Safe set:
$$\mathcal{C} = \{x : \phi(x) \geq 0\}$$

safe

unsafe

X_t

System dynamics
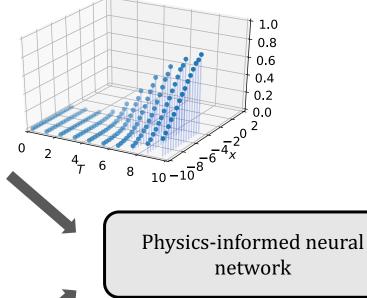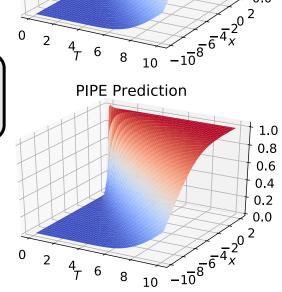$$dx = f(x, u)dt + \sigma dw$$

Monte Carlo estimation data
$$\tilde{F} = \mathbb{P}(x \notin \mathcal{C}) = \frac{N_{\text{unsafe}}}{N}$$

Training Data

Inaccurate sub-sampled data

Physics-informed neural network

Governing risk probability PDE
$$\frac{\partial F}{\partial T} = f(x, u)\frac{\partial F}{\partial x} + \frac{1}{2}\text{tr}\left(\sigma^2 \frac{\partial^2 F}{\partial x^2}\right)$$

Physics model

Ground Truth

PIPE Prediction