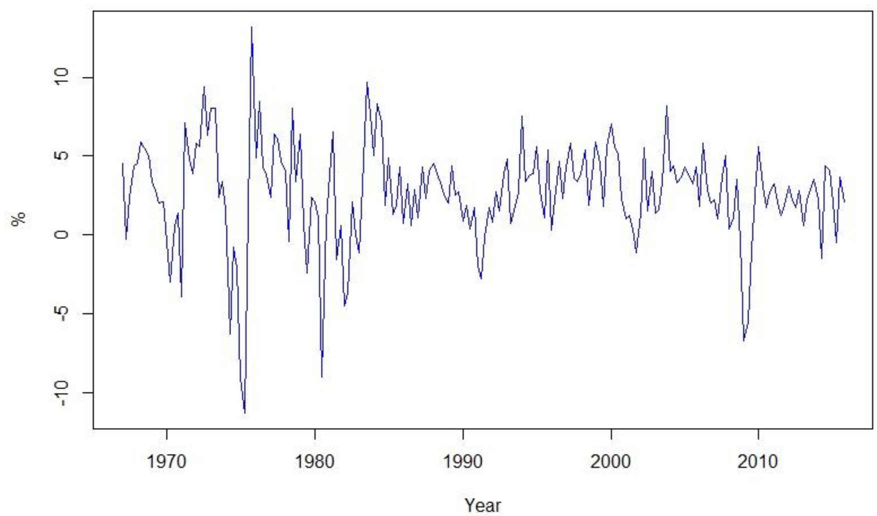
1.

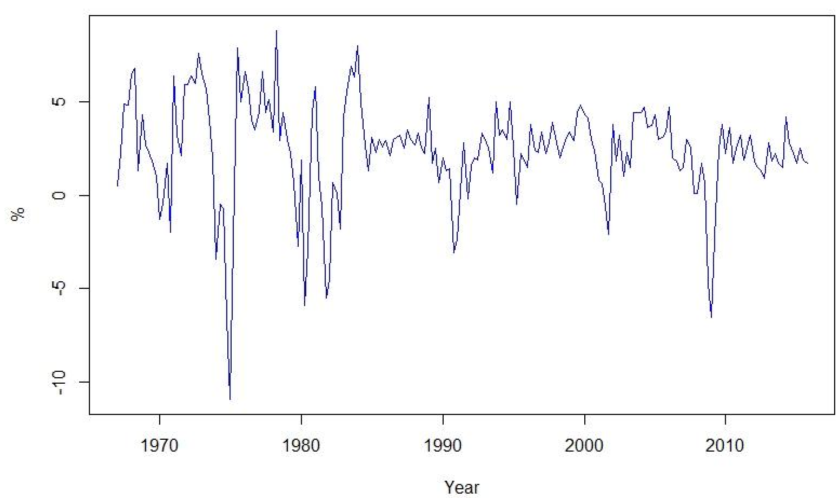
Exercise 6

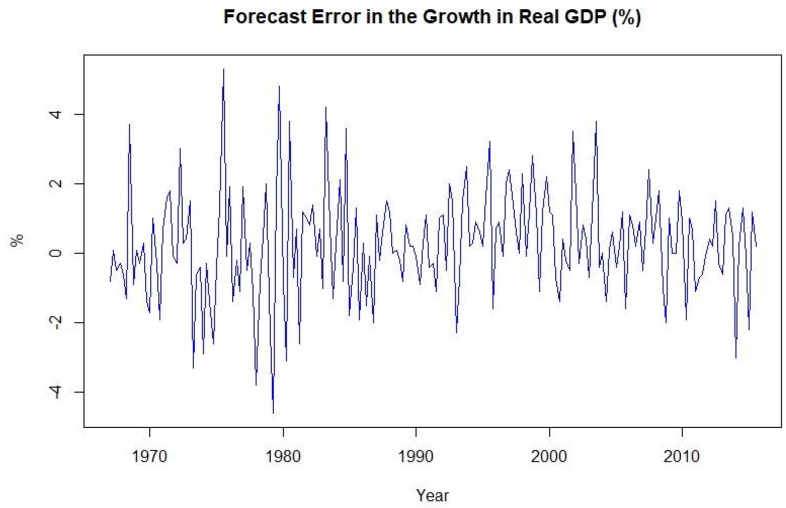
The following depicts the time series of realized values, forecasts and forecast errors in Real GDP Growth.

Realized Growth in Real GDP (%)



Forecasted Growth in Real GDP (%)

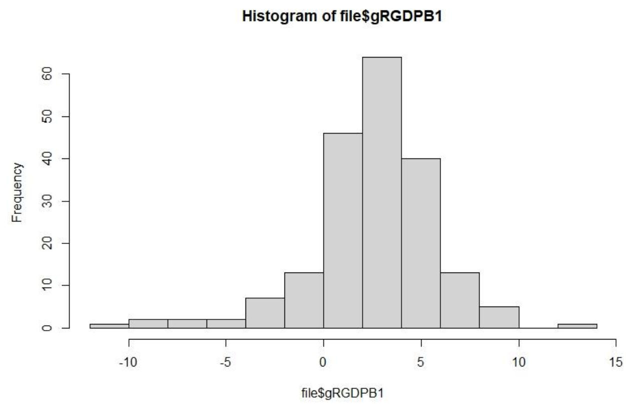




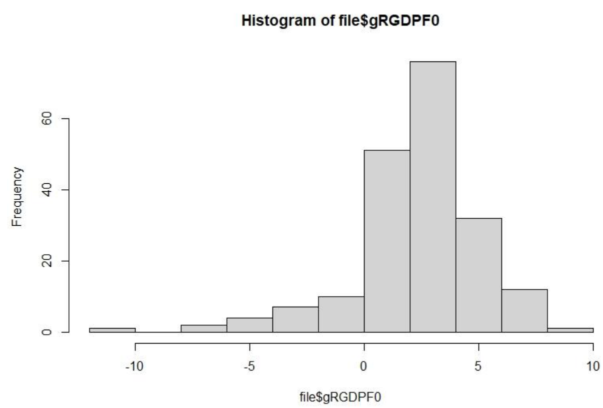
In these time series, we can observe that they are first order weakly stationary as the random variables for the years 1967 onwards fluctuate around a value and thus, have the same mean. Furthermore, in both the realized and forecasted growth, we observe a larger variance between 1967 and 1985 followed by a smaller variance thereafter.

The following depicts the descriptive statistics of the three series.

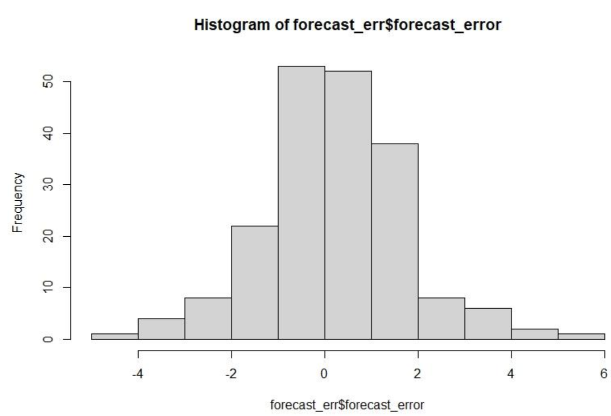
```
realized_values (%)
Min.   :-11.300
1st Qu.:  1.275
Median :  2.700
Mean   :  2.639
3rd Qu.:  4.400
Max.   : 13.200
```




```
forecast_values (%)
Min.   :-10.900
1st Qu.:  1.500
Median :  2.500
Mean   :  2.371
3rd Qu.:  3.800
Max.   :  8.800
```




```
forecast_error (%)
Min.   :-4.6000
1st Qu.:-0.6000
Median : 0.2000
Mean   : 0.2554
3rd Qu.: 1.1000
Max.   : 5.3000
```




```
> sd(file$gRGDPB1)
[1] 3.234817
> sd(file$gRGDPF0)
[1] 2.726851
```

The mean Real GDP Growth rate is determined to be 2.64 whereas the forecasted mean is 2.37 which is relatively close as it only differs by 0.27. Furthermore, it is determined that the standard deviation of the realized GDP growth is equal to 3.23 which is higher than the standard deviation of the forecasted GDP growth equal to 2.73. We would expect a higher standard deviation in the realized GDP growth as there are innovation and technology shocks contributing to the GDP growth that cannot be accounted for in the forecast. Thus, the higher standard deviation is accurately reflected in the data presented. In the data of the forecast error, we observe a mean of 0.26. Thus, we would expect the forecast to underestimate the actual realized data of the GDP growth.

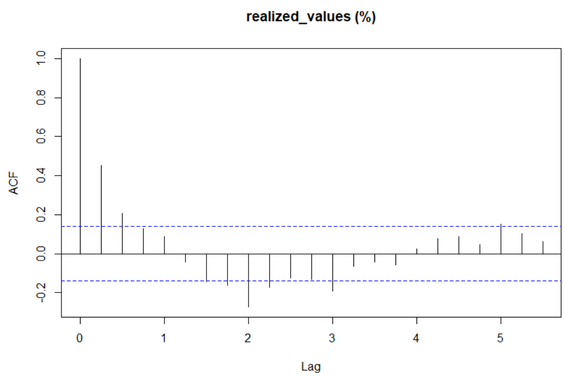
The following depicts the ACF and PACF of the realized values, forecasts and forecast errors.

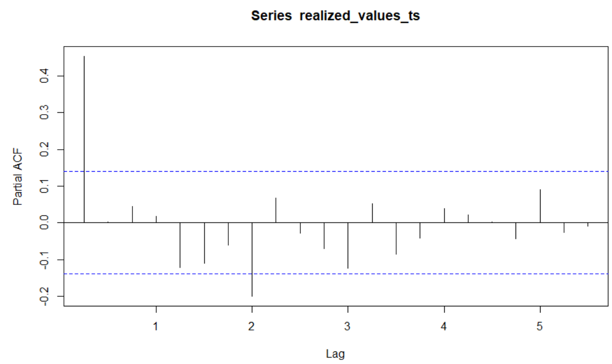
```
Autocorrelations of series ‘realized_values_ts’, by lag

  0.00   0.25   0.50   0.75   1.00   1.25   1.50   1.75   2.00   2.25   2.50   2.75
 1.000  0.454  0.207  0.130  0.089 -0.044 -0.143 -0.162 -0.273 -0.171 -0.124 -0.133
  3.00   3.25   3.50   3.75   4.00   4.25   4.50   4.75   5.00   5.25   5.50
-0.190 -0.065 -0.044 -0.056  0.025  0.076  0.088  0.046  0.153  0.103  0.062
```

```
Partial autocorrelations of series ‘realized_values_ts’, by lag

  0.25   0.50   0.75   1.00   1.25   1.50   1.75   2.00   2.25   2.50   2.75   3.00
 0.454  0.002  0.044  0.017 -0.122 -0.110 -0.061 -0.200  0.067 -0.028 -0.069 -0.124
  3.25   3.50   3.75   4.00   4.25   4.50   4.75   5.00   5.25   5.50
 0.051 -0.085 -0.042  0.038  0.022  0.003 -0.044  0.090 -0.026 -0.009
```



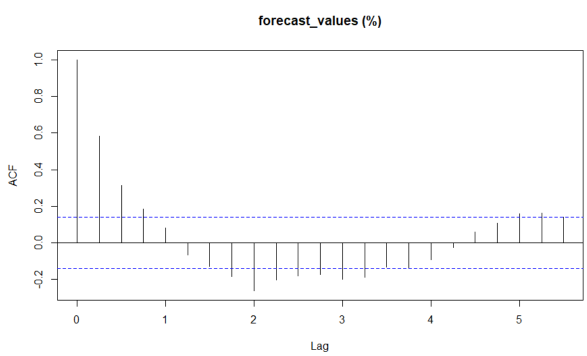


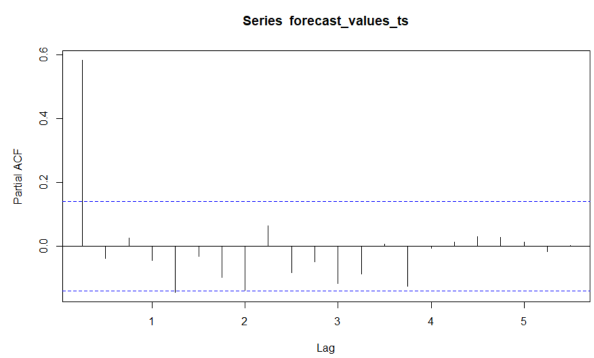

```
Autocorrelations of series ‘forecast_values_ts’, by lag

  0.00   0.25   0.50   0.75   1.00   1.25   1.50   1.75   2.00   2.25   2.50   2.75
 1.000  0.583  0.315  0.185  0.081 -0.066 -0.127 -0.182 -0.260 -0.201 -0.180 -0.173
  3.00   3.25   3.50   3.75   4.00   4.25   4.50   4.75   5.00   5.25   5.50
-0.199 -0.188 -0.132 -0.136 -0.092 -0.026  0.059  0.109  0.160  0.162  0.143
```

```
Partial autocorrelations of series ‘forecast_values_ts’, by lag

  0.25   0.50   0.75   1.00   1.25   1.50   1.75   2.00   2.25   2.50   2.75   3.00
 0.583 -0.039  0.026 -0.045 -0.145 -0.032 -0.097 -0.138  0.063 -0.083 -0.049 -0.116
  3.25   3.50   3.75   4.00   4.25   4.50   4.75   5.00   5.25   5.50
-0.087  0.007 -0.126 -0.007  0.012  0.030  0.029  0.013 -0.017  0.002
```



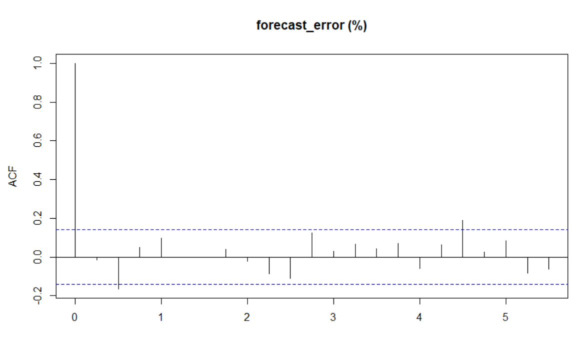


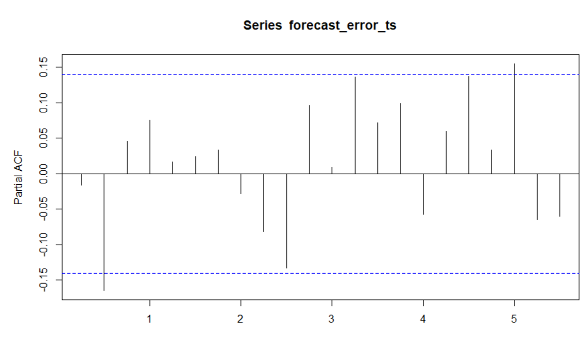

```
Autocorrelations of series ‘forecast_error_ts’, by lag

  0.00   0.25   0.50   0.75   1.00   1.25   1.50   1.75   2.00   2.25   2.50   2.75
 1.000 -0.016 -0.164  0.050  0.099 -0.002 -0.002  0.038 -0.023 -0.086 -0.111  0.124
  3.00   3.25   3.50   3.75   4.00   4.25   4.50   4.75   5.00   5.25   5.50
 0.028  0.066  0.044  0.071 -0.060  0.062  0.188  0.026  0.085 -0.082 -0.062
```

```
Partial autocorrelations of series ‘forecast_error_ts’, by lag

  0.25   0.50   0.75   1.00   1.25   1.50   1.75   2.00   2.25   2.50   2.75   3.00
-0.016 -0.165  0.046  0.075  0.016  0.024  0.033 -0.028 -0.081 -0.133  0.096  0.009
  3.25   3.50   3.75   4.00   4.25   4.50   4.75   5.00   5.25   5.50
 0.136  0.071  0.099 -0.058  0.060  0.137  0.033  0.155 -0.065 -0.060
```

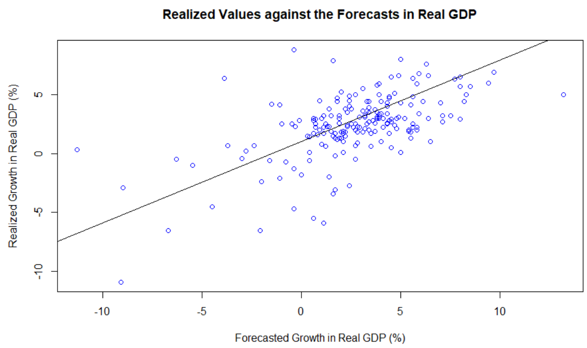




In both the realized and forecasted real GDP growth rates, we observe time dependence. In both the autocorrelation function of the realized and forecasted values, we observe a similar overall trend. In the ACF of the realized values, we observe a statistically significant correlation between the present growth rate and the growth rate of the previous two quarter lags. Similarly in the ACF of the forecasted values, we observe a statistically significant correlation between the present growth rate and the growth rate of the previous three quarter lags. We also observe that the ACF of the forecasted values is smoother than the realized values as expected since forecasts do not include technological and innovation shocks unlike the realized values.

On the other hand, we do not observe time dependence within the growth of the forecast errors. This is observed as there is no statistically significant correlation between the present forecast errors and any lagged forecast errors in the ACF. Since the forecast errors represent a white noise process where each error is defined as a random shock, it is expected that the forecast errors are not time dependent, which is shown in our data generated.

Exercise 7




```
Call:
lm(formula = realized_values_ts ~ forecast_values_ts)

Residuals:
     Min       1Q   Median       3Q      Max 
-12.5104  -1.3136   0.0816   1.4974   8.7467 

Coefficients:
                   Estimate Std. Error t value Pr(>|t|)    
(Intercept)         1.00335    0.24998   4.014 8.53e-05 ***
forecast_values_ts  0.69000    0.06928   9.959  < 2e-16 ***
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 2.638 on 194 degrees of freedom
Multiple R-squared:  0.3383,	Adjusted R-squared:  0.3349 
F-statistic: 99.19 on 1 and 194 DF,  p-value: < 2.2e-16
```

As shown in the graph above, we observe that most of the values in the data are scattered around the line of best fit plotted. The estimates B0_hat and B1_hat are determined to be 1.00335 and 0.69 respectively. In order to determine if the forecast is unbiased, we would need to conduct a t-test for B0_hat = 0 and B1_hat = 1.

Conducting a t-test for B0_hat = 0, we have the null hypothesis to be B0_hat = 0 and the alternative hypothesis to be B0_hat != 0. Calculating the t-statistic using R, we obtain a value of 4.014. The 95% significance level t-value for a two-tailed test is determined to be +/- 1.972268. Therefore, since the t-value is greater than the 95% significance level, we can reject the null hypothesis. Thus, we observe that the intercept of the plot, B0_hat, does not equal to 0 and is statistically significant.

On the other hand, conducting a t-test for B1_hat = 1, we have the null hypothesis to be B1_hat = 1 and the alternative hypothesis to be B1_hat != 1. Calculating the t-statistic ($t = \frac{0.69-1}{0.06928}$) we obtain a value of -4.475. The 95% significance level t-value for a two-tailed test is determined to be +/- 1.972268. Therefore, since the t-value is not within the 95% confidence interval, we can reject the null hypothesis. Thus, we observe that the coefficient of the forecasted values, B1_hat, is statistically significant and not equal to 1.

Conducting an F-test for the joint hypothesis, we have the null hypothesis to be B1_hat = 0 whereas the alternative hypothesis to be B1_hat != 0. The critical value for the F-stat at the 0.05 significance level with a degree of freedom of 1 and 194 yields 3.89. The F-statistic is determined to be 99.19, therefore, since the F-stat is greater than the critical value, we can reject the joint null hypothesis.

Based on the t-tests and F-test conducted, we determine that the forecast is not unbiased.

Exercise 8

The mean of the forecast errors calculated is determined to be 0.2553846. The t-test obtained in R with the null hypothesis being that the forecast errors equal to zero and the alternative hypothesis being that the mean is not equal to 0 is equal to 2.3318. Therefore, since 2.3318 is greater than 1.965, we can reject the null hypothesis at the 5% significance level. Thus, we conclude that the expected value of the forecast errors is not equal to zero. This implies that the forecasts are not 100% accurate when trying to predict the actual values thus, displaying the disparity between the realized and forecasted values as shown in the forecast errors.

```
        One Sample t-test

data:  forecast_error_ts
t = 2.3318, df = 194, p-value = 0.02074
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 0.03938069 0.47138854
sample estimates:
mean of x
0.2553846
```

The regression results of the forecast errors on several lags is as follows.

```
Time series regression with "ts" data:
Start = 1967(4), End = 2015(3)

Call:
dynlm(formula = forecast_error_ts ~ L(forecast_error_ts, 1) +
    L(forecast_error_ts, 2) + L(forecast_error_ts, 3))

Residuals:
    Min      1Q  Median      3Q     Max
-4.6152 -0.8745 -0.0092  0.8364  5.0718

Coefficients:
                        Estimate Std. Error t value Pr(>|t|)
(Intercept)              0.29885    0.11560   2.585   0.0105 *
L(forecast_error_ts, 1) -0.01130    0.07280  -0.155   0.8768
L(forecast_error_ts, 2) -0.16659    0.07186  -2.318   0.0215 *
L(forecast_error_ts, 3)  0.04713    0.07326   0.643   0.5208
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 1.527 on 188 degrees of freedom
Multiple R-squared:  0.03034,   Adjusted R-squared:  0.01487
F-statistic: 1.961 on 3 and 188 DF,  p-value: 0.1214
```

From the results, we observe a value of 0.01487 for the adjusted R-squared which is very low. Therefore, we can interpret that the lagged forecast errors do not explain the present forecast errors. For k = 3, we perform an F-test for the joint hypothesis where H0: B1 = B2 = B3 = 0 and HA: At least one of the beta coefficients != 0. The critical value for the F-stat at the 0.05 significance level with a degree of freedom of 3 and 188 yields 2.653. Therefore, since the F-statistic is equal to 1.961, it is lesser than 2.653, meaning that we do not reject the null hypothesis. Thus, we conclude that the beta coefficients all equal to 0, indicating that the lagged forecast errors do not correlate with the present forecast errors. Hence, this indicates that the forecast errors are not predictable from its own past.

Exercise 9

The regression results are as follows.

```
Time series regression with "ts" data:
Start = 1967(3), End = 2015(3)

Call:
dynlm(formula = forecast_error_ts ~ L(forecast_error_ts, 1) +
    L(forecast_error_ts, 2) + L(forecast_values_ts, 0) + L(forecast_values_ts,
    1))

Residuals:
    Min      1Q  Median      3Q     Max
-4.6205 -0.9281  0.0504  0.8800  4.2635

Coefficients:
                         Estimate Std. Error t value Pr(>|t|)
(Intercept)               0.26028    0.15178   1.715   0.0880 .
L(forecast_error_ts, 1)  -0.04583    0.07231  -0.634   0.5269
L(forecast_error_ts, 2)  -0.15970    0.07225  -2.210   0.0283 *
L(forecast_values_ts, 0)  0.11652    0.04969   2.345   0.0201 *
L(forecast_values_ts, 1) -0.09405    0.04942  -1.903   0.0586 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 1.506 on 188 degrees of freedom
Multiple R-squared:  0.05742,   Adjusted R-squared:  0.03736
F-statistic: 2.863 on 4 and 188 DF,  p-value: 0.02466
```

From the results, we observe that the relationship between the forecast values (no lag and 1 period lag) and the forecast error is statistically significant at the 0.05 significance level. This indicates that there could be a correlation between the lagged forecast values and forecast errors and we will test this further using the F-test.

On the other hand, the relationship between the 2 period lag forecast error and the present forecast error is statistically significant. However, the relationship between the 1 period lag forecast error and the present forecast error is not statistically significant. We would expect that the current forecast error is not time dependent with the lagged forecast errors as displayed in Exercise 6. Thus, by performing an F-test for the joint hypothesis H0: B1 = B2 = B3 = B4 = 0, we determine the p-value of F-statistic to be 0.02466. Hence, we do not reject the null hypothesis which supports our claim that the current forecast error is not time dependent with the lagged forecast errors.

The F statistic for H0: B3 = B4 = 0 is as follows.

```
Linear hypothesis test

Hypothesis:
L(forecast_values_ts, 0) = 0
L(forecast_values_ts,0

Model 1: restricted model
Model 2: forecast_error_ts ~ L(forecast_error_ts, 1) + L(forecast_error_ts,
    2) + L(forecast_values_ts, 0) + L(forecast_values_ts, 1)

  Res.Df    RSS Df Sum of Sq     F  Pr(>F)
1    190 440.17
2    188 426.63  2    13.543 2.984 0.05299 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

The critical value for the F-statistic at the 0.05 significance level with a degree of freedom of 2 and 188 yields 3.044. Since the F-statistic determined in R is equal to 2.984, it is smaller than the critical value and thus, we do not reject the null hypothesis, H0: B3 = B4 = 0. Thus, we would expect that the joint present and 1 period lag forecast values have no correlation with the forecast error. Hence, the general consensus with the t-tests and F-tests indicates that the forecast error is not predictable based on historical data.

2.

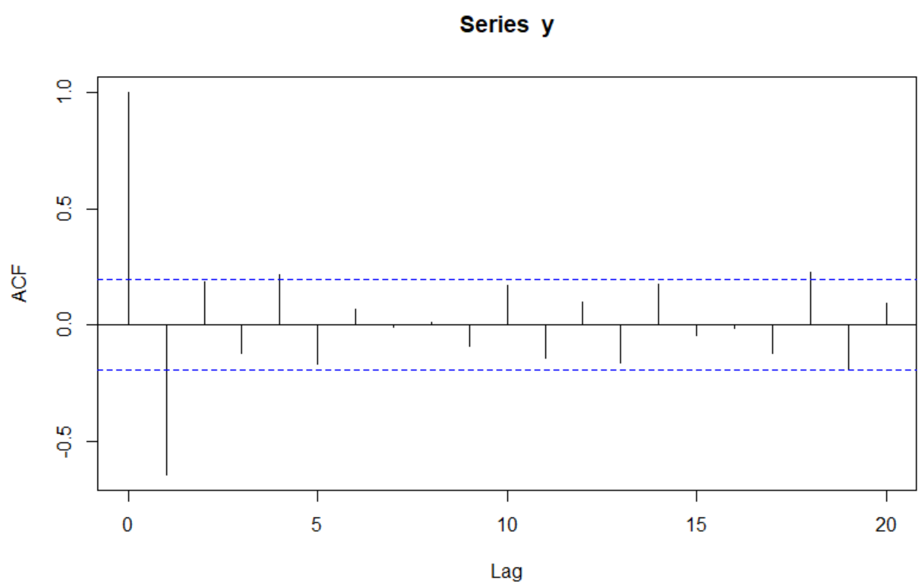
Exercise 4

(b)

The sample autocorrelation function for n = 100 observations is as follows.

```
Autocorrelations of series ‘y’, by lag

     0      1      2      3      4      5      6      7      8      9     10 
 1.000 -0.644  0.184 -0.121  0.214 -0.170  0.069 -0.011  0.010 -0.093  0.170 
    11     12     13     14     15     16     17     18     19     20 
-0.141  0.099 -0.165  0.177 -0.043 -0.016 -0.121  0.226 -0.192  0.092 
```



When compared to my results calculated in part a., the results in part b. are relatively close to the theoretical ACF. For rho 1 and rho 2, the theoretical ACF obtained are -0.688 and 0.198 respectively. Therefore, both the numbers are close to the ACF generated with rho 1 being -0.644 and rho 2 being 0.184. In order to increase the accuracy of our simulated sample autocorrelation function, we could increase the number of observations from 100 to 1000. This would produce sample ACF values that are much closer to the theoretical values in part a.