Feel free to consult reference sheets that cover material from *past* courses. No need for a calculator on this exam. Show all your work as clearly as you can, making it clear which techniques you are using. Evaluate the integrals completely but don't fuss over simplifying your answers. *Do*, however, remember to substitute back to the original variable! Remember to breathe and focus, and you'll be just fine :-)

1. Evaluate

$$\int \theta \sec^2\theta \tan\theta \, d\theta.$$

IBP: $u = \theta \Rightarrow du = d\theta$

$v = \int \sec^2\theta \tan\theta \, d\theta = \int \sec\theta \sec\theta \tan\theta \, d\theta$

u-sub: $w = \sec\theta \Rightarrow dw = \sec\theta \tan\theta \, d\theta$

$\Rightarrow v = \int w \, dw = \frac{1}{2} w^2 = \frac{1}{2}\sec^2\theta$

So $\int \theta \sec^2\theta \tan\theta \, d\theta = uv - \int v \, du$

$= \frac{1}{2}\theta\sec^2\theta - \frac{1}{2}\int \sec^2\theta \, d\theta$

$= \frac{1}{2}\theta \sec^2\theta - \frac{1}{2}\tan\theta + C$

2. Evaluate

$$\int \frac{\cos\theta}{\sqrt{1 - sin\theta}}\, d\theta.$$

u-sub: $u = 1 - \sin\theta \Rightarrow -du = \cos\theta\, d\theta$

$$\int \frac{\cos\theta\, d\theta}{\sqrt{1-\sin\theta}} = -\int \frac{du}{\sqrt{u}}$$

$$= -\int u^{-1/2}\, du$$

$$= -\frac{u^{1/2}}{1/2} + C$$

$$= -2\sqrt{u} + C$$

$$= -2\sqrt{1-\sin\theta} + C$$

3. Evaluate

$$\int \frac{\sqrt{\log t}}{t}\, dt.$$

u-sub: $u = \log t \Rightarrow du = \frac{1}{t} dt$

$$\int \frac{\sqrt{\log t}}{t}\, dt = \int \sqrt{\log t}\, \frac{1}{t}\, dt$$

$$= \int \sqrt{u}\, du$$

$$= \int u^{1/2}\, du$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} (\log t)^{3/2} + C$$

4. Evaluate

$$\int y^2 e^{2y}\, dy.$$

IBP: $u = y^2 \Rightarrow du = 2y\,dy$

$v = \int e^{2y}\,dy = \frac{1}{2}e^{2y}$

$$\int y^2 e^{2y}\,dy = uv - \int v\,du$$

$$= \frac{1}{2}y^2 e^{2y} - \int \left(\frac{1}{2}e^{2y}\right)(2y)\,dy$$

$$= \frac{1}{2}y^2 e^{2y} - \int y e^{2y}\,dy$$

IBP on $\int y e^{2y}\,dy$:

$u = y \Rightarrow du = dy$

$v = \int e^{2y}\,dy = \frac{1}{2}e^{2y}$

so

$$\int y^2 e^{2y}\,dy = \frac{1}{2}y^2 e^{2y} - \left(\frac{1}{2}y e^{2y} - \frac{1}{2}\int e^{2y}\,dy\right)$$

$$= \frac{1}{2}y^2 e^{2y} - \frac{1}{2}y e^{2y} + \frac{1}{4}e^{2y} + C$$

$$= \frac{1}{4}e^{2y}\left(2y^2 - 2y + 1\right) + C$$

5. Evaluate

$$\int \frac{2x^4 + 8x^2 + x + 1}{x^2(x^2+4)}\, dx.$$

Order of numerator = order of denominator
so divide:

$$\frac{2x^4+8x^2+x+1}{x^2(x^2+4)} = \frac{2x^4+8x^2+x+1}{x^4+4x^2} = \frac{2(x^4+4x^2)+x+1}{x^4+4x^2} = 2 + \frac{x+1}{x^2(x^2+4)}$$

Partial Fractions: $\frac{x+1}{x^2(x^2+4)} = \frac{A}{x} + \frac{B}{x^2} + \frac{Cx+D}{x^2+4}$

$$\Rightarrow x+1 = Ax(x^2+4) + B(x^2+4) + (Cx+D)x^2 = Ax^3 + 4Ax + Bx^2 + 4B + Cx^3 + Dx^2$$

$$= (A+C)x^3 + (B+D)x^2 + 4Ax + 4B \Rightarrow 4B = 1,\ 4A = 1,\ A+C = 0 = B+D$$

$$\Rightarrow B = \tfrac{1}{4},\ A = \tfrac{1}{4},\ C = \tfrac{-1}{4},\ D = \tfrac{-1}{4}$$

So $\int 2 + \frac{x+1}{x^2(x^2+4)}\,dx = 2\int dx + \frac{1}{4}\int \frac{dx}{x} + \frac{1}{4}\int \frac{dx}{x^2} - \frac{1}{4}\int \frac{x+1}{x^2+4}\,dx$

$$= 2\int dx + \frac{1}{4}\int\frac{dx}{x} + \frac{1}{4}\int\frac{dx}{x^2} - \frac{1}{4}\int\frac{x\,dx}{x^2+4} - \frac{1}{4}\int\frac{dx}{x^2+4}$$

Let $\int \frac{x\,dx}{x^2+4} = I$, u-sub: $u = x^2+4 \Rightarrow \frac{1}{2}du = x\,dx$ so

$$I = \frac{1}{2}\int\frac{du}{u} = \frac{1}{2}\log|u| = \frac{1}{2}\log(x^2+4)$$

Let $\int\frac{dx}{x^2+4} = J$ trig-sub: $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$ & $x^2+4 = 4(\tan^2\theta+1) = 4\sec^2\theta$

$$J = \int \frac{2\sec^2\theta\, d\theta}{4\sec^2\theta} = \frac{1}{2}\int d\theta = \frac{1}{2}\theta = \frac{1}{2}\arctan x$$

So $\int 2 + \frac{x+1}{x^2(x^2+4)}\,dx = 2x + \frac{1}{4}\log|x| - \frac{1}{4x} - \frac{1}{8}\log(x^2+4) - \frac{1}{8}\arctan x + C$

6. Evaluate

$$\int \frac{w^2}{\sqrt{9-25w^2}}\,dw.$$

Trig-sub : $w = \frac{3}{5}\sin\theta \Rightarrow dw = \frac{3}{5}\cos\theta\,d\theta$ & $\sqrt{9-25w^2} = \sqrt{9-9\sin^2\theta} = 3\cos\theta$

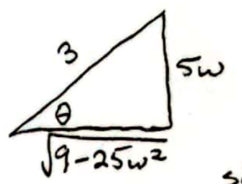
$$\int \frac{w^2}{\sqrt{9-25w^2}}\,dw = \int \frac{\frac{9}{25}\sin^2\theta}{3\cos\theta}\,\frac{3}{5}\cos\theta\,d\theta$$

$$= \frac{9}{125}\int \sin^2\theta\,d\theta \qquad \sin^2\theta = \frac{1-\cos 2\theta}{2}$$

$$= \frac{9}{250}\int 1-\cos 2\theta\,d\theta$$

$$= \frac{9}{250}\int d\theta - \frac{9}{250}\int \cos 2\theta\,d\theta$$

$$= \frac{9}{250}\theta - \frac{9}{500}\sin 2\theta + C$$



$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin\theta = \frac{5w}{3}$

$\cos\theta = \frac{\sqrt{9-25w^2}}{3}$

$\sin 2\theta = \frac{10}{9}w\sqrt{9-25w^2}$

$$= \frac{9}{250}\arcsin\left(\frac{5w}{3}\right) - \frac{1}{50}w\sqrt{9-25w^2} + C$$