ASEN 5044 | HW 8

Problem 1

C) i) Implement a KF to estimate the joint augmented aircraft states.

First, combine the Aircraft A and Aircraft B DT LTI matrices.

$\underline{\underline{F_S}}$
$$F_S = \left[\begin{array}{c|c} F_A & 0 \\ \hline 0 & F_B \end{array}\right] \qquad A_S = \left[\begin{array}{c|c} A_A & 0 \\ \hline 0 & A_B \end{array}\right] \qquad W_S = \left[\begin{array}{c|c} W & 0 \\ \hline 0 & W \end{array}\right] \qquad \Gamma_S = \left[\begin{array}{c|c} \Gamma_A & 0 \\ \hline 0 & \Gamma_B \end{array}\right]$$

Now, form a new Q_S matrix:

$$Z_S = dt \left[\begin{array}{c|c} -A_S & \Gamma_S W_S \Gamma_S' \\ \hline 0 & A_S' \end{array}\right]$$

$$e^{Z_S} = \exp(Z_S) = \left[\begin{array}{c|c} (\dots) & F_S^{-1} Q_S \\ \hline 0 & F' \end{array}\right] \Rightarrow Q_S = (F_S')' \left[F_S^{-1} Q_S\right] \text{ (from } e^{Z_S} \text{ matrix)}$$

$\underline{\underline{Q_S}}$
$$Q_S = \begin{bmatrix} 0.83 & 2.50 & 0.03 & 0.10 & 0 & 0 & 0 & 0 \\ 2.50 & 10.00 & 0.07 & 0.33 & 0 & 0 & 0 & 0 \\ 0.03 & 0.07 & 0.21 & 0.63 & 0 & 0 & 0 & 0 \\ 0.10 & 0.33 & 0.63 & 2.51 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.83 & 2.50 & 0.02 & 0.03 \\ 0 & 0 & 0 & 0 & 2.50 & 10.00 & 0.03 & 0.17 \\ 0 & 0 & 0 & 0 & 0.02 & 0.03 & 0.21 & 0.67 \\ 0 & 0 & 0 & 0 & 0.03 & 0.17 & 0.62 & 2.50 \end{bmatrix}$$

$\underline{\underline{H_S}}$
$$H_S = \left[\begin{array}{c|c} H & 0 \\ \hline H & -H \end{array}\right] \Rightarrow Y_S(k) = H x_S(k) + V_D(k)$$

Top "row" handles original Aircraft A "direct" measurements, bottom "row" is the result of the Aircraft B measurements being a sum of Aircraft A measurement plus transponder measurements.

$$y_D(k) = \underset{\hookrightarrow H}{r_A(k)} - \underset{\hookrightarrow -H}{r_B(k)}$$

$\underline{\underline{R_S}}$
$$R_S = \left[\begin{array}{c|c} R_A & 0 \\ \hline 0 & R_D \end{array}\right]$$

$$R_S = \begin{bmatrix} 20 & 0.05 & 0 & 0 \\ 0.05 & 20 & 0 & 0 \\ 0 & 0 & 10 & 0.15 \\ 0 & 0 & 0.15 & 10 \end{bmatrix} (m^2)$$