Relativistic Non-Ideal Flows

Jacopo Tissino

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The Schwarzschild metric

Flat metric:

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi\right).$$

Schwarzschild metric:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \frac{1}{1 - \frac{2M}{r}}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi\right),$$

where $c = G = 1$.

The Schwarzschild metric

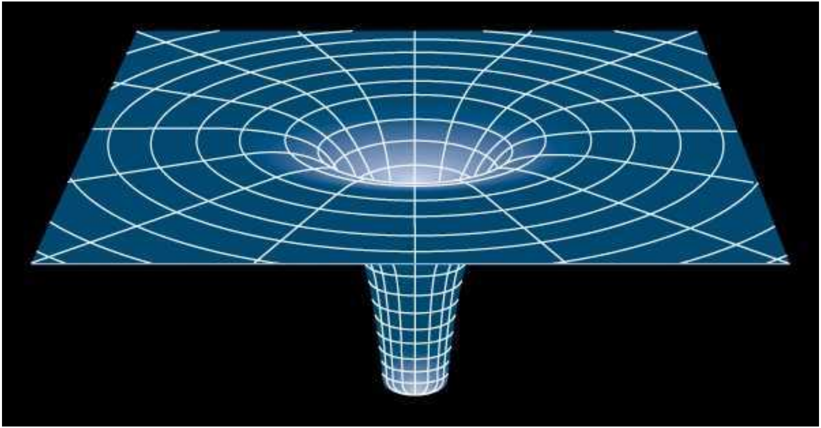

Image credit: `https://www.physicsforums.com/insights/schwarzschild-metric-part-1-gps-satellites/`.

The stress-energy tensor

The component $T^{\mu\nu}$ is the flux of μ-th component of the four-momentum p^{μ} through a surface of constant coordinate x^{ν}. For an ideal fluid ($\eta = \xi = \kappa = 0$) in the Local Rest Frame:

$$T^{\mu\nu}_{\text{ideal fluid}} = \begin{bmatrix} \rho & & & \\ & p & & \\ & & p & \\ & & & p \end{bmatrix}_{\text{fid}} ,$$

where $\rho = \rho_0(1 + \varepsilon)$

The Local Rest Frame

A set of vectors $V^{\mu}_{(\alpha)}$ such that

$$g_{\mu\nu} V^{\mu}_{(\alpha)} V^{\nu}_{(\beta)} = \eta_{(\alpha)(\beta)} \, .$$

We assume spherical symmetry and stationarity.
TODO: vector visualization.

The radiation moments

$$w_0 = \int I \,\mathrm{d}\Omega \qquad \text{radiation energy density}$$

$$w_1 = \int I \cos\theta \,\mathrm{d}\Omega \qquad \text{radiation energy flux}$$

$$w_2 = \int I \left(\cos^2\theta - \frac{1}{3}\right) \mathrm{d}\Omega \qquad \text{radiation shear stress.}$$

TODO: add image.

The full stress-energy tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal fluid}} + \begin{bmatrix} w_0 & w_1 & 0 & 0 \\ w_1 & \frac{1}{3}w_0 + w_2 & 0 & 0 \\ 0 & 0 & \frac{1}{3}w_0 - \frac{1}{2}w_2 & \\ 0 & 0 & & \frac{1}{3}w_0 - \frac{1}{2}w_2 \end{bmatrix}_{\text{fid}}$$

The equations to solve are:

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= 0 && \text{2 equations} \\ \nabla_\mu (\rho_0 u^\mu) &= 0 && \text{1 equation} \\ \text{conservation of photon number} & && \text{2 equations.} \end{aligned}$$

Singularities

The Euler equation becomes:

$$(v^2 - v_s^2)\frac{(yv)'}{yv} - 2v_s^2 + \frac{M}{y^2 r} = -\frac{r}{yv(p+\rho)}\left((\Gamma - 1)s_0 + vs_1\right)$$

TODO: include singularity with optical thickness dependence?

Accretion efficiency

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