Lecture 06

Typos:

- Problem 1

  (i) Either A and B $\longrightarrow$ Either A or B.

- Problem 2

  (b) ... of at least of seeing at least ...

  $\downarrow$

  ... of seeing at least ...

Sampling w/out replacement & w/ ordering

k-permutations

of ways to choose k values from a set of n values w/out replacement w/ ordering:

$$(n) \times (n-1) \times (n-2) \times \dots (n-k+1) = \frac{n!}{(n-k)!}$$

Example 1:

$n = 20$

$k = 2$

$$\frac{n!}{(n-k)!} \Rightarrow P(\text{2 heads in 20 coin flips})$$

$$\Rightarrow \frac{\frac{n!}{(n-k)!}}{2^{20}}$$

(H, H, T, H, T, ... T)

$\hookrightarrow$ len = 20

h_i = position where heads occured:

$h_i \in \{0, 1, 2, .. 19\}$

(0, 1, 3)

$$\binom{n-k+1}{k} = C_k^{n+k-1} = \frac{(n+k+1)!}{k!\,(n+k-1-k)!}$$

$$= \frac{(n+k-1)!}{k!\,(n-1)!}$$

Number of ways to choose k values from n values:

Ordering \ Replacement	Yes	No
Yes	n^k	k-permutations $\frac{n!}{(n-k)!}$
No	partitions $C_k^{n+k-1} = \frac{(n+k-1)!}{k!(n-1)!}$	$C_k^n = \frac{n!}{k!(n-k)!}$

The Bayes' Theorem / Bayes Rule

$A, B \in \mathcal{F}$

Chain Rules:

$$P(A \cap B) = P(A|B) \cdot P(B)$$

$$\parallel$$

$$P(B \cap A) = P(B|A) \cdot P(A)$$

$$\Rightarrow P(A|B) P(B) = P(B|A) \cdot P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

$B \equiv$ effect

$A \equiv$ cause

$$\underline{\underline{P(\text{cause}|\text{effect})}} = \frac{P(\text{effect}|\text{cause}) \cdot P(\text{cause})}{P(\text{effect})}$$

likelihood prob.

$$\underbrace{P(A \mid B)}_{\text{posterior or a posteriori prob.}} = \frac{P(B \mid A) \cdot P(A)}{P(B)}$$

prior or a priori prob.

evidence or effect prob.

Example 3:

Prob. that die selected is 12-sided if we roll a 5?

$S \equiv$ fair 6-sided die was selected

$D_i \equiv$ face i was rolled

$$P(\bar{S} \mid D_5) = \frac{P(D_5 \mid \bar{S}) \cdot P(\bar{S})}{P(D_5)}$$

Bayes!

Law of total prob.

$$= \frac{P(D_5 \mid \bar{S}) \cdot P(\bar{S})}{P(D_5 \mid \bar{S})\,P(\bar{S}) + P(D_5 \mid S)\,P(S)}$$

$$= \frac{\frac{1}{12} \cdot \frac{1}{2}}{\frac{1}{12} \cdot \frac{1}{2} + \frac{1}{6} \cdot \frac{1}{2}}$$

$$= \frac{1}{3}$$

$$P(\text{fair coin} \mid \overbrace{\{H, H, H, H, H\}}^{E})$$

Prior $= \frac{1}{2}$

$$= \frac{P(E \mid \text{fair coin}) \cdot P(\text{fair})}{P(E \mid \text{fair}) P(\text{fair}) + P(E \mid \text{unfair}) P(\text{unfair})}$$

$$= \frac{\left(\frac{1}{2}\right)^5 \cdot \frac{1}{2}}{\left(\frac{1}{2}\right)^5 \cdot \frac{1}{2} + 1^5 \cdot \frac{1}{2}} \approx 0.03$$

$$P(\text{fair coin} \mid E) \approx 0.0588$$

Prior $= \frac{2}{3}$ fair coin

A = event that ~~the~~ person has disease

B = " " test is positive

$P(B|\underline{A}) = 0.95$

Binary Hypothesis Test

H_0 (Null Hypothesis): The observation (diff. of means) is due to random sampling.

H_1 (Alternative Hypothesis): Avg. firearms mortality rate for 2014 is larger than the average firearms mortality rate for 2005

Pooling set