Basic statistical orbit determination - final project description

ASEN 5044

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1 Introduction

Spacecraft missions such as TOPEX-POSEIDON, Jason I, II and III measure the global sea level height with ever improving precision. Jason III manages to measure it with an accuracy of an inch. In order to do so, the position of the spacecraft must be determined to at least similar accuracy. This field, called statistical orbit determination knows many challenges, but can be as simple or complex as one chooses it to be. In this project, a highly simplified example of statistical orbit determination will be set up.

2 Dynamical system

The spacecraft position in the orbital plane can be represented by Earth-centered coordinates X and Y, and the rates of change $\dot{X}$ and $\dot{Y}$. Let $r = \sqrt{X^2 + Y^2}$ represent the distance of the spacecraft from Earth's center.

If we assume a simple gravity point-mass force model, which obeys a simple inverse square law, we can write the full non-linear equations of motion as:

$$\ddot{X} = -\frac{\mu X}{r^3} + u_1 + \tilde{w}_1$$
$$\ddot{Y} = -\frac{\mu Y}{r^3} + u_2 + \tilde{w}_2$$

where μ is the standard gravitational parameter (which is 398600 $\mathrm{km}^3/\mathrm{s}^2$ for this assignment), u_1 and u_2 are control accelerations, and $\tilde{w}_1$ and $\tilde{w}_2$ are disturbances.

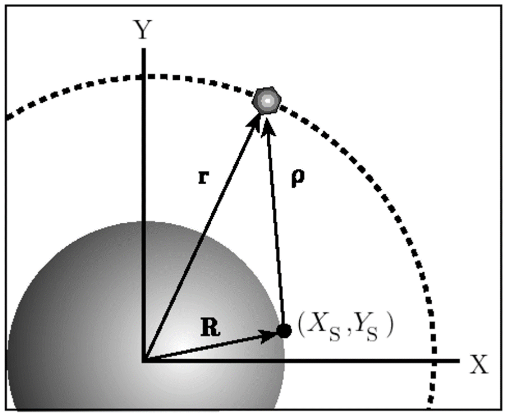

Figure 1: Basic orbit determination problem setup.

3 Nominal state and parameters

The spacecraft starts in a circular orbit at 300 km altitude. Hence, the initial states are $X(0) = 6678$ km (the nominal orbit radius r_0), $Y(0) = 0$ km, $\dot{X}(0) = 0$ km/s, and $\dot{Y}(0) = r_0 \cdot \sqrt{\frac{\mu}{r_0^3}}$ km/s.

The full set of states, inputs and disturbances are thus

$$\mathbf{x}(t) = [X, \dot{X}, Y, \dot{Y}]^T$$
$$\mathbf{u}(t) = [u_1, u_2]^T$$
$$\tilde{\mathbf{w}}(t) = [\tilde{w}_1, \tilde{w}_2]^T$$

The nonlinear measurements are given by noisy relative range, range rate, and elevation angle observations from tracking stations i located at positions $(X_s^i(t), Y_s^i(t))$ in the orbital plane on the Earth's surface (which is rotating counterclockwise completely in the XY plane). The data are modeled as

$$\mathbf{y}^i(t) = \begin{bmatrix} \rho^i(t) \\ \dot{\rho}^i(t) \\ \phi^i(t) \end{bmatrix} + \tilde{\mathbf{v}}^i(t)$$

where $\tilde{\mathbf{v}}^i(t) \in \mathbb{R}^3$ is the measurement error vector at station i and

$$\rho^i(t) = \sqrt{(X(t) - X_s^i(t))^2 + (Y(t) - Y_s^i(t))^2}$$
$$\dot{\rho}^i(t) = \frac{[X(t) - X_s^i(t)] \cdot [\dot{X}(t) - \dot{X}_s^i(t)] + [Y(t) - Y_s^i(t)] \cdot [\dot{Y}(t) - \dot{Y}_s^i(t)]}{\rho^i(t)}$$
$$\phi^i(t) = \tan^{-1}\left(\frac{Y(t) - Y_s^i(t)}{X(t) - X_s^i(t)}\right).$$

Assume the tracking stations' positions are perfectly known for all time t, such that

$$
\begin{aligned}
X^i(t) &= R_E \cos(\omega_E t + \theta^i(0)), \\
Y^i(t) &= R_E \sin(\omega_E t + \theta^i(0)),
\end{aligned}
$$

where $R_E = 6378$ km, $\omega_E = \frac{2\pi}{86400}\ \frac{rad}{s}$, and each station $i = \{1, 2, 3, \cdots, 12\}$ is initially located at

$$
\begin{aligned}
(X^i(0), Y^i(0)) &= \left(R_E \cos\theta^i(0), R_E \sin\theta^i(0)\right), \\
\theta^i(0) &= (i-1) \cdot \frac{\pi}{6}
\end{aligned}
$$

Note that, due to the relative motion of the spacecraft and each station, each station i only produces valid observation vectors $\mathbf{y}^i(t)$ for a limited time-varying range of possible $\phi^i(t)$ angles (the spacecraft cannot be sensed through the Earth). That is, station i only generates a data vector $\mathbf{y}^i(t)$ at time t if

$$
\begin{aligned}
\phi^i(t) &\in [-\frac{\pi}{2} + \theta^i(t), \frac{\pi}{2} + \theta^i(t)], \\
\text{where} \quad \theta^i(t) &= \tan^{-1}\left(\frac{Y^i(t)}{X^i(t)}\right).
\end{aligned}
$$

Hence, in general, the full observation vector $\mathbf{y}(t)$ consists of vertically stacked $\mathbf{y}^i(t)$ vectors (in ascending order with smallest i on top), where the number of $\mathbf{y}^i(t)$ terms stacked together (and hence the total length of $\mathbf{y}(t)$) varies according to which stations i can observe the satellite at time t. Note that there may be some small temporary 'blind spots' where no station can pick up the spacecraft, in which case $\mathbf{y}(t)$ will be empty.