Jacob Windsor - Bayesian Statistics Exam 2019/2020

This is a compiled document containing the code and output. All files can be found at https://github.com/jacobwindsor/BIST_SDA_BAYESIAN

Code

```
import pandas as pd
import matplotlib.pyplot as plt
import statsmodels.api as sm
import scipy.stats as stats
import numpy as np
from pathlib import Path
import math
import pymc3 as pm
from statsmodels.stats.multicomp import MultiComparison
from statsmodels.sandbox.stats.multicomp import tukeyhsd
from statsmodels.graphics.factorplots import interaction_plot
from functools import partial

# Ex 1
x_data = np.loadtxt(Path.cwd() / "michel.out")
plt.hist(x_data, bins=50)
plt.title("Michel distribution from experimental data")
figpath = Path.cwd() / "histogram.png"
print(f"Histogram saved to {figpath}")
plt.savefig(figpath)

# Ex 2

def michel_equation(n, p, x):
    part_1 = (4 / (1+(2*n)))
    part_2_a = (3*(x**2))*(1-x)
    part_2_b = (2/3)*(p*(x**2))*((4*x)-3)
    part_2_c = (3*n*x)*(1-x)

    return part_1 * (part_2_a + part_2_b + part_2_c)

def log_posterior(n, p):
    if n < 0:
        return -np.inf

    return np.sum(
        np.log(
            michel_equation(n, p, x_data)
        )
    )


# Ex 3-4
```

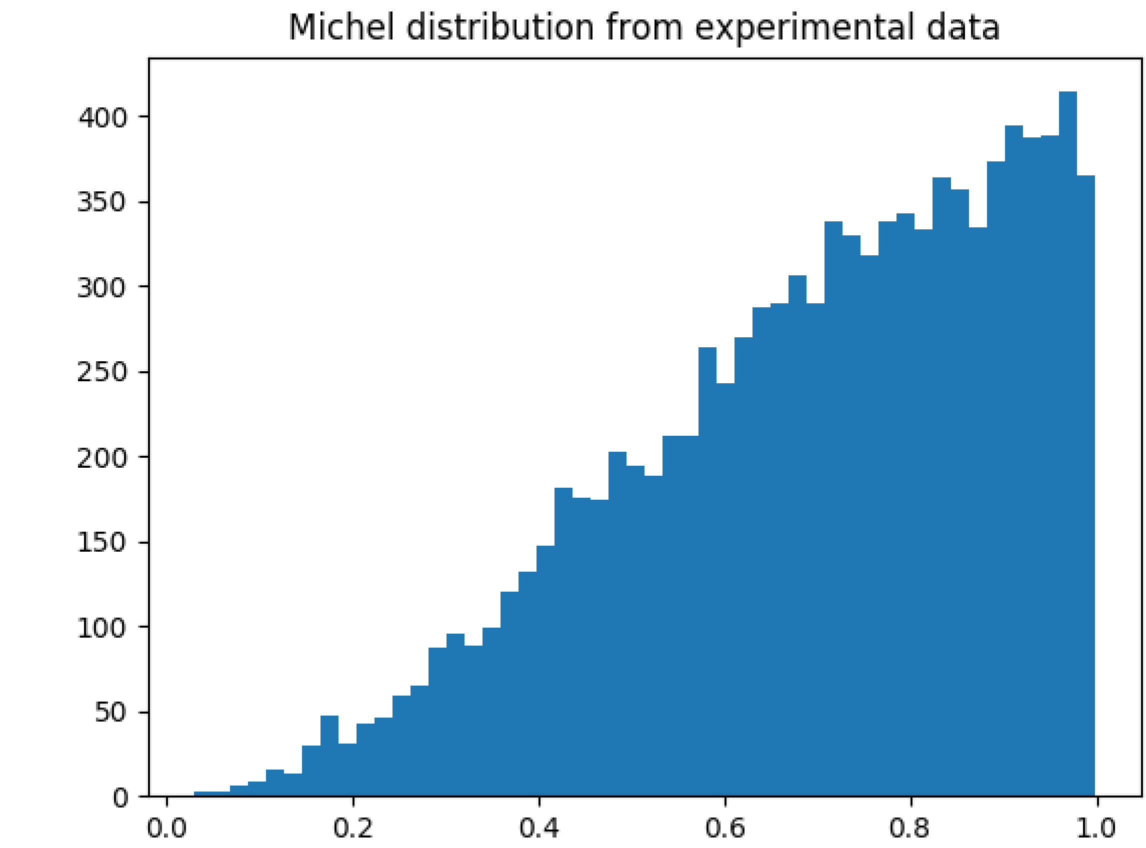
```
smu_1 = 0.08
smu_2 = 0.05

def proposal(n, p):
    p_n = np.random.normal(n, smu_1)
    p_p = np.random.normal(p, smu_2)

    return (p_n, p_p)
NC = 10**6
NB = 10**5
chain = np.empty((NC, 3))

# Set initial n and p
init_n = 0.4
init_p = 0.1

chain[0] = init_n, init_p, log_posterior(init_n, init_p)

for i in range(0, NC-1):
    prop = proposal(chain[i,0], chain[i,1])
    log_post_1 = log_posterior(*prop)
    log_ratio = log_post_1 - chain[i,2]

    if (log_ratio >= 0) or (log_ratio > np.log(np.random.random())):
        chain[i+1] = prop[0], prop[1], log_post_1
    else:
        chain[i+1] = chain[i]

chain = chain[NB:]
sample_ns = chain[:,0]
sample_ps = chain[:,1]

predicted_p = np.mean(sample_ps)
predicted_n = np.mean(sample_ns)

n_sd = np.std(sample_ns)
p_sd = np.std(sample_ps)


# Ex 5
print(f"Sample mean of n: {predicted_n}")
print(f"Sample mean of p {predicted_p}")

print(f"Sample standard deviations of n: {n_sd}")
print(f"Sample standard deviations of p: {p_sd}")

print(f"Correlation coefficient: {np.corrcoef(sample_ps, sample_ns)[0,1]}")

plt.figure()
plt.scatter(sample_ps, sample_ns)
plt.title("Scatter plot for rho and eta")
plt.xlabel("Rho")
plt.ylabel("Eta")
plt.savefig("scatter.png")
```

```
plt.figure()
plt.hist(sample_ps, bins=30)
plt.title("Histogram for rho")
plt.savefig("rho_hist.png")

plt.figure()
plt.hist(sample_ns, bins=30)
plt.title("Histogram for eta")
plt.savefig("eta_hist.png")

predicted_p = np.mean(sample_ps)
predicted_n = np.mean(sample_ns)

n_confirmed = predicted_n <= n_sd and predicted_n >= -n_sd
n_text = "confirms" if n_confirmed else "does not confirm"
print(f"The predicted value for eta {n_text} the validity of the standard
model.")


p_confirmed = predicted_p <= predicted_p + p_sd and predicted_p >= -p_sd
p_text = "confirms" if p_confirmed else "does not confirm"
print(f"The predicted value for eta {p_text} the validity of the standard
model.")
```

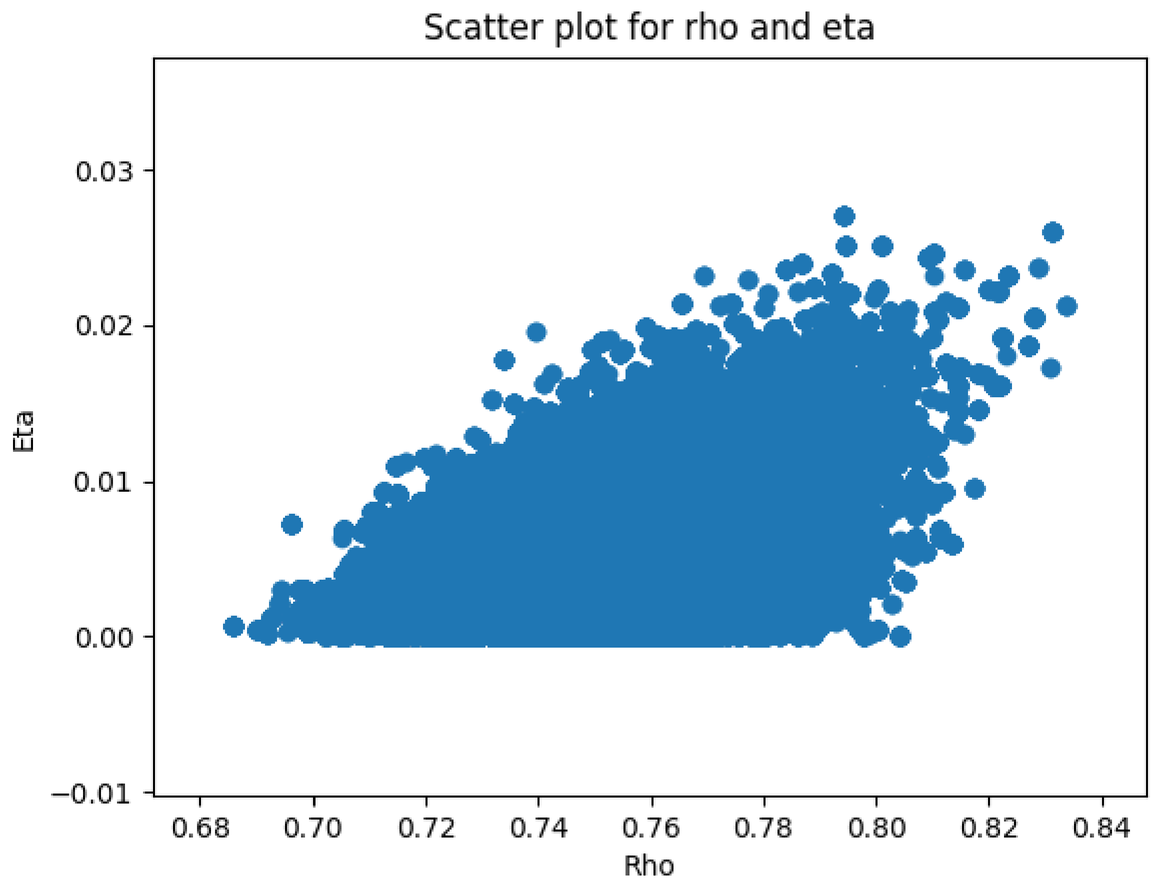
Output

```
Sample mean of n: 0.004044820836289604
Sample mean of p 0.7515884604980703

Sample standard deviations of n: 0.003402260546943624З
Sample standard deviations of p: 0.017568480847059545

Correlation coefficient: 0.47013443243948283

The predicted value for eta does not confirm the validity of the standard
model.

The predicted value for eta confirms the validity of the standard model.
```

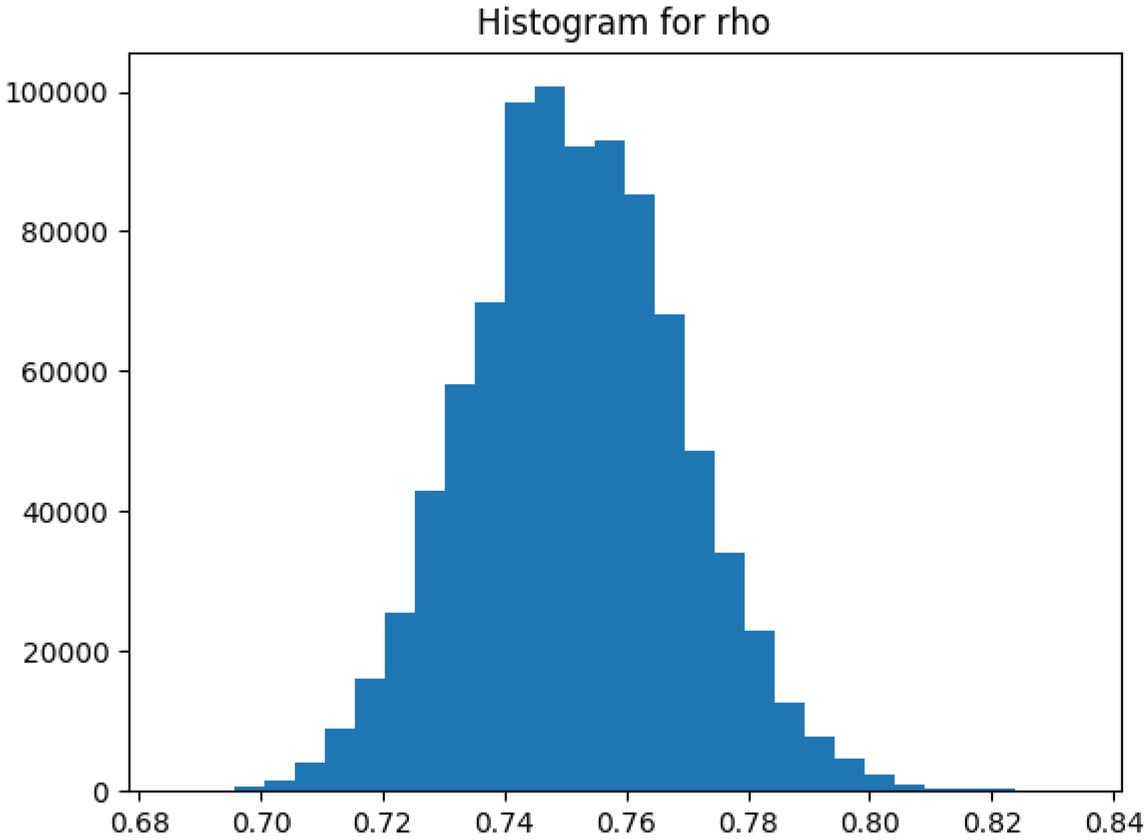
Histogram of input data



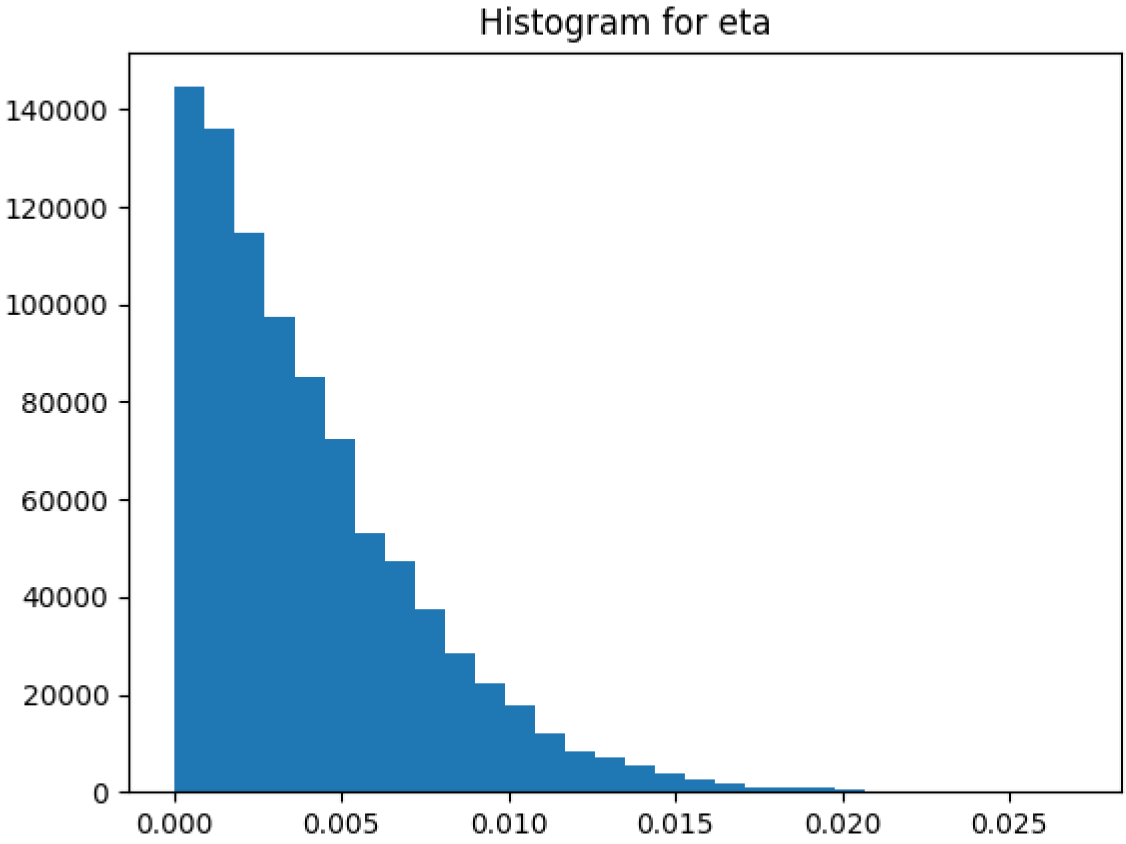
Scatter plot of inferred rho and eta



Histogram of inferred rho values



Histogram of inferred eta values


Histogram for eta
140000
120000
100000
80000
60000
40000
20000
0
0.000
0.005
0.010
0.015
0.020
0.025