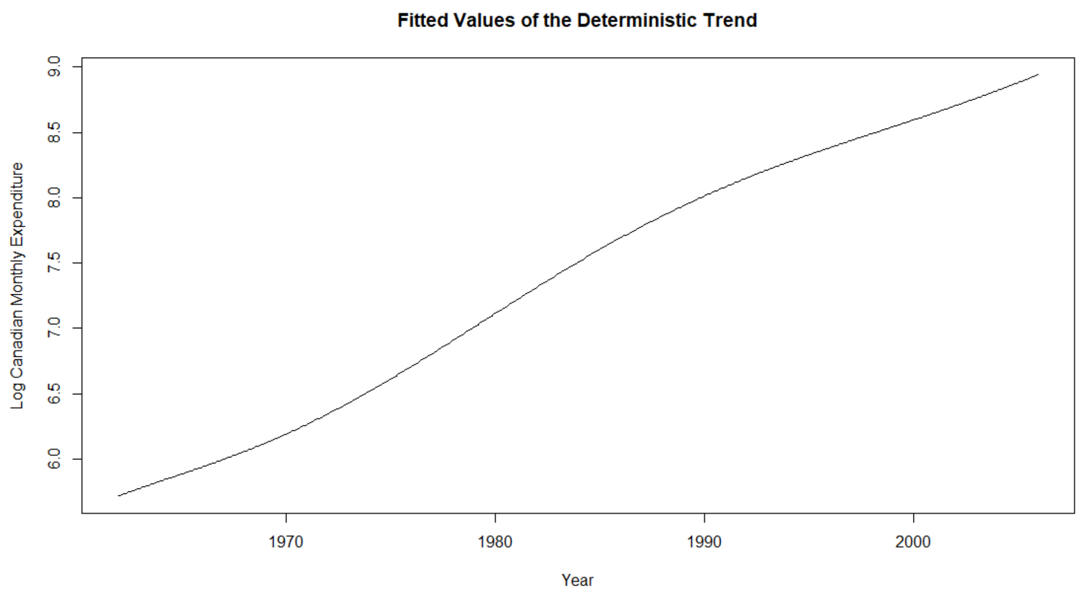
1.

a.
I have decided on model 6 since it minimizes the AIC to an extent where there aren't drastic changes in the AIC as we include more time regressors of higher polynomials.

The estimation results of the deterministic component is as follows.

```
> m6

Call:
lm(formula = y_ts ~ t + t2 + t3 + t4 + t5 + t6)

Coefficients:
(Intercept)            t           t2           t3           t4           t5           t6
  5.712e+00    5.270e-03   -2.667e-05    3.494e-07   -1.305e-09    1.971e-12   -1.063e-15
```

The fitted value is plotted as follows.



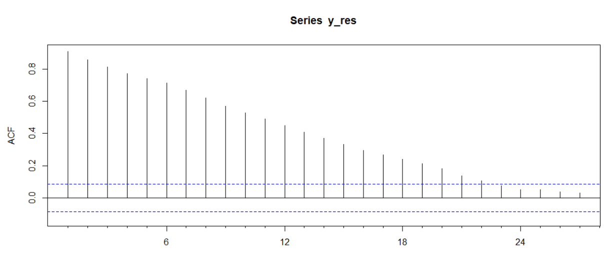
b.

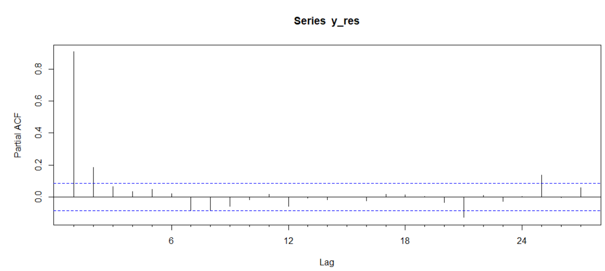
A Dickey-Fuller Test is conducted on the residual series and the result is as follows.

```
> adf.test(y_res)
Augmented Dickey-Fuller Test
alternative: stationary

Type 1: no drift no trend
     lag   ADF p.value
[1,]   0 -4.94    0.01
[2,]   1 -3.92    0.01
[3,]   2 -3.59    0.01
[4,]   3 -3.35    0.01
[5,]   4 -3.21    0.01
[6,]   5 -3.15    0.01
Type 2: with drift no trend
     lag   ADF p.value
[1,]   0 -4.94  0.0100
[2,]   1 -3.92  0.0100
[3,]   2 -3.58  0.0100
[4,]   3 -3.35  0.0145
[5,]   4 -3.21  0.0212
[6,]   5 -3.14  0.0244
Type 3: with drift and trend
     lag   ADF p.value
[1,]   0 -4.93  0.0100
[2,]   1 -3.91  0.0129
[3,]   2 -3.58  0.0339
[4,]   3 -3.34  0.0632
[5,]   4 -3.21  0.0870
[6,]   5 -3.14  0.0981
----
Note: in fact, p.value = 0.01 means p.value <= 0.01
```

The p value for lags 0 in Type 1, 2 and 3 tests are lesser than 0.05. Furthermore, the ADF values are all less than 1. Therefore, we reject the null hypothesis indicating that the residual series is stationary. Hence, we only need to determine an ARMA model instead of an ARIMA model.

To determine the model appropriate for the residuals, we observe the acf and pacf of the residuals.





Since the ACF dies down and the PACF cuts off after lag 2, we take the AR(2) model into consideration.

Additionally, observing the PACF indicates several spikes at lags 7, 8, 21 and 25. Thus, we could also consider the ARMA(2,1) model since there is some decay toward zero in both the ACF and the PACF and no clear cutoff in ACF or PACF. Since the first two spikes in the PACF seem to be the most dominant in the PACF, it reflects the AR(2) component and the remaining dependence can be modelled by the MA(1) component

Computing the ARMA models of AR(2) and ARMA(2,1) yields the following.

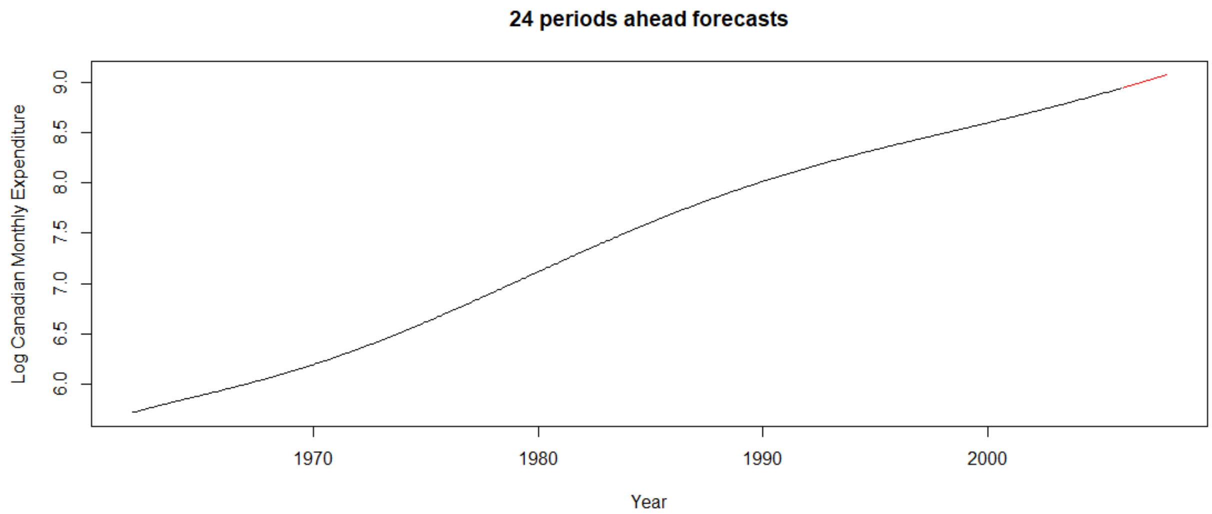
```
> arima(y_res, order=c(2,0,0))

Call:
arima(x = y_res, order = c(2, 0, 0))

Coefficients:
         ar1     ar2  intercept
      0.7256  0.2024    -0.0002
s.e.  0.0426  0.0428     0.0032

sigma^2 estimated as 2.929e-05:  log likelihood = 2005.54,  aic = -4003.09
> arima(y_res, order=c(2,0,1))

Call:
arima(x = y_res, order = c(2, 0, 1))

Coefficients:
         ar1      ar2      ma1  intercept
      1.1214  -0.1572  -0.4173    -0.0004
s.e.  0.1532   0.1428   0.1418     0.0037

sigma^2 estimated as 2.904e-05:  log likelihood = 2007.87,  aic = -4005.73
```

Thus, since the ARMA(2,1) has a lower AIC, we use the ARMA(2,1) model to represent the residuals as a lower AIC represents a better model fit.

c.

The forecasts for 24 periods are as follows.

```
> m6_f
          Jan      Feb      Mar      Apr      May      Jun      Jul      Aug      Sep      Oct      Nov
2006 8.945000 8.950356 8.955725 8.961106 8.966499 8.971903 8.977318 8.982743 8.988179 8.993625 8.999081
2007 9.010019 9.015501 9.020990 9.026488 9.031992 9.037503 9.043020 9.048542 9.054069 9.059601 9.065136
          Dec
2006 9.004545
2007 9.070675
```



2.

The following below are the AIC and MAPE generated from the various thresholds.

7%: AIC = -2164, MAPE = 11.74%

6%: AIC = -2159, MAPE = 11.81%

5%: AIC = -2154, MAPE = 11.67%

For this dataset, the most suitable threshold would be the 7% threshold model. The AIC measures the goodness of fit of the model whereas the MAPE measures the forecasting performance. Thus, the best model would have the lowest AIC and MAPE. Therefore, since the 7% threshold model has the lowest AIC and MAPE, it is most suitable for this dataset.

On the other hand, the 5% threshold model has a higher AIC but lower MAPE compared to the 7% threshold model. Therefore, if we were to prioritise forecasting performance over the model fit, we would use the 5% threshold model since it has a lower MAPE. On the other hand, if we were to prioritise model fit, we would use the 7% threshold model since it has a lower AIC.