2)

AR(3) model: $y_t = C + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t$

Unconditional Mean:

$$E(y_t) = E(C + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \phi_3 y_{t-3} + \varepsilon_t)$$

$$E(y_t) = E(C) + \phi_1 E(y_{t-1}) + \phi_2 E(y_{t-2}) + \phi_3 E(y_{t-3}) + E(\varepsilon_t)$$

$$E(y_t) = C + \phi_1 E(y_{t-1}) + \phi_2 E(y_{t-2}) + \phi_3 E(y_{t-3}) + 0$$

Since $\varepsilon_t \sim N(0,1)$

$$E(y_t)(1 - \phi_1 - \phi_2 - \phi_3) = C$$

$$E(y_t) = \frac{C}{(1 - \phi_1 - \phi_2 - \phi_3)}$$

Optimal forecast for $h=1$:

$$f_{t,1} = \mu_{t+1|t}$$

$$= E\left(y_{t+1} \mid I_t\right)$$

$$= E\left(c + \phi_1 y_t + \phi_2 y_{t-1} + \phi_3 y_{t-2} + \varepsilon_{t+1} \mid I_t\right)$$

$$= E(c) + \phi_1 E(y_t) + \phi_2 E(y_{t-1}) + \phi_3 E(y_{t-2}) + E(\varepsilon_{t+1})$$

$$= c + \phi_1 y_t + \phi_2 y_{t-1} + \phi_3 y_{t-2}$$

Optimal forecast for $h=2$:

$$f_{t,2} = \mu_{t+2|t}$$

$$= E\left(Y_{t+2} \mid I_t\right)$$

$$= E\left(c + \phi_1 y_{t+1} + \phi_2 y_t + \phi_3 y_{t-1} + \varepsilon_{t+2} \mid I_t\right)$$

$$= E(c) + \phi_1 E\left(y_{t+1} \mid I_t\right) + \phi_2 E(y_t) + \phi_3 E\left(y_{t-1}\right) + E(\varepsilon_{t+2})$$

$$= c + \phi_1 \left[c + \phi_1 y_t + \phi_2 y_{t-1} + \phi_3 y_{t-2}\right] + \phi_2 y_t + \phi_3 y_{t-1}$$

$$= c(1 + \phi_1) + \phi_1^2 y_t + \phi_1 \phi_2 y_{t-1} + \phi_1 \phi_3 y_{t-2} + \phi_2 y_t + \phi_3 y_{t-1}$$

$$= c + \phi_1 \left(f_{t,1}\right) + \phi_2 y_t + \phi_3 y_{t-1}$$

Optimal forecast for h=3 :

$$f_{t,3} = \mu_{t+3|t}$$

$$= E\left(Y_{t+3} \mid I_t\right)$$

$$= E\left(C + \phi_1 Y_{t+2} + \phi_2 Y_{t+1} + \phi_3 Y_t + \varepsilon_{t+3} \mid I_t\right)$$

$$= E(C) + \phi_1 E(y_{t+2} \mid I_t) + \phi_2 E(y_{t+1} \mid I_t) + \phi_3 E(y_t) + E(\varepsilon_{t+3})$$

$$= C + \phi_1\left(C(1+\phi_1) + \phi_1^2 y_t + \phi_1\phi_2 y_{t-1} + \phi_1\phi_3 y_{t-2} + \phi_2 y_t + \phi_3 y_{t-1}\right)$$
$$+ \phi_2\left(C + \phi_1 y_t + \phi_2 y_{t-1} + \phi_3 y_{t-2}\right) + \phi_3 y_t$$

$$= C + \phi_1\left(f_{t,2}\right) + \phi_2\left(f_{t,1}\right) + \phi_3 y_t$$