Assignment6_AY23_24
Group 14

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Contents

1 Introduction

The case study involved the analysis and hedging of a structured bond issued by Bank XX on 16 February 2024 at 10:45 C.E.T. We began by bootstrapping market discounts for the issue date and then constructed a complete set of swap rates. Next, we determined the upfront pricing of the structured bond and computing spot volatilities. We calculated risk measures, such as Delta-bucket sensitivities and total Vega, to assess how the bond responded to changes in interest rates and volatilities. For portfolio risk management, we hedged the Delta risk with a selection of swaps and adjusted their notionals to minimize risk across specific time intervals. Additionally, we hedged Vega using an ATM 5-year cap and evaluated Vega-bucket sensitivities. Lastly, we considered course-grained buckets for Vega and implemented hedging strategies using 5-year and 15-year caps to manage bucketed Vega risk effectively.

2 Bootstrap

To find the market discounts up to February 16, 2024, since market data for swaps were partial, we initially created a complete set of swaps rates, with maturities from 2 years up to 50 years. To do this, we exploited the spline interpolation on mid rates, with business days as query points and the act/365 year fraction convention. Finally, we bootstrapped this complete set of rates to obtain all discounts we wanted.

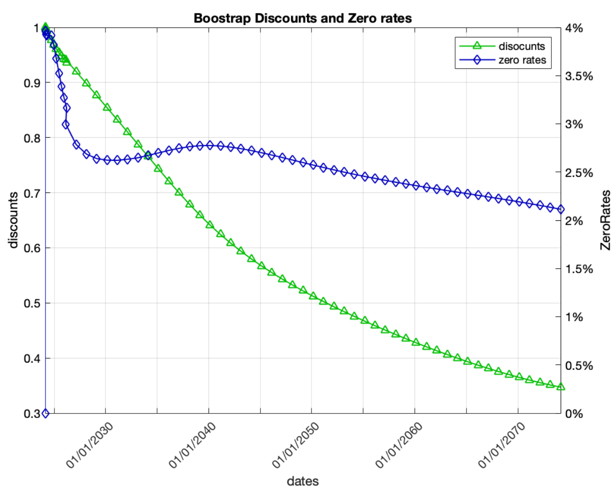


Figure 1: Bootstrap.

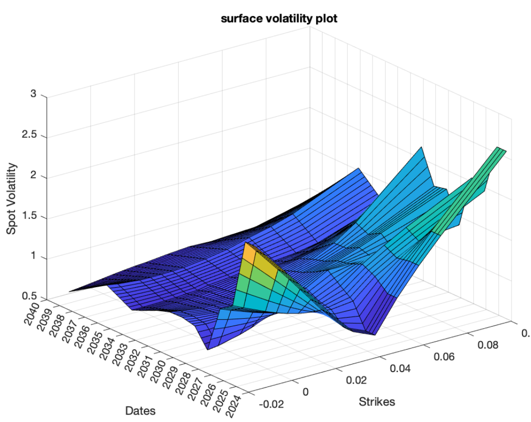
Note: The observed graph shows a descending zero rate curve, except for the initial portion. This behaviour may be due to high starting rates from products emerging in 2024, a time of elevated interest rates. This higher starting point suggests an expectation of declining future rates as the market anticipates rate normalization over the time.

3 Upfront

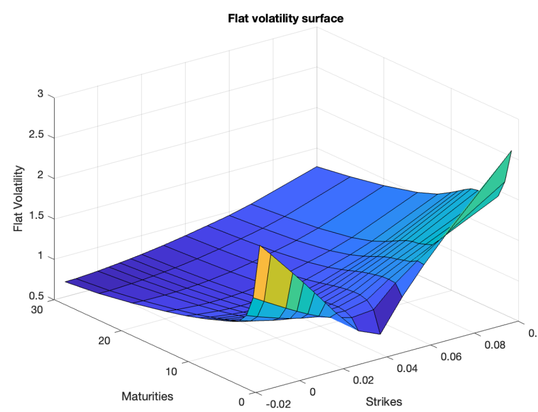
First thing to do is compute the caplet spot volatilities. We suppose all caplet flat volatilities under 1 year are equal to each other. By means of linear dependency and the definition of Cap in terms of the sum of caplets, we can find the volatilities for the following years. In other words, we followed an iterative procedure where at each step, the following system of equations has been solved:

$$\begin{cases} \Delta C = \sum_{j=t_j+1}^{t_{j+1}} \text{caplet}(T_j, \sigma_j) \\ \sigma_i = \sigma_{t_j} + \frac{T_i - T_{t_j}}{T_{t_{j+1}} - T_{t_j}} (\sigma_{t_{j+1}} - \sigma_{t_j}) \end{cases}$$

Here are the graphs illustrating both the flat volatility surface and the spot volatility surface:



(a) spot volatility surface



(b) flat volatility surface

Note: We can observe the volatility smile (volatility wrt strikes), which is very pronounced at T0, diminishes as time increases. Another notable observation is the distinction between the flat and spot volatility graphs: it is evident that the flat volatility, being inherently smoother by definition, maintains a more consistent pattern across the time horizon.

Subsequently, to find the upfront value, we simply calculated the Net Present Value of our cash flows, and put it equal to zero. The termsheet describes a swap agreement between Party A and Party B with a principal amount of 50 million EUR. Party A pays a quarterly interest rate of Euribor 3-month plus a fixed spread of 2.00%, while Party B pays a fixed coupon starting at 3% in the first quarter. After the first quarter, the coupon is based on Euribor 3-month plus 1.10%, capped at different rates depending on the time period (4.30% for the first five years, 4.60% for the next five years, and 5.10% for the remaining years). One notable simplification arises from the relationship between the payments of Party A and Party B. Since both parties' payments are based on the Euribor 3-month rate, these Euribor components can offset each other, effectively negating themselves. This simplifies the net payment flow between the parties, focusing the analysis on the fixed spreads and caps. We obtain:

$$X = s_{\text{spread}} \cdot \text{BPV} - s \cdot \text{BPV}_s + 1 - B(t_0, t_{\text{end}}) - 1 \cdot B(t_0, t_1) + B(t_0, t_{\text{end}}) + \sum_{i=1}^{4} \text{Cap}_i - c \cdot \delta_1 \cdot B(t_0, t_1)$$

where:

- s_{spread} = spol of the bank XX = 2%

- s = spol of the I.B. = 1.1%
- BPV = BPV starting from 3 months
- BPV_s = BPV starting from 6 months
- c = initial coupon
- Cap_i = price of the i^{th} Cap,
  where the four caps are the ones between 0-2 years, 2-5 years, 5-10 years, and 10-15 years.

We found X $= 14,66\%$

4 Delta-bucket sensitivities

To assess risk, we calculated the Delta-Bucket sensitivities, which are derived from DV01 computations. The function `delta_bucket_sens` shifts up deposits, futures and swaps by 1 basis point (one by one), subsequently computing the corresponding updated upfront following the same strategy explained above. DV01 is then determined as the difference between the new upfront and the upfront computed using spot volatilities as in the case study. (The difference between the two upfronts is equivalent to compute the difference between the NPV's).

These are the values of DV01 we obtained after the shift of 1bp:

	Dates	**DV01**
depos	21/02/24	6.881×10^{-10}
depos	27/02/24	1.933×10^{-6}
depos	20/03/24	-3.224×10^{-8}
depos	22/04/24	2.039×10^{-5}
futures	24/06/24	2.807×10^{-5}
futures	23/09/24	2.553×10^{-5}
futures	20/12/24	2.066×10^{-5}
futures	20/03/25	1.726×10^{-5}
futures	23/06/25	1.578×10^{-5}
futures	22/09/25	1.314×10^{-5}
futures	19/12/25	9.101×10^{-6}
swaps	20/02/25	-5.622×10^{-13}
swaps	20/02/26	-5.810×10^{-5}
swaps	22/02/27	-6.763×10^{-6}
swaps	21/02/28	1.892×10^{-6}
swaps	20/02/29	1.201×10^{-5}
swaps	20/02/30	5.124×10^{-6}
swaps	20/02/31	3.688×10^{-6}
swaps	20/02/32	4.990×10^{-6}
swaps	21/02/33	2.137×10^{-6}
swaps	21/02/34	2.147×10^{-5}
swaps	20/02/35	-6.888×10^{-6}
swaps	20/02/36	2.067×10^{-5}
swaps	21/02/39	1.214×10^{-4}
swaps	22/02/44	-5.035×10^{-6}
swaps	22/02/49	-1.858×10^{-5}
swaps	20/02/54	-6.193×10^{-6}
swaps	20/02/64	-6.876×10^{-5}
swaps	20/02/74	-2.641×10^{-5}

Note: We noticed that calculating a new upfront for each instrument is very time-consuming. With our computational availability, the calculation required 47.7 seconds.

5 Total Vega

To compute the total Vega, we simply shifted the whole flat volatilities matrix by 1 basis point to acquire a new spot volatility surface (through the function `Spot_volatility_surf`) and then we calculated a new upfront value. The total Vega is the result of the difference between the new upfront value and the old one.

$$\text{total vega} = 8.2196 \times 10^{-4}$$

6 Vega-bucket sensitivities

To compute Vega-bucket sensitivities, the strategy closely resembles the one used for delta bucket sensitivities. Through iterative for loops, we calculated new spot volatility surfaces, incrementing each volatility by 1 bp at a time. We then computed the corresponding new upfront values and determined the risk measure for each bucket as the difference between the new upfront and the previous one. To verify the accuracy of the calculations, we computed the sum of the Vega-bucket sensitivities and ensured it matched the total Vega. We obtained a result that is very close (8.4550×10^{-4}).

Note: As before, the computational time required was very high. The code for this passage ran in 322.6 seconds.

7 Course-grained buckets Delta Hedge

Initially, we developed the `course_grained_bucket_sens` function to compute coarse-grained buckets deltas for four distinct periods: 0-2 years, 2-5 years, 5-10 years and 10-15 years. In this function we shift rates according to some precise weights, as described by the theory. We simplified the weight calculations by using linear interpolation, which yields the same results. Then we compute the DV01's as described above. The results are the following:

DV01	value
2y	1.8957×10^{-4}
5y	4.5987×10^{-4}
10y	8.6039×10^{-4}
15y	5.7410×10^{-4}

Then we computed the course-grained buckets deltas for the swaps using the function `DV01_swap_cg` refining the weights of each swap based on their individual maturity dates. In conclusion, in order to hedge our portfolio delta, we computed the Swap notionals starting from the one with the longest maturity, exploiting the following equations:

$$\text{Notional}_{15Y} = -N \cdot \frac{DV01^{\text{CG}}_{10-15}}{DV01^{10-15}_{15Y}}$$

$$\text{Notional}_{10Y} = -\frac{N \cdot \text{DV01}^{\text{CG}}_{5-10} + \text{Notional}_{15Y} \cdot \text{DV01}^{5-10}_{15Y}}{\text{DV01}^{5-10}_{10Y}}$$

$$\text{Notional}_{5Y} = -\frac{N \cdot \text{DV01}^{\text{CG}}_{2-5} + \text{Notional}_{10Y} \cdot \text{DV01}^{2-5}_{10Y} + \text{Notional}_{15Y} \cdot \text{DV01}^{2-5}_{15Y}}{\text{DV01}^{2-5}_{5Y}}$$

$$\text{Notional}_{2Y} = -\frac{N \cdot \text{DV01}^{\text{CG}}_{0-2} + \text{Notional}_{5Y} \cdot \text{DV01}^{0-2}_{5Y} + \text{Notional}_{10Y} \cdot \text{DV01}^{0-2}_{10Y} + \text{Notional}_{15Y} \cdot \text{DV01}^{0-2}_{15Y}}{\text{DV01}^{0-2}_{2Y}}$$

Here are the notional results expressed in € (positive notionals mean long positions, negative notionals mean short positions):

Notionals	**value**
2y	1.9436×10^6
5y	1.6279×10^7
10y	-1.7960×10^7
15y	1.0982×10^7

8 Vega Hedge

We started by hedging the total Vega of the portfolio using an ATM 5y cap (strike = market euribor at 5 years). Then we considered the new portfolio made of the structured product plus the 5y cap and we hedged the delta applying exactly the same steps as before. The following formulas were used:

$$\text{Notional}_{15Y} = -N \cdot \frac{DV01^{\text{CG}}_{10-15}}{DV01^{10-15}_{15Y}}$$

$$\text{Notional}_{10Y} = -\frac{N \cdot \text{DV01}^{\text{CG}}_{5-10} + \text{Notional}_{15Y} \cdot \text{DV01}^{5-10}_{15Y}}{\text{DV01}^{5-10}_{10Y}}$$

$$\text{Notional}_{5Y} = -\frac{N \cdot \text{DV01}^{\text{CG}}_{2-5} + \text{Notional}_{10Y} \cdot \text{DV01}^{2-5}_{10Y} + \text{Notional}_{15Y} \cdot \text{DV01}^{2-5}_{15Y} + \text{Notional}_{\text{cap\_5y}} \cdot \text{delta}_{\text{cap\_5y\_5y}}}{\text{DV01}^{2-5}_{5Y}}$$

$$\text{Notional}_{2Y} = -\frac{N \cdot \text{DV01}^{\text{CG}}_{0-2} + \text{Notional}_{5Y} \cdot \text{DV01}^{0-2}_{5Y} + \text{Notional}_{10Y} \cdot \text{DV01}^{0-2}_{10Y} + \text{Notional}_{15Y} \cdot \text{DV01}^{0-2}_{15Y} + \text{Notional}_{\text{cap\_5y}} \cdot \text{delta}_{\text{cap\_5y\_2y}}}{\text{DV01}^{0-2}_{2Y}}$$

Here are the notional results expressed in € (positive notionals mean long positions, negative notionals mean short positions):

Notionals	**value**
2y	-2.8862×10^8
5y	1.5225×10^8
10y	-1.7960×10^7
15y	1.0982×10^7

It's noticeable how the 10-year and 15-year notionals remain unchanged from the previous point. This is because we're hedging the delta with a product expiring in 5 years, so its effect is null after 5 years. However, the notionals at 2 and 5 years do change.

9 Course-grained buckets Vega Hedge

We hedged the Vega approximately using the same approach as before, but with a different set of buckets. Indeed, here we considered the 0-5 years and 5-15 years buckets. Therefore, in the function `course_bucket_vega` we computed a new spot volatility surface exploiting a different collection of weights, and hedged the vega with the new flows available. The formulas we derived for the notionals are as follows:

$$\text{Notional}_{15Y} = -\frac{N \cdot \text{DV01\_vega}^{\text{CG}}_{5-15}}{\text{Vega\_cap}_{15Y}}$$

$$\text{Notional}_{5Y} = -\frac{N \cdot \text{DV01\_vega}^{\text{CG}}_{0-5} + \text{Notional}_{15Y} \cdot \text{Vega\_cap}_{15Y}}{\text{Vega\_cap}_{5Y}}$$

Here are the notional results expressed in € (positive notionals mean long positions, negative notionals mean short positions):

Notionals	**value**
5y	-4.1778×10^7
15y	-1.8193×10^7

10 Notes & Issues

During the process of analyzing and hedging the structured bond, we encountered a few issues that we would like to highlight:

1. Flat Volatility Surface Misinterpretation: The flat volatility surface included a line with volatilities at 18 months, which we were instructed not to consider. However, this note was overlooked, and these volatilities were included in our calculations. This oversight resulted in the impossibility of making adjustments later on, as the size of the volatility matrix required changes in several parts of the code. Despite this, the final results should remain relatively consistent.

2. Time-Consuming Spot Volatility Bootstrap: The bootstrap of spot volatilities required a significant amount of time due to our decision to calibrate on 60 dates and then bootstrap on the same 60 dates. This approach entailed solving the same four equations (the one concerning Cap computations with flat versus spot volatilities plus three linear constraints) 60 times, resulting in substantial computation time. A more efficient method could involve calibrating on 15 dates and then bootstrapping the necessary 60 dates, thereby reducing the computational burden and improving efficiency.