Gravitational Waves @ Jena University

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Introduction

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The syllabus can be found here.

Interesting things on the Indico server of Jena university.

In this first lecture, a basic introduction to the theory of gravitational waves: Einstein's first papers, the sticky bead argument by Bondi & Feynman, the quadrupole formula:

$$\overline{h}_{ij}(t,r) = \frac{2G}{c^4 r}\ddot{I}_{ij}(t-r)\,. \tag{0.1}$$

The idea behind the multipole expansion is that we are solving the Poisson equation $\nabla^2\phi = \rho$, so

$$\phi(\vec{r}) = \int \frac{\rho(\vec{x})\,\mathrm{d}^3x}{|\vec{r}-\vec{x}|}\,, \tag{0.2}$$

so as long as we are far away from the source we will see

$$\phi(\vec{r}) = -\frac{q}{r} - \frac{p_i n^i}{r^2} - \frac{Q_{ij}n^i n^j}{r^3} + \dots\,. \tag{0.3}$$

Quiz: which of these are GW sources?

1. spherical star: no, its quadrupole is vanishing;
2. rotating star: no, its quadrupole is constant;
3. star with a mountain: yes, its quadrupole evolves (potential source of continuous GW);
4. supernova explosion: yes, if there is asymmetry (potential source of burst GW);
5. binary system: yes, already detected!

Claim 0.1. *Order of magnitude expression:*

$$h \lesssim \frac{GM}{c^2D}\frac{v^2}{c^2} = \frac{R}{D}\frac{GM}{c^2R}\left(\frac{v}{c}\right)^2, \tag{0.4}$$

where D is the distance to the object, R is the characteristic scale of the object (so that GM/c^2R is the compactness), while v is the characteristic velocity. The quantity we calculate is $h \sim \delta L/L$, the strain.

Proof. To do. □

The Hulse-Taylor pulsar. The two-body problem in GR is difficult.

The typical waveform in the PN region looks like:

$$h_+(t) \approx \frac{4}{r}\left(\frac{GM_c}{c^2}\right)^{5/3}\left(\frac{\pi f_{\mathrm{gw}}(t)}{c}\right)^{2/3}\cos\Big(2\pi f_{\mathrm{gw}}(t)t\Big)\,, \tag{0.5}$$

then we need numerical relativity to simulate the plunge and merger, and finally the ring-down is simulated using BH perturbation methods. The mass scale is

$$h(t) \sim \nu\frac{1}{r/M}(Mf_{\mathrm{gw}})^{2/3}\,, \tag{0.6}$$

while

$$\phi_{\mathrm{gw}}(t) \sim 2\phi_{\mathrm{orb}}(t) = 2M_c^{-5/8}t^{5/8} = 2\nu^{-3/8}\left(\frac{t}{M}\right)^{5/8}, \tag{0.7}$$

where $\nu = \mu/M$, and $\mu = 1/(1/M_1 + 1/M_2)$.

Multiple detectors are crucial for sky localization, as well as for the measurement of polarization.

At leading order, the two-body problem in GR is scale-invariant: the length of the signal can be estimated simply from the mass of the stars involved.

R-process nucleosynthesis might have something to do with BNS mergers, if the stars are torn apart by the collision.