Notes on Calculus I

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October 3, 2016

Chapter 1

Naïve set theory

1.1 Basic sets

$$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R}, \qquad 0 \notin \mathbb{N} \tag{1.1.1}$$

$$\mathbb{R}^+ := \{x \in \mathbb{R} : x > 0\} \tag{1.1.2}$$

Remarks

- "$\in$" is for elements belonging to sets, "$\subseteq$" is for subsets
- $\{x\} \neq x$: the first is a set with x as its only element, and is called a "singlet"
- $\subsetneq$ means "is a subset of, but not equal to"
- the elements of $\mathcal{P}(A)$ are precisely all the subsets of A
- $\sharp A$ is the cardinality of A
- $\sharp\mathcal{P}(A) = 2^{\sharp A}$

The naïve definitions of $A \cup B$, $A \cap B$, $A \smallsetminus B$ are given.

Properties

- $A = (A \cap B) \cup (A \smallsetminus B)$
- $(A \cap B) \cap (A \smallsetminus B) = \emptyset$
- $C \cap (A \cup B) = (C \cap A) \cup (C \cap B)$
- $C \cup (A \cap B) = (C \cup A) \cap (C \cup B)$

Complement

Definition 1.1.1. With respect to a "universe" set U, we define the complement of A as $U \setminus A$, denoted A^C.

The following hold:

- $(A \cup B)^C = A^C \cap B^C$
- $(A \cap B)^C = A^C \cup B^C$

Cartesian product

Definition 1.1.2. An *ordered pair* is a set of the form $\{\{x\}, \{x, y\}\}$, denoted (x, y) (where order matters).

Definition 1.1.3. We define the *cartesian product* $A \times B$ of two sets A and B as:

$$A \times B := \big\{(a, b) : a \in A, b \in B\big\} \tag{1.1.3}$$

1.2 Propositional logic

Implication

Definition 1.2.1.

$$p \implies q \iff (\neg p) \vee q \tag{1.2.1}$$

Double implication

Definition 1.2.2.

$$(p \iff q) \iff (p \implies q \wedge q \implies p) \tag{1.2.2}$$

Quantifiers $P(x)$ is a *predicate*. We say that $\forall x : P(x)$ if $P(x)$ is true independently of x, and that

$$\exists x : P(x) \iff \neg(\forall x : \neg P(x)) \tag{1.2.3}$$

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