**1.**

**(a)**

An example of a redundancy that relation Reservation, combined with FDs S, allow is as follows.

| sID | sName | rating | age | cID | cName | length | day |
|---|---|---|---|---|---|---|---|
| 1 | Hooks | 5 | 30 | 1 | Voyager | 200 | 5/29/2015 5:50 |
| 1 | Hooks | 5 | 30 | 2 | Cruiser | 250 | 5/30/2015 5:50 |

We can observe that there is repeated, redundant information of the skipper when the same skipper reserves different crafts.

**2.**

**(a)**

- $\times$ $KOQ^{+} = KOPQRS$, so KOQ is not a superkey and $KOQ \rightarrow PS$ violates BCNF
- $\times$ $KQ^{+} = KQRS$, so KQ is not a superkey and $KQ \rightarrow RS$ violates BCNF
- $\times$ $L^{+} = KLN$, so L is not a superkey and $L \rightarrow KN$ violates BCNF

All of which violate BCNF

**(b)**

Decompose $R$ using $KOQ \rightarrow PS$

$R1 = KOPQRS$

$R2 = KLMNOQ$

Project the FDs onto $R1 = KOPQRS$

| K | O | P | Q | R | S | closure | FDs |
|---|---|---|---|---|---|---|---|
| ✓ | | | | | | $K^+ = K$ | *nothing* |
| | ✓ | | | | | $O^+ = O$ | *nothing* |
| | | ✓ | | | | $P^+ = P$ | *nothing* |
| | | | ✓ | | | $Q^+ = Q$ | *nothing* |
| | | | | ✓ | | $R^+ = R$ | *nothing* |
| | | | | | ✓ | $S^+ = S$ | *nothing* |
| ✓ | ✓ | | ✓ | | | $KOQ^+ = KOPQRS$ | $KOQ \rightarrow PSR;\ KOQ\ is\ superkey\ of\ R1$ |
| ✓ | | | ✓ | | | $KQ^+ = KQRS$ | $KQ \rightarrow RS:\ violates\ BCNF;\ abort$ |

We must decompose R1 further

Decompose $R1$ using FD $KQ \rightarrow RS$. This yields two relations:

$R3 = KQRS$

$R4 = KOPQ$

Project the FDs onto $R3 = KQRS$

| K | Q | R | S | closure | FDs |
|---|---|---|---|---|---|
| ✓ | | | | $K^+ = K$ | *nothing* |
| | ✓ | | | $Q^+ = Q$ | *nothing* |
| | | ✓ | | $R^+ = R$ | *nothing* |
| | | | ✓ | $S^+ = S$ | *nothing* |
| *All other subsets of KOPQ* | | | | *Irrelevant* | *Only weaker FDs* |

This relation satisfies BCNF.

$KQ^+ = KQRS$ from $R3$

Project the FDs onto $R4 = KOPQ$

| K | O | P | Q | closure | FDs |
|---|---|---|---|---|---|
| ✓ | | | | $K^+ = K$ | $nothing$ |
| | ✓ | | | $O^+ = O$ | $nothing$ |
| | | ✓ | | $P^+ = P$ | $nothing$ |
| | | | ✓ | $Q^+ = Q$ | $nothing$ |
| ✓ | ✓ | | ✓ | $KOQ^+ = KOPQRS$ | $KOQ \rightarrow PSR;\ KOQ\ is\ a\ superkey\ of\ R1$ |
| ✓ | | | ✓ | $KQ^+ = KQRS$ | $KQ \rightarrow RS;\ KQ\ is\ a\ superkey\ of\ R3$ |
| $All\ other\ subsets\ of\ KOPQ$ | | | | $Irrelevant$ | $Only\ weaker\ FDs$ |

This relation satisfies BCNF.

$KQ^+ = KQRS$ from $R4$

$KOQ^+ = KOPQSR$ from $R1$

Return to R2 and project the FDs onto $R2 = KLMNOQ$

| K | L | M | N | O | Q | closure | FDs |
|---|---|---|---|---|---|---|---|
| ✓ | | | | | | $K^+ = K$ | $nothing$ |
| | ✓ | | | | | $L^+ = KLN$ | $L \rightarrow KN:\ violates\ BCNF;\ abort$ |

We must decompose R2 further.

Decompose $R2$ using $L \rightarrow KN$. This yields two relations:
$R5 \ = \ KLN$
$R6 \ = \ LMOQ$

Project the FDs onto $R5 \ = \ KLN$

| L | K | N | closure | FDs |
|---|---|---|---|---|
| ✓ | | | $L^+ = KLM$ | $L \rightarrow KN;\ L\ is\ a\ superkey\ of\ R5$ |
| | ✓ | | $K^+ = K$ | $nothing$ |
| | | ✓ | $N^+ = N$ | $nothing$ |
| ✓ | ✓ | | $LK^+ = KLM$ | $L \rightarrow KN;\ L\ is\ a\ superkey\ of\ R5$ |
| ✓ | | ✓ | $LN^+ = KLM$ | $L \rightarrow KN;\ L\ is\ a\ superkey\ of\ R5$ |
| | ✓ | ✓ | $KN^+ = KN$ | $nothing$ |

This relation satisfies BCNF.
$L^+ \ = \ LKN$ from $R5$

Project the FDs onto $R6 \ = \ LMOQ$

| L | M | O | Q | closure | FDs |
|---|---|---|---|---|---|
| ✓ | | | | $L^+ = KLM$ | $nothing$ |
| | ✓ | | | $M^+ = M$ | $nothing$ |
| | | ✓ | | $O^+ = O$ | $nothing$ |
| | | | ✓ | $Q^+ = Q$ | $nothing$ |
| $All\ other\ subsets\ of\ LMOQ$ | | | | $Irrelevant$ | $Only\ weaker\ FDs$ |

This relation satisfies BCNF.

Final decomposition in alphabetical order:
a) $R5 \ = \ KLN$ with FD $L \rightarrow KN$
b) $R4 \ = \ KOPQ$ with FD $KOQ \rightarrow P$
c) $R3 \ = \ KQRS$ with FD $KQ \rightarrow RS$
d) $R6 \ = \ LMOQ$ with no FDs

**(c)**

Yes, it is. For each of the original FDs in set S, there is a relation that includes all the FDs attributes, except $KOQ \rightarrow PS$ which was shortened to $KOQ \rightarrow P$ since S is preserved in $KQ \rightarrow RS$. This implies that $KOQ \rightarrow S$ can be combined with $KOQ \rightarrow P$. This preserves the relation $KOQ \rightarrow PS$.

**(d)**

From part (b), we decompose into the following relations: KLN, KOPQ, KQRS and LMOQ

The Chase Test demonstrates that it is a lossless-join decomposition as there is an unsubscripted row in the last row of the following table:

| K | L | M | N | O | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|
| **k** | **l** | $m_1$ | **n** | $o_1$ | $p_1$ | $q_1$ | $r_1$ | $s_1$ |
| **k** | $l_2$ | $m_2$ | $n_2$ | **o** | **p** | **q** | r | s |
| **k** | $l_3$ | $m_3$ | $n_3$ | $o_3$ | $p_3$ | **q** | **r** | **s** |
| k | **l** | **m** | n | **o** | p | **q** | r | s |

Therefore, if we were to project this instance onto the relations LMOQ, KLN, KOPQ and KQRS and then natural join the results back together, the result would include the tuple <k, l, m, n, o, p, q, r, s> which does appear above as shown by the Chase Test.

**3.**

**(a)**

**Step 1:** Split the RHSs to get our initial set of FDs, S1:
(a) $ACDE \rightarrow B$
(b) $B \rightarrow C$
(c) $B \rightarrow F$
(d) $CD \rightarrow A$
(e) $CD \rightarrow F$
(f) $BCF \rightarrow A$
(g) $BCF \rightarrow D$
(h) $ABF \rightarrow H$

**Step 2:** For each FD, reduce the LHS:
(a) By finding the closure of CDE, we have $CDE^{+} = CDEAFBH$. Hence, we can reduce the LHS of this FD, yielding the new FD: $CDE \rightarrow B$
(b) This FD has only one attribute on the LHS thus, we cannot reduce the LHS of this FD.
(c) This FD has only one attribute on the LHS thus, we cannot reduce the LHS of this FD.
(d) The closure of C and D are unable to yield A. Thus, we cannot reduce the LHS of this FD
(e) The closure of C and D are unable to yield F. Thus, we cannot reduce the LHS of this FD
(f) By finding the closure of B, we have $B^{+} = BCFADH$. Hence, we can reduce the LHS of this FD, yielding the new FD: $B \rightarrow A$
(g) By the same argument, we can reduce this FD to: $B \rightarrow D$
(h) By the same argument, we can reduce this FD to: $B \rightarrow H$

Our new set of FDs, let's call it S2, is
(a) $CDE \rightarrow B$
(b) $B \rightarrow C$
(c) $B \rightarrow F$
(d) $CD \rightarrow A$
(e) $CD \rightarrow F$
(f) $B \rightarrow A$
(g) $B \rightarrow D$
(h) $B \rightarrow H$

**Step 3:** Try to eliminate each FD.

(a) ${CDE^+}_{S2-(a)} = CDEAF$. We need this FD.

(b) ${B^+}_{S2-(b)} = BFADH$. We need this FD.

(c) ${B^+}_{S2-(c)} = BCADHF$. We can remove this FD.

(d) ${CD^+}_{S2-\{(c),\,(d)\}} = CDF$. We need this FD.

(e) ${CD^+}_{S2-\{(c),\,(e)\}} = CDA$. We need this FD.

(f) ${B^+}_{S2-\{(c),\,(f)\}} = BCDHAF$. We can remove this FD.

(g) ${B^+}_{S2-\{(c),\,(f),\,(g)\}} = BCH$. We need this FD.

(h) ${B^+}_{S2-\{(c),\,(f),\,(h)\}} = BCDAF$. We need this FD.

Our final set of FDs in ascending alphabetical order are:

(a) $B \rightarrow C$

(b) $B \rightarrow D$

(c) $B \rightarrow H$

(d) $CD \rightarrow A$

(e) $CD \rightarrow F$

(f) $CDE \rightarrow B$

**(b)**

| Attribute | Appears on | | Conclusion |
|---|---|---|---|
| | LHS | RHS | |
| G | — | — | Must be in every key |
| E | ✓ | — | Must be in every key |
| F | — | ✓ | Is not in any key |
| H | — | ✓ | Is not in any key |
| A, B, C, D | ✓ | ✓ | Must check |

This means that we have to consider all combinations of A, B, C and D. For each, we must add in G and E, since they are in every key.

- $GEA^{+} = GEA$. This is not a key.
- $GEB^{+} = GEBCDHAF$. So GEB is a key.
- $GEC^{+} = GEC$. This is not a key.
- $GED^{+} = GED$. This is not a key.
- $GEAC^{+} = GEAC$. This is not a key.
- $GEAD^{+} = GEAD$. This is not a key.
- $GECD^{+} = GECDAFBH$. So GECD is a key.
- All other possibilities include GEB and GECD so we're done.

Therefore, the keys for R using my solution for a minimal basis are GEB and GECD (BEG and CDEG - in alphabetical order).

**(c)**

Following the 3NF synthesis algorithm, we would get one relation for each FD. However, we combine the FDs with the same LHS to create a single relation. Merging the final set of FDs from part (a), we have:

(a) $CDE \rightarrow B$
(b) $B \rightarrow CDH$
(c) $CD \rightarrow AF$

The set of relations obtained from the 3NF synthesis algorithm is as follows.

$R1(B,\ C,\ D,\ E)$
$R2(B,\ C,\ D,\ H)$
$R3(A,\ C,\ D,\ F)$

Since there is no relation that is a superkey for the set of attributes, we add a relation whose schema is some key. Hence, our final set of relations are as follows.

$R1(B,\ C,\ D,\ E)$
$R2(B,\ C,\ D,\ H)$
$R3(A,\ C,\ D,\ F)$
$R4(B,\ E,\ G)$

**(d)**

Since we formed each relation from an FD, the LHS of those FDs are indeed superkeys for their relations. However, there may be other FDs that violate BCNF and hence, allow redundancy. The only way to find out is to project the FDs onto each relation.

We can find a relation that violates BCNF without doing all the full projections:
Clearly, $B \rightarrow C$ will project onto the relation R1. By finding the closure of $B^+$we obtain $B^+ = BCDHAF$. However, B is not a superkey of relation R1 since it does not include E.

Hence, this schema allows redundancy.