Machine Learning Gravitational Waveforms for Binary Neutron Star mergers: a working summary

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The detection of a Gravitational Wave signal from the Binary Neutron Star (BNS) merger GW170817 opened the way for the direct study of Neutron Star properties.

This is done through Bayesian parameter estimation algorithms, which require the comparison of the experimental signal with several million [Lac+17] simulated waveforms or more. Speed in the generation of simulated waveforms is, therefore, crucial.

There are three main strategies for the generation of these waveforms. In order of decreasing evaluation time, as well as (typically) decreasing accuracy, they are:

1. Full Numerical Relativity (NR) simulations: these are our most accurate possible avenue for the study of BNS mergers, but they are so computationally expensive that it is reasonable to simulate at most a few tens of initial condition setups — these are then used to validate the other methods;

2. Effective One Body (EOB) simulations: these amount to constructing an effective Hamiltonian for a test particle as a proxy for the two-body system and simulating its evolution — they can be calibrated by comparison to NR simulations;

3. Post-Newtonian (PN) waveforms: here we expand the equations of motion of the system in powers of $1/c^2$ and analytically calculate the evolution of the amplitude and phase of the waveform.

The EOB and PN formalisms can be complemented with a tidal term, which is relevant for BNS mergers: neutron stars (as opposed to black holes) can be deformed by each other's gravitational field in a way which depends on the specifics of their equation of state.

The evaluation times for EOB waveforms are dependent on the duration of the waveforms: GW170817 lasted for about two minutes, and the generation of waveforms of this length takes on the order of 100ms with state-of-the-art systems, but as the noise in ground-based interferometers decreases we expect to be able to see a longer and longer section of the waveform, which might bring the evaluation times back above 1s.

Also, the waveforms ought to be generated in the frequency domain, since that form is the one which is used in data analysis; EOB models, on the other hand, natively generate waveforms in the time domain.

The work in this thesis is an attempt to make progress in this direction, by developing a machine learning system trained on the (Fourier transforms of the) EOB waveforms. This builds on the work of Schmidt et al. [Sch+20], who developed a similar system to generate Binary Black Hole merger waveforms in the time domain.

The algorithm's basic components are as follows:

- a training dataset is generated by considering uniformly distributed tuples of BNS parameters in the allowed ranges and calculating the corresponding waveforms with the EOB model `TEOBResumS` [Nag+18];
- each time-domain waveform is Fourier-transformed and decomposed into phase and amplitude;
- the dimensionality of the training dataset is reduced through Principal Component Analysis (PCA);
- a feed-forward neural network is trained to reconstruct the map between the BNS system parameters and the PCA components of the waveforms.

The system is currently being improved by having it reconstruct only the residual of the EOB waveform from a certain Post-Newtonian expansion, as opposed to reconstructing the full EOB waveform.

The resulting model must be evaluated in terms of both speed and accuracy. Regarding the former, proper benchmarking will begin at a later stage, but the results obtained by Schmidt et al. [Sch+20] indicate the possibility of obtaining a speedup of one to two orders of magnitude from the Effective One Body waveforms.

Preliminary results with this approach indicate the possibility of the model reconstructing EOB waveforms parametrized by the mass ratio q and the two tidal deformabilities Λ_1 and Λ_2 with a maximal unfaithfulness[1] of the order of $\mathcal{F} \simeq 10^{-3}$, which is comparable to the unfaithfulness of the EOB waveform compared to the Numerical Relativity ones ($\mathcal{F} \simeq 2.5 \times 10^{-3}$ [Nag+18]).

This indicates that it is possible for this system to become a part of a gravitational wave data analysis pipeline without the loss of a great deal of accuracy.

References

[Lac+17] Benjamin D. Lackey et al. "Effective-One-Body Waveforms for Binary Neutron Stars Using Surrogate Models". In: *Physical Review D* 95.10 (May 30, 2017), p. 104036. ISSN: 2470-0010, 2470-0029. DOI: 10.1103/PhysRevD.95.104036. arXiv: 1610.04742. URL: http://arxiv.org/abs/1610.04742 (visited on 2020-11-20).

[1] The unfaithfulness is defined as follows:

$$\mathcal{F}[h_1, h_2] = 1 - \max_{t_0, \phi_0} \frac{(h_1|h_2)}{\sqrt{(h_1|h_2)(h_2|h_2)}} \qquad \text{with} \qquad (h_1|h_2) = 4\,\mathrm{Re} \int_{f_{\min}}^{\infty} \frac{h_1^*(f) h_2(f)}{S_n(f)} \, \mathrm{d}f \; , \tag{0.1}$$

where the maximum is evaluated over all possible starting times t_0 and phases ϕ_0 for the waveforms. The product we define between waveforms is known as the Wiener product, and it also appears when computing the signal-to-noise ratio, as well as in the calculation of the likelihood for parameter estimation.

[Nag+18] Alessandro Nagar et al. *Time-Domain Effective-One-Body Gravitational Waveforms for Coalescing Compact Binaries with Nonprecessing Spins, Tides and Self-Spin Effects*. Oct. 28, 2018. DOI: 10.1103/PhysRevD.98.104052. arXiv: 1806.01772 [gr-qc]. URL: http://arxiv.org/abs/1806.01772 (visited on 2021-02-27).

[Sch+20] Stefano Schmidt et al. *Machine Learning Gravitational Waves from Binary Black Hole Mergers*. Nov. 3, 2020. arXiv: 2011.01958 [gr-qc]. URL: http://arxiv.org/abs/2011.01958 (visited on 2020-11-13).