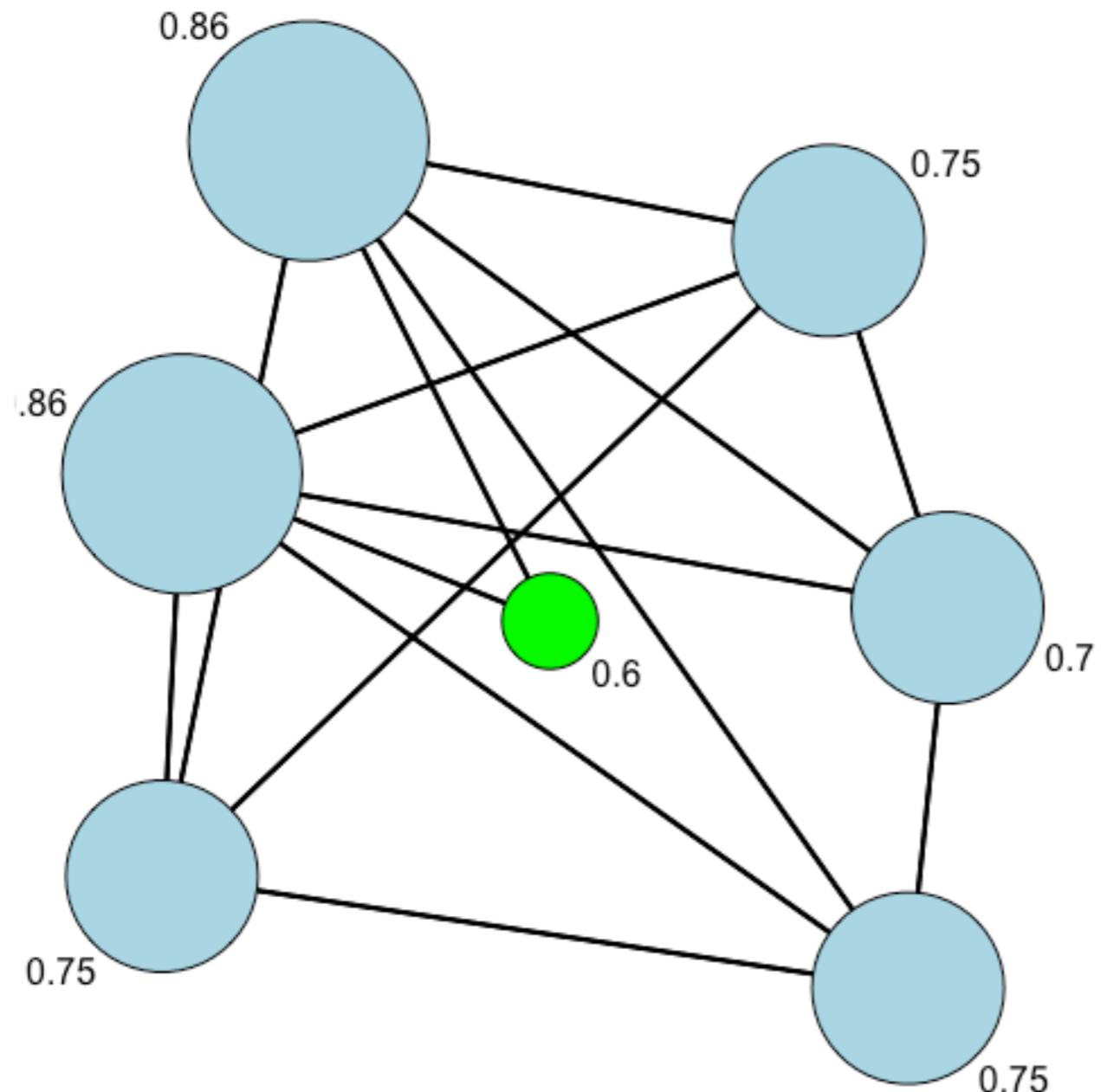
*Nodes sized by  
degree*

*Nodes labeled by  
betweenness  
centrality*

# Comparing Measures of Centrality

	Low Degree	Low Closeness	Low Betweenness
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# High Closeness, Low Degree



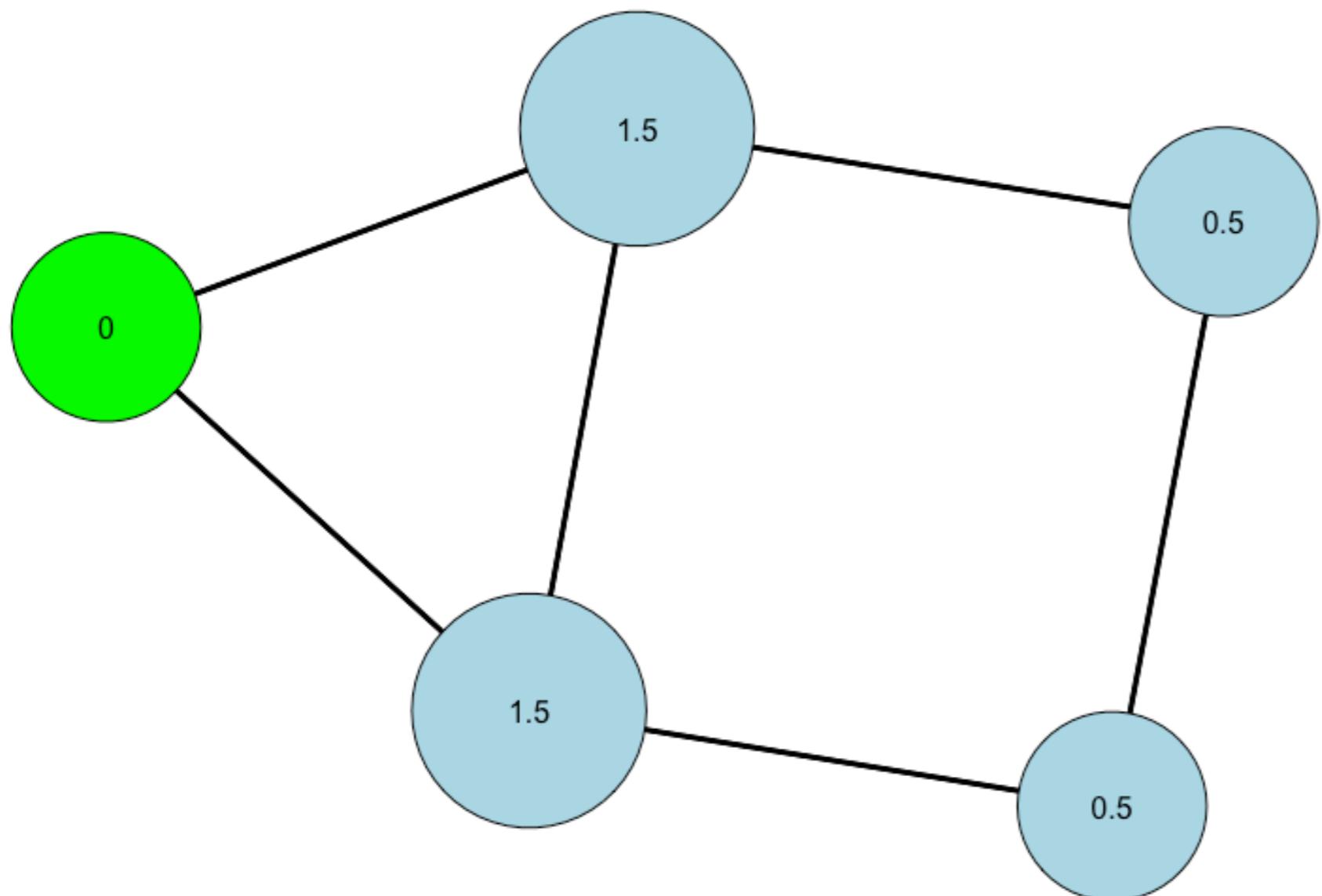
*Nodes sized by  
degree centrality*

*Nodes labeled by  
closeness  
centrality*

# Comparing Measures of Centrality

	Low Degree	Low Closeness	Low Betweenness
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# High Closeness, Low Betweenness



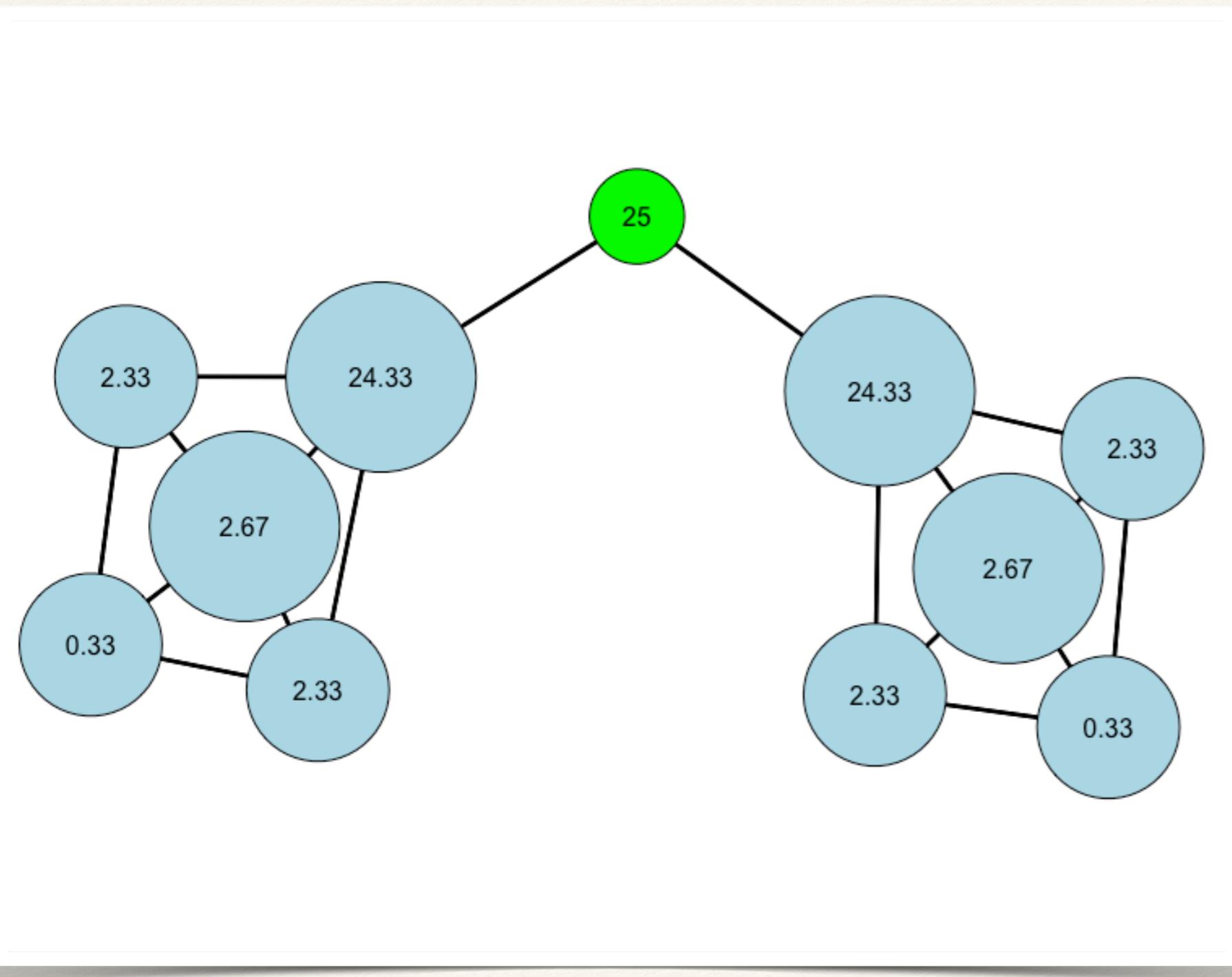
*Nodes sized by  
closeness  
centrality*

*Nodes labeled by  
betweenness  
centrality*

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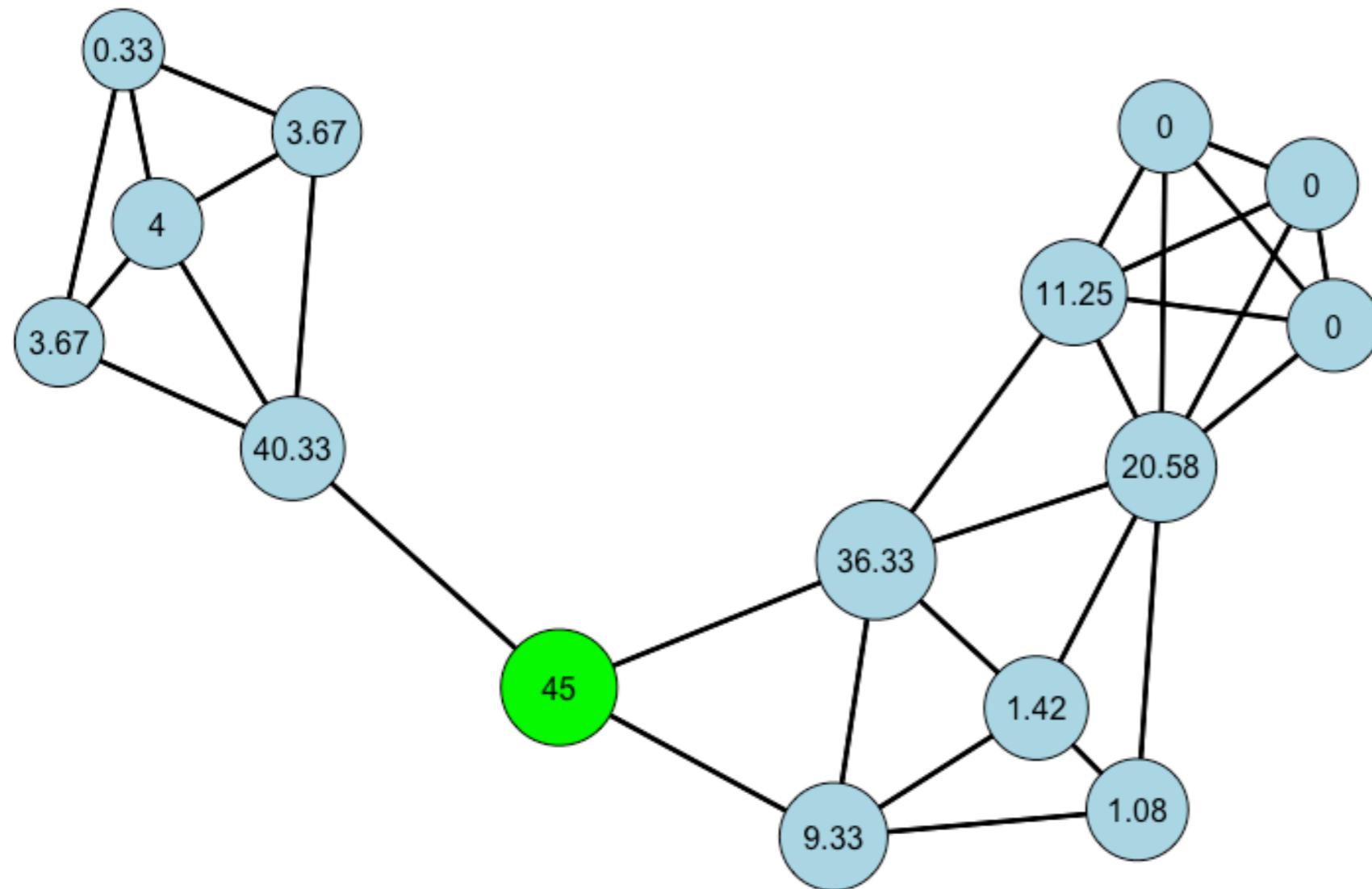
*Nodes sized by  
degree*

*Nodes labeled by  
betweenness  
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# High Betweenness, Low Closeness



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# Learning Goals

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- ❖ At the end of the lecture, you should be able to answer these questions:
  - ❖ What are some different ways we can conceptualize “centrality”?
  - ❖ What is *closeness* and *betweenness* centrality?
  - ❖ How do we calculate these measures for undirected and directed graphs?
  - ❖ What do comparing these measures tell us about the structure of a network?

Questions?