Statistical Analysis of Networks

Centrality

Learning Goals

- Understand the conceptualization of “centrality”.
- Understand calculation of degree centrality.
- Analyze descriptive features of degree centrality.

When we say a *node* is “central,” what do we mean conceptually?

Concepts and Operationalization

- Speaking generally, network *position* should be interesting and important:
  - As a <u>dependent</u> variable (e.g. are taller individuals more likely to be trusted?)
  - As an <u>independent</u> variable (e.g. are more popular adolescents more likely to succeed in school?)
    - As a description of the position of a **node/vertex**
    - *And* as a description of an entire **network**

Conceptualization

- "Everyone agrees, it seems, that centrality is an important structural attribute of networks. All concede that it is related to a high degree to other important group properties and processes. But there consensus ends." (Freeman, 1978/1979: 217)
  - The type of measure we use depends on the substantive question of interest.
    - Various measures of centrality are correlated, but they <u>operationalize</u> different concepts.

Undirected Networks

Conceptualization

- Concepts and unit of analysis:
  - Point centrality (degree, betweenness, closeness)
  - Graph centrality (compactness)

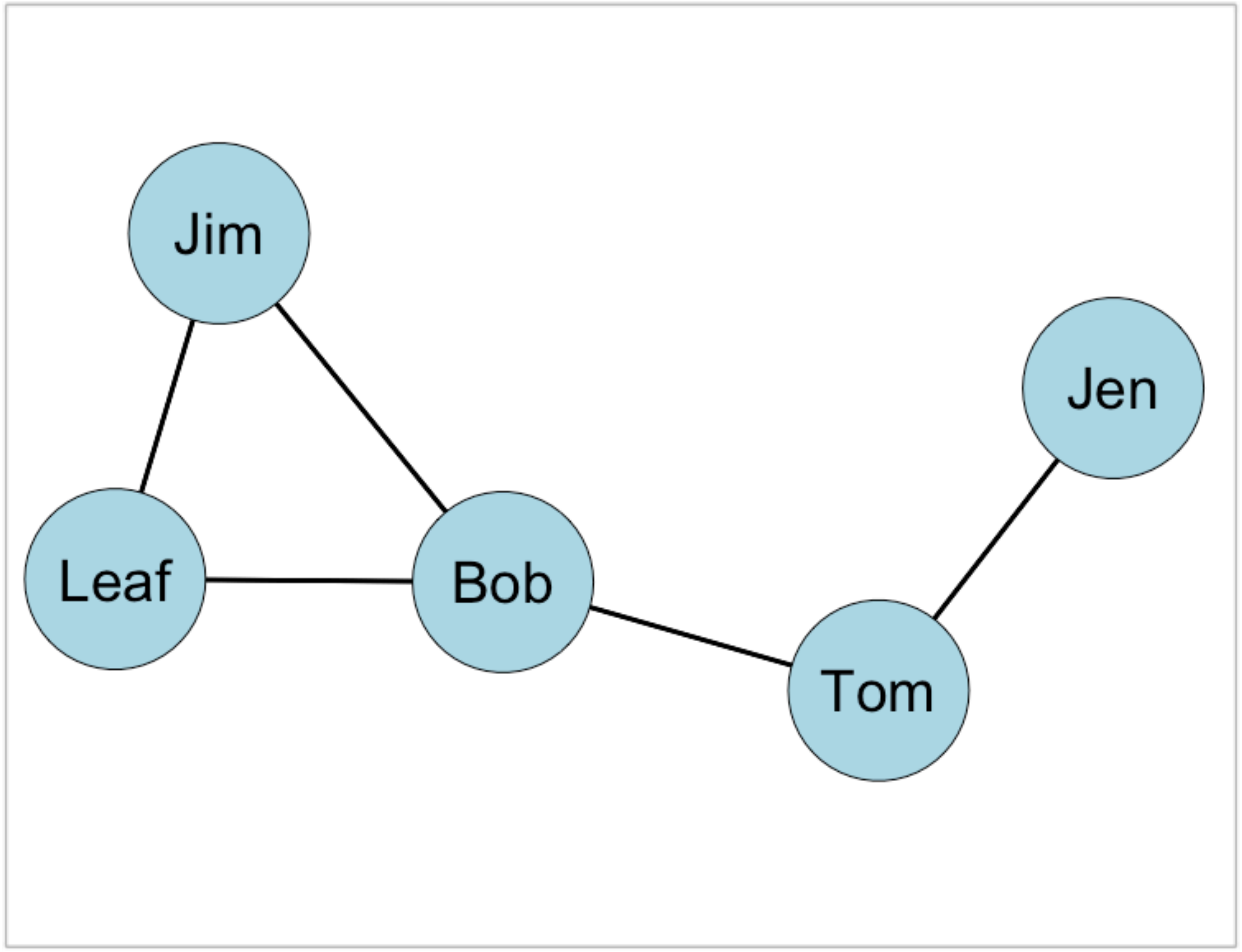
Degree Centrality: Undirected Binary Graphs

- In an undirected binary graph, *actor degree centrality* measures the extent to which a node connects to all other nodes in a social network.
  - In other words, the number of edges incident with a node.
    - This is symbolized as: $d(n_i)$
      - For an undirected binary graph, the degree $d(n_i)$ is the row or column sum.

Degree Centrality: Undirected Binary Graphs

$$C_D(n_i) = d(n_i) = \sum_j x_{ij} = \sum_j x_{ji}$$

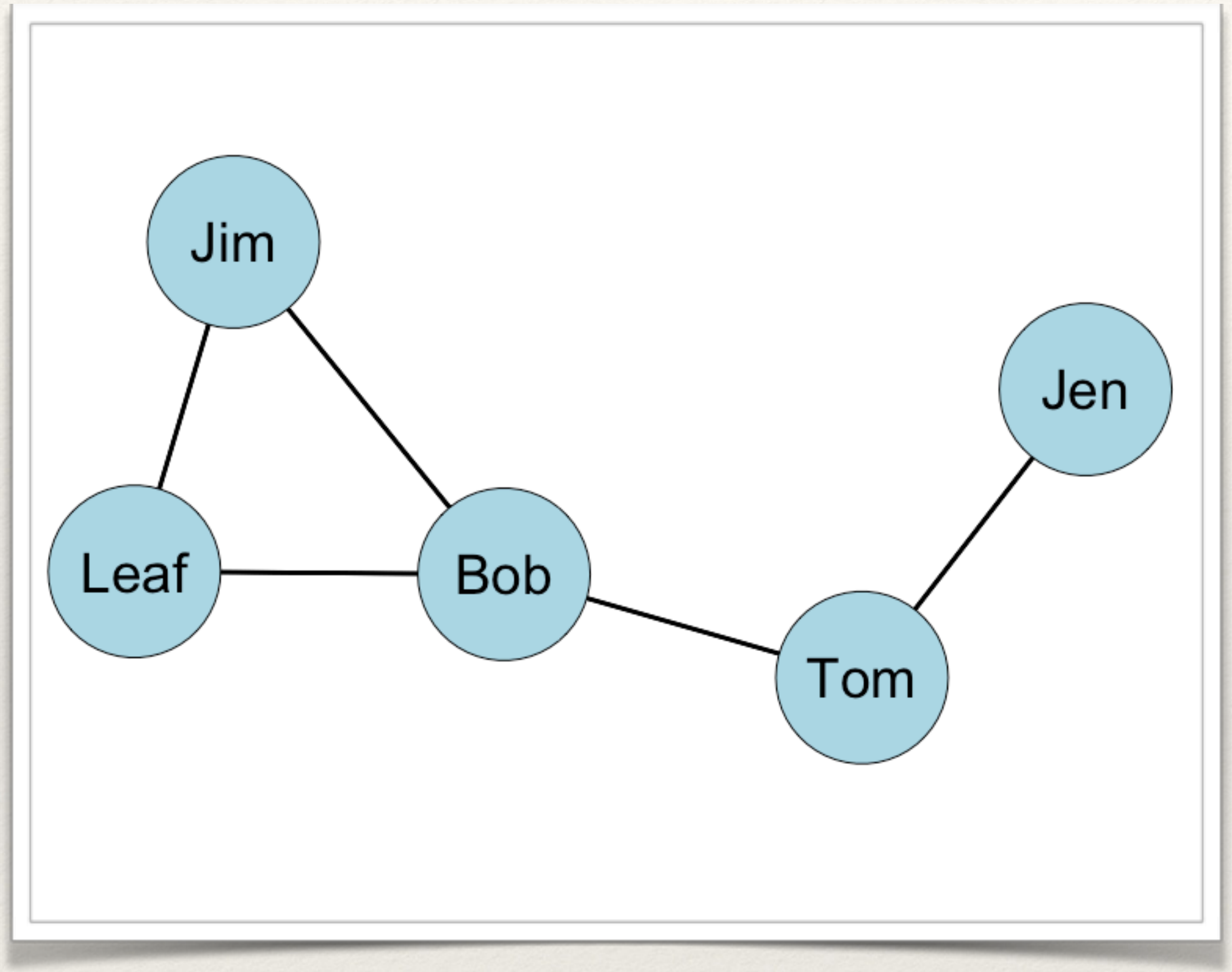
Example: Undirected, Binary Network



	Jen	Tom	Bob	Leaf	Jim
Jen	0	1	0	0	0
Tom	1	0	1	0	0
Bob	0	1	0	1	1
Leaf	0	0	1	0	1
Jim	0	0	1	1	0

What is the degree for each node in this graph?

Example: Undirected, Binary Network



	Jen	Tom	Bob	Leaf	Jim
Jen	0	1	0	0	0
Tom	1	0	1	0	0
Bob	0	1	0	1	1
Leaf	0	0	1	0	1
Jim	0	0	1	1	0

Note that the column sum and row sum are the same.

Degree Centrality: Undirected Binary Graphs

- Actor degree centrality not only reflects each node's connectivity to other nodes but also depends on the size of the network, g.
  - Larger networks will have a higher maximum possible degree centrality value.
    - *Solution?*

Standardized Degree Centrality: Undirected Binary Graphs

- Standardize!
  - Take into account the number of nodes and the maximum possible nodes to which *i* could be connected.
    - That is, *g-1*.

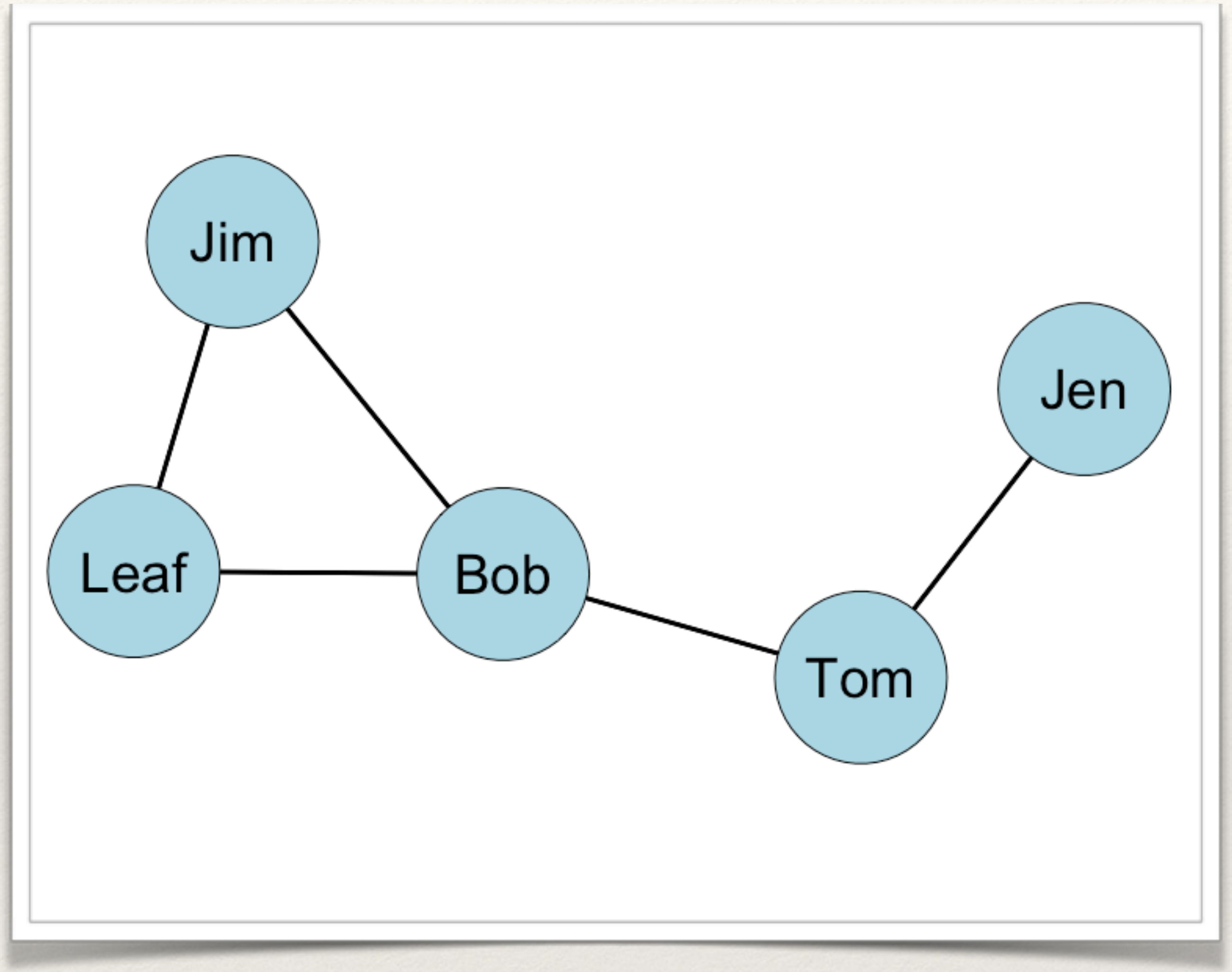
Standardized Degree Centrality: Undirected Binary Graphs

$$C'_D(n_i) = \frac{d(n_i)}{g-1} = \frac{\sum_j x_{ij}}{g-1}$$

Standardized Degree Centrality: Undirected Binary Graphs

- This yields the proportion of the network members with ties to actor *i*.
    - This varies between 0 (no connections; isolate) to 1 (ties to every actor).

Example: Undirected, Binary Network



<u>Raw Degree Centrality</u>

Jen = 1

Tom = 2

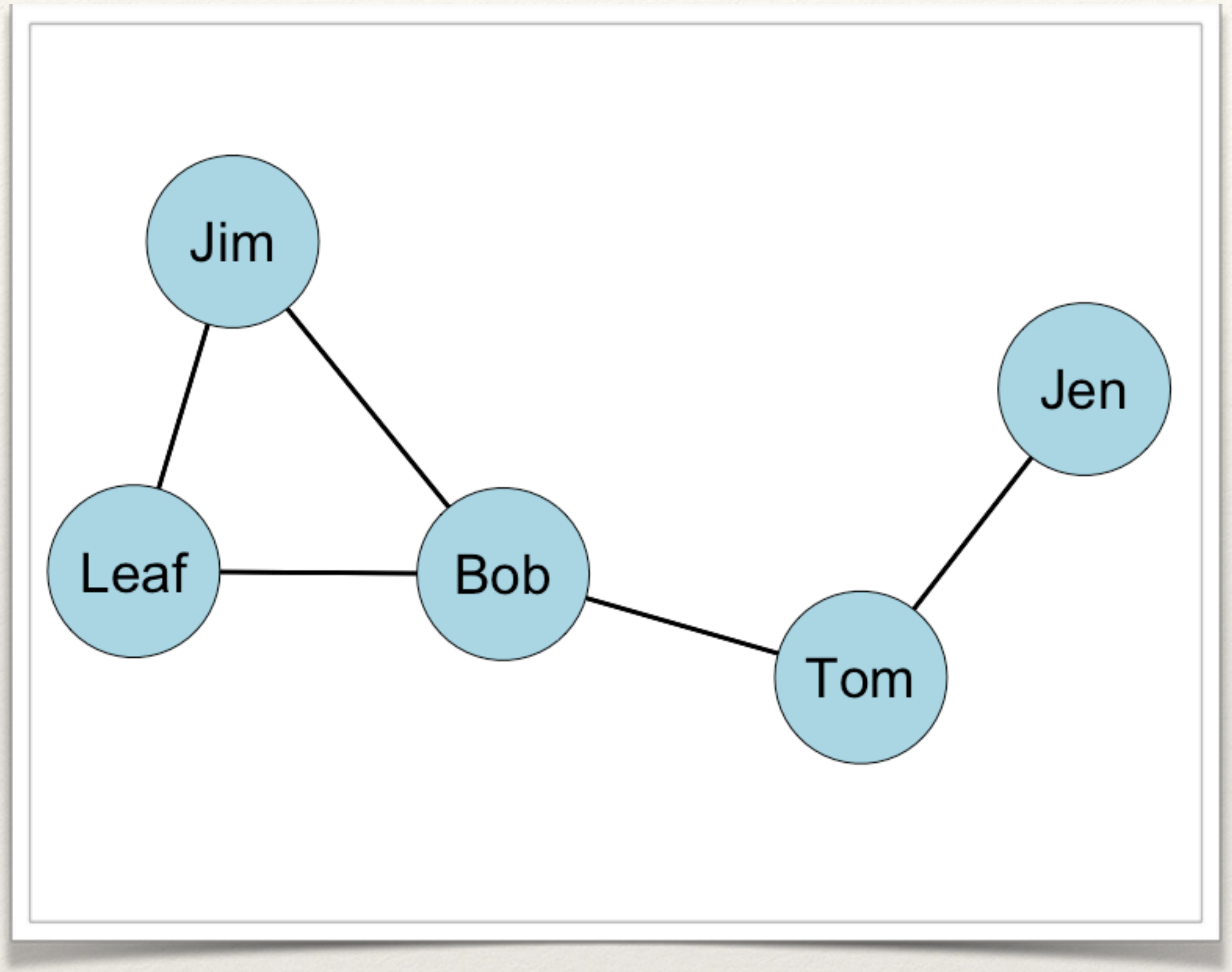
Bob = 3

Leaf = 2

Jim = 2

What is the standardized degree centrality score for each node?

Example: Undirected, Binary Network



Standardized Degree Centrality

Jen = 1/4 = 0.25

Tom = 2/4 = 0.50

Bob = 3/4 = 0.75

Leaf = 2/4 = 0.50

Jim = 2/4 = 0.50

Summarizing Degree Centrality

- We can examine the summary statistics for degree centrality by inspecting the **mean**.
  - The average degree is an important property of a network.
    - *Why? What does a network with a high average degree look like? A low average degree?*

Mean Degree (undirected)

Sum up the degrees for each actor

$$\bar{d} = \frac{\sum_{i=1}^{g} d(n_i)}{g} = \frac{2L}{g}$$

Divide by number of actors

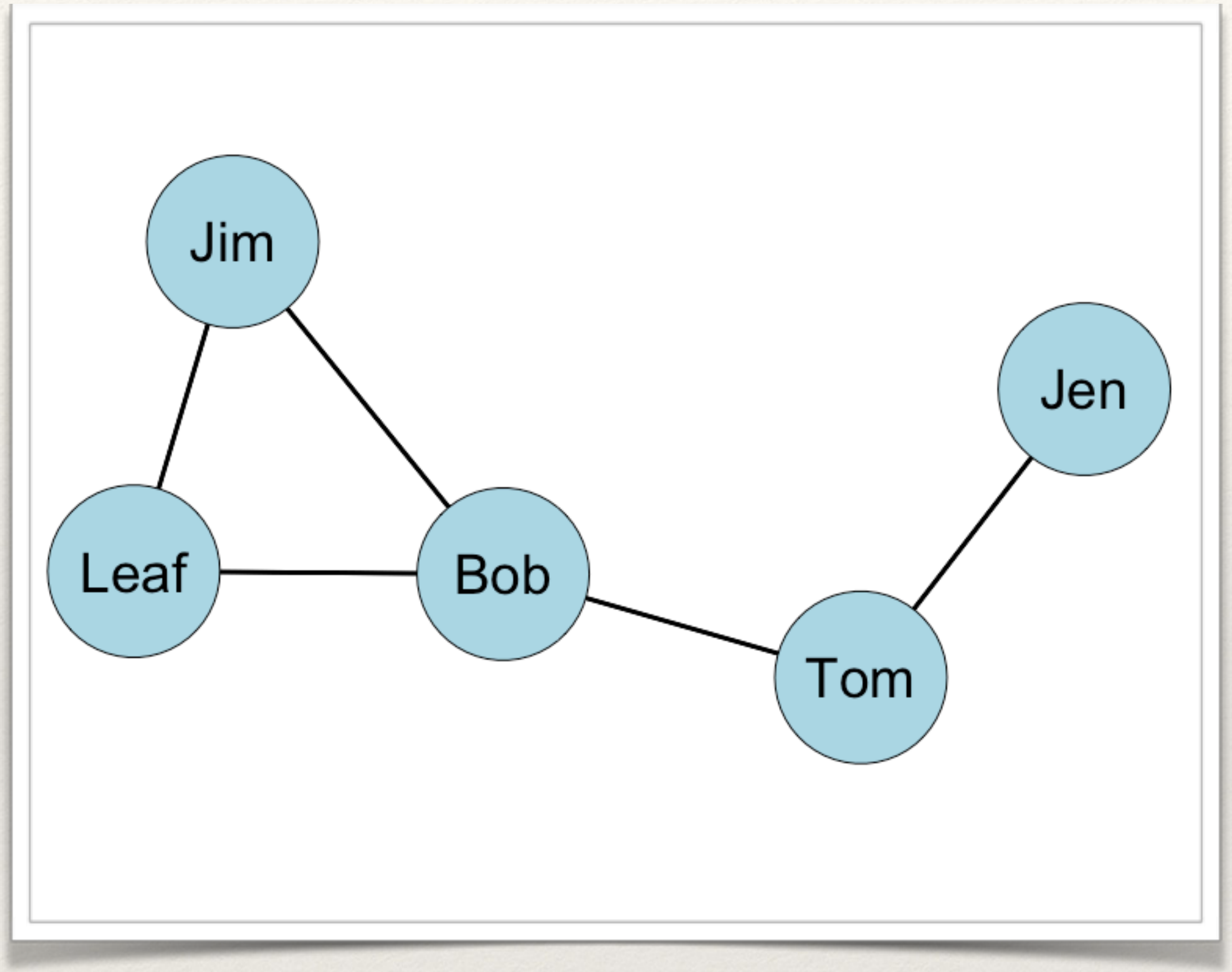
Mean Degree (undirected)

Or, multiply the number of edges by 2.

$$\bar{d} = \frac{\sum_{i=1}^{g} d(n_i)}{g} = \frac{2L}{g}$$

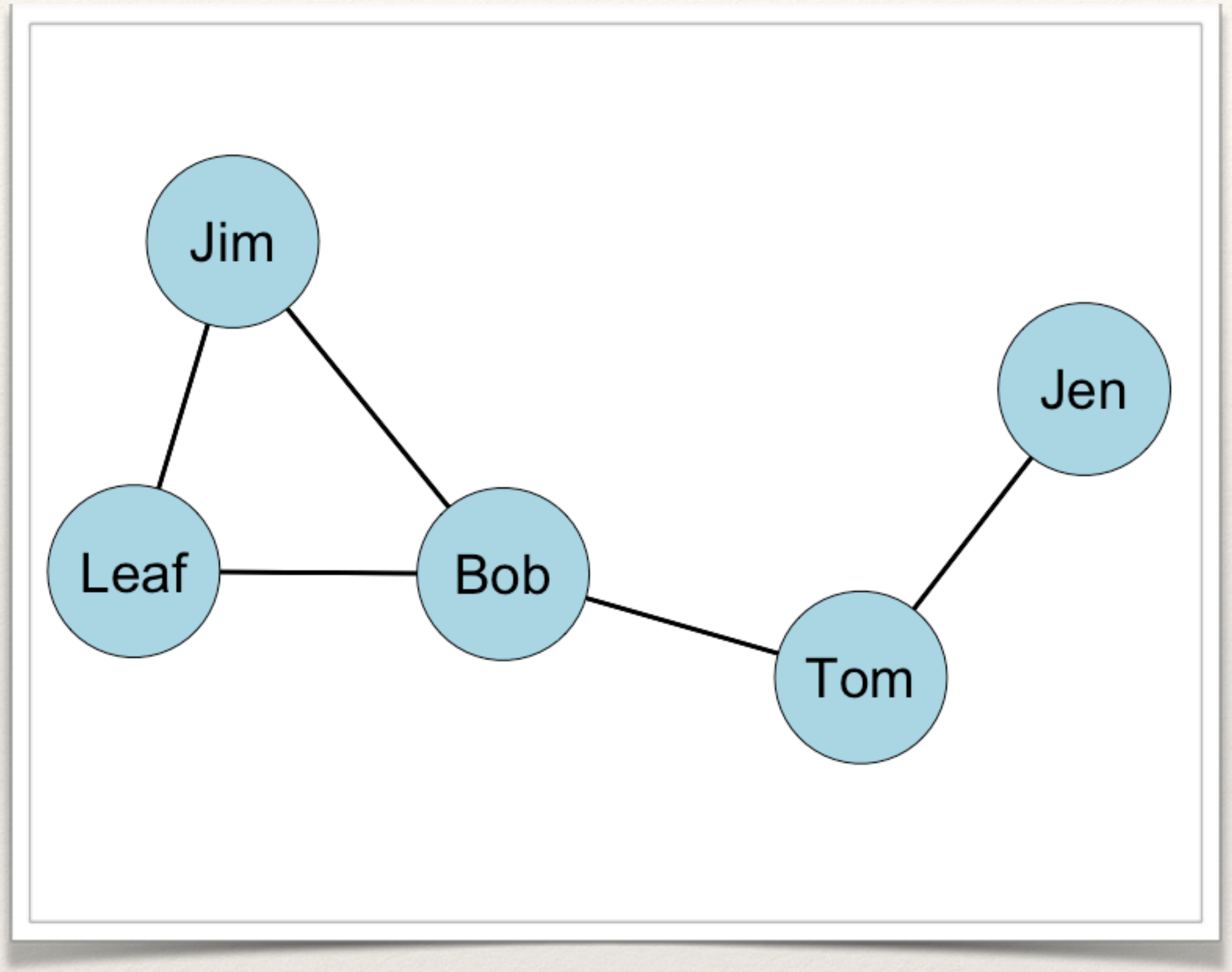
Divide by number of actors

Example: Undirected, Binary Network



What is the mean degree for this graph?

Example: Undirected, Binary Network



$$\bar{d} = \frac{\sum_{i=1}^{g} d(n_i)}{g} = \frac{2L}{g} = \frac{2 * 5}{5} = \frac{10}{5} = 2$$

What is the mean degree for this graph?

Summarizing Degree Centrality

- We can also calculate how centralized the graph itself is.
  - *Group degree centralization* measures the extent to which the actors in a social network differ from one another in their individual degree centralities.

Group Degree Centralization

Largest actor degree centrality scored observed

Degree centrality for actor *i*

$$C_A = \frac{\sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]}{max \sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]}$$

Group Degree Centralization

$$C_A = \frac{\sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]}{max \sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]}$$

Sum of observed differences between the largest actor centrality and all others

Theoretical maximum possible sum of those differences

Index of Group Degree Centralization

$$C_A = \frac{\sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]}{max \sum_{i=1}^{g}[C_A(n^*) - C_A(n_i)]}$$

Sum of observed differences between the largest actor centrality and all others

Theoretical maximum possible sum of those differences

Summarizing Degree Centrality

- Note that this is a generic measure (thanks Freeman, 1979!)
  - We can calculate the denominator as *(g-1)(g-2)* (Thanks Wasserman & Faust, 1994!)

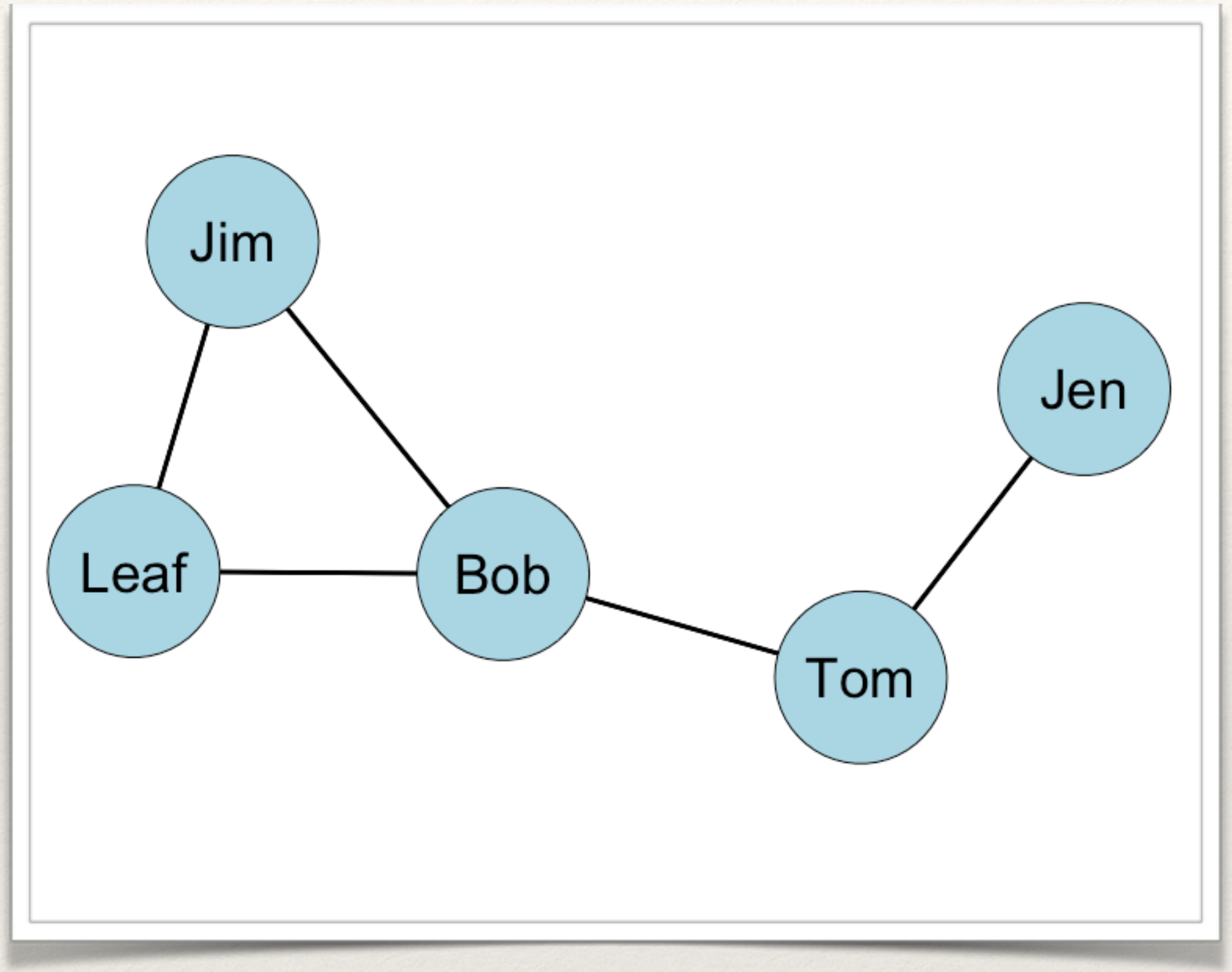
Index of Group Degree Centralization

$$C_D = \frac{\sum_{i=1}^{g} [C_D(n^*) - C_D(n_i)]}{[(g-1)(g-2)]}$$

Sum of observed differences between the largest actor centrality and all others

The maximum possible sum of differences

Example: Undirected, Binary Network



Raw Degree Centrality

Jen = 1

Tom = 2

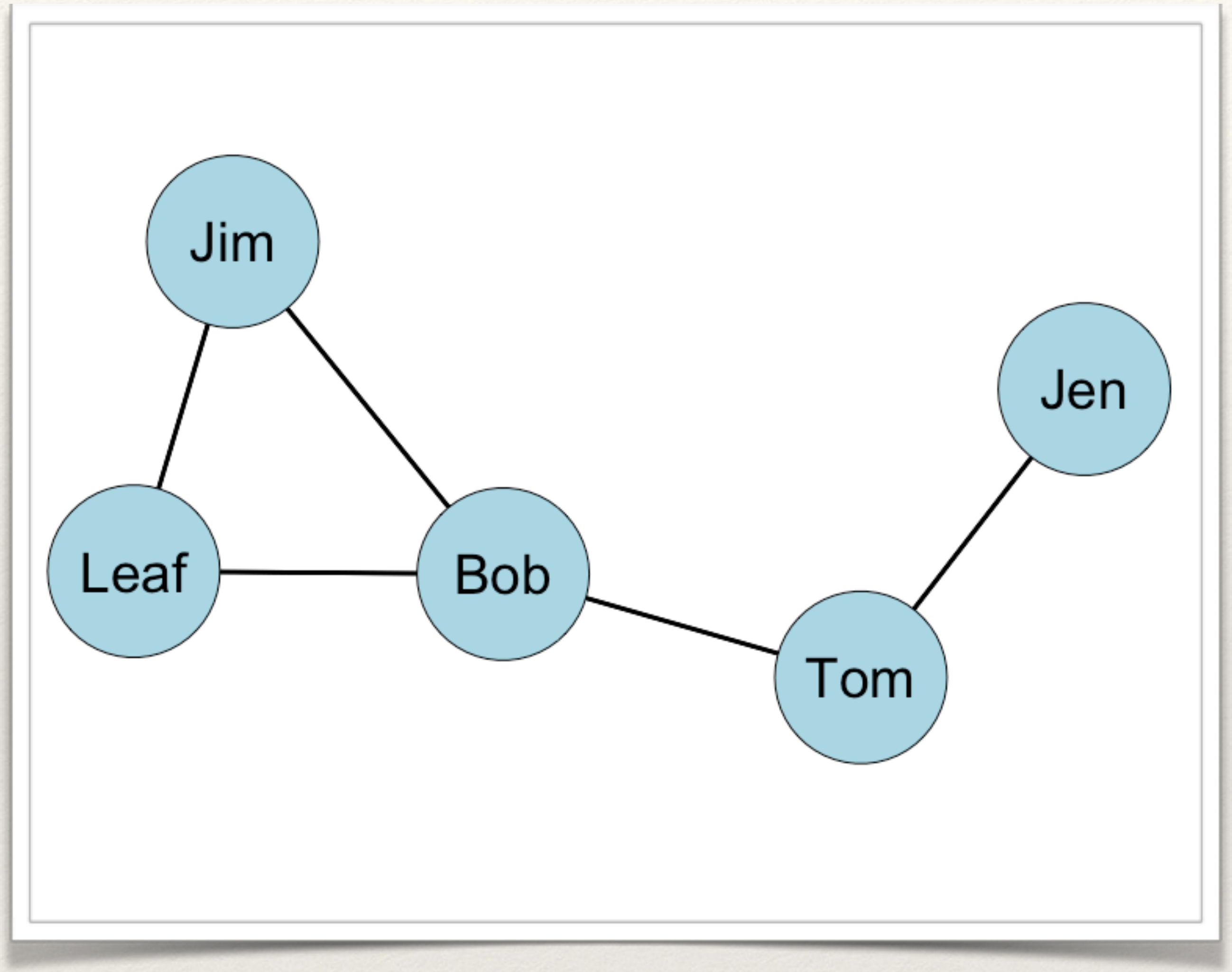
Bob = 3

Leaf = 2

Jim = 2

What is the index of degree centralization for this graph?

Example: Undirected, Binary Network



0.4167

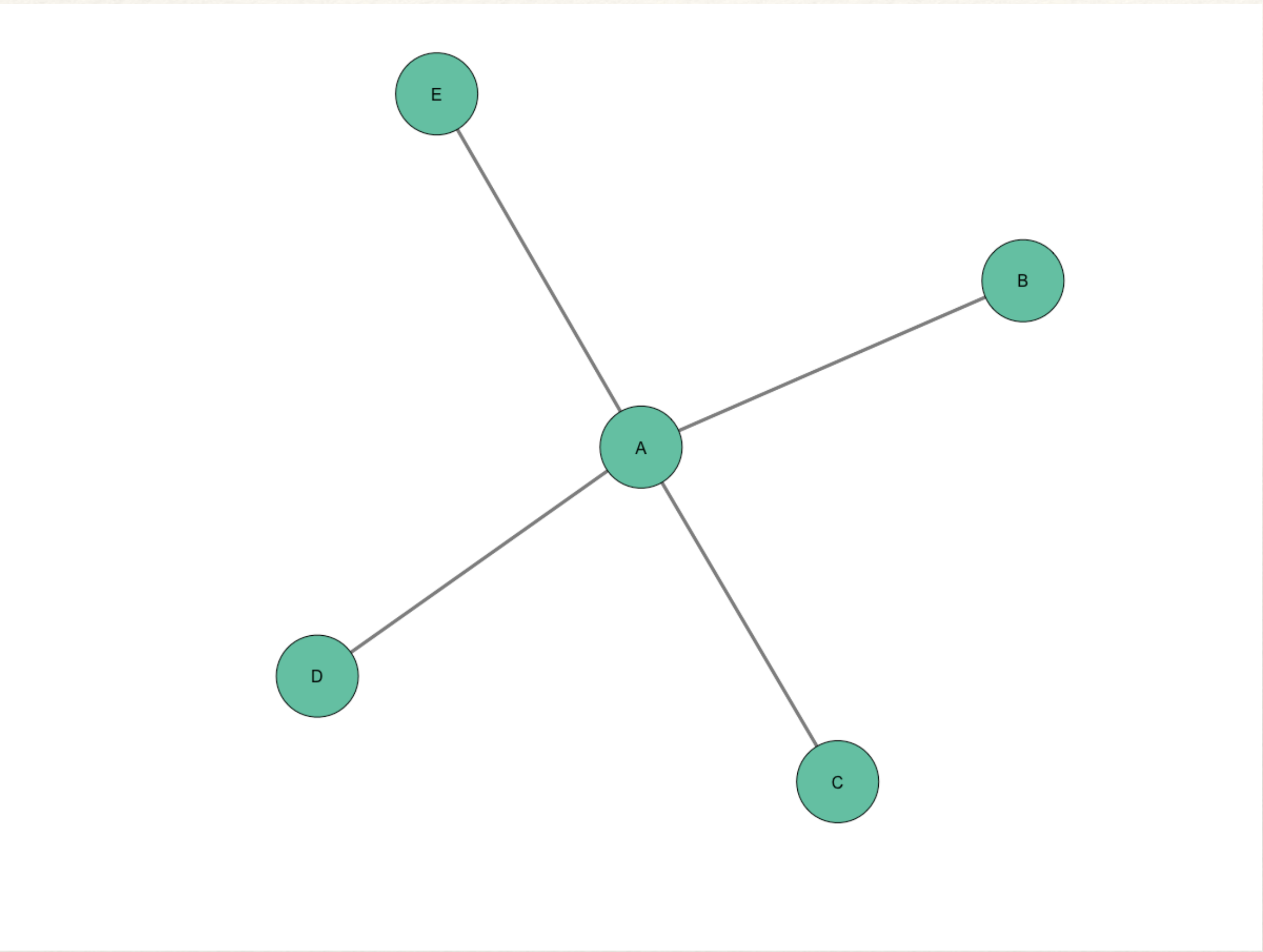
$$C_D = \frac{\sum_{i=1}^{g} [C_D(n^*) - C_D(n_i)]}{[(g-1)(g-2)]}$$

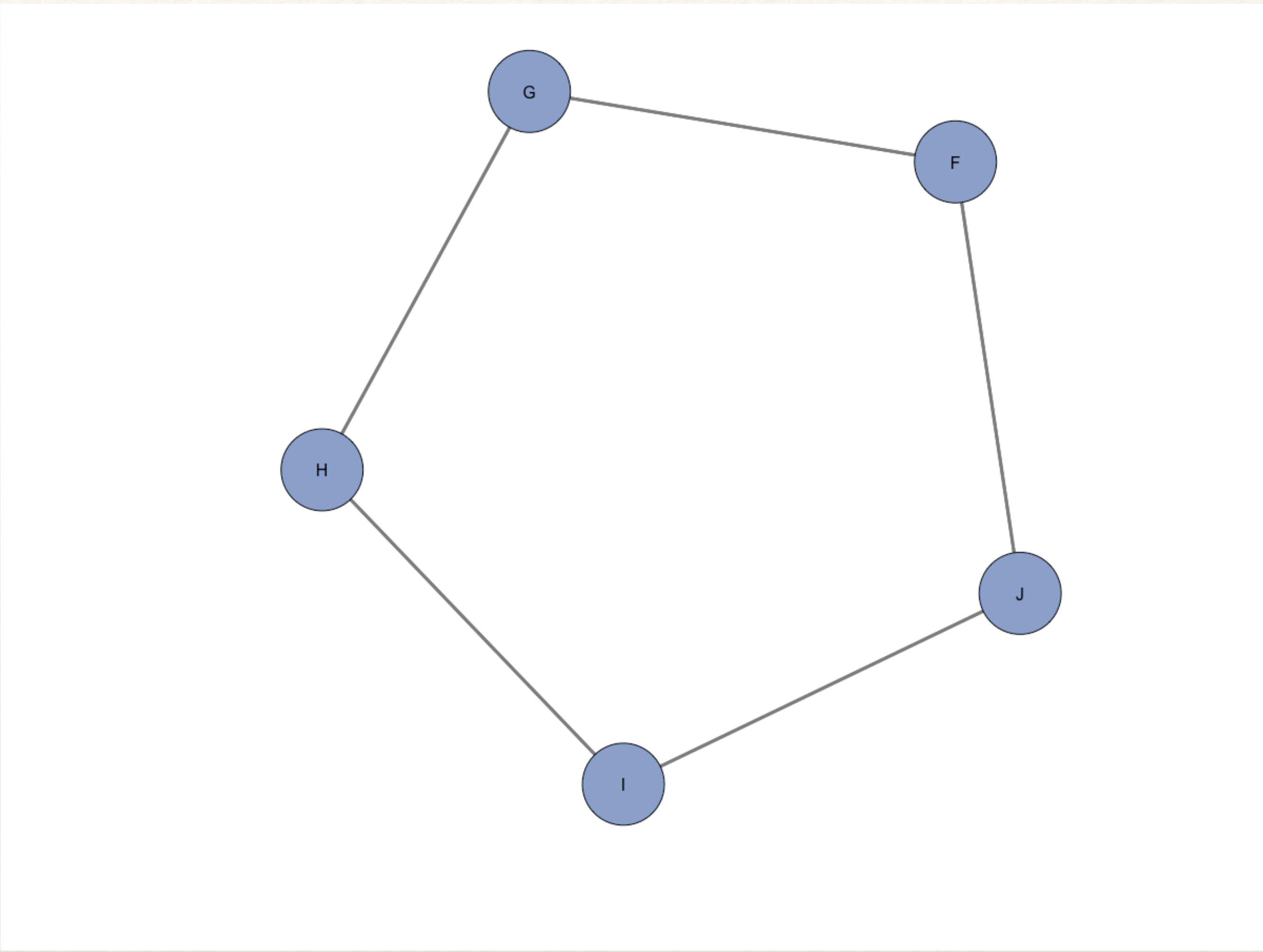
$$= \frac{(3-1)+(3-2)+(3-3)+(3-2)+(3-2)}{(5-1)(5-2)}$$

$$= \frac{2+1+0+1+1}{4 * 3} = \frac{5}{12} = 0.4167$$

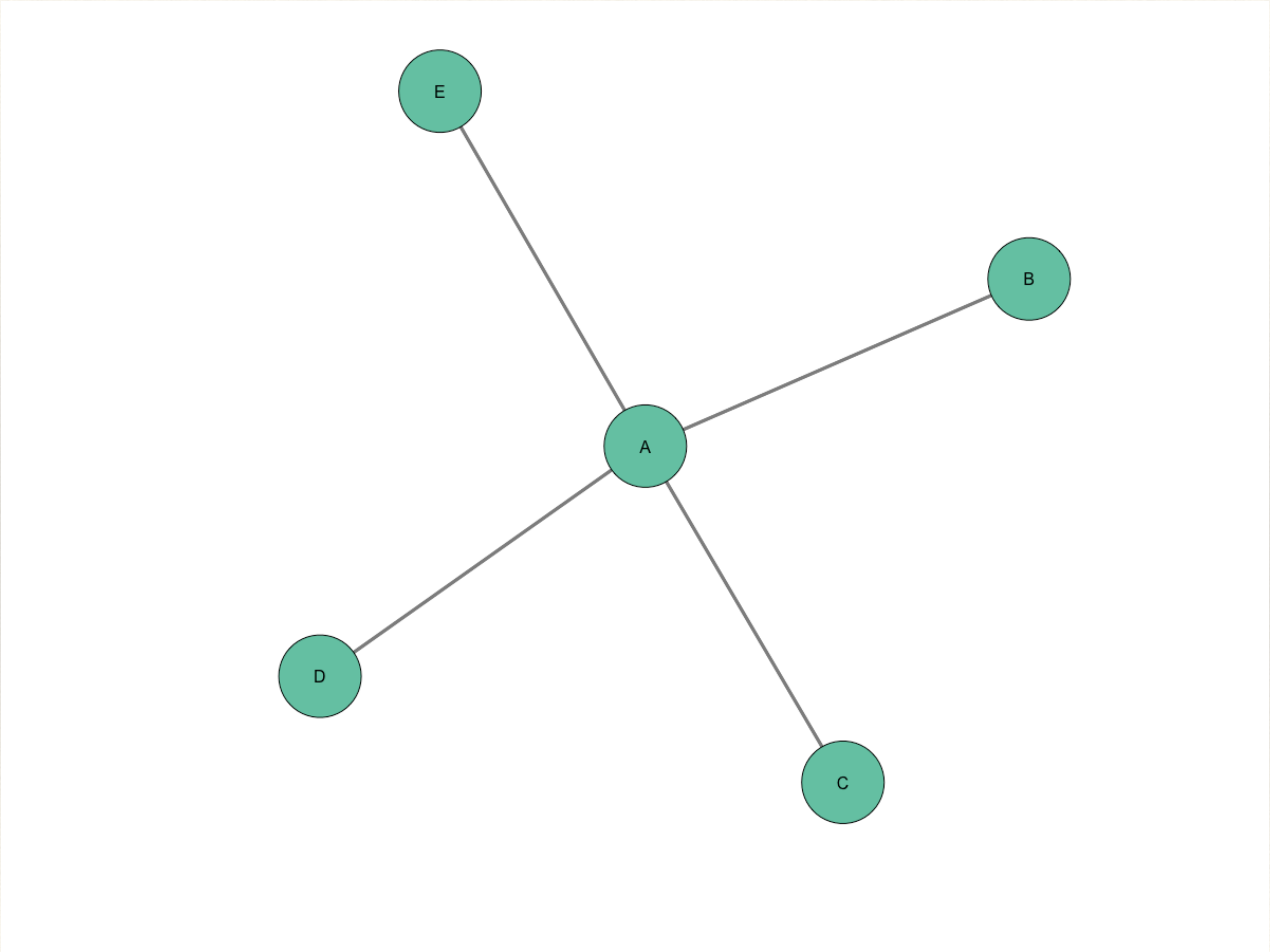
Summarizing Degree Centrality

- When degree centrality is evenly dispersed, the numerator will be zero, and the quotient will be close to 0.
  - When there is considerable inequality in the actor degrees, the quotient will be closer to 1.
    - Thus, closer to 1 indicates that the graph is hierarchically structured.

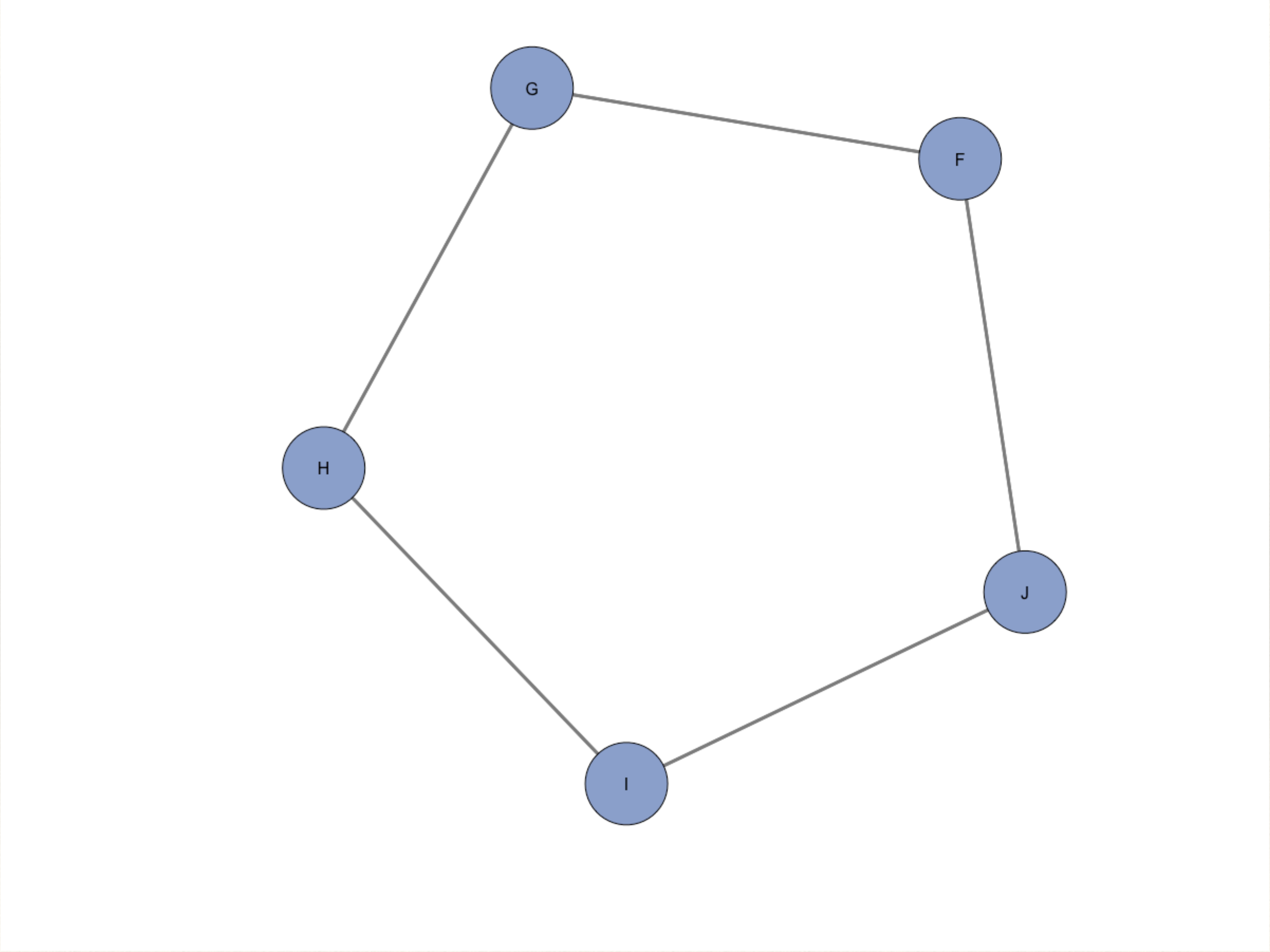

E
B
A
D
C


G
F
H
J
I

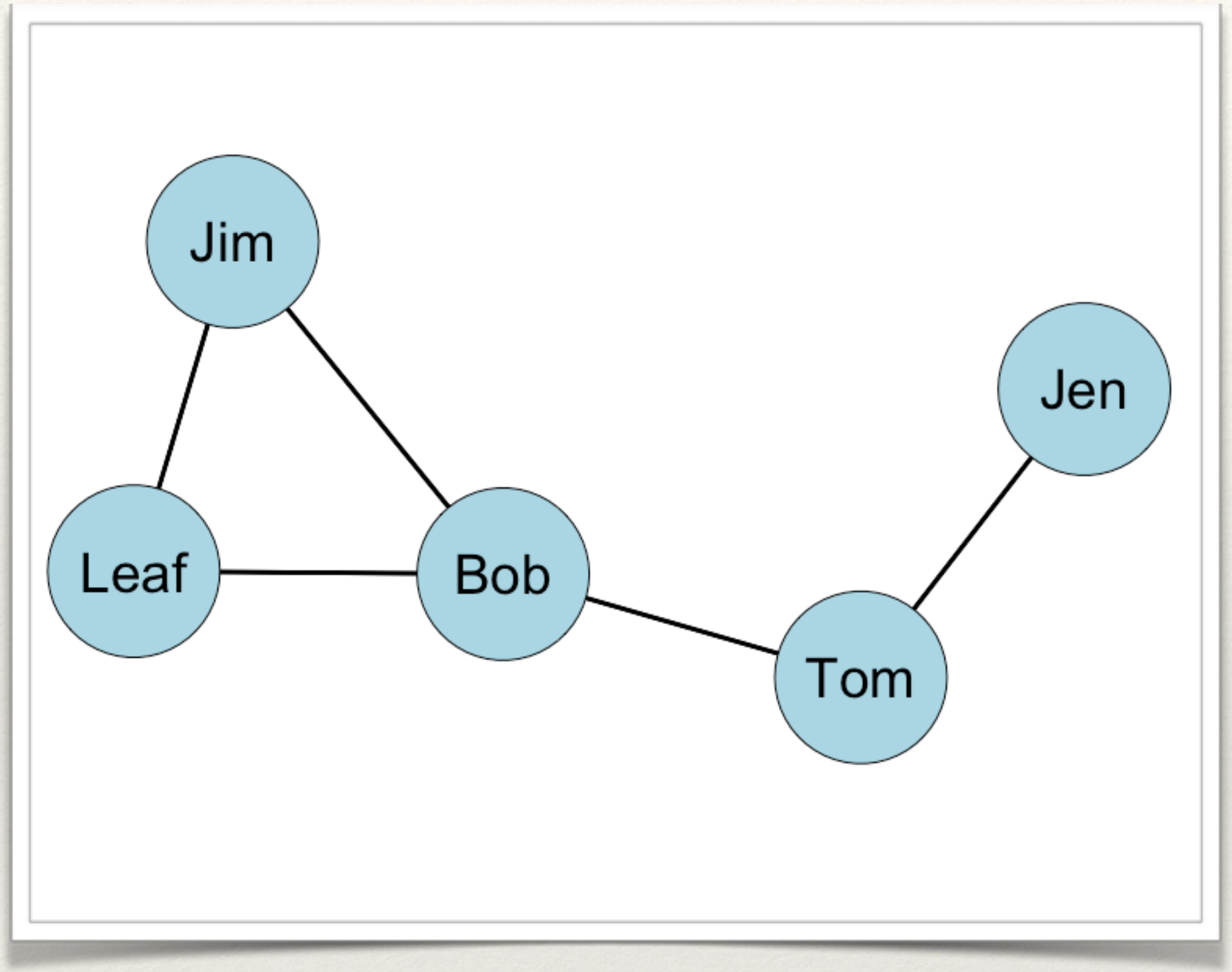
$$C_D = \frac{\sum_{i=1}^{g}[C_D(n^*) - C_D(n_i)]}{[(g-1)(g-2)]} = \frac{(4-4)+(4-1)+(4-1)+(4-1)+(4-1)}{(5-1)(5-2)} = \frac{0+3+3+3+3}{4*3} = \frac{12}{12} = 1.0$$



$$C_D = \frac{\sum_{i=1}^{g}[C_D(n^*) - C_D(n_i)]}{[(g-1)(g-2)]} = \frac{(2-2)+(2-2)+(2-2)+(2-2)+(2-2)}{(5-1)(5-2)} = \frac{0+0+0+0+0}{4*3} = \frac{0}{12} = 0.0$$



Example: Undirected, Binary Network



How should we interpret this value?

0.4167

Directed Networks

Degree Centrality: Directed Binary Graphs

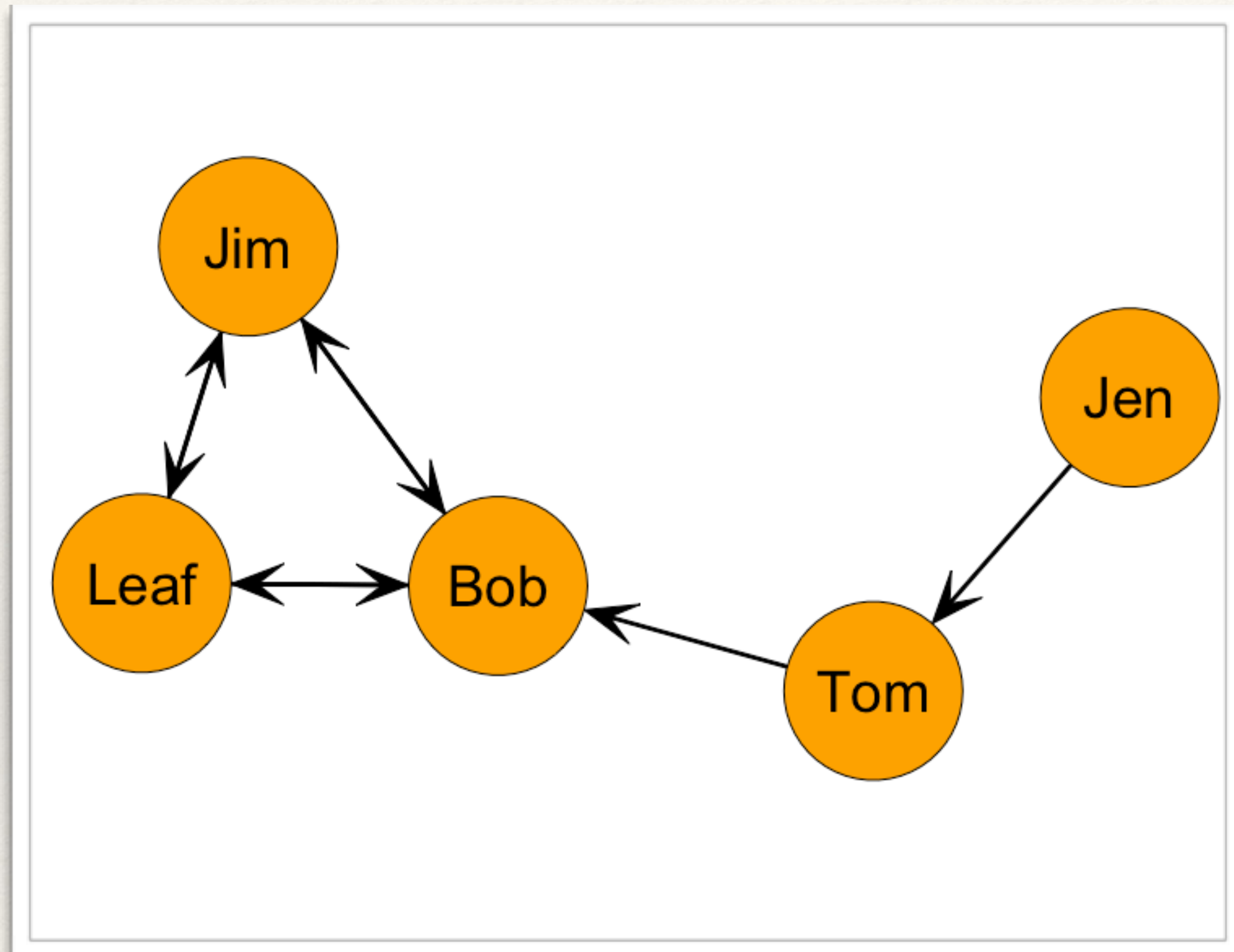
- In a directed binary graph, *actor degree centrality* can be broken down into indegree and oudegree centrality.
  - **Indegree**, $C_I(n_i)$, measures the number of ties that i receives.
    - For the sociomatrix X_{ij}, the indegree for i is the column sum.
  - **Outdegree**, $C_O(n_i)$, measures the number of ties that i sends.
    - For the sociomatrix X_{ij}, the outdegree for i is the row sum.

Degree Centrality: Directed Binary Graphs

$$C_I(n_i) = \sum_j x_{ji}$$

$$C_O(n_i) = \sum_j x_{ij}$$

Example: Directed, Binary Network



	Jen	Tom	Bob	Leaf	Jim
Jen	0	1	0	0	0
Tom	0	0	1	0	0
Bob	0	0	0	1	1
Leaf	0	0	1	0	1
Jim	0	0	1	1	0

What is the indegree and outdegree for each node in the graph?

Example: Directed, Binary Network

NOTE: These both sum to the same value

Raw Indegree Centrality

Jen = 0
Tom = 1
Bob = 3
Leaf = 2
Jim = 2
TOTAL: 8

Raw Outdegree Centrality

Jen = 1
Tom = 1
Bob = 2
Leaf = 2
Jim = 2
TOTAL: 8

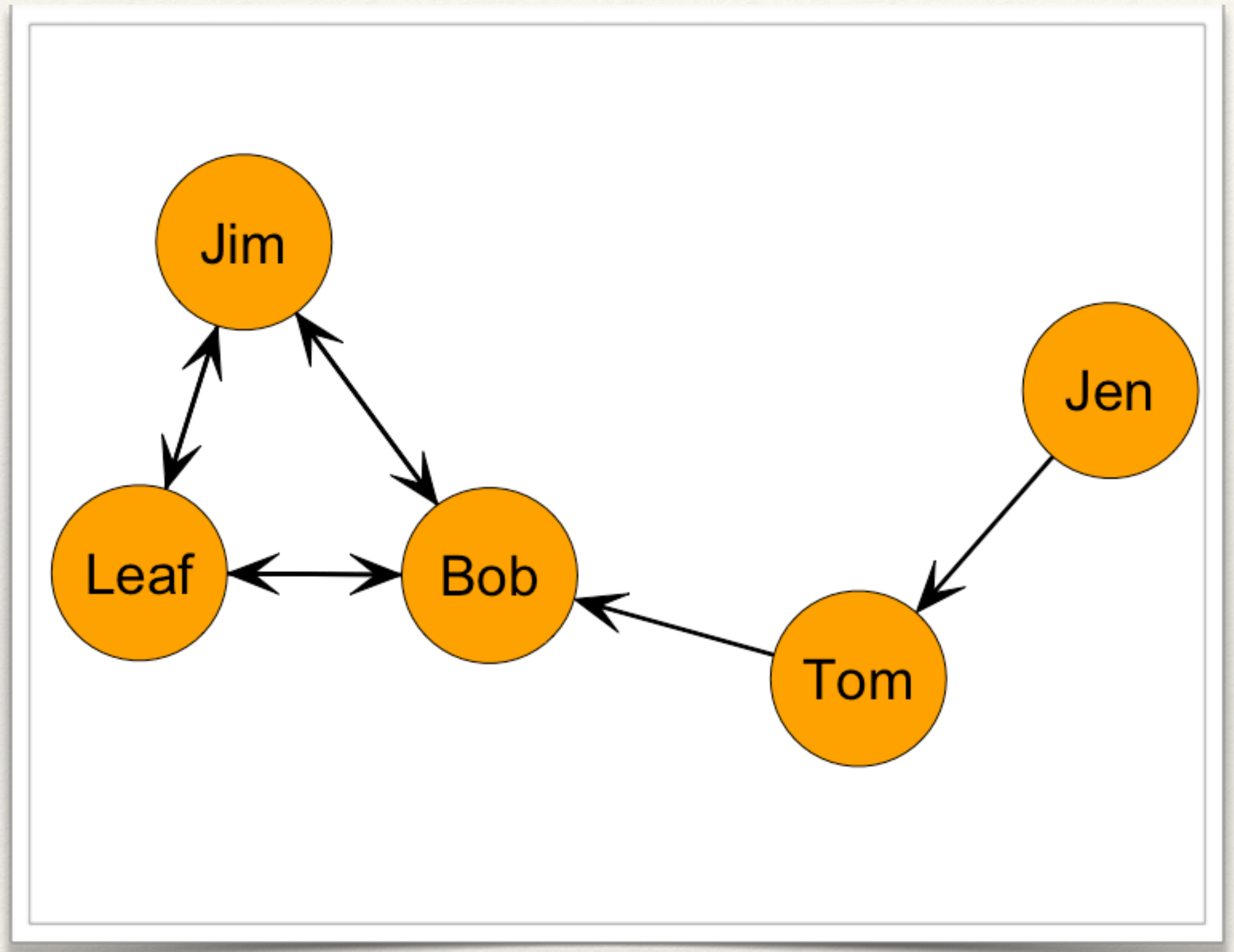
Degree Centrality: Directed Binary Graphs

- Recall that actor degree centrality not only reflects each node's connectivity to other nodes but also depends on the size of the network, *g*.
  - Larger networks will have a higher maximum possible degree centrality value.
    - We can standardize, or normalize, the same way by dividing by *g-1*.

Standardized Degree Centrality: Directed Binary Graphs

$$C'_D(n_i) = \frac{C_I(n_i)}{g-1} = \frac{C_O(n_i)}{g-1}$$

Example: Directed, Binary Network



What is the standardized indegree and outdegree centrality score for each node?

<u>Raw Indegree Centrality</u>

Jen = 0

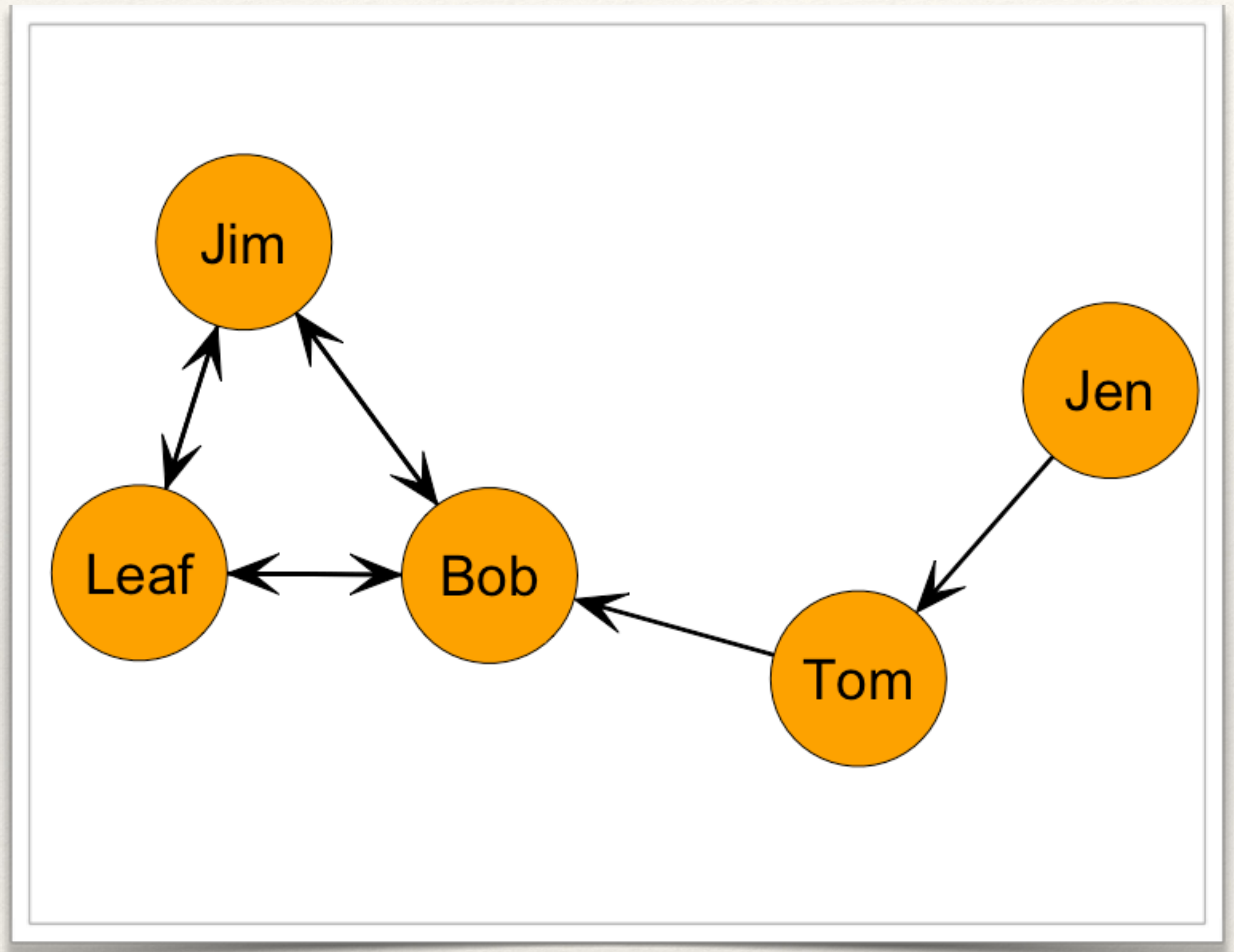
Tom = 1

Bob = 3

Leaf = 2

Jim = 2

Example: Directed, Binary Network



What is the standardized indegree and outdegree centrality score for each node?

Standardized Indegree Centrality

Jen = 0/4 = 0

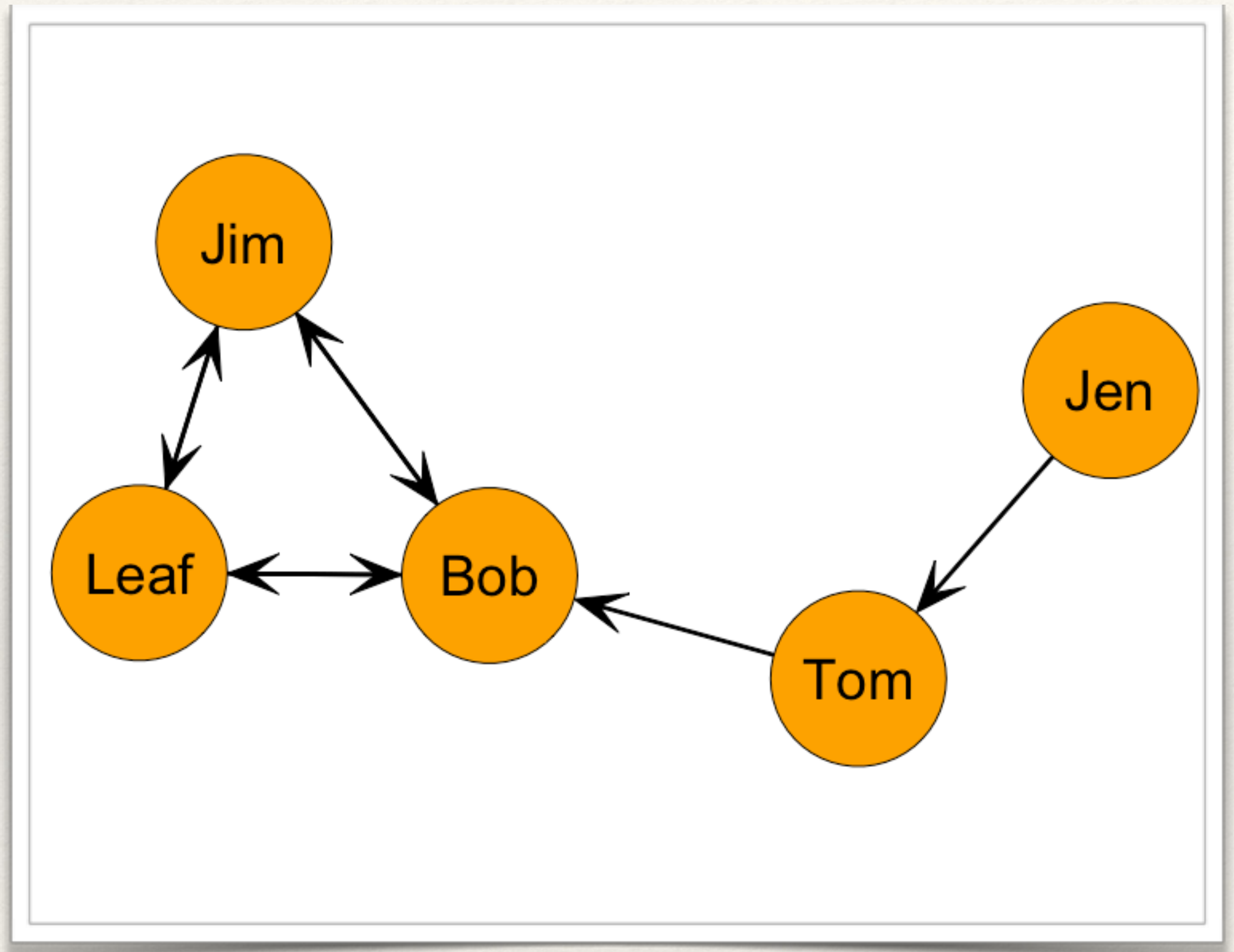
Tom = 1/4 =0.25

Bob = 3/4 = 0.75

Leaf = 2/4 = 0.50

Jim = 2/4 = 0.50

Example: Directed, Binary Network



What is the standardized indegree and outdegree centrality score for each node?

<u>Raw Indegree Centrality</u>

Jen = 0

Tom = 1

Bob = 3

Leaf = 2

Jim = 2

<u>Raw Outdegree Centrality</u>

Jen = 1

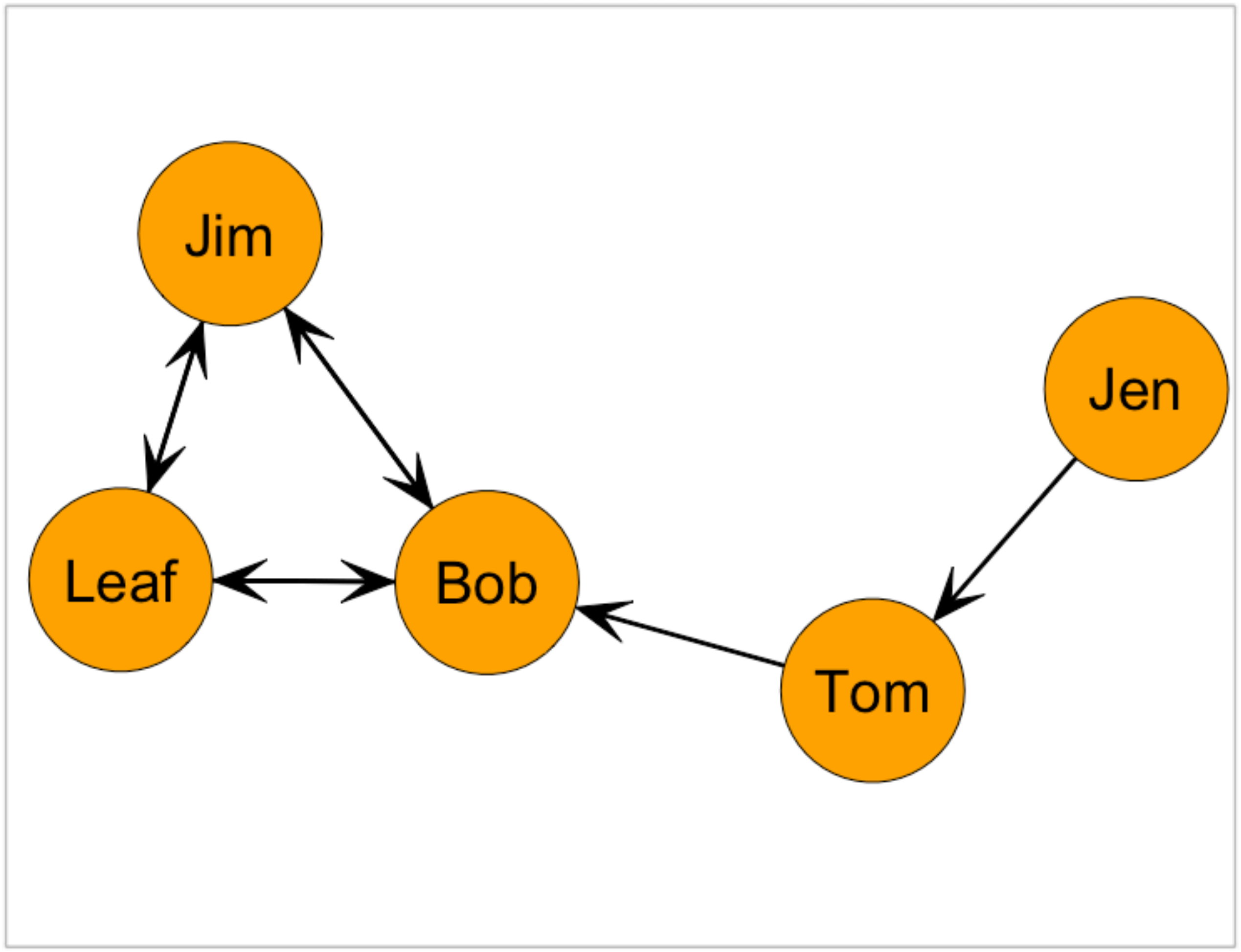
Tom = 1

Bob = 2

Leaf = 2

Jim = 2

Example: Directed, Binary Network



What is the standardized indegree and outdegree centrality score for each node?

Standardized Indegree Centrality

Jen = 0/4 = 0

Tom = 1/4 =0.25

Bob = 3/4 = 0.75

Leaf = 2/4 = 0.50

Jim = 2/4 = 0.50

Standardized Outdegree Centrality

Jen = 1/4 = 0.25

Tom = 1/4 = 0.25

Bob = 2/4 = 0.50

Leaf = 2/4 = 0.50

Jim = 2/4 = 0.50

Summarizing Degree Centrality

- As before, we can examine the summary statistics for degree centrality by inspecting the **mean**.

Mean Degree (directed)

Or, number of edges

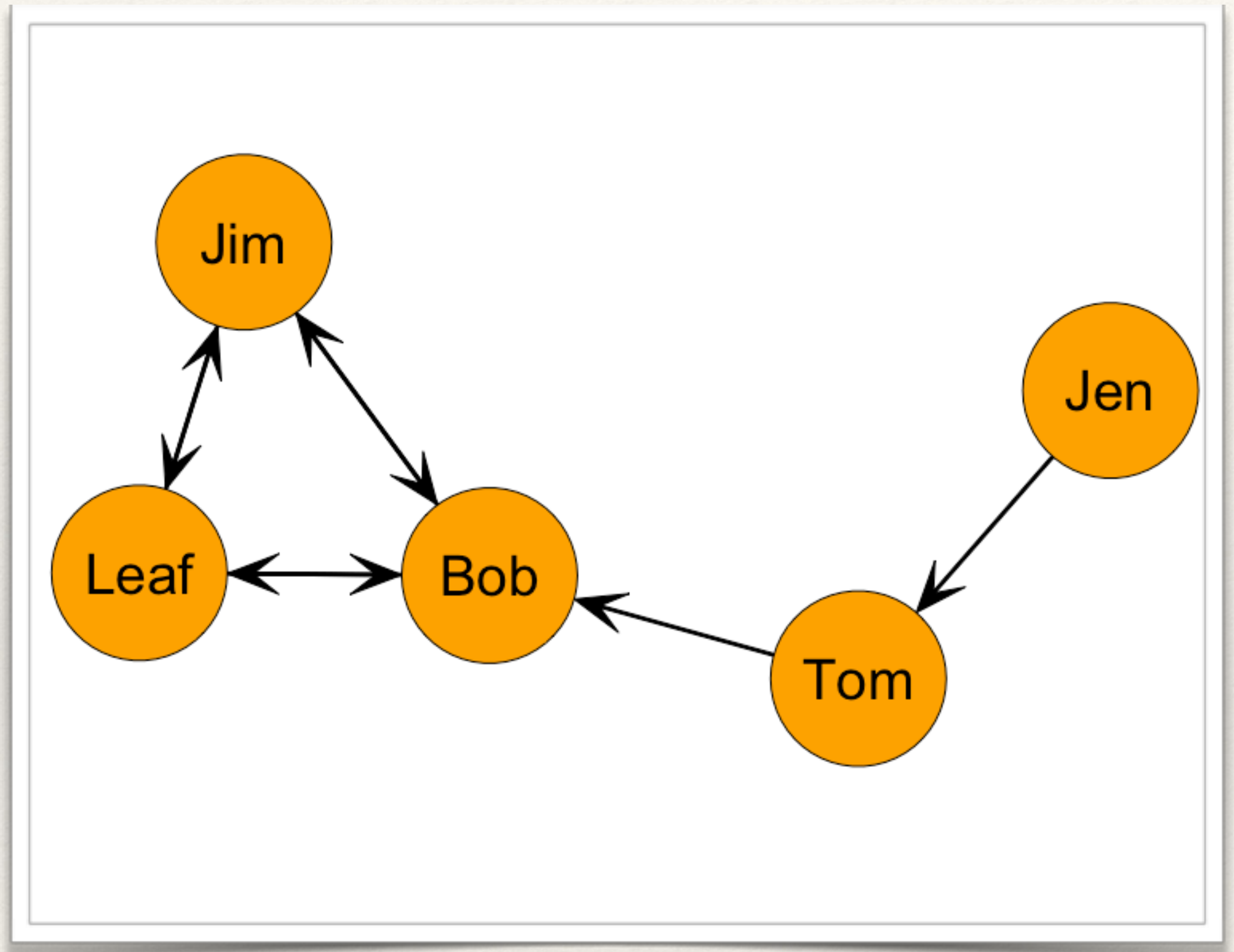
$$\bar{d} = \frac{\sum_{i=1}^{g} C_I(n_i)}{g} = \frac{\sum_{i=1}^{g} C_O(n_i)}{g} = \frac{L}{g}$$

Divide by number of actors

Summarizing Degree Centrality

- The mean indegree is equal to the mean outdegree.
  - *Why?*

Example: Directed, Binary Network



$$\bar{d} = \frac{C_I(n_i)}{g} = \frac{C_O(n_i)}{g} = \frac{L}{g} = \frac{8}{5} = 1.6$$

What is the mean indegree/ outdegree for this graph?

Summarizing Degree Centrality

- We can also calculate how centralized the graph itself is.
  - *Group degree centralization* measures the extent to which the actors in a social network differ from one another in their individual degree centralities.
    - The difference here is that the denominator is $(g-1)^2$ or $(g-1)(g-1)$.
      - Note that the numerator may differ though.

Index of Group Degree Centralization

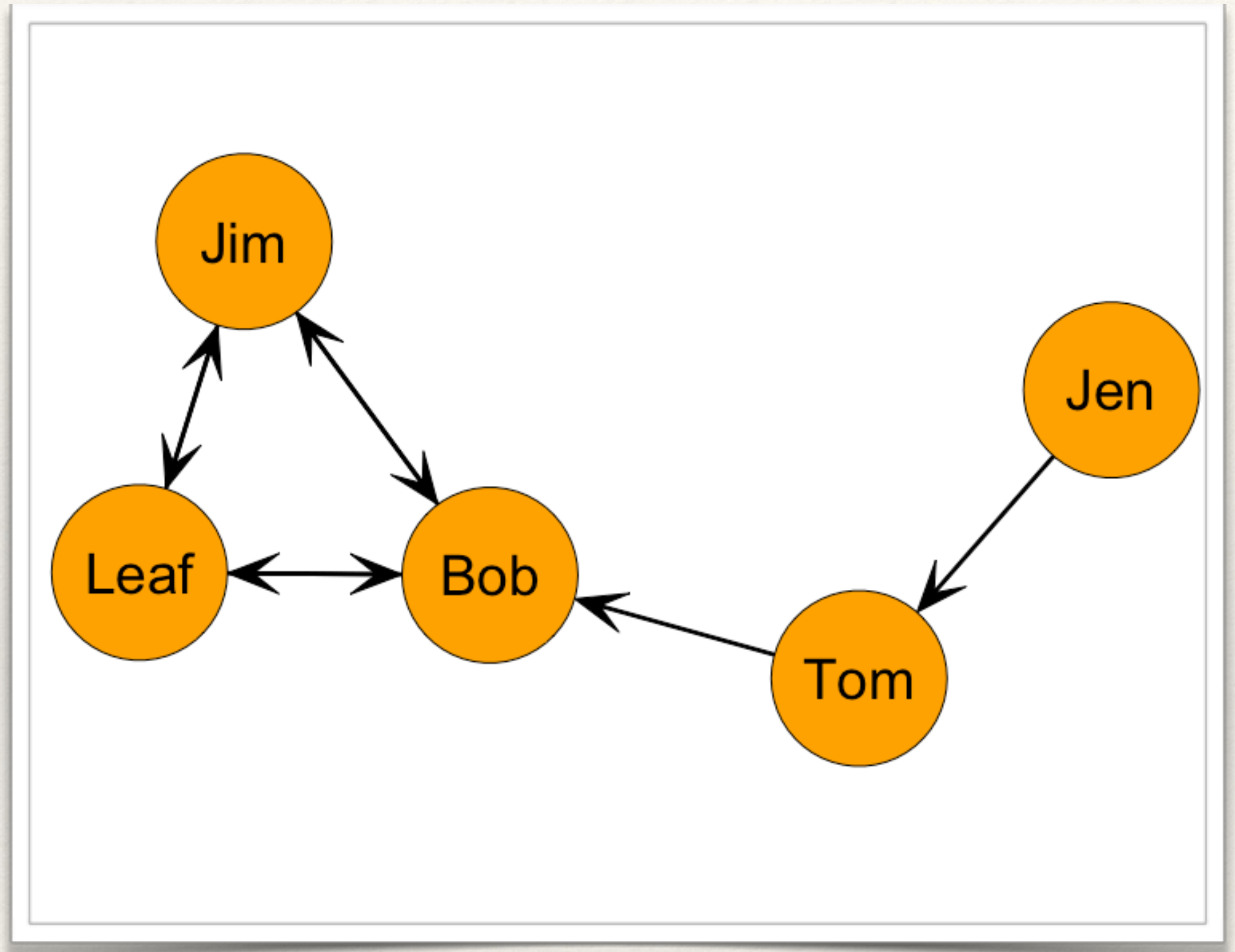
$$C_D = \frac{\sum_{i=1}^{g} [C_D(n^*) - C_D(n_i)]}{[(g-1)^2]}$$

Sum of observed differences between the largest actor centrality and all others

The maximum possible sum of differences

Note the difference (see W&F p. 199)

Example: Directed, Binary Network



Raw Indegree Centrality

Jen = 0

Tom = 1

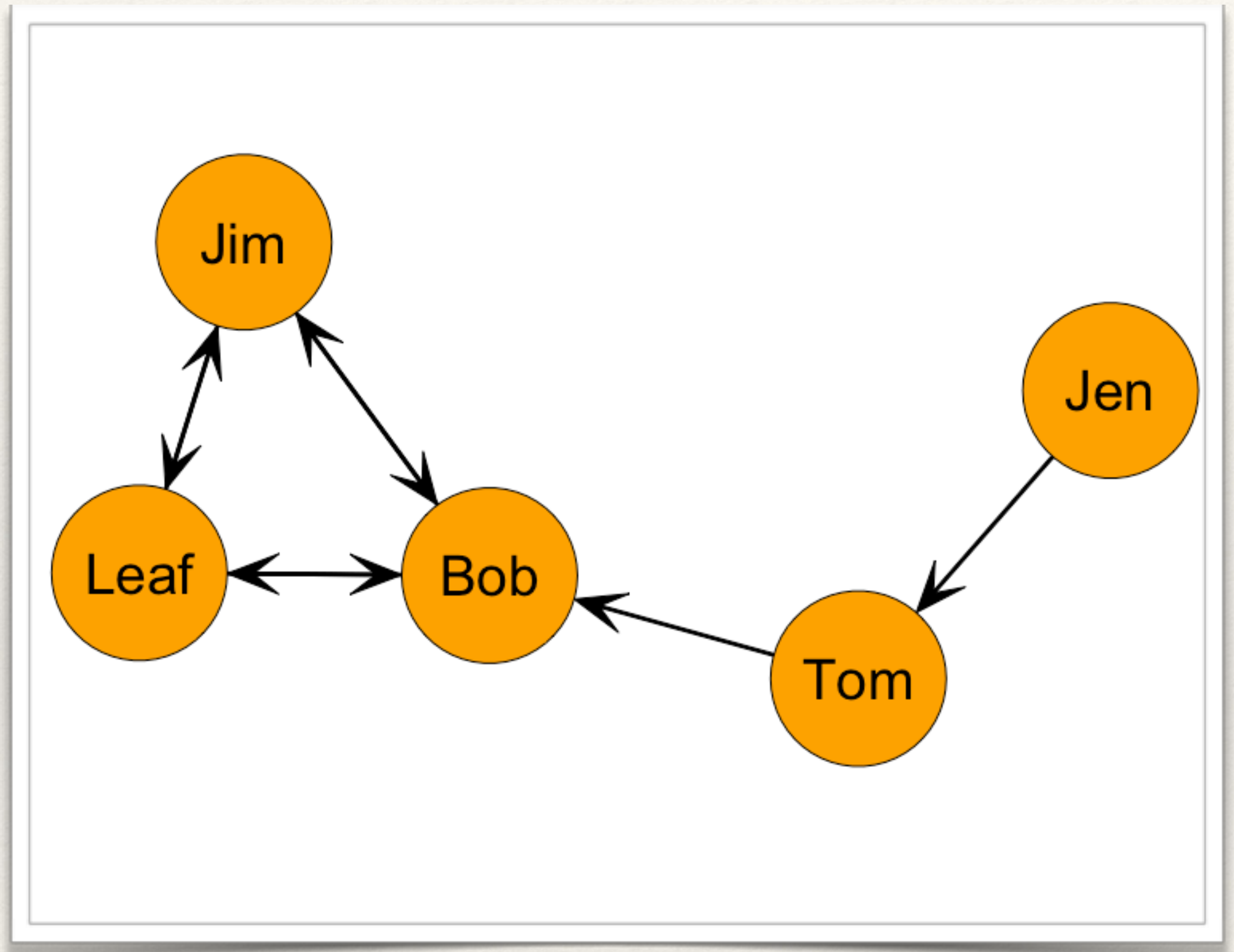
Bob = 3

Leaf = 2

Jim = 2

What is the index of indegree centralization for this graph?

Example: Directed, Binary Network



Raw Indegree Centrality

Jen = 0

Tom = 1

Bob = 3

Leaf = 2

Jim = 2

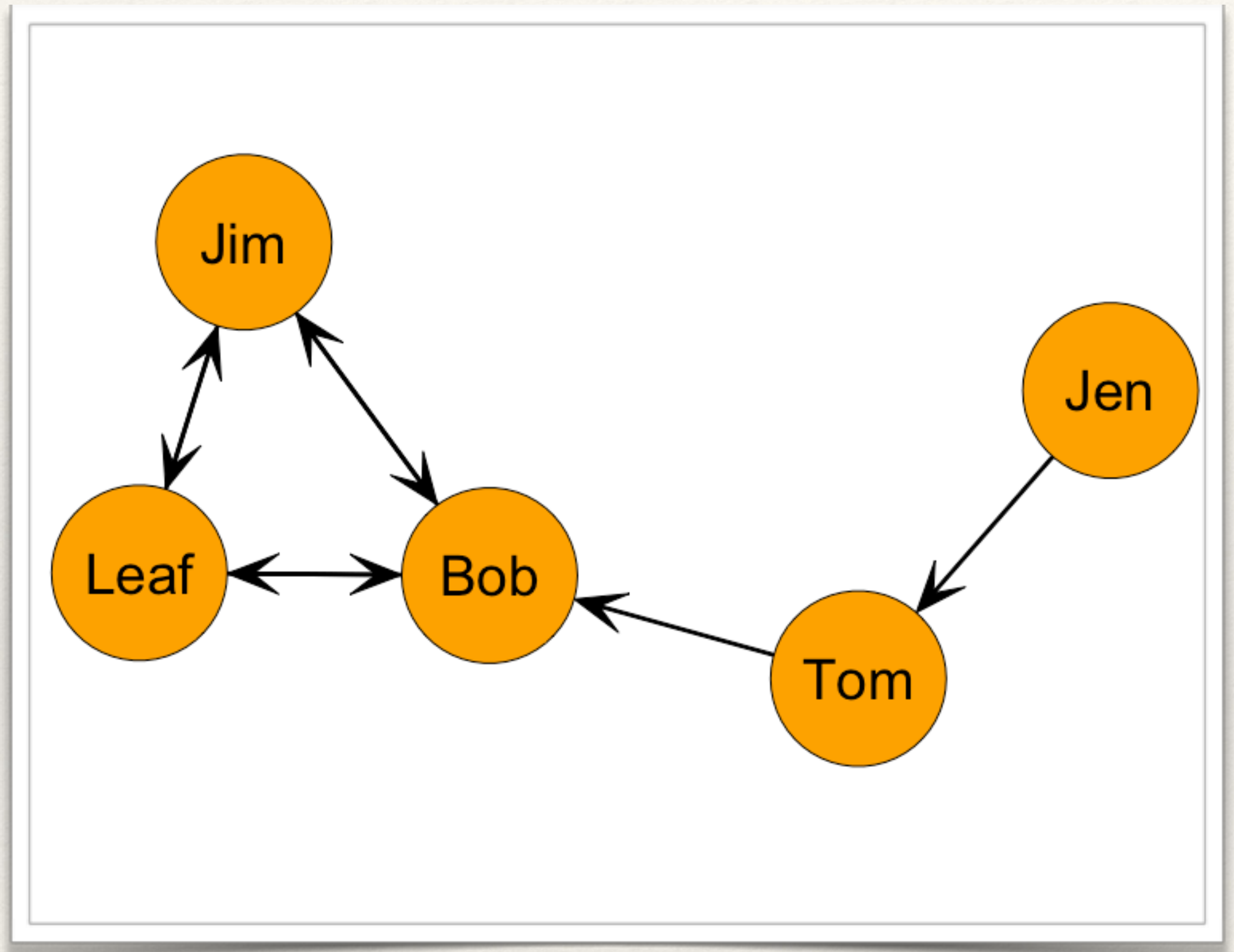
What is the index of indegree centralization for this graph?

0.4375

$$C_I = \frac{\sum_{i=1}^{g} [C_I(n^*) - C_I(n_i)]}{[(g-1)(g-1)]} =$$

$$= \frac{(3-0)+(3-1)+(3-3)+(3-2)+(3-2)}{(5-1)(5-1)} = \frac{3+2+0+1+1}{4*4} = \frac{7}{16} = 0.4375$$

Example: Directed, Binary Network



Raw Outdegree Centrality

Jen = 1

Tom = 1

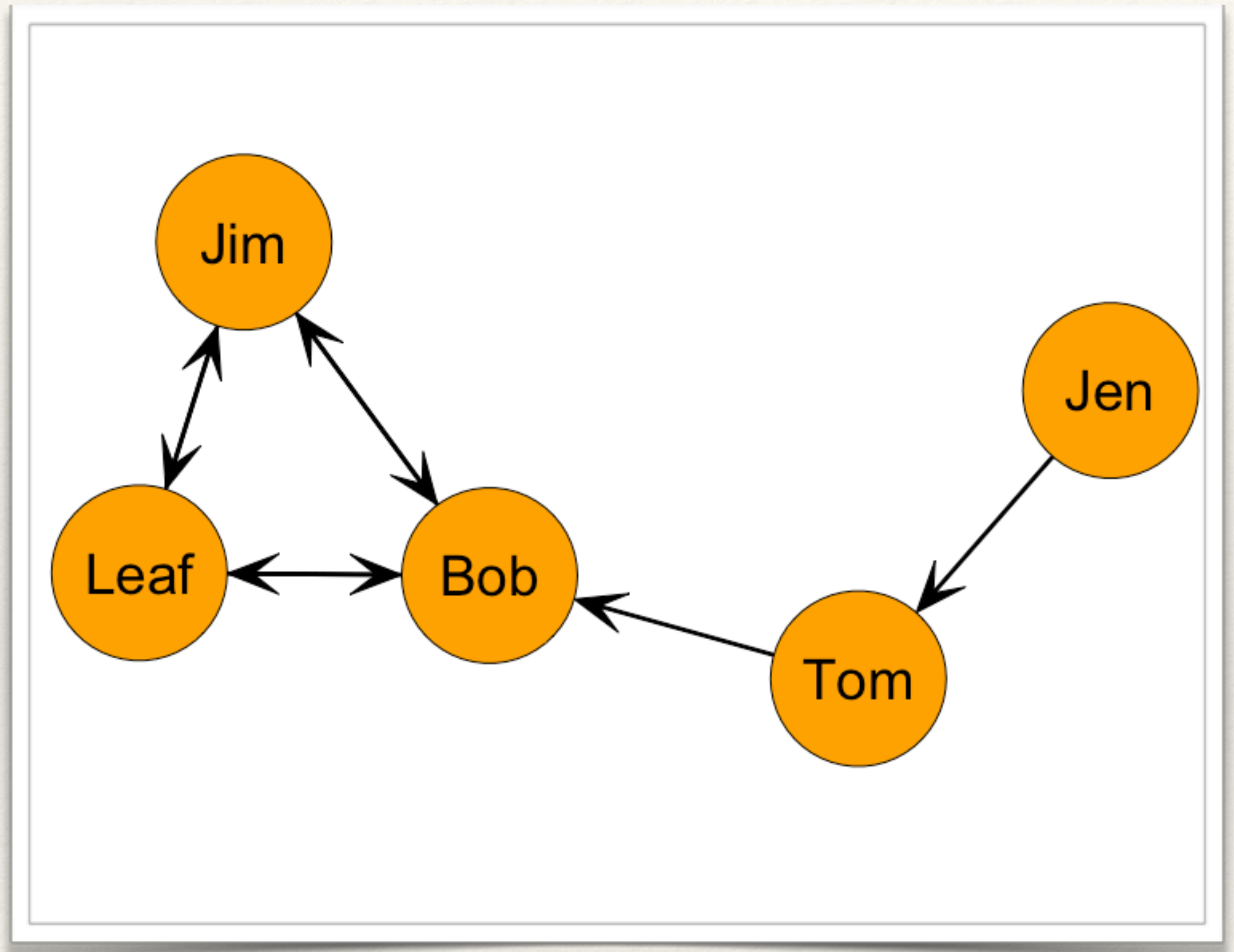
Bob = 2

Leaf = 2

Jim = 2

What is the index of outdegree centralization for this graph?

Example: Directed, Binary Network



<u>Raw Outdegree Centrality</u>

Jen = 1

Tom = 1

Bob = 2

Leaf = 2

Jim = 2

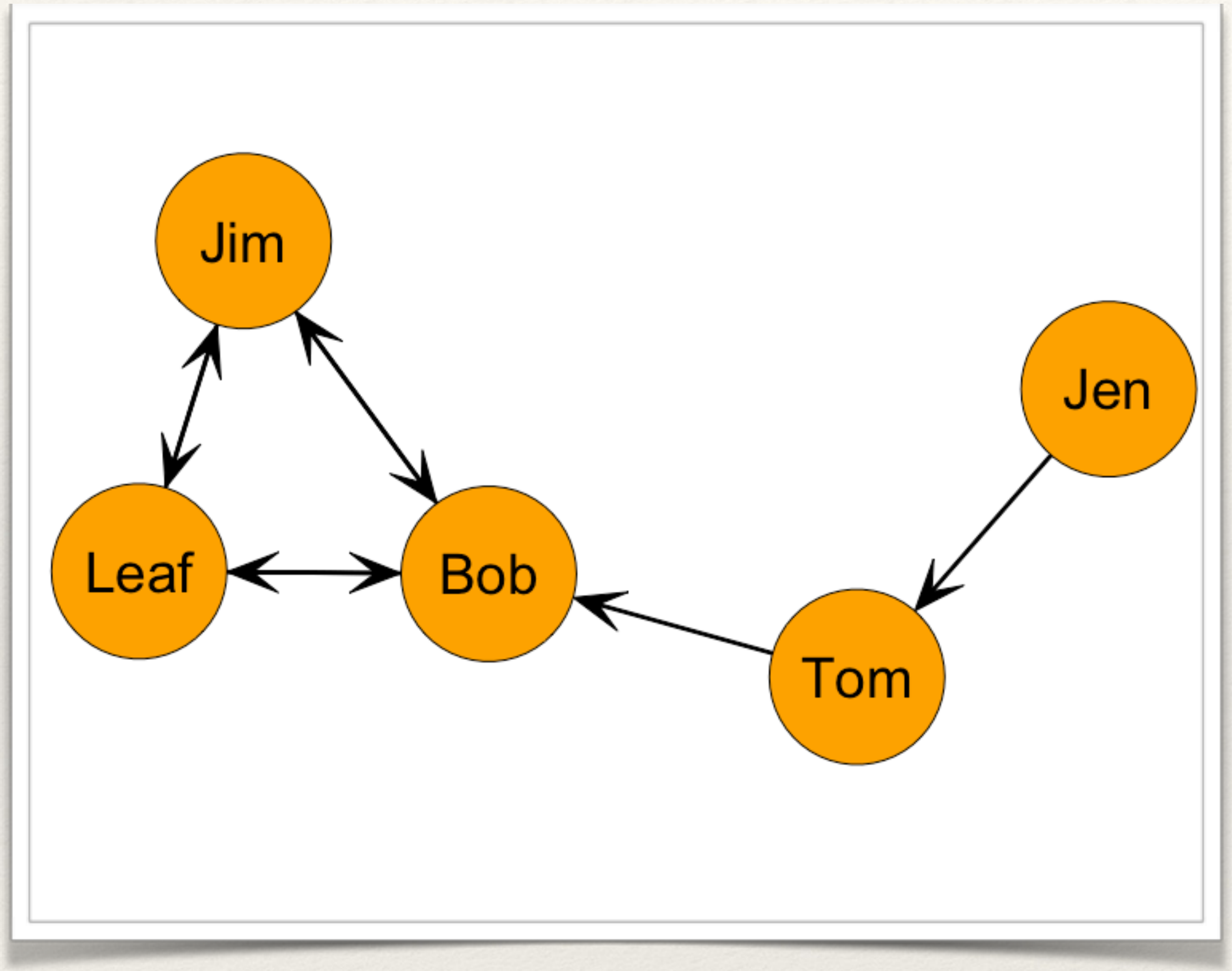
What is the index of outdegree centralization for this graph?

0.125

$$C_O = \frac{\sum_{i=1}^{g}[C_O(n^*) - C_O(n_i)]}{[(g-1)(g-1)]}$$

$$= \frac{(2-1)+(2-1)+(2-2)+(2-2)+(2-2)}{(5-1)(5-1)} = \frac{1+1+0+0+0}{4*4} = \frac{2}{16} = 0.125$$

Example: Directed, Binary Network



$$C_I = 0.4375$$

$$C_O = 0.125$$

What do the differences in the centralization scores tell us about the graph?

Learning Goals

- Understand the conceptualization of “centrality”.
- Understand calculation of degree centrality.
- Analyze descriptive features of degree centrality.

Empirical Example

J Youth Adolescence (2014) 43:104–115
DOI 10.1007/s10964-013-9946-0

EMPIRICAL RESEARCH

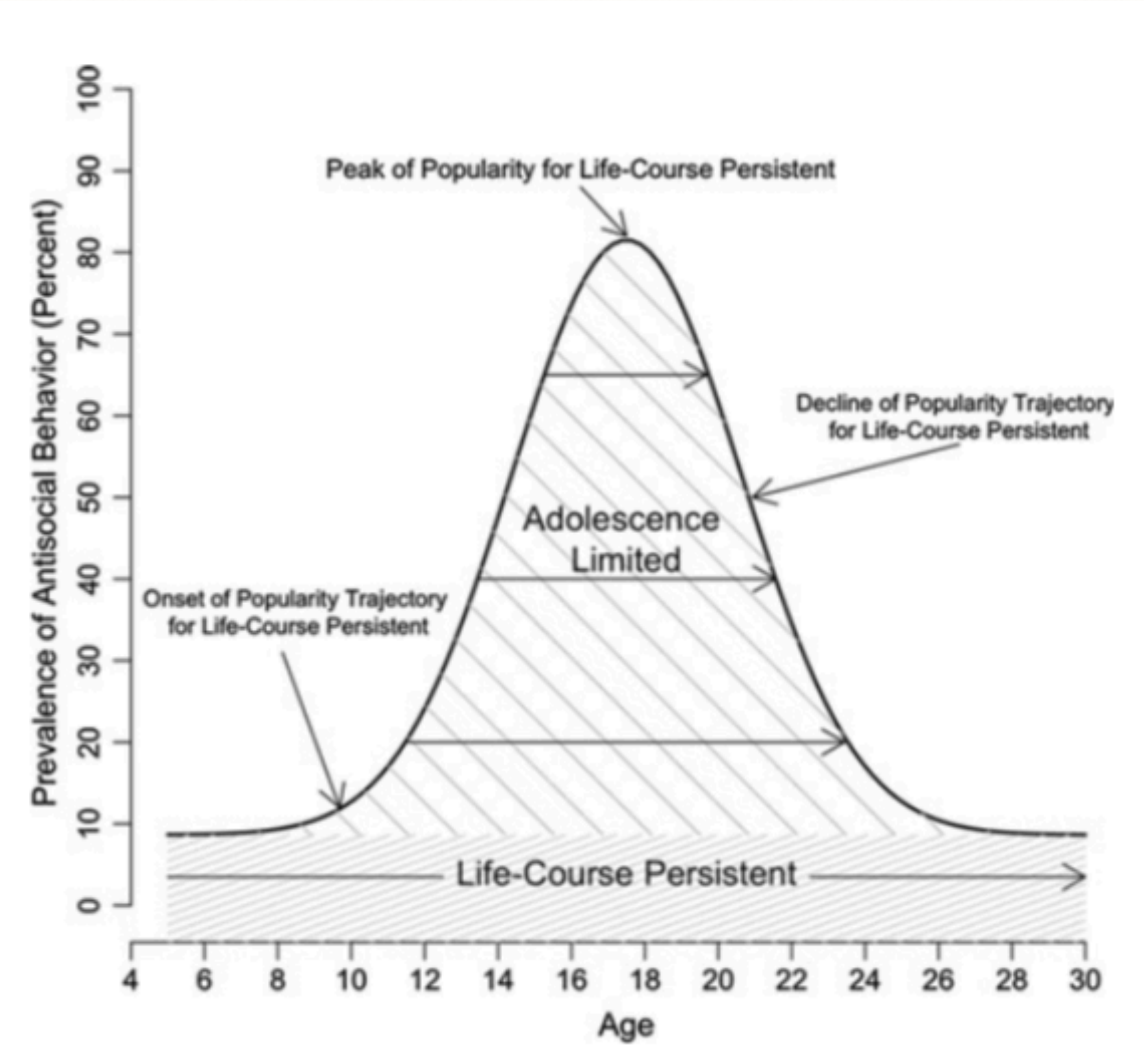
“Role Magnets”? An Empirical Investigation of Popularity Trajectories for Life-Course Persistent Individuals During Adolescence

Jacob T. N. Young

- https://link.springer.com/article/10.1007/s10964-013-9946-0

Empirical Example

- Question: Why is there a dramatic increase in delinquency during adolescence?
    - Argument: Dual-taxonomy theory.

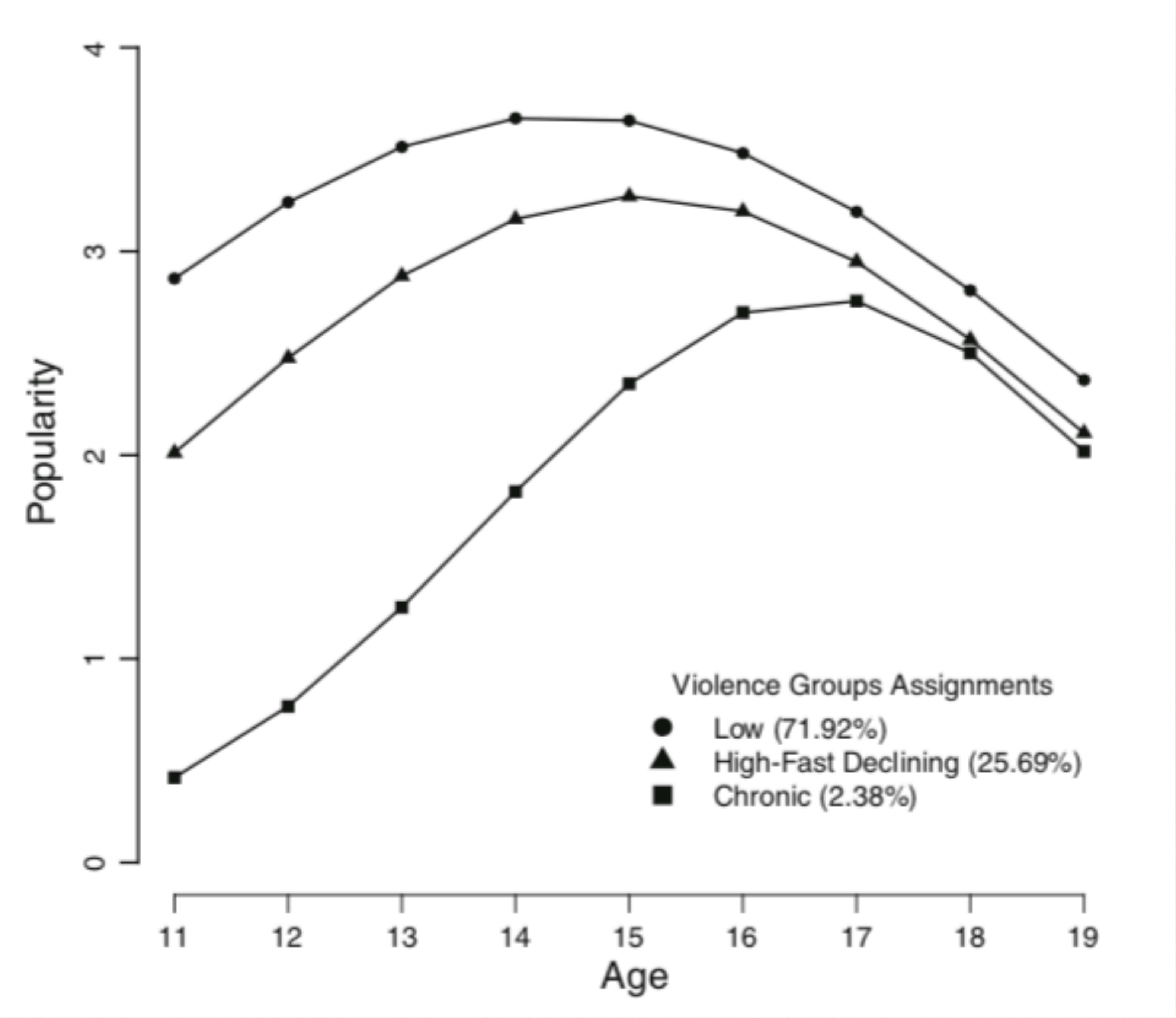


Empirical Example

- Dual-taxonomy theory argues that the causal mechanism generating the dramatic increase in delinquency during adolescence is social mimicry. As a consequence, life-course persistent individuals should occupy "more influential positions in the peer social structure" (Moffitt 1993: 687) and should be "moving toward central positions, during early adolescence" (Moffitt 1997: 28).
  - Concept: Popularity
    - Operationalization: Indegree Centrality
      - Goal: Examine the developmental trajectory of popularity during adolescence for individuals showing persistent violence into young adulthood.

Empirical Example

- Findings: Chronically violent individuals showed a more precipitous increase in indegree centrality during adolescence.



Questions?