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Lecture 6: Time Value of Money – Part 2

The Basics of Time Value of Money

Presentation to Cox Business Students

FINA 3320: Financial Management

Purpose of This Lecture

- **Gain an understanding of the basics of time value of money pertaining to assets with multiple cash flows**
  - (1) Multiple Cash Flows (i.e., PMTs)
  - (2) Ordinary Annuities and Annuities Due
  - (3) Perpetuities
  - (4) Uneven Cash Flows

On Valuation

- Asset: Anything that generates cash flows for its owner

- Valuation: Process of assigning a value to an asset
  - Amounts to finding the present value of cash flows asset generates
  - Examples so far…
    - We valued assets generating single cash flows for their owners
    - But that need not be the case!

Valuation Examples

- A piece of art will generate a future cash flow when it is sold. Should you buy it today?
- Your education will generate a stream of future cash flows for you throughout your career. Is it worth the investment?
- A project generates expected future cash flows for the firm's shareholders. Should you approve the project?

Valuation Examples continued…

- All examples on the prior page require you to:
  - (1) Assess the value of the asset
  - (2) Weigh the value against the cost of the asset
- When the asset value exceeds the purchase price (i.e., cost), wealth is increased
- In each situation, where would you see the effects of the wealth increase?

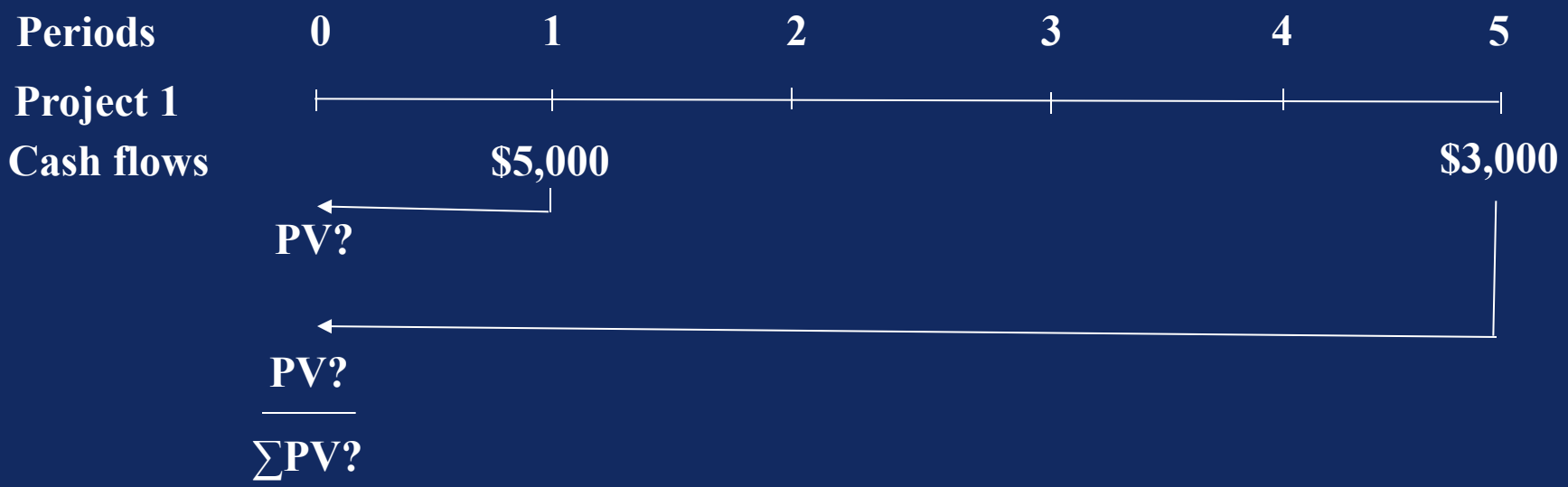
Assets with Multiple Cash Flows

- Suppose an asset generates a series of cash flows for its owners
  - Value each cash flow separately
  - Sum these present values to assign a value to the asset

$$PV_0 = \sum_{t=0}^{\infty} \frac{CF_t}{(1+r)^t} = CF_0 + \frac{CF_1}{(1+r)^1} + \frac{CF_2}{(1+r)^2} + \text{K}$$

Multiple Cash Flows: An Example

- Consider an asset that generates $5,000 in year 1 and $3,000 in year 5; discount rate = 6%
  - View this as a time line that displays the cash flows



Multiple CFs: An Example continued…

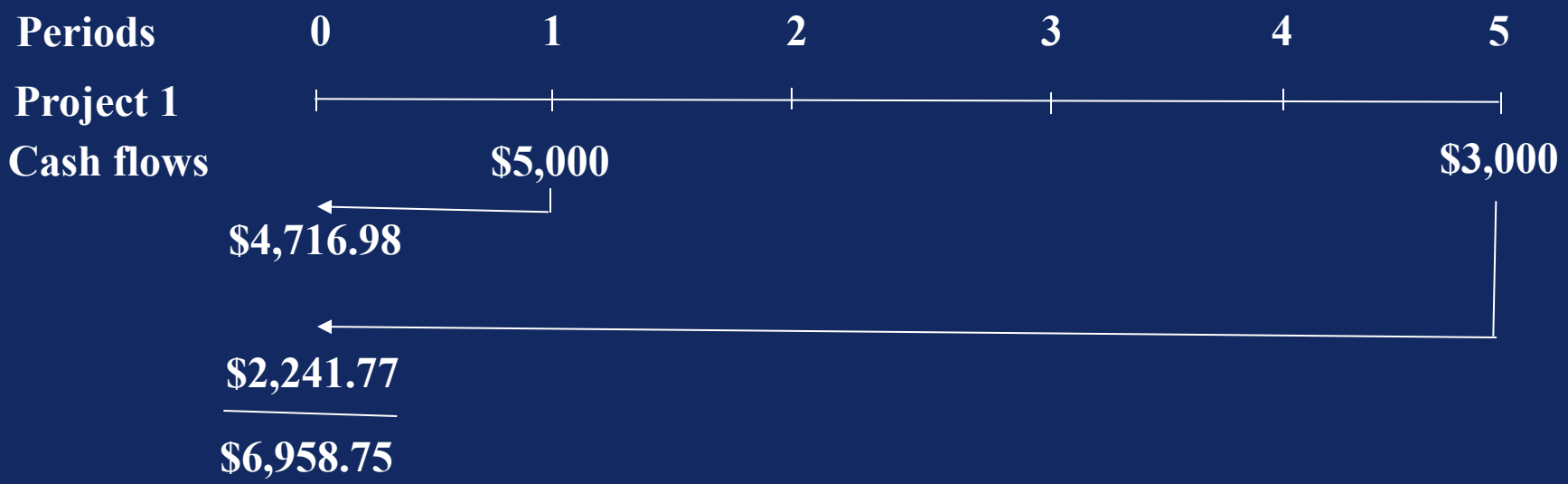
- Consider an asset that generates $5,000 in year 1 and $3,000 in year 5; discount rate = 6% per year
  - To solve, use the basic PV equation and then ∑
    - $PV = FV_t/(1+r)^t = \$5{,}000/(1.06)^1 = \$4{,}716.98$
    - $PV = FV_t/(1+r)^t = \$3{,}000/(1.06)^5 = \$2{,}241.77$

      ∑ = $6,958.75
  - Or, use a financial calculator and then ∑
    - 1n; 6i; 0PMT; 5,000FV; PV = -$4,716.98
    - 5n; 6i; 0PMT; 3,000FV; PV = -$2,241.77

      ∑ = -$6,958.75

Multiple CFs: An Example continued…

- Consider an asset that generates $5,000 in year 1 and $3,000 in year 5; discount rate = 6%
  - To solve, use the basic PV equation



Multiple CFs: An Example continued…

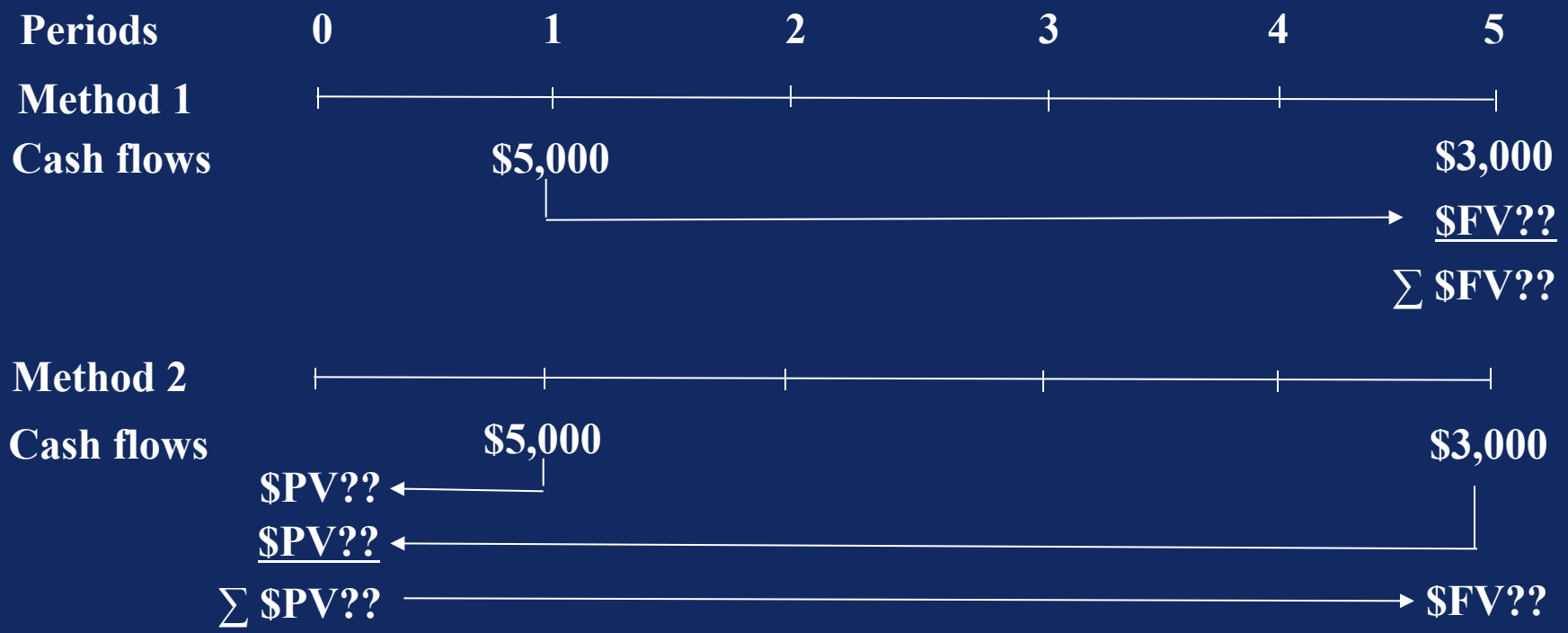
- Consider an asset that generates $5,000 in year 1 and $3,000 in year 5; discount rate = 6%
- **Note**: In finding the PV of this asset, we are simply summing the PVs of each individual cash flow
- **Interpretation**: You are indifferent between receiving the future cash flow stream and having $6,958.75 today

Back to the Future (Values that is)

- What if you wanted to find future value of a stream of cash flows?
- Alternatively, you would be indifferent between receiving the future cash flow stream on the time line and a time period 5 cash flow of ???
- Two ways to answer this question:
  - **Method 1**: Compound each cash flow and sum them up
  - **Method 2**: Find PV of each cash flow, sum them up, and then compound the sum

Back to Future Value (and Time Lines)

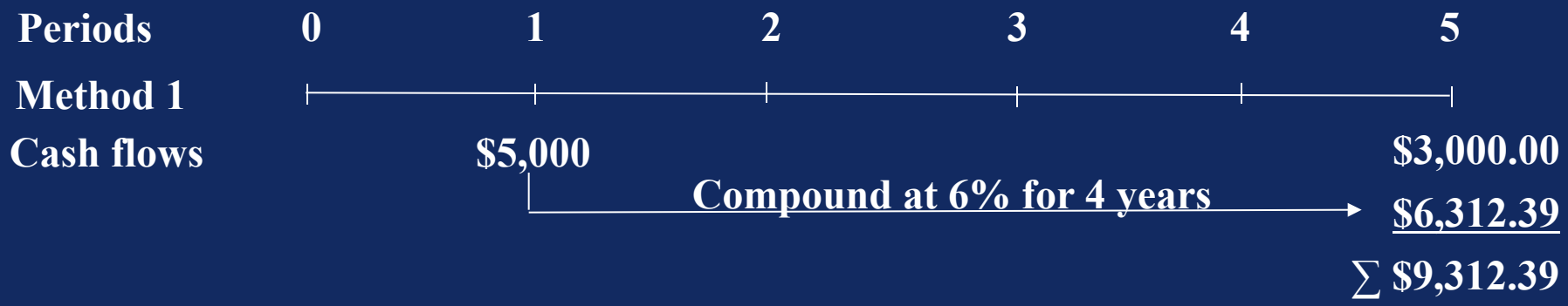
- Consider our example again: Asset generates $5,000 in year 1 and $3,000 in year 5; discount rate = 6%
- What is the future value of this cash flow stream?



Method 1

Formula: $FV_t = PV*(1+r)^t = \$5{,}000*(1.06)^4 = \$6{,}312.39$
Financial Calculator: 4n; 6i; 0PMT; 5,000PV; FV = $6,312.39

- Note: For Method 1 period 5 cash flow is already FV (i.e., at time period 5)



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Method 2

CF 1 **Formula**: $PV = FV_t/(1+r)^t = 5{,}000/(1.06)^1 = \$4{,}716.98$

Financial Calculator: 1n; 6i; 0PMT; 5,000FV; PV = $4,716.98

CF 5 **Formula**: $PV = FV_t/(1+r)^t = 3{,}000/(1.06)^5 = \$2{,}241.78$

Financial Calculator: 5n; 6i; 0PMT; 3,000FV; PV = $2,241.778

FV **Formula**: $FV_t = PV*(1+r)^t = 6{,}958.76*(1.06)^5 = \$9{,}312.39$

Financial Calculator: 5n; 6i; 6,958.76PV; 0PMT; FV = $9,312.39

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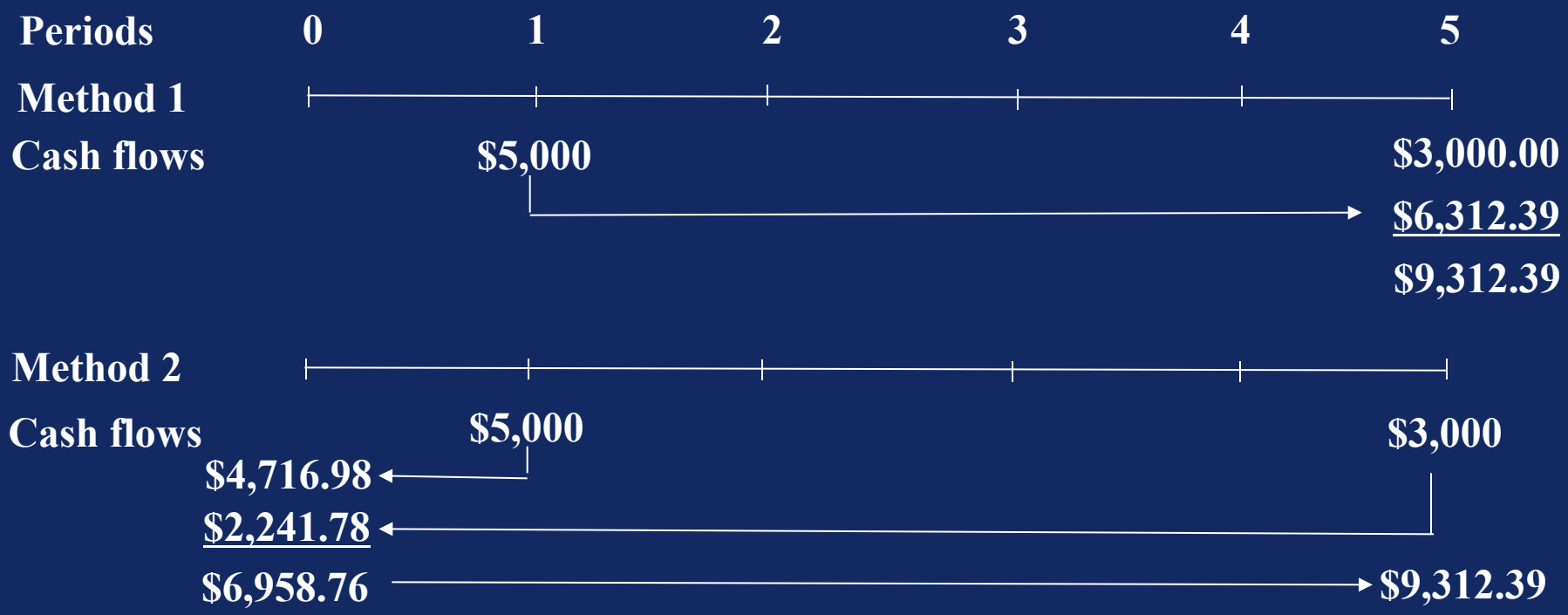
Back to Future Value (and Time Lines)

Method 1 and Method 2 provide the same FV of $9,312.39



Special Cases

- PV formula holds for any stream of cash flows
- Two special cases where the formula simplifies:
  - (1) Perpetuity
  - (2) Annuity

Perpetuity

Present Value of a Perpetuity

$$PVPerpetuity_0 = \sum_{t=0}^{\infty} \frac{CF_t}{(1+r)^t} = CF + \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots$$

Perpetuity

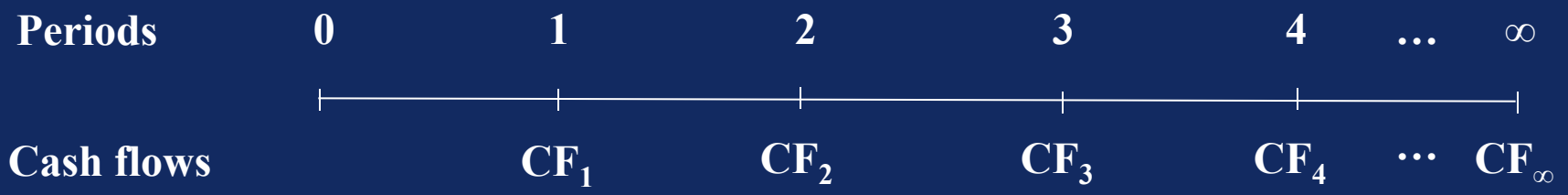
- Definition: Steam of level cash flows that never ends
  - Cash flow is always in the amount of CF
  - First cash flow is in ***period 1*** (i.e., time 0 cash flow is $0)

- What does the time line look like?
  - Draw it!

$$PVPerpetuity_0 = \sum_{t=0}^{\infty} \frac{CF_t}{(1+r)^t} = CF + \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots$$

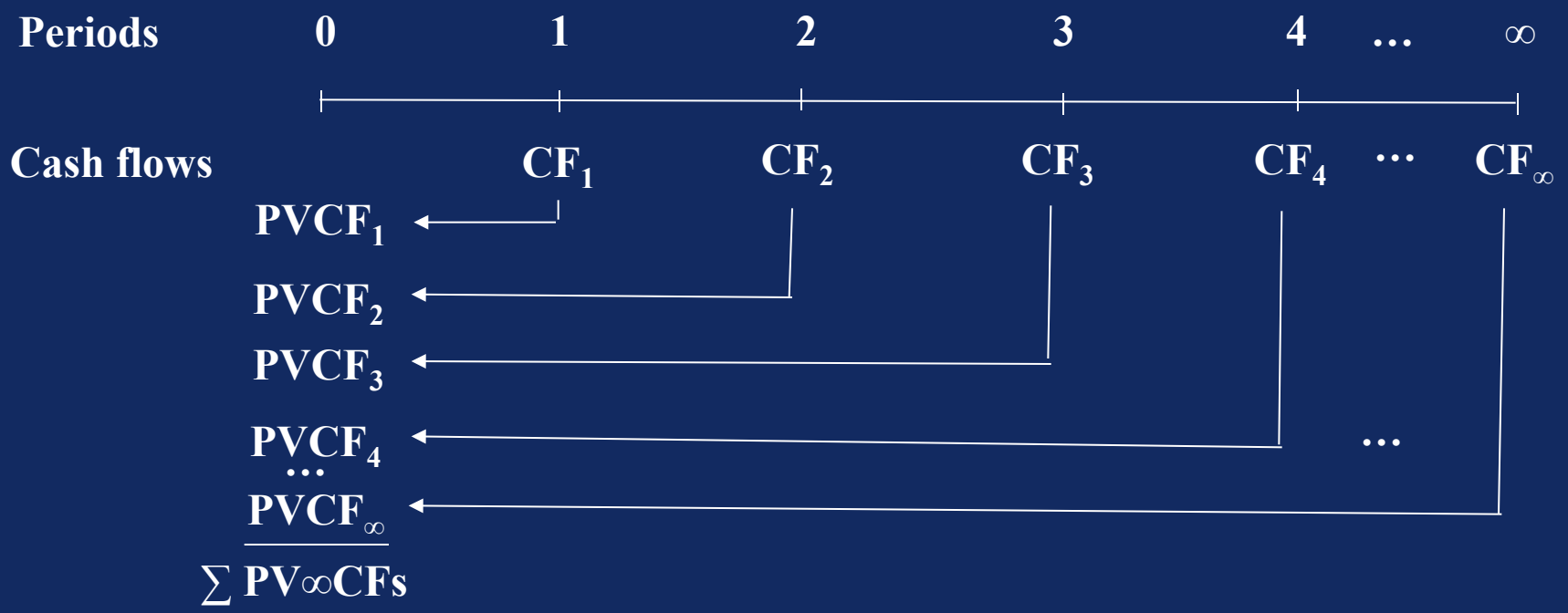
Perpetuities and Time Lines

- Time line for a perpetuity follows…



Perpetuities and Time Lines

- Determining present value for a perpetuity follows…
- Note: You need to determine the present value of an ***infinite*** cash flow stream!



Simplifying a Perpetuities

- A power series take the form:

  $a + ax + ax^2 + ax^3 + ax^4 + \ldots + ax^{\infty}$,

  and is equivalent to

  $a/(1-x)$

- A perpetuity is a power series with:

  $a = CF/(1+r)$

  and

  $x = 1/(1+r)$

- So, $a/(1-x)$ becomes…

Simplifying a Perpetuities continued…

- So, a/(1-x) becomes…

$$a = \frac{CF}{(1+r)} \qquad x = \frac{1}{(1+r)}$$

$$\therefore$$

$$\frac{a}{(1-x)} = \frac{\frac{CF}{(1+r)}}{1-\frac{1}{(1+r)}} = \frac{CF}{r}$$

$$PVPerpetuity_0 = \frac{CF}{r}$$

Think About It…

- What if the first payment of the perpetuity were to occur at time 0?

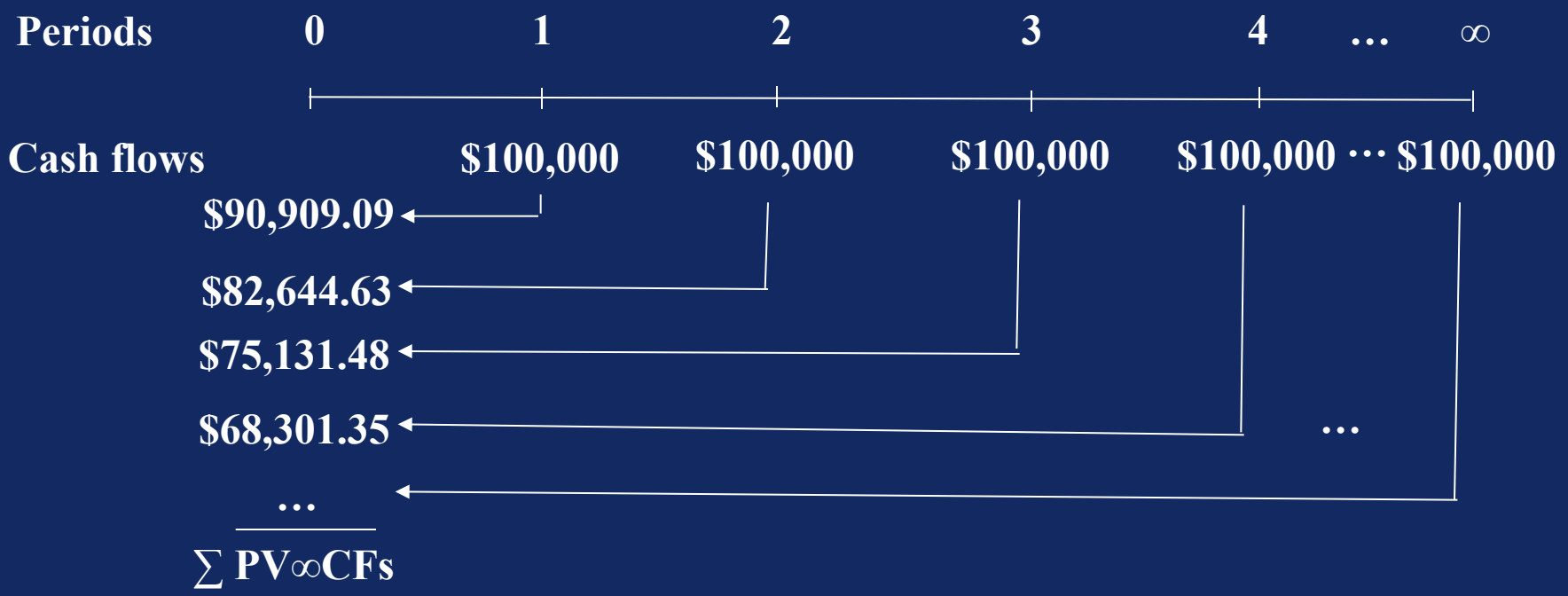
$$PVPerpetuity_0 = \sum_{t=0}^{\infty} \frac{CF_t}{(1+r)^t} = CF + \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots$$

$$PVPerpetuity_0 = (1+r)\left[\frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots\right] = (1+r)\frac{CF}{r}$$

$$PVPerpetuity_0 = CF + \frac{CF}{r}$$

Simplifying a Perpetuities Example continued…

- What is the present value of $100,000 per year in perpetuity at 10% interest?



Simplifying a Perpetuities Example continued…

- What is the present value of $100,000 per year in perpetuity at 10% interest?

$$PVPerpetuity_0 = \frac{CF}{r} = \frac{\$100{,}000}{.10} = \$1{,}000{,}000$$

- You can easily solve for different numbers in the formula!
  - Given PV and CF, what is r? $r = \frac{CF}{PV}$
  - Given PV and r, what is CF? $CF = PV \times r$

Annuity

Present Value of an Annuity (PVA)

$$PVA_0 = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} = CF + \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots + \frac{CF}{(1+r)^T}$$

Future Value of an Annuity (FVA)

$$FVA = \sum_{t=1}^{T} CF_t(1+r)^t = CF_1(1+r)^1 + CF_2(1+r)^2 + \ldots + CF_T(1+r)^T$$

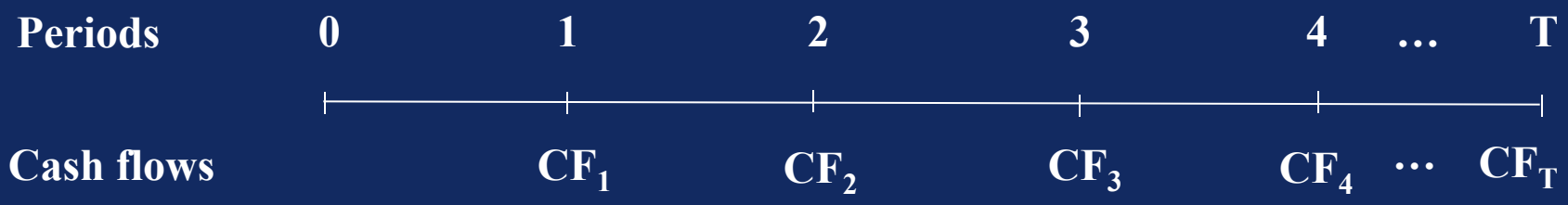
Present Value of an Annuity (PVA)

- Definition: Level stream of cash flows for a limited (finite) period of time discounted back to time 0 at the stated interest rate (i.e., discount rate)
  - Cash flow is always in the amount CF
  - First cash flow is in period 1 (i.e., time 0 cash flow is $0)
  - Last cash flow is in period T
- What does the time line look like for PVA?
  - Draw it!

$$PVA_0 = \sum_{t=0}^{T} \frac{CF_t}{(1+r)^t} = CF + \frac{CF}{(1+r)^1} + \frac{CF}{(1+r)^2} + \ldots + \frac{CF}{(1+r)^T}$$

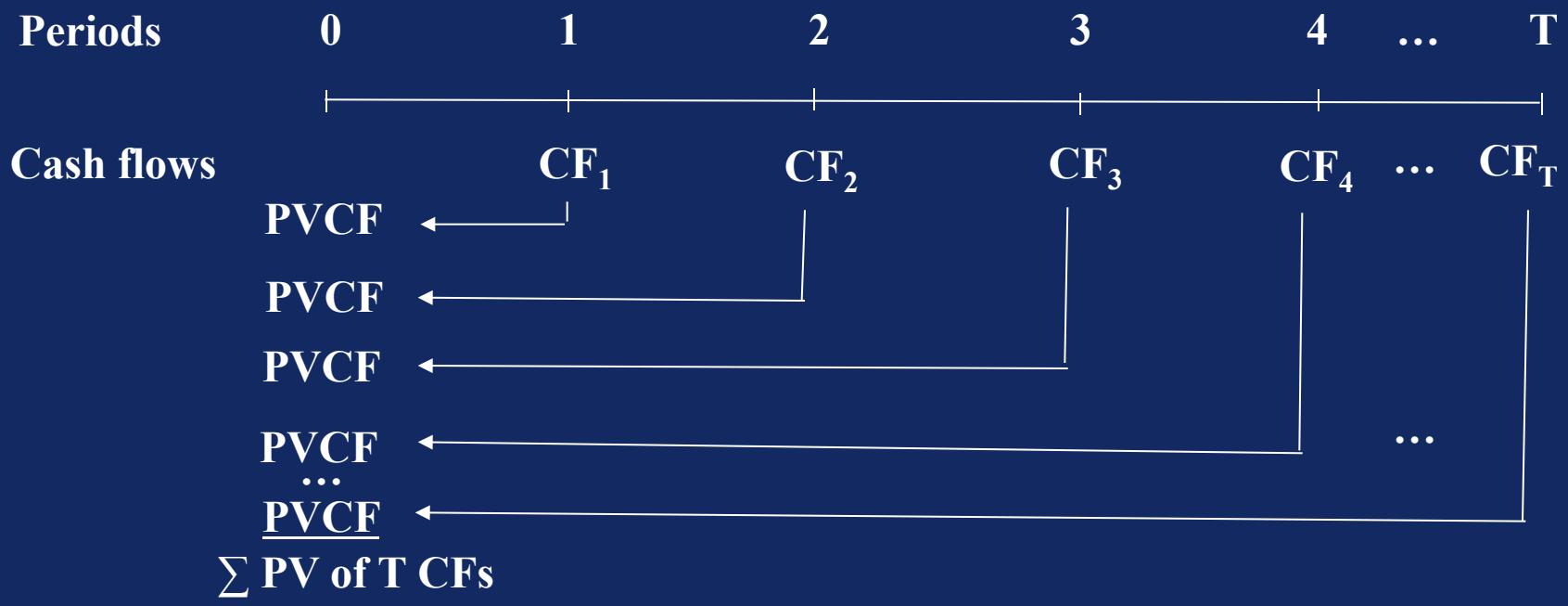
PVAs and Time Lines

- Time line for a present value of an annuity follows…



PVAs and Time Lines

- Determining present value for an annuity follows…
  - Note: You need to determine the present value of a limited (finite) cash flow stream



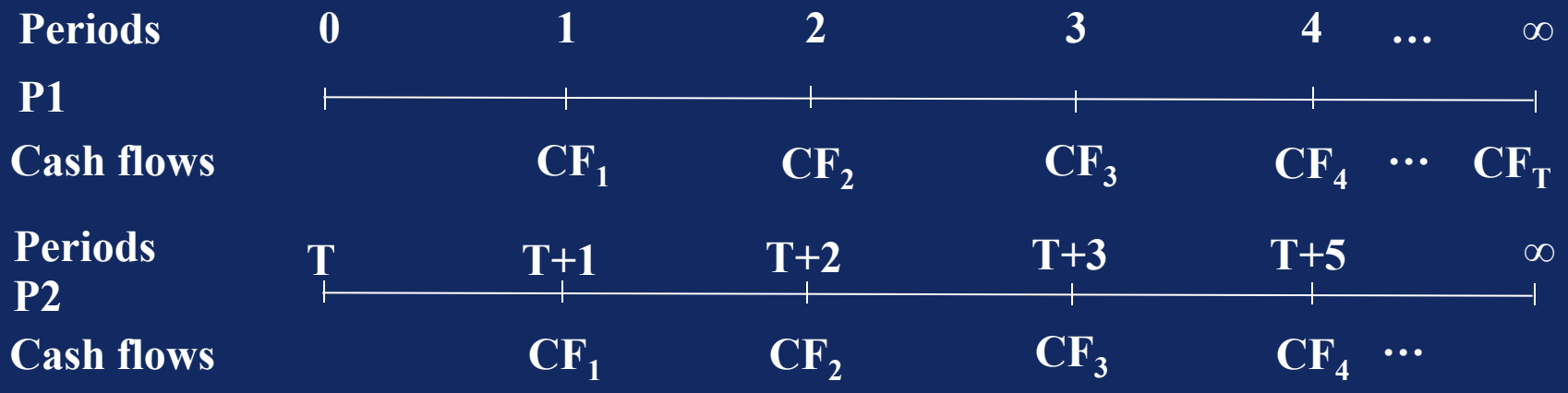
Simplifying an Annuity

- Consider two perpetuities with the same cash flow (call it CF)
  - Perpetuity 1, P1, has first payment at time 1
  - Perpetuity 2, P2, has first payment at time T+1
- What does the time line look like?
  - Draw it!

Simplifying an Annuity continued…

- From the time line, we see that the present value of the T-period annuity is simply:

$$PV(P1)_0 - PV(P2)_0$$



Simplifying an Annuity continued…

- We can easily solve for the present values of the two perpetuities as of the periods before the payments start:
  - $PV(P1)_0 = CF/r$
  - $PV(P2)_T = CF/r$

- But what is $PV(P2)_0$?
  - Just view its present value at time T as a time-T cash flow and discount it to time 0

$$PV(P2)_0 = \frac{PV(P2)_T}{(1+r)^T} = \frac{CF}{r}\left[\frac{1}{(1+r)^T}\right] = CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right]$$

Simplifying an Annuity continued…

- Mathematically…

$$PVA_0 = PV(P1)_0 - PV(P2)_0$$

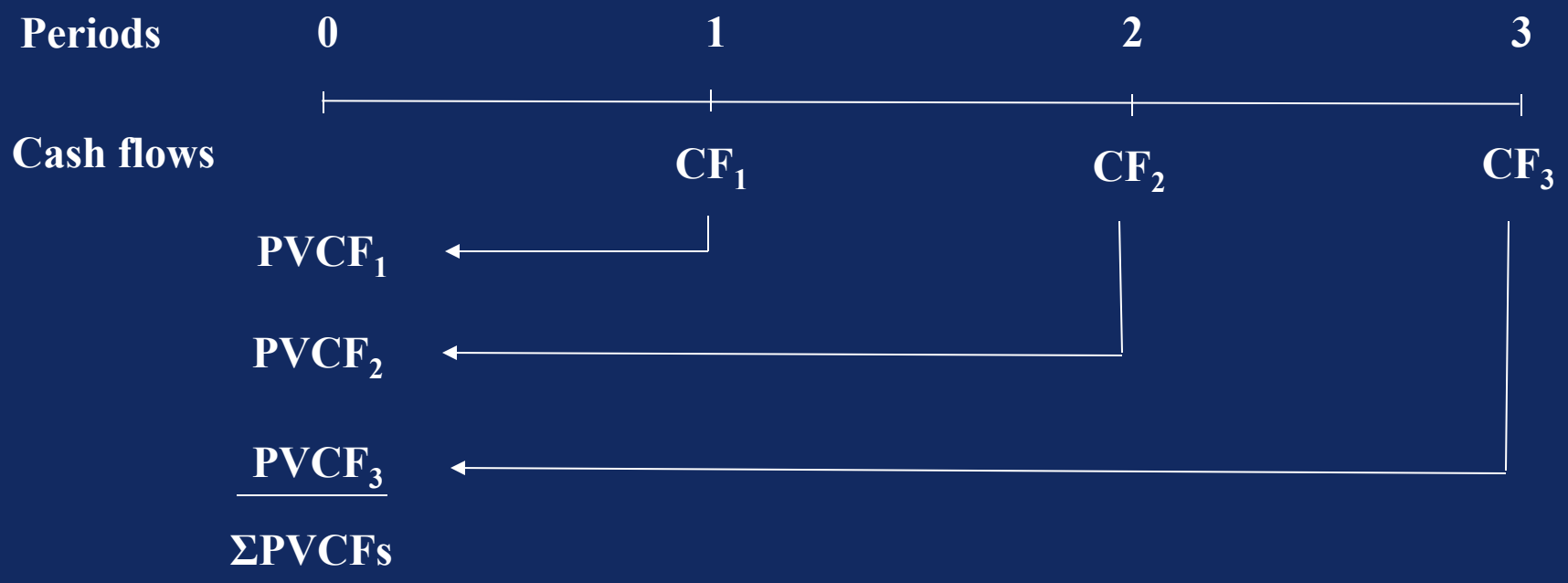
$$PVA_0 = \frac{CF}{r} - \frac{CF}{r}\left[\frac{1}{(1+r)^T}\right]$$

$$PVA_0 = CF\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]$$

$$PVA_0 = CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right]$$

Present Value of an Annuity: Example

- Example: What is the present value of $100,000 per year for 3 years at 10%



Present Value of an Annuity: Example continued…

- Example: What is the present value of $100,000 per year for 3 years at 10%

Formula:

$$PVCF_1 = FVCF_1/(1+r)^1 = \$100{,}000/(1.10)^1 = \$\ 90{,}909.09$$
$$PVCF_2 = FVCF_2/(1+r)^2 = \$100{,}000/(1.10)^2 = \$\ 82{,}644.63$$
$$PVCF_3 = FVCF_3/(1+r)^3 = \$100{,}000/(1.10)^3 = \$\ 75{,}131.48$$
$$\$248{,}685.20$$

Formula:

$$PVA_0 = CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right] = 100{,}000\left[\frac{1-\frac{1}{(1.10)^3}}{0.10}\right] = \$248{,}685.20$$

Financial Calculator: 3n; 10i; 100,000PMT; PV = $248,685.20

Present Value of an Annuity: Example continued…

- Present Value Annuity Factor (PVIF): Factor the payment is multiplied by in order to calculate PV

Formula: $$PVA_0 = CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right] = 100{,}000\left[\frac{1-\frac{1}{(1.10)^3}}{0.10}\right] = \$248{,}685.20$$

PVIF: $$PVA_0 = CF\left[PVIF\right] = 100{,}000\left[\frac{1-\frac{1}{(1.10)^3}}{0.10}\right] = 100{,}000\left[2.486851991\right] = \$248{,}685.20$$

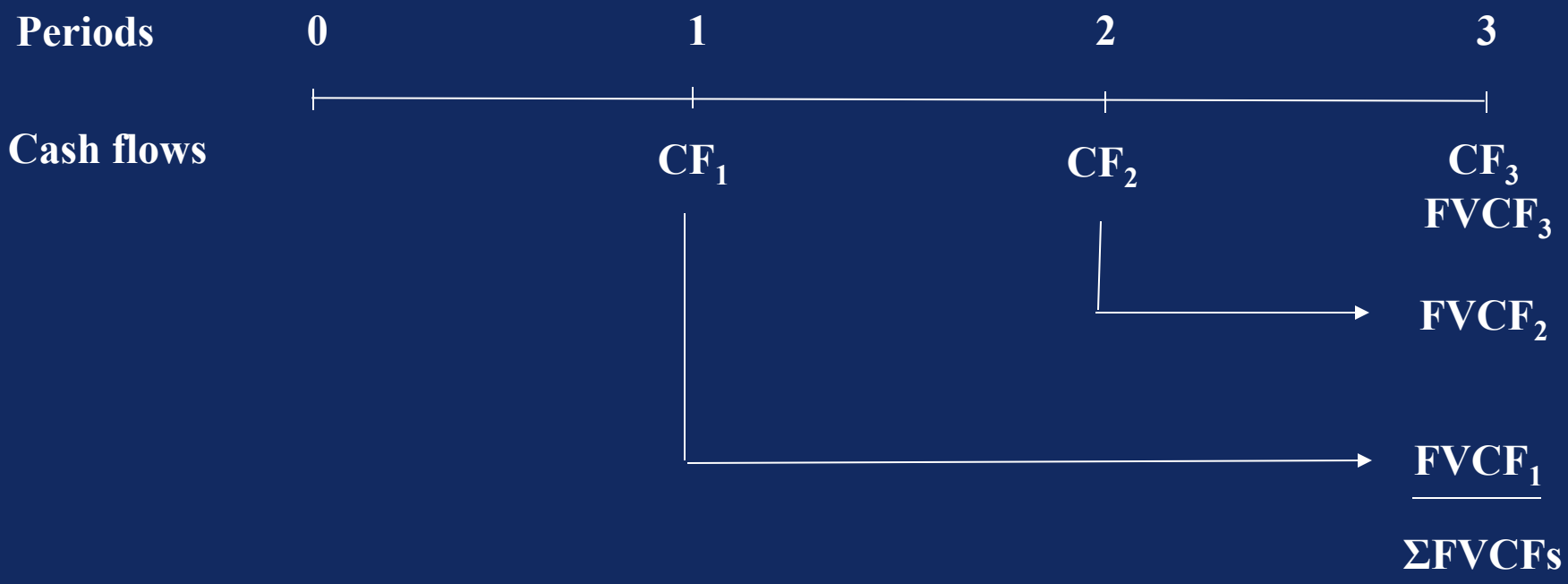
Future Value of an Annuity

- How can you calculate the future value of an annuity?
- Definition: Level stream of cash flows for a limited (finite) period of time compounded to the future at a stated interest rate
  - Cash flow is always in the amount CF
  - First cash flow is in period 1 (i.e., time 0 cash flow is $0)
  - Last cash flow is in period T
    - What does time line look like? Draw it!

$$FVA = \sum_{t=1}^{T} CF_t(1+r)^T = CF_1(1+r)^1 + CF_2(1+r)^2 + \ldots + CF_T(1+r)^T$$

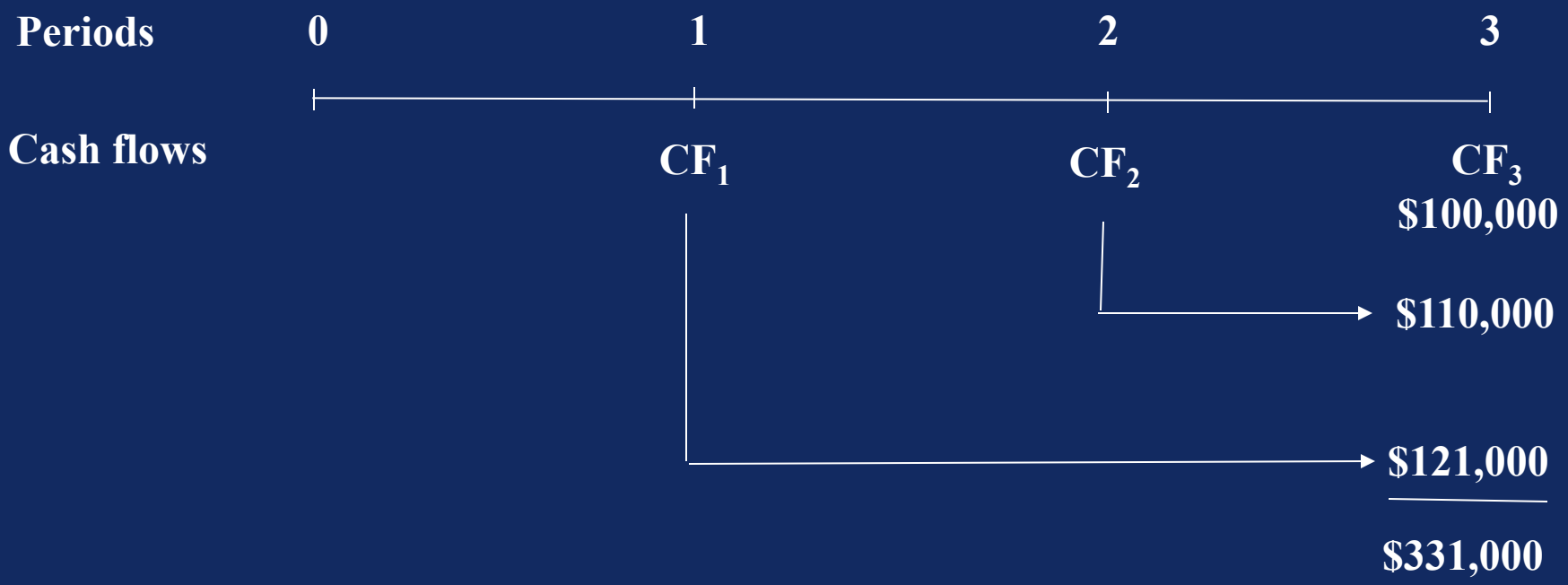
Future Value of an Annuity

- Time line for future value of an annuity follows…



Future Value of an Annuity: An Example

- Example: What is the future value of $100,000 per year for 3 years at 10%



Future Value of an Annuity: An Example

- Example: What is the future value of $100,000 per year for 3 years at 10%

Formula: $FVCF_1 = PVCF_1*(1+r)^2 = \$100,000*(1.10)^2 = \$121,000.00$

$FVCF_2 = PVCF_2*(1+r)^1 = \$100,000*(1.10)^1 = \$110,000.00$

$FVCF_3 = PVCF_3*(1+r)^0 = \$100,000*(1.10)^0 = \underline{\$100,000.00}$

$\$331,000.00$

Formula: $$FVA_T = CF\left[\frac{(1+r)^T - 1}{r}\right] = 100,000\left[\frac{(1.10)^3 - 1}{0.10}\right] = \$331,000$$

Financial Calculator: 3n; 10i; 100,000PMT; FV = $331,000.00

Annuity Due

Present Value of an Annuity Due

$$PVADue_0 = (1+r)CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right]$$

Future Value of an Annuity Due

$$FVADue_T = (1+r)CF\left[\frac{(1+r)^T-1}{r}\right]$$

Present Value of an Annuity Due

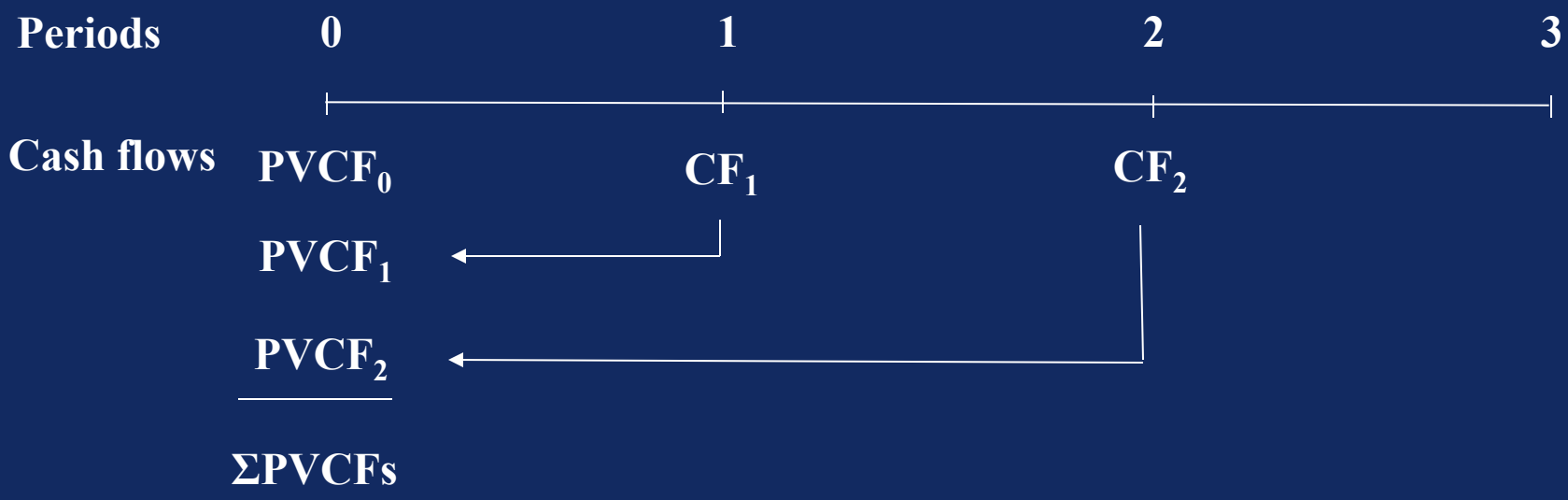
- Definition: Present value of an annuity with first payment at time 0 (instead of time 1)

- Amounts to discounting one less period than with an ordinary annuity
- Multiply present value of an ordinary annuity by (1+r)

$$PVADue_0 = (1+r)CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right]$$

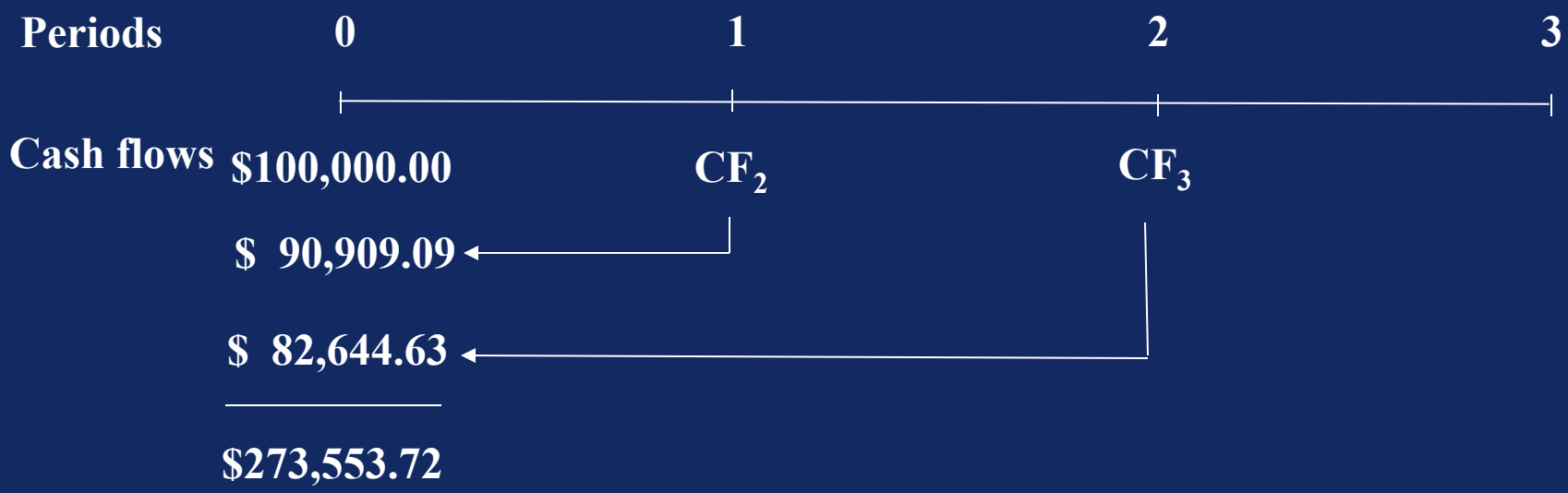
Present Value of an Annuity Due: Time Line

- Time line for the present value of an annuity due follows…



Present Value of an Annuity Due: An Example

- Example: What is the present value of an annuity due with three cash flows of $100,000 at a rate of 10%?



Present Value of an Annuity Due: An Example

- Example: What is the present value of an annuity due with three cash flows of $100,000 at a rate of 10%?

Formula:

$$PVCF_0 = FVCF_0/(1+r)^0 = 100{,}000/(1.10)^0 = \$100{,}000.00$$
$$PVCF_1 = FVCF_1/(1+r)^1 = 100{,}000/(1.10)^1 = \$\ 90{,}909.09$$
$$PVCF_2 = FVCF_2/(1+r)^2 = 100{,}000/(1.10)^2 = \underline{\$\ 82{,}644.63}$$
$$\$273{,}553.72$$

Formula: $$PVADue_0 = (1+r)CF\left[\frac{1-\frac{1}{(1+r)^T}}{r}\right] = (1.10)\$100{,}000\left[\frac{1-\frac{1}{(1.10)^3}}{0.10}\right] = \$273{,}553.72$$

Financial Calculator: Set Due: 3n; 10i; 100,000PMT; PV = $273,553.72

Future Value of an Annuity Due

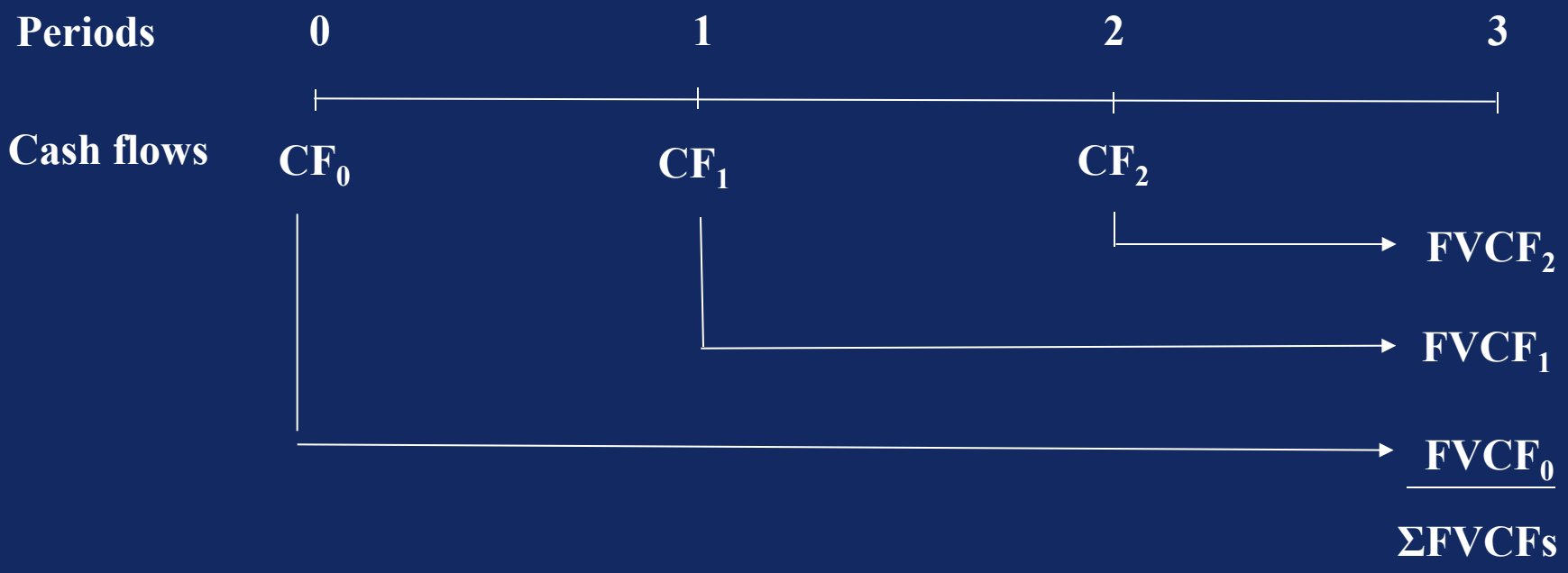
- Definition: Future value of an ordinary annuity with the first payment at time 0 (instead of time 1)

- Amounts to compounding one period more than with an ordinary annuity
- Multiply ordinary annuity FV by (1+r)

$$FVADue_T = (1+r)CF\left[\frac{(1+r)^T - 1}{r}\right]$$

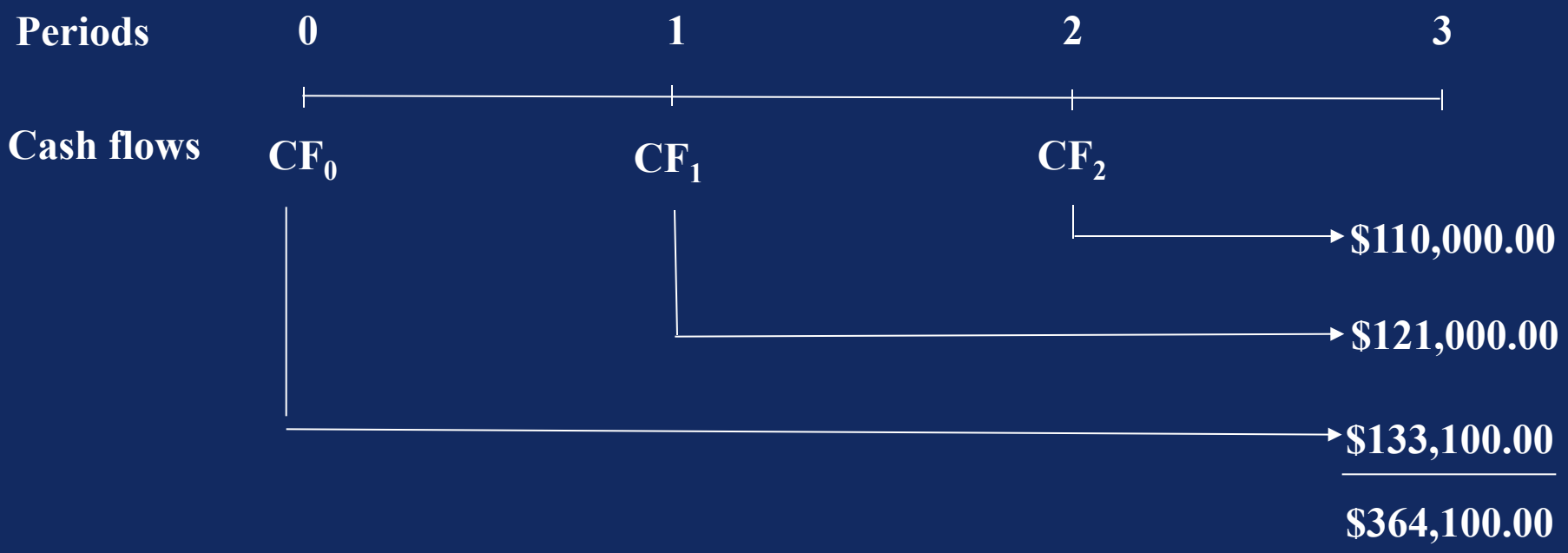
Future Value of an Annuity Due: An Example

- What does the time line look like for the future value of an annuity due? Draw it…



Future Value of an Annuity Due: An Example

- Example: What is the future value of $100,000 per year for 3 years at 10%



Future Value of an Annuity Due: An Example

- Example: What is the future value of $100,000 per year for 3 years at 10%

Formula:

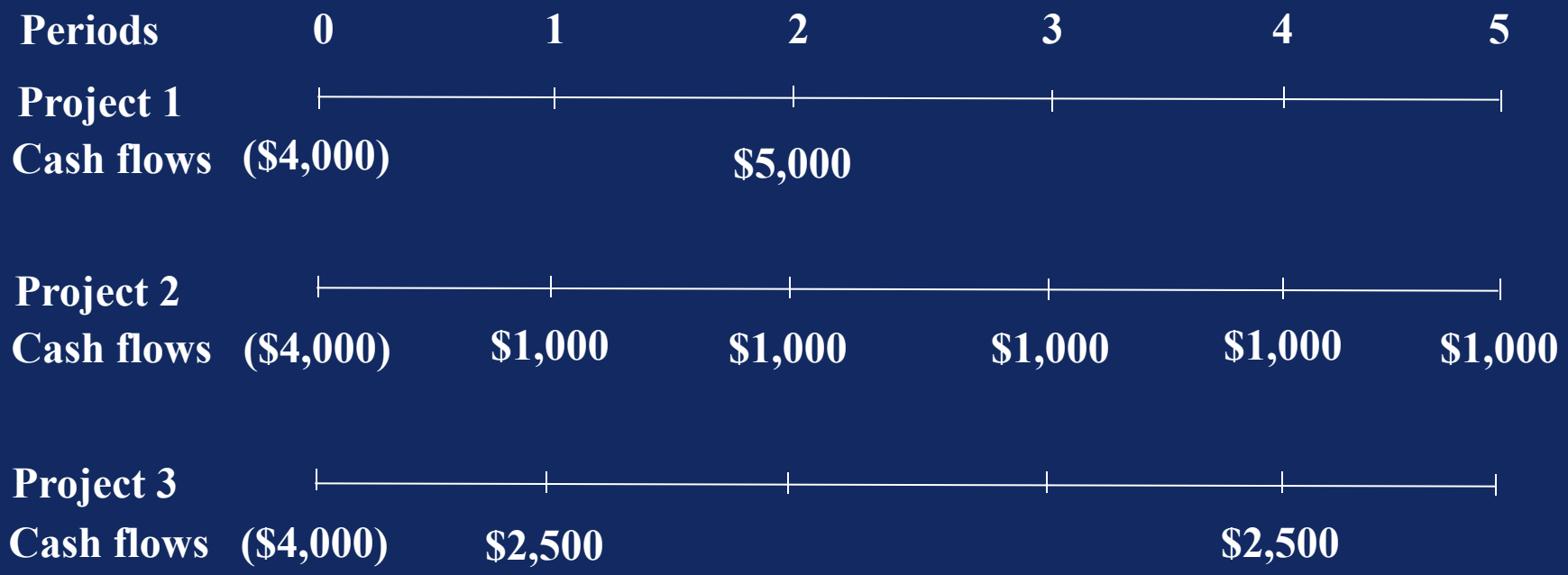
$$FVCF_0 = PV*(1+r)^3 = 100{,}000*(1.10)^3 = \$133{,}100.00$$
$$FVCF_1 = PV*(1+r)^2 = 100{,}000*(1.10)^2 = \$121{,}000.00$$
$$FVCF_2 = PV*(1+r)^1 = 100{,}000*(1.10)^1 = \underline{\$110{,}000.00}$$
$$\$364{,}100.00$$

Formula: $$FVADue_T = (1+r)CF\left[\frac{(1+r)^T - 1}{r}\right] = (1.10)\$100{,}000\left[\frac{(1.10)^3 - 1}{0.10}\right] = \$364{,}100.00$$

Financial Calculator: Set Due: 3n; 10i; 100,000PMT; FV = $364,100.00

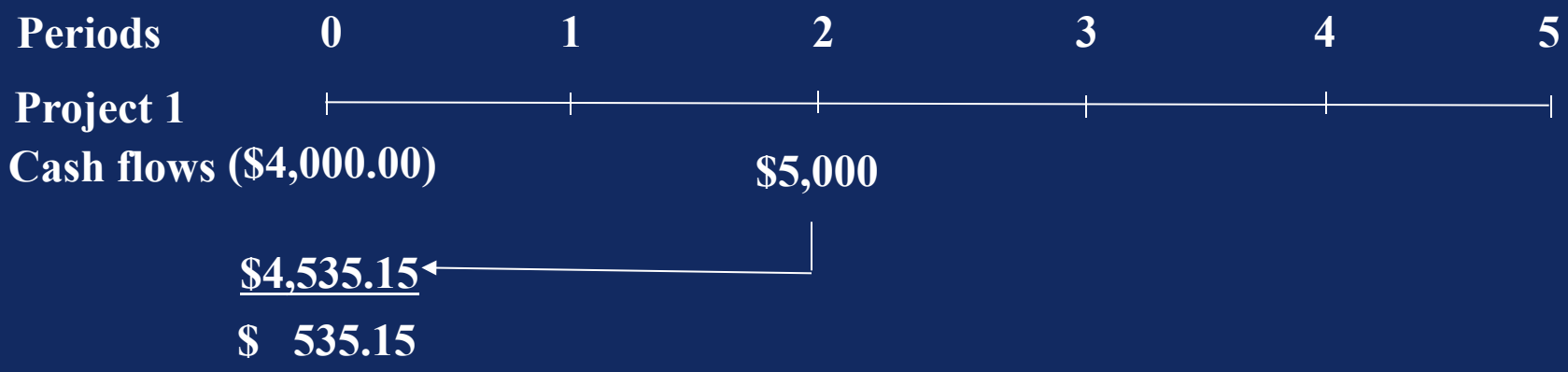
Back to Our Original Three Projects

- Can you determine which project provides the best value creation if all project cash flows are discounted at 5%?



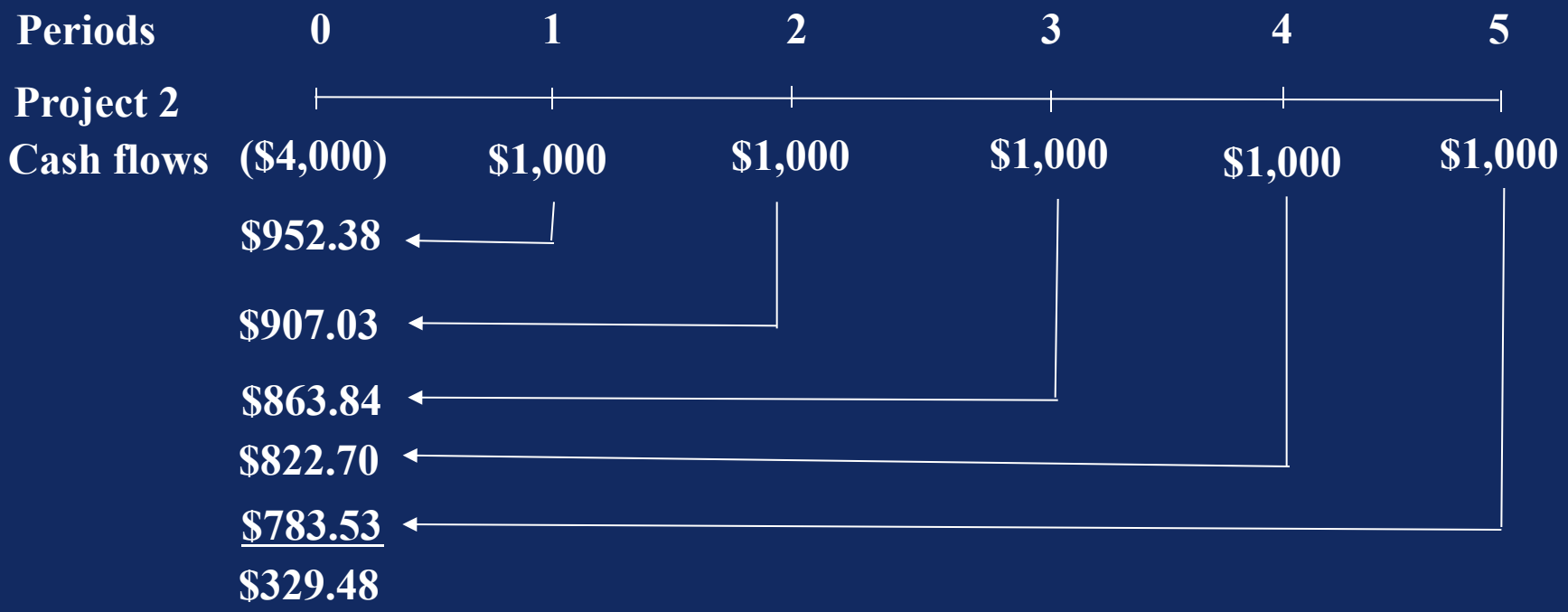
Projects 1

- What is present value of all cash flows for project 1 discounted at 5%?



Project 2

- What is the present value of all cash flows for project 2 discounted at 5%?



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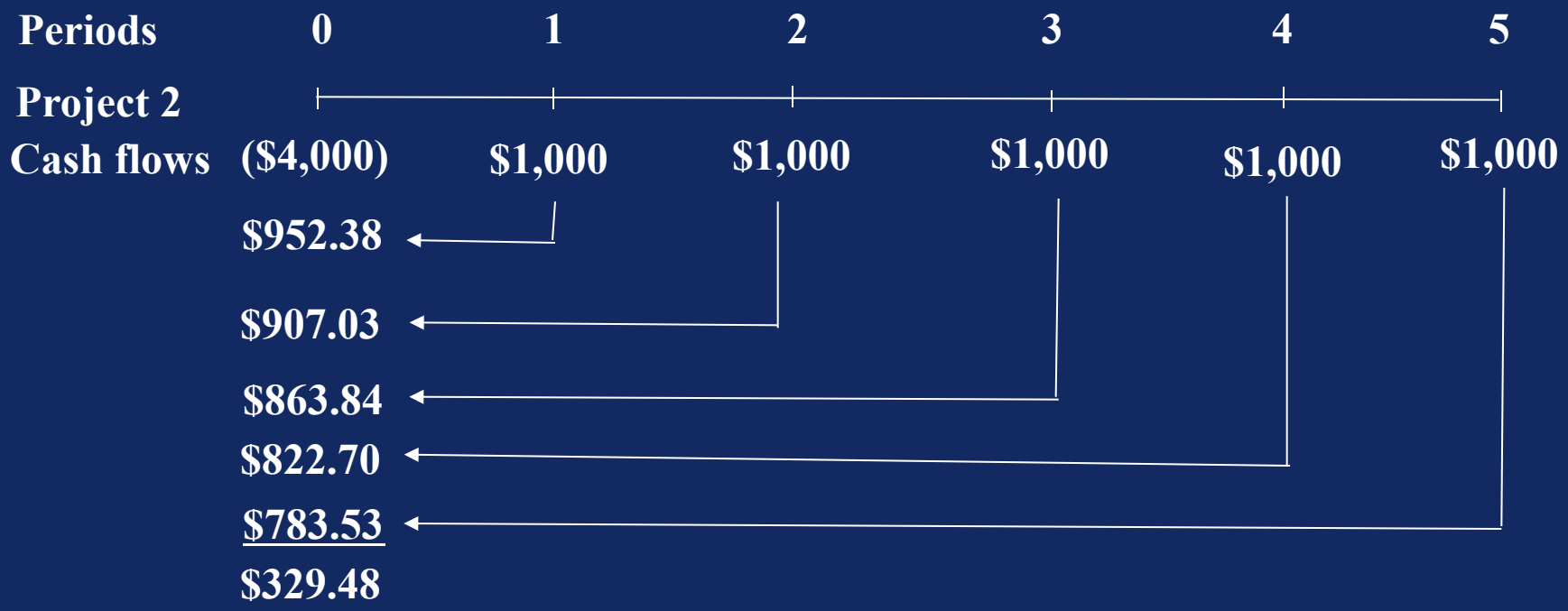
Project 2

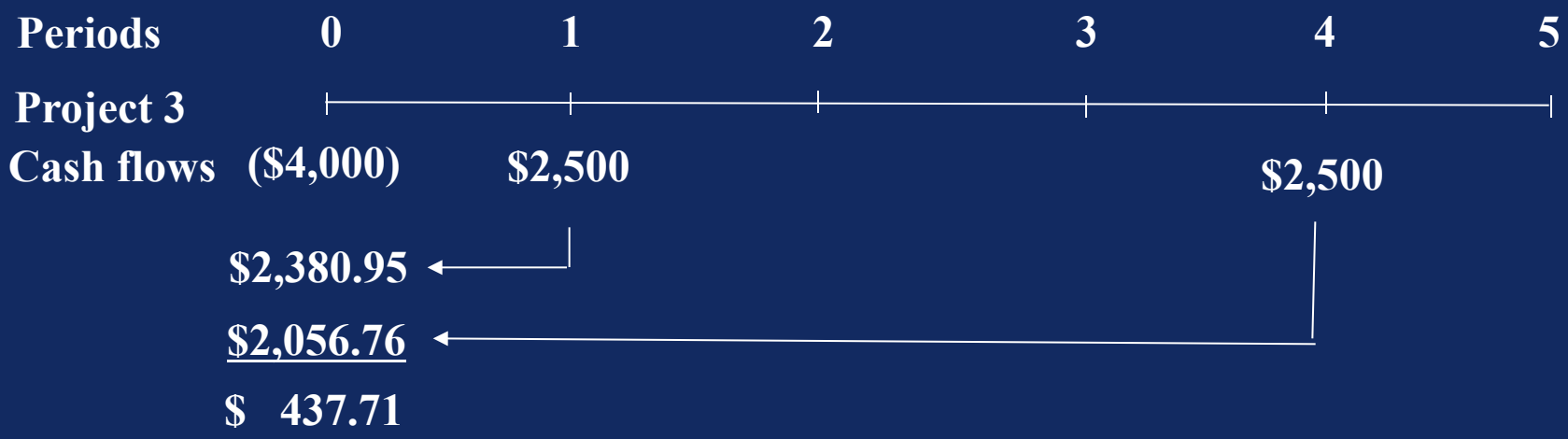
- Note: There is an annuity in project 2!

Financial Calculator: 5n; 5i; 1,000PMT; PV = $4,329.48 - $4,000 = $329.48



Project 3

- What is the present value of all cash flow from project discounted at 5%?



Back to Our Original Three Projects

- Project 1: Net Cash Flows = $535.15
- Project 2: Net Cash Flows = $329.48
- Project 3: Net Cash Flows = $437.71
- **Which project provides the best value creation?**

We now know how to do Present and Future Values of Single Cash flows *and* Multiple Cash Flows!

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Thank You!

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