Let

$$k(n) = 2^{n+1} - 3 \qquad \text{and} \qquad p_N(z) = z + \sum_{k=0}^{N} b_k z^{-k}.$$

Suppose we know the constants $b_0, b_1, \ldots, b_{j-1}$. We also have the recursion relation

$$A_n(z) = (A_{n-1}(z))^2 + z.$$

We know that for a given n, that the coefficient of z^{2^n-j-1} is zero for $j \leq k(n)$. Suppose we know $b_0, b_1, \ldots, b_{j-1}$. Then examining $A_n(p_j(z))$ allows us to compute b_j. For example, since $k(1) = 1$, we can look at

$$A_1(p_1(z)) = z^2 + (2b_0 + 1)z + (2b_1 + b_0^2 + b_0)z^0 + (2b_0 + b_1)z^{-1} + b_1^2 z^{-2},$$

where we know that

$$2b_0 + 1 = 0 \qquad \text{and} \qquad 2b_1 + b_0^2 + b_0 = 0,$$

which gives $b_0 = -1/2$ and $b_1 = 1/8$.