LabRM 1 - Group 14

Jacopo Raffaeli, Nicolò Randon, Giovanni Simone Riondato, Aurora Vido

1 Questions first part

1. Q1: Hazard rate piecewise curves

   We bootstrapped the ZC curve in order to obtain hazard rate curves for IG and HY issuers assuming piece-wise constant hazard rate. We started by computing the risk-free discounts from the ZC rates and the corresponding maturities. Since we were not provided with market data for the ZC risk-free bond with 1y and 1.5y maturity, we evaluated the missing ZC bond discount with linear interpolation. Then we used the risk-neutral pricing of a defaultable bond to find the hazard rate with the following formula:

   $\overline{C}(t,T) = \sum_{n=1}^{N} \left(\bar{c}_n \overline{B}(t,T_n)\right) + \overline{B}(t,T_N) + \pi \sum_{n=1}^{N} \left[P(t,T_{n-1}) - P(t,T_n)\right] B(t,T_n),$

   knowing that $\overline{B}(t,T) = B(t,T) \cdot P(t,T) = B(t,T) \cdot \exp(-h(t,T)(T-t))$.

   We obtained the following results:

	1y	2y
IG Hazard rate (bp)	58.4	53.7
HY Hazard rate (bp)	270.3	197.4

2. Q2: Z-spread (scalar)

   As in the first question we calculated the risk-free discounts from the ZC rates and the respective maturities, then we computed Z-spread by inverting the pricing function for the bond dirty price

   $\overline{C}(t,T) = \sum_{n=1}^{N} \left(\bar{c}_n \overline{B}(t,T_n)\right) + \overline{B}(t,T_N)$ and obtained the following Z-spreads:

	1y	2y
IG Z-spread (bp)	58.2	53.6
HY Z-spread (bp)	269.2	197.7

2 Rating transition matrix

Firstly we derived the default probabilities of IG and HY for 1y and 2y, for the last column of the matrix. Then we constructed a system of four equations and, since Q is a transition matrix, we know that each row must sum to 1 and so we came up with two equations. To complete the system of 4 equations we exploited Chapman Kolmogorov's theorem according to which the transition matrix between 0 and 2 years is equal to the matrix from 0 to 1 year squared, and so we came up with two more equations on the first two rows that must sum to 1. Finally we solved it numerically with matlab and obtained the following matrix:

$$Q = \begin{bmatrix} 0.71 & 0.28 & 0.01 \\ 0.09 & 0.88 & 0.03 \\ 0 & 0 & 1 \end{bmatrix}$$

We also tried to derive Q by solving a linear system, where P are the default probabilities of IG and HY, for both 1y and 2y. By subtracting these two probabilities, we computed the default probability between one year and two year.

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ P_{1y,IG} & P_{1y,HY} & 0 & 0 \\ 0 & 0 & P_{1y,IG} & P_{1y,HY} \end{bmatrix} \begin{pmatrix} q_{IG,IG} \\ q_{IG,HY} \\ q_{HY,IG} \\ q_{HY,HY} \end{pmatrix} = \begin{pmatrix} 1 - P_{1y,IG} \\ 1 - P_{1y,HY} \\ P_{2y,IG} - P_{1y,IG} \\ P_{2y,HY} - P_{1y,HY} \end{pmatrix}$$

which implies

$$Q = \begin{bmatrix} 0.7777 & 0.2193 & 0.0030 \\ 0.4179 & 0.5582 & 0.0239 \\ 0 & 0 & 1 \end{bmatrix}$$