High Energy Experimental Astroparticle Physics

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Introduction

Monday 2021-11-8, compiled 2021-11-23

This first part of this course is given by Ivan De Mitri.

This is not a course on particle detectors: the basics will be assumed, and there will be some short courses about the details. It is more about "experimental tools": how do we design an experiment?

"High energy", here, means roughly considering energies $E \gtrsim 1\,\mathrm{GeV}$, but there will be some things in the MeV range as well. A more apt description of this field is that it is about observing things that come from outside the Earth: particles or high-energy radiation.

The other, "low-energy" course is about the search for rare events, where the issue is the presence of a large background, so we need to go underground.

Then, there is neutrino physics which is at the intersection between these two areas.

The part on gravitation and cosmology, on the other hand, is wholly distinct.

Cosmic rays

We start with a review of some basic concepts.

As a first approximation, the spectrum of cosmic particles/radiation $\phi(E)$ is decreasing with E. From $E \sim 10^8\,\mathrm{eV}$ to $E \sim 10^{20}\,\mathrm{eV}$ there is roughly a powerlaw, $\phi \propto E^{-\gamma}$ with $\gamma \sim 2.7$ at first, then $\gamma \sim 3$ (as shown in figure 1).

With artificial particle accelerators we can probe up to roughly $10^{13}\,\mathrm{eV}$ (since in the center of mass we have $(7+7)\mathrm{TeV}$).

Since the spectrum is roughly a power-law with index 3 spanning 12 orders of magnitude, the number of observed events changes by roughly 12×3 orders of magnitude between its ends.

There is a "knee" in the energy spectrum, where the spectral index γ goes from 2.7 to 3, is at about $3 \times 10^{15}\,\mathrm{eV}$.

Typically, below the knee we can do direct, small experiments, while above it we need to do indirect measurements: they need to be very large, since the presence of a single event becomes very rare.

The particles which have been found to make up **cosmic rays** are protons p, Helium nuclei He, heavy nuclei such as Fe, γ rays, electrons and positrons $e^{\pm}$, antiprotons $\overline{\mathrm{p}}$. Antihelium still has to be discovered, but there has been some progress in this regard lately.

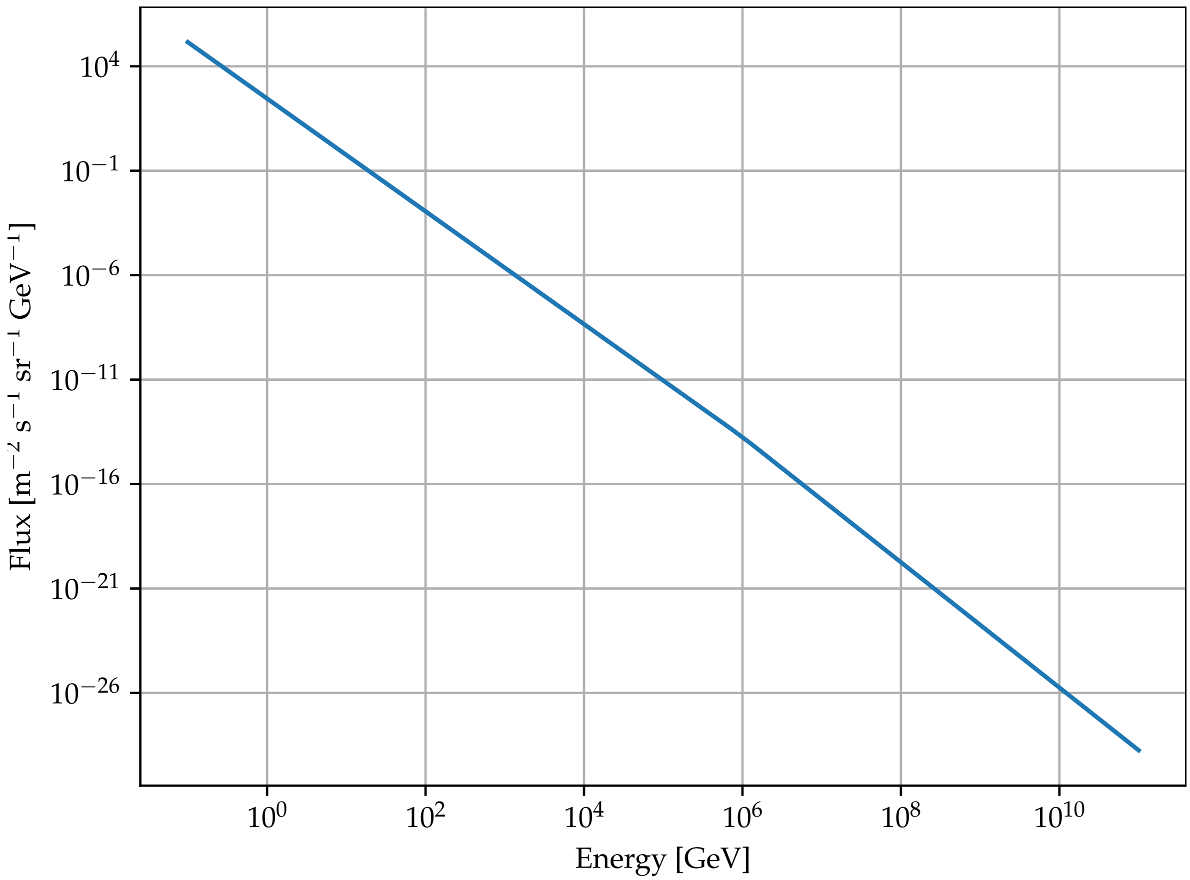

Figure 1: A rough sketch of the cosmic ray flux dependence on the energy.

Also, there are neutrinos: solar ones are on the scale of the MeV, up to 10 MeV. Protons come down the atmosphere and interact with it a few tens of km above, producing other things, such as neutrinos (these are “atmospheric neutrinos”). Alternatively, we can have a source producing neutrinos directly, “astrophysical neutrinos”.

Why are we doing these experiments?

1. We can explore the astrophysical processes allowing for the production of such high-energy particles, and

2. we can probe high-energy particle physics.

In order to match the center-of-mass energy of the LHC collisions we’d need to have 10^{17} eV in a fixed-target experiment.

This is due to the fact that fixed-target experiments have much lower center-of-mass energetics in general.

Here is a quick sketch of why, in natural units, using the formalism detailed in the second half of this lecture. If we have two particles colliding and with equal and opposite momenta the total four-momentum will be $p_{\text{tot}} = (E, \vec{p}) + (E, -\vec{p}) = (2E, \vec{0})$, therefore the total center-of-mass energy will be $\sqrt{s} = 2E$.

On the other hand, in a fixed-target experiment we will have one of them being stationary, so the total four-momentum will be $p_{\text{tot}}(E, \vec{p}) + (m, \vec{0}) = (E + m, \vec{p})$, where the momentum $\vec{p}$ is determined by $m^2 = E^2 - m^2$, so the magnitude will be $\sqrt{s} = \sqrt{(E+m)^2 - (E^2 - m^2)} = \sqrt{2m(E+m)}$.

In terms of the Lorentz factor γ the beam-beam collision is $\sqrt{s} = 2m\gamma$, while the fixed-target one is $\sqrt{s} = m\sqrt{2(1+\gamma)}$. For large values of γ, the first one clearly wins out, by a factor roughly $\sqrt{\gamma}$.

Specifically, if one wants $\sqrt{s} = 14\,\text{TeV}$, this can be achieved with $E \approx 7\,\text{TeV}$ in a beam-beam collision, or with $E \approx 1000\,\text{TeV}$ in a fixed-target collision. The ratio, as expected, is roughly $70 \div 80 \approx \sqrt{\gamma} = \sqrt{7\,\text{TeV}/1\,\text{GeV}}$ when accounting for the fact that we need to accelerate twice as many particles in the beam-beam scenario.

History

Cosmic rays were discovered in the early 1900s, and up to the early 50s most particle physics was done through them.

In 1957 the antiproton was discovered by Segrè and Chamberlain. This was done with accelerators, thanks to proton collisions producing secondaries, in a process like

$$\mathrm{p} + \mathrm{A} \to \mathrm{p} + \mathrm{A} + \mathrm{p} + \overline{\mathrm{p}}\,. \tag{0.1}$$

The cosmic ray discovery of the antiproton was performed only a few months later.

References

A good one is Aloisio et al. [Alo+18].

1 Review

1.1 Relativistic kinematics

Suppose we have a frame O and a different frame O' moving with constant velocity along the $x \sim x'$ direction.

If we define, in terms of the speed of light c and the relative velocity v between the two frames,

$$\beta = \frac{|\vec{v}|}{c} \qquad \text{and} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}\,, \tag{1.1}$$

then all observations in one frame can be connected with ones in the other thanks to the Lorentz transformation law.

The way the four-vector $(ct, x, y, z)^\top$ transforms is

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}\,. \tag{1.2a}$$

This is actually a general law, since we can write the transformation decomposing the position vector into parallel and perpendicular components to the motion:

$$\begin{bmatrix} ct' \\ r'_{\parallel} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ r_{\parallel} \end{bmatrix} \tag{1.3a}$$

$$r'_{\perp} = r_{\perp} \,. \tag{1.3b}$$

We can denote $(ct, x, y, z)^{\top}$ as $(x_0, x_1, x_2, x_3) \equiv x$.

The scalar product used by these vectors is the mixed signature

$$x \cdot y = x^{\mu} \eta_{\mu\nu} x^{\nu} \,, \tag{1.4}$$

where

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} , \tag{1.5a}$$

because we are using the evil particle physicist's mostly-minus convention.

We choose this because of the invariance of the spacetime interval.

Lorentz transformations are "rotations" in the sense that they leave the magnitude of vectors unchanged.

We need to define a *velocity* four-vector. In the nonrelativistic case, $\vec{v} = \mathrm{d}\vec{r}/\mathrm{d}t$; how do we extend it to the relativistic case?

We could define this as $\mathrm{d}x_{\mu} \big/ \mathrm{d}t$, but this is not invariant since time is not invariant (i.e. it is not a scalar).

Instead, we define the *proper time* as the time *as measured in the reference frame of the particle*. We know that $\Delta t = \gamma \Delta\tau$ by the Lorentz transformation law: therefore, we can define the relativistic velocity as

$$u_{\mu} = \lim_{\Delta\tau \to 0} \frac{\Delta x_{\mu}}{\Delta\tau} = \gamma \frac{\mathrm{d}x_{\mu}}{\mathrm{d}t} \,. \tag{1.6}$$

This is indeed a four-vector. Its components are $(\gamma c, \gamma v_x, \gamma v_y, \gamma v_z)^{\top}$, where $\vec{v}$ is the non-relativistic velocity $\mathrm{d}\vec{x}/\mathrm{d}t$.

The magnitude of this four-vector must be a Lorentz invariant: it comes out to be

$$u^2 = \gamma^2 \left(c^2 - |v|^2 \right) = c^2 \,. \tag{1.7}$$

In the nonrelativistic case the momentum is $\vec{p} = m\vec{v}$, while in the relativistic case we can define $p = mu$.

Its components will be

$$p^{\mu} = (\gamma mc, \gamma m\vec{v})^{\top} \,. \tag{1.8}$$

The term γmc is the (kinetic + rest) energy divided by the speed of light:

$$\gamma mc = \frac{mc}{\sqrt{1-\beta^2}} \approx mc\left(1 - \frac{1}{2}\beta^2\right) \tag{1.9a}$$

$$\approx \frac{1}{c}\left(mc^2 + \frac{1}{2}mv^2 + \mathcal{O}\left(\beta^4\right)\right). \tag{1.9b}$$

Therefore, we can write $p = (E/c, \vec{p})^\top$, where $\vec{p} = \gamma m\vec{v}$ and $E = \gamma mc^2$. This is a four-vector, and transforms as such, since the mass is a scalar. Its magnitude is $p^2 = m^2c^2$. This relation means that

$$E^2 = (mc^2)^2 + (\left|p\right|^2 c^2)^2 . \tag{1.10}$$

The conventional way to define the kinetic energy is $E_{\text{kin}} = E - mc^2$.

If we have a system of particles with four-momenta p_i we can compute the total energy and the total three-momentum by adding them all together.

If these particles scatter the four-momentum is conserved. What is the variable associated with the modulus of the total momentum?

$$p_{\text{tot}}^2 = \left(\frac{\sum_i E_i}{c}\right)^2 - \left|\sum_i \vec{p}_i\right|^2 \stackrel{\text{def}}{=} M^2c^2s . \tag{1.11}$$

The variable M is called *invariant mass*, and it is also written in terms of the Mandelstam variable s as $\sqrt{s}$ in the case of a two-two process.

1.2 The J/ψ discovery

Professor Samuel Ting got the Nobel Prize in 1976. His experiments can be used as an illustration for the formalism outlined here.

They were doing fixed-target experiments in Brookhaven: protons against Beryllium targets. The process looked like

$$\mathrm{p} + \mathrm{N} \to x + e^+ + e^- , \tag{1.12}$$

where the kinematic properties of the electron and positron were measured by spectrometers.

So, they knew $E^\pm$ and $\vec{p}^\pm$ (where $+$ and $-$ denote the positron and electron respectively).

This allows one to compute

$$m_{e^+e^-} = \frac{1}{c}\sqrt{(p_{e^+} + p_{e^-})} \tag{1.13a}$$

$$= \frac{1}{c}\sqrt{\left(\frac{E^- + E^+}{c}\right)^2 - \left|\vec{p}_+ + \vec{p}_-\right|^2} . \tag{1.13b}$$

If we plot a histogram of these quantities, we get a big spike at $m_{e^+e^-}c^2 = 3.1\,\text{GeV}$.

Therefore, we have a good suggestion of the fact that there was an intermediary:

$$\mathrm{p} + \mathrm{N} \to Y + x \tag{1.14a}$$
$$Y \to e^+ + e^- \,. \tag{1.14b}$$

Also, we know that $m_Y \approx 3.1\,\mathrm{GeV}/c^2$.

At the same time, on the opposite coast of the US, a different group was looking at e^+e^- collisions, and considering the number of reactions as a function of the center-of-mass energy: they also saw a peak in the effective cross-section at 3.1 GeV.

This was happening in 1974, and this particle was dubbed the J/ψ.

For one event they saw a "ψ" shape. It was later discovered that the J/ψ particle was a meson made of charm quarks, $c\bar{c}$.

The width of the peak is related to the decay time of the particle: $\tau \Delta E \sim \hbar$. Very narrow peaks mean that τ is quite large — there are kinematic reasons why this particle has a hard time decaying, which we will not go into here.

A similar experiment was done in Frascati a few months later, with ADONE (which is a larger ADA, "Anello di Accumulazione").

1.2.1 A $1 \to 2$ decay example

Suppose we have a particle with mass M decaying into two particles with masses m_1 and m_2.

In the lab system M will be moving, but we can look at the decay in the *center-of-mass* frame, in which p_{tot} is purely timelike. This is typically denoted with a star: $p^*_{\text{tot}} = (Mc, \vec{0})^\top$.

After the decay, the two particles are produced with momenta $\vec{p}^*_1 = -\vec{p}^*_2$ by conservation of momentum.

The angle θ^* is the one made by the two particles 1 and 2 with respect to the propagation direction of M, as measured in the CoM frame.

The energy conservation law reads

$$E^*_1 + E^*_2 = Mc^2 \tag{1.15a}$$
$$E^*_1 = \sqrt{(m_1 c^2)^2 + (|p^*_1| c)^2} \tag{1.15b}$$
$$E^*_2 = \sqrt{(m_2 c^2)^2 + (|p^*_2| c)^2}\,, \tag{1.15c}$$

where $p^*_1 = -p^*_2$.

We can compute $p_1^2 = m_1^2 c^2$ as the square of $p - p_2$: this yields

$$(p - p_2)^2 = p^2 + p_2^2 - 2p \cdot p_2 \tag{1.16a}$$
$$m_1^2 c^2 = M^2 c^2 + m_2^2 c^2 - 2\left(\frac{E^*}{c}\frac{E^*_2}{c} - \vec{p}^* \cdot \vec{p}^*_2 \right) \tag{1.16b}$$
$$= M^2 c^2 + m_2^2 c^2 - 2ME^*_2\,, \tag{1.16c}$$

therefore

$$E^*_2 = \frac{M^2 c^4 + m_2^2 c^4 - m_1^2 c^4}{2Mc^2}\,. \tag{1.17}$$

We know that this will be $E_2^* \geq m_2 c^2$, which can be found to be equivalent to $M \geq m_1 + m_2$. The same reasoning applies for E_1^*, swapping $1 \leftrightarrow 2$.

This is independent of the center-of-mass emission angle θ^*.

A two-output decay like this is a very "constrained" problem: in the CoM frame the energies of the particles are fully determined.

We can recover the lab-frame quantities by a Lorentz transformation with velocity $-\beta$ (opposite to the motion of M). The decomposition of the momentum in the CoM frame is

$$
\begin{aligned}
p_\parallel^* &= p^* \cos\theta^* && (1.18a) \\
p_\perp^* &= p^* \sin\theta^* \,. && (1.18b)
\end{aligned}
$$

Consider a positively charged pion: it decays as

$$
\pi^+ \to \mu^+ + \nu_\mu \,. \tag{1.19}
$$

We can compute the value E_μ^* in this case, for example, approximating the mass of the neutrino as 0. In natural units, the mass of the pion is $m_{\pi^+} \approx 139.57\,\text{MeV}$, while the mass of the muon is $m_{\mu^+} \approx 105.66\,\text{MeV}$ [Gro+20]; this means that in the center of mass we will have

$$
\begin{aligned}
E_\mu^* &\approx \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \approx 109.78\,\text{MeV} && (1.20a) \\
E_\nu^* &\approx \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \approx 29.79\,\text{MeV} \,. && (1.20b)
\end{aligned}
$$

As expected, $m_\pi = E_\mu^* + E_\nu^*$.

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As a reminder, the Lorentz factor can be written as $\gamma = E/mc^2$, while the factor β can be written as $\beta = |\vec{p}|c/E$.

Now we move to natural units: $\hbar = c = 1$.

We come back to our decay $M \to m_1 + m_2$. We have computed the CoM energies and momenta, now we need to move back to the lab frame. The Lorentz transformation with $-\beta$ reads

$$
\begin{aligned}
\begin{bmatrix} E_i \\ p_{i,\parallel} \end{bmatrix} &= \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} E_i^* \\ p_i^* \cos\theta^* \end{bmatrix} && (1.21a) \\
&= \begin{bmatrix} \gamma\big(E_i^* + \beta p^* \cos\theta^*\big) \\ \gamma\big(\beta E_i^* + p^* \cos\theta^*\big) \end{bmatrix} \,. && (1.21b)
\end{aligned}
$$

The emission angles may change depending on the interaction, so the θ^* dependence is relevant.

We can recover the emission angle in the lab frame by

$$
\tan\theta_i = \frac{p_{i,\perp}}{p_{i,\parallel}} = \frac{p_i^* \sin\theta^*}{\gamma\big(\beta E_i^* + p_i^* \cos\theta^*\big)} \,. \tag{1.22}
$$

Even if the emission is isotropic in the CoM frame, it will not be in the lab frame; qualitatively, it will be peaked in the forward direction.

If $\theta^* = 0$, we get $\theta = 0$. If $\theta^* = \pi/2$, we get

$$\tan\theta = \frac{1}{\gamma}\frac{p^*}{\beta E_i^*} = \frac{1}{\gamma}\frac{\beta_i^*}{\beta^*}\,. \tag{1.23}$$

In the ultrarelativistic limit, this will be $\sim 1/\gamma$: therefore, we will get $\theta \sim \arctan(1/\gamma) \sim 1/\gamma$.

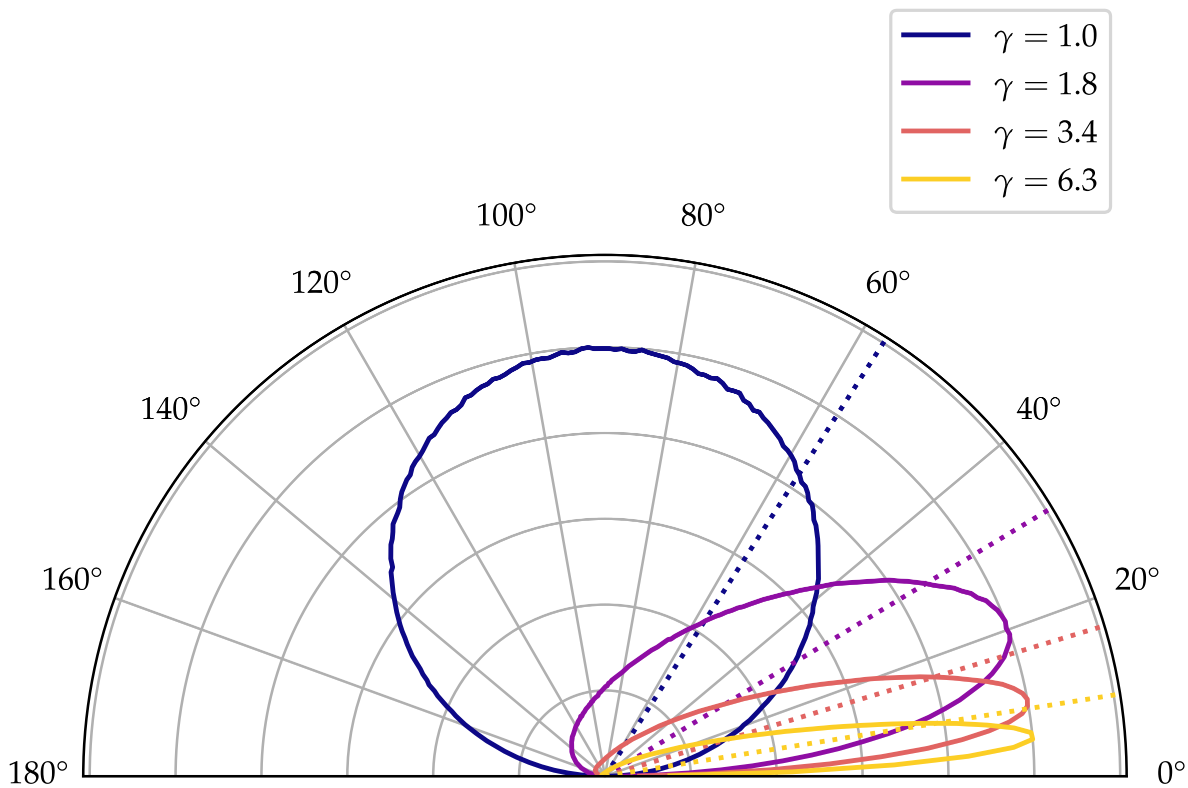

Figure 2: Angular distribution of the emitted radiation if the starting emission is isotropic, which means that the probability distribution for θ^* is $\mathrm{d}p = \sin\theta^*\,\mathrm{d}\theta^*$, which corresponds in the plot to the $\gamma = 1$ case. Here we are assuming that the decay looks like $1 \to 2 + 3$, where both 2 and 3 are massless. We also show the $\theta \sim 1/\gamma$ approximation as a dashed line. In the high γ limit, this is the median of the distribution.

Let us consider the decay of a neutral pion:

$$\pi^0 \to \gamma + \gamma \,. \tag{1.24}$$

In the CoM system, both photons will get $E^* = M/2 = p^*$, where $M = m_{\pi^0} \approx 134.98\,\text{MeV}$.

The lab-frame energy of one of these is given by

$$\begin{aligned}
E &= \gamma\big(E^* + \beta p^* \cos\theta^*\big) && \text{(1.25a)} \\
&= \gamma E^* (1 + \beta \cos\theta^*) && \text{(1.25b)} \\
&= \frac{E_\pi}{m_\pi} \frac{m_\pi}{2} \big(1 + \beta \cos\theta^*\big) && \text{(1.25c)} \\
&= \frac{E_\pi}{2} \big(1 + \beta \cos\theta^*\big) \,. && \text{(1.25d)}
\end{aligned}$$

The maximum of this quantity is $E_\pi(1+\beta)/2$; its minimum is $E_\pi(1-\beta)/2$. This is just a bit smaller than the distribution $E \in [0, \pi]$.

What is the distribution of the photons we get in the lab frame? How wide will the detector need to be? Well, we can frame it by selecting the size we need to get a certain high percentage of the emitted photons.

An order-of-magnitude estimate is: half of the photons are emitted with $\theta^* < \pi/2$, and their angle will be $\lesssim 1/\gamma$.

The original J/ψ papers are in the drive — add citation.

1.2.2 A $1 \to 3$ decay example

Now the three emitted particle momenta in the CoM frame will satisfy

$$\vec{p}^*_{\text{tot}} = p^*_1 + p^*_2 + p^*_3 = 0 \,. \tag{1.26}$$

We define θ^* as θ^*_1; the trick to solving this problem is to "bunch" particles 2 and 3 into one, with momentum $\vec{p}^*_{23} = p^*_2 + p^*_3$. The invariant mass of this system will read

$$\begin{aligned}
m_{23} &= \sqrt{p_2^2 + p_3^2 + 2 p_2 \cdot p_3} && \text{(1.27a)} \\
&= \sqrt{m_2^2 + m_3^2 + 2 E^*_2 E^*_3 - 2 \big|p^*_2\big| \big|p^*_3\big| \cos\theta^*_{23}} \,, && \text{(1.27b)}
\end{aligned}$$

where θ^*_{23} is the angle between 2 and 3 in the CoM frame.

We can recurse back to the 2 body formulas for the behaviour of the 1 and 23 system: the energy of 1 will be

$$E^*_1 = \frac{M^2 + m_1^2 - m_{23}^2}{2M} \,. \tag{1.28}$$

Now, even in the CoM frame the energy of particle 1 (which is arbitrary) depends on the emission angle θ^*_{23}. We can, however, compute the minimum and maximum values of E^*_1.

There is a "trick" which allows us to skip the whole computation. The idea is that m_{23} will be maximum when the most energy will be given to the 23 system; so this happens when m_1 is produced at rest, while the other two are back-to-back.

Then, we will have $p_2^* = -p_3^*$.

In this case, we will have $m_{23}^{\max} = M - m_1$. It's as if both particles (1 and the "combined particle" 23) were produced at rest.

In this case, then,

$$E^*_{1,\min} = \frac{M^2 + m_1^2 - (M^2 + m_1^2 - 2Mm_1)}{2M} = m_1\,, \tag{1.29}$$

as expected.

The other extreme happens when $m_{23} = m_2 + m_3$; in this case, then, there must exist a frame in which both particles are at rest. This means that they must travel with the same velocity. This does not mean that the momenta are equal, in general $p_2^* \neq p_3^*$, but $\vec{v}_2^* = \vec{v}_3^*$.

The maximum energy is therefore

$$E^*_{1,\max} = \frac{M^2 + m_1^2 - (m_2 + m_3)^2}{2M}\,. \tag{1.30}$$

The most famous example here is β decay: from a nuclear perspective it is

$$N(A, Z) \to N(A, Z+1) + e^- + \overline{\nu}_e\,, \tag{1.31}$$

since from a microphysical perspective what happened was

$$\mathrm{n} \to \mathrm{p} + e^- + \overline{\nu}_e\,. \tag{1.32}$$

Proton decay has not been observed, but β^+ decay can happen since the nucleus is also involved.

In the lab system, which is also the CoM system, we will have a minimum energy of 511 keV, and a maximum energy of roughly 1.29 MeV, which comes from the approximation

$$E_{e,\max} \approx \frac{M^2(A, Z) - M^2(A, Z+1)}{2M(A, Z)}\,. \tag{1.33}$$

If the neutrino were massive, this maximum of the distribution will be shifted down a bit. This is an experimental challenge, since the mass of the neutrino is at most on the eV scale.

Let us consider the decay $\pi^- \to \mu^- + \overline{\nu}_\mu$.

The decay $\pi^- \to e^- + \overline{\nu}_e$ is forbidden by the handedness of weak interactions. The helicity is conventionally positive for antiparticles and negative for particles.

As a first approximation, if the antineutrino is going to the left and the electron is going to the right, both spins will be to the left, summing to 1 to the left, while the pion has spin 0.

This is also the case for decay towards muons, but $m_\mu \gg m_e$, therefore the probability for the muon to have a spin component to the right is much higher.

A $2 \to 3$ process Before looking at three-body decays, let us consider a $2 \to 3$ process. Consider

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$$\mathrm{K}^- + p \to \pi^+ + \pi^- + \Lambda\,, \tag{1.34}$$

where the masses are $m_\pi \approx 1\,\mathrm{GeV}$, $m_\pi \approx 140\,\mathrm{GeV}$, $m_K \approx 500\,\mathrm{MeV}$, and $m_\Lambda \approx 1.1\,\mathrm{GeV}$.

The experimental setup is a kaon beam impacting upon a proton target.

The invariant mass of the starting particles will be

$$m_{Kp} = \frac{1}{c}\sqrt{\left(\sum p_i\right)^2} = \frac{1}{c}\sqrt{(p_k + p_p)^2}\,. \tag{1.35}$$

We could also define a corresponding energy $m_{Kp}c^2$.

If we extend this sum to all the particles in the initial state, $m_{\text{initial}}c^2 = m_{\text{final}}c^2 = \sqrt{s}$ is the total invariant mass. The variable s is written as $s = c^2 p_{\text{tot}}^2$. Its meaning is that it is the total energy in the center-of-mass frame.

We can then think of this as an equivalent decay of a particle with m_{Kp}, therefore the kinematic threshold is $m_{Kp} \geq m_{\pi^+} + m_{\pi^-} + m_\Lambda$. This is a general statement:

$$\sqrt{s} \geq \sum_{i \in \text{products}} m_i c^2\,. \tag{1.36}$$

Let us approximate the protons as stationary, and we have a kaon with E_K.

The total four-momentum reads

$$p_{\text{tot}} = (E_K + m_p, \vec{p}_k)^\top\,, \tag{1.37}$$

so we get

$$\begin{aligned}\sqrt{s} &= \sqrt{(E_k + m_p)^2 - \left|\vec{p}_k\right|^2} && (1.38)\\ &= \sqrt{m_K^2 + m_p^2 + 2E_k m_p}\,. && (1.39)\end{aligned}$$

We can then straightforwardly compute the energy threshold for E_k. In this case we have $E_k^{\text{threshold}} < m_k$, so we have no real lower limit.

This reasoning can be applied, for example, when looking at the reaction

$$\mathrm{p} + \mathrm{p} \to 3\mathrm{p} + \overline{p}\,. \tag{1.40}$$

The threshold for the energy of the single proton impacting on the target comes out to be $7m_p \approx 6.6\,\mathrm{GeV}$.

This is counterintuitive! It is due to the fact that we lose a lot of energy when boosting back to the CoM frame.

Let us go back to the $2 \to 3$ reaction. We can always compute the invariant mass of a subsystem such as $m_{\pi^\pm\Lambda}$:

$$m_{\pi^\pm\Lambda} = \sqrt{(E_{\pi^\pm} + E_\Lambda)^2 - \left|\vec{p}_{\pi^\pm} + \vec{p}_\Lambda\right|^2}\,, \tag{1.41}$$

which we can compute once we have the measurement from our detector.

See "dalitz-pi-lambda" figure in the drive.

We can look at the histogram of $m^2_{\pi^+\Lambda}$ versus $m^2_{\pi^-\Lambda}$.

There are peaks! This suggests the presence of something decaying into $\pi^\pm\Lambda$. This particle is a $\Sigma^\pm$, and what happened was

$$\mathrm{K}^- + p \to \Sigma^\pm + \pi^\mp \tag{1.42}$$

$$\Sigma^\pm \to \pi^\pm + \Lambda\,. \tag{1.43}$$

We have not detected the Σ particle, which was short-lived.

We know that there should be a range $m_2 + m_2 \leq m_{23} \leq M - m_1$.

This can be extended to a constraint on the two invariant masses.

This is called a **Dalitz plot**, a common trick to find peaks for new particles.

The (true?) width of the peak is about 36 MeV, which corresponds to 1.8×10^{-23} s. Is this a $\Sigma(1385)$?

But we should also consider experimental error in the determination of the energy…

An alternative way to measure lifetimes is to look at track length: the path length we expect is

$$\lambda = \beta c \tau \gamma\,, \tag{1.44}$$

where we need the Lorentz factor since the particle decays in time τ in its own rest frame.

In this case the track length would be about 5.5 fm; however we also have the γ factor: if that is large enough we may get something measurable.

We'd want to see at least a few points. If σ_x is our spatial resolution, we need a track length which is at least $\gtrsim 5\sigma_x$.

The best spatial trackers are silicon detectors or emulsion trackers, on the order of 10 µm. This means that we need to increase our path length by a factor 10^{10}… The γ factor is surely not that large: it would mean having a beam with energy on the order of 10^{19} eV.

Let us consider another example: $\mathrm{p} + \gamma \to \Delta^+$, which can decay into $\pi^+ + \mathrm{n}$ or $\pi^0 + \mathrm{p}$.

What is the kinematic threshold for this reaction? This is a cosmic ray proton interacting with a CMB photon.

These photons have an average energy of 2.7 K $\approx$ 230 µeV.

This does not precisely match $\Omega_{0\gamma}\rho_c c^2$ when multiplied by $n_{0\gamma}$, since we'd need to integrate the Planckian.

The value of $\sqrt{s}$ is

$$\sqrt{s} = \sqrt{m_p^2 + 2\left(E_p E_\gamma - \vec{p}_p \cdot \vec{p}_\gamma\right)} \tag{1.45}$$

$$= \sqrt{m_p^2 + 2E_p E_\gamma\left(1 - \cos\theta_{p\gamma}\right)} \geq m_{\Delta^+}\,. \tag{1.46}$$

This means that

$$E_p \geq \frac{m^2_{\Delta^+} - m_p^2}{2E_\gamma\left(1 - \cos\theta_{p\gamma}\right)}\,. \tag{1.47}$$

If the collision is head-on, we get $1 - \cos = 2$, while in the other case the threshold diverges since the proton cannot reach the photon.

We should average the formula over all values of $\theta_{p\gamma}$.

The threshold is about 10^{19} eV. This is the GZK cutoff, or GZK effect. In order to properly study this phenomenon we would need to also look at the probability: [Gro+20] says it is of the order of 10^{-1} mb for the γp process.

We can study this in the lab simply, we only need 200 MeV photons colliding on stationary protons.

With this, we can find that a typical proton above the threshold will travel for only tens of Mpc before losing energy to this process.

The products from protons being annihilated in this way will produce many secondary particles: photons, neutrinos, electrons, positrons, muons and more, all being very energetic.

Let us do another possibility: could photons from a high-energy source be absorbed by the CMB? For example, you can have pair production.

We need $\sqrt{s}$ for the process to be larger or equal than about 1 MeV. This comes out to be about 10^{14} eV.

Let's see how that works. We are looking at the process $\gamma + \gamma \to e^+ + e^-$; in the lab frame (which is the frame in which the CMB has vanishing dipole moment, the universal rest frame) one photon will have a typical energy of $E_{\gamma_{\text{CMB}}} \approx 235\,\mu\text{eV}$, while the other will have a large energy E_γ.

The invariant mass of the scattering will read

$$\sqrt{s} = \sqrt{(p_\gamma + p_{\gamma_{\text{CMB}}})^2} \tag{1.48}$$

$$= \sqrt{(E_\gamma + E_{\gamma_{\text{CMB}}})^2 - (\vec{p}_\gamma + \vec{p}_{\gamma_{\text{CMB}}})^2} \tag{1.49}$$

$$= \sqrt{2E_\gamma E_{\gamma_{\text{CMB}}}(1 - \cos\theta_{\gamma\gamma})}\,, \tag{1.50}$$

$E_\gamma^2 - \vec{p}_\gamma^{\,2} = 0$ for both photons.

which we want to be $\geq 2m_e \approx 1$ MeV. This condition reads

$$E_\gamma \geq \frac{(1\,\text{MeV})^2}{2E_{\gamma_{\text{CMB}}}(1 - \cos\theta_{\gamma\gamma})} \approx \frac{2.2\,\text{PeV}}{(1 - \cos\theta_{\gamma\gamma})}\,. \tag{1.51}$$

Tuesday 2021-11-16

Last time we compute the threshold for reactions with a CMB photon interacting with some high-energy particles.

We can plot the mean free path with varying energy. It is roughly infinite below a certain threshold (we can have some interactions with the tails of the distribution near it), then it asymptotes to $\Lambda \sim 100$ Mpc.

Around 5×10^{19} eV the $\text{p} + \gamma_{\text{CMB}} \to \Delta^\pm$ reaction kicks in, while at around 10^{14} eV we get the $\gamma + \gamma_{\text{CMB}} \to e^+e^-$ reaction. There, however, the energy affects the cross-section of the reaction, so we expect to see a minimum.

The all-particle energy spectrum has an ultra-high-energy cutoff around 10^{20} eV because of these reactions.

At LHC we have center-of-mass energies of 14 TeV, which is found by adding together the energies of the bins. This is true in this particular case since we have a symmetric configuration, meaning that the total momentum in the lab frame as well vanishes.

In a fixed target context (such as a cosmic ray coming in) instead we get

$$\sqrt{s} = \sqrt{2m(E+m)}\,, \tag{1.52}$$

so the correct comparison reads

$$\sqrt{2m(E+m)} = 2E^*\,, \tag{1.53}$$

where $E^* = 7\,\text{TeV}$. Therefore,

$$E = \frac{(2E^*)^2}{2m} - m\,. \tag{1.54}$$

It depends on E^* quadratically! In our case, with protons, we get $E \sim 10^{17}\,\text{eV}$.

How are the angles changed? The invariant mass of the total system is

$$m_{\text{inv}} = \sqrt{\left(\sum_i E_i\right)^2 - \left|\sum_i p_i\right|^2}\,. \tag{1.55}$$

We also know that $\gamma = E/m$. The total γ can be computed by looking at this expression with the total energy E_{tot} and $\sqrt{s} = m_{\text{inv}}$ as the total mass.

In the case of our cosmic rays, this reads

$$\gamma_{\text{CMS}} \sim \sqrt{\frac{10^{17}\,\text{eV}}{2\times 10^{9}\,\text{eV}}} \approx 7\times 10^3\,, \tag{1.56}$$

which corresponds to an angle of $1/\gamma_{\text{CMS}} \sim 30\,\text{arcsec}$.

This is a good thing, in a way: instead of having to make a detector all around the event, we can make a smaller one in the forward direction only.

We can define a new kinematic variable in order to better understand these angles.

Consider a Lorentz transformation with a velocity $v_0 = \beta_0 c$ along the x axis. The transformation for a velocity v reads

$$v = v' + v_0 \tag{1.57}$$

in the NR case, while for the relativistic case it will be

$$v = \frac{v' + v}{1 + v'v_0/c^2}\,. \tag{1.58}$$

This only holds for the x component, along the boost.

Therefore, the velocities are not additive. A trick is to introduce a new quantity, the *rapidity*:

$$y' = \frac{1}{2}\log\frac{1+\beta'}{1-\beta'} = \text{arctanh}(\beta')\,, \tag{1.59}$$

and rapidities are indeed additive. These are typically considered only looking at the component parallel to the boost:

$$y = \operatorname{arctanh}(\beta_\parallel) \qquad \text{where} \qquad \beta_\parallel = \beta\cos\theta = \frac{p_\parallel}{E} \,. \tag{1.60}$$

This formulation holds in 3D as well. The rapidity can also be written as

$$y = \frac{1}{2}\log\left(\frac{E+p_\parallel}{E-p_\parallel}\right) . \tag{1.61}$$

The Lorentz parameters can be written in terms of hyperbolic functions:

$$\beta = \tanh(y) \tag{1.62}$$
$$\gamma = \cosh(y) \tag{1.63}$$
$$\beta\gamma = \sinh(y) \,, \tag{1.64}$$

so that the Lorentz matrix reads

$$\begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} = \begin{bmatrix} \cosh(y) & -\sinh(y) \\ -\sinh(y) & \cosh(y) \end{bmatrix} . \tag{1.65}$$

In the ultrarelativistic limit, the rapidity reads

$$y \approx \log\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} \tag{1.66}$$
$$\approx \log\frac{\sqrt{(1+\cos\theta)/2}}{\sqrt{(1-\cos\theta)/2}} \tag{1.67}$$
$$\approx -\log\tan(\theta/2) = \eta \,, \tag{1.68}$$

where η is called the pseudo-rapidity.

make figure

At LHC we can only detect particles which are not emitted very forward or very backward.

What about the rapidities of the particles for the case of a 10^{17} eV cosmic ray? They are additive, so $y_{\text{lab}} = y' + y_0$, where

$$y_0 = \frac{1}{2}\log\left(\frac{1+\beta_{\text{CMS}}}{1-\beta_{\text{CMS}}}\right) \tag{1.69}$$
$$\beta_{\text{CMS}} = \frac{|p'|}{E+m} \approx \frac{E}{E+m} \tag{1.70}$$
$$y_0 \approx \frac{1}{2}\log\left(\frac{2E+m}{m}\right) \approx \frac{1}{2}\log\frac{2E}{m} \,. \tag{1.71}$$

Let us consider an energy $E \approx 10^{15}$ eV. Then, $y_0 \approx 7$.

We can divide the CMS depending on the sign of η^*,the pseudo-rapidity in the CMS. Then the total rapidity will read $\eta = \eta^* + y_0$.

The boundary between these in the lab frame is found when $\eta^* = 0$, so we get $\eta = -\log\tan\theta/2$: θ is about 6 arcminutes for $\eta = 7$.

For a collision 20 km in the air, we expect about half of the particles to be found within a 40 m radius.

Collider experiments typically cover $|\eta| \lesssim 5$ (the figure refers to ATLAS); this only covers the central region, we do not get all of them.

The definition of the pseudo-rapidity is convenient because PDFs in the form $\mathrm{d}N/\mathrm{d}\eta$ are only shifted under Lorentz boosts.

The **transverse mass** can also be defined:

$$E^2 = m^2 + p^2 = \underbrace{m^2 + p_T^2}_{m_T^2} + p_\parallel^2\,, \tag{1.72}$$

and $m_T^2 = m^2 + p_T^2$ defined as above is Lorentz invariant under boosts in the $\parallel$ direction. We can also write this relation like

$$\left(\frac{E}{m_T}\right)^2 - \left(\frac{p_\parallel}{m_T}\right)^2 = 1 \tag{1.73}$$

$$\cosh^2 y - \sinh^2 y = 1\,. \tag{1.74}$$

The rapidity can also be expressed as

$$y = \frac{1}{2}\log\left(\frac{E + p_\parallel}{m_T}\right). \tag{1.75}$$

Exercise: suppose we have an incoming particle with mass M impacting upon a particle with mass m; what is the maximum transfer of energy from particle M to particle m, if the collision is elastic?

The result should be

$$\Delta E = \frac{2m\gamma^2\beta^2}{1 + 2\gamma m/M + (m/M)^2}\,. \tag{1.76}$$

The case we are interested in will be a proton interacting with a medium and interacting with an electron. There, and for a relativistic proton, this will read $\Delta E \approx 2m\gamma^2$.

Next time, we will discuss some matter-radiation interactions.

A particle of mass M with Lorentz factor γ and velocity β is impacting a stationary particle of mass m.
The total invariant mass will be

$$\sqrt{s} = \sqrt{(E_M + m)^2 - p_M^2} \tag{1.77}$$

$$= \sqrt{M^2 + m^2 + 2\gamma Mm}\,, \tag{1.78}$$

where $E_M = \gamma M$ while $p_M = \beta E_M$.
The Lorentz factor γ_{CoM} of the transformation required to move between the center of mass frame and the lab one can be computed from the total energy:

$$\gamma_{\text{CoM}} = \frac{E_{\text{tot}}}{\sqrt{s}} = \frac{\gamma M + m}{\sqrt{M^2 + m^2 + 2\gamma M m}} \,. \tag{1.79}$$

As one would expect, this approaches 1 as $\gamma \to 1$.
In the CoM frame we will have

$$\sqrt{s} = E_M^* + E_m^* = E_M^{*\prime} + E_m^{*\prime} \,, \tag{1.80}$$

where the prime denotes a value computed after the collision.
Let us denote the momentum of particle m after the collision in the center of mass frame as $\vec{p}^*$, so that

$$p_M^{*\prime} = [E_M^{*\prime}, -\vec{p}^*] \tag{1.81}$$

$$p_m^{*\prime} = [E_m^{*\prime}, \vec{p}^*] \,, \tag{1.82}$$

the sum of which will equal $[\sqrt{s}, 0]^\top = p_M^{*\prime} + p_m^{*\prime}$. Isolating the M momentum and computing the modulus of this equation yields the following expression for the energy of m after the collision:

$$\begin{aligned} E_m^{*\prime} &= \frac{s + m^2 - M^2}{2\sqrt{s}} && (1.83) \\ &= \frac{M^2 + m^2 + 2\gamma M m + m^2 - M^2}{2\sqrt{M^2 + m^2 + 2\gamma M m}} && (1.84) \\ &= \frac{m^2 + \gamma M m}{\sqrt{M^2 + m^2 + 2\gamma M m}} \,. && (1.85) \end{aligned}$$

As expected, this equals m when $\gamma = 1$. Also, this precisely equals $m\gamma_{\text{CoM}}$: this is correct, since what we have done is boosting particle m from stationarity with γ_{CoM}; the kinematics before and after the collision are exactly the same, so the energy of particle m in the center of mass frame will be the same before and after it.
In the CoM frame this is fixed, but the angle of emission is arbitrary, and can lead to differing values for the energy of m in the lab frame.
Specifically, the expression for the energy and momentum along the boost direction of mass m in the lab frame reads

$$\begin{bmatrix} E_m \\ p_{m\parallel} \end{bmatrix} = \begin{bmatrix} \gamma_{\text{CoM}} & -\beta_{\text{CoM}}\gamma_{\text{CoM}} \\ -\beta_{\text{CoM}}\gamma_{\text{CoM}} & \gamma_{\text{CoM}} \end{bmatrix} \begin{bmatrix} E_m^{*\prime} \\ p_\parallel^* \end{bmatrix} \,, \tag{1.86}$$

where $p_\parallel^* = p^* \cos\theta$ is the momentum along the boost direction for mass m. The value of p^* is fixed: we know $\gamma_m^* = E_m^*/m$, and the corresponding value for β_m^* is readily computed as $\beta_m^* = \sqrt{1 - (1/\gamma_m^*)^2}$. Then, we will have $p_m^* = \beta_m^* \gamma_m^* m$.

Further, we can simplify the expression by making use of the fact that $\gamma_{\text{CoM}} = \gamma_m^*$, and similarly for β_{CoM}. With all this, the energy E_m in the lab frame reads

$$E_m = \gamma_{\text{CoM}} \gamma_m^* m - \gamma_{\text{CoM}} \beta_{\text{CoM}} \gamma_m^* \beta_m^* m \cos\theta \tag{1.87}$$

$$= \left(1 - \beta_{\text{CoM}}^2 \cos\theta\right) \gamma_{\text{CoM}}^2 m \tag{1.88}$$

$$= \left(1 - \beta_{\text{CoM}}^2 \cos\theta\right) \gamma_{\text{CoM}}^2 m \,. \tag{1.89}$$

This will be maximized when $\cos\theta$ is as small as possible, so we require $\cos\theta = -1$;[a] this means we get

$$E_m = \left(1 + \beta_{\text{CoM}}^2\right) \gamma_{\text{CoM}}^2 m \tag{1.90}$$

$$= \left(2 - \frac{1}{\gamma_{\text{CoM}}^2}\right) \gamma_{\text{CoM}}^2 m \tag{1.91}$$

$$= \left(2\gamma_{\text{CoM}}^2 - 1\right) m \,. \tag{1.92}$$

Before proceeding, we should remember that the quantity we are after is not the final energy E_m but the *change* in energy of particle m, which starts out stationary, so $\Delta E = E_m - m$:

$$\Delta E = 2\left(\gamma_{\text{CoM}}^2 - 1\right) m \tag{1.93}$$

$$= 2\left(\frac{(\gamma M + m)^2}{M^2 + m^2 + 2\gamma M m} - 1\right) m \tag{1.94}$$

$$= 2\frac{\left(\gamma^2 M^2 + m^2 + 2\gamma M m\right) - M^2 - m^2 - 2\gamma M m}{M^2 + m^2 + 2\gamma M m} m \tag{1.95}$$

$$= 2\frac{M^2\left(\gamma^2 - 1\right)}{M^2 + m^2 + 2\gamma M m} m \tag{1.96}$$

$$= 2\frac{\gamma^2 \beta^2}{1 + 2\gamma m/M + (m/M)^2} m \,. \tag{1.97}$$

[a] While setting $\theta = 0$ yields the minimum, $E_m = m$: the collision is perfectly elastic and time-symmetric, m is bounced straight back in the CoM frame, so it returns stationary in the lab frame.

2 Matter-radiation interaction

Tuesday 2021-11-23

The binding energy of electrons to atoms is typically of the order of $1 \div 10\,\text{eV}$; the maximum change in energy of an electron struck by a high-energy particle is $\Delta E \approx 2 m_e \beta^2 \gamma^2$.

This is a lot for the electron, but a small amount compared to the initial energy of the high-energy particle.

The trajectory of the large particle is also deflected but only by a small amount. The quantity we are interested in computing is the average change in energy per unit length: $\mathrm{d}E/\mathrm{d}x$.

Suppose we have a beam of particles with a certain flux Φ, defined as

$$\Phi = \frac{\#\text{ incoming particles}}{\text{time} \times \text{surface}}\,. \tag{2.1}$$

After hitting a target, these particles will go along a trajectory defined by angles θ, φ from the interaction point.

The differential amount of particles emitted in a certain direction, $\Delta N_s(\theta, \varphi)$, will satisfy

$$\Delta \dot{N}_s(\theta, \varphi) \propto \Phi\,, \tag{2.2}$$

so we define

$$\frac{\Delta \dot{N}_s}{\Delta\Omega} = \Phi \frac{\Delta\sigma}{\Delta\Omega}\,, \tag{2.3}$$

where $\Delta\Omega$ is a small solid angle. This allows us to compute the differential cross-section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{\Phi}\frac{\mathrm{d}\dot{N}_s(\theta, \varphi)}{\mathrm{d}\Omega}\,. \tag{2.4}$$

We can further define the total, or integral cross-section:

$$\sigma = \int \frac{1}{\Phi}\frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega}\,\mathrm{d}\Omega\,. \tag{2.5}$$

This is all written with respect to a single target, but in practice we will have a certain slab of matter with thickness δx, and a particle flux impacting on it within an area A. In this case, then, we also need to account for the number of targets N_T:

$$\frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} = \Phi N_T \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,. \tag{2.6}$$

If n is the number density of the targets, and ρ is the matter density there, the number of targets will read $N_T = nA\delta x$. With this, we find

$$\frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} = \underbrace{\Phi A}_{\dot{N}_b} n\delta x \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}\,, \tag{2.7}$$

where we can identify the incoming beam particle rate $\dot{N}_b$. The definitions for the differential and total cross-sections will then read

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{\dot{N}_b}\frac{1}{n\delta x}\frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} \tag{2.8}$$

$$\sigma = \frac{1}{\dot{N}_b}\frac{1}{n\delta x}\underbrace{\int \frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega}\,\mathrm{d}\Omega}_{=\dot{N}_s}\,. \tag{2.9}$$

There is an alternative way to compute this in terms of QFT quantities, but we will not concern ourselves with it.

If we know the cross-section for, say, electron-proton interactions, we can compute the number of scatterings as

$$\dot{N}_s = \dot{N}_b (n \, \mathrm{d}x) \sigma \,. \tag{2.10}$$

These quantities all depend on $n\delta x$, never on the number density or the thickness singularly.

How does the intensity of a particle beam decrease as it travels through a medium? We know $I(x = 0)$, and we want to compute the dependence $I(x)$. In a certain thickness Δx, we will have a certain number of interactions $\Delta N_s = N_b (n \Delta x) \sigma$, so we can define the probability of interaction

$$\frac{\Delta N_s}{N_b} = \Delta P = n\sigma\Delta x \,. \tag{2.11}$$

The probability of *not* having an interaction is $1 - n\sigma\Delta x$. The survival probability $P(x)$ is the probability of a particle surviving at least until x. We can write a relation for $P(x + \Delta x)$:

$$P(x + \Delta) = P(x)(1 - n\sigma\Delta x) \,, \tag{2.12}$$

since this means that in this Δx the particle still has not interacted. Then,

$$P(x + \Delta x) - P(x) = \Delta P = -P(x) n\sigma\Delta x \tag{2.13}$$

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -n\sigma P \tag{2.14}$$

$$P(x) \propto \exp(-n\sigma x) \,. \tag{2.15}$$

We can also compute the average x at which an interaction occurs:

$$\langle x \rangle = \frac{\int_0^\infty x \exp(-n\sigma x) \, \mathrm{d}x}{\int_0^\infty \exp(-n\sigma x) \, \mathrm{d}x} = \frac{1}{n\sigma} = \Lambda \,, \tag{2.16}$$

which can therefore be interpreted as the *mean free path*.

References

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