Lecture 16

- Library intro – highlight
- More reading the code.
- More boilerplate code
- More examples in the code to try later.
- Analytical – could we do more examples?

Example 2 :

The 95% confidence interval is the interval $[a, b]$ for which $\boxed{P(T \leq a) = 0.025}$

and $\boxed{P(T \geq b)}$

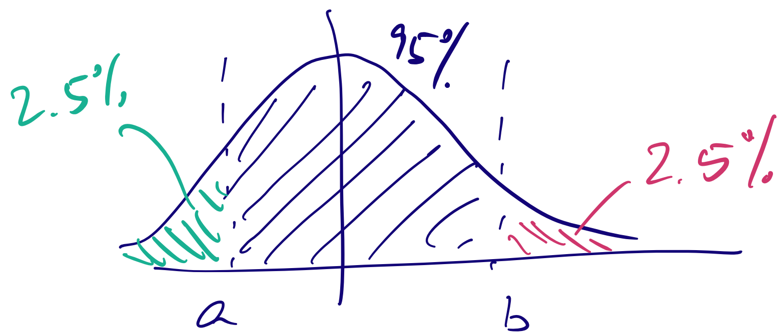
$$\boxed{= 0.025}$$

$$P(T \leq a) = F_T(a) = 0.025$$

$$a = F_T^{-1}(0.025)$$

$$P(T \leq b) = 0.975$$

$$b = F_T^{-1}(0.975)$$



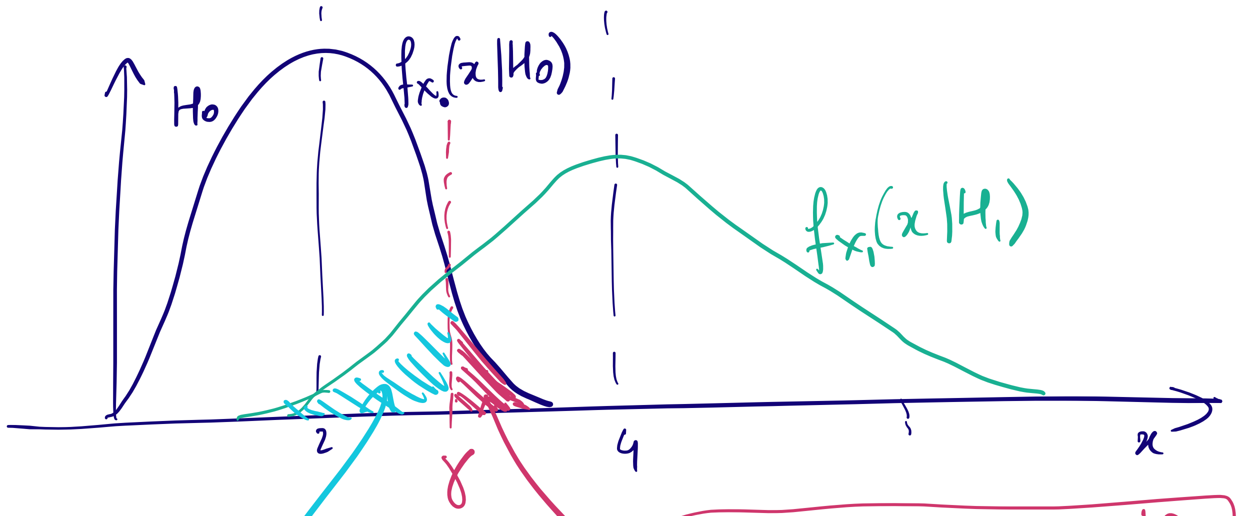
Example 1 :

$\ominus$ H_0 : man in their 60s <u>does not</u> have cancer

$\oplus$ H_1 : man in their 60s has cancer.

Under H_0 : $X_0 \sim G(2, \underbrace{1}_{\sigma^2})$

Under H_1 : $X_1 \sim G(4, 2)$



$\underline{P_m}$ | FALSE NEGATIVE OR MISS

When we reject H_1

↳ Type II error

FALSE POSITIVE / FALSE ALARM $\boxed{P_{fa}}$

Falsely rejected H_0.

↳ Type I error.

1. For $\underbrace{P_{fa}}_{\text{false positives/ false alarms}} = 10\%$, find $\underbrace{P_m}_{\text{misses}}$

$$P_{fa} = 0.1 \iff P(X_0 \geq \gamma \mid H_0) = 0.1$$

$\mathcal{N}(X_0)$

$$Q\left(\frac{\gamma - 2}{\sqrt{1}}\right) = 0.1$$

$\sigma^2(X_0)$

$$\Leftrightarrow \gamma - 2 = Q^{-1}(0.1)$$

$$\Leftrightarrow \gamma = 2 + Q^{-1}(0.1)$$

$$\gamma \approx 2 + 1.28 = 3.28$$

$$P_m = P(X_1 < \gamma \mid H_1)$$

$$= Q\left(\frac{\mu_{H_1} - \gamma}{\sigma_{H_1}}\right)$$

$$= Q\left(\frac{4 - 3.28}{\sqrt{2}}\right) \simeq 0.3$$

$\Rightarrow$ For 10% False positives, we obtain 30% false negatives.

2) $P_m = 10\%$

Find γ s.t. $P_m = 10\%$

$$P_m = Q\left(\frac{4-\gamma}{\sqrt{2}}\right) = 0.1$$

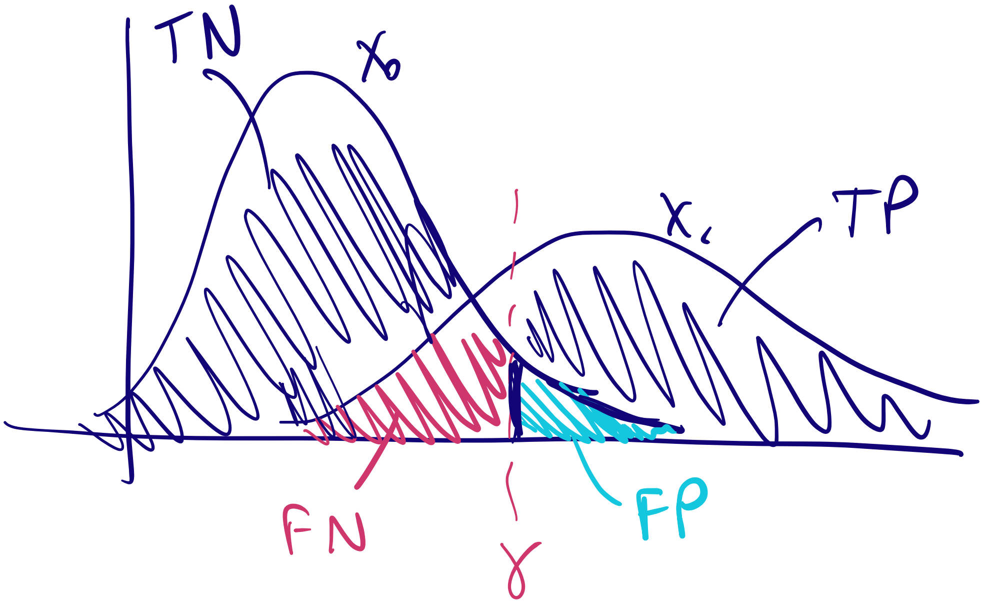
$$\frac{4-\gamma}{\sqrt{2}} = Q^{-1}(0.1)$$

$$\gamma \approx 4 - 1.28\sqrt{2}$$
$$= 2.19$$

$\gamma_{new} = 2.19$:

$$P_{fa} = P(X_0 \geq \gamma \mid H_0)$$
$$= Q\left(\frac{2.19-2}{\sqrt{1}}\right)$$
$$= Q(0.19)$$
$$\approx 0.425$$

For False negatives of 10%, we have 42.5% false positives!!



Confusion Matrix

	Prediction	
True Label / Ground Truth	Class H_0 (-ve)	Class H_1 (+ve)
Class H_0 (-ve)	TN	FP (type 1)
Class H_1 (+ve)	FN	TP

Want ↑

False Positive Rate (FPR)

$$\boxed{P_{fa} = FPR = \frac{FP}{FP + TN}} \quad \left(\begin{array}{c}\text{ideally} \\ FPR = 0\end{array}\right)$$

True Positive Rate (TPR)

$$1 - P_m = \boxed{TPR = \frac{TP}{TP + FN}} \quad \left(\begin{array}{c}\text{ideally} \\ TPR = 1\end{array}\right)$$