Counting: The Sum and Product Principles

1. Consider the following code.

Problem 1 Code

```
1     for i = 2 to n
2         j = i
3         while (j ≥ 2)(A[j] < A[j-1])
4             swap A[j] and A[j-1]
5             j = j-1
```

What is the maximum number of times (considering all lists of n items that you could be asked to sort) the program makes the comparison $A[j] < A[j-1]$? Describe as succinctly as you can those lists that require this number of comparisons.

2. Five schools are going to send their badminton teams to a tournament in which each team must play each other team exactly once. How many games are required?

3. In how many ways can a 10-person club select a president and a secretary-treasurer from among its members (assuming that a member cannot fill both positions)?

4. In how many ways can a 10-person club select a two-person executive committee from among its members?

5. In how many ways can a 10-person club select a president and a two-person executive advisory board from among its members (assuming that the president is not on the advisory board)?

6. Using the formula for $\binom{n}{2}$, it is straightforward to show that

$$n\binom{n-1}{2} = \binom{n}{2}(n-2).$$

However, this proof simply uses blind substitution and simplification. Find a more conceptual explanation of why this formula is true. (Hint: Think in terms of officers and committees in a club.)