Relativistic Non-Ideal Flows

Laureando: Jacopo Tissino
Relatore: prof. Roberto Turolla

24/09/2019

The Schwarzschild metric

The Schwarzschild metric is given by:

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 + \frac{1}{1 - \frac{2M}{r}}\,\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi\right),$$

while the flat metric is:

$$\mathrm{d}s^2 = -\,\mathrm{d}t^2 + \mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\varphi\right).$$

where $c = G = 1$.

Tetrads: the Local Rest Frame

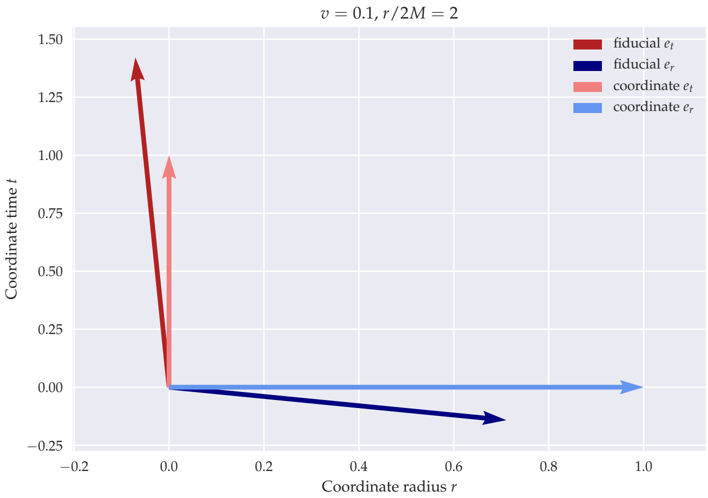
The *comoving* reference frame for spherically symmetric accretion is:

$$\begin{aligned}
\hat{e}_t &= u^\mu \\
\hat{e}_r &= a^\mu / \sqrt{a^\nu a_\nu} \\
\hat{e}_\theta &= (0, 0, 1/r, 0) \\
\hat{e}_t &= \left(0, 0, 0, 1/(r \sin\theta)\right) .
\end{aligned}$$

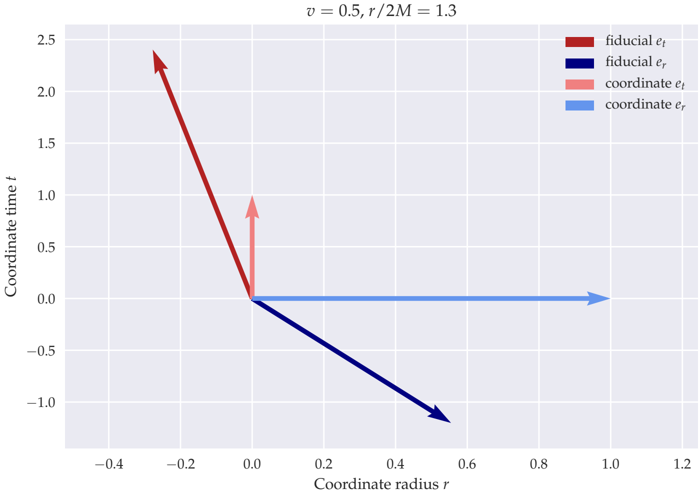
This basis satisfies:

$$g_{\mu\nu} \hat{e}^\mu_{(\alpha)} \hat{e}^\nu_{(\beta)} = \eta_{(\alpha)(\beta)} .$$

The Local Rest Frame



The Local Rest Frame



The stress-energy tensor

The component $T^{\mu\nu}$ is the flux of μ-th component of the four-momentum p^μ through a surface of constant coordinate x^ν. For an ideal fluid ($\eta = \xi = \kappa = 0$) in the Local Rest Frame:

$$T^{\mu\nu}_{\text{ideal fluid}} = \begin{bmatrix} \rho & & & 0 \\ & p & & \\ & & p & \\ 0 & & & p \end{bmatrix}_{\text{fid}} ,$$

where $\rho = \rho_0(1 + \varepsilon)$.

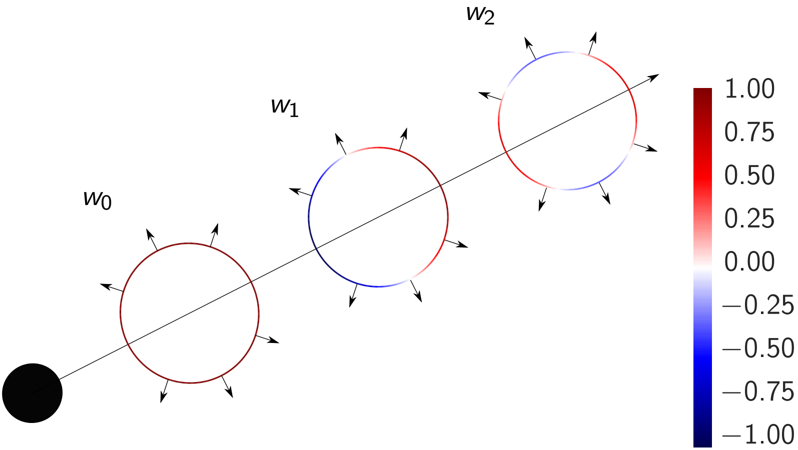
The radiation moments

$$w_0 = \int I \,\mathrm{d}\Omega \qquad \text{radiation energy density}$$

$$w_1 = \int I \cos\theta \,\mathrm{d}\Omega \qquad \text{radiation energy flux}$$

$$w_2 = \int I \left(\cos^2\theta - \frac{1}{3}\right) \mathrm{d}\Omega \qquad \text{radiation shear stress.}$$

The radiation moments



The full stress-energy tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\text{ideal fluid}} + \begin{bmatrix} w_0 & w_1 & 0 & 0 \\ w_1 & \frac{1}{3}w_0 + w_2 & 0 & 0 \\ 0 & 0 & \frac{1}{3}w_0 - \frac{1}{2}w_2 & 0 \\ 0 & 0 & 0 & \frac{1}{3}w_0 - \frac{1}{2}w_2 \end{bmatrix}_{\text{fid}}$$

The equations to solve are:

$$\begin{aligned} \nabla_\mu T^{\mu\nu} &= 0 && \text{2 equations} \\ \nabla_\mu \left(\rho_0 u^\mu\right) &= 0 && \text{1 equation} \\ \text{moments of the photon transfer equation} & && \text{2 equations.} \end{aligned}$$

Critical point

There is a critical point in the Euler equation at $v = v_s$:

$$(v^2 - v_s^2)\frac{(yv)'}{yv} - 2v_s^2 + \frac{M}{y^2 r} = -\frac{r}{yv(p+\rho)}\left((\Gamma - 1)s_0 + v s_1\right).$$

where:

- y is the energy at infinity per unit rest mass,
- v is the velocity of the fluid,
- $v_s = \sqrt{\left(\partial p / \partial \rho\right)_s}$ is the adiabatic speed of sound,
- s_0 and s_1 are the source moments,
- Γ is the local adiabatic exponent.

Accretion efficiency

The Eddington luminosity is attained when the radiation pressure on a test electron-proton pair equals the gravitational pull on it:

$$\frac{L_{\mathrm{Edd}}}{M} = \frac{4\pi c G m_p}{\sigma_T} \approx 3.27 \times 10^4 \frac{L_\odot}{M_\odot}\,.$$

In the numerical simulations we fix the accretion rate $\dot{M}$ and get $L = 4\pi r^2 w_1$.

We can adimensionalize these with the Eddington luminosity L_{Edd} and accretion rate $\dot{M}_{\mathrm{Edd}} = L_{\mathrm{Edd}}/c^2$: we define $l = L/L_{\mathrm{Edd}}$ and $\dot{m} = \dot{M}/\dot{M}_{\mathrm{Edd}}$.

Accretion efficiency

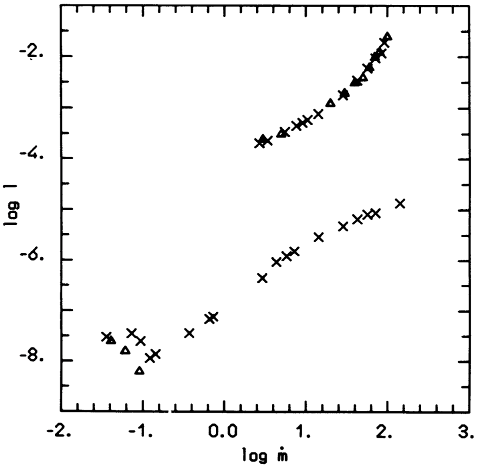


Image credit: L. Nobili, R. Turolla, L. Zampieri. In: *ApJ* 383 (Dec. 1991).