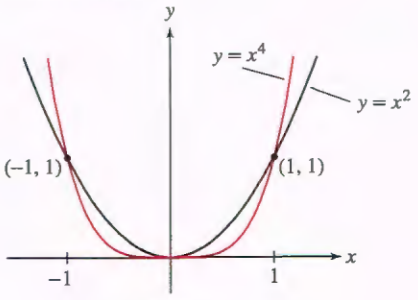
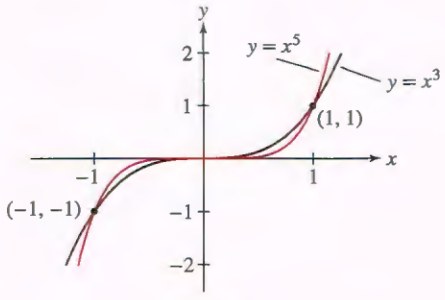
ELEMENTARY FUNCTIONS

Power Functions $f(x) = x^a$

$f(x) = x^n$, n a positive integer

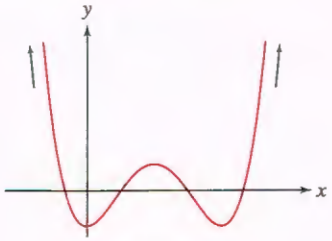


n even



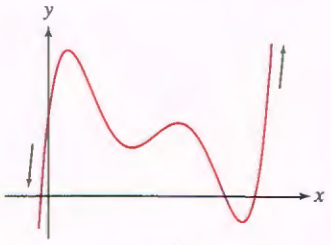
n odd

Asymptotic behavior of a polynomial function of even degree and positive leading coefficient



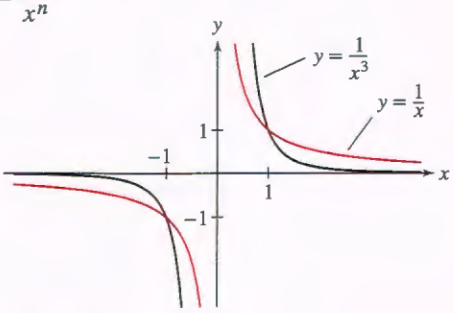
n even

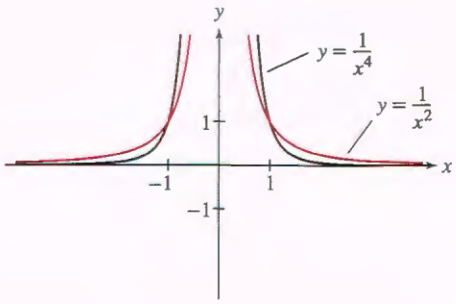
Asymptotic behavior of a polynomial function of odd degree and positive leading coefficient



n odd

$f(x) = x^{-n} = \dfrac{1}{x^n}$

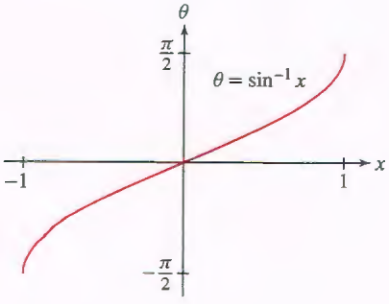




Inverse Trigonometric Functions

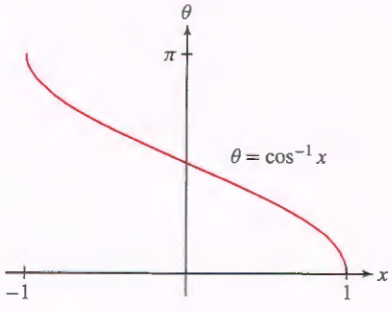
$\arcsin x = \sin^{-1} x = \theta$

$\Leftrightarrow \quad \sin\theta = x, \quad -\dfrac{\pi}{2} \le \theta \le \dfrac{\pi}{2}$



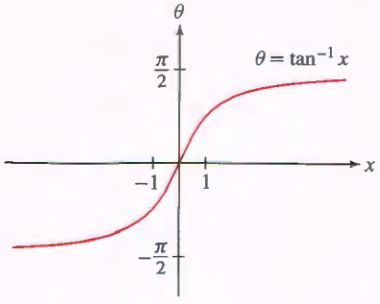
$\arccos x = \cos^{-1} x = \theta$

$\Leftrightarrow \quad \cos\theta = x, \quad 0 \le \theta \le \pi$



$\arctan x = \tan^{-1} x = \theta$

$\Leftrightarrow \quad \tan\theta = x, \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$

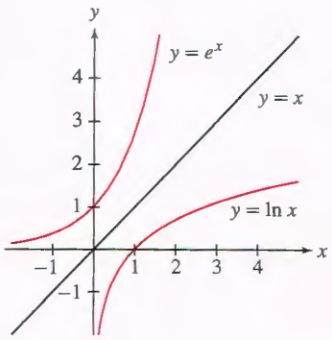


Exponential and Logarithmic Functions

$$\log_a x = y \quad \Leftrightarrow \quad a^y = x$$

$$\log_a(a^x) = x \qquad a^{\log_a x} = x$$

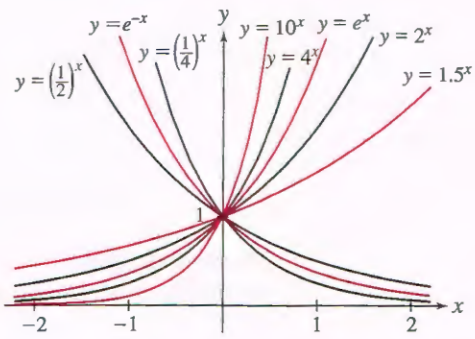
$$\log_a 1 = 0 \qquad \log_a a = 1$$



$$\ln x = y \quad \Leftrightarrow \quad e^y = x$$

$$\ln(e^x) = x \qquad e^{\ln x} = x$$

$$\ln 1 = 0 \qquad \ln e = 1$$



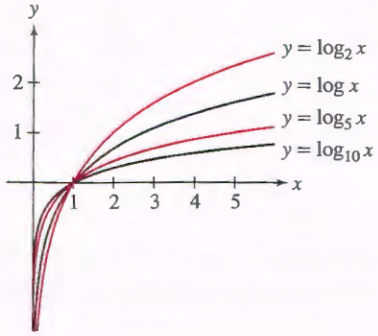
$$0 < a < 1: \lim_{x \to -\infty} a^x = \infty, \quad \lim_{x \to \infty} a^x = 0$$

$$a > 1: \lim_{x \to -\infty} a^x = 0, \quad \lim_{x \to \infty} a^x = \infty$$

$$\log_a(xy) = \log_a x + \log_a y$$

$$\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y$$

$$\log_a(x^r) = r \log_a x$$



$$\lim_{x \to 0^+} \log_a x = -\infty$$

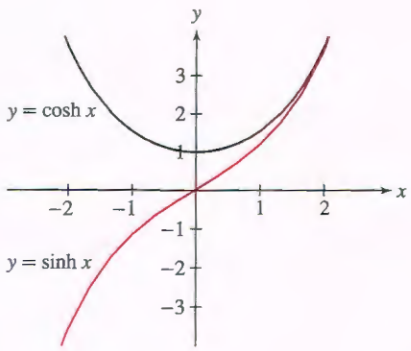
$$\lim_{x \to \infty} \log_a x = \infty$$

Hyperbolic Functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

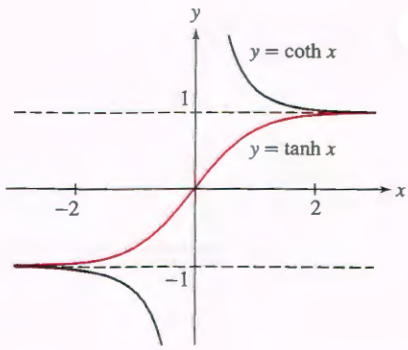
$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{\cosh x}{\sinh x}$$



$$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$$

$$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$



$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

Inverse Hyperbolic Functions

$$y = \sinh^{-1} x \quad \Leftrightarrow \quad \sinh y = x$$

$$y = \cosh^{-1} x \quad \Leftrightarrow \quad \cosh y = x \text{ and } y \geq 0$$

$$y = \tanh^{-1} x \quad \Leftrightarrow \quad \tanh y = x$$

$$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$$

$$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right) \quad x > 1$$

$$\tanh^{-1} x = \frac{1}{2} \ln\left(\frac{1 + x}{1 - x}\right) \quad -1 < x < 1$$