Spectral Graphs on Calabi–Yau Manifolds: A Research Hypothesis and Program

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September 2025

Abstract

This paper presents a research hypothesis and program concerning the spectral properties of graphs derived from Calabi–Yau (CY) manifolds. The central claim is that discrete Laplacians on CY graphs capture nontrivial topological and geometric information related to Hodge numbers and Euler characteristics. We propose a systematic study of CY-inspired spectral models as a step toward building the theory of CYbits and their meta-quantum extensions. Although the present work does not include numerical experiments, it establishes the theoretical framework and outlines a detailed roadmap of research tasks.

1 Introduction and Motivation

Studying computation in higher-dimensional frameworks can provide new insights into lower-dimensional models. Calabi–Yau manifolds are well known from string theory and complex geometry as rich mathematical structures with highly constrained topology. We hypothesize that spectral graph theory on discretizations of CY spaces can reveal computational features not accessible in ordinary Euclidean settings. This document should be read as a programmatic research note: it makes conjectures and proposes a systematic plan for verification.

2 Definitions and Setup

Let M be a Calabi–Yau manifold of complex dimension k. We define a discretization $\{p_1, \ldots, p_N\} \subset M$ with edges given by k-nearest neighbors. The discrete Laplacian is

$$(Lf)(p_i) = \sum_{j:(i,j)\in E} w_{ij}(f(p_i) - f(p_j)),$$

where w_{ij} depends on the CY metric. We are interested in the spectrum $\{\lambda_\alpha\}$ of L and its large-N asymptotics.

3 Central Hypothesis

We conjecture that:

- The eigenvalue distribution of L converges, as $N \to \infty$, to the spectrum of the Laplace–Beltrami operator on M.
- Spectral gaps and degeneracies encode information about the Hodge numbers $(h^{1,1}, h^{2,1})$ of M.
- These spectral features can be used to define a *computational capacity* $\mathcal{C}(M)$ for CY-inspired quantum models.

4 Research Program

The proposed research program is structured into several stages:

4.1 Stage 1: Discretization Methods

1. Develop numerical discretizations of simple Calabi–Yau spaces: 2-torus T^2, 3-torus T^3, and the K3 surface.
2. Implement both random sampling and lattice-based point sets.
3. Define edge weights using approximate Ricci-flat metrics (as in Donaldson's algorithm).

4.2 Stage 2: Spectral Analysis

1. Compute spectra of discrete Laplacians for increasing N.
2. Study convergence properties toward continuum spectra.
3. Analyze spectral density, spectral gaps, and degeneracy patterns.

4.3 Stage 3: Topological Correspondence Tests

1. Compare degeneracy structures with known Hodge numbers.
2. Explore correlations between low-lying eigenvalues and Euler characteristics.
3. Test robustness of these correspondences under different discretization schemes.

4.4 Stage 4: Computational Interpretation

1. Define and refine the notion of *computational capacity* $\mathcal{C}(M)$ as a function of spectral invariants.
2. Interpret $\mathcal{C}(M)$ in terms of possible quantum state encodings.
3. Explore the analogy with qubits and propose CYbits as computational primitives.

4.5 Stage 5: Toward Meta-CYbits

1. Extend the framework to higher-dimensional Calabi–Yau 3-folds, such as the quintic hypersurface in $\mathbb{CP}^4$.
2. Study spectral stability under deformations of complex structure.
3. Define Meta-CYbits as composite structures built from families of CYbits.

5 Expected Impact

Confirming these hypotheses would establish a quantitative link between CY topology and computational capacity. This would form the first step in a larger program leading to CYbits and Meta-CYbits, providing a foundation for a new class of meta-quantum computation models.

Acknowledgements

The author thanks the LLC VOSCOM ONLINE Research Initiative for providing the conceptual framework.

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