Chapter 6

3) $y_t = 2.3 - 0.95\varepsilon_{t-1} + \varepsilon_t \qquad \varepsilon_t \to \text{iid.}$

Assume: $\varepsilon_t = 0.4$
$\varepsilon_{t-1} = -1.2$
Quadratic loss function.

Optimal Forecasts

$h=1$:

$$f_{t,1} = E(Y_{t+1} \mid I_t)$$

$$= E(2.3 - 0.95\varepsilon_t + \varepsilon_{t+1} \mid I_t)$$

$$= E(2.3 \mid I_t) - 0.95E[\varepsilon_t \mid I_t] + E[\varepsilon_{t+1} \mid I_t]$$

$$= 2.3 - 0.95 \times 0.4$$

$$= 1.92$$

$h:2$

$$f_{t,2}=E(Y_{t+2}|I_t)$$
$$=E(2.3-0.95\varepsilon_{t+1}+\varepsilon_{t+2}|I_t)$$
$$=E(2.3|I_t)-0.95E[\varepsilon_{t+1}|I_t]+E[\varepsilon_{t+2}|I_t]$$
$$=2.3$$

$h:3$

$$f_{t,3}=E(Y_{t+3}|I_t)$$
$$=E(2.3-0.95\varepsilon_{t+2}+\varepsilon_{t+3}|I_t)$$
$$=E(2.3|I_t)-0.95E[\varepsilon_{t+2}|I_t]+E[\varepsilon_{t+3}|I_t]$$
$$=2.3$$

Variance of forecast error

$h: 1$

$$\sigma^2_{t+1|t} = Var(Y_{t+1} | I_t)$$

$$= E[(Y_{t+1} - f_{t,1})^2 | I_t]$$

$$e_{t,1} = Y_{t+1} - f_{t,1}$$

$$= 2.3 - 0.95\varepsilon_t + \varepsilon_{t+1} - f_{t,1}$$

$$= 2.3 - 0.95(0.4) - 1.92 + \varepsilon_{t+1}$$

$$= E[(\varepsilon_{t+1})^2 | I_t] = E(\varepsilon_{t+1})^2$$

$$= 1$$

since $\varepsilon_t \overset{iid}{\sim} N(0,1)$

$Var(\varepsilon_t) = 1$

$= E(\varepsilon_t^2) - \underbrace{E(\varepsilon_t)}_{0}^2$

$\therefore E(\varepsilon_t^2) = 1$

$h=2$

$$e_{t,2} = y_{t+2} - f_{t,2}$$

$$= 2.3 - 0.95\varepsilon_{t+1} + \varepsilon_{t+2} - 2.3$$

$$= -0.95\varepsilon_{t+1} + \varepsilon_{t+2}$$

$$E\left[(-0.95\varepsilon_{t+1} + \varepsilon_{t+2})^2\right]$$

$$= E\left[\left(-0.95^2\varepsilon_{t+1}^2 + \varepsilon_{t+2}^2 + 2(-0.95\varepsilon_{t+1}\varepsilon_{t+2})\right)\right]$$

$$= -0.95^2 E[\varepsilon_{t+1}^2] + E[\varepsilon_{t+2}^2] + 2\cdot -0.95 E[\varepsilon_{t+1}\varepsilon_{t+2}]$$

$$= -0.95^2 Var(\varepsilon_{t+1}) + Var(\varepsilon_{t+2}) + 2\cdot -0.95 Cov(\varepsilon_{t+1}, \varepsilon_{t+2})$$

$$= -0.95^2 \cdot 1 + 1 + 0$$

$$= 1.90$$

h : 3

Similarly for $E[e_{t,3}^2]$, the variance for the forecast errors is 1.90 which is the same as $E[e_{t,2}^2]$

Density forecasts

$h:1 \rightarrow N(1.92, 1)$

$h:2 \rightarrow N(2.3, 1.90)$

$h:3 \rightarrow N(2.3, 1.90)$

4)

(a) $y_t = 0.7 - 2\varepsilon_{t-1} + 1.35\varepsilon_{t-2} + \varepsilon_t$

$\text{mean}(y_t) = E(y_t)$

$= E(0.7) - E(2\varepsilon_{t-1}) + E(1.35\varepsilon_{t-2}) + E(\varepsilon_t)$

$= 0.7$

$Var(y_t) = Var(0.7 - 2\varepsilon_{t-1} + 1.35\varepsilon_{t-2} + \varepsilon_t)$

$= 2^2\, Var(\varepsilon_{t-1}) + 1.35^2\, Var(\varepsilon_{t-2}) + Var(\varepsilon_t)$

$= 6.8225$

$cov(y_t, y_{t-1}) = \underbrace{E[y_t y_{t-1}]} - E[y_t]E[y_{t-1}]$

$\Rightarrow E[(0.7 - 2\varepsilon_{t-1} + 1.35\varepsilon_{t-2} + \varepsilon_t)(0.7 - 2\varepsilon_{t-2} + 1.35\varepsilon_{t-3} + \varepsilon_{t-1})]$

$\Rightarrow E[0.7 \times 0.7 - 2\varepsilon_{t-1}^2 - 2 \cdot 1.35\,\varepsilon_{t-2}^2] = -4.21$

$= -4.21 - 0.7 \times 0.7$

$= -4.7$

$\therefore \rho_1 = \frac{\gamma_0}{\gamma_1} = \frac{-4.7}{6.8225} = -0.688$

$cov(y_t, y_{t-2}) = \underbrace{E[y_t y_{t-2}]} - E[y_t]E[y_{t-2}]$

$\Rightarrow E[(0.7 - 2\varepsilon_{t-1} + 1.35\varepsilon_{t-2} + \varepsilon_t)(0.7 - 2\varepsilon_{t-3} + 1.35\varepsilon_{t-4} + \varepsilon_{t-2})]$

$\Rightarrow E[0.7 \times 0.7 + 1.35\varepsilon_{t-2}^2] = 1.84$

$= 1.84 - 0.7 \times 0.7$

$= 1.35$

$\therefore \rho_2 = \frac{\gamma_0}{\gamma_1} = \frac{1.35}{6.8225} = 0.198$

$\rho_3 = \rho_4 = \ldots = \rho_{10} = 0$

since for MA(2) ρ_{2+1} onwards $= 0$