question 1: Is the product $C^T u$ well-defined when C is 3×5 and u is 5×1?
solution 1: C^T is 5×3. A product $5\times 3\cdot 5\times 1$ is impossible because the inner dimensions (3 and 5) differ, so the expression is undefined.
answer 1: undefined

question 2: Compute $C^T(2v+5w)$.
solution 2:

1. $2v+5w=(21,13,14)^T$.
2. $C^T=\begin{pmatrix}1 & 1 & 0.5\\ 3 & 1 & 0\\ 2 & 1 & 0.5\\ 0 & 1 & 0.5\\ 1 & 0 & 1\end{pmatrix}$.
3. Multiply to get $(41,76,62,20,35)^T$.

answer 2: $\begin{pmatrix}41\\ 76\\ 62\\ 20\\ 35\end{pmatrix}$

question 3: Is $(2C^T+3A)u$ defined?
solution 3: $2C^T+3A$ is 5×3, u is 5×1; inner sizes 3 and 5 mismatch ⇒

undefined.
answer 3: undefined

question 4: Compute AB for
$A=\begin{pmatrix}1 & 0 & 1\\ 0.5 & 3 & 0\\ 0 & 1.4 & 6\\ 1 & 2 & 0\\ 3 & 4 & 5\end{pmatrix}$,
$B=\begin{pmatrix}1 & 3\\ 2 & 1\end{pmatrix}$.
solution 4: Only the top two rows of A can multiply B; result
$\begin{pmatrix}3 & 3\\ 7 & 3\end{pmatrix}$.
answer 4: $\begin{pmatrix}3 & 3\\ 7 & 3\end{pmatrix}$

question 5: Compute BA with the same A,B.
solution 5:

$BA=\begin{pmatrix}-1 & 3 & 1\\ 3 & 2 & 3\end{pmatrix}$.

answer 5: $\begin{pmatrix}-1 & 3 & 1\\ 3 & 2 & 3\end{pmatrix}$

question 6: Is the product CA defined for C 3×5 and A 5×3?
solution 6: C is 3×5; A is 5×3; CA is 3×3 so it is defined. (In the sheet they considered C 2×3 therefore undefined.)
answer 6: defined (3×3)

question 7: Compute $A-B$ using the 2×2 versions of A,B above.
solution 7: Subtract entry-wise: $\begin{pmatrix}0 & 2\\ 0 & 1\end{pmatrix}$.

answer 7: $\begin{pmatrix}0 & 2\\ 0 & 1\end{pmatrix}$

question 8: Compute $C^T A$.
solution 8: Multiply 5×3 by 5×3 not possible ⇒ undefined.
answer 8: undefined

question 9: Compute BA where
$A=¿$ and $B=(\ldots)$ (full 5×5 sheet).
solution 9: Use row–column dot products as in sheet; resulting matrix is

shown in the solution PDF.
answer 9: (matrix from PDF)

question 10: Compute $(A^T)^T B$.
solution 10: $(A^T)^T=A$; thus same as AB already found.
answer 10: $\begin{pmatrix}3 & 3\\ 7 & 3\end{pmatrix}$

question 11: Does a non-zero 2×2 matrix A with $A^2=0$ exist?
solution 11: Yes. Take $A=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$; then $A^2=0$ but $A\neq0$.
answer 11: yes; example shown.

question 12: Find a non-zero B such that $DB=BD^2$ for $D=\begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}$.
solution 12: Let $B=\begin{pmatrix}10 & 3\\ 0 & 5\end{pmatrix}$; direct multiplication verifies the equality.
answer 12: $B=\begin{pmatrix}10 & 3\\ 0 & 5\end{pmatrix}$

question 13: Write $U_{\pi/4}$.

solution 13: Insert $\cos\pi/4=\sin\pi/4=\sqrt{2}/2$: $\begin{pmatrix}\sqrt{2}/2 & -\sqrt{2}/2\\ \sqrt{2}/2 & \sqrt{2}/2\end{pmatrix}$.

answer 13: $\begin{pmatrix}\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2}\\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2}\end{pmatrix}$

question 14: Write $U_{2\pi/3}$.
solution 14: $\cos 2\pi/3=-1/2, \sin 2\pi/3=\sqrt{3}/2$.
answer 14: $\begin{pmatrix}-\frac{1}{2} & -\frac{\sqrt{3}}{2}\\ \frac{\sqrt{3}}{2} & -\frac{1}{2}\end{pmatrix}$

question 15: Write $U_{3\pi/2}$.
solution 15: $\cos 3\pi/2=0, \sin 3\pi/2=-1$.
answer 15: $\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$

question 16: Prove $U_\alpha U_\beta=U_{\alpha+\beta}$.
solution 16: Multiply the two 2×2 rotation matrices; trigonometric addition formulas give the result.
answer 16: property holds.

question 17: Show $U_\alpha^n=U_{n\alpha}$ for natural n.
solution 17: Induction: base $n=1$ trivial; induction step uses previous property.
answer 17: proven true for all $n\in N$.

question 18: Given $U_\alpha=\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}$ find α.

solution 18: Matrix corresponds to rotation by $\alpha=\pi/2$ (or 90°).

answer 18: $\alpha=\frac{\pi}{2}$.

question 19: Let $A=\begin{pmatrix}1 & 1\\ 0 & 1\end{pmatrix}$. Compute A^3.

solution 19: $A^2=\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}$; multiply once more to get $A^3=\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$.

answer 19: $\begin{pmatrix}1 & 3\\ 0 & 1\end{pmatrix}$

question 20: Find $det\begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}$.
solution 20: $2\cdot7-5\cdot3=14-15=-1$.
answer 20: -1

question 21: Does $\begin{pmatrix} 2 & 5 \\ 3 & 7 \end{pmatrix}$ have an inverse?
solution 21: Yes, determinant $-1\neq0$.
answer 21: yes

question 22: Compute $rank\begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix}$.
solution 22: Rows are linearly dependent ⇒ rank 1.
answer 22: 1

question 23: Give eigenvalues of $\begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}$.
solution 23: Diagonal matrix ⇒ eigenvalues 4 and 9.
answer 23: 4, 9

question 24: Is $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ orthogonal?
solution 24: Columns are orthonormal; $Q^T Q=I$.
answer 24: yes

question 25: Compute A^{-1} for $A=\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.
solution 25: $det=-2$; inverse $-1/2\begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix}$.
answer 25: $\begin{pmatrix} -2 & 1 \\ 1.5 & -0.5 \end{pmatrix}$

question 26: Determine if $B=\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ is nilpotent.
solution 26: $B^1=0$; nilpotent of index 1.
answer 26: yes

question 27: Provide a non-zero 2×2 idempotent matrix.
solution 27: $P=\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$; $P^2=P$.
answer 27: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

question 28: Compute $trace\begin{pmatrix}5 & -1\\2 & 7\end{pmatrix}$.

solution 28: $5+7=12$.
answer 28: 12

question 29: Evaluate $det\begin{pmatrix}a & 0\\0 & b\end{pmatrix}$.

solution 29: ab.
answer 29: ab

question 30: For $A=\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$ find minimal polynomial.

solution 30: $A^2=0\neq A$; minimal polynomial t^2.
answer 30: t^2

question 31: Does $A=\begin{pmatrix}3 & 1\\0 & 3\end{pmatrix}$ diagonalise over R?

solution 31: Single eigenvalue 3 with single eigenvector ⇒ no.
answer 31: no

question 32: Compute $(AB)^T$ for $A=\begin{pmatrix}1 & 2\\0 & 1\end{pmatrix}, B=\begin{pmatrix}2 & 0\\3 & 1\end{pmatrix}$.

solution 32: $AB=\begin{pmatrix}8 & 2\\3 & 1\end{pmatrix}$; transpose $\begin{pmatrix}8 & 3\\2 & 1\end{pmatrix}$.

answer 32: $\begin{pmatrix}8 & 3\\2 & 1\end{pmatrix}$

question 33: Compute $A^T A$ for $A=\begin{pmatrix}1 & 0\\1 & 1\end{pmatrix}$.

solution 33: $A^T A=\begin{pmatrix}2 & 1\\1 & 1\end{pmatrix}$.

answer 33: $\begin{pmatrix}2 & 1\\1 & 1\end{pmatrix}$

question 34: Find determinant of rotation matrix U_θ.
solution 34: Always 1.
answer 34: 1

question 35: Is the set $\{(x,y) \mid x+y=0\} \subset R^2$ a subspace?
solution 35: Closed under addition/scalar ⇒ yes.
answer 35: yes

question 36: Basis for that subspace?
solution 36: $\{(1,-1)\}$.
answer 36: $\{(1,-1)\}$

question 37: Compute $det\begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}$.
solution 37: $0\cdot0-(-1)\cdot1=1$.
answer 37: 1

question 38: For $A=\begin{pmatrix}2 & 0\\0 & -3\end{pmatrix}$ compute A^5.
solution 38: Raise diagonals: $\begin{pmatrix}32 & 0\\0 & -243\end{pmatrix}$.
answer 38: $\begin{pmatrix}32 & 0\\0 & -243\end{pmatrix}$

question 39: Does $AB=BA$ for $A=\begin{pmatrix}1 & 0\\0 & 2\end{pmatrix}, B=\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$?
solution 39: $AB=\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$, $BA=\begin{pmatrix}0 & 2\\0 & 0\end{pmatrix}$; unequal.
answer 39: no

question 40: Evaluate $det\begin{pmatrix}1 & 1 & 1\\0 & 1 & 1\\0 & 0 & 1\end{pmatrix}$.
solution 40: Upper triangular ⇒ product of diagonal =1.
answer 40: 1

question 41: Rank of same matrix?
solution 41: 3 (all pivots).
answer 41: 3

question 42: Give an eigenvalue of nilpotent $N=\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$.
solution 42: Only eigenvalue 0.
answer 42: 0

question 43: Find N^3 for $N=\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$.
solution 43: $N^2=0$; thus $N^3=0$.
answer 43: zero matrix

question 44: Is $P=\frac{1}{2}\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}$ a projection?
solution 44: $P^2=P$; yes.
answer 44: yes

question 45: Determine $trace(P)$.
solution 45: 1.
answer 45: 1

question 46: Solve $x^2-5x+6=0$.
solution 46: $(x-2)(x-3)=0$.
answer 46: $x=2,3$

question 47: Let $A=\begin{pmatrix} 0 & 2 \\ -1 & 0 \end{pmatrix}$. Compute A^2.
solution 47: $A^2=\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$.
answer 47: $\begin{pmatrix} -4 & 0 \\ 0 & -4 \end{pmatrix}$

question 48: Is A diagonalizable over R?
solution 48: Eigenvalues $\pm 2i$ imaginary ⇒ no real diagonalization.
answer 48: no

question 49: Compute $det(A^2)$ for the matrix in 47.
solution 49: $det(A^2)=(-4)(-4)=16$.
answer 49: 16

question 50: Give a 2×2 matrix with determinant -6.
solution 50: $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$.
answer 50: shown

question 51: For $Q=\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ verify $Q^T Q=I$.
solution 51: Multiply to obtain identity.
answer 51: verified

question 52: Compute $det(3I_2)$.
solution 52: $3^2=9$.
answer 52: 9

question 53: Does $3I_2$ commute with every 2×2 matrix?
solution 53: Yes; scalar multiple of identity.
answer 53: yes

question 54: Find the minimal polynomial of $3I_2$.
solution 54: $t-3$.
answer 54: $t-3$

question 55: Provide a 2×2 matrix whose minimal polynomial equals its characteristic polynomial.
solution 55: $A=\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$; char poly t^2+1 irreducible.
answer 55: example given

question 56: Compute trace of A in 55.
solution 56: 0.
answer 56: 0

question 57: Determine if $A=\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ is diagonalizable over C.
solution 57: Jordan block size 2 ⇒ not diagonalizable.
answer 57: no

question 58: Compute e^{At} for matrix in 57.
solution 58: $e^{At}=\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$.
answer 58: $\begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix}$

question 59: Give determinant of upper-triangular matrix product equals product of determinants.
solution 59: Property of determinants; holds.
answer 59: true

question 60: Let V be span$\{(1,0,0),(0,1,0)\}$. Dimension?
solution 60: 2.
answer 60: 2

question 61: Is $(1,1,1)$ in V?
solution 61: No; third coordinate non-zero.
answer 61: no

question 62: Compute cross product $(1,0,0)\times(0,1,0)$.
solution 62: $(0,0,1)$.
answer 62: $(0,0,1)$

question 63: Find $det\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$.
solution 63: 1.
answer 63: 1

question 64: Rank of zero matrix 3×3.
solution 64: 0.
answer 64: 0

question 65: Provide a 3×3 nilpotent matrix of index 3.
solution 65: $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$.
answer 65: shown

question 66: Compute its square.
solution 66: $\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.
answer 66: shown

question 67: Compute its cube.
solution 67: zero matrix.
answer 67: zero matrix

question 68: For diagonal matrix $D = diag(1,2,3)$ compute D^{-1}.
solution 68: $diag(1,1/2,1/3)$.
answer 68: shown

question 69: Trace of D above.
solution 69: 6.
answer 69: 6

question 70: Determine if D diagonalises (trivial).
solution 70: Already diagonal.
answer 70: yes

question 71: Compute determinant of D^2.
solution 71: $(1 \cdot 2 \cdot 3)^2 = 36$.
answer 71: 36

question 72: Let $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$. Show $A^4 = I$.
solution 72: $A^2 = -I$; square again gives I.
answer 72: property proven

question 73: Is A in 72 orthogonal?
solution 73: Yes.
answer 73: yes

question 74: Provide characteristic polynomial of A in 72.
solution 74: $t^2 + 1$.
answer 74: $t^2 + 1$

question 75: Compute A^{10} for same A.
solution 75: $A^4 = I$; $10 = 4 \cdot 2 + 2$; $A^{10} = A^2 = -I$.
answer 75: $-I$

question 76: Let $B = \begin{pmatrix} 2 & -2 \\ 1 & 3 \end{pmatrix}$. Trace?
solution 76: $2 + 3 = 5$.
answer 76: 5

question 77: Determinant of B.
solution 77: $2\cdot 3-(-2)\cdot 1=8$.
answer 77: 8

question 78: Eigenvalues of B.
solution 78: Solve $t^2-5t+8=0$; roots $5\pm i\sqrt{-8+25}/2=5\pm i\sqrt{-7}/2$ complex.
answer 78: $\frac{5\pm i\sqrt{7}}{2}$

question 79: Is B diagonalizable over C?
solution 79: Distinct eigenvalues ⇒ yes.
answer 79: yes

question 80: Find $det(kA)$ for $A=2\times 2$ and scalar k.
solution 80: $k^2 det\,A$.
answer 80: $k^2 det\,A$

question 81: Compute $det(A^{-1})$.
solution 81: $1/det\,A$.
answer 81: $det\,A^{-1}=(det\,A)^{-1}$

question 82: Give rank of identity $n\times n$.
solution 82: n.
answer 82: n

question 83: Is the zero vector orthogonal to every vector?
solution 83: Yes (dot product zero).
answer 83: yes

question 84: Compute $\|(3,4)\|$.
solution 84: $\sqrt{3^2+4^2}=5$.
answer 84: 5

question 85: Determine if $(1,2,3)$ and $(4,5,6)$ are orthogonal.
solution 85: Dot product 32 ≠0.
answer 85: no

question 86: Cross product of those two vectors?
solution 86: $(-3,6,-3)$.
answer 86: $(-3,6,-3)$

question 87: Area of parallelogram spanned by them.
solution 87: Norm of cross product $\sqrt{(-3)^2+6^2+(-3)^2}=\sqrt{54}=3\sqrt{6}$.
answer 87: $3\sqrt{6}$

question 88: Find eigenvalues of $J=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$.
solution 88: Solve $t^2-1=0$; eigenvalues ±1.
answer 88: ±1

question 89: Is J symmetric?
solution 89: Yes ($J^T=J$).
answer 89: yes

question 90: Determine if J orthogonally diagonalises.
solution 90: Symmetric ⇒ yes.
answer 90: yes

question 91: Compute J^3.
solution 91: $J^2=I$; $J^3=J$.
answer 91: J

question 92: Provide rank of projection P in 27.
solution 92: 1.
answer 92: 1

question 93: Is every projection diagonalizable?
solution 93: Yes; eigenvalues 0,1.
answer 93: yes

question 94: Determine $det\, P$ from 27.
solution 94: 0.
answer 94: 0

question 95: Let $A=\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{pmatrix}$. Rank?
solution 95: Rows scalar multiples ⇒ rank 1.
answer 95: 1

question 96: Trace of that A.
solution 96: $1+4+9=14$.
answer 96: 14

question 97: Does A have an inverse?
solution 97: No (rank<3).
answer 97: no

question 98: Compute A^T.
solution 98: swap rows and columns.
answer 98: $\begin{pmatrix}1 & 2 & 3\\ 2 & 4 & 6\\ 3 & 6 & 9\end{pmatrix}$

question 99: Is A symmetric?
solution 99: Yes ($A^T = A$).
answer 99: yes

question 100: Eigenvalues of rank-1 symmetric A above.
solution 100: One non-zero eigenvalue equal to trace 14, two zeros.
answer 100: $14, 0, 0$