SMU COX SCHOOL OF BUSINESS

Lecture 6: Time Value of Money – Part 1

The Basics of Time Value of Money

Presentation to Cox Business Students

FINA 3320: Financial Management

Purpose of This Lecture

- **Gain an understanding of the basics of time value of money**

  - (1) Rationale behind why money has time value

  - (2) Difference between simple and compound interest
    - Become familiar with annual percentage rate (APR) and effective annual rate (EAR)

  - (3) Present value and future value of a single cash flow

Decisions, Decisions…

- Each project requires an upfront investment of $4,000
  - Project 1: Returns $5,000 in 2 years
  - Project 2: Returns $1,000 per year for the next 5 years
  - Project 3: Returns $2,500 in year 1 and $2,500 in year 4

- All project cost $4,000 and return a total of $5,000
  - Are they to be considered different from one another?
  - Which, if any, would be acceptable to shareholders?
  - As a financial manager, which one (ones) would you consider undertaking? Why?

A Simple Offer

- Would you rather have…
  - $1,000 right now?
  - Or
  - $1,000 one year from today?
- Why?
- What about $1,001 in a year? Or maybe $1,002?
  - What would it take for you to consider taking the money one year from today as opposed to $1,000 now?

Money Has a Time Value

- A dollar is a dollar is a dollar…or is it?
  - Cash in hand today can be invested to generate additional cash in the future
  - Cash flows at ***different times*** are to be viewed as ***different quantities***

- **Bottom line**: A dollar today is worth more than a dollar tomorrow

- What can we say about our three projects?

Time Value and Time Lines

- Time lines enable financial managers to visualize time value problems
- A time line for each of the 3 projects follows…

Periods	0	1	2	3	4	5
Project 1 Cash flows	($4,000)		$5,000			
Project 2 Cash flows	($4,000)	$1,000	$1,000	$1,000	$1,000	$1,000
Project 3 Cash flows	($4,000)	$2,500			$2,500	

Future Values

The amount to which a cash flow (or series of cash flows) will grow over a given period of time when compounded at a given interest rate

"A dollar in hand today is worth more than a dollar to be received in the future"

Compounding

The arithmetic process of determining the final value of a cash flow (or series of cash flows) when compound interest is applied

The Rate of Interest

- "A dollar today is worth more than a dollar tomorrow"
  - This is a qualitative statement
  - But how can we ***quantify*** the tradeoff?

- Suppose you are indifferent between \$1,000 today and \$1,050 one year from now
  - Ratio of period 1 to period 0 "equivalent" cash flows is 1.05/1
  - In other words, next period's cash flow must be 5% greater than this period's cash flow to establish the equivalence

- The rate of interest (in this case, 5% per annum) quantifies the tradeoff between cash flows across time

The Rate of Interest continued…

- "A dollar today is worth more than a dollar tomorrow"

- What does the above statement imply about the rate of interest?

- What if the interest rate were zero? Negative?

A “Bank Account” Framework

- Consider a bank account:
  - Money can be deposited
  - Money can be withdrawn
  - At any time, there exists an account balance
  - The bank pays the owner interest based on the account balance

- Interest represents the ***rent*** paid for money
  - Owner desires compensation for giving up the current use of the money
  - Bank is willing to pay since it desires the funds now

A Basic Case

- Scenario: Make a \$100 deposit and leave the money in the account for 1 year
  - Call the initial \$100 the ***present value*** or **PV**
  - Call the account balance in 1 year the ***future value*** or **FV**

- If the annual rate (r) is 6%, what is the value of the account in 1 year?

- Two components of the FV
  - \$100 initially invested (principal)
  - \$6 in interest
  - Or
  - FV = PV*(1+r) = 100*(1.06) = \$106

A Basic Case continued…

- **Formula approach: $FV_t = PV*(1+r)^t$**
  - What would be the value of the account after various numbers of years?
  - In 1 Year: $FV_1 = PV*(1+r)^1 = 100*(1.06)^1 = \$106.0000$
  - In 2 Years: $FV_2 = PV*(1+r)^2 = 100*(1.06)^2 = \$112.3600$
  - In 3 Years: $FV_3 = PV*(1+r)^3 = 100*(1.06)^3 = \$119.1016$

A Basic Case continued…

- **Financial calculators**
  - Five (5) keys for the five variable in the basic time value equations
    - n = number of periods
    - i = interest rate per period
    - PV = present value
    - PMT = payment
    - FV = future value

A Basic Case continued…

- **Financial calculators**
  - What would be the value of the account after various numbers of years?
  - In 1 Year: 1n; 6i; -100PV; 0PMT; FV = $106.0000
  - In 2 Years: 2n; 6i; -100PV; 0PMT; FV = $112.3600
  - In 3 Years: 3n; 6i; -100PV; 0PMT; FV = $119.1016

Note: Negative for PV indicates it is a cash outflow

Interest Calculations (Multiple Periods)

- **Simple interest**: Interest paid on initial principal only
- **Compound interest**: Interest paid on prior period's ending balance
  - Each period's interest is added to the principal
  - Interest is paid on interest
  - FV_t of \$1 = $1*(1+r)^t$
    - r = periodic interest rate
    - t = number of periods

Examples

- **Account 1**: Invest $1,000 for 4 years at 6% ***simple interest***

- **Account 2**: Invest $1,000 for 4 years at 6% interest ***compounded annually***

- Which is worth more after 4 years?

Simple vs. Compound Interest

	Account 1			Account 2		
Year	Beg. Bal.	Interest	End Bal.	Beg. Bal.	Interest	End Bal.
1	1,000	60	1,060	1,000	60	1,060
2	1,060	60	1,120	1,060	64	1,124
3	1,120	60	1,180	1,124	67	1,191
4	1,180	60	1,240	1,191	71	1,262

Compounding Periods

- Account 3: Invest $1,000 for 4 years at 6% interest ***compounded semi-annually***
  - Semi-annual rate = 3%
  - 8 semi-annual periods

- Account 4: Invest $1,000 for 4 years at 6% interest ***compounded monthly***
  - Monthly rate = 0.5%
  - 48 monthly periods

Compounding Periods continued…

- Account 3: Invest $1,000 for 4 years at 6% interest ***compounded semi-annually***
  - **Formula**: $FV_8 = PV*[1+(0.06/2)]^8 = 1000*[(1.03)]^8 = \$1266.770$
  - **Financial Calculator**: 8n; 3i; -1000PV; 0PMT; FV = $1266.770

- Account 4: Invest $1,000 for 4 years at 6% interest ***compounded monthly***
  - **Formula**: $FV_{48} = PV*[1+(0.06/12)]^{48} = 1000*[(1.005)]^{48} = \$1270.489$
  - **Financial Calculator**: 48n; 0.5i; -1000PV; 0PMT; FV = $1270.489

Compounding Periods continued…

- Annual percentage rate (APR) = periodic interest rate multiplied by periods per year
  - "Typical" interest rate quotes
  - All rates in past examples are APRs
- FV of \$1 in 1 year = $\$1*(1+APR/m)^m$
  - Increase m…Increase FV
  - Increase m to infinity? – ***Continuous compounding***
  - Continuous compounding: $FV = 1*e^{APR}$
- For t years: $1*(1+APR/m)^{mt}$ and $1*e^{APR*t}$

Compounding Periods continued…

- How can we compare rates with different compounding periods?
  - **Effective annual rate (EAR)**: Annual rate that accounts for compounding
  - Return over 1 year = $(1+APR/m)^m = 1 + EAR$

- **Interpretation**: For a given APR and compounding period, EAR is the rate that will earn the same interest under annual compounding

Effective Annual Rates (EAR)

- Compute EAR for an APR of 12% compounded:
  - Annually
  - Semi-annually
  - Monthly
  - Daily
  - Continuously

- “Increasing m increases FV” and “Increasing m increases EAR” are equivalent statements

Effective Annual Rates (EAR)

- Compute EAR for an APR of 12% compounded:
  - Annually: $[1+(.12/1)]^{1} - 1 =$ 12.0000%
  - Semi-annually: $[1+(.12/2)]^{2} - 1 =$ 12.3600%
  - Monthly: $[1+(.12/12)]^{12} - 1 =$ 12.6825%
  - Daily: $[1+(.12/365)]^{365} - 1 =$ 12.7475%
  - Continuously: $e^{0.12} - 1 =$ 12.7497%

Using the EAR

- Which has higher FV – (A) 6% compounded monthly or (B) 6.1% compounded semi-annually?
- Step 1: Compute the EAR for each
- Step 2: Compare across accounts

Using the EAR

- Which has higher FV – (A) 6% compounded monthly or (B) 6.1% compounded semi-annually?

- Step 1: (A) $[1+(0.06/12)]^{12} - 1 = 6.1678\%$
  (B) $[1+(0.061/2)]^{2} - 1 = 6.1930\%$

- Step 2: Compare across accounts
  (B) has higher EAR than (A)

Present Values

The value today of a future cash flow (or series of cash flows) discounted at a given interest rate

Discounting

The process of finding the present value of a cash flow (or series of cash flows)

Discounting is the reverse (i.e., reciprocal) of compounding

Present Values

- Given PV and r, we can compute an equally valuable cash flow from any other time period
- But much of finance seeks today's value of expected future cash flows
  - Example 1: A stock generates future cash flows for its owner, but how much is the stock worth today?
  - Example 2: An investment project generates future cash flows for shareholders, but how much is the project worth today?
- Use same concept as before:
  - Begin with: $FV_t = PV_0*(1+r)^t$
  - Simply solve for present value: $PV_0 = FV_t/(1+r)^t$

Relevant Terminology

- Discount rate: Interest rate used in the denominator of present value calculations

- Discount factor: Factor by which a cash flow is multiplied to calculate a present value
  - $1/(1+r)^t$ in formulas
  - Less than 1 if positive r

Present Value Example

- Someone promises you $3,000 in 3 years. How much is this worth to you ***today*** if you can invest money at 10% compounded annually?

- **Interpretation**: You are indifferent between receiving $3,000 in 3 years and what $ amount today if you required a 10% rate of return compounded annually?

Present Value Example

- Someone promises you \$3,000 in 3 years. How much is this worth to you ***today*** if you can invest money at 10% compounded annually?

  - **Formula**: $PV = FV_t/(1+r)^t = \$3{,}000/(1.10)^3 = \$2{,}253.94$
  - **Financial Calculator**: 3n; 10i; 0PMT; 3,000FV; PV = -\$2,253.94
    Note: Negative for PV indicates it is a cash outflow

- **Interpretation**: You are indifferent between receiving \$3,000 in 3 years and \$2,253.94 today

Present Value Example

- Suppose you own the claim to $3,000 in 3 years
  - For how much should you be able to sell this claim in a secondary market?
  - ANSWER: ***$2,253.94*** if purchaser in secondary market required a 10% rate of return compounded annually

  - What if your debtor wanted to settle the claim today? How much would you require?
  - ANSWER: ***$2,253.94*** if you require a 10% rate of return compounded annually

More Examples

- You know that you will need $5,000 in 4 years. How much would you have to invest today at 4% compounded annually in order to reach your goal?

Periods	0	1	2	3	4
Cash flows					$5,000

Discounted at 4% for 4 years

PV?

- How much would you invest if you could invest at 5% instead of 4%?

Cash flows					$5,000

Discounted at 5% for 4 years

PV?

More Examples

- You know that you will need \$5,000 in 4 years. How much would you have to invest today at 4% compounded annually in order to reach your goal?

  - **Formula**: $PV = FV_t /(1+r)^t = \$5,000/(1.04)^4 = \$4,274.02$
  - **Financial Calculator**: 4n; 4i; 0PMT; 5,000FV; PV = \$4,274.02

- How much would you invest if you could invest at 5% instead of 4%?

- **Formula**: $PV = FV_t /(1+r)^t = \$5,000/(1.05)^4 = \$4,113.51$
- **Financial Calculator**: 4n; 5i; 0PMT; 5,000FV; PV = -\$4,113.51

Present Value Sensitivities

- How does present value change when the future cash flow changes?

- How does present value change when the discount rate is changed?

- How does present value change when the timing of the cash flow changes?

Present Value Sensitivities

- How does present value change when the future cash flow changes?
  - ***Same Direction***: When FV goes up (down) so does PV

- How does present value change when the discount rate is changed?
  - ***Opposite Direction***: When r goes up (down), PV goes down (up)

- How does present value change when the timing of the cash flow changes?
  - ***Opposite Direction***: When t goes up (down), PV goes down (up)

Solving for Interest Rate (r)

- Just takes a little algebra…
- Example: Invest $2,572 today and get $3,000 in 3 years. What is the interest rate under annual compounding?
- Rule of 72: Approximate the interest rate that will double the investment by dividing 72 by the length of the investment period (in years)

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Solving for Interest Rate (r)

- Just takes a little algebra…

- Example: Invest $2,572 today and get $3,000 in 3 years. What is the interest rate under annual compounding?

  - Using algebra: $r = (FV/PV)^{1/t} - 1$
    $r = (\$3{,}000/\$2{,}572)^{1/3} - 1 = 5.2649\%$

- Rule of 72: 72/3years = 24%

Solving for t

- Again, just a little algebra (but easier with calculator)…

- Example: How long does it take $2,500.08 to grow to $4,406 at 12% with annual compounding?

- Rule of 72: Approximate the time it will take to double the investment by dividing 72 by the annual percentage interest rate (r)

Solving for t

- Again, just a little algebra (but easier with calculator)…

- Example: How long does it take $2,500.08 to grow to $4,406 at 12% with annual compounding?

  - **Financial Calculator**: 12i; -2,500PV; 0PMT; 4,406FV; n = 5
  - Formula: $FV/(1+r)^t = PV$; $\$4,406/(1.12)^5 = \$2,500.08$

- Rule of 72: 72/12% = 6 years

We now know how to do Present and Future Values of Single Cash Flows!

But what if an asset has multiple cash flows?

Thank You!

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