

Relativistic Non-Ideal Flows

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The Schwarzschild metric

Flat metric:

$$ds^2 = -dt^2 + dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi).$$

Schwarzschild metric:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{1}{1 - \frac{2M}{r}} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\varphi),$$

where $c = G = 1$.

The Schwarzschild metric

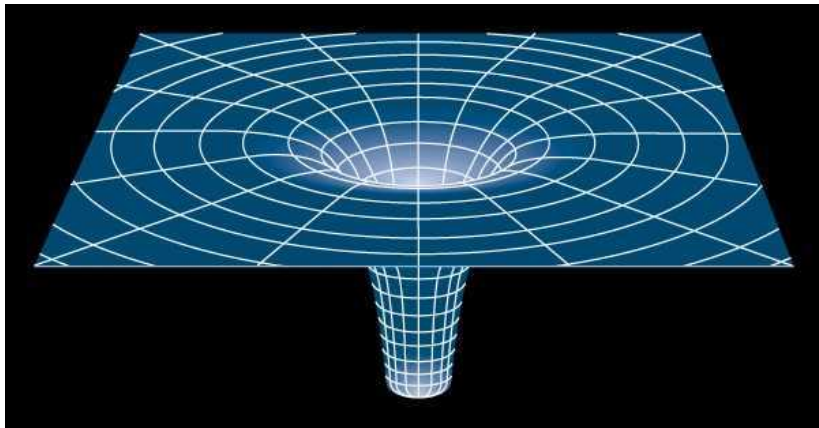


Image credit: <https://www.physicsforums.com/insights/schwarzschild-metric-part-1-gps-satellites/>.

The stress-energy tensor

The component $T^{\mu\nu}$ is the flux of μ -th component of the four-momentum p^μ through a surface of constant coordinate x^ν .
For an ideal fluid ($\eta = \xi = \kappa = 0$) in the Local Rest Frame:

$$T_{\text{ideal fluid}}^{\mu\nu} = \begin{bmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{bmatrix}_{\text{fid}},$$

where $\rho = \rho_0(1 + \varepsilon)$.

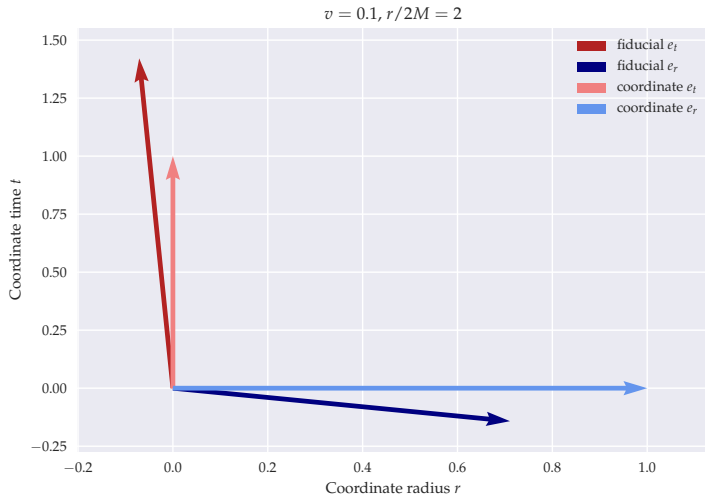
The Local Rest Frame

A set of vectors $V_{(\alpha)}^\mu$ such that

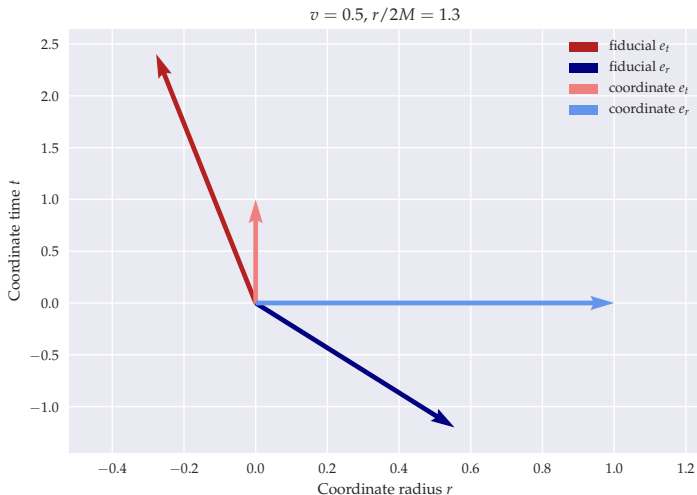
$$g_{\mu\nu} V_{(\alpha)}^\mu V_{(\beta)}^\nu = \eta_{(\alpha)(\beta)} .$$

If we assume spherical symmetry and stationarity, we can determine the whole tetrad if we have the normalized radius $r/2M$ and the velocity v .

The Local Rest Frame



The Local Rest Frame



The radiation moments

$$w_0 = \int I \, d\Omega$$

radiation energy density

$$w_1 = \int I \cos \theta \, d\Omega$$

radiation energy flux

$$w_2 = \int I \left(\cos^2 \theta - \frac{1}{3} \right) d\Omega$$

radiation shear stress.

TODO: add image.

The full stress-energy tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal fluid}} + \begin{bmatrix} w_0 & w_1 & 0 & 0 \\ w_1 & \frac{1}{3}w_0 + w_2 & 0 & 0 \\ 0 & 0 & \frac{1}{3}w_0 - \frac{1}{2}w_2 & \\ 0 & 0 & & \frac{1}{3}w_0 - \frac{1}{2}w_2 \end{bmatrix}_{\text{fid}}$$

The equations to solve are:

$$\nabla_\mu T^{\mu\nu} = 0 \quad 2 \text{ equations}$$

$$\nabla_\mu (\rho_0 u^\mu) = 0 \quad 1 \text{ equation}$$

conservation of photon number 2 equations.

Singularities

There is a singularity in the Euler equation at $v = v_s$:

$$(v^2 - v_s^2) \frac{(yv)'}{yv} - 2v_s^2 + \frac{M}{y^2 r} = -\frac{r}{yv(\rho + \rho)} ((\Gamma - 1)s_0 + vs_1) .$$

Also, since we assume $w_2 = f(\tau)w_0$, there is another singularity at the zeroes of:

$$v^2 - vf(\tau) - \frac{1}{3}$$

Accretion efficiency

The Eddington luminosity is attained when the radiation pressure on a test hydrogen atom equals the gravitational pull on it. We have approximately:

$$\frac{L_{\text{Edd}}}{M} = \frac{4\pi c G m_p}{\sigma_T} \approx 3.27 \times 10^4 \frac{L_{\odot}}{M_{\odot}}.$$

From the fit, fixing \dot{M} , we get $L = 4\pi r^2 w_1$.

We look at $l = L/L_{\text{Edd}}$ in terms of $\dot{m} = \dot{M}c^2/L_{\text{Edd}}$.

Accretion efficiency

