Important Discrete Random Variables
EEL 3850 Data Science for ECE

1 The Bernoulli Random Variable

An event $A \in \mathcal{F}$ is considered a ”success”.

- A Bernoulli RV X is defined by

$$X = \begin{cases} 1, & s \in A \\ 0, & s \notin A \end{cases}$$

- The PMF for a Bernoulli RV X is defined by

$$p_X(x) = P(X = x) = \begin{cases} p, & x = 1 \\ 1 - p, & x = 0 \\ 0, & x \neq \{0, 1\} \end{cases}$$

- We have seen this PMF before when we considered *data likelihood* for a coin flip. Remember for the toss of a coin, which comes up heads with probability p, and a tail with probability $1 - p$.
- **Engineering examples/applications:** whether a bit is 0 or 1, whether a bit is in error, whether a component has failed, whether something has been detected.

2 The Binomial Random Variable

- A Binomial RV represents the number of successes on n independent Bernoulli trials.
  - Example: a coin is tossed n times.
- Thus, a Binomial RV can also be defined as the sum of n independent Bernoulli RVs.
  - Example: At each toss, the coin comes up heads with probability p and a tail with probability $1 - p$, independent of prior tosses.

- Let X be the # of successes.
  - Example: X is the number of heads in the n-toss sequence.
- We refer to X as the **Binomial** RV with parameters n and p.
- The PMF of X is given by

$$p_X(x) = P(X = x) = \begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, \ldots, n \\ 0, & \text{o.w.} \end{cases}$$

- **Engineering examples/applications:** The number of bits in error in a packet, the number of defective items in a manufacturing run.

3 The Geometric Random Variable

- A Geometric RV occurs when independent Bernoulli trials are conducted until the first success
  - Example: repeatedly and independently toss a coin with probability of a head equal to p, where $0 < p < 1$.
- X is the number of trials required.
  - Example: The Geometric RV is the number X of tosses needed for a head to come up for the first time.
- The PMF of X is given by

$$p_X(x) = P(X = x) = \begin{cases} p(1-p)^{x-1}, & x = 1, 2, \ldots \\ 0, & \text{o.w.} \end{cases}$$

- **Engineering examples/applications:** The number of retransmissions required for a packet, number of white dots between black dots in the scan of a black and white document.

4 The Poisson Random Variable

- A Poisson RV models events that occur randomly in space or time.
- Let $\lambda = \#$ of events/(unit of space or time)

- Consider observing some period of time or space of length t and let $\alpha = \lambda t$.
- Let $X =$ the # of events in time (or space) t
- The PMF of the Poisson RV is given by

$$p_X(x) = P(X = x) = \begin{cases} \frac{\alpha^x}{x!} e^{-\alpha}, & x = 0, 1, \ldots \\ 0, & \text{o.w.} \end{cases}$$

- For large α, the Poisson PMF has a bell shape. For example, see the PMF when $\alpha = 20$.
- **Engineering examples/applications:** calls coming in to a switching center, packets arriving at a queue in a network, processes being submitted to a scheduler, # of misprints on a group of pages in a book, # of people in a community that live to be 100 years old, # of wrong telephone numbers that are dialed in a day, # of α-particles discharged in a fixed period of time from some radioactive material, # of earthquakes per year, # of computer crashes in a lab in a week.