Lists, Permutations, Subsets

1. List all the functions from the three-element set $\{1, 2, 3\}$ to the set $\{a, b\}$. Which functions, if any, are one-to-one? Which functions, if any, are onto? Do the same for all functions $\{1, 2\} \to \{a, b, c\}$.

2. Suppose that $|S| = s$ and $|T| = t$. How many functions $S \to T$ are there?

3. Assuming we pass out all the fruit, if $k \leq n$, in how many ways can we pass out k identical pieces of fruit to n children if each child may get at most one? What if $k > n$? Now answer the same question, but suppose each piece of fruit is distinct.

4. List in lexicographic (like alphabetic order, so for instance $\{1, 3, 5\}$, $\{1, 5, 3\}$, $\{3, 1, 5\}$ are in lexicographic order) order all three-element permutations of the five-element set $\{1, 2, 3, 4, 5\}$. Note that this problem is probably much easier to do by writing a program to do the listing and print the permutation for you. Underline those elements that correspond to the set $\{1, 3, 5\}$. Draw a rectangle around those that correspond to the set $\{2, 4, 5\}$. How many three-element permutations of $\{1, 2, 3, 4, 5\}$ correspond to a given three-element set? How many three-element subsets does the set $\{1, 2, 3, 4, 5\}$ have?

5. In how many ways can a class of 20 students choose a group of three students from among themselves to go to the professor to explain that the 3-hour labs actually take 10 hours?

6. Suppose you are organizing a panel discussion on allowing alcohol on campus. Participants will sit behind a table in the order in which you list them. You must choose four administrators from a group of 10 and four students from a group of 20. If the administrators must sit together in a group and the students must sit together in a group, in how many ways can you choose and list the eight people? If you must alternate students and administrators, in how many ways can you choose and list them?

7. A basketball team has 12 players. However, only five players play at any given time during a game. In how may ways can the coach choose the five players? To be more realistic, the five players playing a game normally consist of two guards, two forwards, and one center. If there are five guards, four forwards, and three centers on the team, in how many ways can the coach choose two guards, two forwards, and one center? What if one of the centers is equally skilled at playing forward?

8. Explain why a function from an n-element set to an n-element set is injective if and only if it is surjective. Is this true for sets with infinite cardinality? Why or why not?

9. The function g is called an *inverse* to the function f if the domain of g is the range of f, if $g(f(x)) = x$ for every x in the domain of f, and if $f(g(y)) = y$ for each y in the range of f.

a. Explain why a function is a bijection if and only if it has an inverse function.

b. Explain why a function that has an inverse function has only one inverse function.