# Assignment 3 - Group 14

Jacopo Raffaeli, Nicolò Randon, Giovanni Simone Riondato, Aurora Vido

## 1 Esercizio 1

Using the discounts obtained from the case-study via bootstrap on the 15th of February 2008 at 10:45 C.E.T., we computed the discount at 1 year through interpolation and constructed the vector of the asw discounts containing the discounts for 1y, 2y and 3y. With this data, we computed the price of the risk-free coupon bond, and with the following formula, we determined the asset swap spread.

$$S^{asw} = \frac{C(0)-\overline{C}(0)}{BPV(0)}$$

The obtained result is -0.3510.

## 2 Esercizio 2

Firstly, we created a complete set of CDS spreads and dates with spline interpolation, and obtained the following results:

| Dates | CDS spreads |
|---|---|
| 19-Feb-2009 | 29.00 bp |
| 19-Feb-2010 | 32.00 bp |
| 21-Feb-2011 | 35.00 bp |
| 20-Feb-2012 | 39.00 bp |
| 19-Feb-2013 | 40.00 bp |
| 19-Feb-2014 | 39.54 bp |
| 19-Feb-2015 | 41.00 bp |

Then, with these CDS spreads, we did the bootstrap to compute the survival probabilities and also the piecewise constant $\lambda$ for each time step. We computed the probabilities and intensities both neglecting and considering the accrual term. Finally we computed them also using the Jarrow-Turnball approximation. We obtained the following results:

Table 1: Probability of survival

| Approx | Exact | JT |
|---|---|---|
| 0.995374677819331 | 0.995363956222352 | 0.995178328426267 |
| 0.990094175552130 | 0.990069467095952 | 0.989884839193471 |
| 0.984341949445781 | 0.984300626702870 | 0.984095427557063 |
| 0.977980572543836 | 0.977918895666192 | 0.977737204753128 |
| 0.971503449434747 | 0.971420662280821 | 0.971258622116167 |
| 0.965142938506002 | 0.965039805375482 | 0.964879214477073 |
| 0.958592232012425 | 0.958467494782513 | 0.958308349203054 |

Table 2: Intensities

| Approx | Exact | JT |
|---|---|---|
| 0.004636052082259 | 0.004646823558530 | 0.004833333333333 |
| 0.005319161475178 | 0.005333345972896 | 0.005333333333333 |
| 0.005794527322095 | 0.005811458237606 | 0.005833333333333 |
| 0.006501600923804 | 0.006522746244213 | 0.006500000000000 |
| 0.006663495634179 | 0.006685708937078 | 0.006666666666667 |
| 0.006568606264793 | 0.006590250723969 | 0.006589851822114 |
| 0.006810429952621 | 0.006833700261346 | 0.006833333333333 |

We can observe that the survival probabilities and the intensities bootstrapped with the approximated version (neglecting the accrual term) and the exact one (considering the accrual term) are quite similar, indeed their difference in norm2 is negligible (respectively 0.000198 and 0.000051). We can say the same also for the survival probabilities and the intensities bootstrapped with the exact version and the JT approximation, indeed the order of magnitude of the difference is $10^{-4}$ (0.000470 and 0.000190).

We can also compare graphically in the figures below the probabilities and intensities:

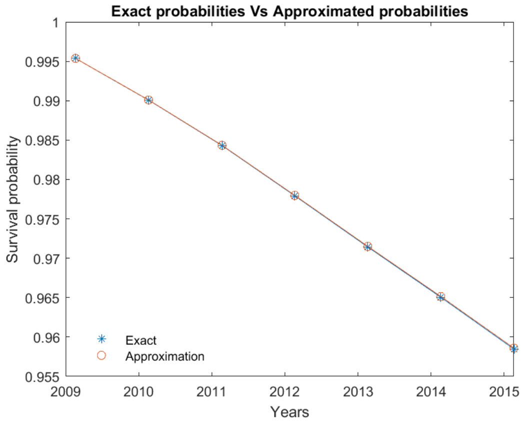


Figure 1: Survival probabilities

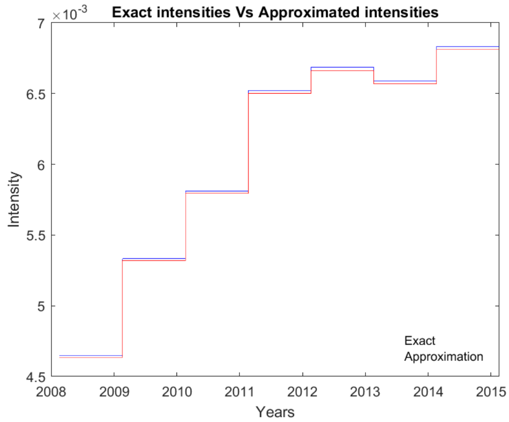


Figure 2: Intensities

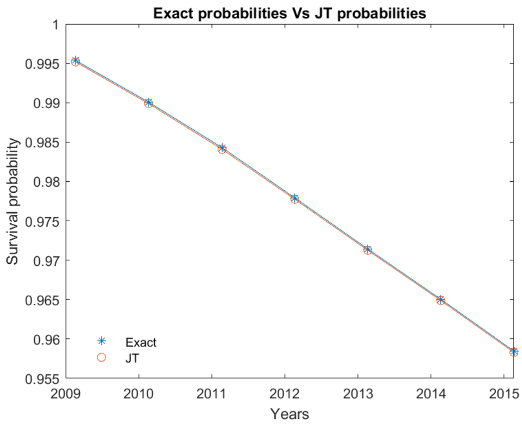


Figure 3: Survival probabilities

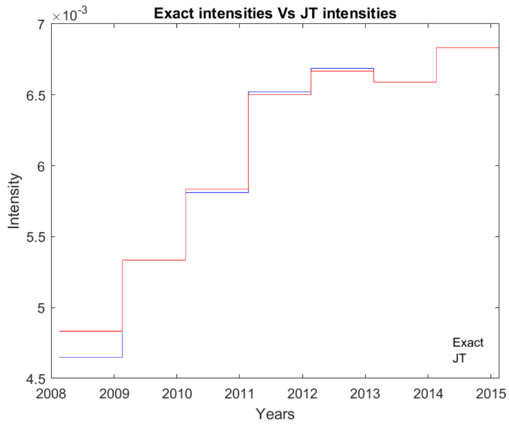


Figure 4: Intensities

# 3 Esercizio 3

As in previous exercise we started by creating a complete set of CDS spreads ans dates with spline interpolation, and obtained: Then we proceeded by computing the survival probabilities for the obligor UCG using the

| Dates | CDS spreads |
|---|---|
| 19-Feb-2009 | 34.00 bp |
| 19-Feb-2010 | 39.00 bp |
| 21-Feb-2011 | 45.00 bp |
| 20-Feb-2012 | 46.00 bp |
| 19-Feb-2013 | 47.00 bp |
| 19-Feb-2014 | 48.01 bp |
| 19-Feb-2015 | 47.00 bp |

approximation that neglect accrual term. We calculated the price of a First To Defaut (FTD), with UCG and ISP as observed obligors, via Li model considering a Gaussian copula. We started by computing N simulations of the standard Gaussian random variable, N simulations of the correlated Gaussian variable with zero mean and covariance matrix obtained via Cholesky factorization and, lastly, N simulations of the vector $u$ distributed according to the Gaussian copula.

At this point, we should invert the marginal survival probability functions to compute the default time $\tau$ for both obligors. However we do not have any analytical formula for the survival probability functions. For this reason we used a complete set of survival probabilities from 1yr up to 50yr both for ISP and UCG, obtained from the function "bootstrapCDS" written in the previous exercise, to operate an interpolation. So we computed the two $\tau$ for both ISP and UCG via reverse interpolation. Moreover, we evaluated the CDS spread for each simulations of the default time using the data of the obligor linked to the minimum default time. Finally, we computed the mean of all the CDS spreads previously obtained to define the MC approximation for the fair price (i.e. the fee) of the First To Default contract. The result we obtained is $FTD_{fee} = 85.45bp$ and the fee value standard deviation is 5.32%.

Then we plotted the First to default price w.r.t. different values of the correlation rho.

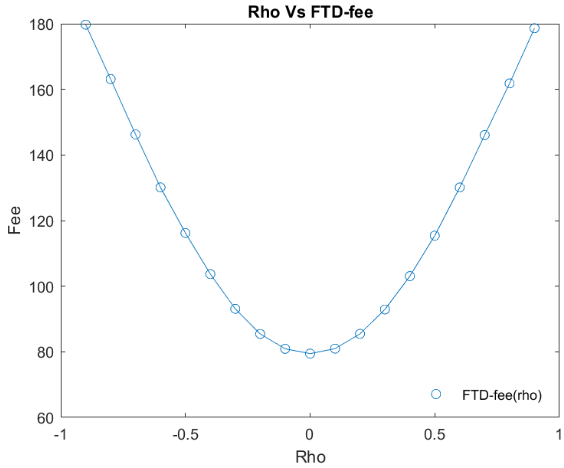


Figure 5: FTD Price

We observe that $FTD_{fee}$ increases as the absolute value of rho increases too, while $FTD_{fee}$ reaches its minimum value as rho = 0. So this plot demonstrates a trend where increasing rho leads to a higher probability of simultaneous defaults, resulting in higher prices for $FTD_{fee}$ and, it indicates that the correlation parameter significantly impacts the FPD price.