Problem 1

a) Specify the full DT LTI stochastic dynamics model for each Ak.

$$\Gamma_A = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad A_A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\Omega_A \\ 0 & 0 & 0 & 1 \\ 0 & \Omega_A & 0 & 0 \end{bmatrix} \qquad dt = 0.5 \text{ sec}, \quad \Omega_A = 0.045 \text{ rad/s}$$

$$Z_A = dt \cdot \begin{bmatrix} -A_A & \Gamma_A W \Gamma_A' \\ 0 & A_A' \end{bmatrix} \Rightarrow e^{Z_A} = \exp(Z_A)$$

$$e^{Z_A} = \begin{bmatrix} \cdots & F^{-1}Q \\ 0 & F^T \end{bmatrix} \Rightarrow Q_A = (F_A^T)^T \cdot [F_A^{-1} Q_A] \quad (\text{from } e^{Z_A} \text{ matrix})$$

$$Q_A = \begin{bmatrix} 0.83 & 2.50 & 0.03 & 0.10 \\ 2.50 & 10.00 & 0.07 & 0.33 \\ 0.03 & 0.07 & 0.21 & 0.63 \\ 0.10 & 0.33 & 0.63 & 2.51 \end{bmatrix}$$

$$W = q_w \cdot \begin{bmatrix} 2 & 0.05 \\ 0.05 & 0.5 \end{bmatrix}, \quad q_w = 10 \ (m/s)^2$$

$$F_A = \begin{bmatrix} 1 & \sin(\Omega_A dt)/\Omega_A & 0 & -(1-\cos(\Omega_A dt))/\Omega_A \\ 0 & \cos(\Omega_A dt) & 0 & -\sin(\Omega_A dt) \\ 0 & (1-\cos(\Omega_A dt))/\Omega_A & 1 & \sin(\Omega_A dt)/\Omega_A \\ 0 & \sin(\Omega_A dt) & 0 & \cos(\Omega_A dt) \end{bmatrix}$$

$$F_A = \begin{bmatrix} 1 & 0.50 & 0 & -0.01 \\ 0 & 1.00 & 0 & -0.02 \\ 0 & 0.01 & 1 & 0.50 \\ 0 & 0.02 & 0 & 1.00 \end{bmatrix}$$

$$\Gamma_B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \qquad A_B = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\Omega_B \\ 0 & 0 & 0 & 1 \\ 0 & \Omega_B & 0 & 0 \end{bmatrix} \qquad dt = 0.5 \text{ sec}, \quad \Omega_B = -0.045 \text{ rad/s}$$

$$Z_A = dt \cdot \begin{bmatrix} -A_B & \Gamma_B W \Gamma_B' \\ 0 & A_B' \end{bmatrix} \Rightarrow e^{Z_B} = \exp(Z_B)$$

$$Q_B = (F_B^T)^T [F_B^{-1} Q_B] = \begin{bmatrix} 0.83 & 2.50 & 0.02 & 0.03 \\ 2.50 & 10.00 & 0.05 & 0.17 \\ 0.02 & 0.05 & 0.21 & 0.62 \\ 0.03 & 0.17 & 0.62 & 2.50 \end{bmatrix} = Q_B$$

$$F_B = \begin{bmatrix} 1 & \sin(\Omega_B dt)/\Omega_B & 0 & -(1-\cos(\Omega_B dt)/\Omega_B \\ 0 & \cos(\Omega_B dt) & 0 & -\sin(\Omega_B dt) \\ 0 & (1-\cos(\Omega_B dt))/\Omega_B & 1 & \sin(\Omega_B dt)/\Omega_B \\ 0 & \sin(\Omega_B dt) & 0 & \cos(\Omega_B dt) \end{bmatrix} = \begin{bmatrix} 1 & 0.50 & 0 & 0.01 \\ 0 & 1.00 & 0 & 0.02 \\ 0 & -0.01 & 1 & 0.50 \\ 0 & -0.02 & 0 & 1.00 \end{bmatrix} = F_B$$