High Energy Experimental Astroparticle Physics

Jacopo Tissino

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Introduction

This first part of this course is given by Ivan De Mitri.

This is not a course on particle detectors: the basics will be assumed, and there will be some short courses about the details. It is more about "experimental tools": how do we design an experiment?

"High energy", here, means roughly considering energies $E \gtrsim 1\,\text{GeV}$, but there will be some things in the MeV range as well. A more apt description of this field is that it is about observing things that come from outside the Earth: particles or high-energy radiation.

The other, "low-energy" course is about the search for rare events, where the issue is the presence of a large background, so we need to go underground.

Then, there is neutrino physics which is at the intersection between these two areas.

The part on gravitation and cosmology, on the other hand, is wholly distinct.

Cosmic rays

We start with a review of some basic concepts.

As a first approximation, the spectrum of cosmic particles/radiation $\phi(E)$ is decreasing with E. From $E \sim 10^8\,\text{eV}$ to $E \sim 10^{20}\,\text{eV}$ there is roughly a powerlaw, $\phi \propto E^{-\gamma}$ with $\gamma \sim 2.7$ at first, then $\gamma \sim 3$ (as shown in figure 1).

With artificial particle accelerators we can probe up to roughly $10^{13}\,\text{eV}$ (since in the center of mass we have $(7+7)\,\text{TeV}$).

Since the spectrum is roughly a power-law with index 3 spanning 12 orders of magnitude, the number of observed events changes by roughly 12×3 orders of magnitude between its ends.

There is a "knee" in the energy spectrum, where the spectral index γ goes from 2.7 to 3, is at about $3 \times 10^{15}\,\text{eV}$.

Typically, below the knee we can do direct, small experiments, while above it we need to do indirect measurements: they need to be very large, since the presence of a single event becomes very rare.

The particles which have been found to make up **cosmic rays** are protons p, Helium nuclei He, heavy nuclei such as Fe, γ rays, electrons and positrons $e^{\pm}$, antiprotons $\overline{\text{p}}$. Antihelium still has to be discovered, but there has been some progress in this regard lately.
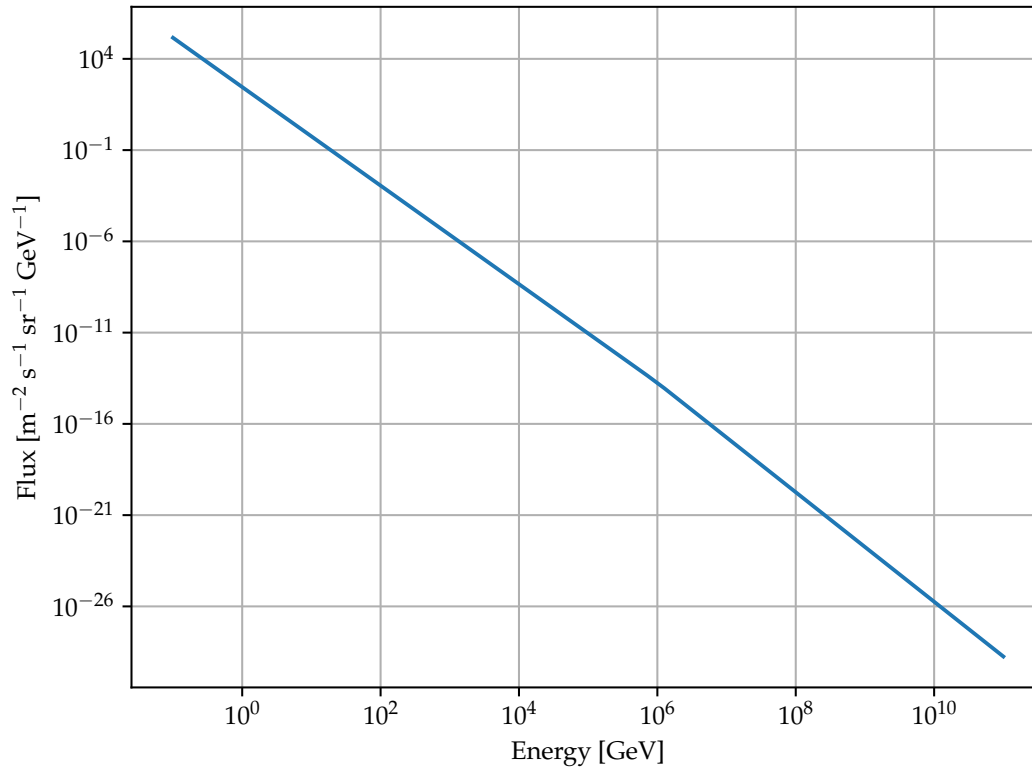
Figure 1: A rough sketch of the cosmic ray flux dependence on the energy.

Also, there are neutrinos: solar ones are on the scale of the MeV, up to $10\,\text{MeV}$. Protons come down the atmosphere and interact with it a few tens of km above, producing other things, such as neutrinos (these are "atmospheric neutrinos"). Alternatively, we can have a source producing neutrinos directly, "astrophysical neutrinos".

Why are we doing these experiments?

1. We can explore the astrophysical processes allowing for the production of such high-energy particles, and

2. we can probe high-energy particle physics.

In order to match the center-of-mass energy of the LHC collisions we'd need to have $10^{17}\,\text{eV}$ in a fixed-target experiment.

This is due to the fact that fixed-target experiments have much lower center-of-mass energetics in general.

Here is a quick sketch of why, in natural units, using the formalism detailed in the second half of this lecture. If we have two particles colliding and with equal and opposite momenta the total four-momentum will be $p_\text{tot} = (E, \vec{p}) + (E, -\vec{p}) = (2E, \vec{0})$, therefore the total center-of-mass energy will be $\sqrt{s} = 2E$.

On the other hand, in a fixed-target experiment we will have one of them being stationary, so the total four-momentum will be $p_{\text{tot}}(E, \vec{p}) + (m, \vec{0}) = (E + m, \vec{p})$, where the momentum $\vec{p}$ is determined by $m^2 = E^2 - m^2$, so the magnitude will be $\sqrt{s} = \sqrt{(E + m)^2 - (E^2 - m^2)} = \sqrt{2m(E + m)}$.

In terms of the Lorentz factor γ the beam-beam collision is $\sqrt{s} = 2m\gamma$, while the fixed-target one is $\sqrt{s} = m\sqrt{2(1 + \gamma)}$. For large values of γ, the first one clearly wins out, by a factor roughly $\sqrt{\gamma}$.

Specifically, if one wants $\sqrt{s} = 14\,\text{TeV}$, this can be achieved with $E \approx 7\,\text{TeV}$ in a beam-beam collision, or with $E \approx 1000\,\text{TeV}$ in a fixed-target collision. The ratio, as expected, is roughly $70 \div 80 \approx \sqrt{\gamma} = \sqrt{7\,\text{TeV}/1\,\text{GeV}}$ when accounting for the fact that we need to accelerate twice as many particles in the beam-beam scenario.

History

Cosmic rays were discovered in the early 1900s, and up to the early 50s most particle physics was done through them.

In 1957 the antiproton was discovered by Segrè and Chamberlain. This was done with accelerators, thanks to proton collisions producing secondaries, in a process like

$$p + A \rightarrow p + A + p + \bar{p}. \tag{0.1}$$

The cosmic ray discovery of the antiproton was performed only a few months later.

References

A good one is Aloisio et al. [Alo+18].

1 Review

1.1 Relativistic kinematics

Suppose we have a frame O and a different frame O' moving with constant velocity along the $x \sim x'$ direction.

If we define, in terms of the speed of light c and the relative velocity v between the two frames,

$$\beta = \frac{|\vec{v}|}{c} \qquad \text{and} \qquad \gamma = \frac{1}{\sqrt{1 - \beta^2}}, \tag{1.1}$$

then all observations in one frame can be connected with ones in the other thanks to the Lorentz transformation law.

The way the four-vector $(ct, x, y, z)^\top$ transforms is

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}. \tag{1.2}$$

This is actually a general law, since we can write the transformation decomposing the position vector into parallel and perpendicular components to the motion:

$$\begin{bmatrix} ct' \\ r'_{\parallel} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ r_{\parallel} \end{bmatrix} \tag{1.3}$$

$$r'_{\perp} = r_{\perp} . \tag{1.4}$$

We can denote $(ct, x, y, z)^{\top}$ as $(x_0, x_1, x_2, x_3) \equiv x$.

The scalar product used by these vectors is the mixed signature

$$x \cdot y = x^{\mu} \eta_{\mu\nu} x^{\nu} , \tag{1.5}$$

where

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} , \tag{1.6}$$

because we are using the evil particle physicist's mostly-minus convention.

We choose this because of the invariance of the spacetime interval.

Lorentz transformations are "rotations" in the sense that they leave the magnitude of vectors unchanged.

We need to define a *velocity* four-vector. In the nonrelativistic case, $\vec{v} = \mathrm{d}\vec{r}/\mathrm{d}t$; how do we extend it to the relativistic case?

We could define this as $\mathrm{d}x_{\mu}/\mathrm{d}t$, but this is not invariant since time is not invariant (i.e. it is not a scalar).

Instead, we define the *proper time* as the time *as measured in the reference frame of the particle*. We know that $\Delta t = \gamma \Delta \tau$ by the Lorentz transformation law: therefore, we can define the relativistic velocity as

$$u_{\mu} = \lim_{\Delta\tau \to 0} \frac{\Delta x_{\mu}}{\Delta \tau} = \gamma \frac{\mathrm{d}x_{\mu}}{\mathrm{d}t} . \tag{1.7}$$

This is indeed a four-vector. Its components are $(\gamma c, \gamma v_x, \gamma v_y, \gamma v_z)^{\top}$, where $\vec{v}$ is the non-relativistic velocity $\mathrm{d}\vec{x}/\mathrm{d}t$.

The magnitude of this four-vector must be a Lorentz invariant: it comes out to be

$$u^2 = \gamma^2 \left(c^2 - |v|^2 \right) = c^2 . \tag{1.8}$$

In the nonrelativistic case the momentum is $\vec{p} = m\vec{v}$, while in the relativistic case we can define $p = mu$.

Its components will be

$$p^{\mu} = (\gamma mc, \gamma m\vec{v})^{\top} . \tag{1.9}$$

The term γmc is the (kinetic + rest) energy divided by the speed of light:

$$\gamma mc = \frac{mc}{\sqrt{1 - \beta^2}} \approx mc\left(1 - \frac{1}{2}\beta^2\right) \tag{1.10}$$

$$\approx \frac{1}{c}\left(mc^2 + \frac{1}{2}mv^2 + \mathcal{O}\left(\beta^4\right)\right). \tag{1.11}$$

Therefore, we can write $p = (E/c, \vec{p})^\top$, where $\vec{p} = \gamma m\vec{v}$ and $E = \gamma mc^2$. This is a four-vector, and transforms as such, since the mass is a scalar. Its magnitude is $p^2 = m^2c^2$. This relation means that

$$E^2 = (mc^2)^2 + (|p|^2c^2)^2. \tag{1.12}$$

The conventional way to define the kinetic energy is $E_{\text{kin}} = E - mc^2$.

If we have a system of particles with four-momenta p_i we can compute the total energy and the total three-momentum by adding them all together.

If these particles scatter the four-momentum is conserved. What is the variable associated with the modulus of the total momentum?

$$p_{\text{tot}}^2 = \left(\frac{\sum_i E_i}{c}\right)^2 - \left|\sum_i \vec{p}_i\right|^2 \overset{\text{def}}{=} M^2c^2s. \tag{1.13}$$

The variable M is called *invariant mass*, and it is also written in terms of the Mandelstam variable s as $\sqrt{s}$ in the case of a two-two process.

1.2 The J/ψ discovery

Professor Samuel Ting got the Nobel Prize in 1976. His experiments can be used as an illustration for the formalism outlined here.

They were doing fixed-target experiments in Brookhaven: protons against Beryllium targets. The process looked like

$$\text{p} + \text{N} \to x + e^+ + e^-, \tag{1.14}$$

where the kinematic properties of the electron and positron were measured by spectrometers.

So, they knew $E^\pm$ and $\vec{p}^\pm$ (where $+$ and $-$ denote the positron and electron respectively). This allows one to compute

$$m_{e^+e^-} = \frac{1}{c}\sqrt{(p_{e^+} + p_{e^-})} \tag{1.15}$$

$$= \frac{1}{c}\sqrt{\left(\frac{E^- + E^+}{c}\right)^2 - |\vec{p}_+ + \vec{p}_-|^2}. \tag{1.16}$$

If we plot a histogram of these quantities, we get a big spike at $m_{e^+e^-}c^2 = 3.1\,\text{GeV}$.

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Therefore, we have a good suggestion of the fact that there was an intermediary:

$$p + N \rightarrow Y + x \tag{1.17}$$

$$Y \rightarrow e^+ + e^- . \tag{1.18}$$

Also, we know that $m_Y \approx 3.1 \, \mathrm{GeV}/c^2$.

At the same time, on the opposite coast of the US, a different group was looking at e^+e^- collisions, and considering the number of reactions as a function of the center-of-mass energy: they also saw a peak in the effective cross-section at 3.1 GeV.

This was happening in 1974, and this particle was dubbed the J/ψ.

For one event they saw a "ψ" shape. It was later discovered that the J/ψ particle was a meson made of charm quarks, $c\bar{c}$.

The width of the peak is related to the decay time of the particle: $\tau \Delta E \sim \hbar$. Very narrow peaks mean that τ is quite large — there are kinematic reasons why this particle has a hard time decaying, which we will not go into here.

A similar experiment was done in Frascati a few months later, with ADONE (which is a larger ADA, "Anello di Accumulazione").

1.2.1 A $1 \rightarrow 2$ decay example

Suppose we have a particle with mass M decaying into two particles with masses m_1 and m_2.

In the lab system M will be moving, but we can look at the decay in the *center-of-mass* frame, in which p_{tot} is purely timelike. This is typically denoted with a star: $p^*_{\text{tot}} = (Mc, \vec{0})^\top$.

After the decay, the two particles are produced with momenta $\vec{p}^*_1 = -\vec{p}^*_2$ by conservation of momentum.

The angle θ^* is the one made by the two particles 1 and 2 with respect to the propagation direction of M, as measured in the CoM frame.

The energy conservation law reads

$$E^*_1 + E^*_2 = Mc^2 \tag{1.19}$$

$$E^*_1 = \sqrt{(m_1 c^2)^2 + (|p^*_1|c)^2} \tag{1.20}$$

$$E^*_2 = \sqrt{(m_2 c^2)^2 + (|p^*_2|c)^2} , \tag{1.21}$$

where $p^*_1 = -p^*_2$.

We can compute $p^2_1 = m^2_1 c^2$ as the square of $p - p_2$: this yields

$$(p - p_2)^2 = p^2 + p^2_2 - 2p \cdot p_2 \tag{1.22}$$

$$m^2_1 c^2 = M^2 c^2 + m^2_2 c^2 - 2 \left(\frac{E^*}{c} \frac{E^*_2}{c} - \vec{p}^* \cdot \vec{p}^*_2 \right) \tag{1.23}$$

$$= M^2 c^2 + m^2_2 c^2 - 2ME^*_2 , \tag{1.24}$$

therefore

$$E^*_2 = \frac{M^2 c^4 + m^2_2 c^4 - m^2_1 c^4}{2Mc^2} . \tag{1.25}$$

We know that this will be $E_2^* \geq m_2 c^2$, which can be found to be equivalent to $M \geq m_1 + m_2$. The same reasoning applies for E_1^*, swapping $1 \leftrightarrow 2$.

This is independent of the center-of-mass emission angle θ^*.

A two-output decay like this is a very "constrained" problem: in the CoM frame the energies of the particles are fully determined.

We can recover the lab-frame quantities by a Lorentz transformation with velocity $-\beta$ (opposite to the motion of M). The decomposition of the momentum in the CoM frame is

$$p_\parallel^* = p^* \cos \theta^* \tag{1.26}$$

$$p_\perp^* = p^* \sin \theta^* . \tag{1.27}$$

Consider a positively charged pion: it decays as

$$\pi^+ \to \mu^+ + \nu_\mu . \tag{1.28}$$

We can compute the value E_μ^* in this case, for example, approximating the mass of the neutrino as 0. In natural units, the mass of the pion is $m_{\pi^+} \approx 139.57\,\text{MeV}$, while the mass of the muon is $m_{\mu^+} \approx 105.66\,\text{MeV}$ [Gro+20]; this means that in the center of mass we will have

$$E_\mu^* \approx \frac{m_\pi^2 + m_\mu^2}{2 m_\pi} \approx 109.78\,\text{MeV} \tag{1.29}$$

$$E_\nu^* \approx \frac{m_\pi^2 - m_\mu^2}{2 m_\pi} \approx 29.79\,\text{MeV} . \tag{1.30}$$

As expected, $m_\pi = E_\mu^* + E_\nu^*$.

As a reminder, the Lorentz factor can be written as $\gamma = E/mc^2$, while the factor β can be written as $\beta = |\vec{p}|c/E$.

Now we move to natural units: $\hbar = c = 1$.

We come back to our decay $M \to m_1 + m_2$. We have computed the CoM energies and momenta, now we need to move back to the lab frame. The Lorentz transformation with $-\beta$ reads

$$\begin{bmatrix} E_i \\ p_{i,\parallel} \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} E_i^* \\ p_i^* \cos \theta^* \end{bmatrix} \tag{1.31}$$

$$= \begin{bmatrix} \gamma \left(E_i^* + \beta p^* \cos \theta^* \right) \\ \gamma \left(\beta E_i^* + p^* \cos \theta^* \right) \end{bmatrix} . \tag{1.32}$$

The emission angles may change depending on the interaction, so the θ^* dependence is relevant.

We can recover the emission angle in the lab frame by

$$\tan \theta_i = \frac{p_{i,\perp}}{p_{i,\parallel}} = \frac{p_i^* \sin \theta^*}{\gamma \left(\beta E_i^* + p_i^* \cos \theta^* \right)} . \tag{1.33}$$

Even if the emission is isotropic in the CoM frame, it will not be in the lab frame; qualitatively, it will be peaked in the forward direction.

If $\theta^* = 0$, we get $\theta = 0$. If $\theta^* = \pi/2$, we get

$$\tan \theta = \frac{1}{\gamma} \frac{p^*}{\beta E_i^*} = \frac{1}{\gamma} \frac{\beta_i^*}{\beta^*}. \tag{1.34}$$

In the ultrarelativistic limit, this will be $\sim 1/\gamma$: therefore, we will get $\theta \sim \arctan(1/\gamma) \sim 1/\gamma$.
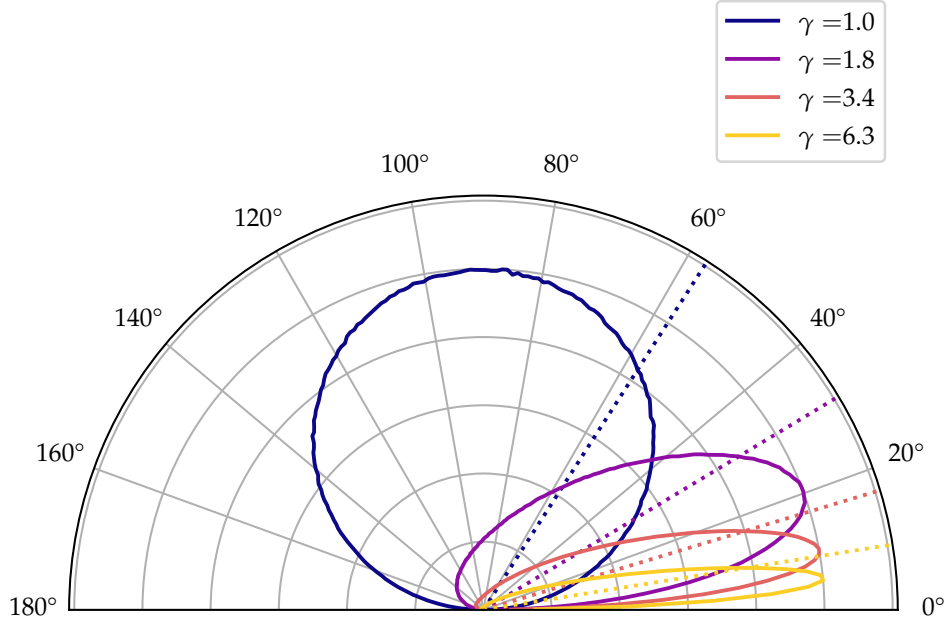

Figure 2: Angular distribution of the emitted radiation if the starting emission is isotropic, which means that the probability distribution for θ^* is $\mathrm{d}p = \sin \theta^* \, \mathrm{d}\theta^*$, which corresponds in the plot to the $\gamma = 1$ case. Here we are assuming that the decay looks like $1 \to 2 + 3$, where both 2 and 3 are massless. We also show the $\theta \sim 1/\gamma$ approximation as a dashed line. In the high γ limit, this is the median of the distribution.

Let us consider the decay of a neutral pion:

$$\pi^0 \to \gamma + \gamma. \tag{1.35}$$

In the CoM system, both photons will get $E^* = M/2 = p^*$, where $M = m_{\pi^0} \approx 134.98 \, \text{MeV}$.

The lab-frame energy of one of these is given by

$$E = \gamma(E^* + \beta p^* \cos \theta^*) \tag{1.36}$$

$$= \gamma E^*(1 + \beta \cos \theta^*) \tag{1.37}$$

$$= \frac{E_\pi}{m_\pi} \frac{m_\pi}{2} (1 + \beta \cos \theta^*) \tag{1.38}$$

$$= \frac{E_\pi}{2} (1 + \beta \cos \theta^*). \tag{1.39}$$

The maximum of this quantity is $E_\pi(1 + \beta)/2$; its minimum is $E_\pi(1 - \beta)/2$. This is just a bit smaller than the distribution $E \in [0, \pi]$.

What is the distribution of the photons we get in the lab frame? How wide will the detector need to be? Well, we can frame it by selecting the size we need to get a certain high percentage of the emitted photons.

An order-of-magnitude estimate is: half of the photons are emitted with $\theta^* < \pi/2$, and their angle will be $\lesssim 1/\gamma$.

The original J/ψ papers are in the drive — add citation.

1.2.2 A $1 \to 3$ decay example

Now the three emitted particle momenta in the CoM frame will satisfy

$$\vec{p}_{\text{tot}}^* = p_1^* + p_2^* + p_3^* = 0. \tag{1.40}$$

We define θ^* as θ_1^*; the trick to solving this problem is to "bunch" particles 2 and 3 into one, with momentum $\vec{p}_{23}^* = p_2^* + p_3^*$. The invariant mass of this system will read

$$m_{23} = \sqrt{p_2^2 + p_3^2 + 2p_2 \cdot p_3} \tag{1.41}$$

$$= \sqrt{m_2^2 + m_3^2 + 2E_2^* E_3^* - 2|p_2^*||p_3^*| \cos \theta_{23}^*}, \tag{1.42}$$

where θ_{23}^* is the angle between 2 and 3 in the CoM frame.

We can recurse back to the 2 body formulas for the behaviour of the 1 and 23 system: the energy of 1 will be

$$E_1^* = \frac{M^2 + m_1^2 - m_{23}^2}{2M}. \tag{1.43}$$

Now, even in the CoM frame the energy of particle 1 (which is arbitrary) depends on the emission angle θ_{23}^*. We can, however, compute the minimum and maximum values of E_1^*.

There is a "trick" which allows us to skip the whole computation. The idea is that m_{23} will be maximum when the most energy will be given to the 23 system; so this happens when m_1 is produced at rest, while the other two are back-to-back.

Then, we will have $p_2^* = -p_3^*$.

In this case, we will have $m_{23}^{\max} = M - m_1$. It's as if both particles (1 and the "combined particle" 23) were produced at rest.

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In this case, then,

$$E^*_{1,\min} = \frac{M^2 + m_1^2 - (M^2 + m_1^2 - 2Mm_1)}{2M} = m_1 \,, \tag{1.44}$$

as expected.

The other extreme happens when $m_{23} = m_2 + m_3$; in this case, then, there must exist a frame in which both particles are at rest. This means that they must travel with the same velocity. This does not mean that the momenta are equal, in general $p_2^* \neq p_3^*$, but $\vec{v}_2^* = \vec{v}_3^*$.

The maximum energy is therefore

$$E^*_{1,\max} = \frac{M^2 + m_1^2 - (m_2 + m_3)^2}{2M} \,. \tag{1.45}$$

The most famous example here is β decay: from a nuclear perspective it is

$$N(A, Z) \to N(A, Z+1) + e^- + \overline{\nu}_e \,, \tag{1.46}$$

since from a microphysical perspective what happened was

$$\mathrm{n} \to \mathrm{p} + e^- + \overline{\nu}_e \,. \tag{1.47}$$

Proton decay has not been observed, but β^+ decay can happen since the nucleus is also involved.

In the lab system, which is also the CoM system, we will have a minimum energy of 511 keV, and a maximum energy of roughly 1.29 MeV, which comes from the approximation

$$E_{e,\max} \approx \frac{M^2(A, Z) - M^2(A, Z+1)}{2M(A, Z)} \,. \tag{1.48}$$

If the neutrino were massive, this maximum of the distribution will be shifted down a bit. This is an experimental challenge, since the mass of the neutrino is at most on the eV scale.

Let us consider the decay $\pi^- \to \mu^- + \overline{\nu}_\mu$.

The decay $\pi^- \to e^- + \overline{\nu}_e$ is forbidden by the handedness of weak interactions. The helicity is conventionally positive for antiparticles and negative for particles.

As a first approximation, if the antineutrino is going to the left and the electron is going to the right, both spins will be to the left, summing to 1 to the left, while the pion has spin 0.

This is also the case for decay towards muons, but $m_\mu \gg m_e$, therefore the probability for the muon to have a spin component to the right is much higher.

References

[Alo+18]   Roberto Aloisio et al. "Selected Topics in Cosmic Ray Physics". In: *Multiple Messengers and Challenges in Astroparticle Physics*. Ed. by Roberto Aloisio, Eugenio Coccia, and Francesco Vissani. Cham: Springer International Publishing, 2018, pp. 1–95. ISBN: 978-3-319-65423-2 978-3-319-65425-6. DOI: 10.1007/978-3-319-65425-6_1. URL: http://link.springer.com/10.1007/978-3-319-65425-6_1 (visited on 2021-11-08).

[Gro+20]   Particle Data Group et al. "Review of Particle Physics". In: *Progress of Theoretical and Experimental Physics* 2020.8 (Aug. 14, 2020). DOI: 10.1093/ptep/ptaa104. URL: https://academic.oup.com/ptep/article/2020/8/083C01/5891211 (visited on 2021-11-08).