Machine Learning for Gravitational Waveforms from Binary Neutron Star mergers

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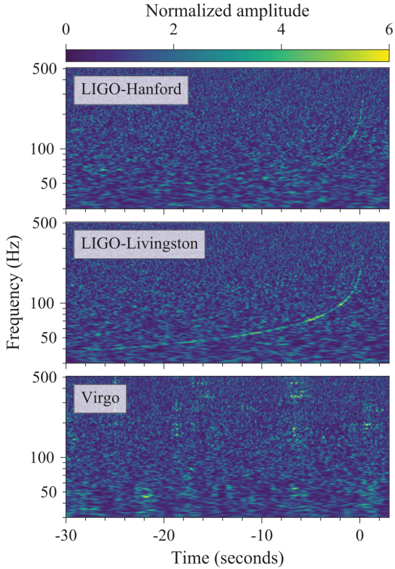
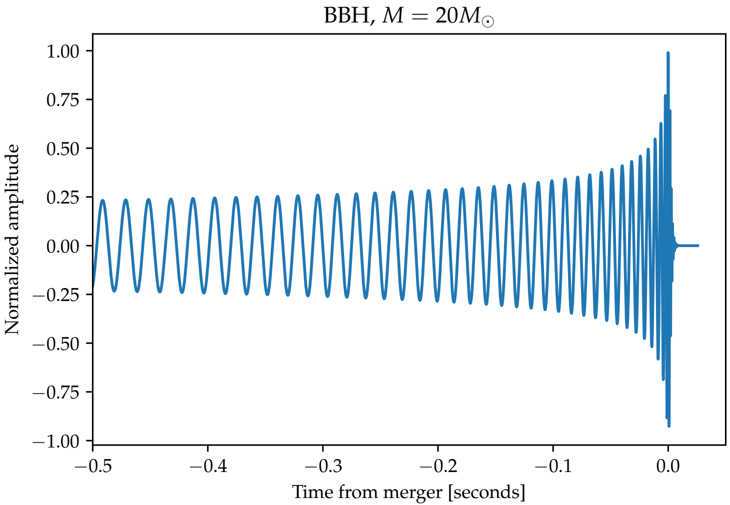
GW170817: the first BNS merger detection

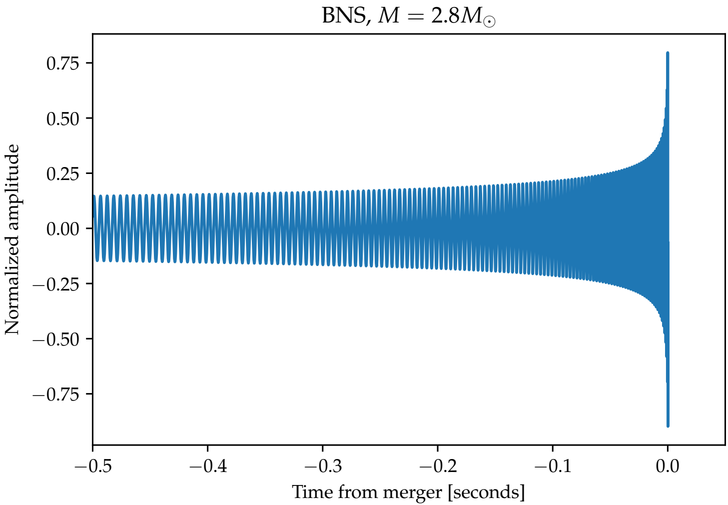


Time-frequency representation of the chirping waveform (Abbott et al. 2017).

BBH merger waveform



BNS merger waveform



The basics of GW data analysis

The signal is modelled as $s(t) = h_\theta(t) + n(t)$, where:

- the noise $n(t)$ is taken to be stationary, with zero mean, and Gaussian with power spectral density $S_n(f)$;
- the signal $h_\theta(t)$ from a binary neutron star merger depends on:
  - intrinsic parameters: mass ratio $q = m_1/m_2$, spins $\vec{\chi}_1$ and $\vec{\chi}_2$, tidal polarizabilities Λ_1 and Λ_2;
  - extrinsic parameters: total mass $M = m_1 + m_2$, luminosity distance D_L, inclination ι...

The Wiener distance

The likelihood used in parameter estimation reads (Maggiore 2007):

$$\Lambda(s|\theta) \propto \exp\left((h_\theta|s) - \frac{1}{2}(h_\theta|h_\theta) \right) , \tag{1}$$

where $(a|b)$ is the Wiener product:

$$(a|b) = 4\,\mathrm{Re} \int_0^\infty \frac{\widetilde{a}^*(f)\widetilde{b}(f)}{S_n(f)}\,\mathrm{d}f \ . \tag{2}$$

Models and accuracy

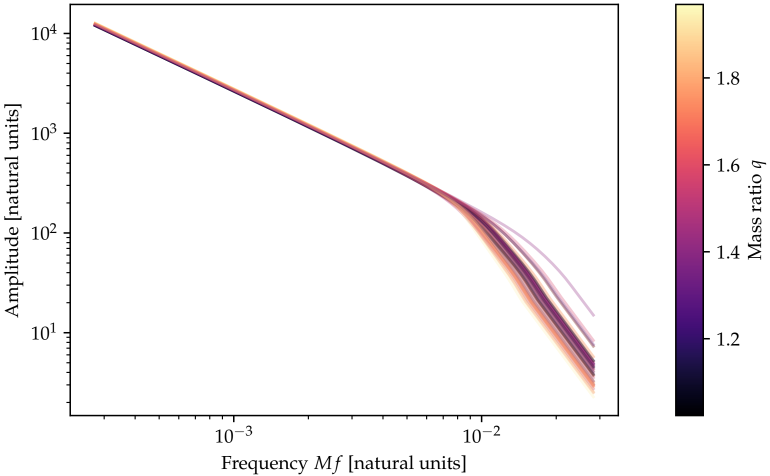
The main strategies for the generation of theoretical waveforms are:

- numerical relativity;
- effective one body;
- post-Newtonian.

Their accuracy is computed according to the **mismatch**

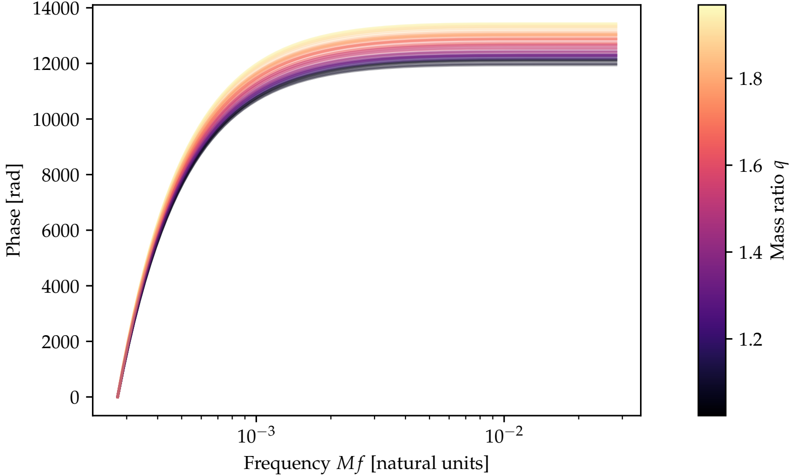
$$\mathcal{F}[h_1, h_2] = 1 - \max_{t_0,\phi_0} \underbrace{\frac{(h_1|h_2(t_0,\phi_0))}{\sqrt{(h_1|h_1)(h_2|h_2)}}}_{\cos\theta} . \qquad (3)$$

Amplitudes



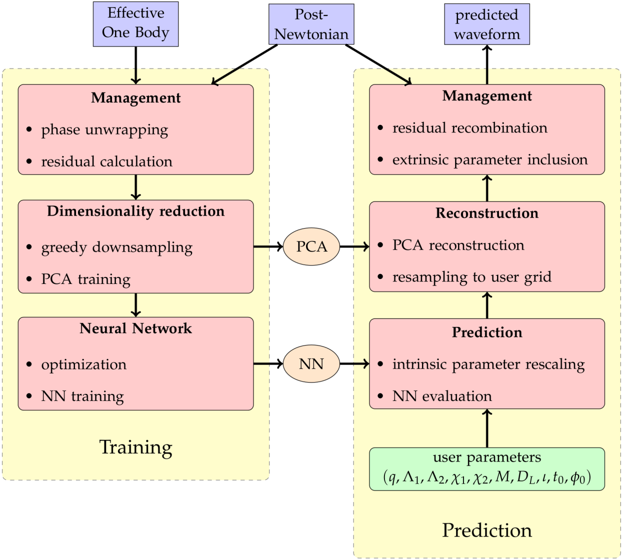
10^4
10^3
10^2
10^1
Amplitude [natural units]
10^{-3}
10^{-2}
Frequency Mf [natural units]
1.8
1.6
1.4
1.2
Mass ratio q

Phases

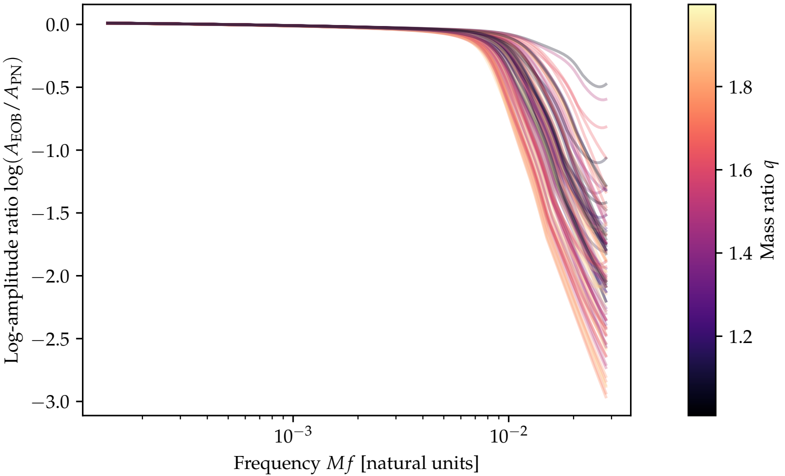


14000
12000
10000
8000
6000
4000
2000
0
Phase [rad]
10^{-3}
10^{-2}
Frequency Mf [natural units]
1.8
1.6
1.4
1.2
Mass ratio q

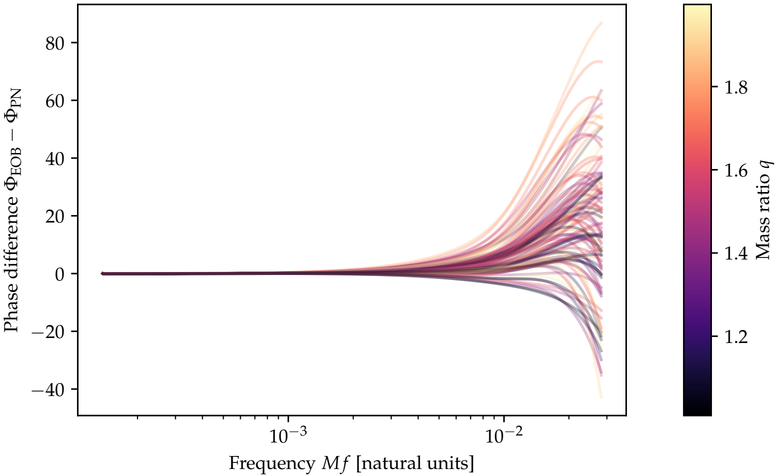
mlgw_bns structure



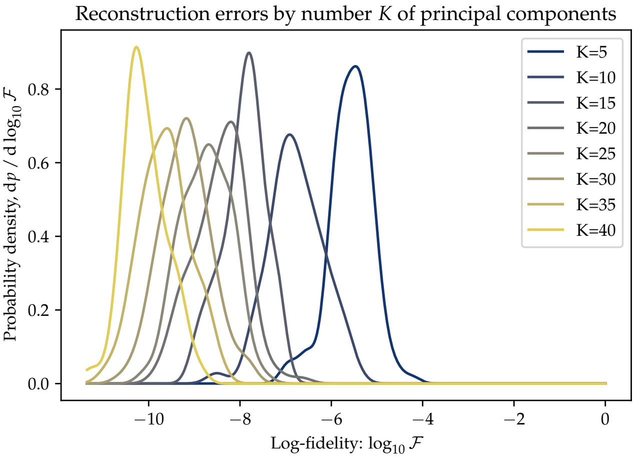
Residuals: amplitude



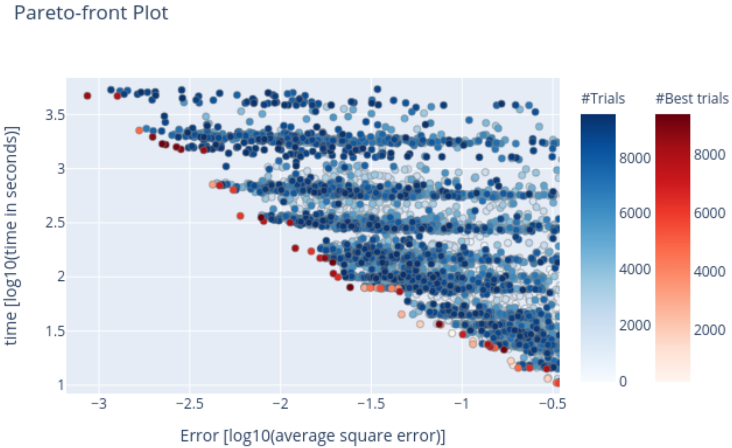
Residuals: phase



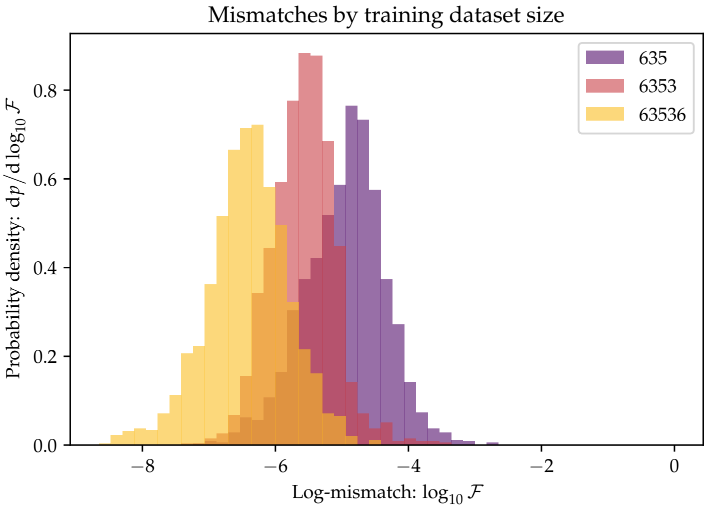
PCA mismatches



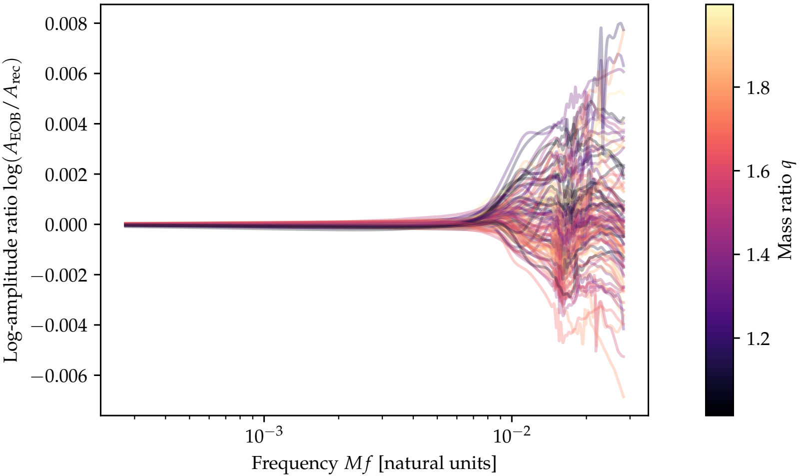
Hyperparameter optimization



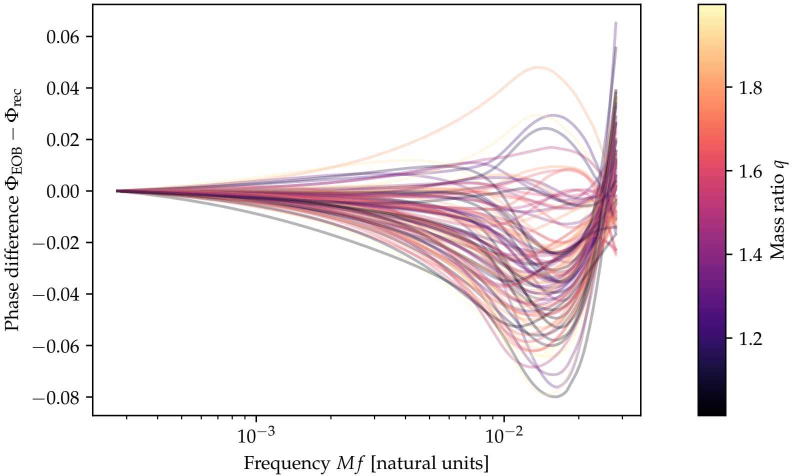
Fidelity


Mismatches by training dataset size
635
6353
63536
Probability density: $\mathrm{d}p/\mathrm{d}\log_{10}\mathcal{F}$
0.0
0.2
0.4
0.6
0.8
−8
−6
−4
−2
0
Log-mismatch: $\log_{10}\mathcal{F}$

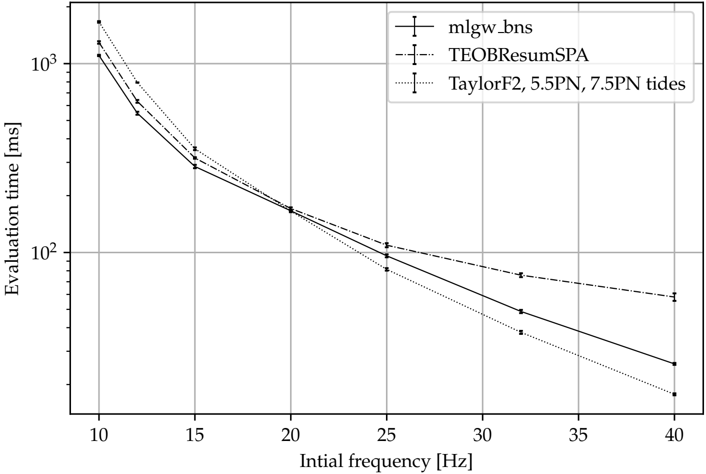
Amplitude reconstruction residuals



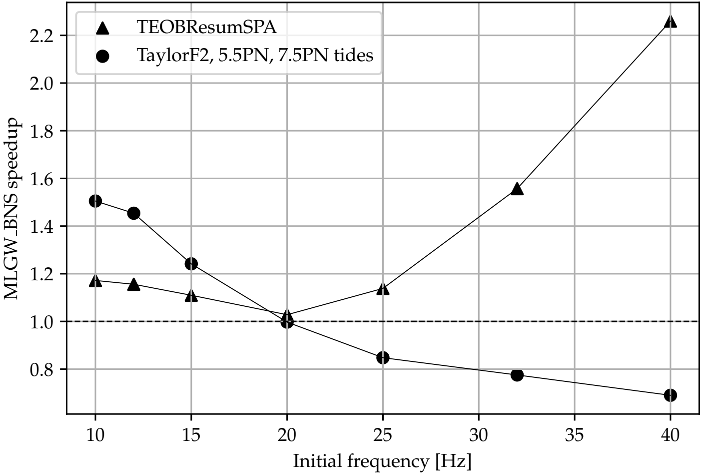
Phase reconstruction residuals



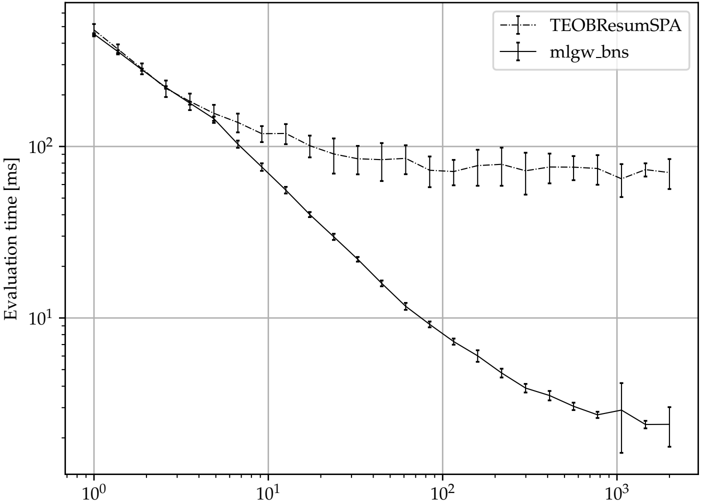
Evaluation time, varying initial frequency.



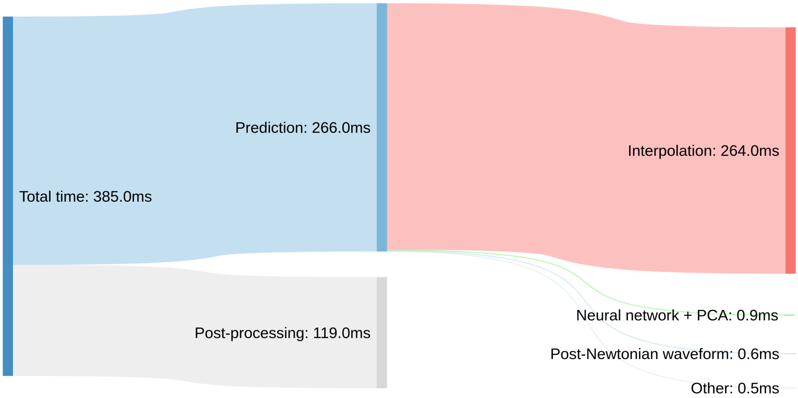
Evaluation time, varying initial frequency.



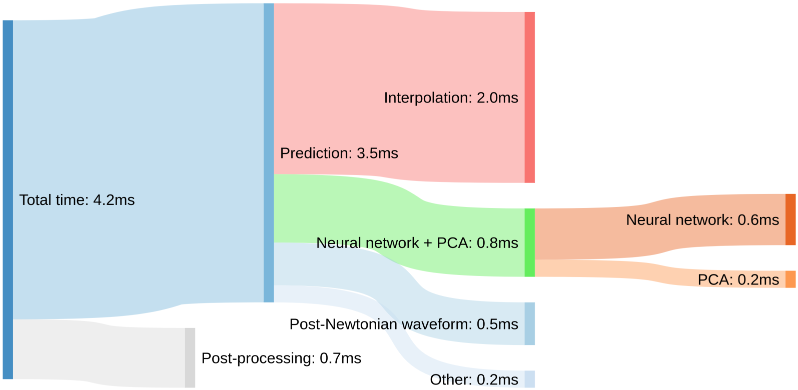
Evaluation time: $f_0 = 12\,\text{Hz}$, downsampling.



Profiling the evaluation: 2×10^6 interpolation points



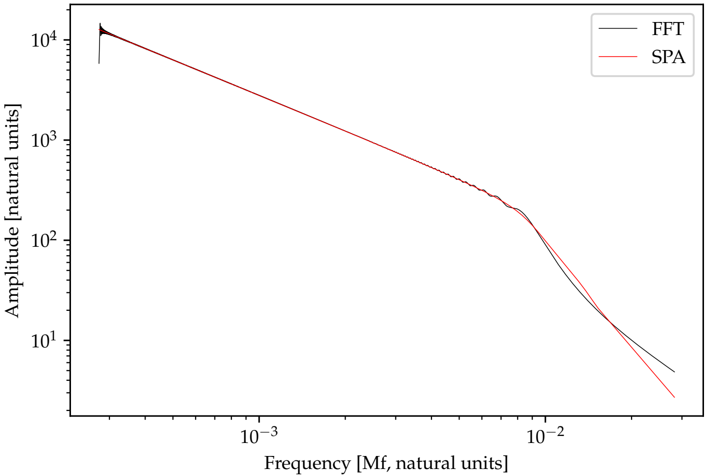
Profiling the evaluation: 8×10^3 interpolation points



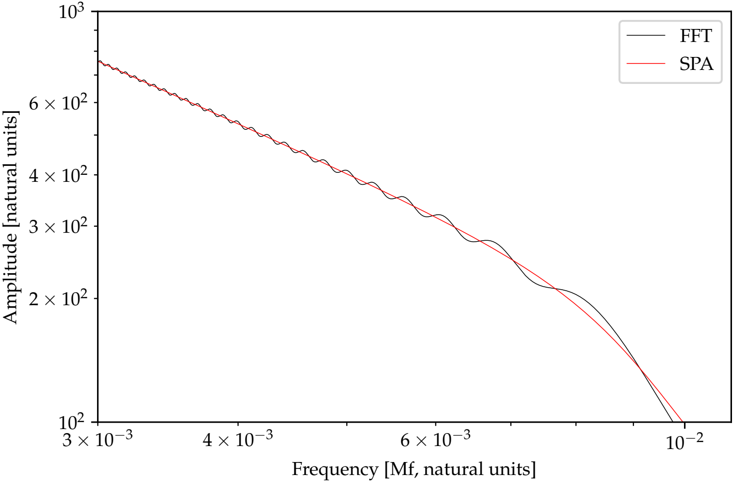
Conclusions

- `mlgw_bns` is a surrogate model trained on effective one body waveforms;
- it is accurate (enough not to bias parameter estimation);
- it is fast(er than the state-of-the-art model it is trained on).

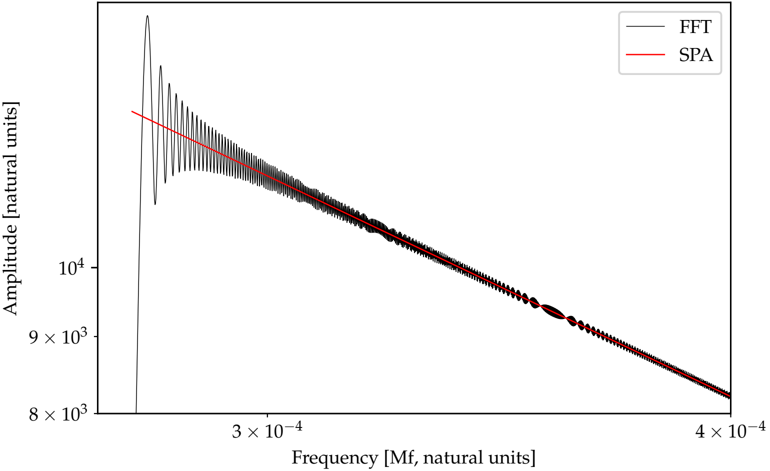
Fourier transform issues



Fourier transform issues



Fourier transform issues



Natural units and quadrupolar approximation

$$\frac{\left|\widetilde{h}(f)\right|}{M} \approx \frac{1}{\pi^{2/3}}\sqrt{\frac{5}{24}}(Mf)^{-7/6}\frac{M}{r}\sqrt{\frac{m_1 m_2}{M^2}} \tag{4}$$

$$\left|h_+(f)\right| = \frac{1+\cos^2\iota}{2}\left|h(f)\right| \tag{5}$$

$$\left|h_\times(f)\right| = \cos\iota\left|h(f)\right|. \tag{6}$$

Fidelity requirements

For detection: with a mismatch $\mathcal{F}$ we miss approximately a fraction

$$1-(1-\mathcal{F})^3 \tag{7}$$

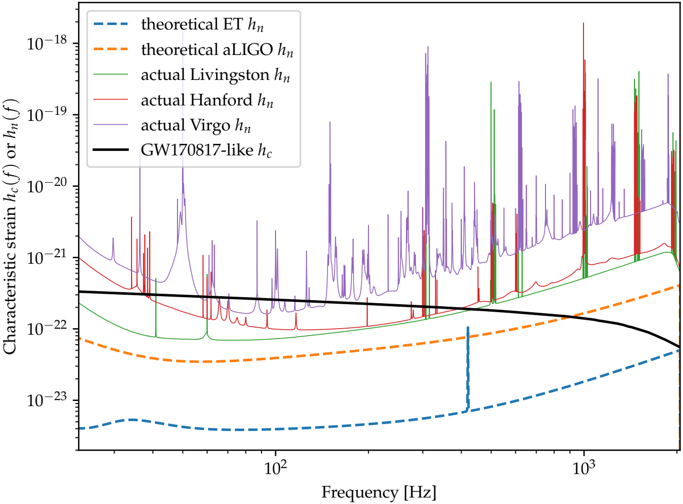
of the signals.

For parameter estimation: in order for the modelling error to be at a 1σ level we need

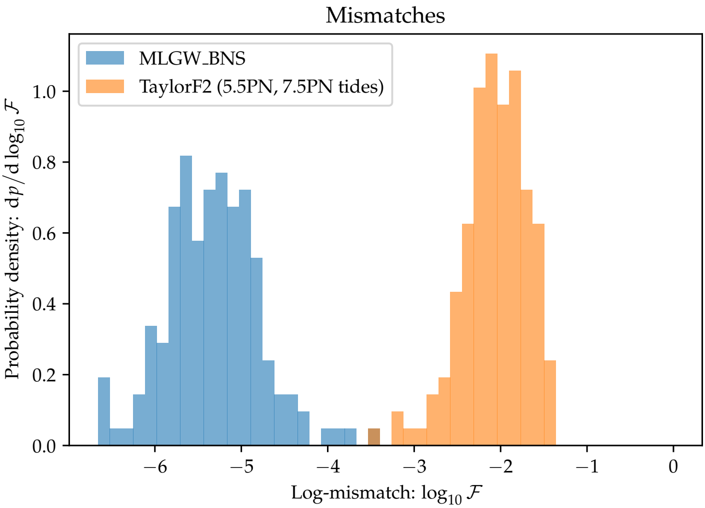
$$\mathcal{F} \lesssim \frac{D}{2\mathsf{SNR}^2}\,, \tag{8}$$

where D is the number of intrinsic parameters we estimate.

Power Spectral densities and GW170817



Fidelity compared to TF2



Bibliography

Abbott, B. P. et al. (Oct. 16, 2017). "GW170817: Observation of Gravitational Waves from a Binary Neutron Star Inspiral". In: *Physical Review Letters* 119.16, p. 161101.

Maggiore, Michele (Nov. 24, 2007). *Gravitational Waves: Volume 1: Theory and Experiments*. 1 edition. Oxford: Oxford University Press. 576 pp.