

Statistical Analysis of Networks

Closeness & Betweenness Centrality

When we say a *node* is “central,”
what do we mean conceptually?

Degree Centrality

- ❖ Nodes that have many edges are “central”.
 - ❖ We can operationalize this using the degree (for undirected graphs) or the indegree / outdegree (for directed graphs).

Motivating Example

J Youth Adolescence (2014) 43:104–115
DOI 10.1007/s10964-013-9946-0

EMPIRICAL RESEARCH

“Role Magnets”? An Empirical Investigation of Popularity Trajectories for Life-Course Persistent Individuals During Adolescence

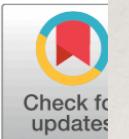
Jacob T. N. Young

PNAS

RESEARCH ARTICLE

POLITICAL SCIENCES

OPEN ACCESS



Disrupting hate: The effect of deplatforming hate organizations on their online audience

Daniel Robert Thomas^{a,1} and Laila A. Wahedi^a

Edited by Timothy Wilson, University of Virginia, Charlottesville, VA; received August 17, 2022; accepted January 20, 2023

Conceptualization

- ❖ “Everyone agrees, it seems, that centrality is an important structural attribute of networks. All concede that it is related to a high degree to other important group properties and processes. But there consensus ends.” (Freeman, 1978/1979: 217)
- ❖ The type of measure we use depends on the substantive question of interest.
 - ❖ Various measures of centrality are correlated, but they operationalize different concepts.

What are some other ways a node
can be “central”?

Motivating Example

- ❖ Problem: Bullying
- ❖ Question: Who is likely to be a bully?
 - ❖ Typical explanation...
 - ❖ Faris & Felmlee's argument...

Status Struggles: Network Centrality and Gender Segregation in Same- and Cross-Gender Aggression

Robert Faris^a and Diane Felmlee^a

American Sociological Review
76(1) 48–73
© American Sociological
Association 2011
DOI: 10.1177/0003122410396196
<http://asr.sagepub.com>


Motivating Example

Faris and Felmlee

61

- ❖ Findings?

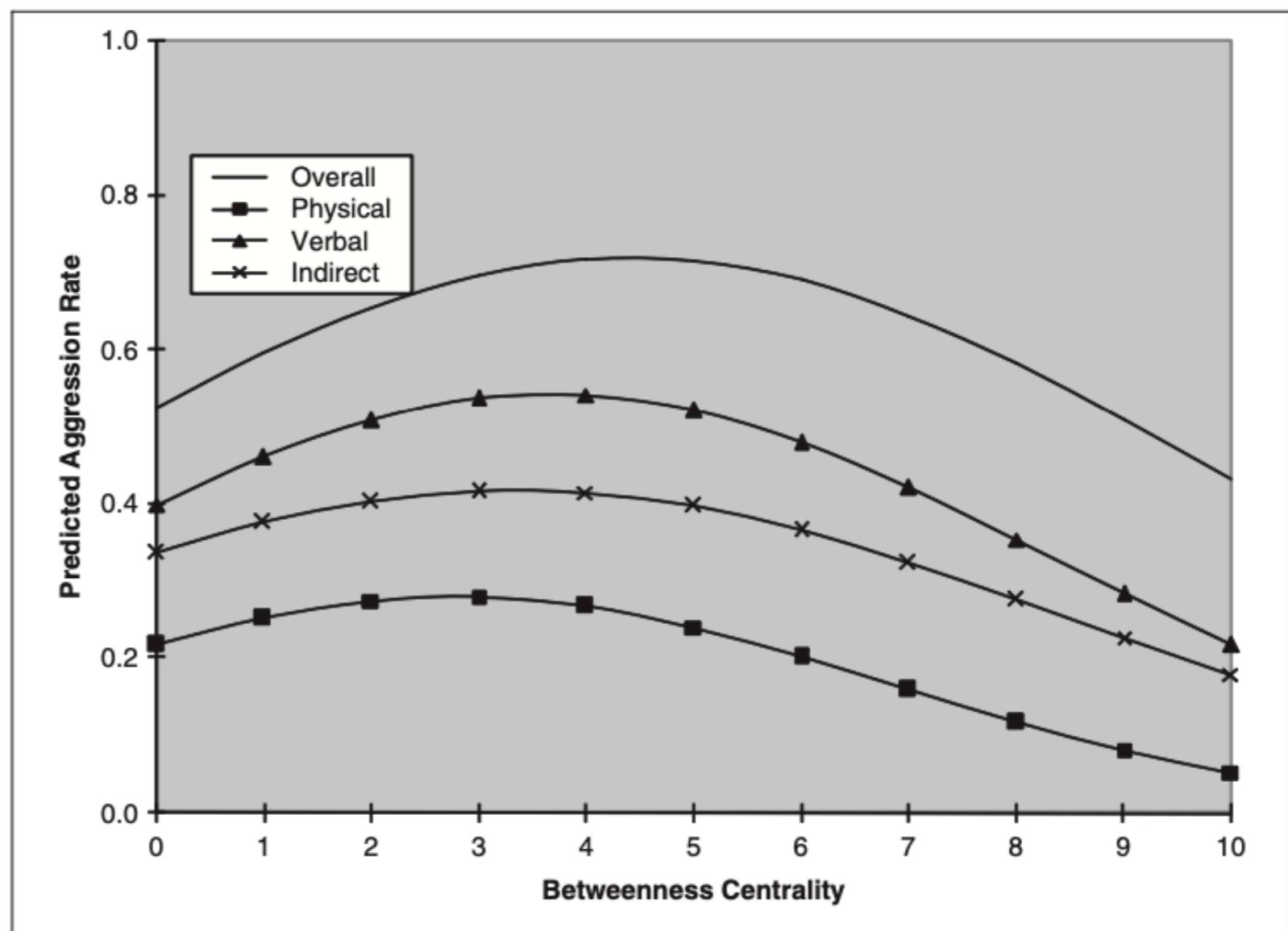


Figure 2. Predicted Aggression Rate by Social Network Centrality

Motivating Example

- ❖ Findings?

Faris and Felmlee

63

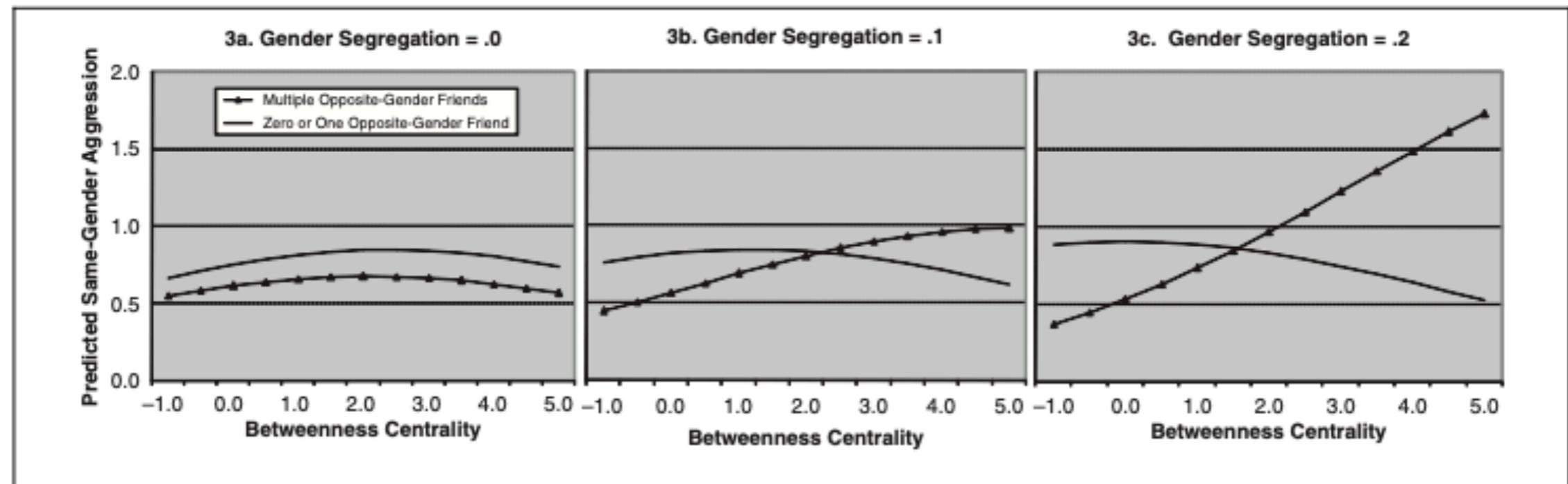


Figure 3. Predicted Rate of Same-Gender Aggression, by Centrality, Gender Segregation, and Cross-Gender Friendships

Learning Goals

- ❖ At the end of the lecture, you should be able to answer these questions:
 - ❖ What are some different ways we can conceptualize “centrality”?
 - ❖ What is *closeness* and *betweenness* centrality?
 - ❖ How do we calculate these measures for undirected and directed graphs?
 - ❖ What do comparing these measures tell us about the structure of a network?

Closeness Centrality

- ❖ How “close” is a node to other nodes?
 - ❖ In a graph, *closeness centrality* measures how near a node is to the other nodes in the network.
 - ❖ Nodes that are central are those that can reach other nodes in short distances.

Closeness Centrality

- ❖ Closeness is based on the inverse of the distance of each actor to every other actor.
- ❖ Terminology:
 - ❖ A **geodesic** is the shortest path between two nodes.
 - ❖ The **distance**, $d(n_i, n_j)$, is the length of the path between i and j .

Closeness Centrality: Undirected Binary Graphs

$$C_C(n_i) = \left[\sum_{j=1}^g d(n_i, n_j) \right]^{-1}$$

Closeness Centrality: Undirected Binary Graphs

$$C_C(n_i) = \left[\sum_{j=1}^g d(n_i, n_j) \right]^{-1}$$

↑
How far is i from
every node?

Closeness Centrality: Undirected Binary Graphs

$$C_C(n_i) = \left[\sum_{j=1}^g d(n_i, n_j) \right]^{-1}$$

How far is i from
every node?

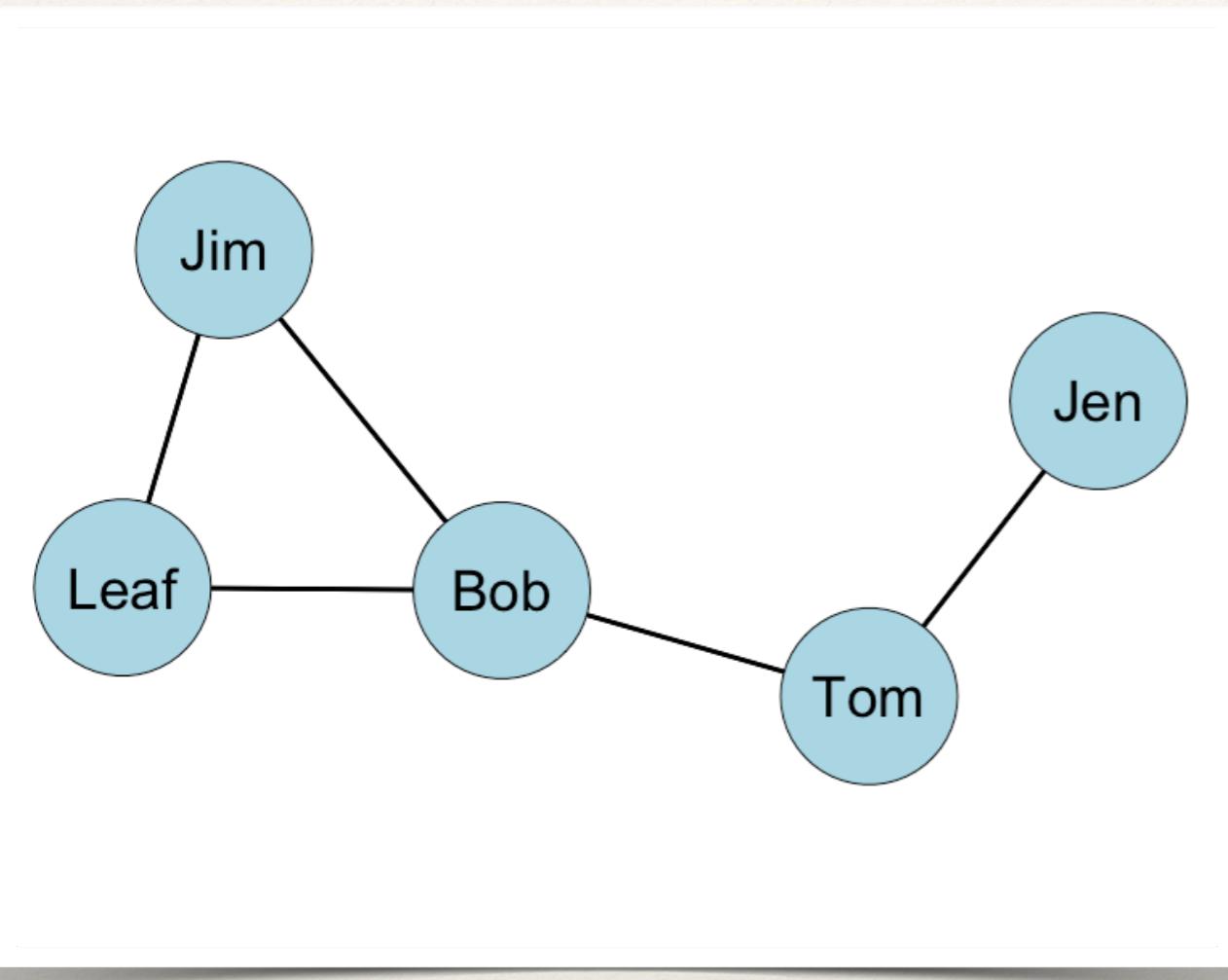
Then take the
inverse of the sum.

Closeness Centrality: Undirected Binary Graphs

$$C_C(n_i) = \left[\sum_{j=1}^g d(n_i, n_j) \right]^{-1} = \frac{1}{\left[\sum_{j=1}^g d(n_i, n_j) \right]}$$

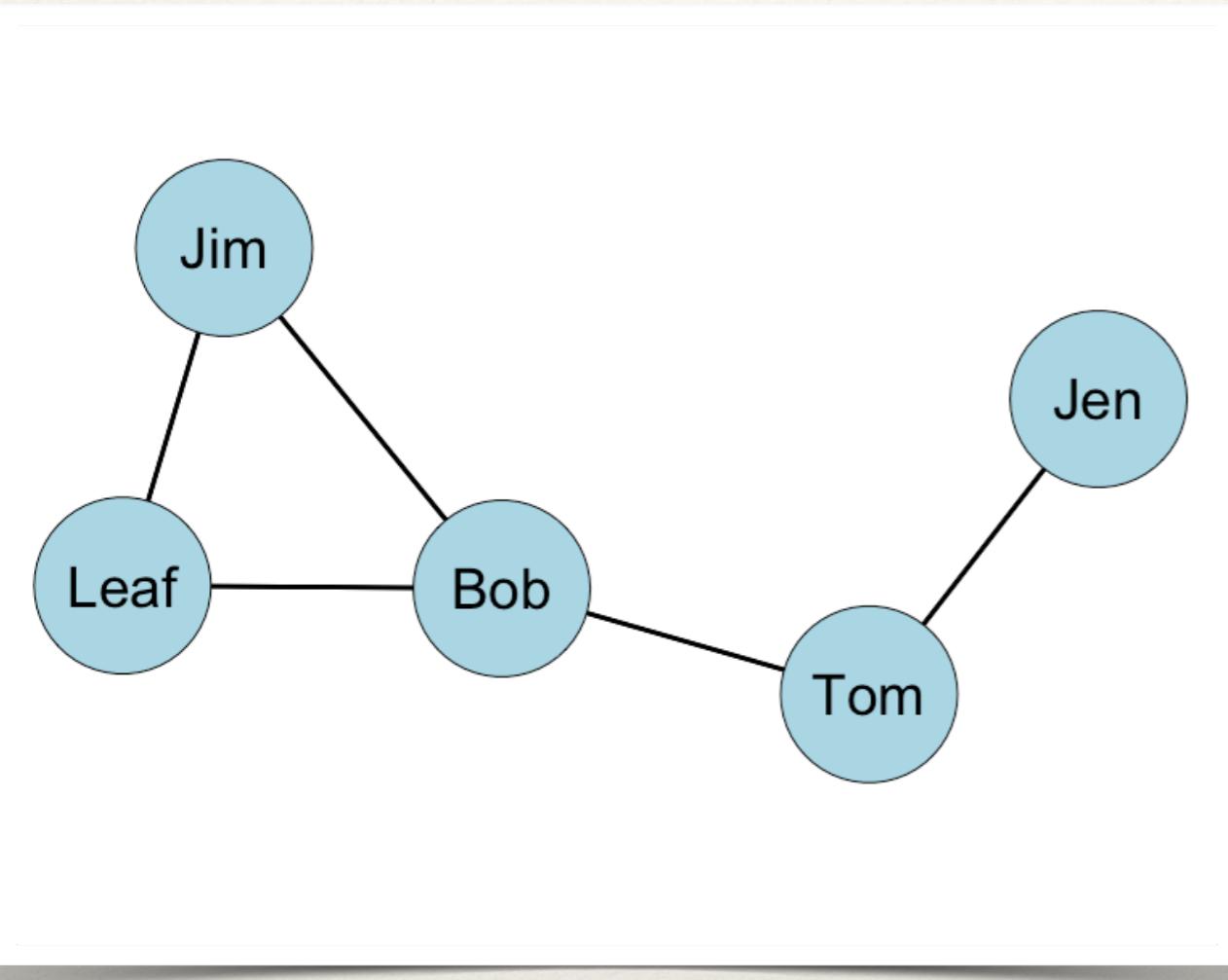
Undirected Networks

Example: Undirected, Binary Network



We want a matrix of the distances between each node.

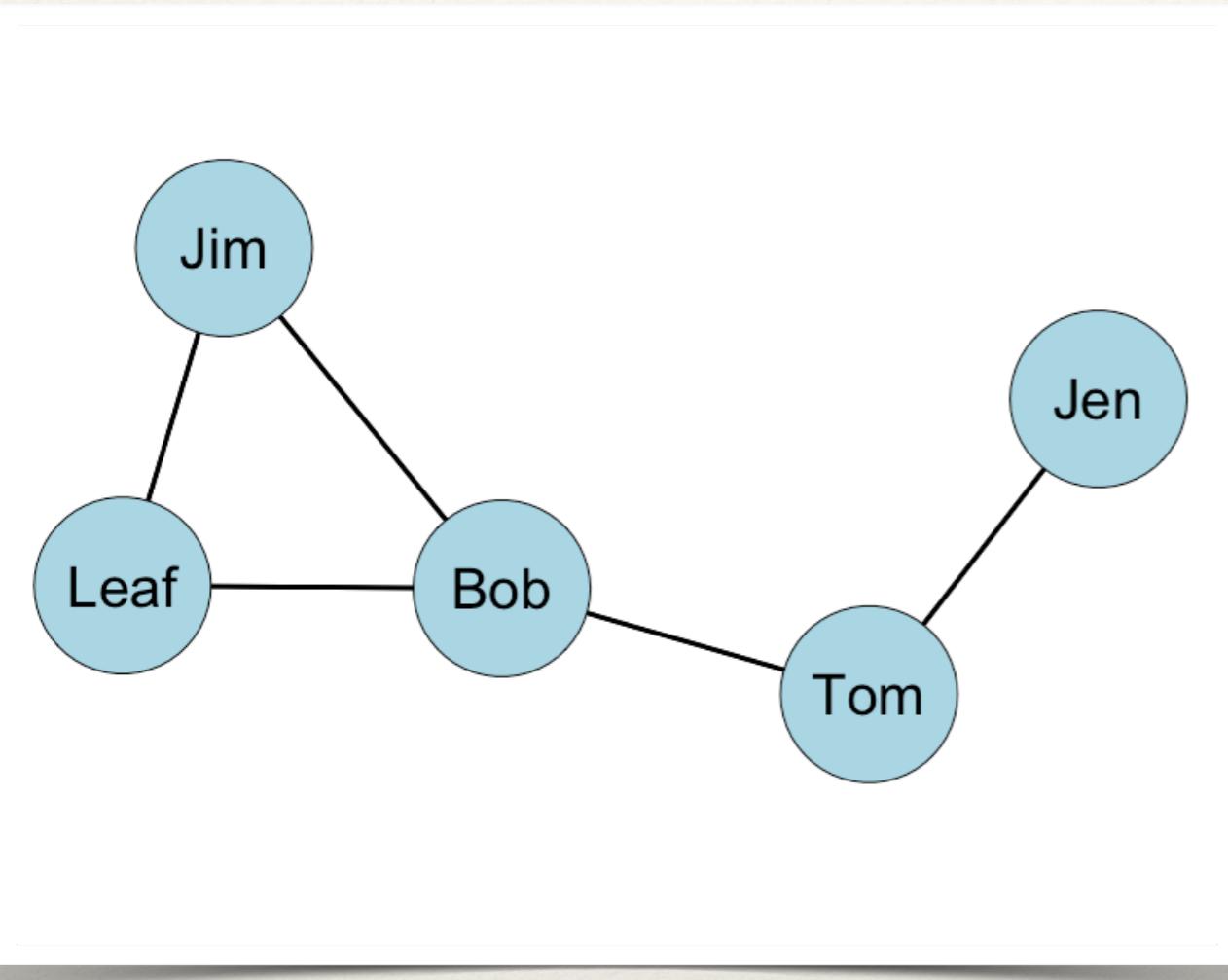
Example: Undirected, Binary Network



How far is Jen from Tom? From Bob?

		Distance Matrix				
		Jen	Tom	Bob	Leaf	Jim
		Jen	?	?		
		Tom				
		Bob				
		Leaf				
		Jim				

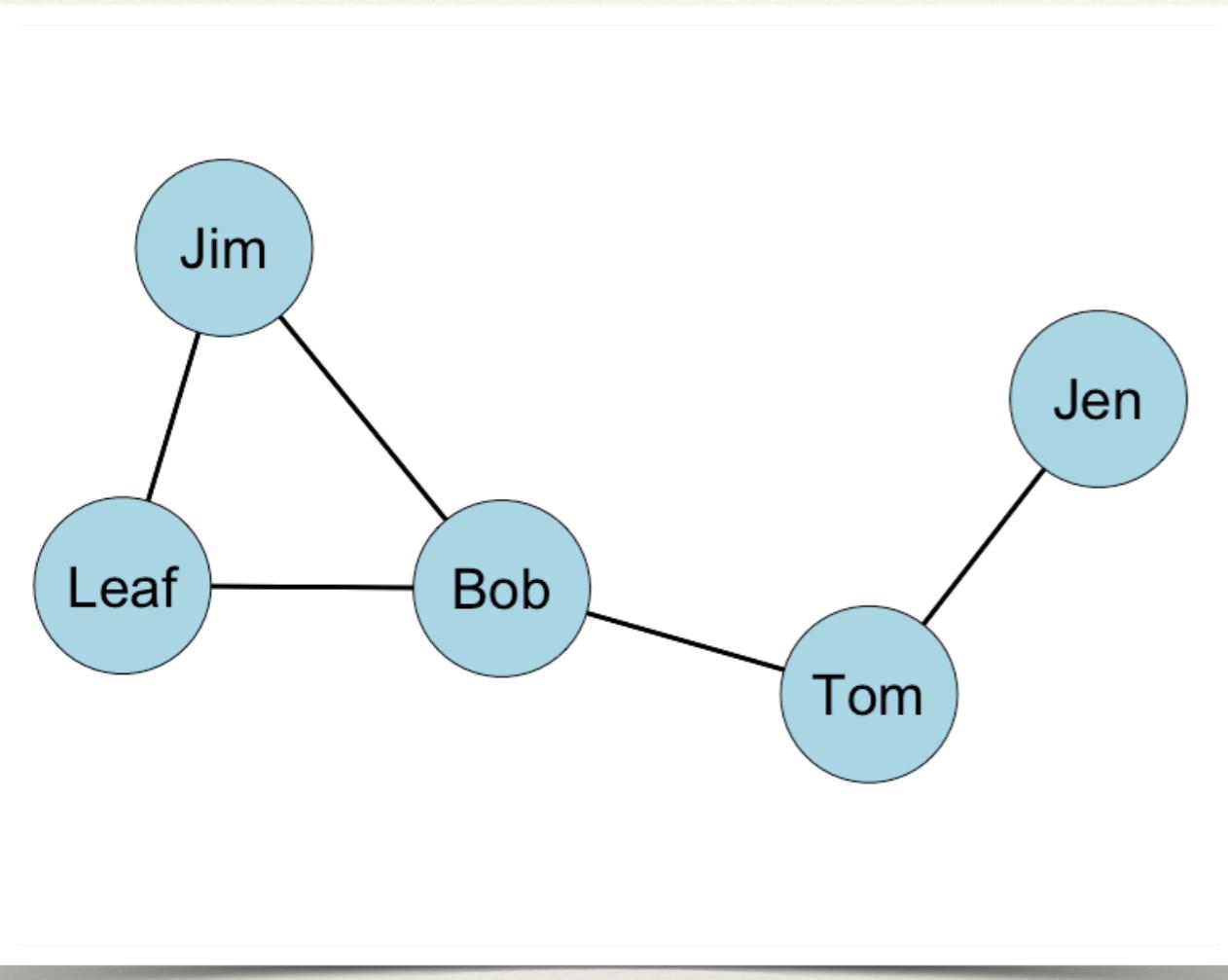
Example: Undirected, Binary Network



How far is Jen from Tom? From Bob?

		Jen	Tom	Bob	Leaf	Jim
Jen			1			
Tom						
Bob						
Leaf						
Jim						

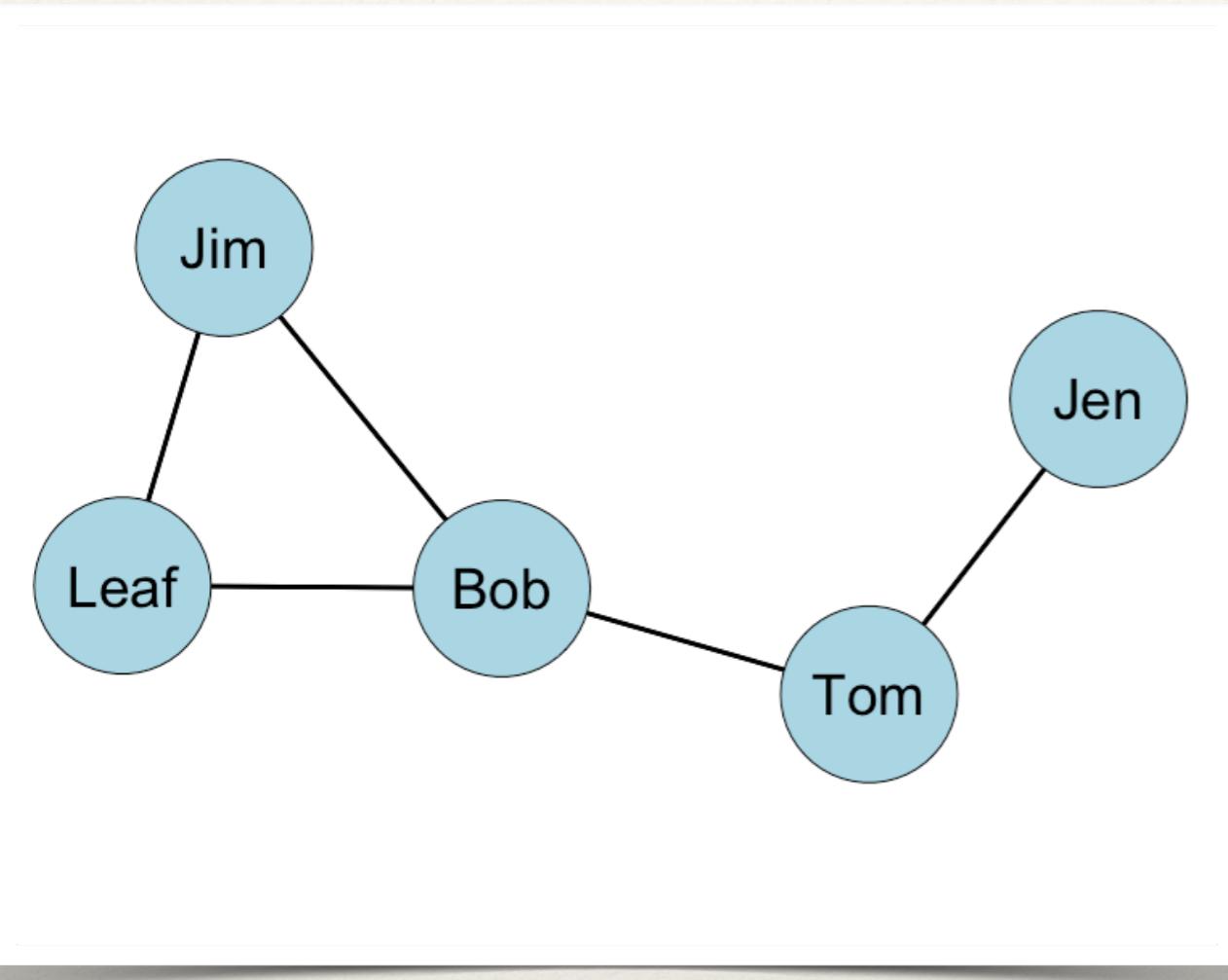
Example: Undirected, Binary Network



How far is Jen from Tom? From Bob?

		Jen	Tom	Bob	Leaf	Jim
Jen			1	2		
Tom						
Bob						
Leaf						
Jim						

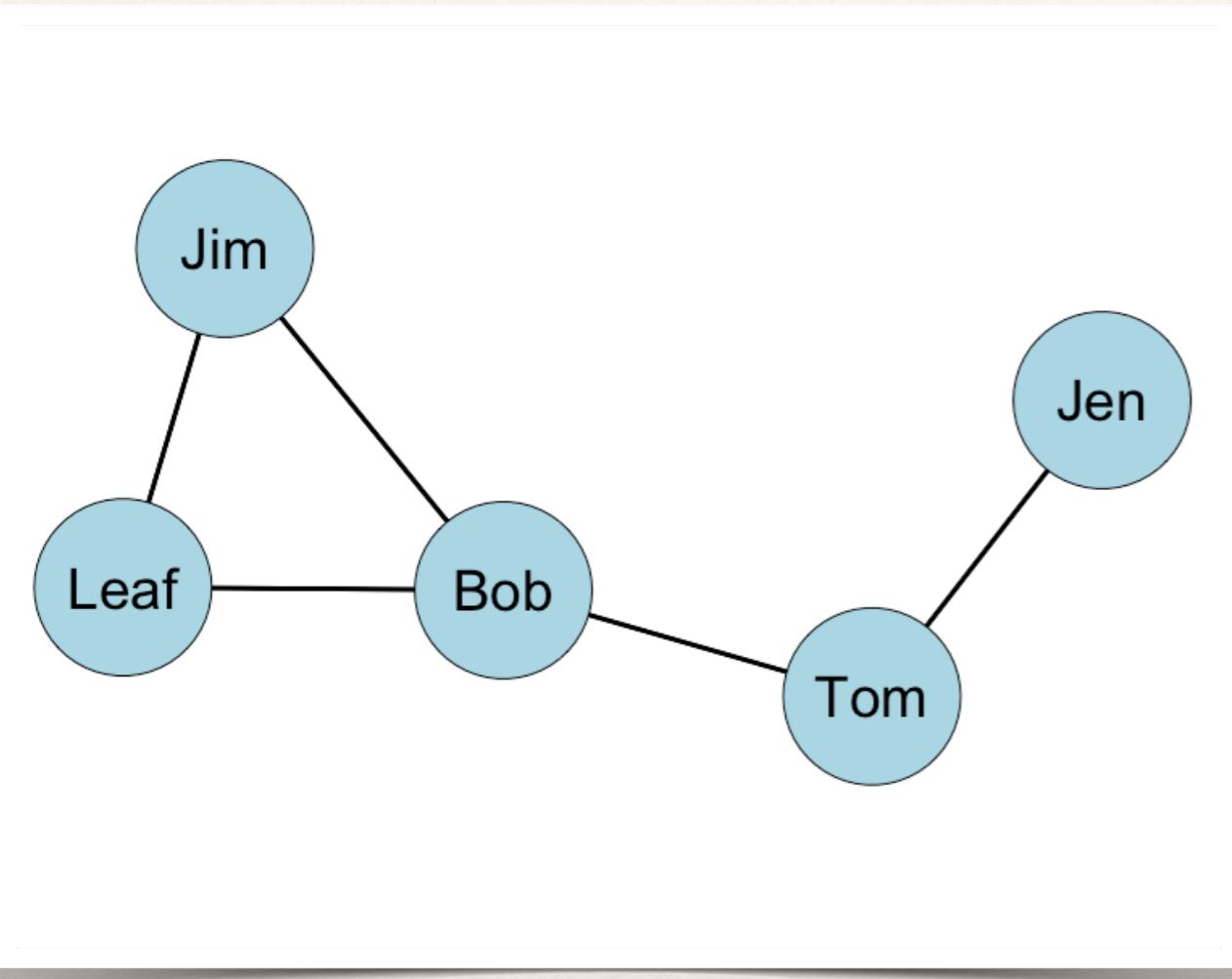
Example: Undirected, Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	?	?	?
Tom			?	?	?	?
Bob				?	?	?
Leaf					?	?
Jim						?

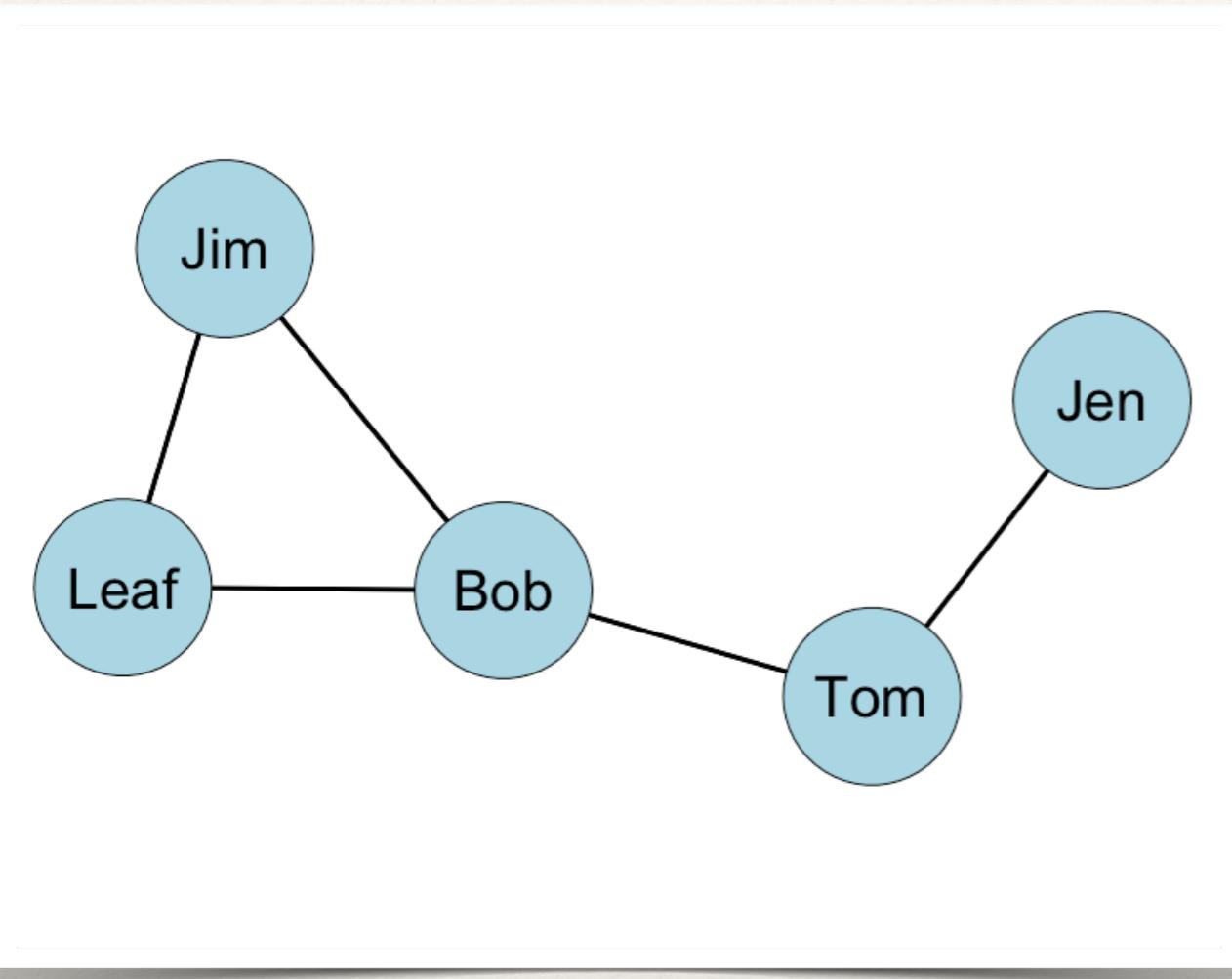
Now, fill in the rest...

Example: Undirected, Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3	
Tom			1	2	2	
Bob				1	1	
Leaf					1	
Jim						1

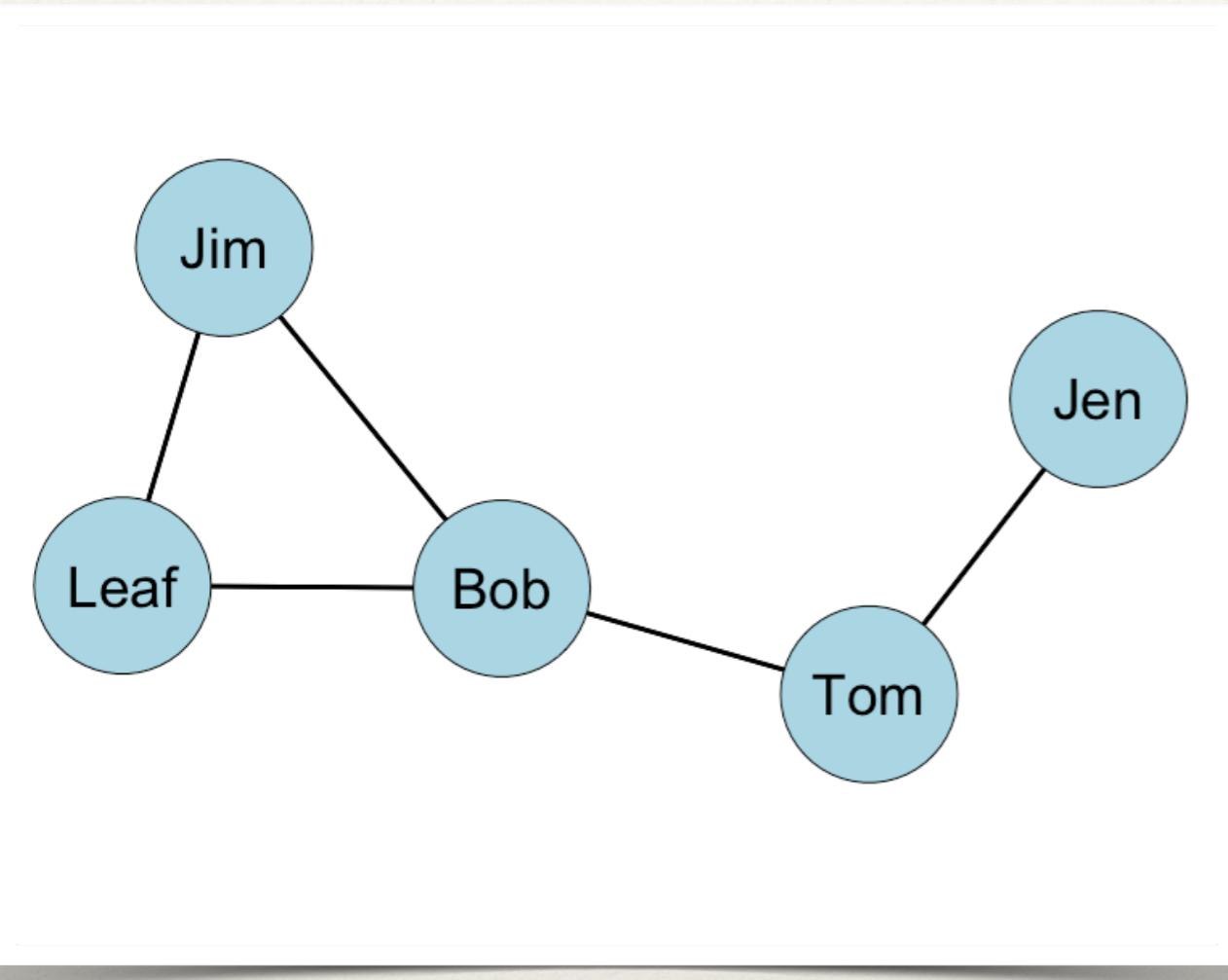
Example: Undirected, Binary Network



Since the graph is undirected, the distance matrix is symmetric about the diagonal.

		Jen	Tom	Bob	Leaf	Jim
Jen			1	2	3	3
Tom	1			1	2	2
Bob	2			1	1	1
Leaf	3	2	1			1
Jim	3	2	1	1		

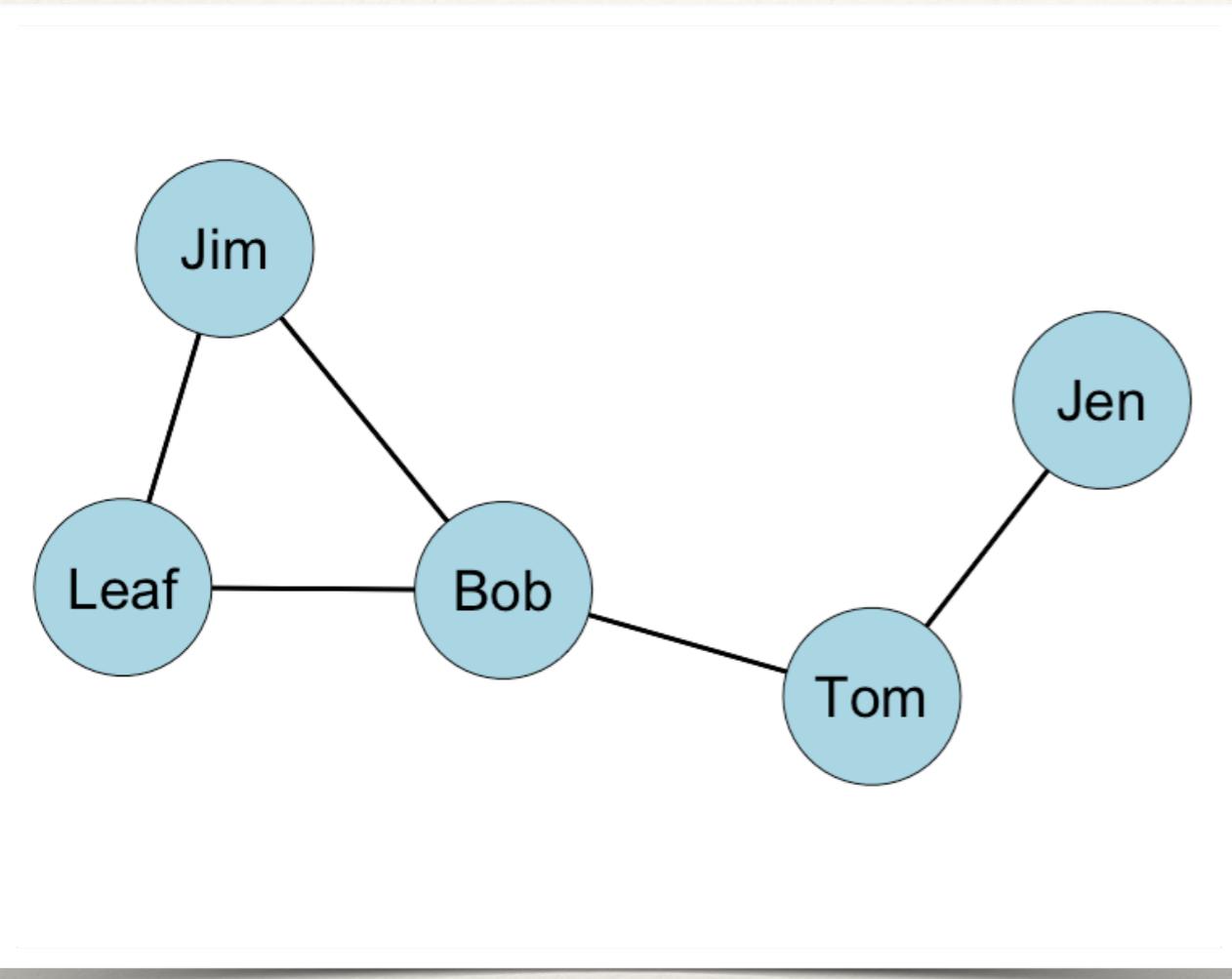
Example: Undirected, Binary Network



The row sum of these is the distances.

Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen	1	2	3	3	3	9
Tom	1		1	2	2	6
Bob	2	1		1	1	5
Leaf	3	2	1		1	7
Jim	3	2	1	1		7

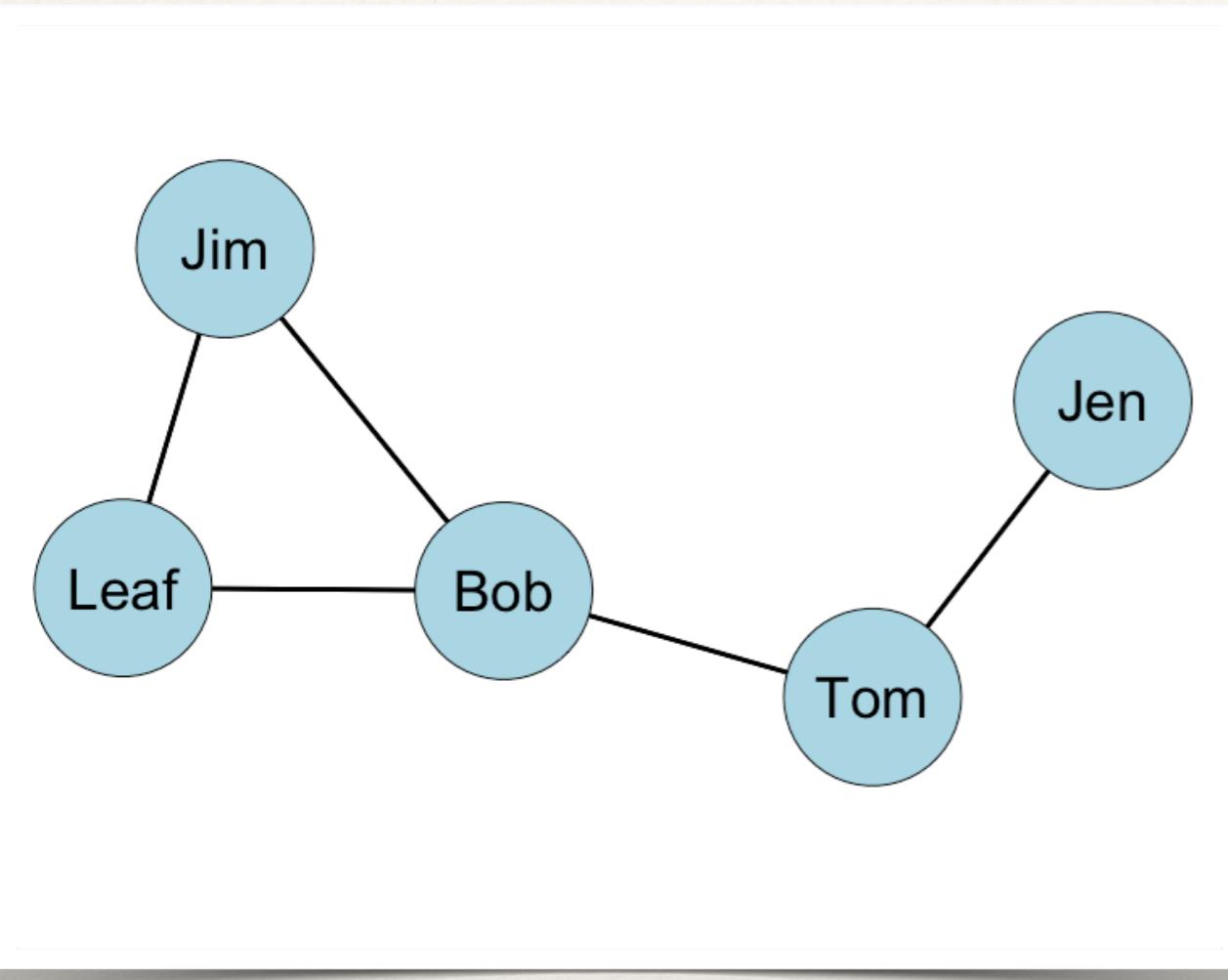
Example: Undirected, Binary Network



Since the graph is undirected, the column sum reports the same information.

Distance Matrix						
	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom	1		1	2	2	6
Bob	2	1		1	1	5
Leaf	3	2	1		1	7
Jim	3	2	1	1		7
Sum	9	6	5	7	7	

Example: Undirected, Binary Network



By taking the reciprocal, we get the closeness centrality score.

Closeness Centrality

$$\text{Jen} = 1/9 = 0.111$$

$$\text{Tom} = 1/6 = 0.167$$

$$\text{Bob} = 1/5 = 0.200$$

$$\text{Leaf} = 1/7 = 0.143$$

$$\text{Jim} = 1/7 = 0.143$$

$$C_C(n_i) = \frac{1}{\left[\sum_{j=1}^g d(n_i, n_j) \right]}$$

Closeness Centrality: Undirected Binary Graphs

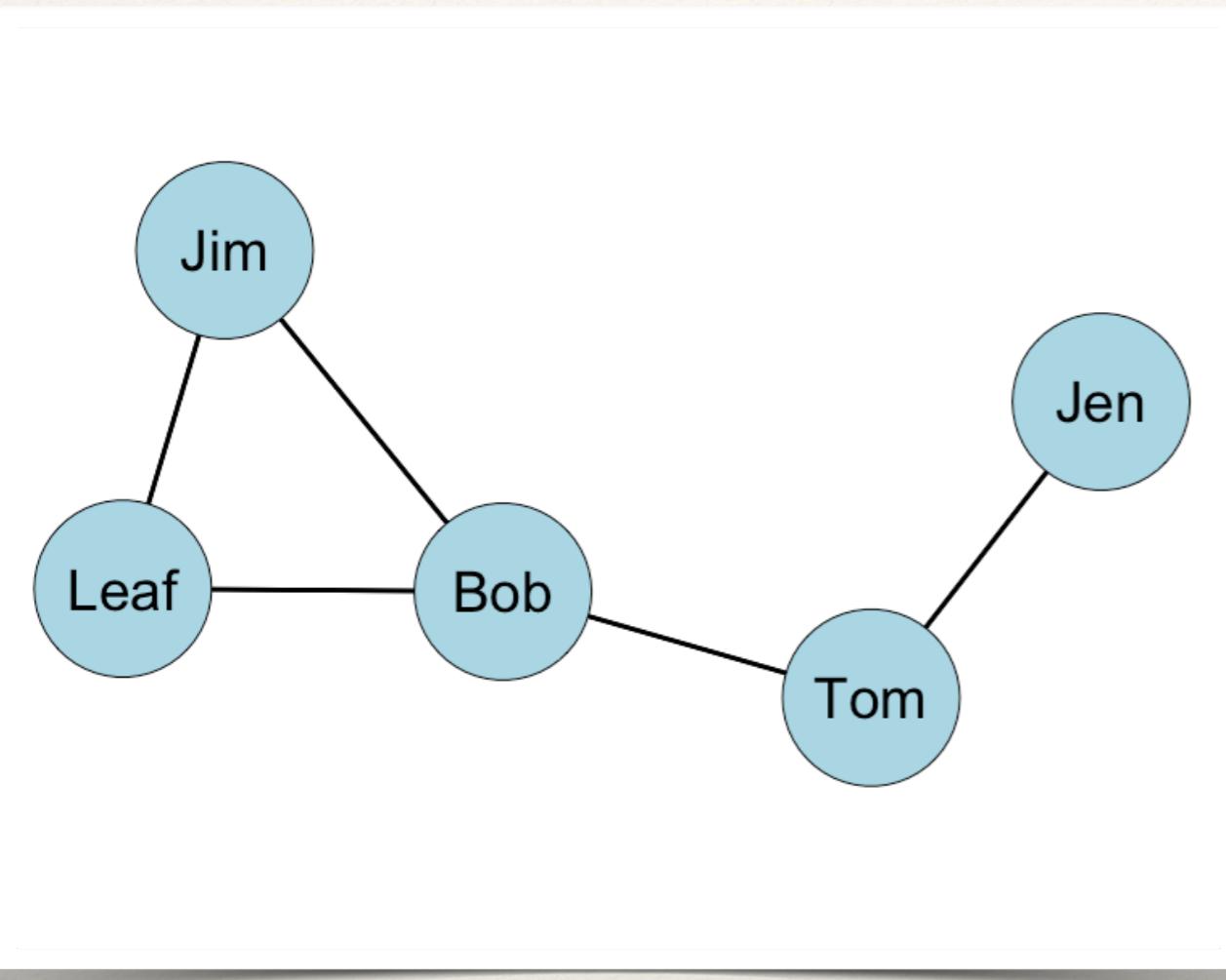
- ❖ Actor closeness centrality not only reflects each node's connectivity to other nodes but also depends on the size of the network, g .
- ❖ Summing over more nodes will push scores closer to zero.
 - ❖ *Solution?*

Standardized Closeness Centrality

$$C'_C(n_i) = \left[\sum_{j=1}^g d(n_i, n_j) \right]^{-1} \times [g - 1]$$

Multiply by $g - 1!$

Example: Undirected, Binary Network



Closeness Centrality

$$\text{Jen} = 1/9 = 0.111$$

$$\text{Tom} = 1/6 = 0.167$$

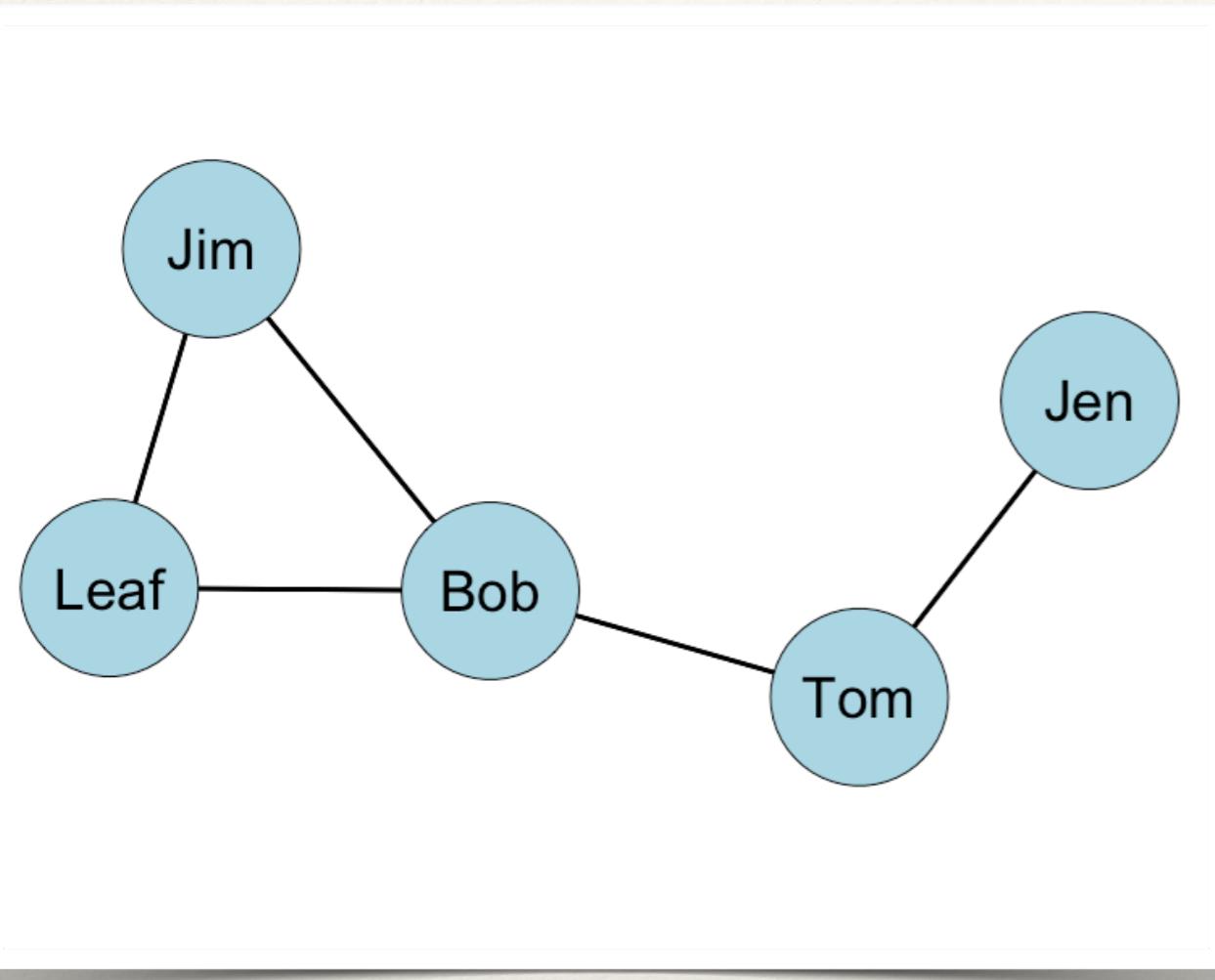
$$\text{Bob} = 1/5 = 0.200$$

$$\text{Leaf} = 1/7 = 0.143$$

$$\text{Jim} = 1/7 = 0.143$$

*Multiplying by $g-1$ gives the
standardized value*

Example: Undirected, Binary Network



Standardized Closeness

Centrality

$$\text{Jen} = 0.111^*4 = 0.444$$

$$\text{Tom} = 0.167^*4 = 0.668$$

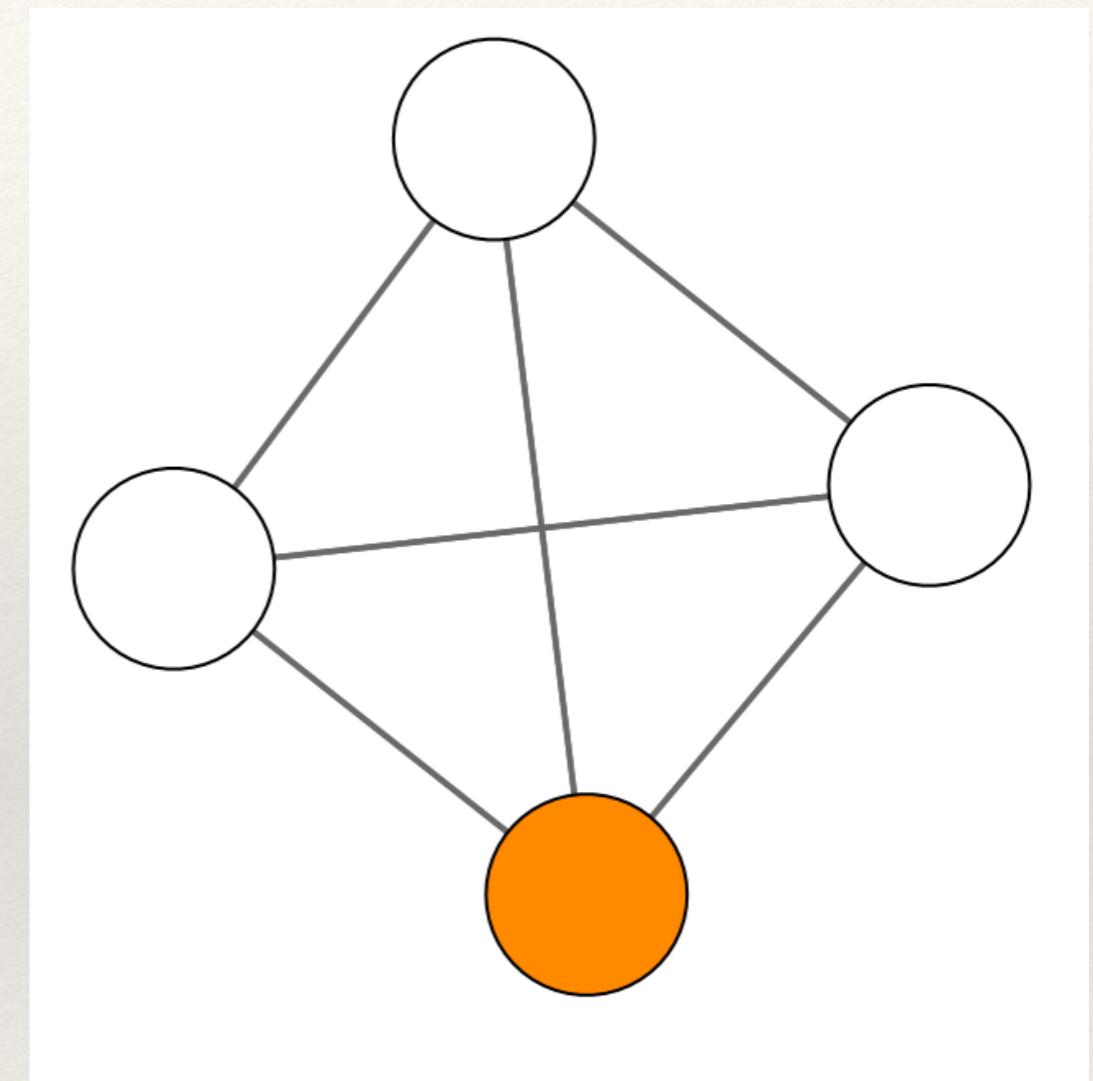
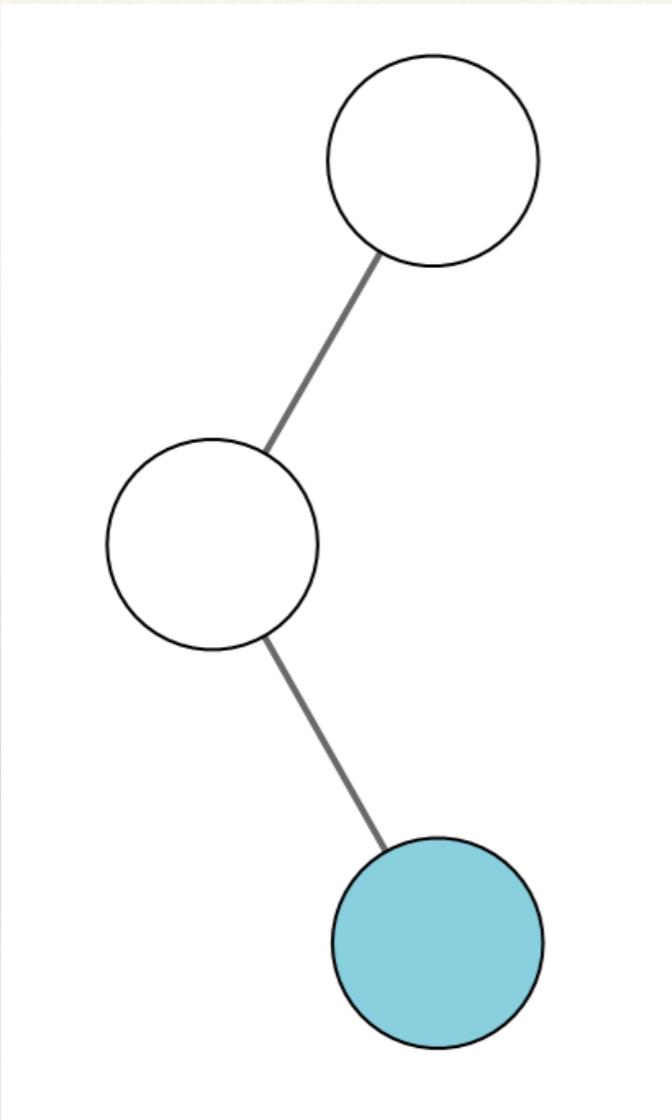
$$\text{Bob} = 0.200^*4 = 0.800$$

$$\text{Leaf} = 0.143^*4 = 0.572$$

$$\text{Jim} = 0.143^*4 = 0.572$$

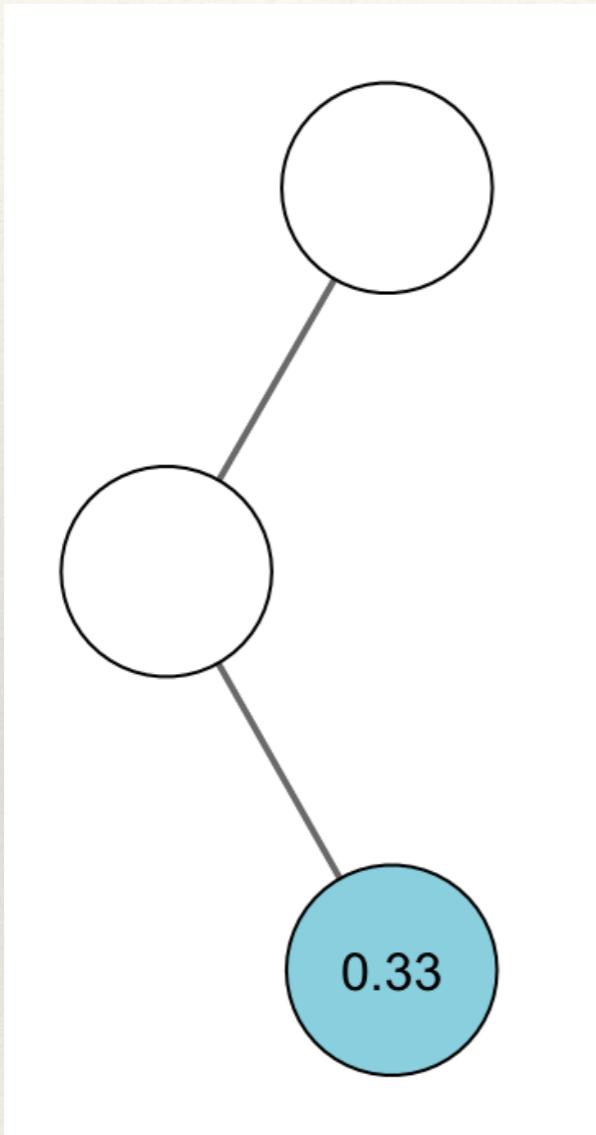
*Multiplying by g-1 gives the
standardized value*

Why standardization matters

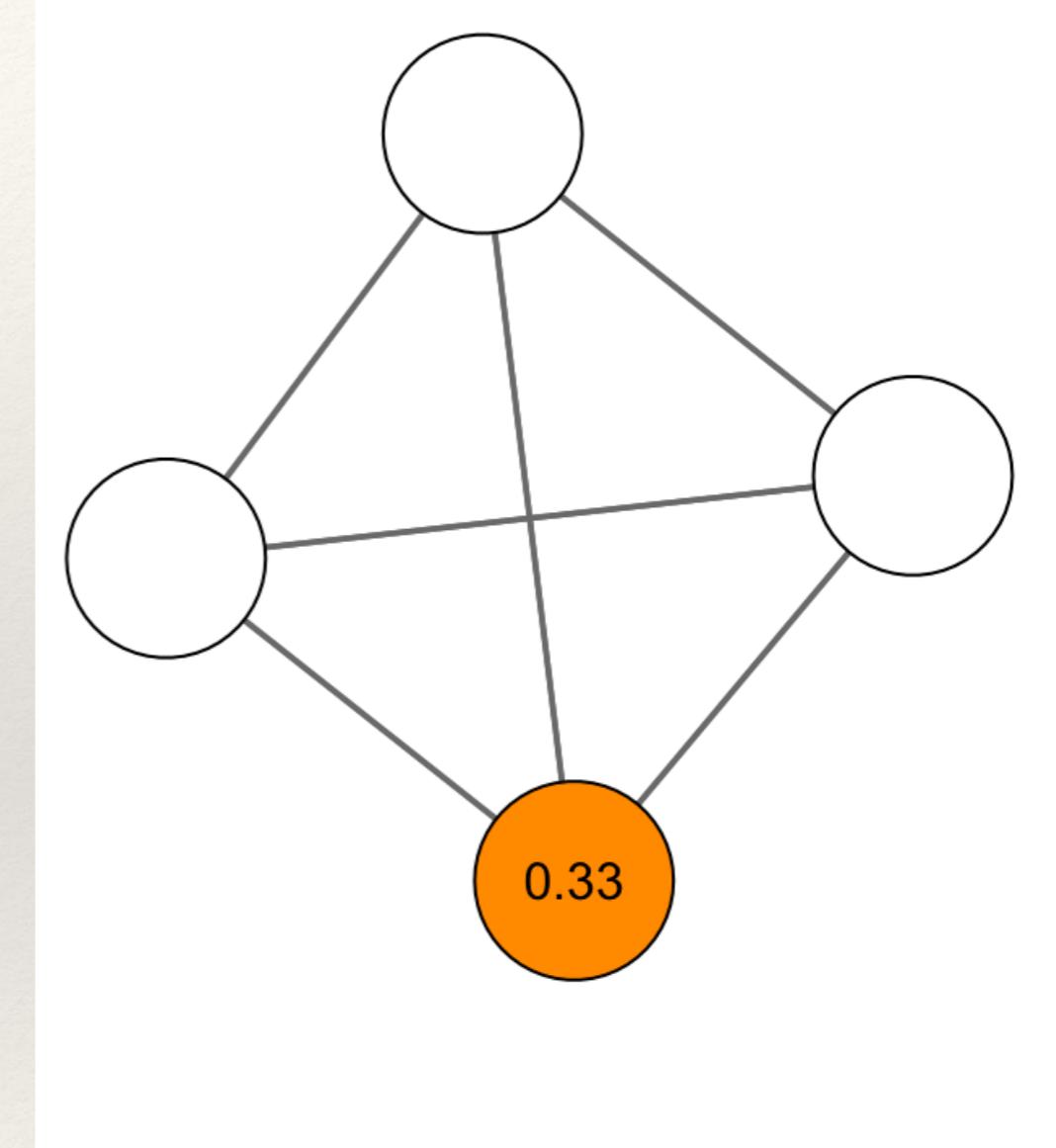


Which node has a higher closeness score?

Why standardization matters

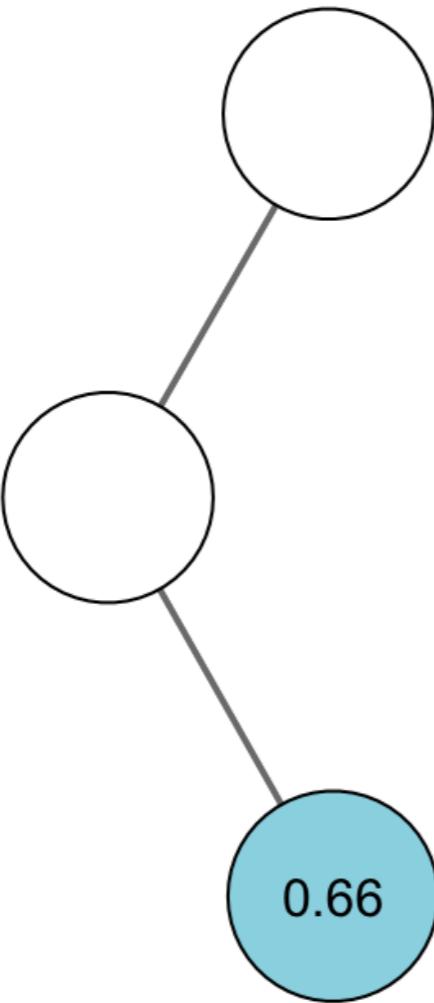


Network A: Raw

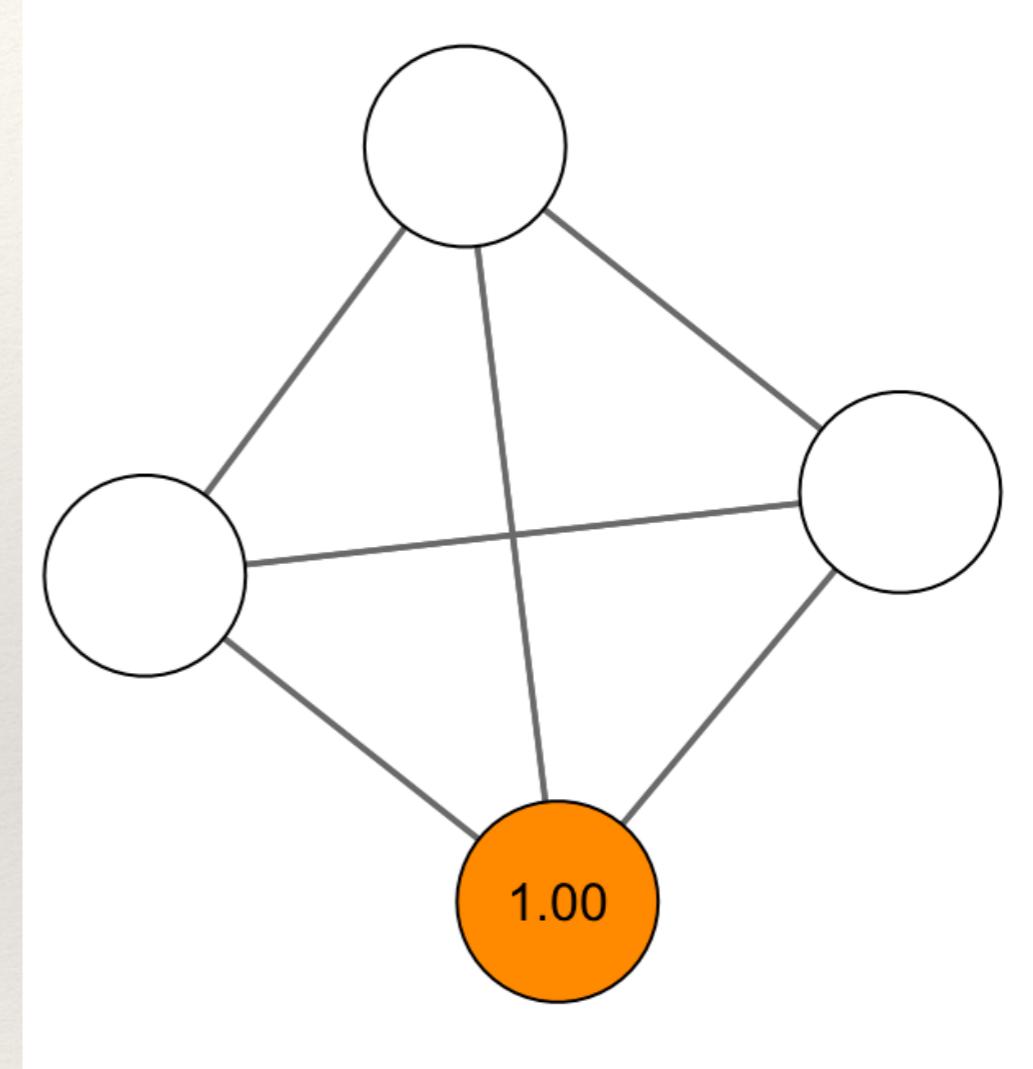


Network B: Raw

Why standardization matters



Network A: Standardized



Network B: Standardized

Centralization

- ❖ Recall that we can describe the dispersion in the centrality scores.
- ❖ This is referred to as *centralization*.



Group Closeness Centralization: Undirected Binary Graphs

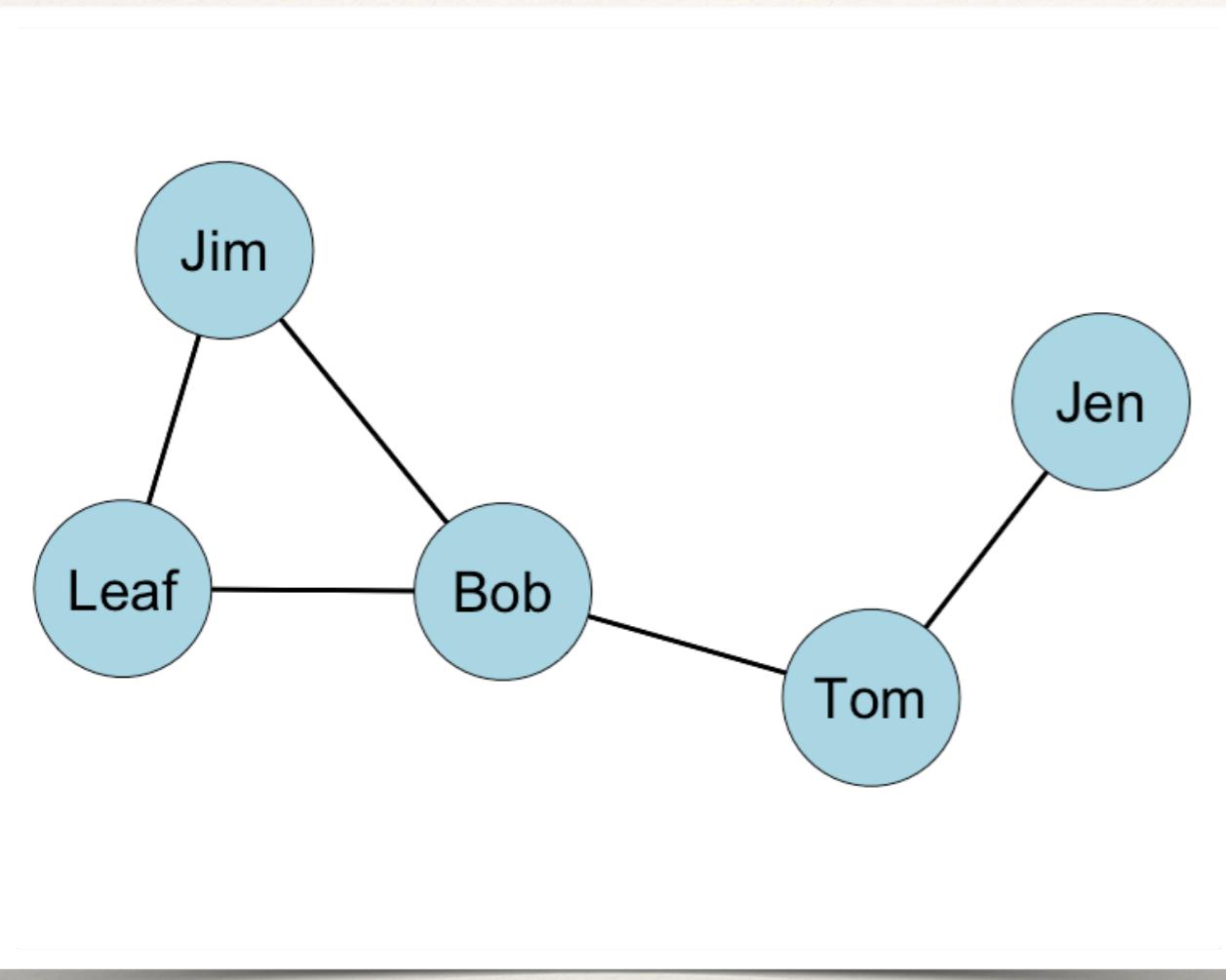
- ❖ Group *closeness* centralization describes the dispersion in the closeness scores between the nodes.
- ❖ The values are between 0 and 1 (as before).
 - ❖ Where scores closer to 0 indicate a decentralized structure and scores closer to 1 indicate a hierarchical structure.

Group Closeness Centralization: Undirected Binary Graphs

$$C_C = \frac{\sum_{i=1}^g [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

Largest
standardized
closeness score Standardized
closeness score for
actor i

Example: Undirected, Binary Network



Standardized Closeness

Centrality

$$\text{Jen} = 0.111^*4 = 0.444$$

$$\text{Tom} = 0.167^*4 = 0.668$$

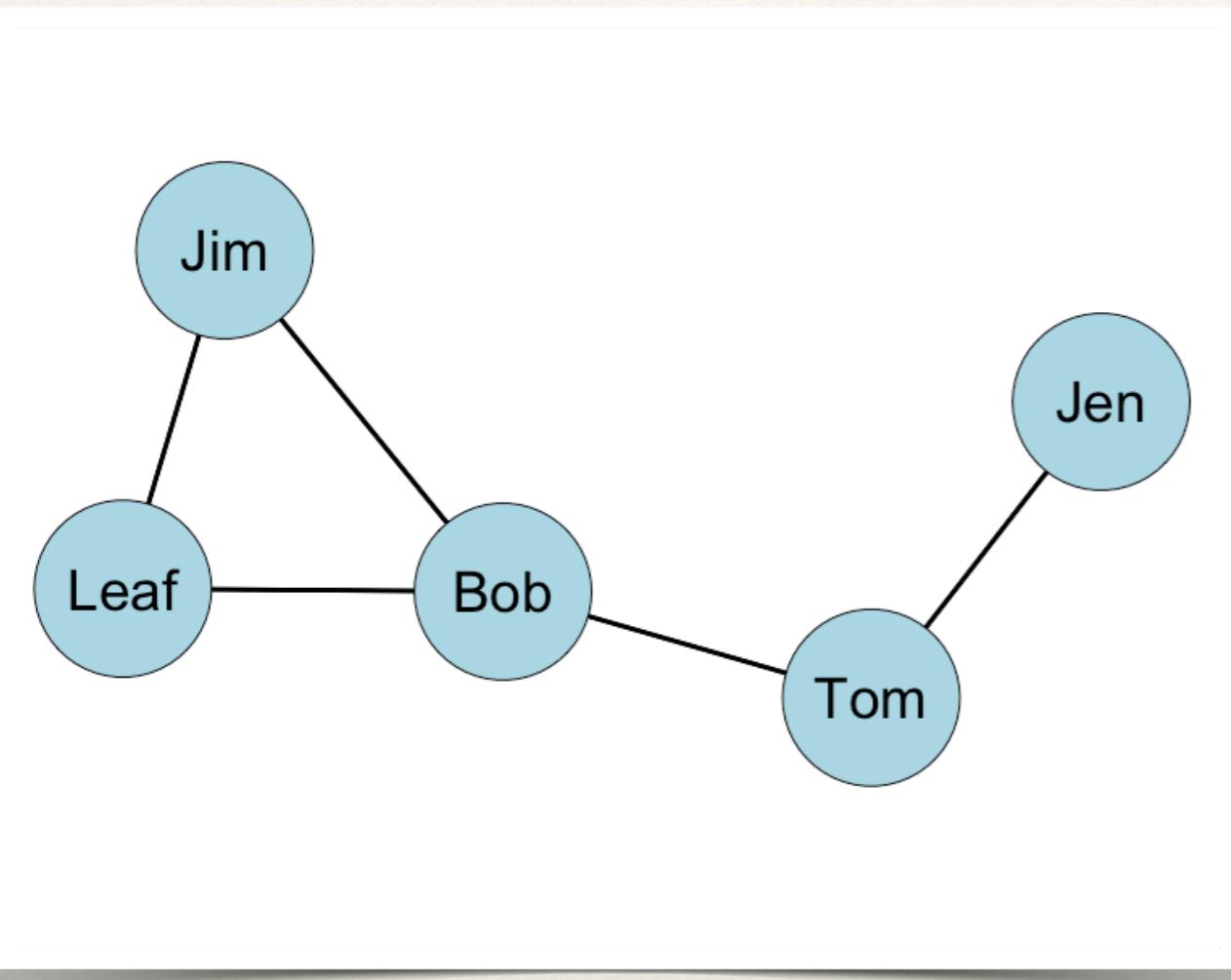
$$\text{Bob} = 0.200^*4 = 0.800$$

$$\text{Leaf} = 0.143^*4 = 0.572$$

$$\text{Jim} = 0.143^*4 = 0.572$$

What is the group closeness centralization score for this graph?

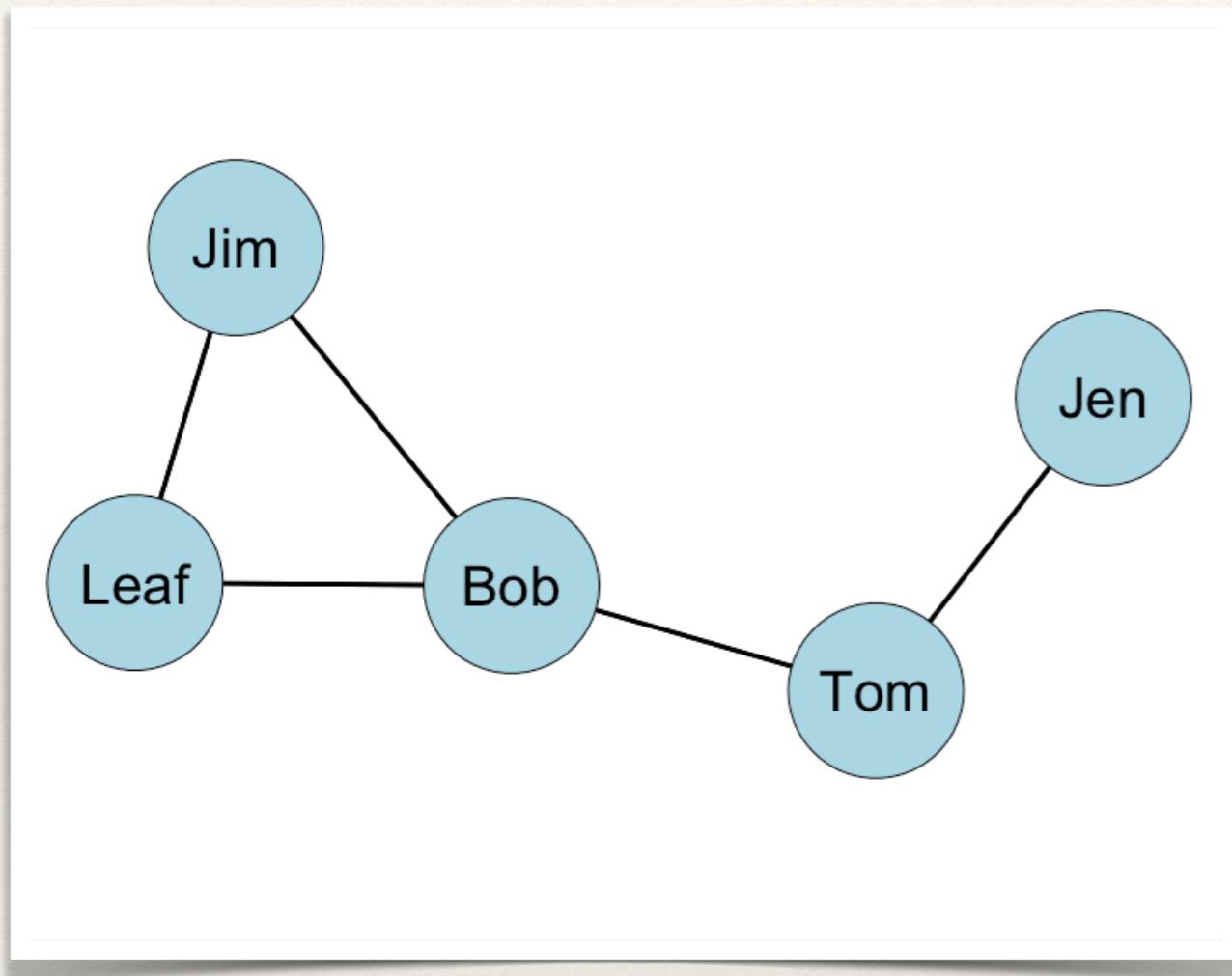
Example: Undirected, Binary Network



Deviations of Standardized Closeness Centrality Scores

Jen = $0.800 - 0.444 = 0.356$
Tom = $0.800 - 0.668 = 0.132$
Bob = $0.800 - 0.800 = 0.000$
Leaf = $0.800 - 0.572 = 0.228$
Jim = $0.800 - 0.572 = 0.228$

Example: Undirected, Binary Network



Deviations of
Standardized Closeness

Centrality Scores

$$\text{Jen} = 0.800 - 0.444 = 0.356$$

$$\text{Tom} = 0.800 - 0.668 = 0.132$$

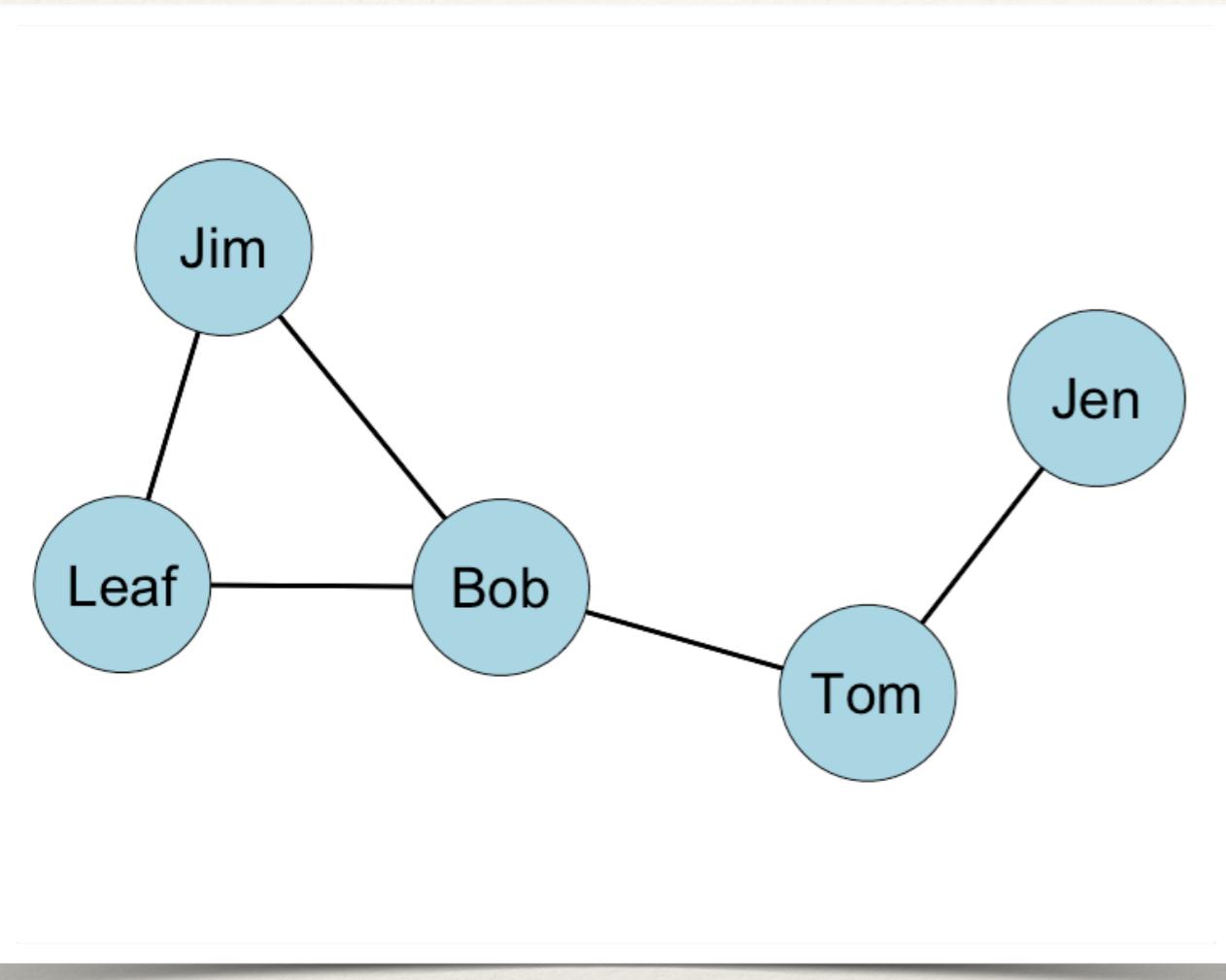
$$\text{Bob} = 0.800 - 0.800 = 0.000$$

$$\text{Leaf} = 0.800 - 0.572 = 0.228$$

$$\text{Jim} = 0.800 - 0.572 = 0.228$$

$$\text{Sum} = 0.944$$

Example: Undirected, Binary Network



Deviations of Standardized Closeness Centrality Scores

$$\text{Jen} = 0.800 - 0.444 = 0.356$$

$$\text{Tom} = 0.800 - 0.668 = 0.132$$

$$\text{Bob} = 0.800 - 0.800 = 0.000$$

$$\text{Leaf} = 0.800 - 0.572 = 0.228$$

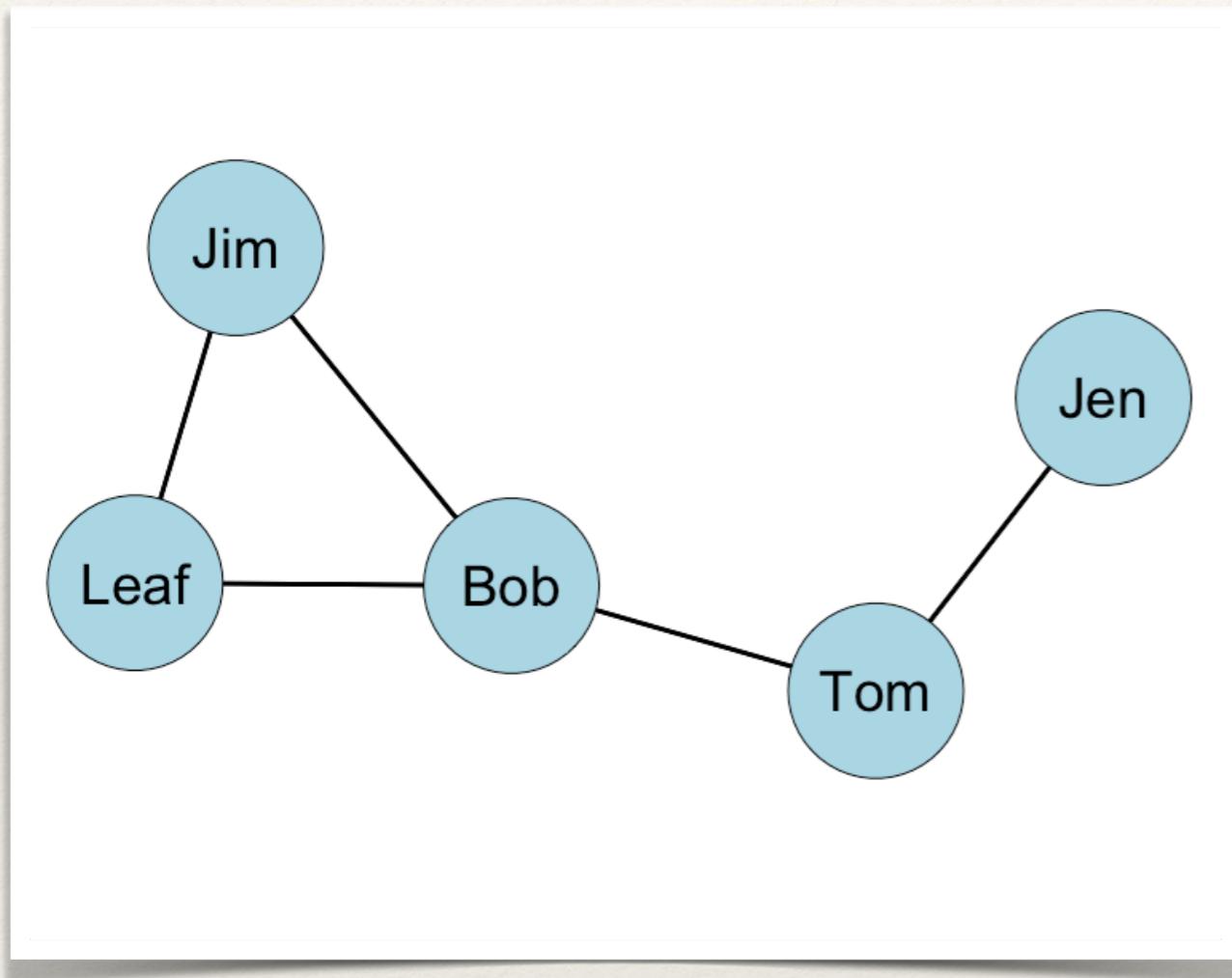
$$\text{Jim} = 0.800 - 0.572 = 0.228$$

$$\text{Sum} = 0.944$$

$$C_C = \frac{\sum_{i=1}^g [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

$$\begin{aligned} \text{Denominator} &= \\ &[(5-2)(5-1)/(2*5-3)] = 1.714 \end{aligned}$$

Example: Undirected, Binary Network



Approximately 0.551

Deviations of
Standardized Closeness
Centrality Scores

$$\text{Jen} = 0.800 - 0.444 = 0.356$$

$$\text{Tom} = 0.800 - 0.668 = 0.132$$

$$\text{Bob} = 0.800 - 0.800 = 0.000$$

$$\text{Leaf} = 0.800 - 0.572 = 0.228$$

$$\text{Jim} = 0.800 - 0.572 = 0.228$$

$$\text{Sum} = 0.944$$

$$\begin{aligned}\text{Denominator} &= \\ &[(5-2)(5-1) / (2 \cdot 5 - 3)] = 1.714\end{aligned}$$

Example: Undirected, Binary Network

$$C_C = \frac{\sum_{i=1}^g [C'_C(n^*) - C'_C(n_i)]}{[(g-2)(g-1)]/(2g-3)}$$

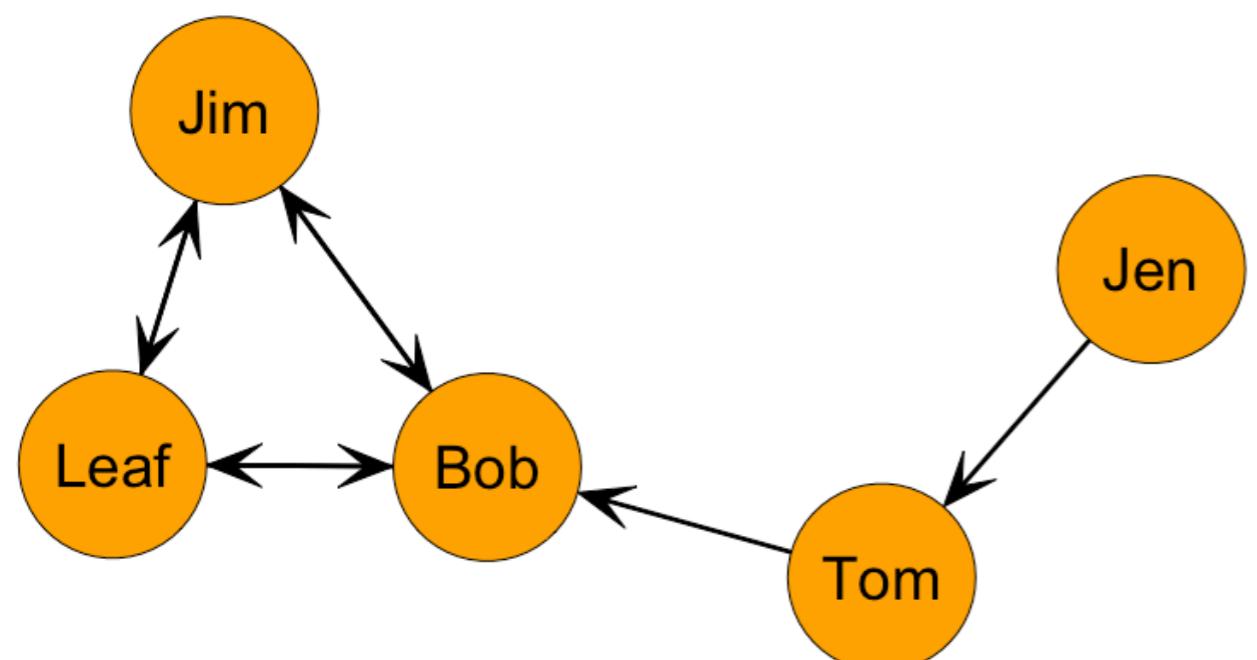
$$= \frac{0.356 + 0.132 + 0.000 + 0.228 + 0.228}{[(5-2)(5-1)]/(2*5-3)} = \frac{0.944}{1.714} = 0.551$$

Directed Networks

Closeness Centrality: Directed Binary Graphs

- ❖ Recall that in a directed network, the directionality matters.
 - ❖ As a result, we have to consider how this might influence our measures.

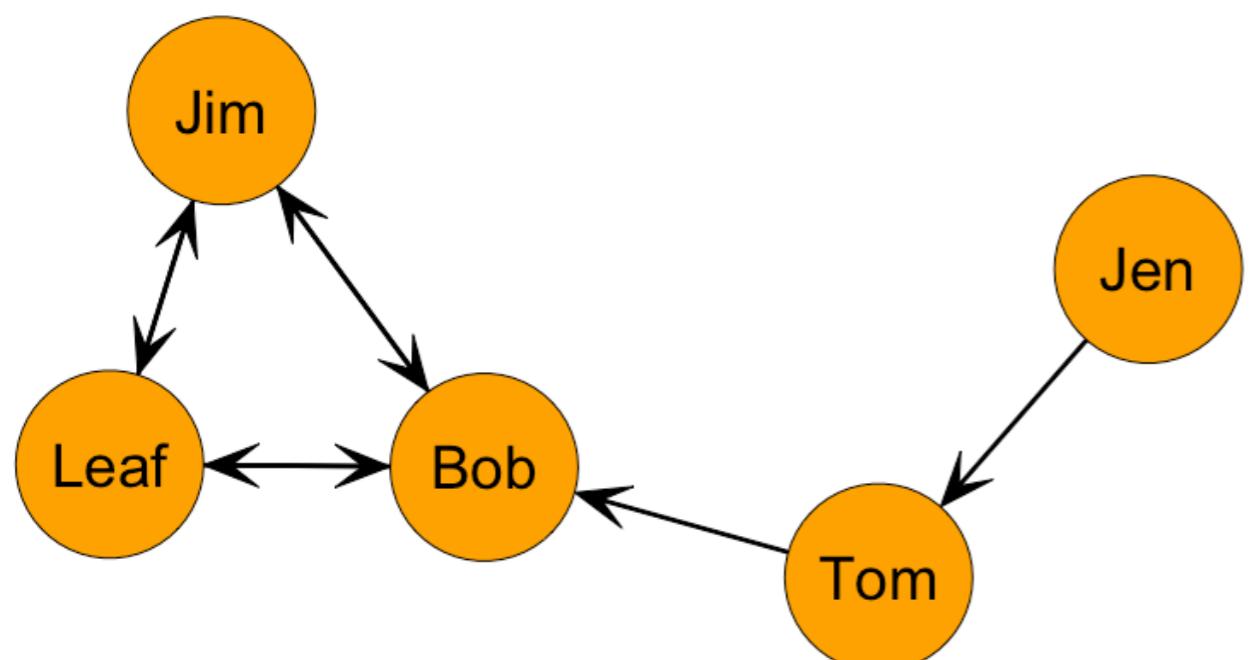
Example: Closeness Centrality for Directed Binary Network



How far is Jen from Tom? From Bob?

		Jen	Tom	Bob	Leaf	Jim
Jen						

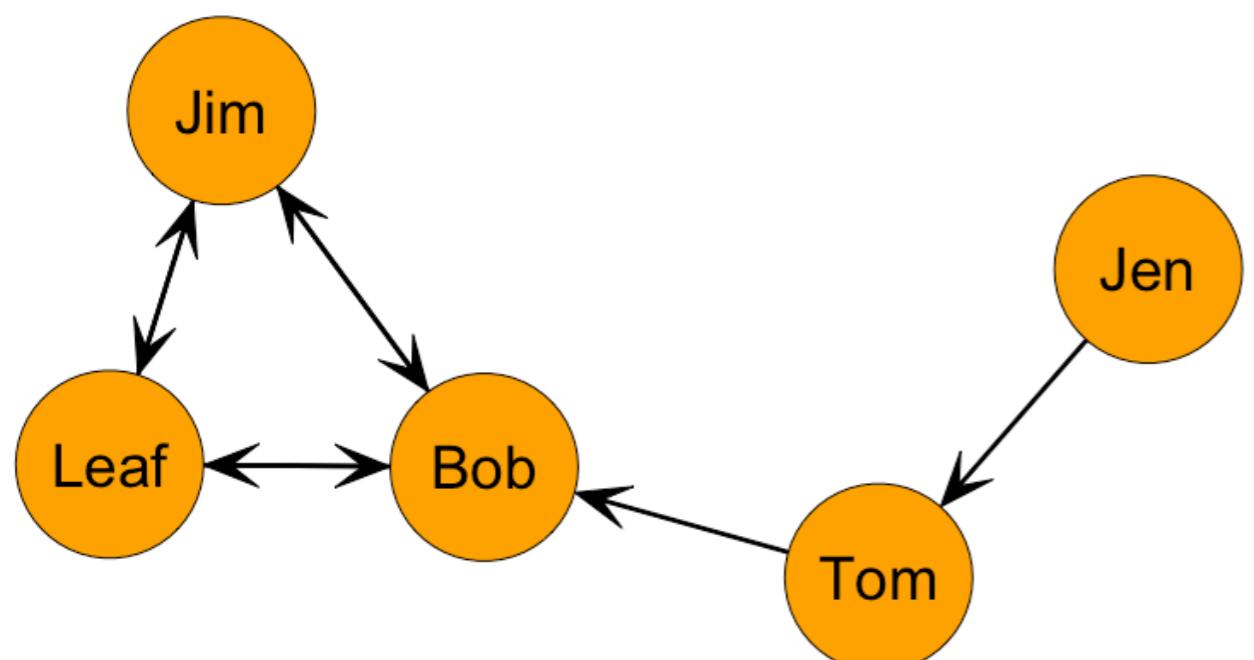
Example: Closeness Centrality for Directed Binary Network



How far is Jen from Tom? From Bob? Depends on direction.

		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob						
Leaf						
Jim						

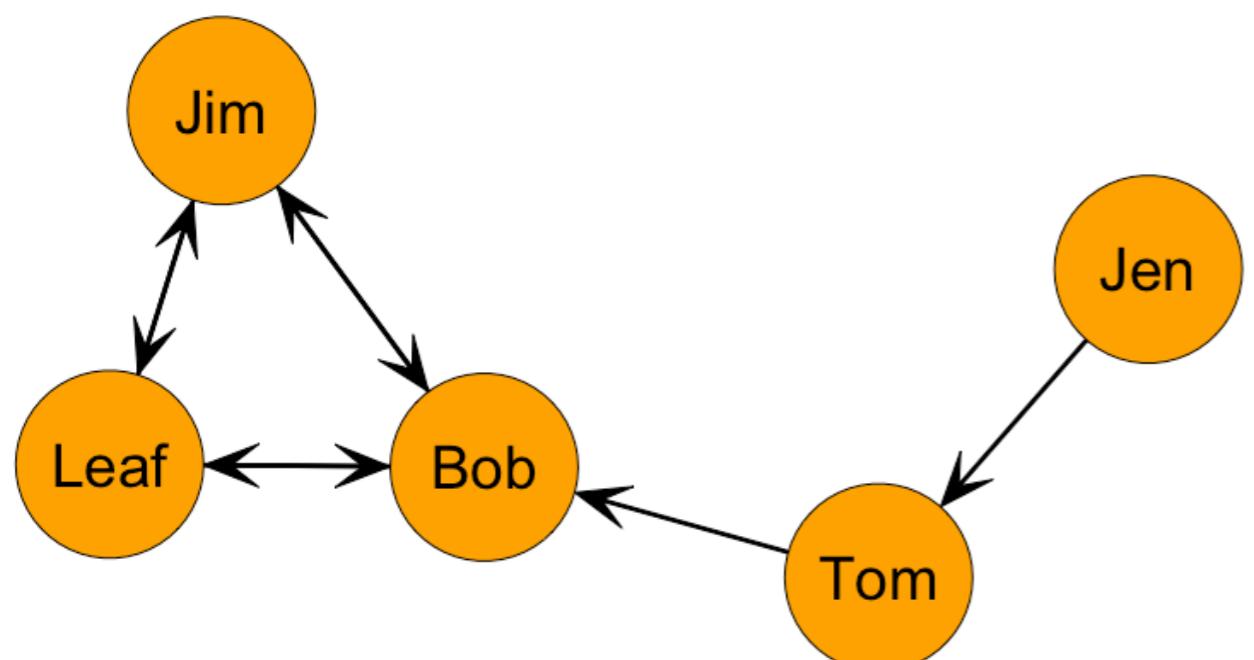
Example: Closeness Centrality for Directed Binary Network



Closeness centrality in directed graphs usually focuses on the send network (i.e. outgoing ties).

		Distance Matrix				
		Jen	Tom	Bob	Leaf	Jim
Jen	Jen					
	Tom					
	Bob					
	Leaf					
	Jim					

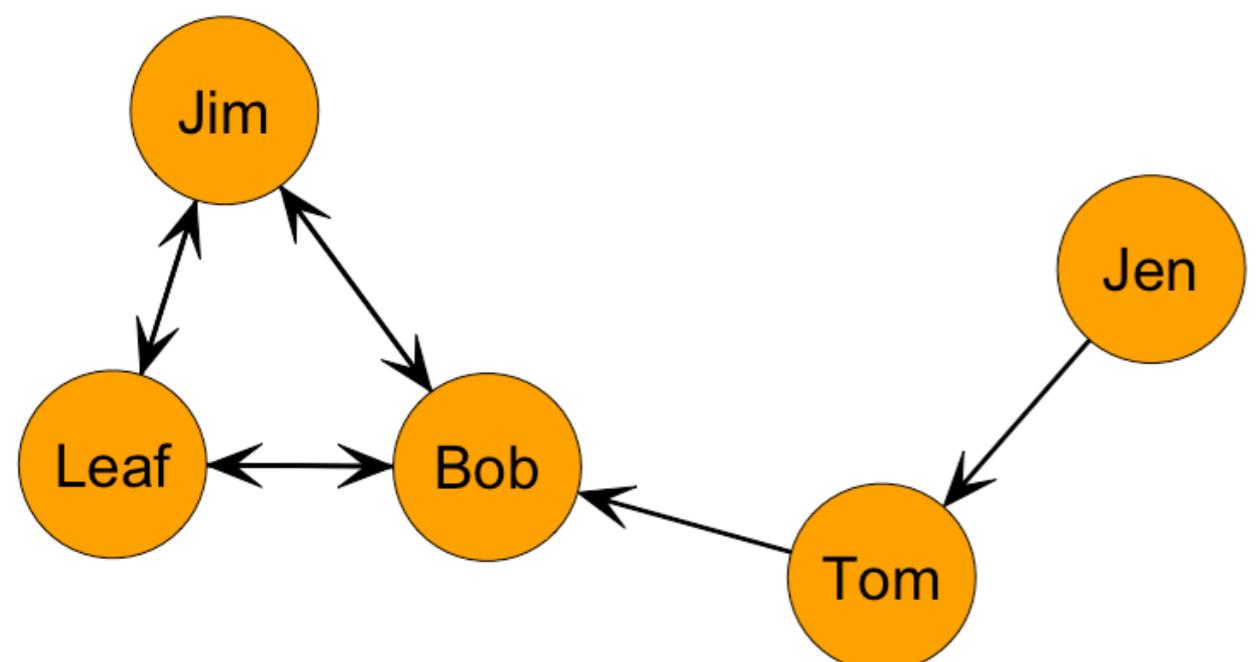
Example: Closeness Centrality for Directed Binary Network



Remember, row sums are the outdegree, so row distance tells you how close you are to others.

		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob						
Leaf						
Jim						

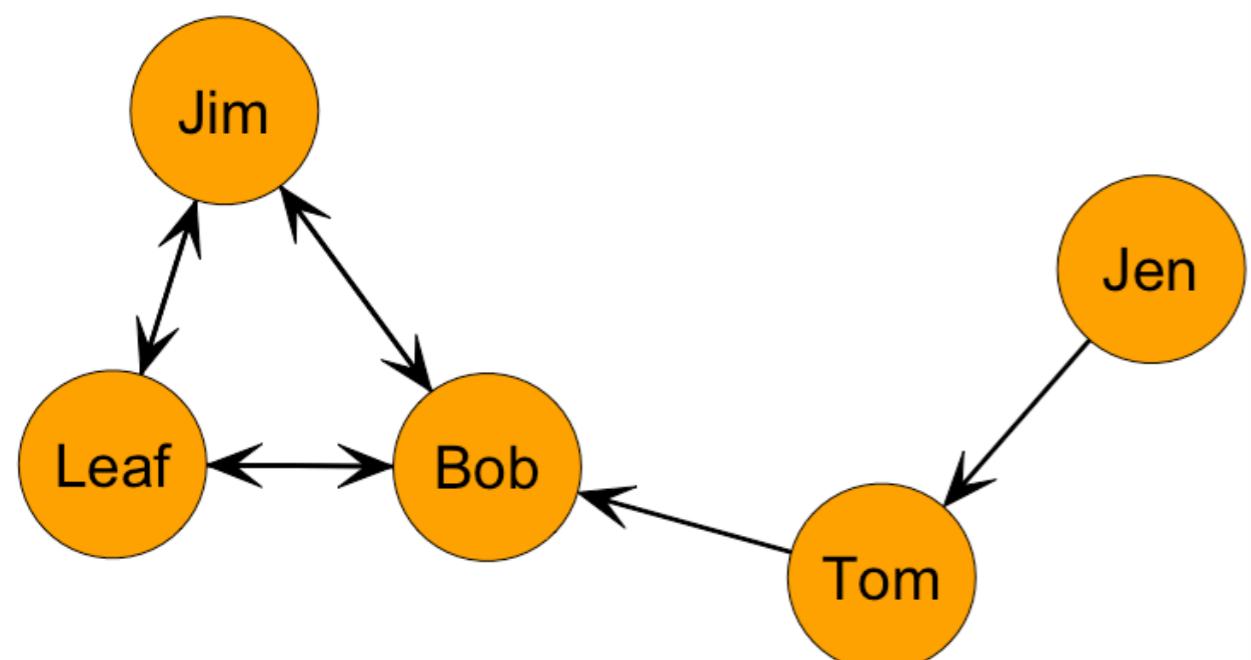
Example: Closeness Centrality for Directed Binary Network



How far is Jen from Tom? From Bob?

		Jen	Tom	Bob	Leaf	Jim
Jen		?	?			
Tom						
Bob						
Leaf						
Jim						

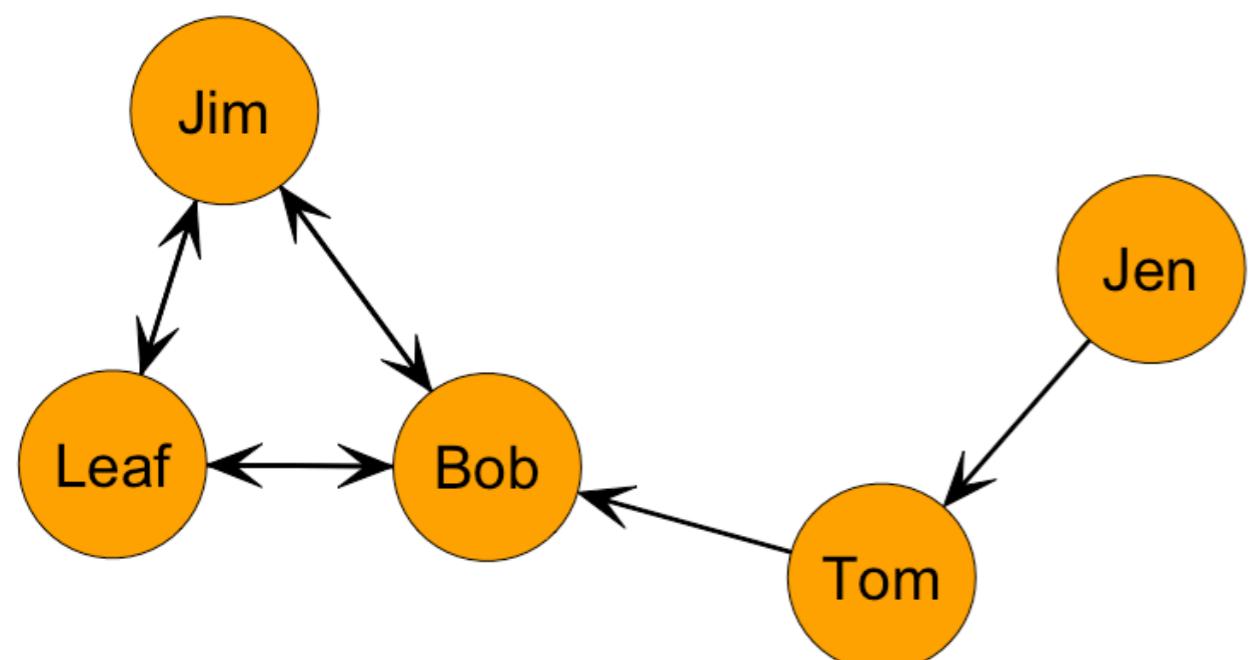
Example: Closeness Centrality for Directed Binary Network



How far is Jen from Tom? From Bob?

		Jen	Tom	Bob	Leaf	Jim
Jen		1	2			
Tom						
Bob						
Leaf						
Jim						

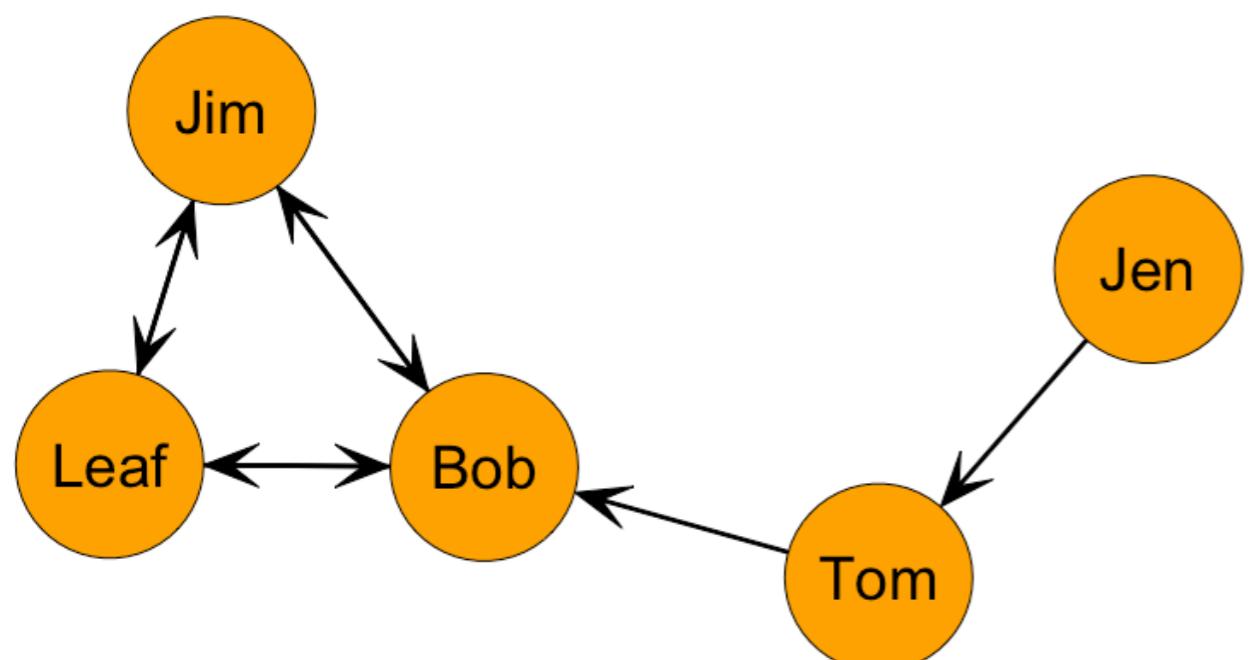
Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	?	?	
Tom						
Bob						
Leaf						
Jim						

What about the rest?

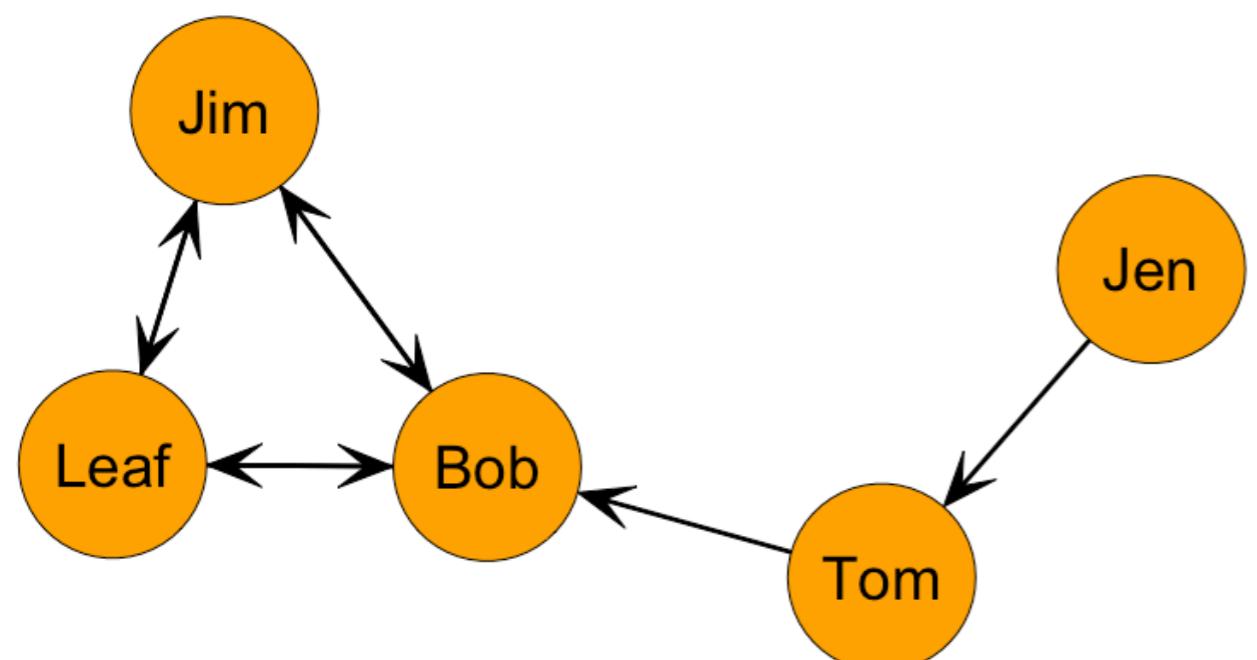
Example: Closeness Centrality for Directed Binary Network



What is Jen's closeness in the graph?

		Jen	Tom	Bob	Leaf	Jim
Jen		1	2	3	3	
Tom						
Bob						
Leaf						
Jim						

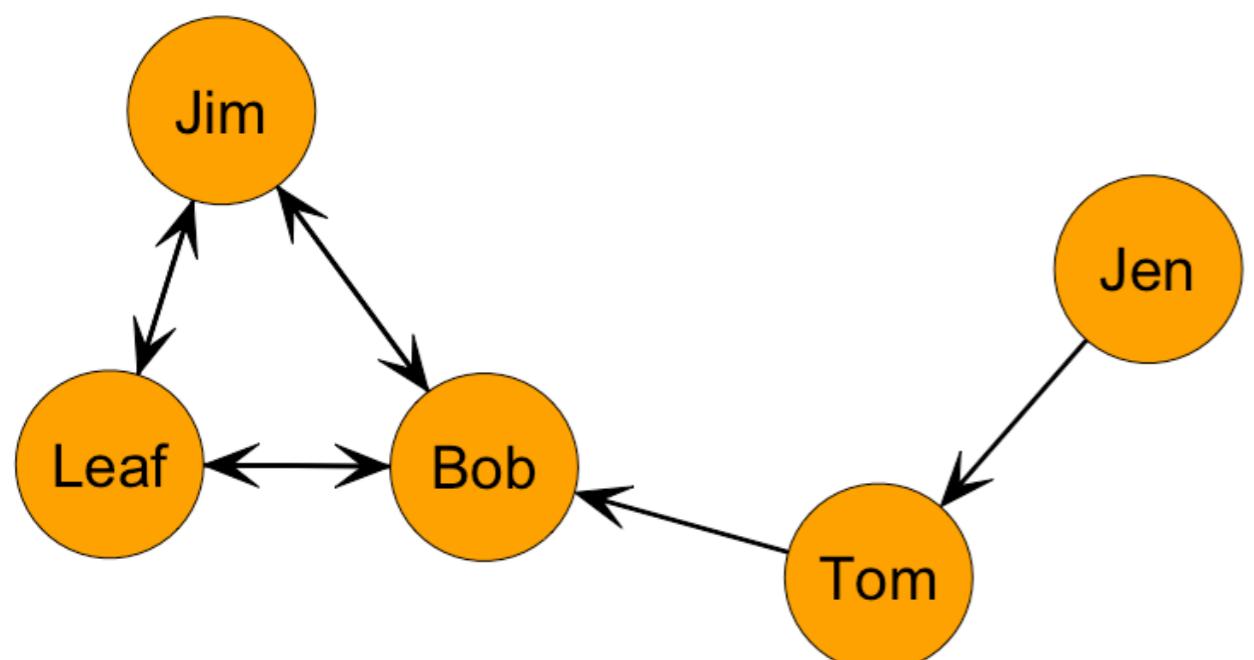
Example: Closeness Centrality for Directed Binary Network



What is Jen's closeness in the graph?

		Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9	
Tom							
Bob							
Leaf							
Jim							

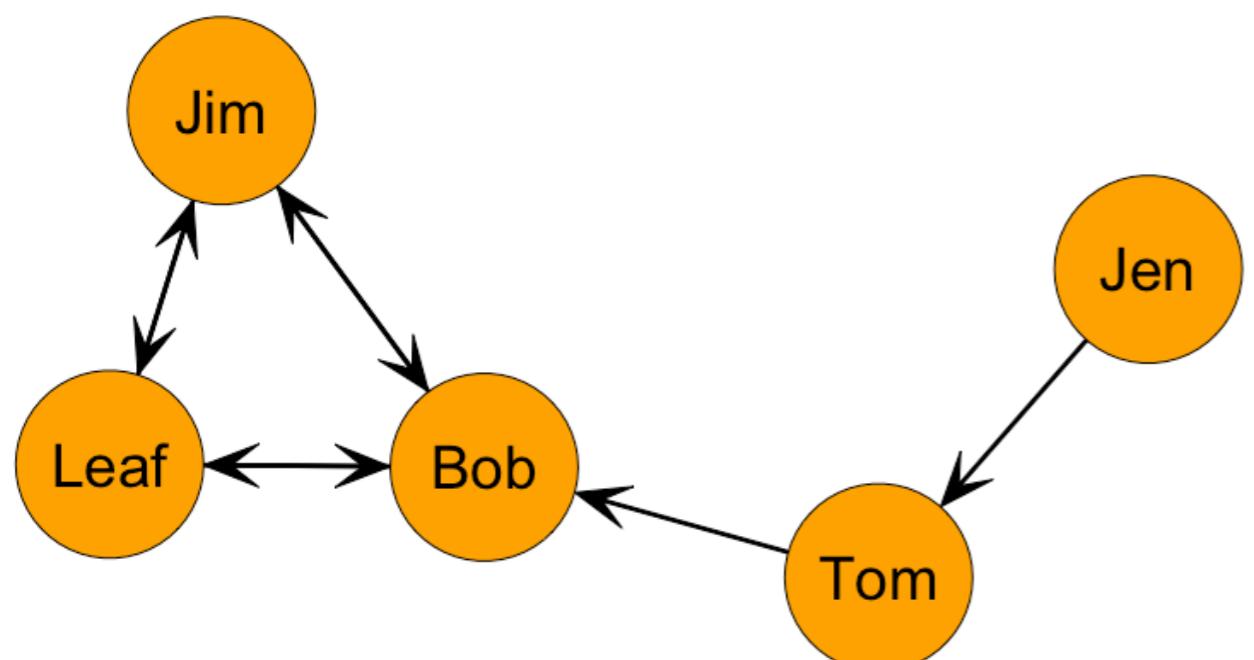
Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9	
Tom							
Bob							
Leaf							
Jim							

*Just take the inverse, which is $1/9$
= 0.111.*

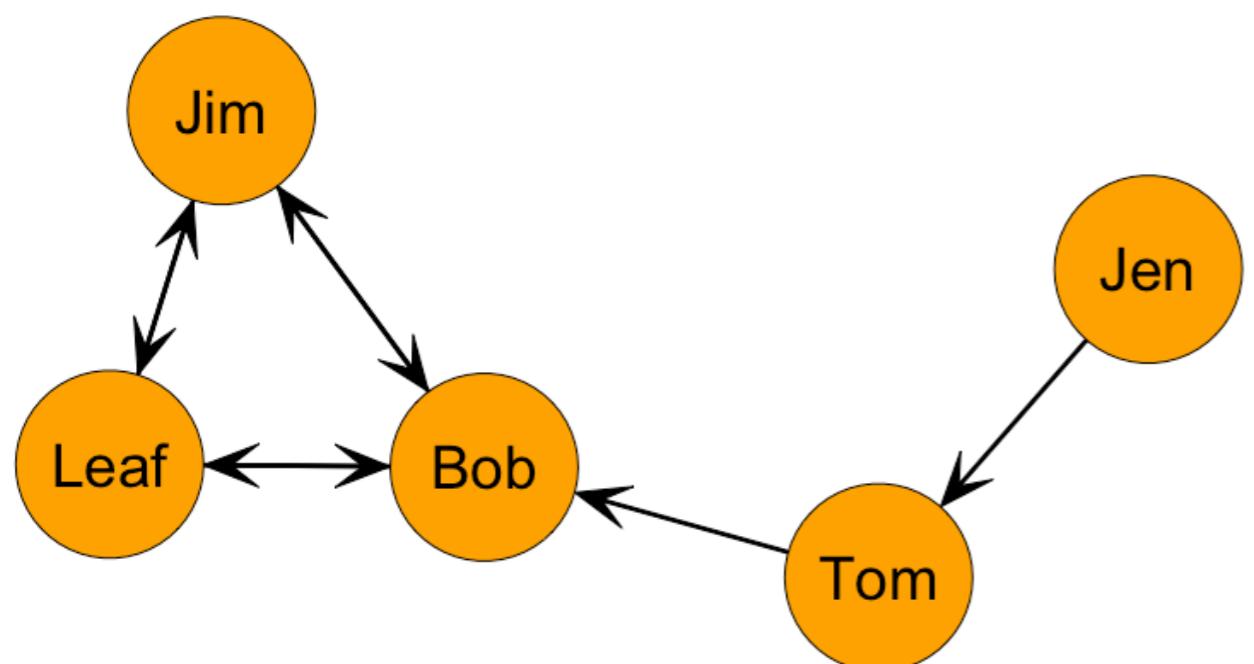
Example: Closeness Centrality for Directed Binary Network



	Jen	Tom	Bob	Leaf	Jim	Sum
Jen		1	2	3	3	9
Tom						
Bob						
Leaf						
Jim						

Then, Jen's standardized score is
 $(1/9) * (g-1) = 0.111 * 4 = 0.444$

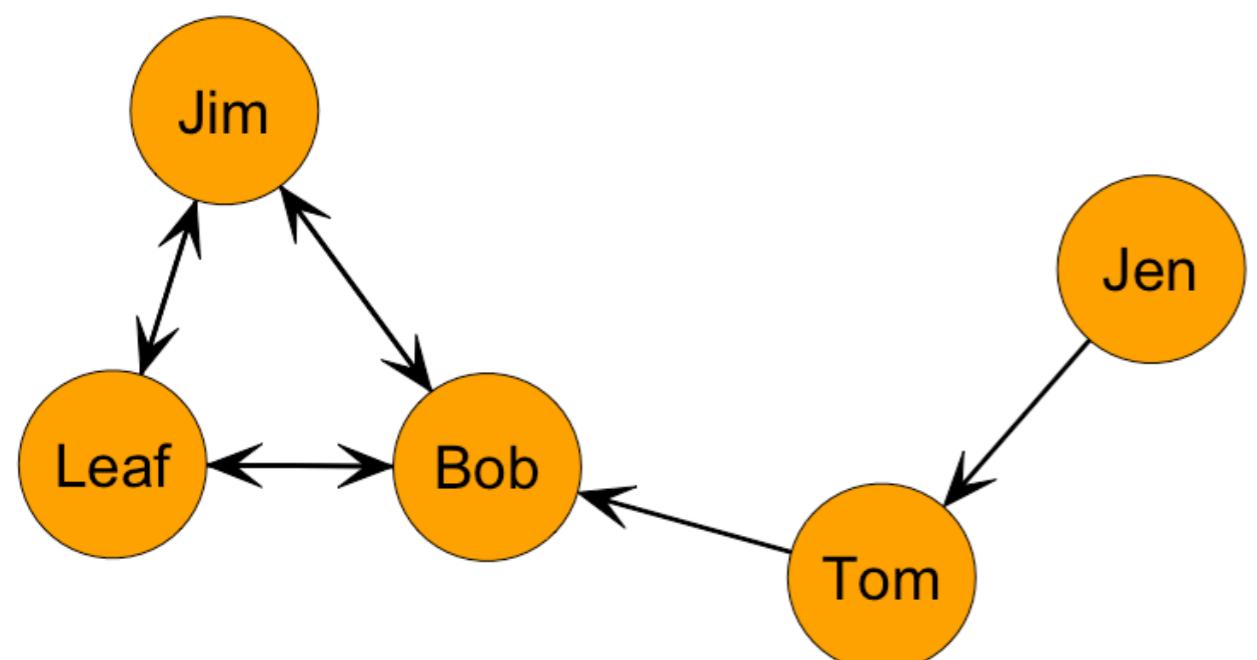
Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob	?	?			?	?
Leaf						
Jim						

What about Bob? How far is Bob from everyone?

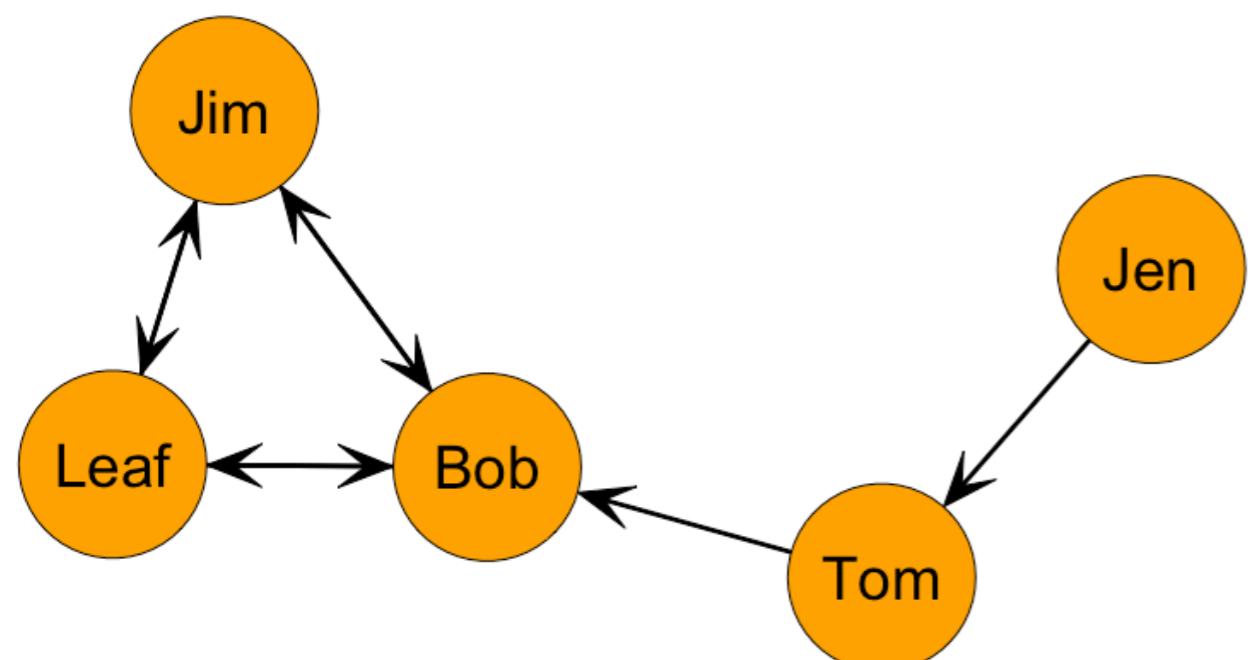
Example: Closeness Centrality for Directed Binary Network



		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob	Inf	Inf			?	?
Leaf						
Jim						

Why is Bob's distance to Tom infinite?

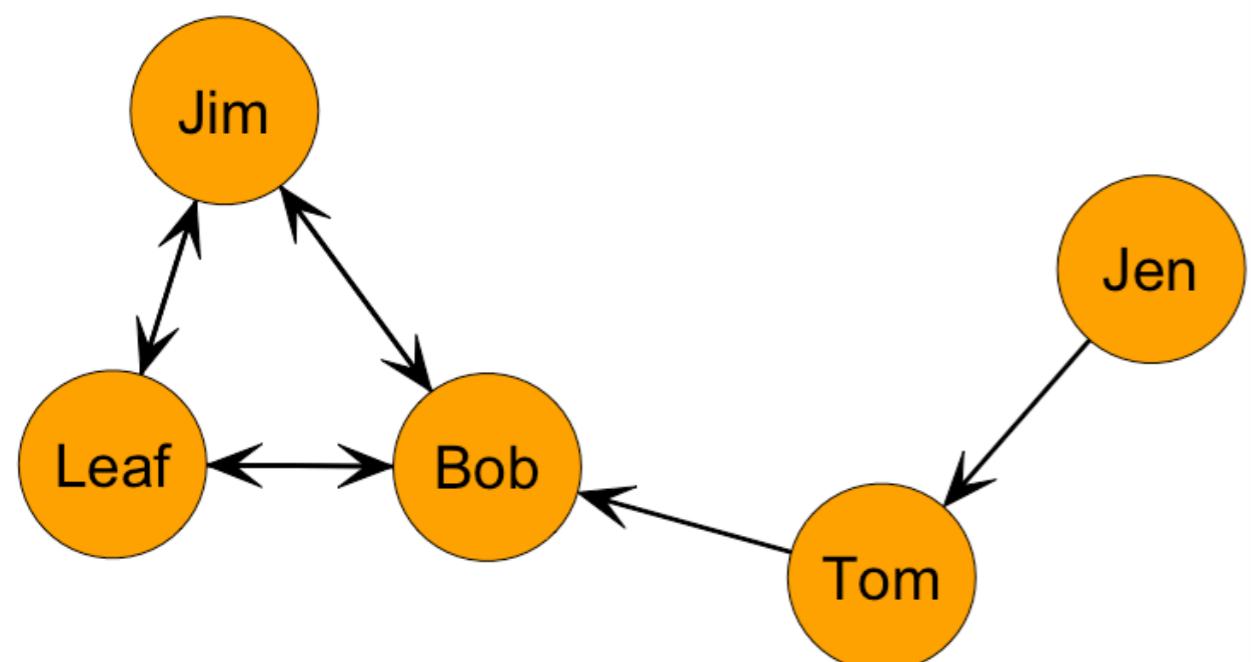
Example: Closeness Centrality for Directed Binary Network



What is Bob's closeness?

		Jen	Tom	Bob	Leaf	Jim	Sum
Jen							
Tom							
Bob	Inf	Inf			1	1	2
Leaf							
Jim							

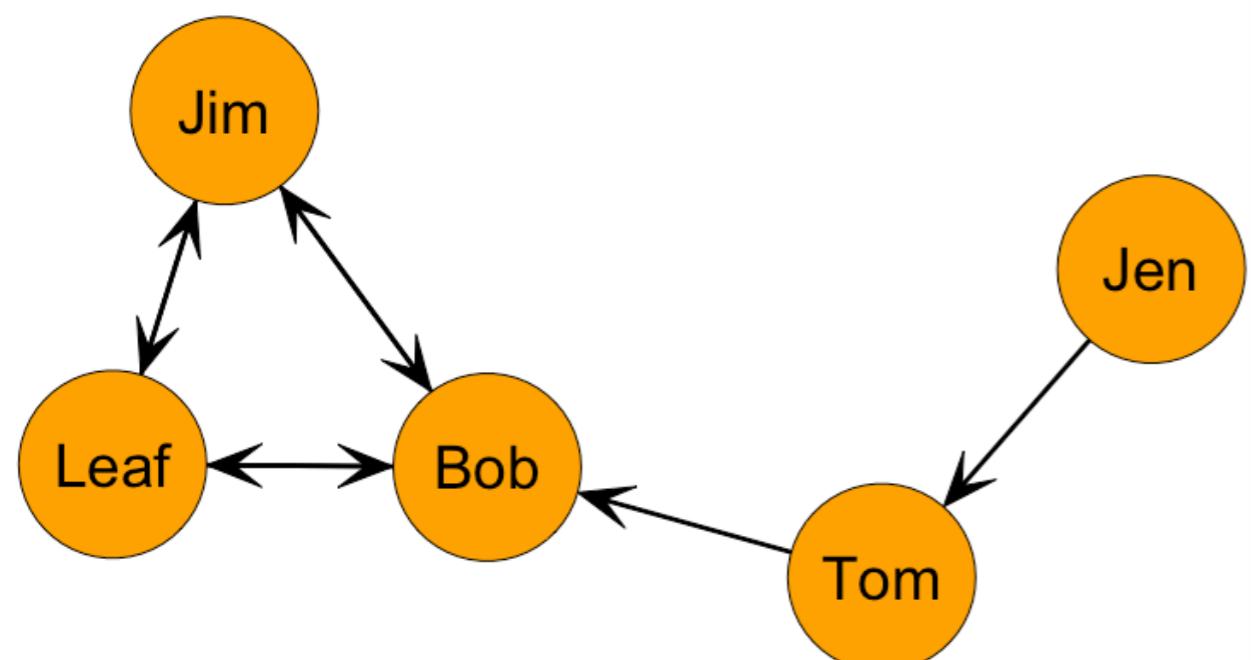
Example: Closeness Centrality for Directed Binary Network



*Take the inverse and Bob's score
is: $1/2 = 0.5$*

		Jen	Tom	Bob	Leaf	Jim	Sum
Jen							
Tom							
Bob		Inf	Inf			1	1
Leaf							
Jim							

Example: Closeness Centrality for Directed Binary Network

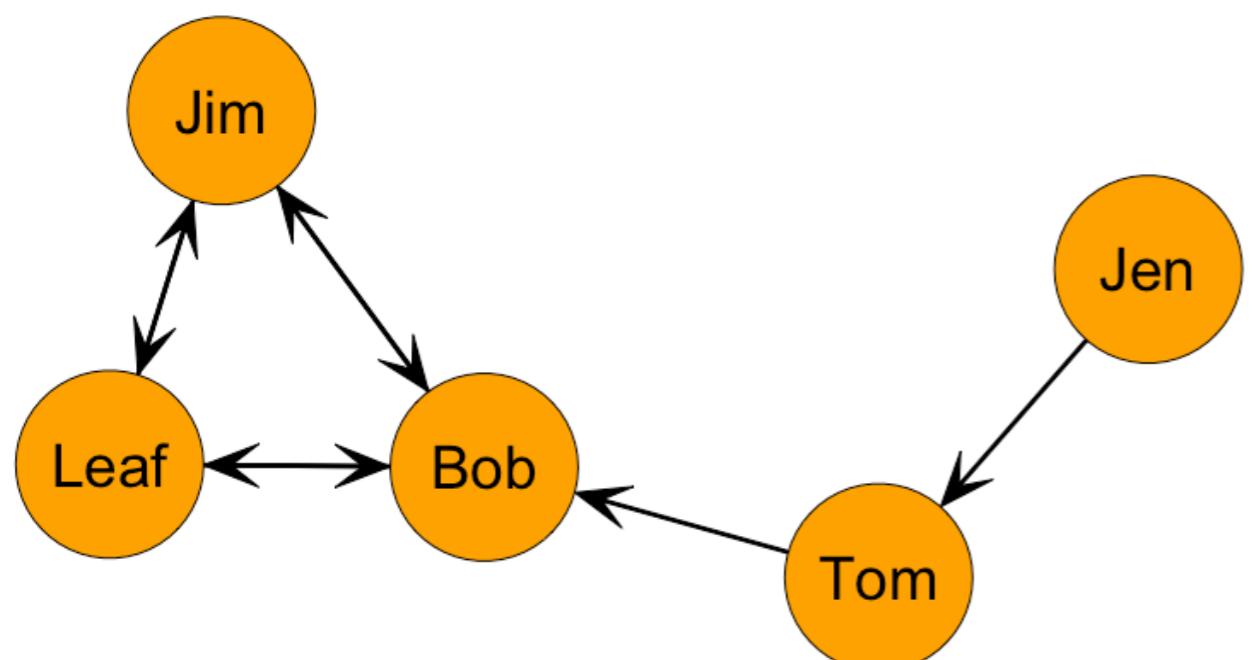


		Jen	Tom	Bob	Leaf	Jim	Sum
Jen							
Tom							
Bob		Inf	Inf		1	1	2
Leaf							
Jim							

Then, Bob's standardized score is

$$(1/2)^*(g-1) = 0.5^*4 = 0.2$$

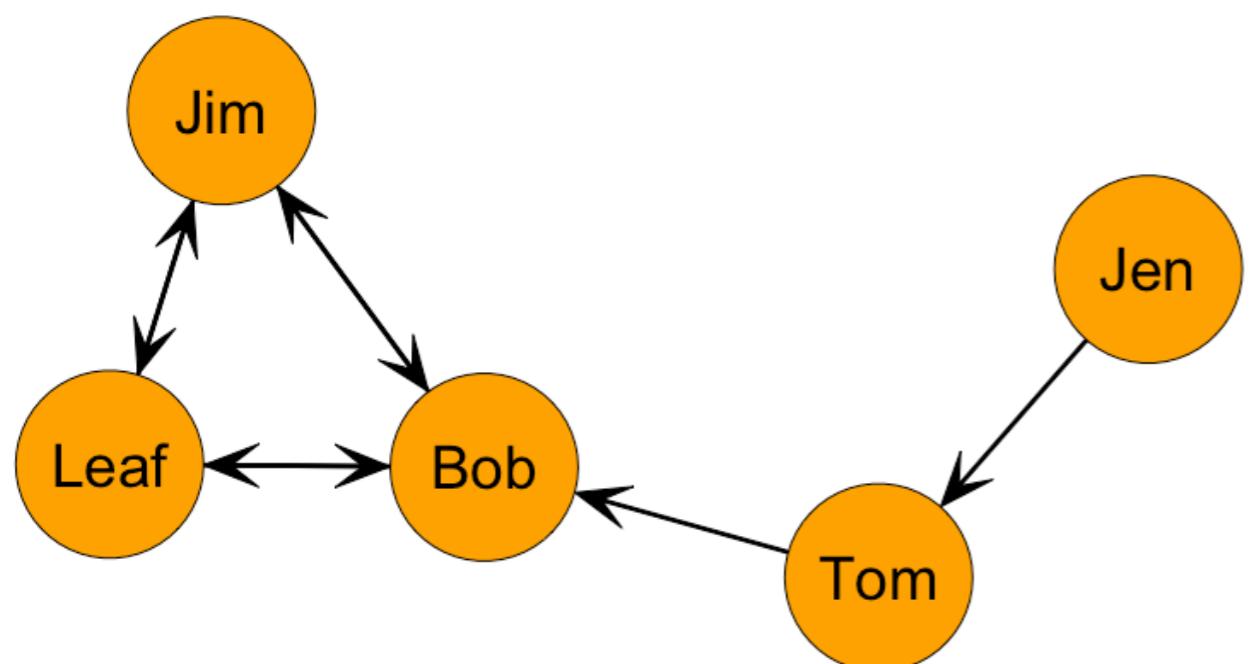
Example: Closeness Centrality for Directed Binary Network



Closeness centrality in directed graphs could focus on the receive network (i.e. incoming ties).

		Distance Matrix				
		Jen	Tom	Bob	Leaf	Jim
Jen	Jen					
	Tom					
	Bob					
	Leaf					
	Jim					

Example: Closeness Centrality for Directed Binary Network



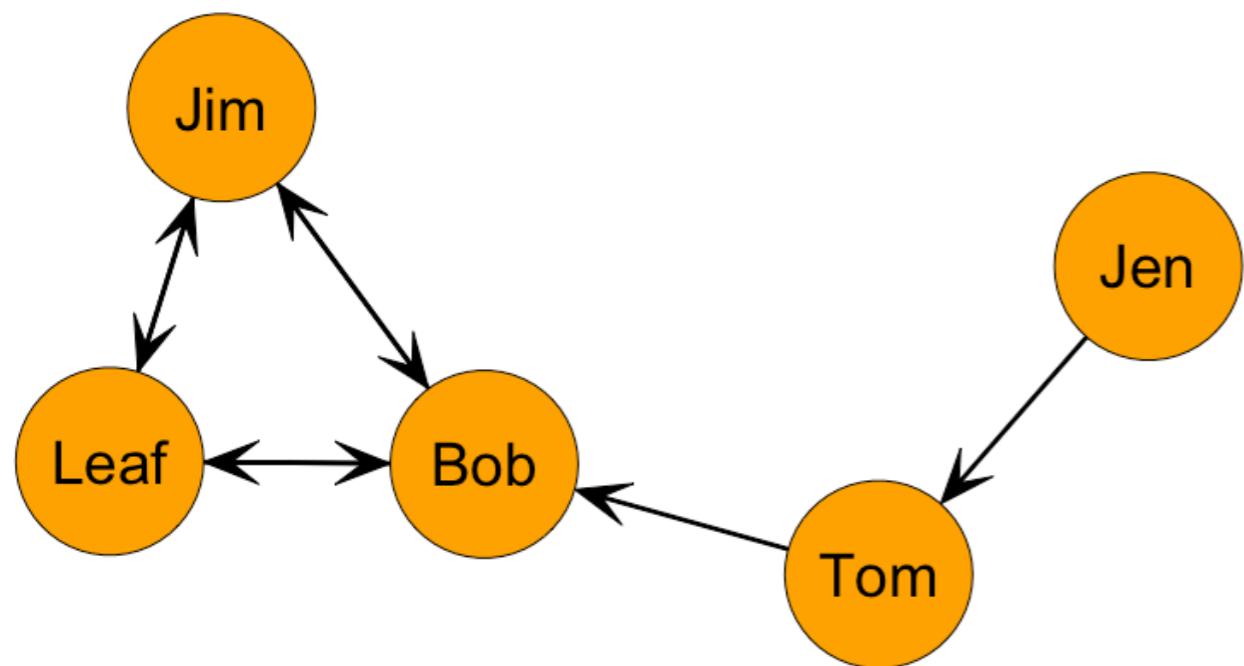
		Jen	Tom	Bob	Leaf	Jim
Jen						
Tom						
Bob						
Leaf						
Jim						

Remember, column sums are the indegree, so column distance tells you how close others are to you.

Group Closeness Centralization: Directed Binary Graphs

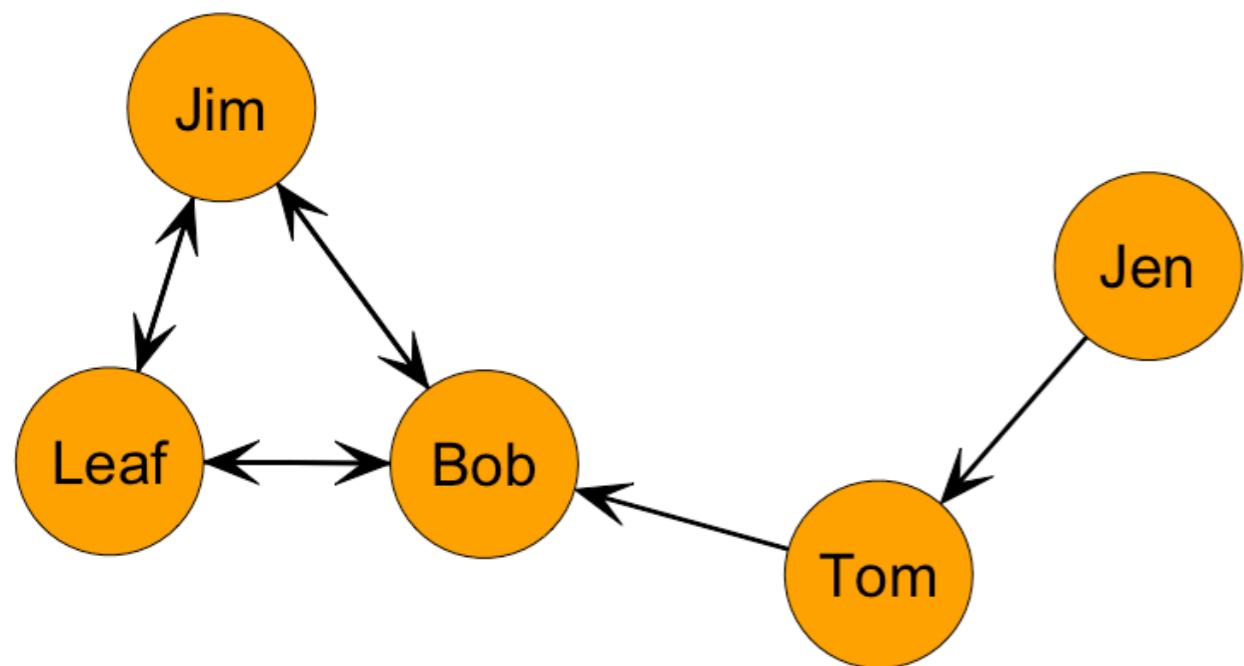
- ❖ The denominator for group closeness centralization is not defined for directed graphs (see Wasserman & Faust 1994: p. 200).
- ❖ But, we could just treat it as undirected and calculate a score.

Example: Undirected, Binary Network



Approximately 0.556

Example: Undirected, Binary Network



Compare the centralization scores:

Indegree = 0.438

Outdegree = 0.125

Closeness = 0.556

What can we say about the differences in the centralization scores for each type of centrality?

Betweenness Centrality

- ❖ We have seen how centrality can be conceptualized as:
 - ❖ Having a high number of ties
 - ❖ Being close to others in the network
 - ❖ We can also conceptualize centrality as a node that lies on a particular path between other nodes.
 - ❖ *Betweenness centrality* is based on the number of **shortest** paths between j and k that actor i resides on.

Betweenness Centrality

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

Betweenness Centrality

The number of
geodesics linking j
to k .

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / \downarrow g_{jk}$$

Betweenness Centrality

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / \downarrow g_{jk}$$

The number of geodesics linking j to k .

The number of geodesics linking j and k that contain i .

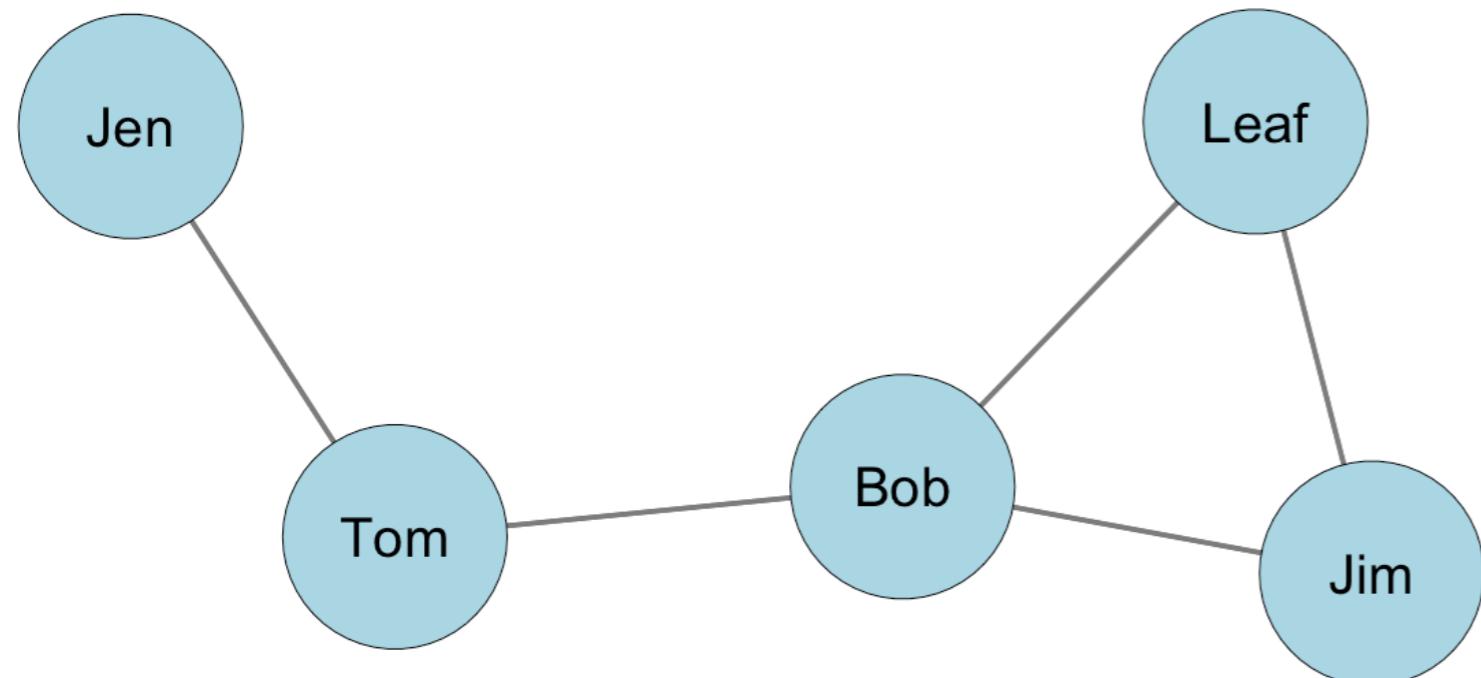
Betweenness Centrality

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

So, betweenness centrality is the ratio of the geodesics between j and k that contain i .

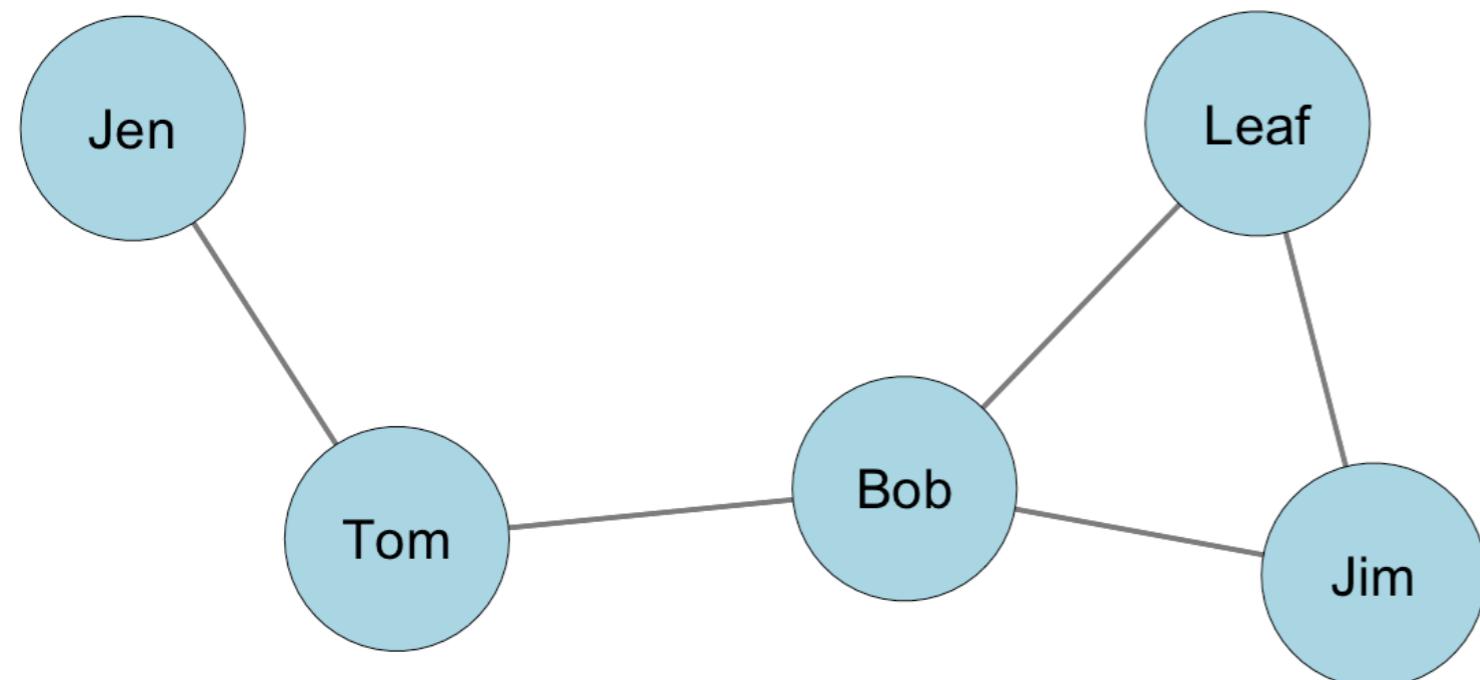
Undirected Networks

Example: Undirected, Binary Network



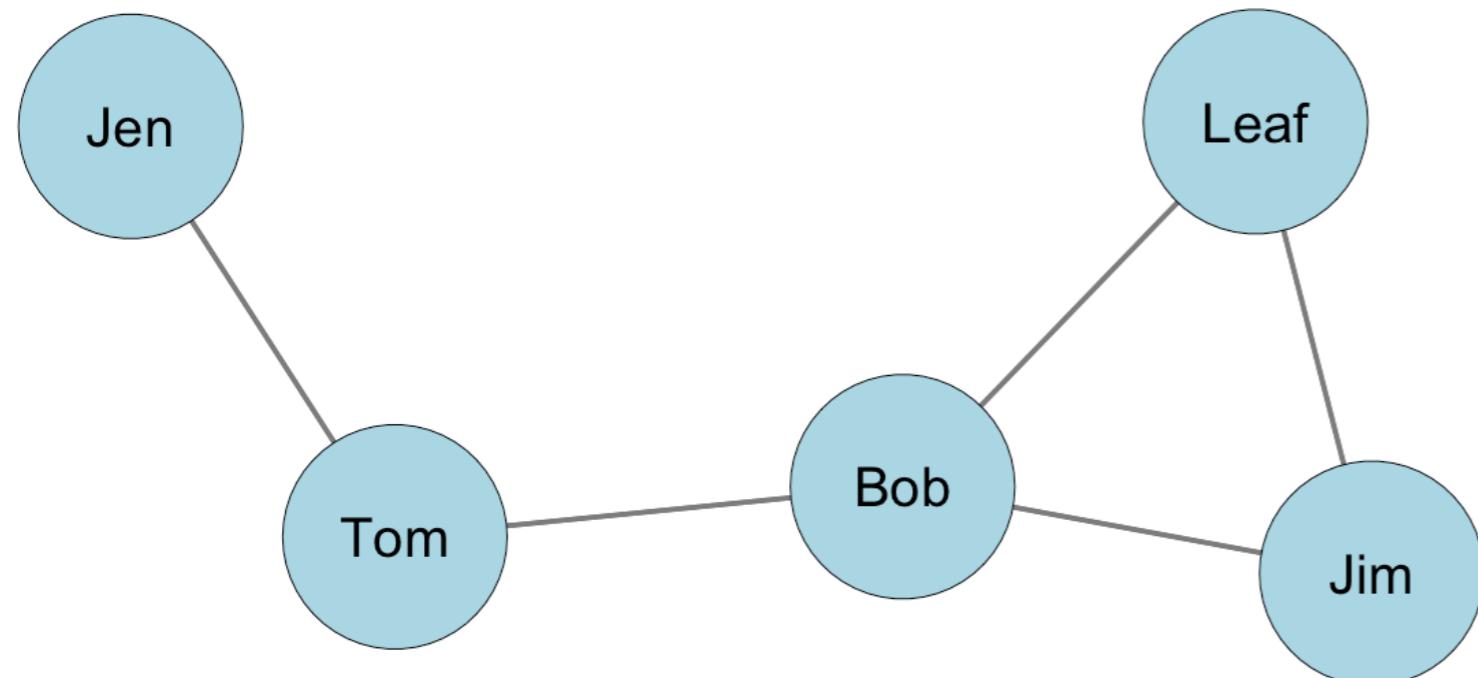
What are the paths
between Jen and Jim?

Example: Undirected, Binary Network



Let's calculate the betweenness centrality for **Bob**.

Example: Undirected, Binary Network



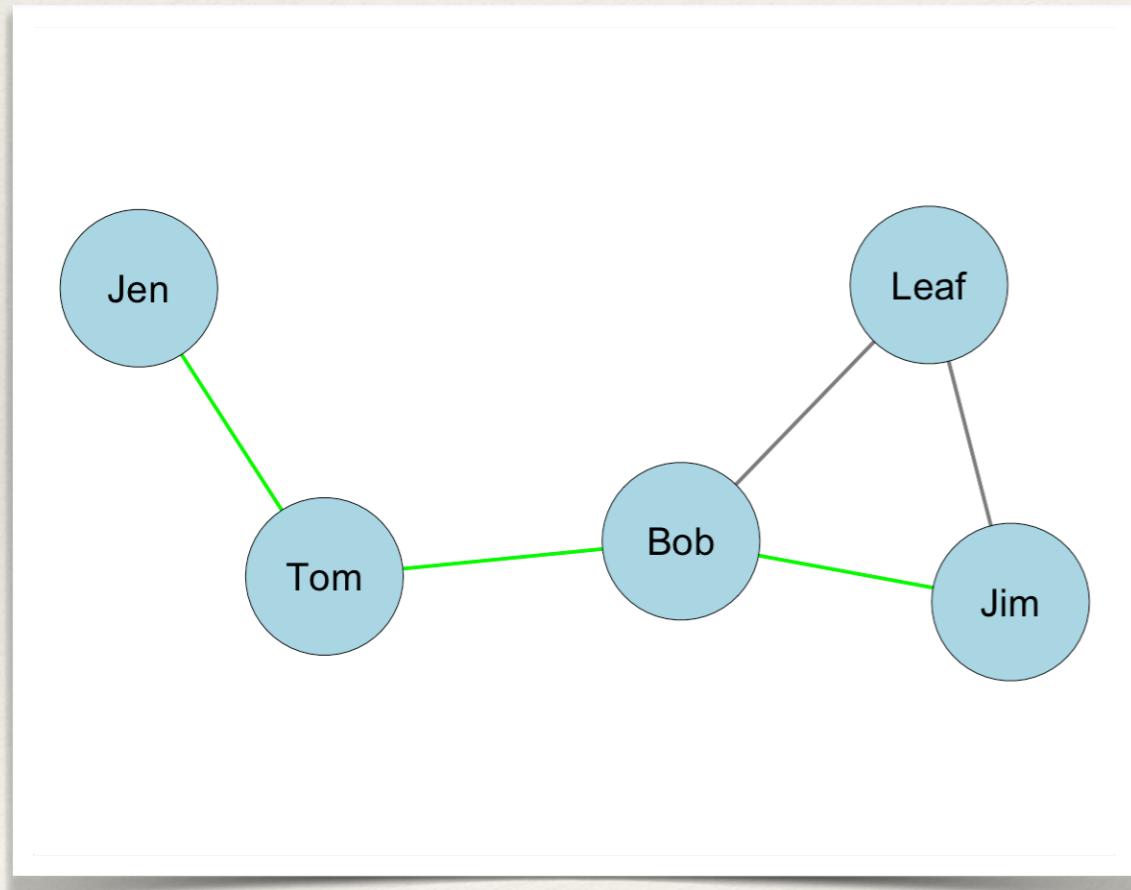
Let's calculate the betweenness centrality for **Bob**.

First, we need to find the geodesics for other nodes.

Then, figure out how many **Bob** occupies.

Example: Undirected, Binary Network

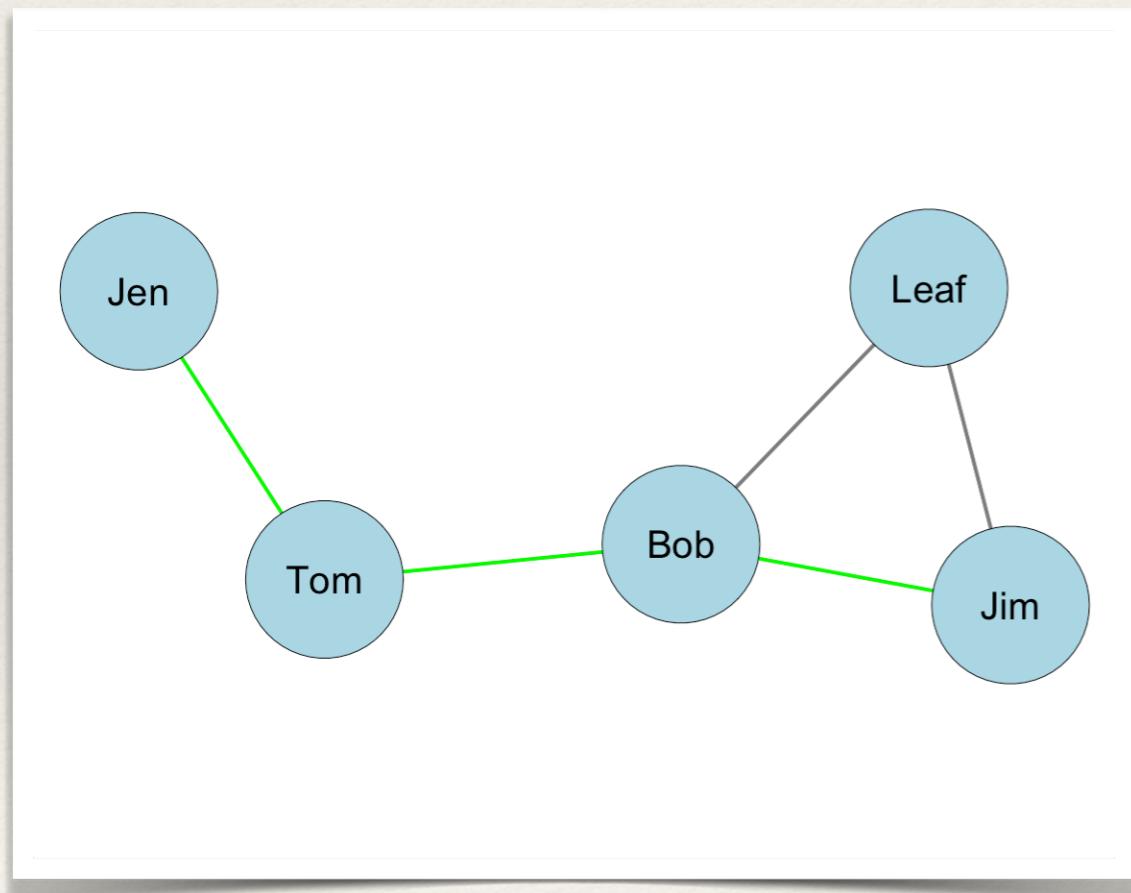
What are the paths between Jen and Jim?



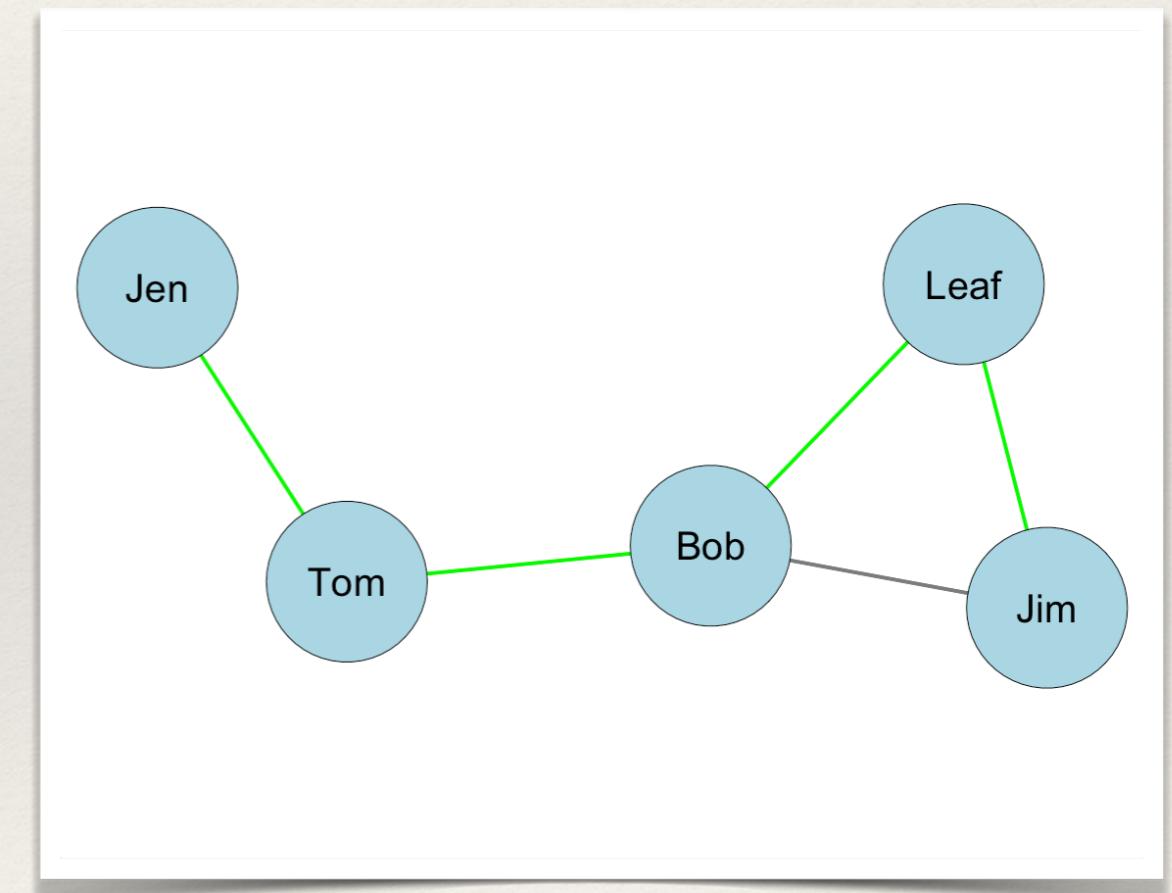
Jen-Tom-Bob-Jim

Example: Undirected, Binary Network

What are the paths between Jen and Jim?

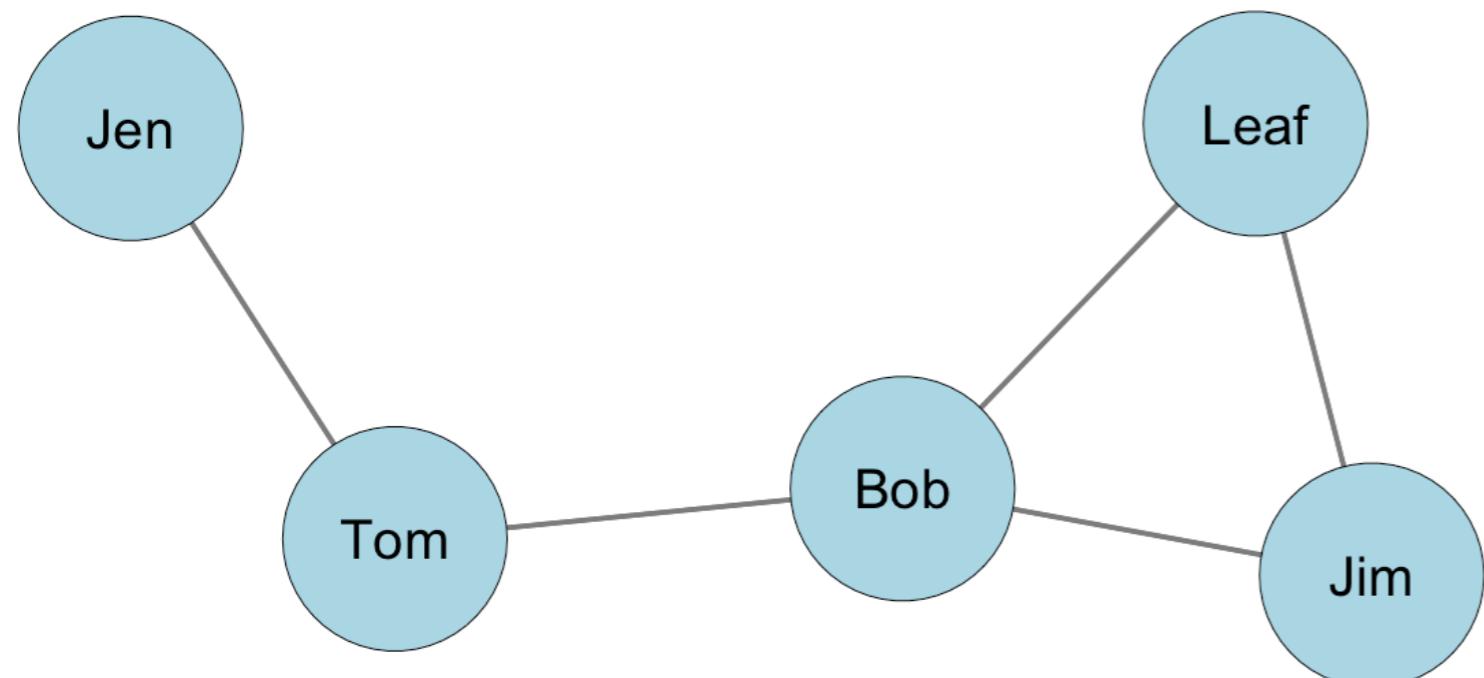


Jen-Tom-Bob-Jim



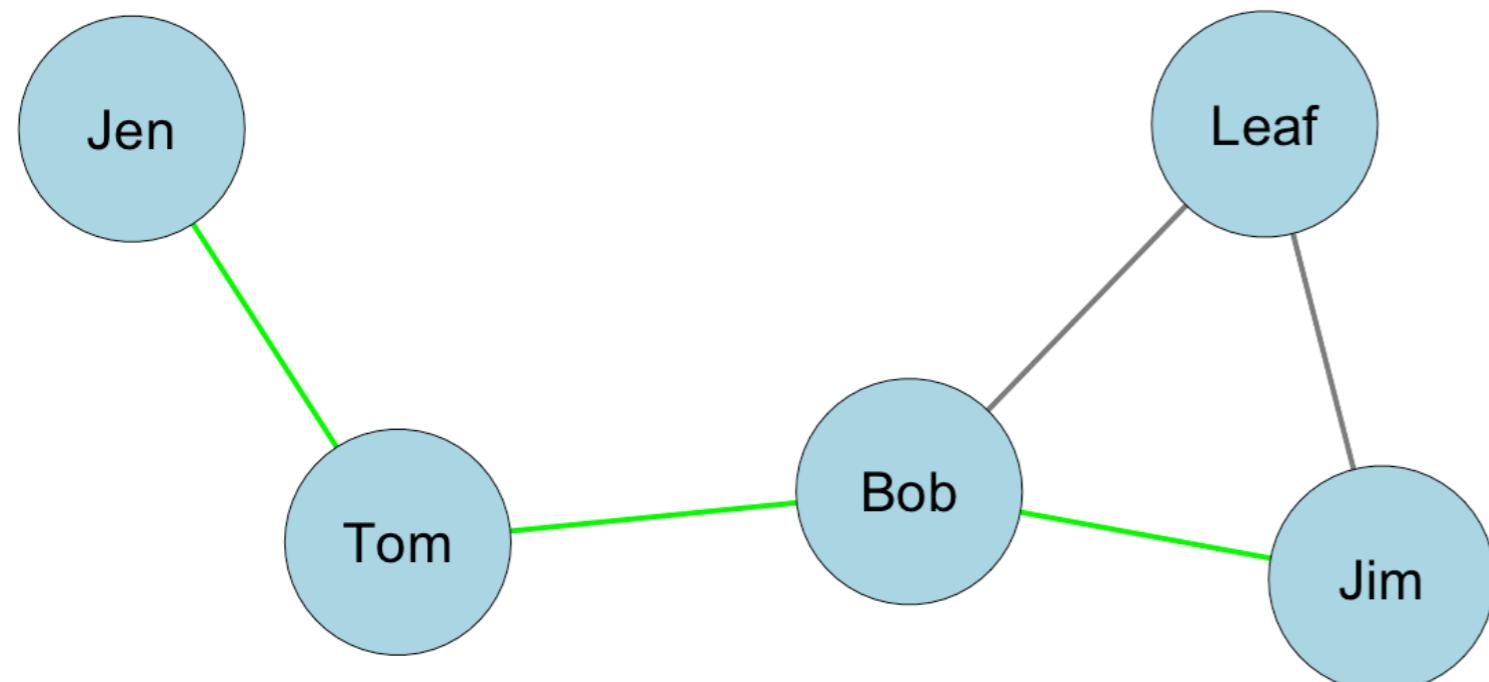
Jen-Tom-Bob-Leaf-Jim

Example: Undirected, Binary Network



What are the geodesic paths between Jen and Jim?

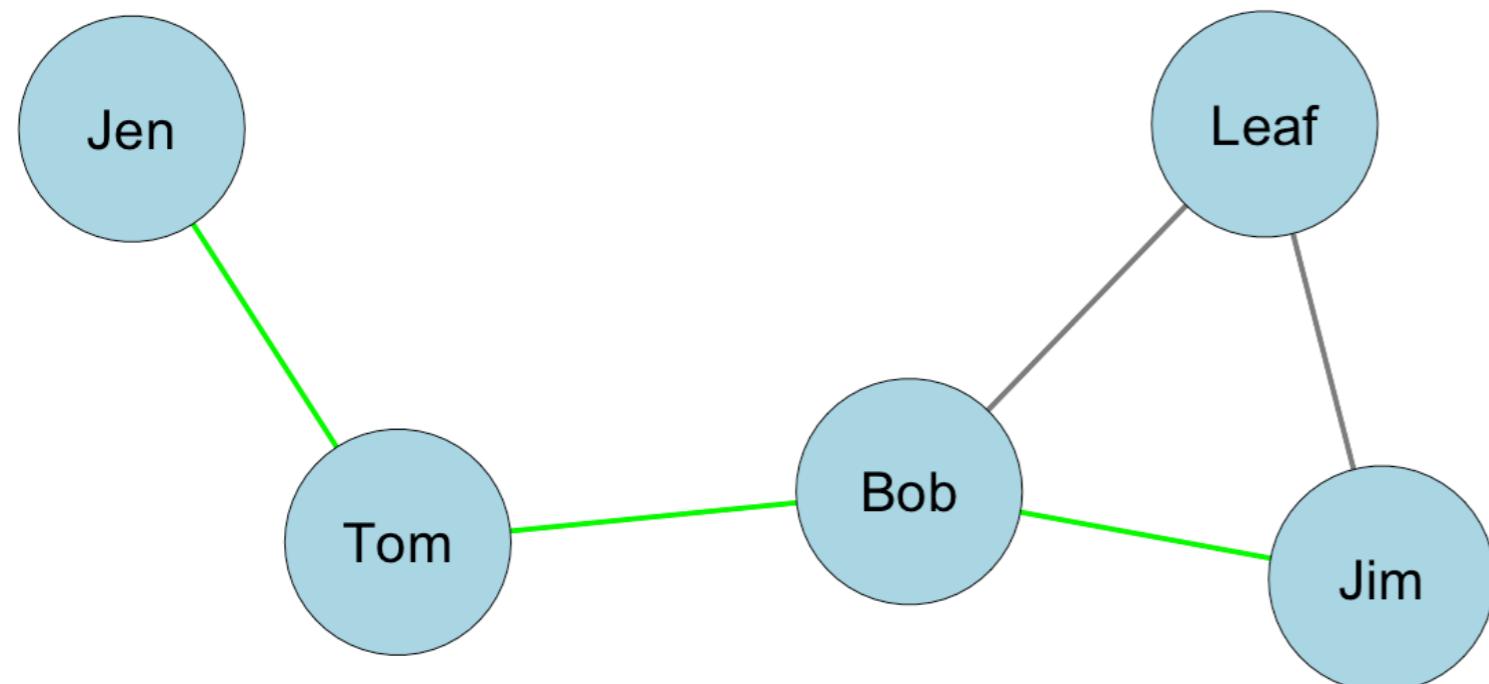
Example: Undirected, Binary Network



What are the geodesic paths between Jen and Jim?

Jen-Tom-Bob-Jim

Example: Undirected, Binary Network

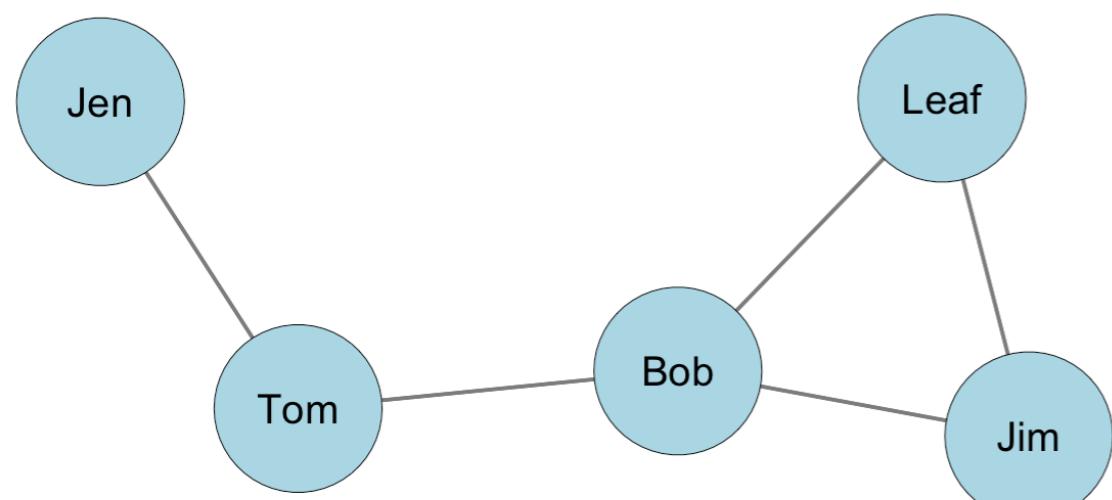


What are the geodesic paths between Jen and Jim?

Jen-Tom-Bob-Jim

Is there anyone on the geodesic between Jen and Jim?

Example: Undirected, Binary Network

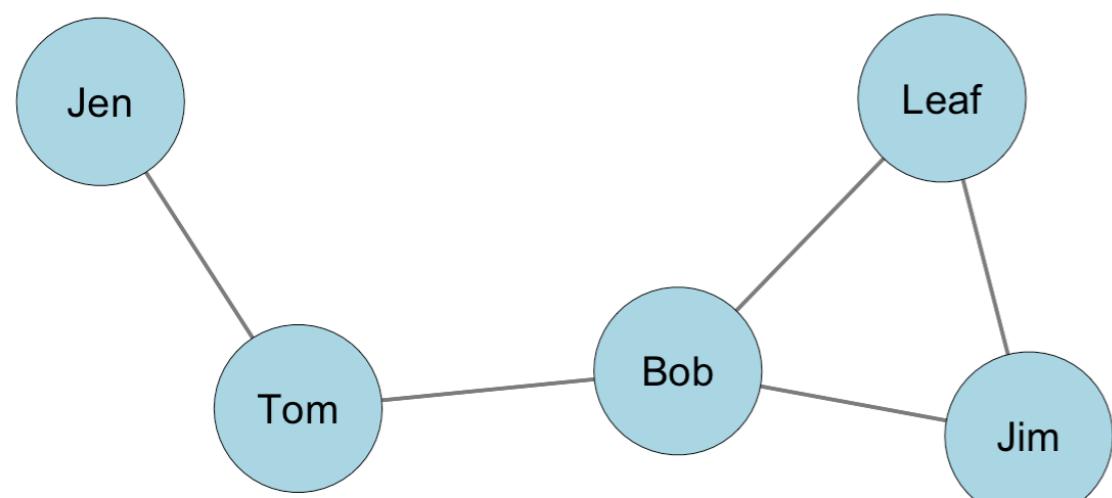


How many geodesics
from Jen to Jim?

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen				
				/?

Example: Undirected, Binary Network

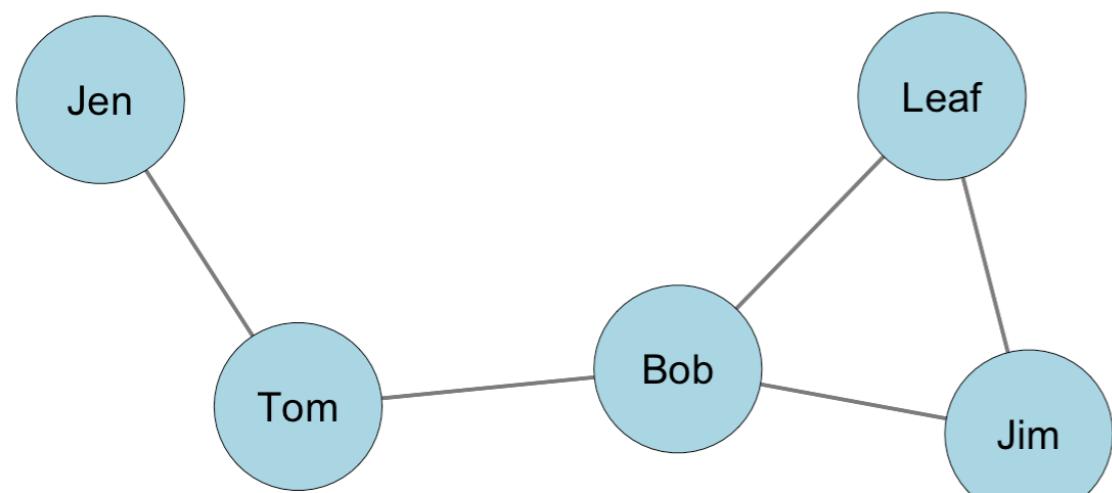


There is 1 geodesic
from **Jen** to **Jim**

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

Example: Undirected, Binary Network

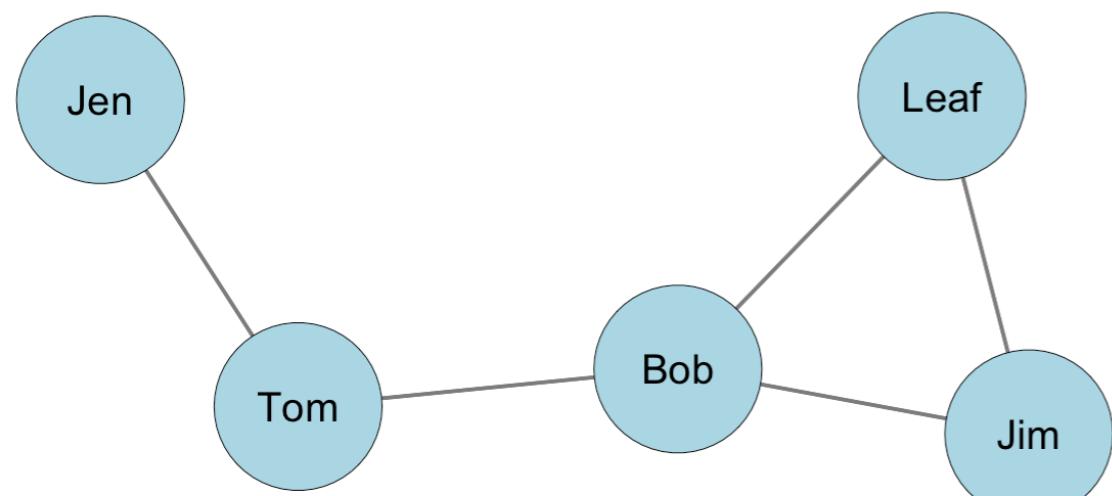


How many geodesics
from **Jen** to **Jim**
include **Bob**?

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				?
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

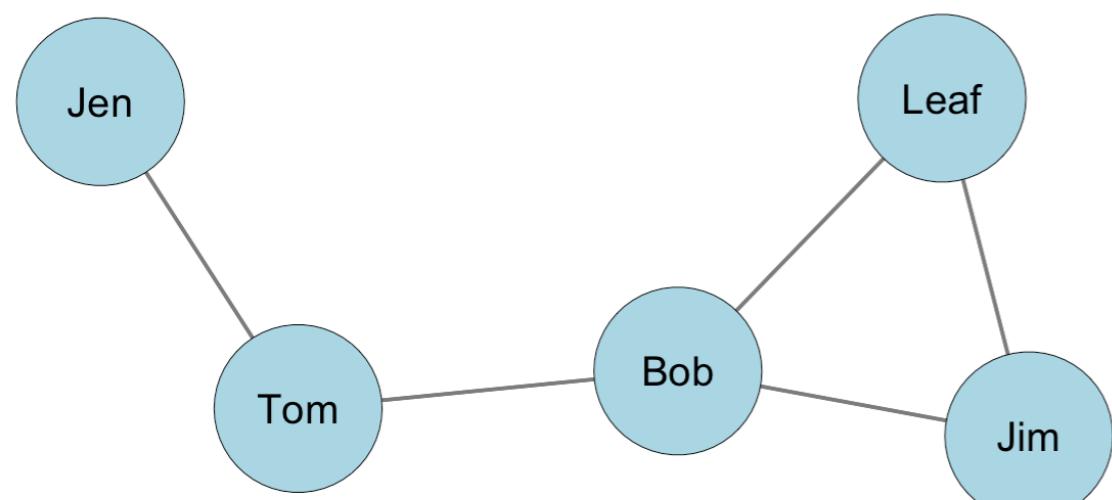
Example: Undirected, Binary Network



Bob is on the
only geodesic
from **Jen** to **Jim**

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				1/1

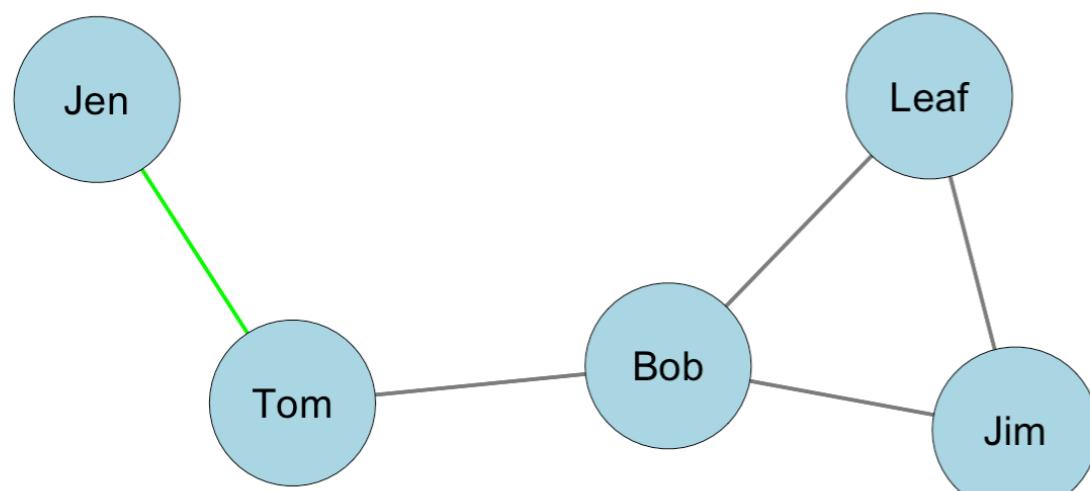
Example: Undirected, Binary Network



What about Jen
to Tom?

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

Example: Undirected, Binary Network

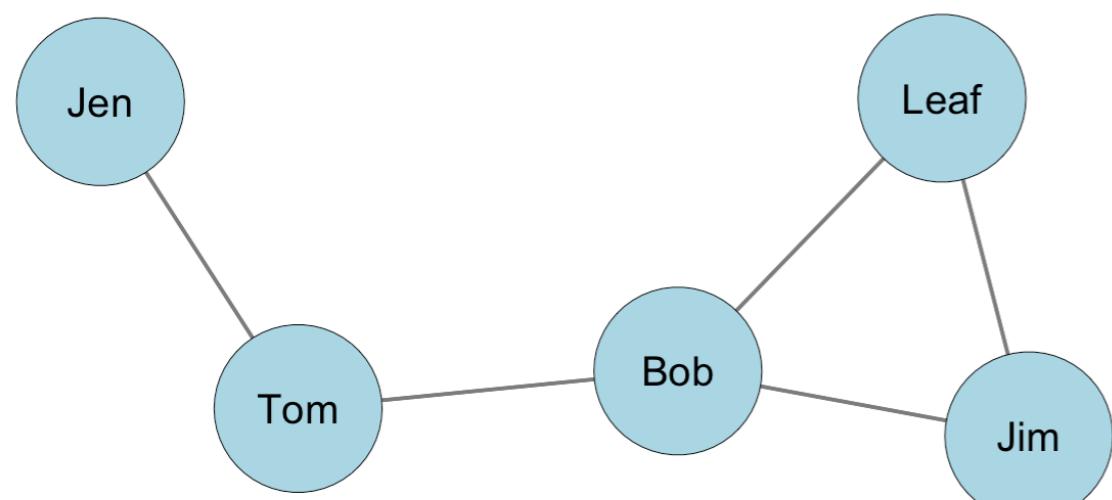


Bob is not on
the only
geodesic from
Jen to Tom

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1		1/1
Tom				
Leaf				
Jim				

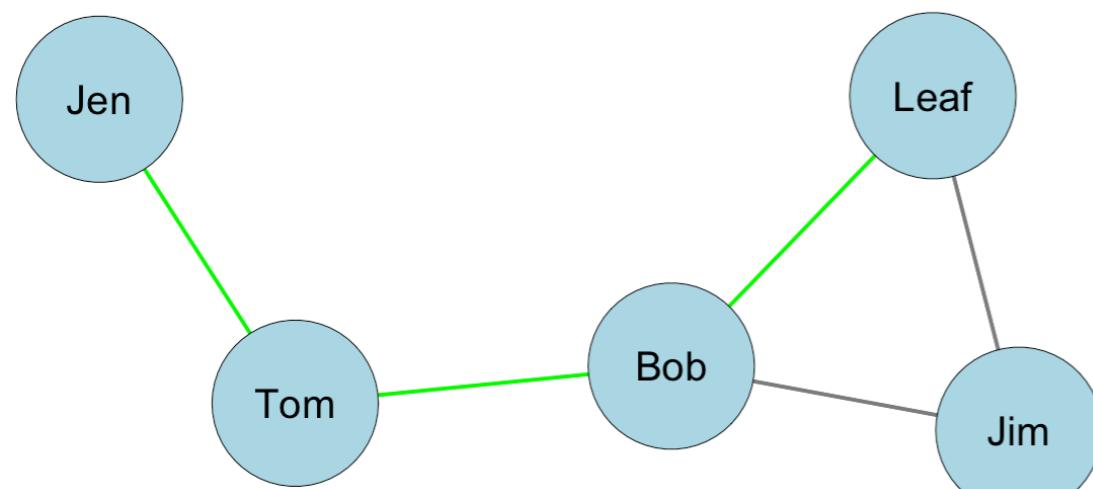
Example: Undirected, Binary Network



What about Jen
to Leaf?

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	/?	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

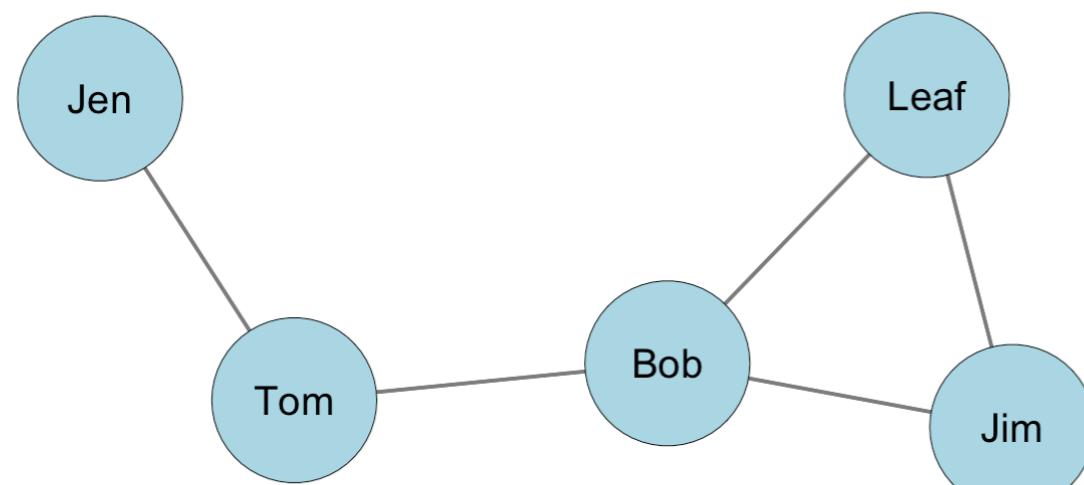
Example: Undirected, Binary Network



Bob is on the
geodesic from
Jen to **Leaf**

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

Example: Undirected, Binary Network

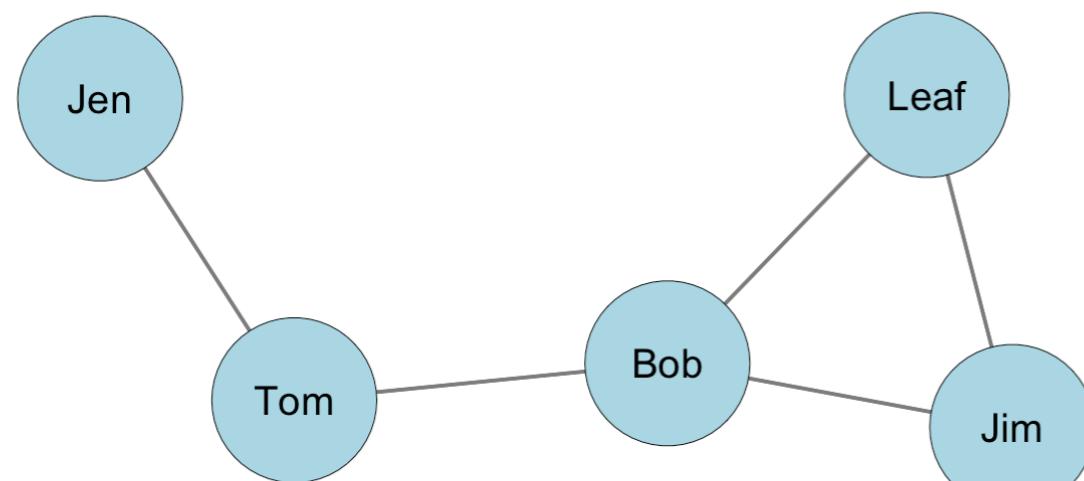


Of the geodesics between
Jen, Tom
Jen, Leaf
Jen, Tom
how many include Bob?

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

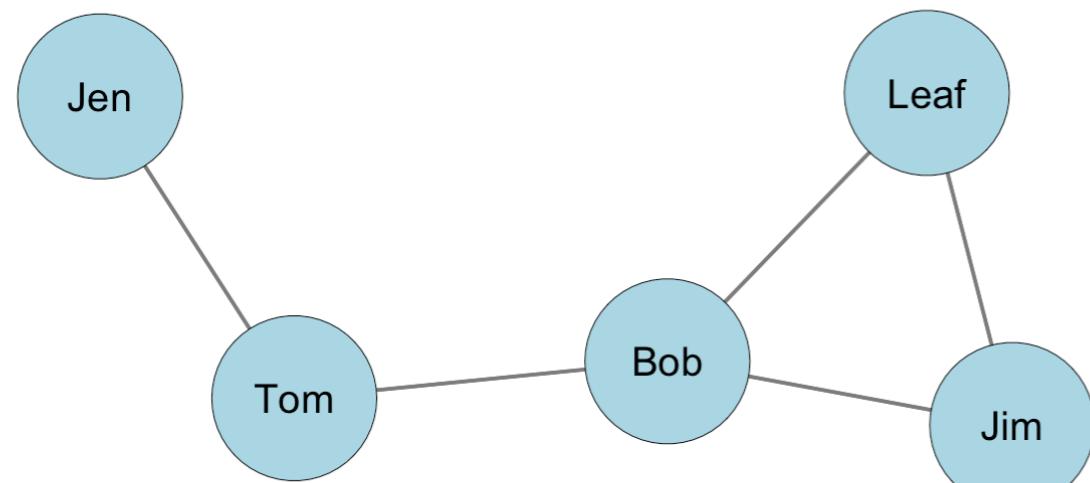
Example: Undirected, Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1

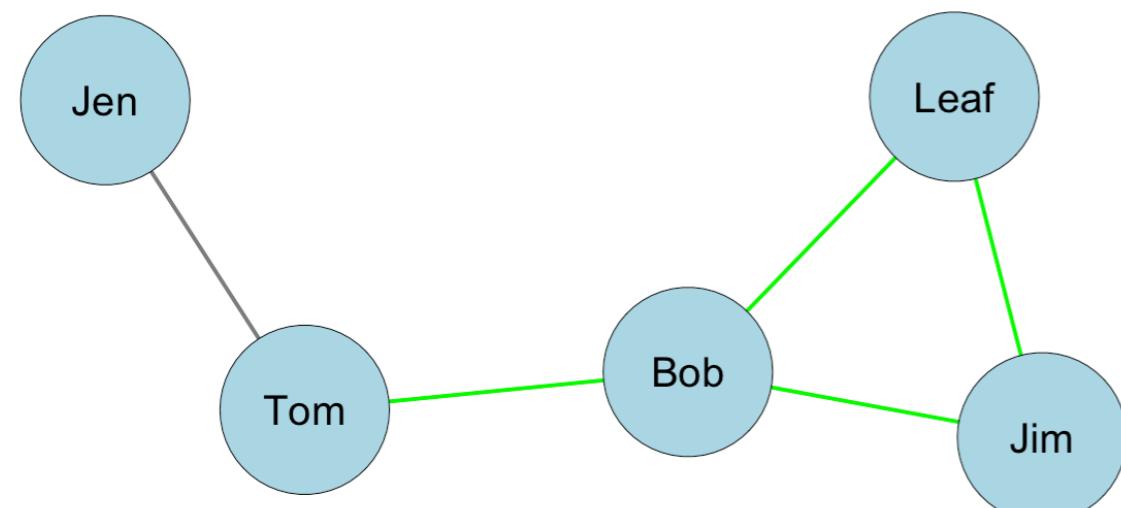
Example: Undirected, Binary Network



To finish, we need to calculate the geodesics for the rest of the matrix for Bob

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			?/?	?/?
<i>Leaf</i>				?/?
<i>Jim</i>				

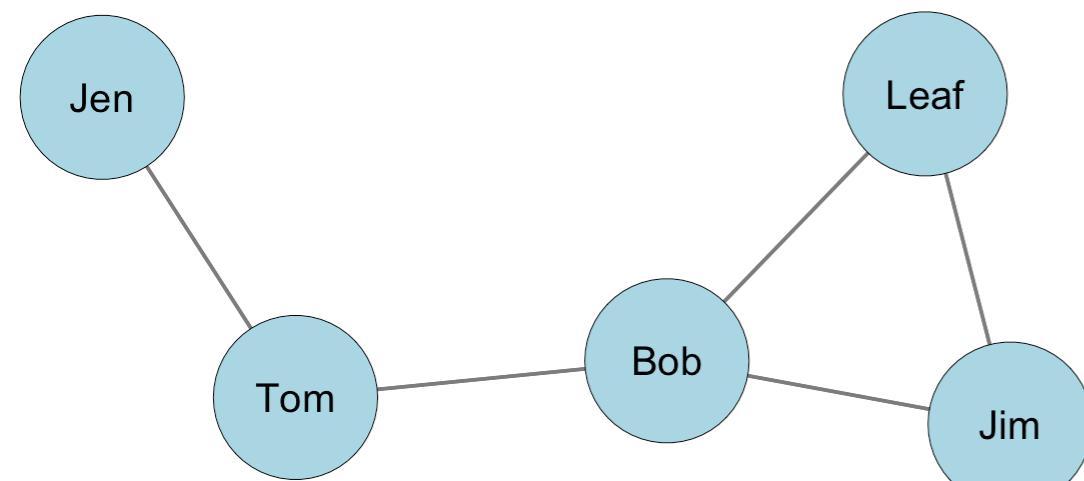
Example: Undirected, Binary Network



To finish, we need to calculate the geodesics for the rest of the matrix for Bob

Geodesic Proportions for <i>Bob</i>				
	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

Example: Undirected, Binary Network



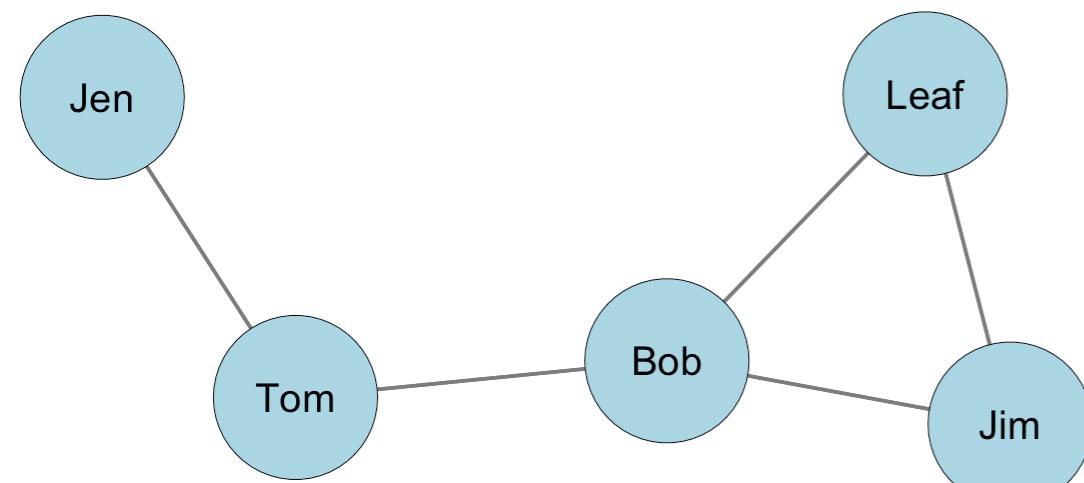
The sum of all these ratios is **Bob's** betweenness centrality score.

What is Bob's score?

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

Example: Undirected, Binary Network

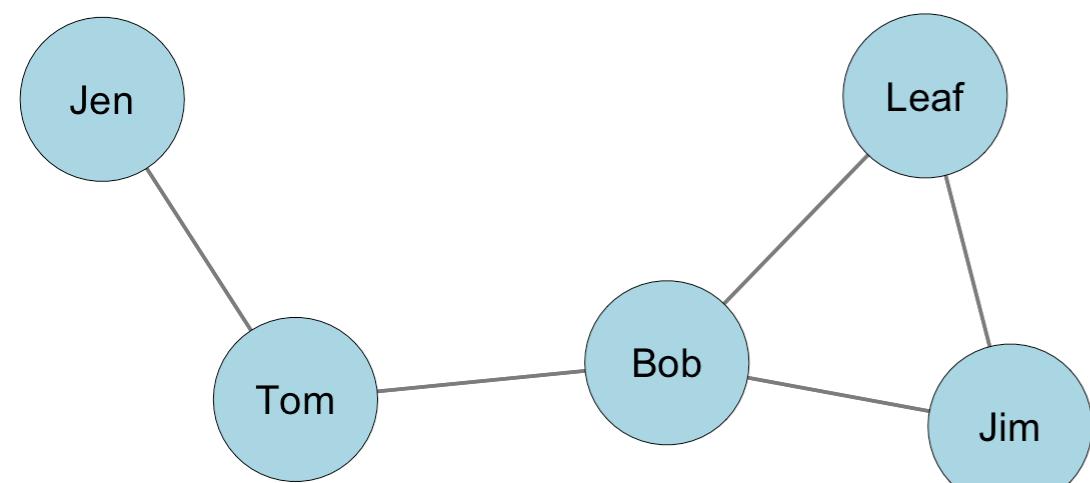


Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

$$C_B(n_i) = \sum_{j < k} g_{jk}(n_i) / g_{jk}$$

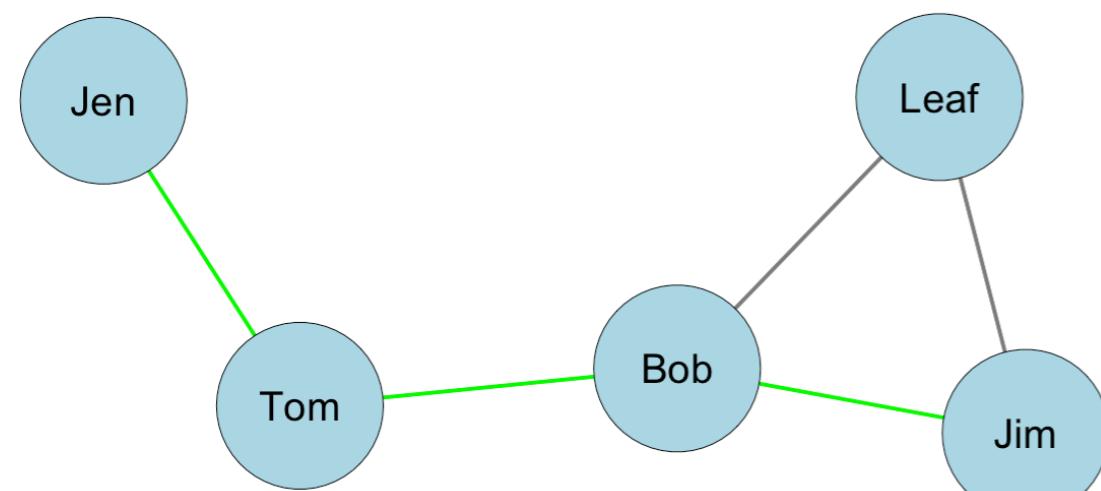
Example: Undirected, Binary Network



Let's do this now for **Leaf**. *What is Leaf's betweenness centrality?*

Geodesic Proportions for Leaf				
	Jen	Tom	Bob	Jim
Jen		??/?	??/?	??/?
Tom			??/?	??/?
Bob				??/?
Jim				

Example: Undirected, Binary Network



Leaf's betweenness centrality is 0



Geodesic Proportions for Leaf

	Jen	Tom	Bob	Jim
Jen		0/1	0/1	0/1
Tom			0/1	0/1
Bob				0/1
Jim				

Betweenness Centrality: Undirected Binary Graphs

- ❖ Actor betweenness centrality not only reflects each node's connectivity to other nodes but also depends on the size of the network, g .
- ❖ Suppose we wanted to compare across networks of different sizes...
 - ❖ *Solution?*

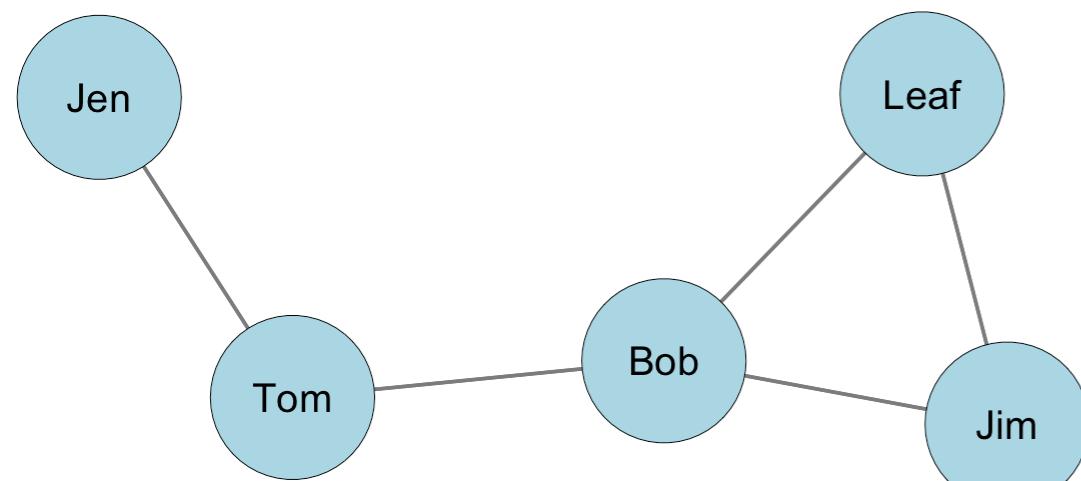
Standardized Betweenness Centrality: Undirected Binary Graphs

$$C'_B(n_i) = \frac{\sum_{j < k} g_{jk}(n_i)/g_{jk}}{[(g-1)(g-2)/2]} = \frac{C_B(n_i)}{[(g-1)(g-2)/2]}$$

The maximum
number of pairs of
actors not including n_i



Example: Undirected, Binary Network



Unstandardized (raw) for Bob: 4

Standardized for Bob:

$$4 / [(5-1)(5-2)/2] = 0.667$$

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>			1/1	1/1
<i>Leaf</i>				0/1
<i>Jim</i>				

Group Betweenness Centralization: Undirected Binary Graphs

- ❖ We can also summarize the entire network, in terms of how close nodes are to each other.
 - ❖ *Group betweenness centralization* tells us how much variation there is in the closeness scores.

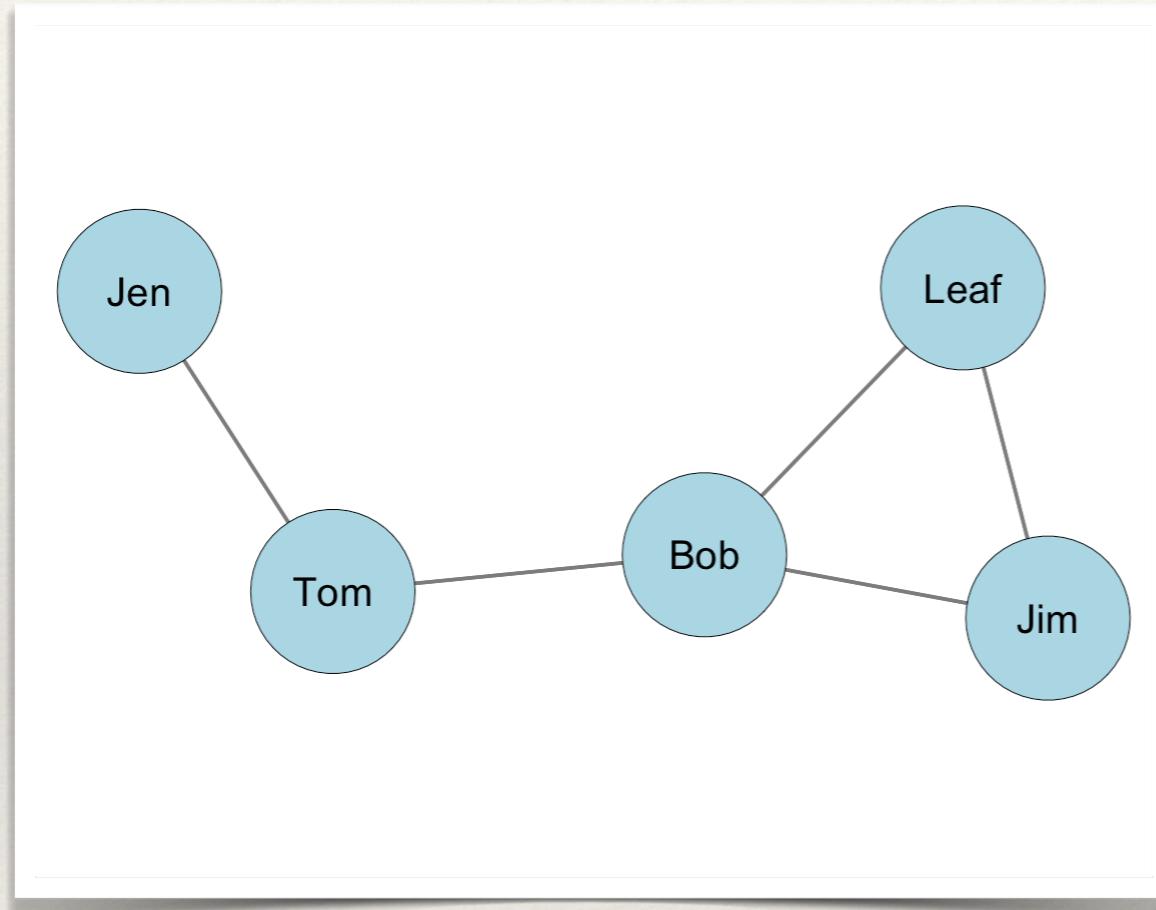
Group Betweenness Centrality: Undirected Binary Graphs

$$C_B = \frac{\sum_{i=1}^g [C'_B(n^*) - C'_B(n_i)]}{(g - 1)}$$

Largest value of the standardized betweenness for the network

Standardized betweenness score for actor i

Example: Undirected, Binary Network



Standardized Betweenness
Centrality Scores

Jen = 0.000

Tom = 0.500

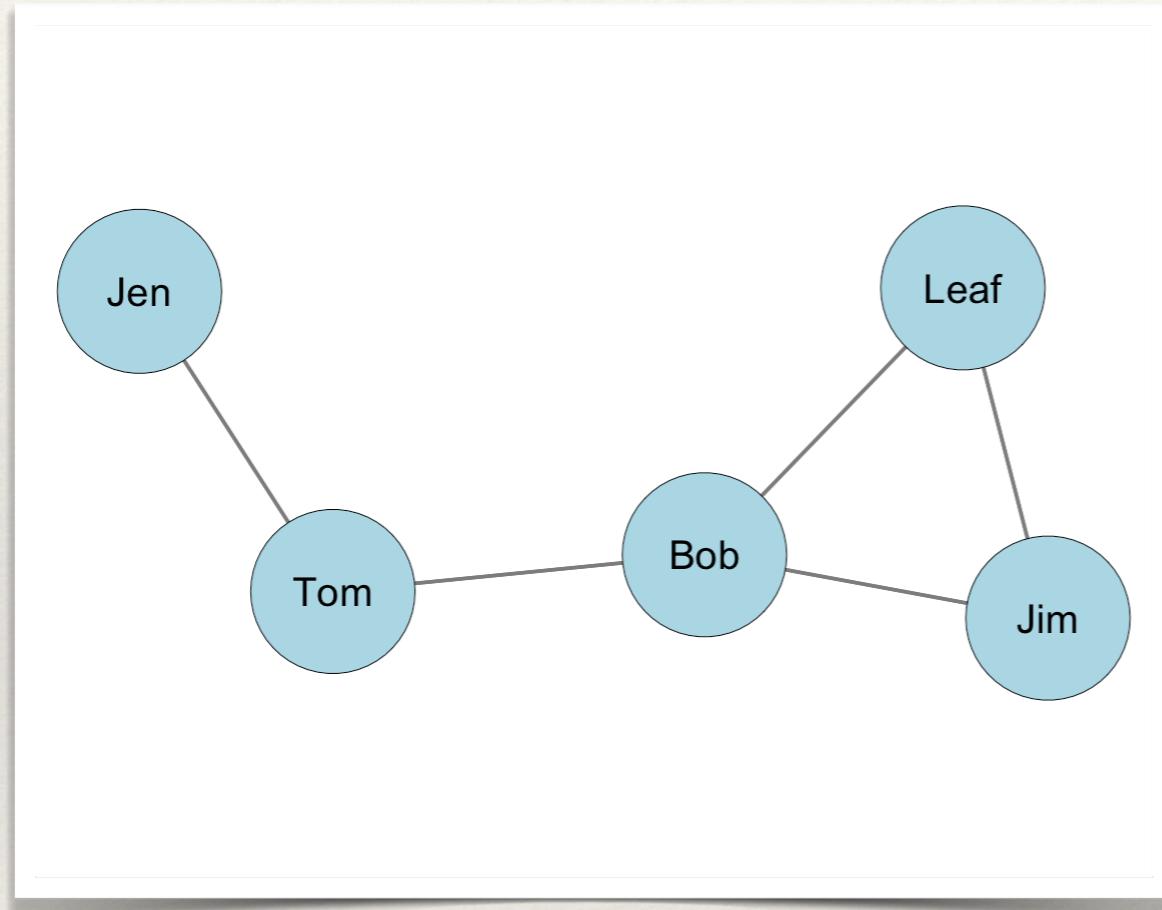
Bob = 0.667

Leaf = 0.000

Jim = 0.000

What is the group betweenness centralization score for this graph?

Example: Undirected, Binary Network



What is the group betweenness centralization score for this graph?

Deviated Standardized Betweenness Centrality

Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

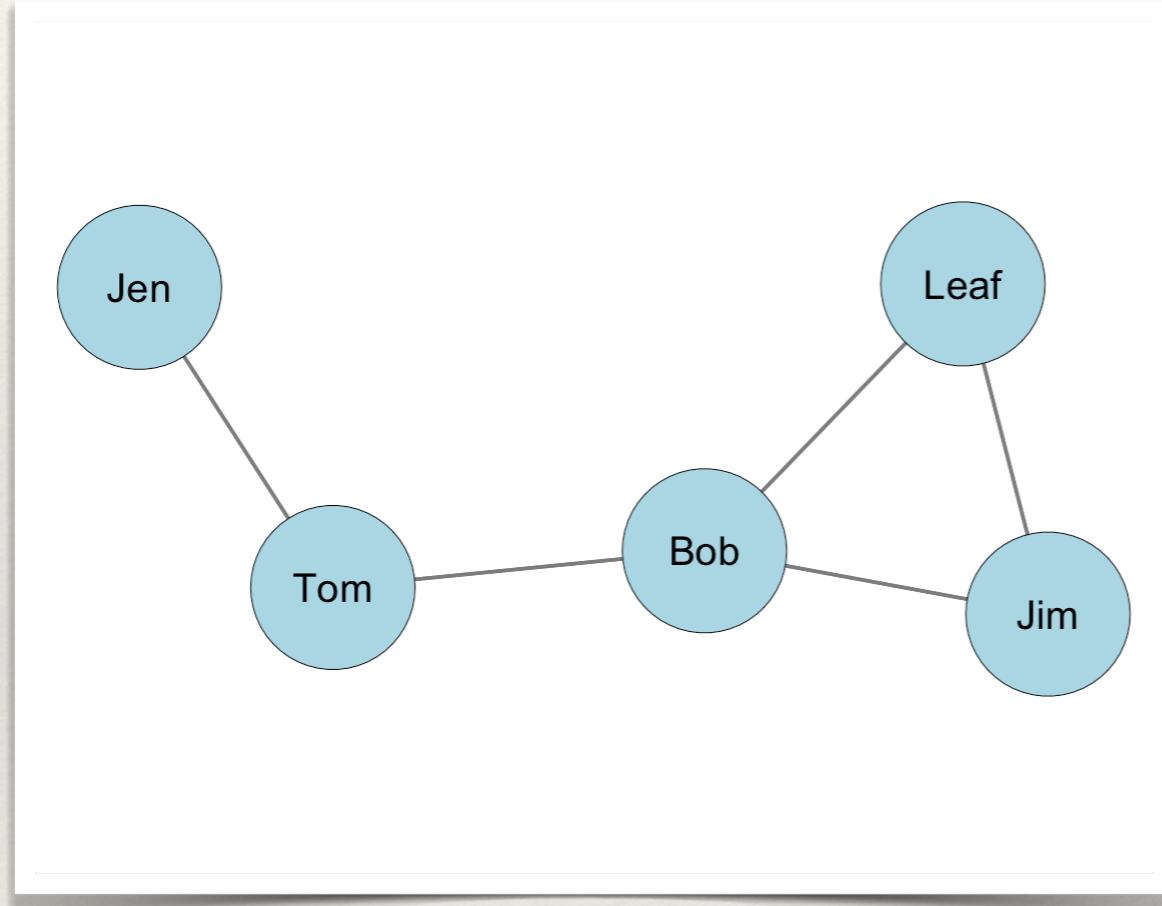
$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

Example: Undirected, Binary Network



Deviated Standardized
Betweenness Centrality
Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

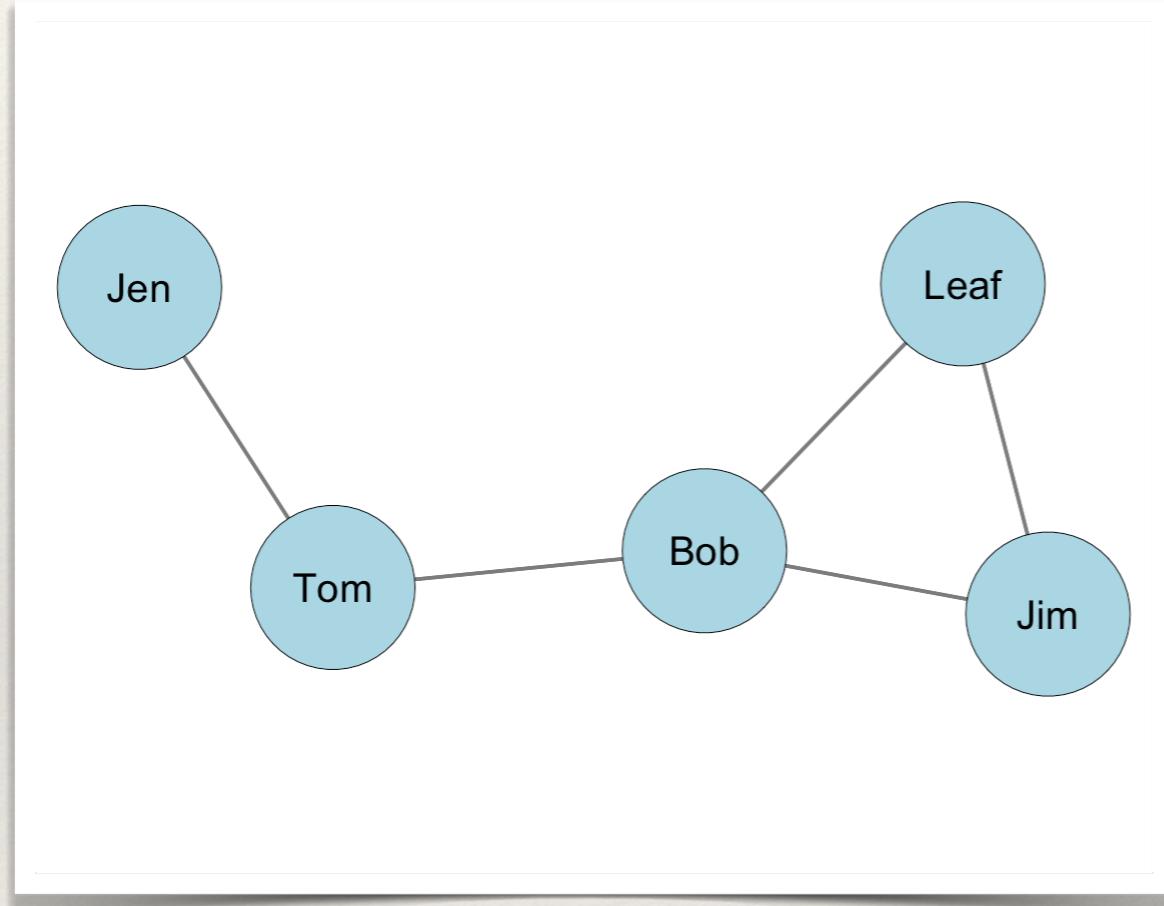
$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

$$\text{Sum} = 2.168$$

What is the group betweenness centralization score for this graph?

Example: Undirected, Binary Network



$$C_B = \frac{\sum_{i=1}^g [C'_B(n^*) - C'_B(n_i)]}{(g - 1)}$$

Deviated Standardized
Betweenness Centrality
Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

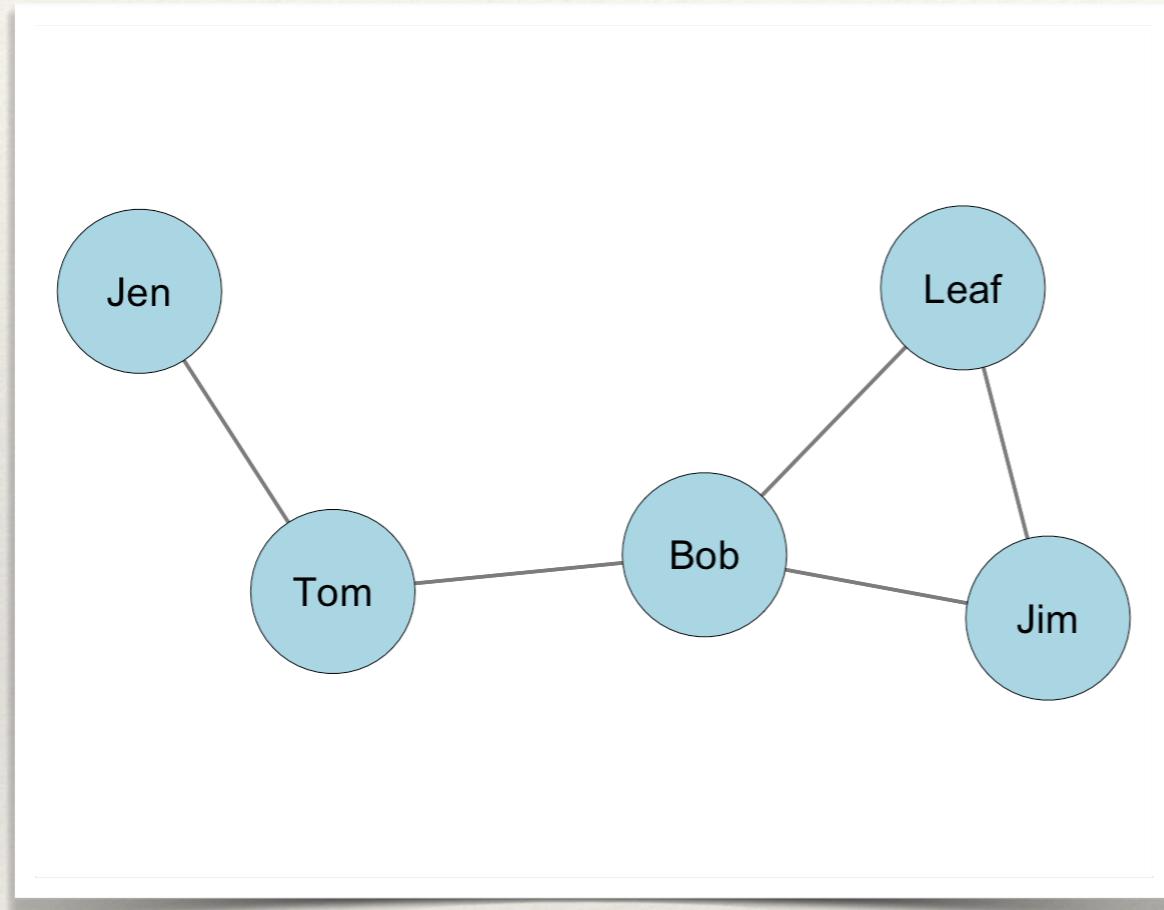
$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

$$\text{Jim} = 0.667 - 0.000 = 0.667$$

$$\text{Sum} = 2.168$$

$$\text{Denominator} = 4$$

Example: Undirected, Binary Network



Deviated Standardized
Betweenness Centrality
Scores

$$\text{Jen} = 0.667 - 0.000 = 0.667$$

$$\text{Tom} = 0.667 - 0.500 = 0.167$$

$$\text{Bob} = 0.667 - 0.667 = 0.000$$

$$\text{Leaf} = 0.667 - 0.000 = 0.667$$

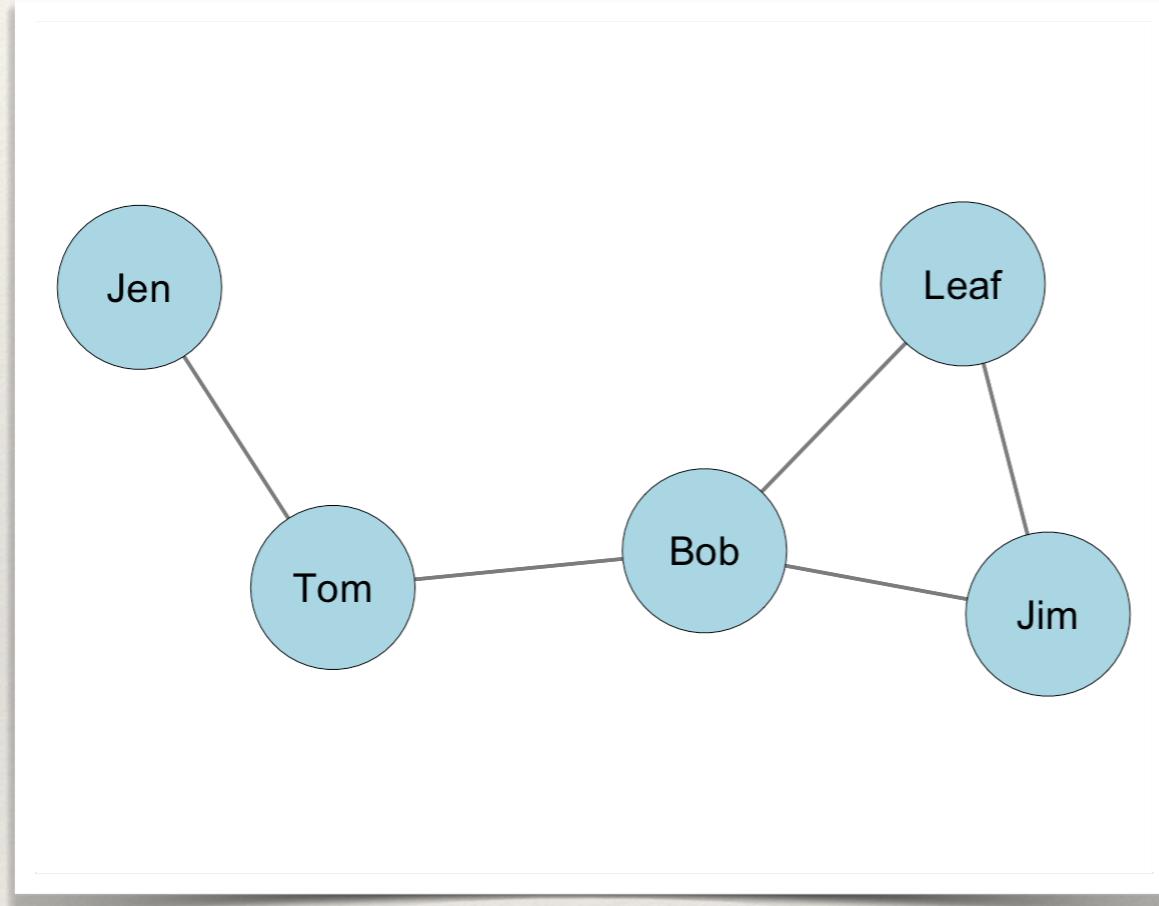
$$\text{Jim} = 0.667 - 0.000 = 0.667$$

$$\text{Sum} = 2.168$$

$$2.168 / 4 = 0.542$$

$$\text{Denominator} = 4$$

Example: Undirected, Binary Network



Compare the centralization scores:

Degree = 0.416

Closeness = 0.551

Betweenness = 0.542

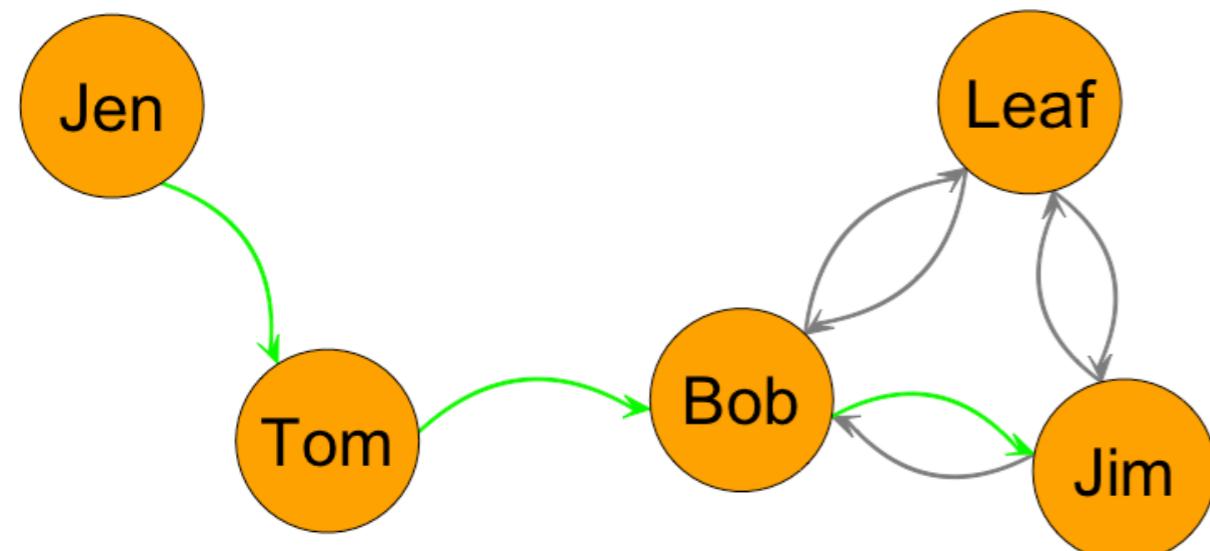
What can we say about the differences in the centralization scores for each type of centrality?

Directed Networks

Betweenness Centrality: Directed Binary Graphs

- ❖ Recall that in a directed network, the directionality matters.
 - ❖ As a result, we have to consider how this might influence our measures.

Example: Betweenness Centrality for Directed Binary Network

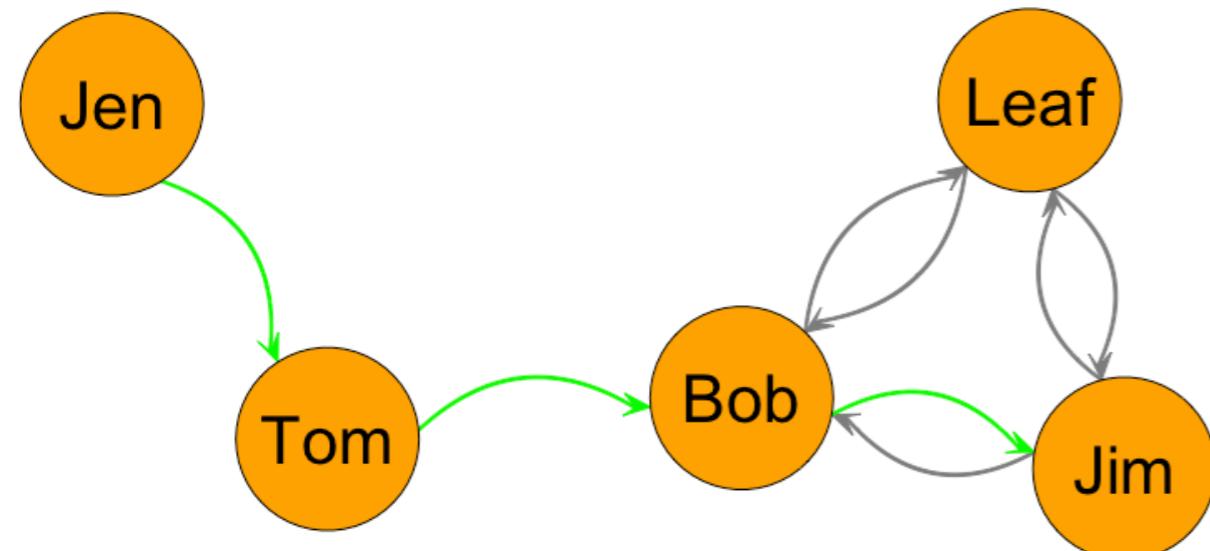


Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen				
			/?	

As with closeness, we
look at the rows

Example: Betweenness Centrality for Directed Binary Network

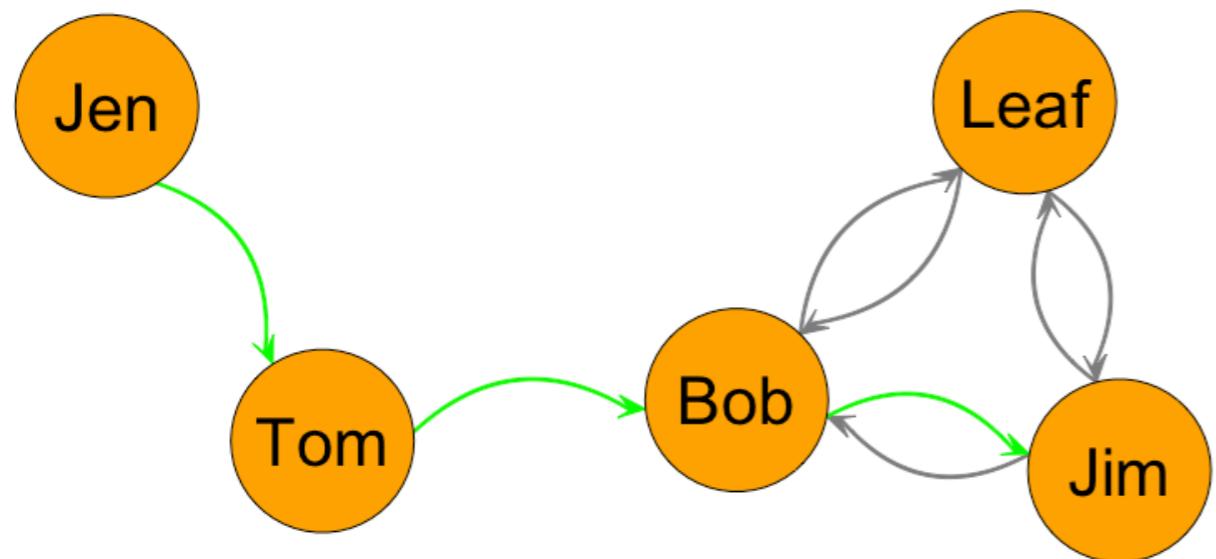


Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen				
			/?	

The rows show sending behavior

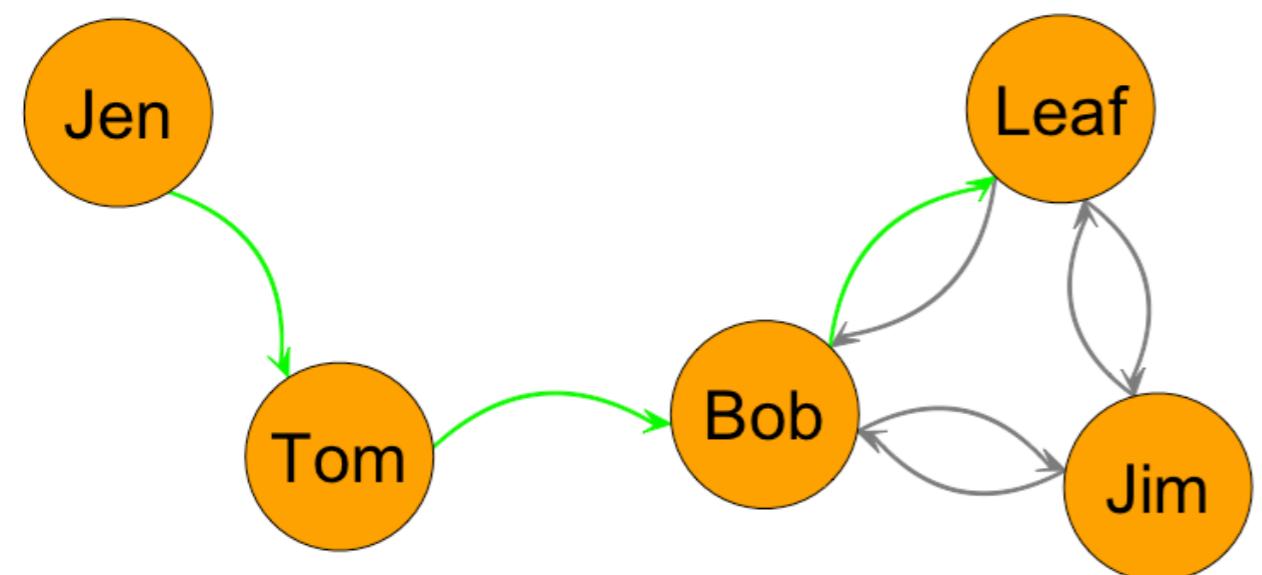
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>				1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

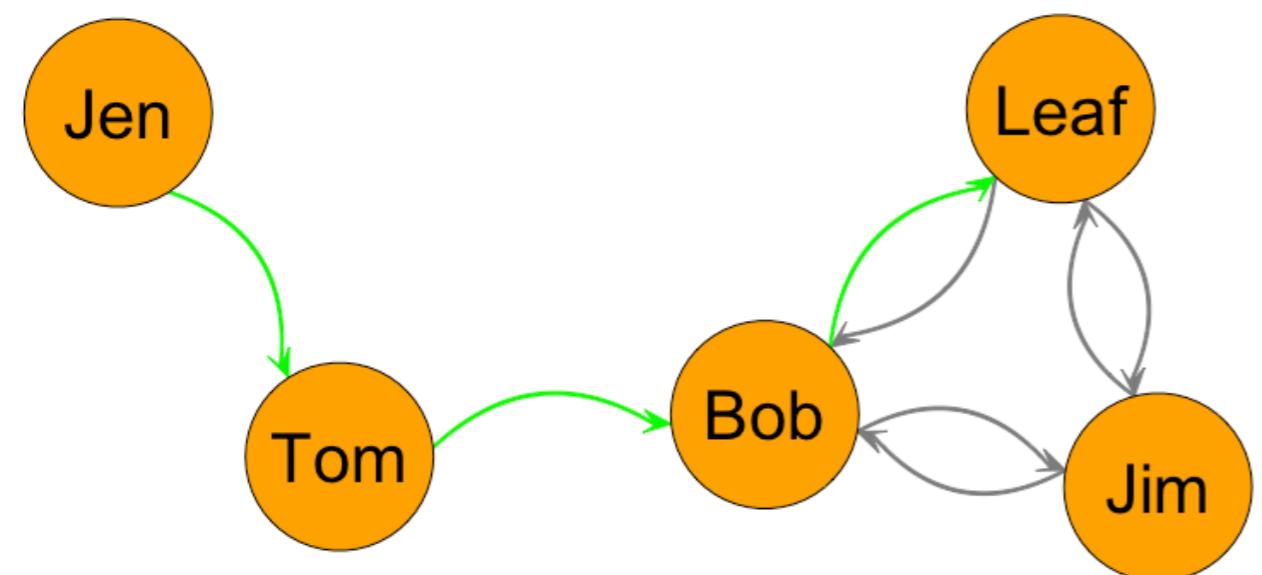
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen			?	1/1
Tom				
Leaf				
Jim				

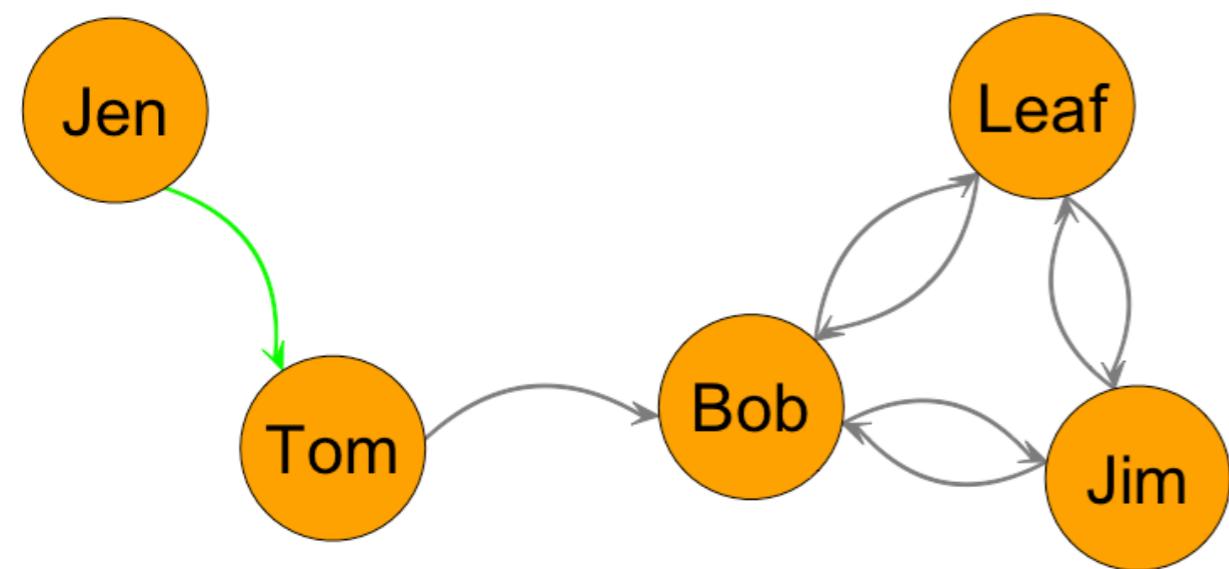
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen			1/1	1/1
Tom				
Leaf				
Jim				

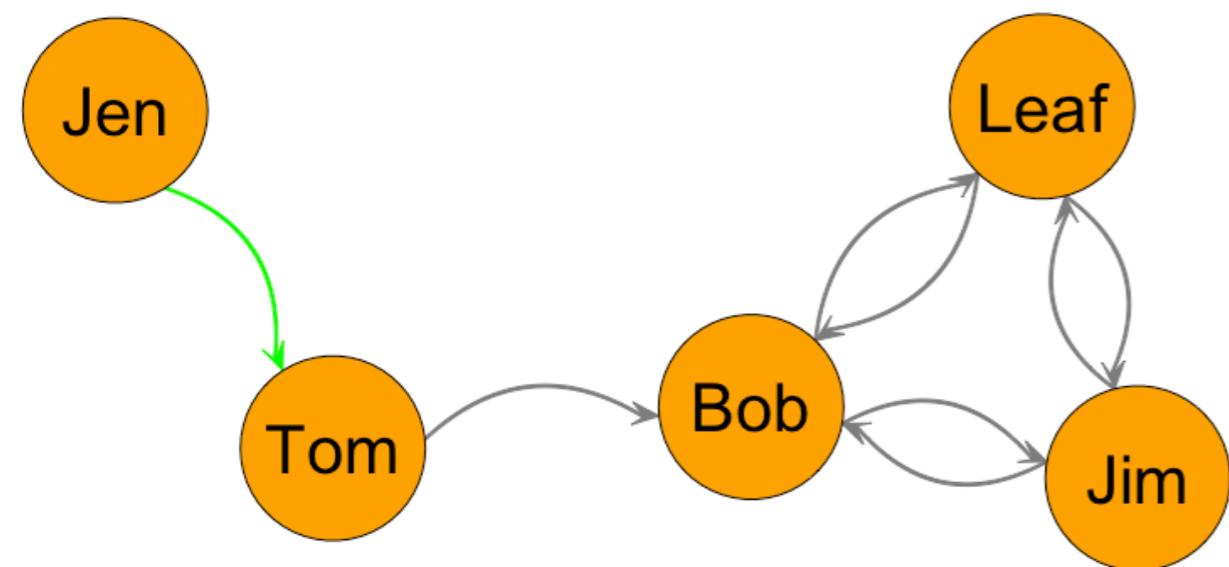
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		?	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

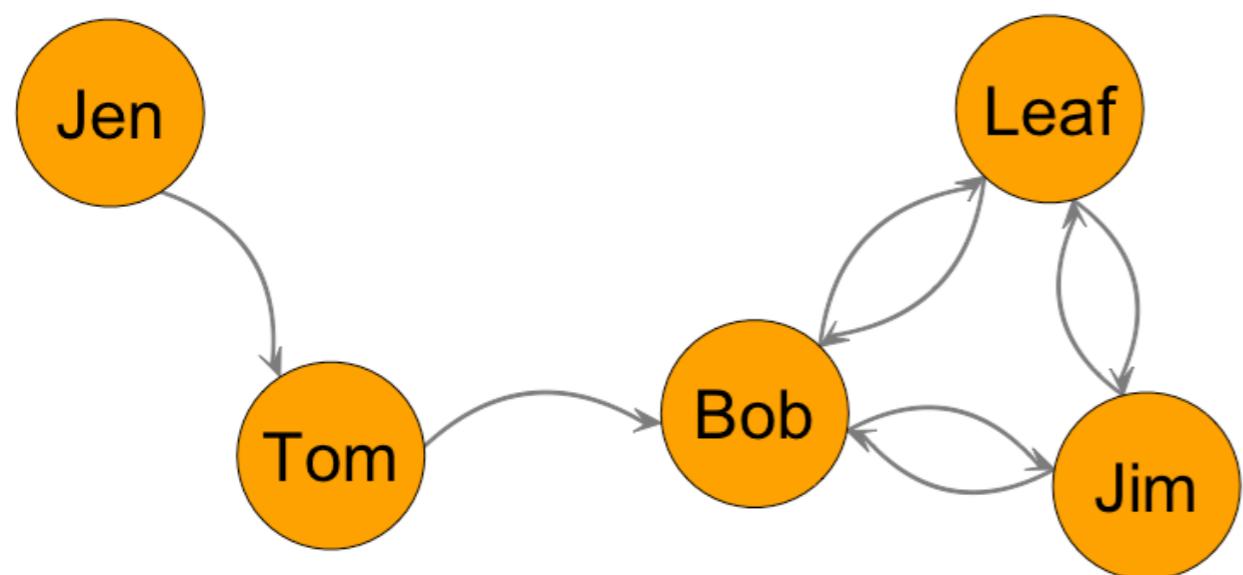
Example: Betweenness Centrality for Directed Binary Network



Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>				
<i>Leaf</i>				
<i>Jim</i>				

Example: Betweenness Centrality for Directed Binary Network

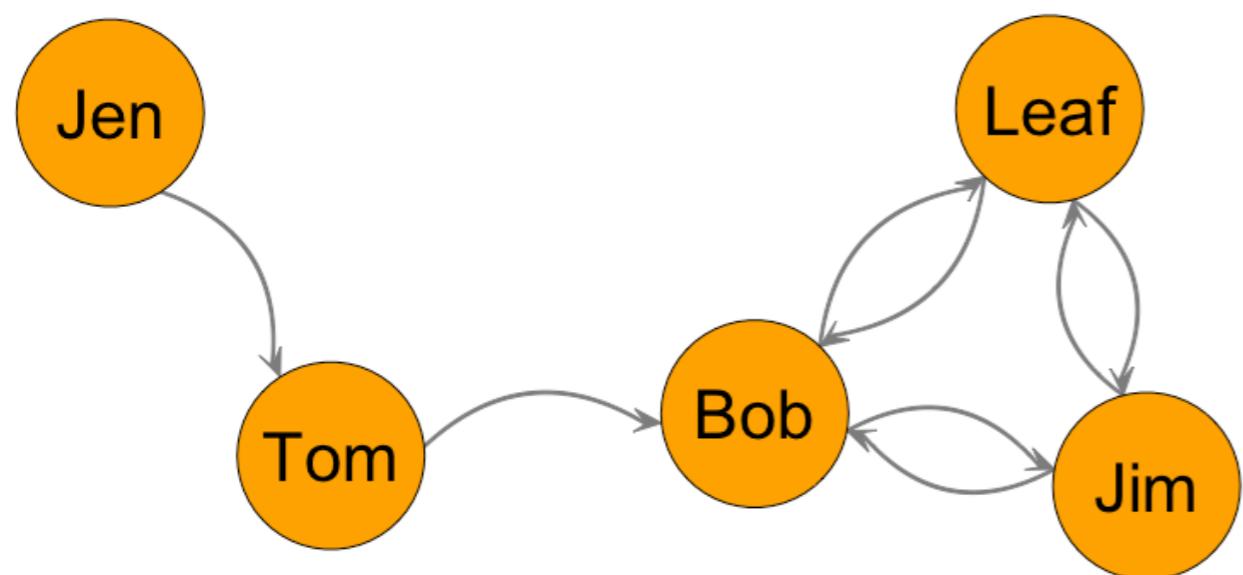


Then, we just complete the matrix.

Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>	0/1	1/1	1/1	
<i>Tom</i>	0/0		1/1	1/1
<i>Leaf</i>	0/0	0/0		0/1
<i>Jim</i>	0/0	0/0	0/1	

Example: Betweenness Centrality for Directed Binary Network

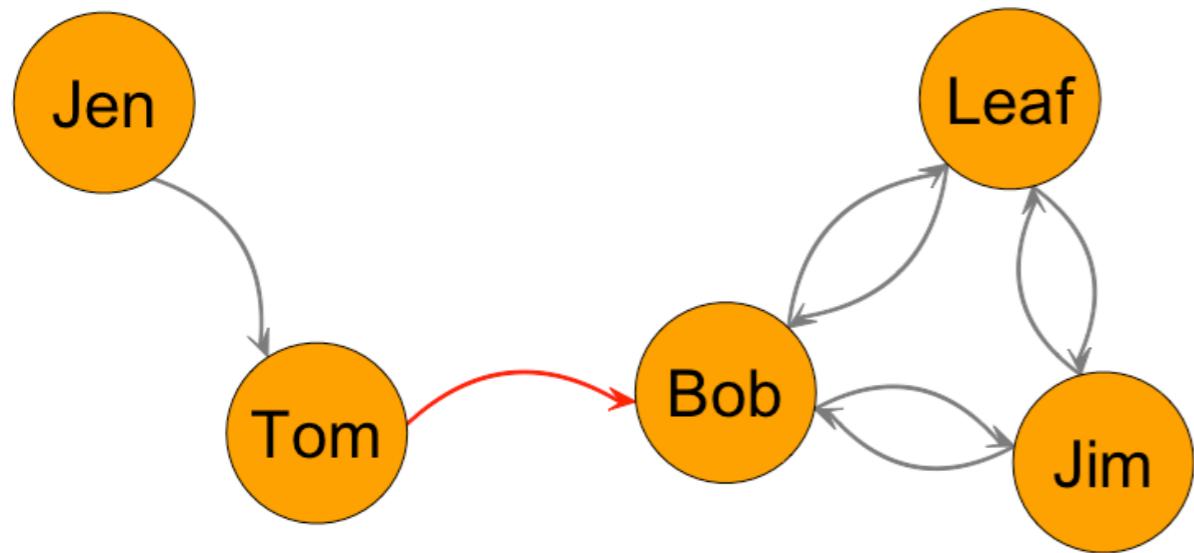


Geodesic Proportions for *Bob*

	<i>Jen</i>	<i>Tom</i>	<i>Leaf</i>	<i>Jim</i>
<i>Jen</i>		0/1	1/1	1/1
<i>Tom</i>	0/0		1/1	1/1
<i>Leaf</i>	0/0	0/0		0/1
<i>Jim</i>	0/0	0/0	0/1	

Why no geodesics?

Example: Betweenness Centrality for Directed Binary Network

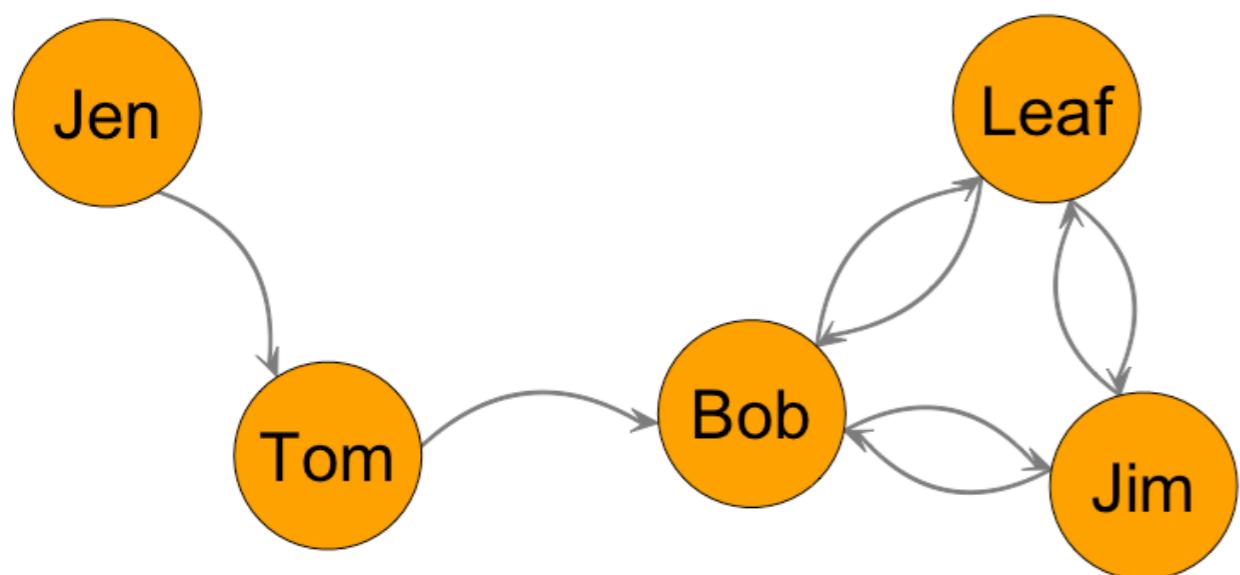


Tom and Jen cannot be reached past Bob, because there are no outgoing ties from Bob.

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

Example: Betweenness Centrality for Directed Binary Network



Unstandardized (raw) for Bob: 4

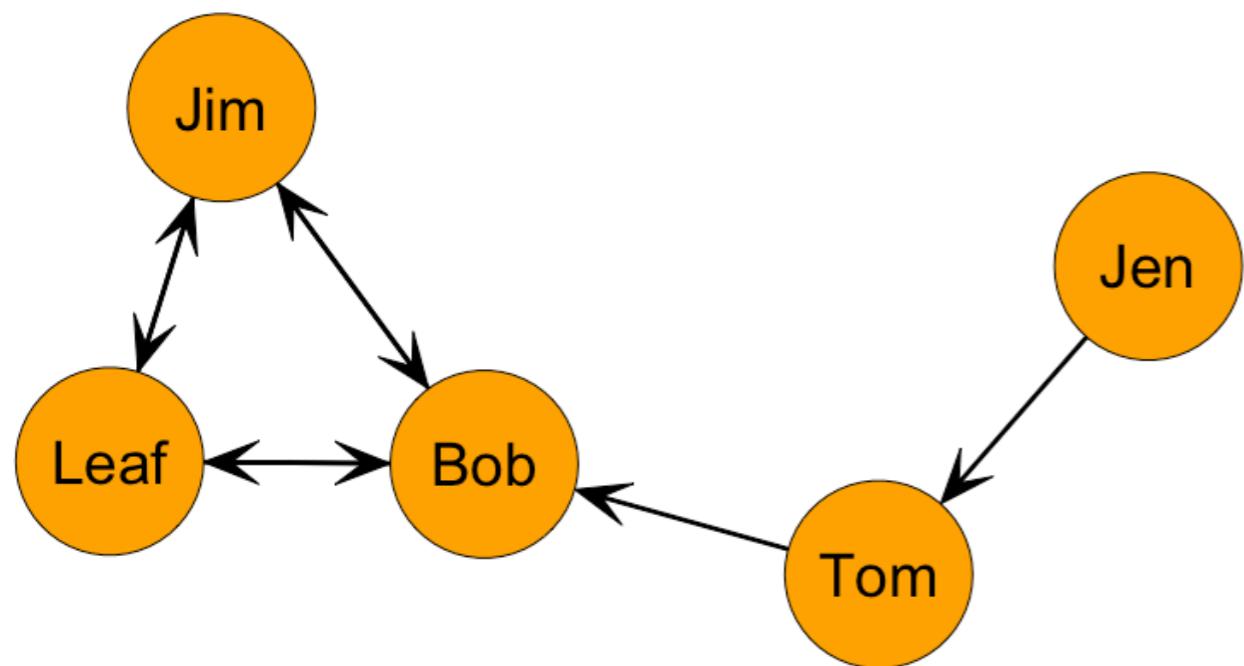
Standardized for Bob:

$$4 / [(5-1)(5-2)/2] = 0.667$$

Geodesic Proportions for *Bob*

	Jen	Tom	Leaf	Jim
Jen		0/1	1/1	1/1
Tom	0/0		1/1	1/1
Leaf	0/0	0/0		0/1
Jim	0/0	0/0	0/1	

Example: Undirected, Binary Network



Compare the centralization scores:

Indegree = 0.438

Outdegree = 0.125

Closeness = 0.555

Betweenness = 0.270

What can we say about the differences in the centralization scores for each type of centrality?

Comparing Measures of Centrality

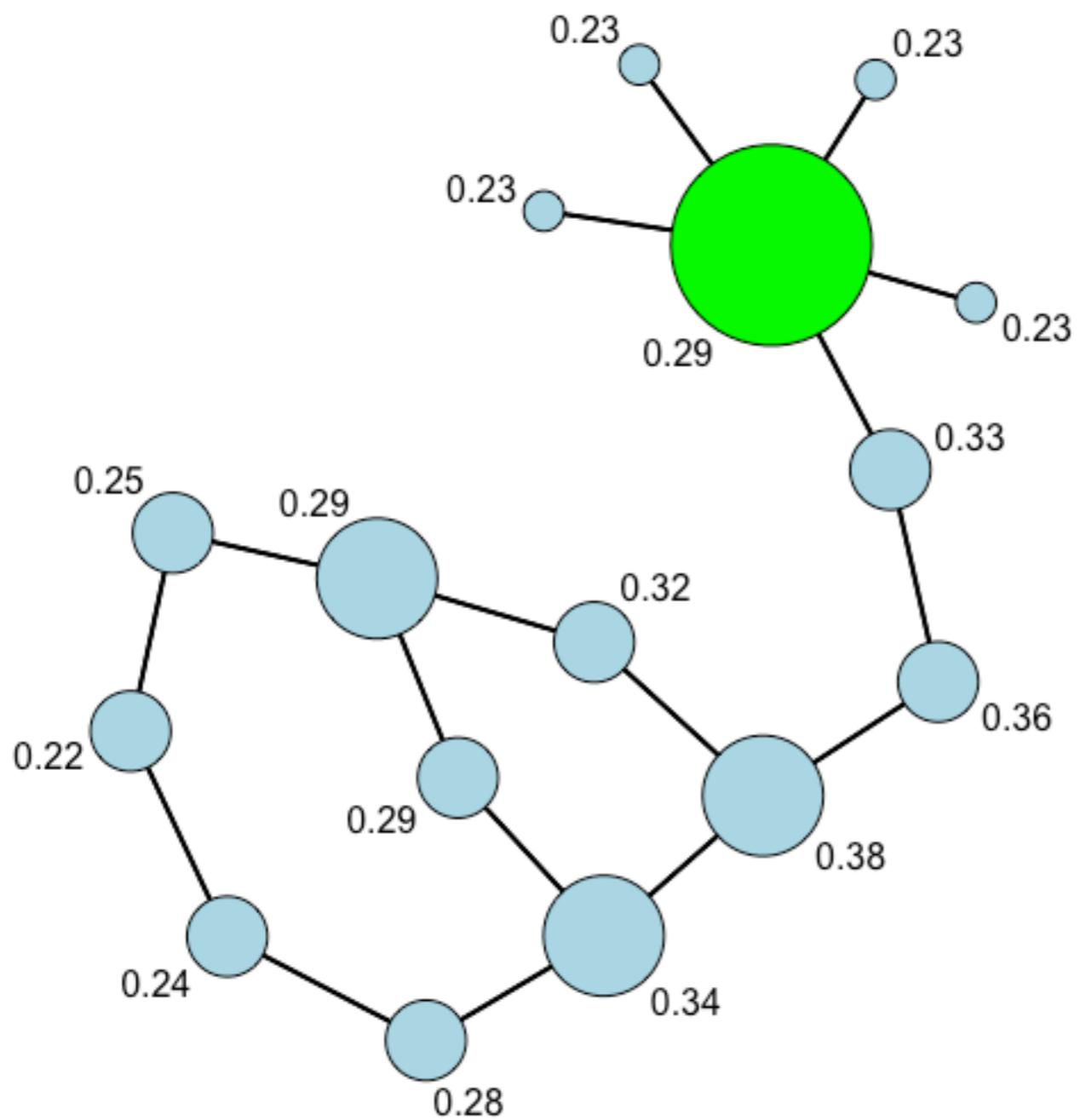
Comparing Measures of Centrality

	Low Degree	Low Closeness	Low Betweenness
High Degree		Embedded in cluster that is far from the rest of the network	Ego's connections are redundant - communication bypasses him/her
High Closeness	Key player tied to important/active alters		Probably multiple paths in the network, ego is near many people, but so are others
High Betweenness	Ego's few ties are crucial for network flow	Very rare. Would mean that ego monopolizes the ties from a small number of people to many others	

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High Degree, Low Closeness



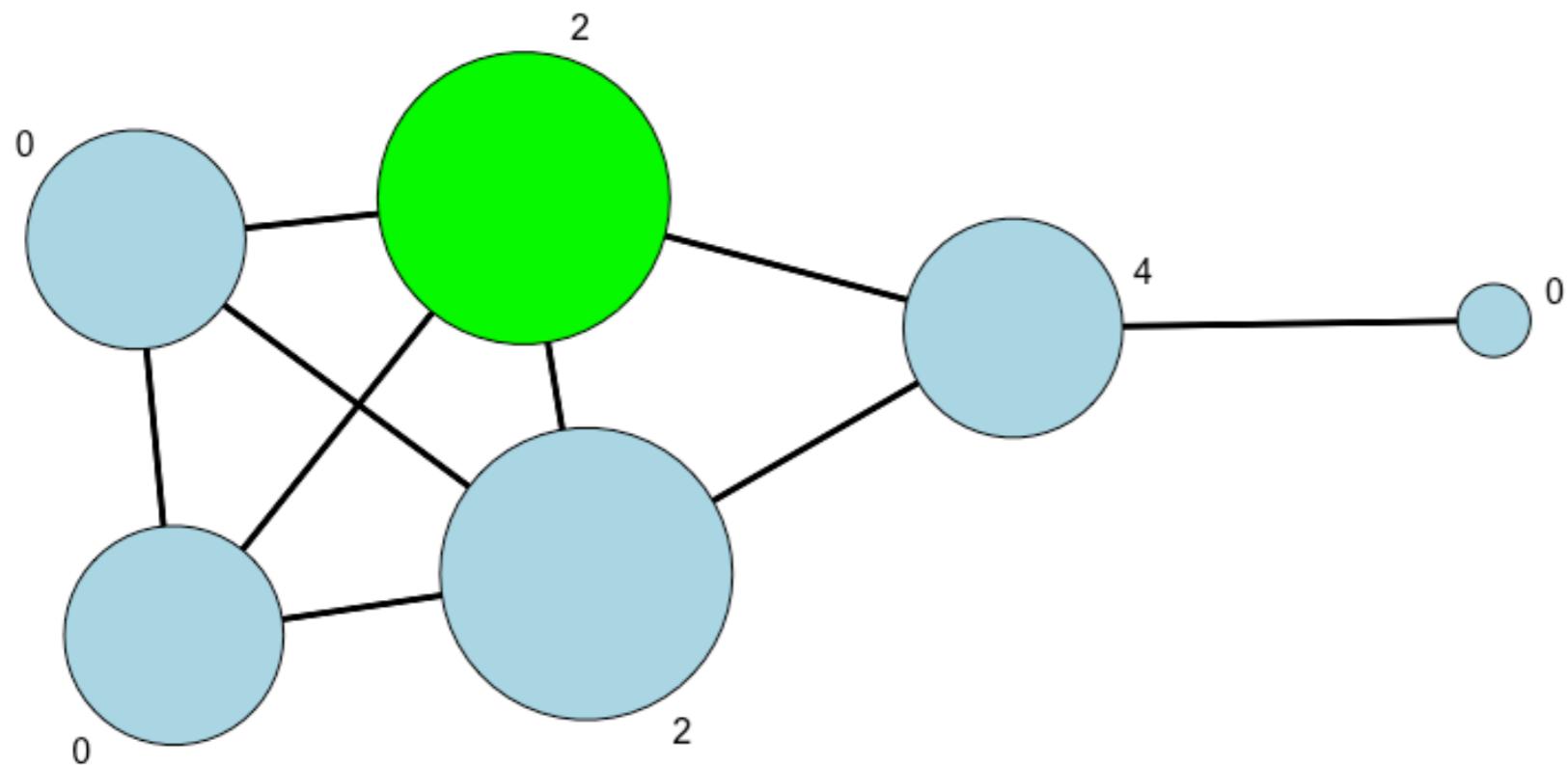
*Nodes sized by
degree*

*Nodes labeled by
closeness
centrality*

Comparing Measures of Centrality

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High Degree, Low Betweenness



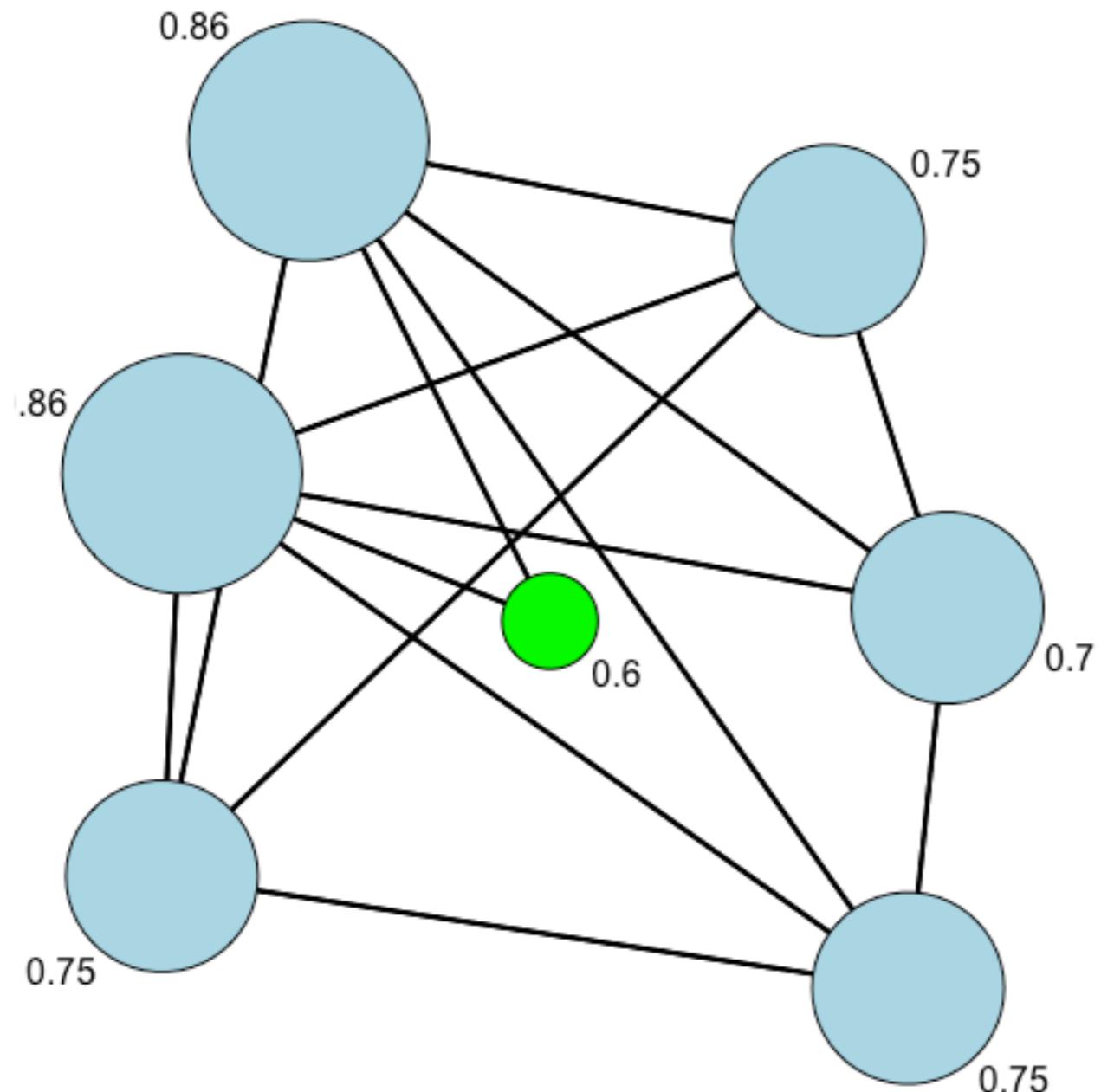
*Nodes sized by
degree*

*Nodes labeled by
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Comparing Measures of Centrality

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High Closeness, Low Degree



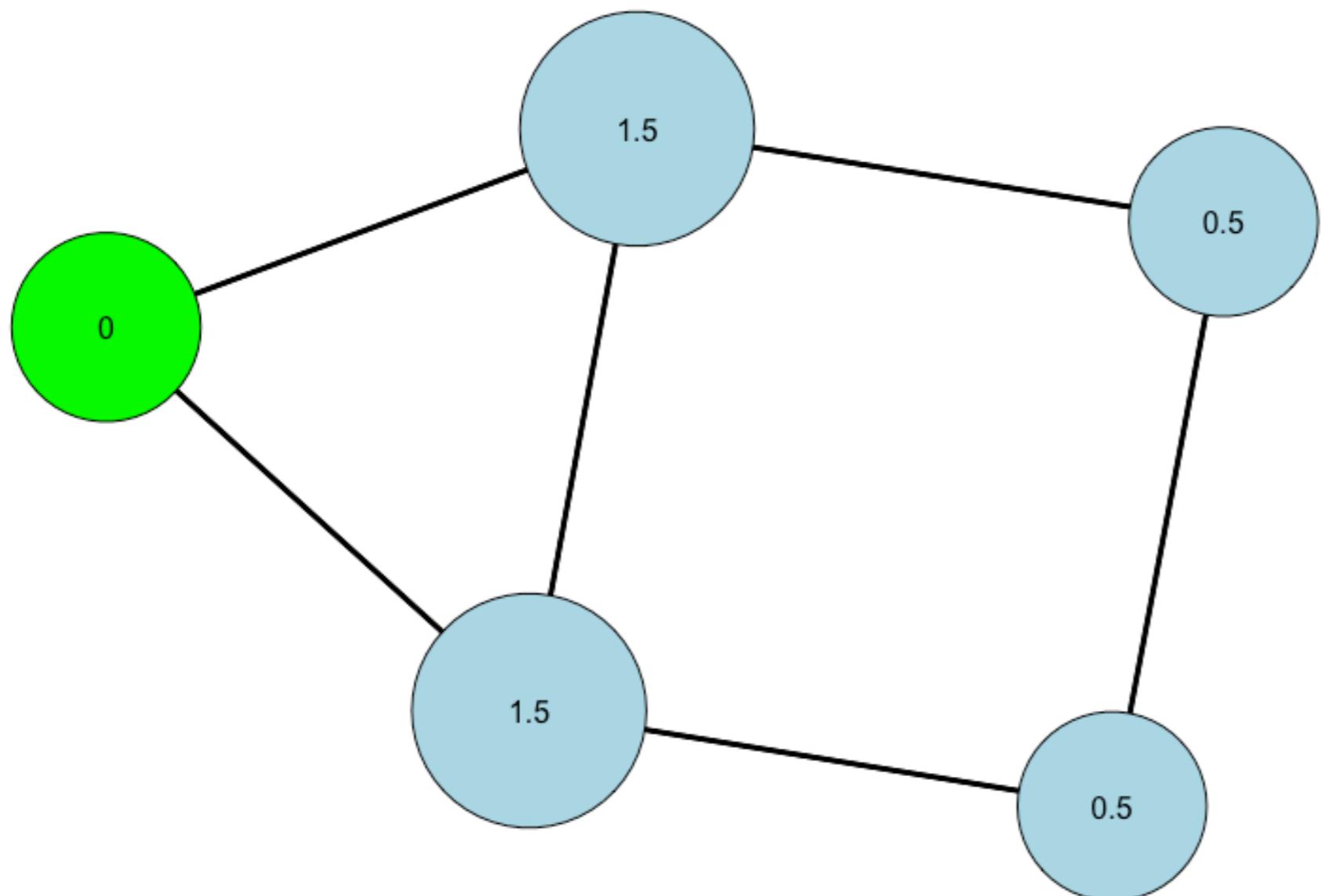
*Nodes sized by
degree centrality*

*Nodes labeled by
closeness
centrality*

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High Closeness, Low Betweenness



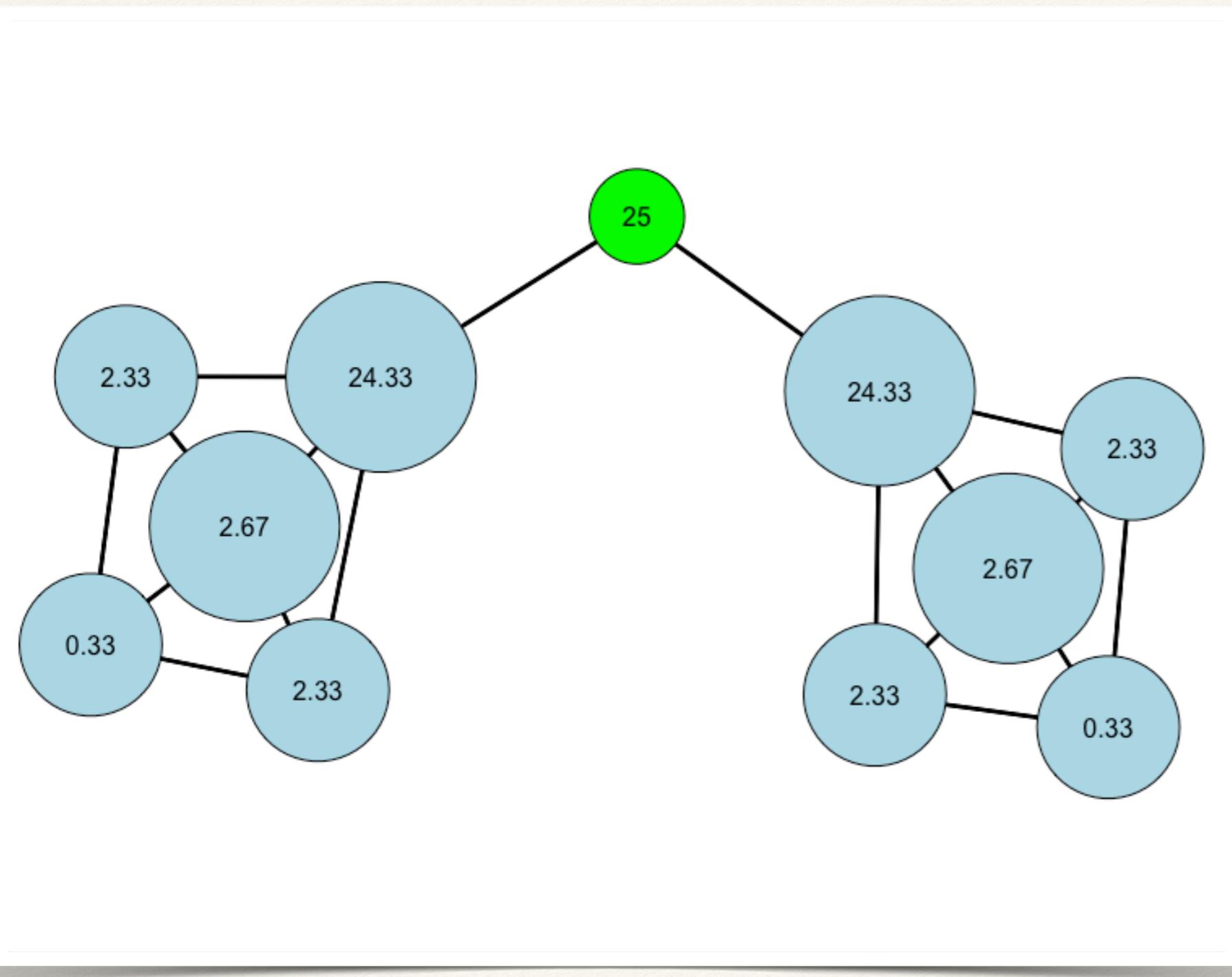
*Nodes sized by
closeness
centrality*

*Nodes labeled by
betweenness
centrality*

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High Betweenness, Low Degree



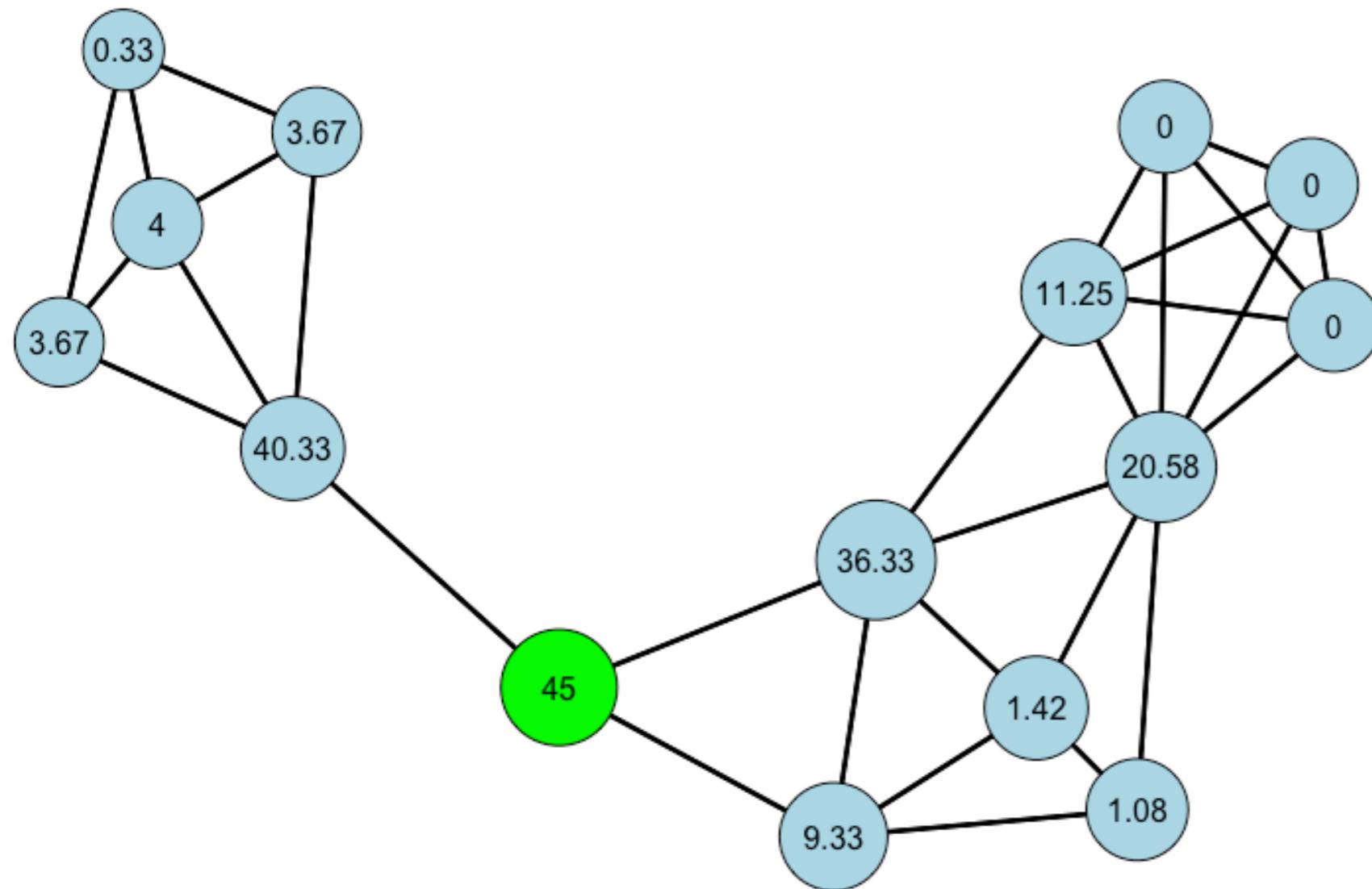
*Nodes sized by
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High Betweenness, Low Closeness



Learning Goals

- ❖ At the end of the lecture, you should be able to answer these questions:
 - ❖ What are some different ways we can conceptualize “centrality”?
 - ❖ What is *closeness* and *betweenness* centrality?
 - ❖ How do we calculate these measures for undirected and directed graphs?
 - ❖ What do comparing these measures tell us about the structure of a network?

Questions?