OPTION PRICING

ASSIGNMENT 1 – GROUP 14

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1. **Option pricing:**
   We priced a European Call Option via Black Formula, Cox-Ross-Rubinstein (CRR) and Monte Carlo (MC) methods with the following characteristics:

   Strike price: 1 Euro
   Value date: 15th of February 2008
   Time-to-maturity (TTM): 3 month (consider a yearfrac 1/4)
   Volatility: 22% (per year)
   TTM-zero-rate: 3%
   Underlying: equity stock
   Dividend Yield: 6%
   Settlement: physical delivery
   Number of contracts: 1 Mln
   Underlying price: 1 Euro

   Disclaimer:
   The data seems to contain the zero-rate expressed already on a 3 months basis, however in the Matlab file "runAssign_TBM.m" it is written that the B = exp(-r*T), it looks like the rate r is discounted another time since it is multiplied by T = 0.25. In our opinion the right formula should be B = exp(-r), however we decided to follow the instructions given to us and use B = exp(-r*T), clearly, this may affect the result obtained.

   We used the given function for the closed formula and implemented those for CRR and MC methods to price the option at maturity.
   For MC we generated a random vector g distributed as a standard normal random variable, we evaluated the payoff for every g and computed the price by discounting the sample mean of the payoff vector.
   In the case of CRR, we calculated the option's payoff at the terminal branches of the tree, then proceeded to evaluate the payoff at each preceding node using the expected value formula, decreasing the number of steps at each iteration until we arrived at the initial node.

   We obtained the following results for one single contract:

   Black_price = 0.0398 Eur
   CRR_price = 0.0398 Eur
   MC_price = 0.0393 Eur

   Since the Notional = 1 Mln we got:
   Black_price = 39800 Eur
   CRR_price = 39800 Eur
   MC_price = 39300 Eur

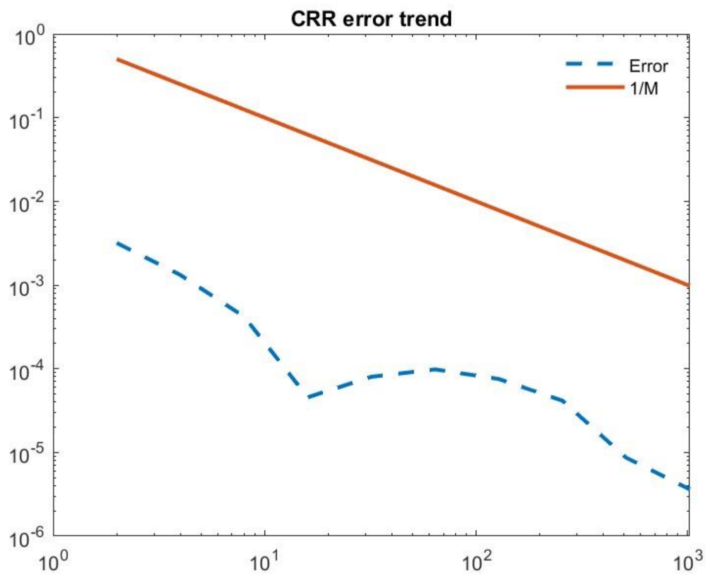
2. **CRR and MC convergence:**

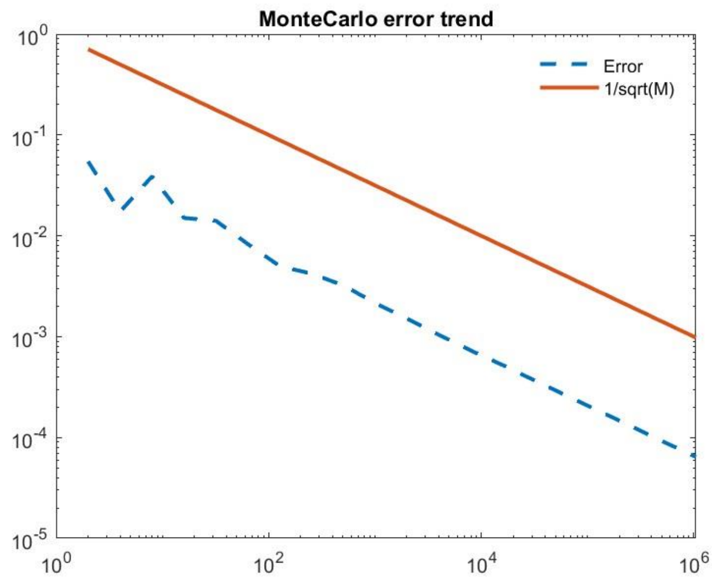
As error for the CRR we considered the difference in absolute value of the payoff of the call obtained with the CRR method and the exact value of the payoff obtained through the closed formula.
For Monte Carlo method we have calculated the standard error based on the estimation of the unbiased standard deviation.
The errors we obtained shows that CRR converges in 15 time steps, while MC converges in about 430000 simulations. So, Cox-Ross-Rubinstein seems to be more precise than Monte Carlo.

3. **Errors:**
We observed that the error in both cases is lower than the basis point (10e-4), therefore we consider the previous iterations correct.

CRR Error = 9.207460e-05
Monte Carlo Error = 9.985436e-05

In order to plot the error for CRR and MC, we implemented two functions which plot the errors varing, respectively, the number of steps or the number of simulations N.
By plotting the error on a loglog scale, with a different number N of time steps or simulations, we can show that the error varies as 1/M for Cox-Ross-Rubinstein and as 1/sqrt(M) for Monte Carlo.





We can see that, for smaller values of -log(sqrt(M)) in MC, the curve oscillates. Hence, for a small number of iterations the method is instable. For CRR, approximately, the trend varies as 1/M, and achieves its optimal approximation in about 15 time steps.

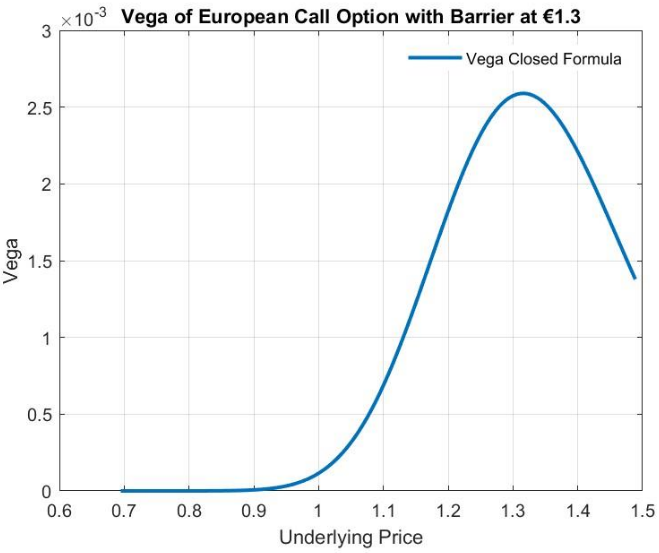
4. **Barrier pricing:**
Now consider a European Call Option with European barrier at KI=1.3 Euros (Up&In), with same feautures of the original option.
We evaluated the barrier with the two numerical techniques (CRR and MC) and with Black formula but with strike KI.
If the barrier is crossed, the option becomes active, and its payoff is calculated as max(F-K,0). Finally, we discounted the payoff to present value to obtain the option price.

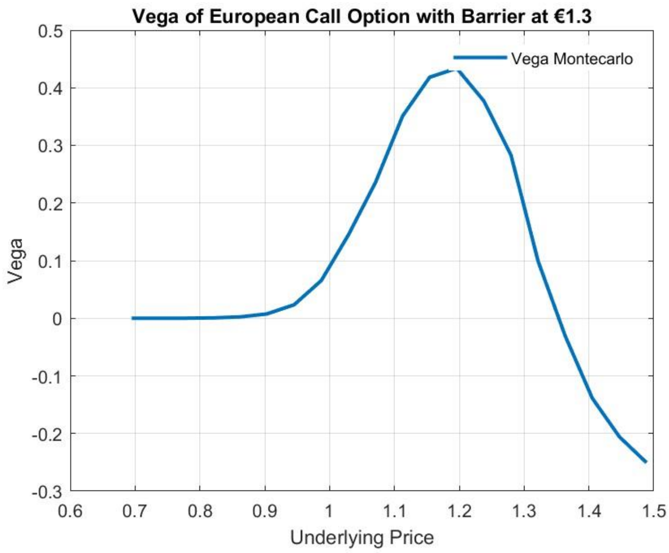
We obtained the following results for one single contract, which are very closed to each other:

CRR_price_Barrier = 0.0022 Eur
MC_price_Barrier = 0.0023 Eur
Closed_price_Barrier = 0.0021 Eur

The Notional = 1Mln so we got:
CRR_price_Barrier = 2200 Eur
MC_price_Barrier = 2300 Eur
Closed_price_Barrier = 2100 Eur

5. **Vega for the barrier option:**
To evaluate the Vega we used the closed analytical formula and the numerical estimate via Monte Carlo where we evaluated the first derivative with the finite differences method.
Then we plotted Vega and obtained the following curves:





The curve of the Vega calculated with finite differences method is accurate by comparison with the one of the Closed formula. We can observe that the Vega reaches its peak at the strike price.

6. **Antithetic**

In the Antithetic variables technique, in order to speed up the computation, we consider the vector of N observations g, as in the classic MC, and also the vector of the observations changed in sign, -g. In this way, by N trials, we obtain 2N observations. Since the standard normal distribution is symmetric in 0 we are sure that also the vector -g contains valid values for our purpose. In a second moment we compute the vector of the final value of the underlying asset by the well know formula both with the vector g and -g, for every i we compute the average of the final values obtained with g_i and -g_i and we use these values to compute the final payoff and the standard deviation. In the end we can observe that this second method need a lot less simulations to obtain the desired precision.

Number of simulation (MC classic) needed to achieve the desired accuracy ~ 430100.

Number of simulation (MC antithetic) needed to achieve the desired accuracy ~ 2040 .

7. Bermudan

We priced the Bermudan option with CRR method, where the holder has the right to exercise the option at the end of every month. Thus we obtained that the price option is 0.0391 Eur.

We expected the Bermudan option to get a higher value of the classic European one since the Bermudan offer more privileges than the European one, probably the computation used contains some error.