Machine learning for data science I

31 August 2023

Surname, name (all caps) _____

Student ID: _____

This is a closed book exam.

Write clearly and justify your answers.

Time limit: 105 min.

Question:	1	2	3	4	5	Total
Points:	20	20	20	20	20	100
Score:						

1. Explain your answers to the following questions related to the k-nearest neighbors method:

[4] (a) What are the time and space complexities of *building* a kNN model of a data set with n instances and f features?

[4] (b) What is the time complexity of making a *prediction* for a single instance using the Euclidean distance?

[4] (c) Propose two approaches (one exact and one approximate) for making predictions with a sublinear time complexity in terms of the number of instances n.

[4] (d) Can you design a better approach for the special case where $k = n$? Comment on its time and space complexity of building the model and making predictions with it.

[4] (e) Describe a data set where the kNN method with $k = 1$ would perform poorly in terms of the accuracy of its predictions.

2. Consider a regression problem where two features are perfectly correlated (suppose that $x_2 = ax_1$, $a \geq 1$). We will model the process with a regularized linear regression and will use an *extremely small* regularization parameter λ (without regularizing the intercept). Assume that the optimal parameters β_i of a non-regularized model are all positive. Explain your answers to the following questions about the parameters β_i' of a regularized model.

[4] (a) What is the effect of using an extremely small regularization parameter λ in linear regression (ignoring potential precision errors in the computation) compared to not using regularization at all?

[8] (b) How will L1 regularization (Lasso) affect the model weights in the correlated case with an extremely small regularization weight mentioned above; which β_1', β_2' are chosen?

[8] (c) What about L2 regularization (Ridge); which β_1', β_2' are chosen?

DATA : $SSE + \lambda R$

L1)

$$\beta_0 + \beta_1 x_1 + \beta_2 x_2 = \beta_0 + x_1 \underbrace{(\beta_1 + \alpha \beta_2)}_{C}$$

($\beta_2 \alpha x_1$)

$\min |\beta_1'| + |\beta_2'|$ s.t. $\beta_1' + \alpha\beta_2' = C = \beta_1 + \alpha\beta_2$

β_1' HAS A STRONGER EFFECT

SUPPOSE TO TAKE OUT 1 OVER β_1' => $\frac{\beta_1'}{1} + \frac{\beta_2'}{\alpha} = C$ → BECAUSE C NEED TO REMAIN THE SAME

$\beta_1' = 0$

$\beta_2' = \frac{C - \beta_1'}{\alpha}$

L2) $\min \beta_1^2 + \beta_2^2 = (C - \alpha\beta_2')^2 + \beta_2'^2 = C^2 - 2\alpha C\beta_2' + \alpha^2\beta_2'^2 + \beta_2^2$

$\frac{dL}{d\beta_2'} = -2\alpha C + 2(\alpha^2+1)\beta_2' = 0 \Rightarrow \beta_2' = \frac{\alpha C}{\alpha^2+1}$

3. Consider a binary classification data set with $n = 1000$ instances. Somehow, we found another data instance, but we don't know its class value. We decided to include it into the data set as two instances - one with a positive class value and the other with a negative class value. Explain what effects can such addition have on different classification models (compared to the same model built on the original data set).

[4] (a) k-nearest neighbors

[4] (b) logistic regression CAN CHANGE

[4] (c) support vector machine (with hard and soft margins)

[4] (d) decision tree ROBUST, BUT CAN BE DIFFERENT

[4] (e) random forest ROBUST

BUILDING
- time $O(nf)$, $O(1)$
- SPACE

PREDICTION
- TIME $O(nf)$

EXACT : SPACE PART.
APPROX : SUBSAMPLE

k = n
BUILD < TIME $O(n)$ / SPACE $O(1)$
PRED - TIME $O(1)$

+ - - + -

4. Answer the following questions about Laplace Approximation (LA) of posterior probability distribution $p(x)$ (let $p(x)$ be a multivariate density).

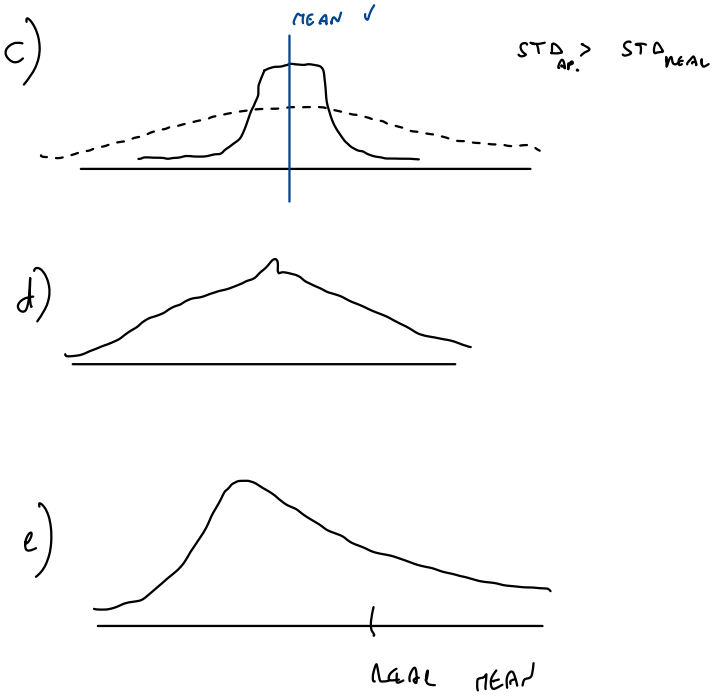
[4] (a) What is the purpose of LA - why do we need it?

[4] (b) Describe the algorithm of LA.

[4] (c) LA can fit the mean well, but severely overestimate the variance. Draw an example univariate density where this would happen and the corresponding LA to that density. Let the drawing be approximately to scale (densities have to integrate to the same area).

[4] (d) Draw an example where LA fits the mean well, but severely underestimates the variance.

[4] (e) Draw an example where LA doesn't approximate the mean well.



5. Explain your answers to the following questions related to the k-means clustering method:

[4] (a) Describe the purpose of clustering methods - what do we want to achieve with them?

[6] (b) Briefly describe the two most popular clustering methods: k-means and hierarchical clustering. Explain one advantage of each of these two methods.

[5] (c) K-means method is solving an optimization problem. Does it find the optimal solution? Explain why it does find the optimum or provide an illustration of a counter-example.

[5] (d) How would you adapt the k-means method to handle a data set with weighted instances (without creating duplicates of data points)? How does your method compare to increasing the data set with copies of data points proportional to their weights?

b) H CLUSTER DOES NOT REQUIRE KNOWING THE NUMBER OF CLUSTERS

K MEANS IS FAST

c) IT IS MINIMIZING THE SQUARED ROOT OF THE DISTANCES BETWEEN THE POINTS OF A CLUSTER AND ITS CENTROID

WRONG

OPTIMAL

IT CAN FIND ALSO THIS THAT IS NOT OPTIMAL