Lecture 15

H_0: The true means are the same; $\mu_X = \mu_Y$

H_1: $\mu_X \neq \mu_Y$

Under H_0:

$$T = \hat{\mu}_X - \hat{\mu}_Y$$

$$\boxed{T \sim G\left(0, \sigma^2\left(\frac{1}{N} + \frac{1}{M}\right)\right)}$$

$\mu_T = E[\hat{\mu}_X - \hat{\mu}_Y]$

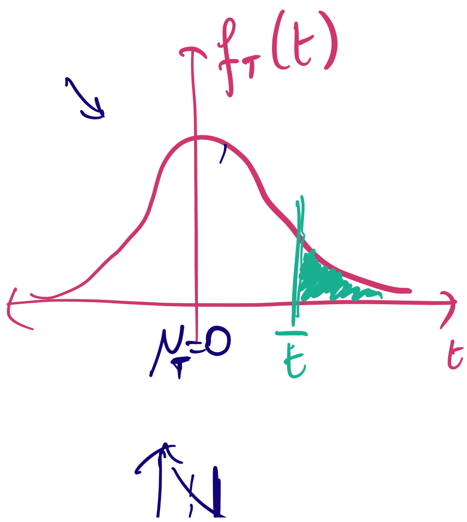
sample size of X

sample size of Y

Suppose that we observe a sample mean difference $\bar{t} = \bar{x} - \bar{y} > 0$ $\Leftarrow$ H_1: $\mu_X > \mu_Y$

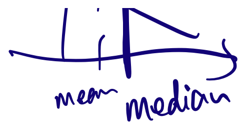
ONE-SIDED HYPOTHESIS TEST:

$$P(T \geq \bar{t} \mid H_0) = P(T > \bar{t} \mid H_0)$$



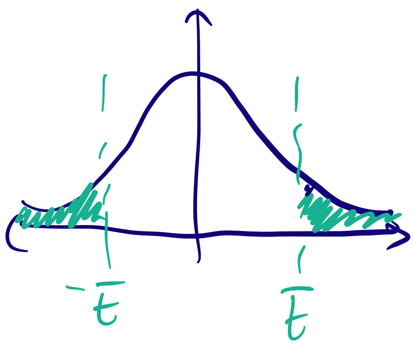
$$= Q\left(\frac{\bar{t} - \mu_T}{\sigma_T}\right)$$

$$= Q\left(\frac{\bar{t}}{\sqrt{\sigma^2\left(\frac{1}{N} + \frac{1}{M}\right)}}\right)$$



$$\mu_T = \int_{-\infty}^{\infty} x \, f_T(x) \, dx$$

TWO - SIDED HYPOTHESIS TEST:



$H_0: \mu_X = \mu_Y$

$H_1: \mu_X \neq \mu_Y$

$\mu_X > \mu_Y$

$\mu_X < \mu_Y$

$$P(|T| \geq \bar{t} \mid H_0) = 2 \cdot P(T \geq \bar{t} \mid H_0)$$

$$= 2 \cdot Q\left(\frac{t}{\sqrt{\sigma^2 \cdot \left(\frac{1}{N} + \frac{1}{M}\right)}}\right)$$

Example 1

1. Null Hypothesis: H_0:

$\mu_G = 23 \qquad \sigma_G^2 = 50$

$$\boxed{G_i \sim \text{Gaussian}(23, \sigma_G^2 = 50)}$$

2. Compute sample mean

$$\hat{\mu} = \frac{1}{10}\sum_{i=1}^{10} G_i$$: sample mean estimator

$$= 27$$

$$E\{\hat{\mu}\} = 23$$

$$\mathrm{Var}(\hat{\mu}) = \sigma^2_{\mu_G} = \frac{\sigma_G^2}{10} = \frac{50}{10} = 5$$

$$T = \hat{\mu} - \mu_G$$

$$T \sim \text{Gaussian}\left(0, \frac{\sigma_G^2}{10}\right)$$

$$= \text{Gaussian}\left(0, 5\right)$$