Experimental Gravitation and Cosmology

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Wednesday 2021-11-17

GW detection! This part is given by Jan Harms.

Marica will talk more about GW science and the multimessenger approach.

The first slide is the north arm of Virgo, Einstein Telescope and the LGWA.

Outline:

1. overview of GW detection;
2. detector responses to GW;
3. noise spectra, filtering, SNR, transfer functions;
4. fluctuation-dissipation, thermal noise;
5. quantum noise & squeezing;
6. passive and active seismic isolation, Newtonian noise;
7. GW detector concepts.

The interferometer is operated at the dark fringe

but not exactly there?

At a frequency scale of $\sim H_0$ we can do measurements thanks to CMB observations.

At the nHz scale (corresponding to lyr wavelengths) we can use Pulsar Timing Arrays, with radio telescopes. Pulsars being the "best clocks in the universe" is not really true. For now, they've discovered red noise, which could potentially be a GW signal.

At the mHz scale we might have space detectors such as LISA: here the GW wavelengths are of the order of 1 AU.

At the 100 Hz scale we have ground-based detectors.

Current infrastructure is expected to reach $z \sim 2$; we expect future infrastructure to reach $z \sim 100$. This goes all the way back to the dark ages in the Universe's history.

This means that ET + CE would see something like 90 % of BNS mergers, and basically all BBH mergers.

It is good to have a long baseline.

Jan disses Cosmic Explorer a bit. 40 km is ambitious because of the cross-couplings due to the fact that the mirrors will not be perpendicular to the suspension system, which will point straight down at both edges. Also, with such lengths there is a loss in sensitivity in the kHz band.

High performance sensing will require cold temperatures, which however also means that we need to redesign basically all the infrastructure.

The dissipation in the system dictates the thermal noise.

We need in-vacuum optical systems; people were extremely worried before the construction of LIGO about this. Now in Livingston they found a hole. The idea to find it is like finding a hole in a bike tire, with liquid helium around the detector and helium sensors inside.

Current detectors have a finite lifetime. Virgo is slowly sinking into the ground, since it is built on soft soil.

There are disturbances caused by powerful lightning strikes. It is crucial to monitor the environment. In the future, there will be online background removal.

Einstein Telescope needs very big underground chambers, with roofs on the order of 25 m. Do we have enough people in Europe to build ET? Not clear.

Kagra taught us some lessons. The spring melt filled up water reservoirs there, so there was a waterfall inside the arms there. That means a lot of humidity. The water stream near the test masses was gravitationally coupled to them; Newtonian noise is bad.

If ET is built in Sardinia it will be really dry; maybe not so for the Netherlands. We need to be careful that the ventilation and cryogenic systems do not create noise.

Ventilation is needed for both humidity and radioactivity management. The experiments in LNGS, say, are much more sensitive.

LISA will be able to locate sources thanks to their amplitude modulation during the year. At these frequencies going underground would not help.

There is a requirement for drag-free navigation, though.

The beam to another satellite is on the order of 25 km in size.

Now we talk about LGWA. Weber invented everything when it comes to GW detection technologies. He invented the resonant bar detectors.

He tried detecting GWs from quadrupolar seismic vibrations on the Earth! We now know that their amplitude is much too small for that. They did, however, put the first upper limits on the amplitude of GWs.

Apollo 17 brought the Lunar Surface Gravimeter. A design flaw limited its sensitivity. The problem of having 14 days of lunar night without solar power is large.

A Russian team is probing microwave beaming for the transmission of power. Nuclear power is possible, but how do we shield from it? They used plutonium there. However, nobody's producing plutonium anymore... But it might be produced again thanks to a decision by the Trump administration.

That gravimeter failed on the Moon because of an arithmetic mistake which failed to account for the decreased gravity there.

One can bring stuff to the Moon at a price of about $\$10^6/\mathrm{kg}$. This might decrease in the future.

LGWA: four seismometers at $\sim$ km separation.

LSGA: interferometers connected to the ground, measuring the deformation of the surface of the Moon. They'd need to be deployed at 10 km distance, but the Moon curves: maybe put them at the rims of a crater? this is very difficult.

GLOC: basically Cosmic Explorer on the Moon. By Jani and Loeb, two theorists. These detectors do require a lot of maintenance.

The spectrum of noise on the Moon is on the order of $10^{-10}\,\mathrm{m}/\sqrt{\mathrm{Hz}}$ between 0.1 Hz and 1 Hz.

This is much lower than what is observed on the Earth, and which is mostly due to the ocean.

Micro-meteoroid impacts have a small impact.

The Moon might be the quietest place in the Solar System: it being tidally locked helps a lot. It is also near the coldest: there are permanently shadowed regions at the poles. These have never seen sunlight, neither direct nor indirect, for a billion years or more.

It might even be colder than Uranus or Neptune: the absence of radioactivity helps a lot.

We can use superconductors for free there.

How do we get power there? An option is beaming from solar panels at the rim. Another option is beaming from satellites in orbit.

Nuclear power? Europe will not do it, since ESA does not launch for Europe (as opposed to NASA and the Chinese space agency): therefore, any issues would be dumped onto a South American country, a huge political issue.

The presence of gravitational waves from inflation, at the tensor-to-scalar ratio given by simple, single-field inflationary models, would be incredibly important.

A direct observation for this kind of thing would be the Big Bang Observer. This would be a proof of quantum gravity!

This would be a 12-satellite configuration, two triangles and a hexagon, smaller than LISA, sensitive to the deciHertz regime. Why not the milliHertz band? LISA is also limited by GW foregrounds!

Foreground removal is computationally difficult.

1 GW detectors

Monday 2021-11-22

The basic design of a GW detector is that found in LIGO Scientific Collaboration and Virgo Collaboration et al. [LIG+16, fig. 3].

It is roughly a Michelson-Morley interferometer, with 10 kW coming in from a power-recycling mirror plus a Fabry-Perot cavity amplifying that power to 100 kW.

These cavities are roughly 3 km long.

The frequency of the laser is on the order of $f \sim 10^{15}$ Hz, and it has a small dispersion in frequency space.

What then happens is that when the length of the cavities is perturbed, some power in each cavity is shifted into the *sidebands* $f_0 \pm \Delta f$ of the carrier frequency f_0, where $\Delta f = f_{\mathrm{GW}} \ll f_0$.

We can fully control the length of the arms, to get our preferred interference condition.

What can happen to laser light? It can be dissipated, it can come back to the mirror, but we need a device to dump the beam and prevent it from coming back to the laser.

A DC offset: we want a small fraction of the light, roughly 0.1 %, to get to the photodetector. We add a bit of the laser light to the sidebands.

This is a homodyne detection scheme, and the laser is called a local oscillator.

Correlated signals go back to the power recycling mirror, while anticorrelated signals go to the detector.

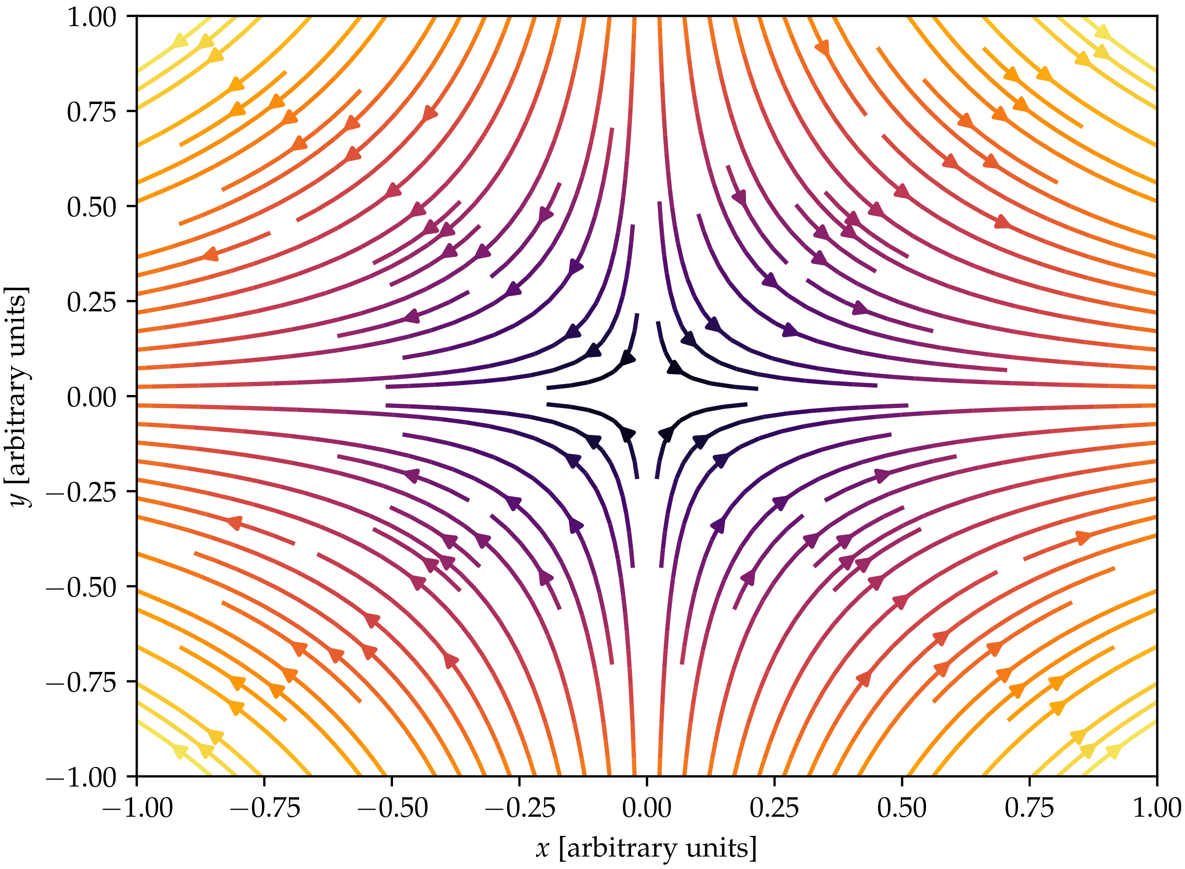

Figure 1: The effect of the h_+ polarization of gravitational waves. Color indicates the magnitude of the field (weaker in the middle).

Noise sources

The main *environmental* noise sources are

1. scattered light;
2. seismic / vibrational noise;
3. Newtonian noise;
4. electromagnetic noise.

Basically all GW detectors have a strain sensitivity curve, which is roughly speaking the noise referred to the quantity "strain": $h = 2\Delta L/L$. The "bucket" is centered around 100 Hz.

The true signal we measure is the current from the photodiode. The noise in the detector could be reported in Ampères, but plotting things in this way allows us to include the response.

A rough way to say this is

$$\text{sensitivity} = \frac{\text{noise}}{\text{response}}\,. \tag{1.1}$$

The laser is tuned to the resonant frequency of the cavity, but the same does not hold for the sidebands, which are therefore not amplified to the same amount!

However, we can do resonant amplification of the sideband signal as well!

There are around a hundred different relevant noise sources. There are "fundamental" noises and "technical" noises. The former are a limit set by the detector configuration, nothing can be done about them. The technical noises can be gradually reduced by the crew working on the detector.

At high frequencies, the problem is quantum noise; at mid-low frequencies we have thermal noise, while at low frequencies the dominant contribution is environmental noise.

Isolation from the environment means:

1. seismic isolation;
2. reducing susceptibility to EM fields;
3. picking a quiet environment;
4. dealing with scattered light;
5. vacuum system.

Building underground helps! The seismic fields are quieter there, and there is more insulation.

Why do we need to build an interferometer in Europe? We (Europeans) have optical telescopes in Chile, for example.

Telescopes only need a small amount of people to man them, while GW interferometers need tens of people, and not that many people want to work in an extremely remote location.

We only want the zeroth-order, Gaussian mode in the spatial cross-section of the decomposition of the beam.

However, since the mirrors are not perfectly spherical higher modes are also excited, with magnitudes of the order of 20 ppm. Out of the fraction, then, a tiny fraction scatters on the edge of the vacuum tube (10 ppm) and re-enters the beam. The vacuum tube is not seismically isolated! This means that that light picks up a huge amount of noise.

We therefore need to insert a *baffle*, which absorbs any light which hits it. What people now do is make detailed models of the system, use raytracers to figure out where the light is going and block it. People have also though about just coating the whole interior — then, the issue becomes maintenance and coating lifetime.

Thermal noise: it is mainly about thermal vibrations of our suspensions, our mirrors (coating and substrate), and the electronics.

Changing the mirror's thermal noise is hard, electronics could be made superconductive...

Quantum noise is quite simple: it has only two components,

1. shot noise;
2. radiation pressure noise.

Once we know how the fluctuations enter our system, we can control them! What defines the quantum state of the detector?

The scaling of the high-frequency part is mostly shot noise, RP noise has a lower frequency, and we can currently manage to make it negligible.

There are methods to manipulate quantum states in order to reduce quantum noise. The broad topic here is "quantum nondemolition techniques".

There are all kinds of other noises which enter our system, but the most important ones are the ones we mentioned.

How do we cool our experiment? We have roughly 0.5 ppm absorption, which means about a Watt of power going to the optics! A thermal link is dangerous, since it can introduce vibrations. Voyager wants to do radiative cooling, since it introduces no extra vibrations. However, getting to very low temperatures is basically impossible because of the $\sim T^4$ temperature.

So, ET needs a thermal link to get to 10 K to 20 K. Maybe superfluid helium could work for this purpose...

Timeseries analysis

The basic tenet of timeseries analysis is to work with the spectral representation of the data.

This timeseries will typically be uniformly sampled.

We define the autocorrelation as

$$C(y; t, t') = \left\langle y(t) y(t') \right\rangle \,, \tag{1.2}$$

where the brackets denote an ensemble average. If the noise is stationary, this will just be a function of $\tau = t' - t$, so we get $C(y; \tau) = \left\langle y(t) y(t+\tau) \right\rangle$.

Even things like thermal and quantum noise are not really stationary, while environmental noise is *definitely* not stationary.

Fourier transforms diverge for infinitely-extending timeseries, but we can take an alternative approach, which is consistent with the fact that our signals are finite in time.

We can only transform a section of length T; its transform will be $\widetilde{y}_T(f)$. In the case of stationary noise, we can define the single-sided Power Spectral Density as

$$\mathrm{PSD}(f) = S(f) = \lim_{T \to \infty} \frac{1}{T} \left| \widetilde{y}_T(f) \right| \,. \tag{1.3}$$

This converges for stationary noise. We can also include a 2 in the definition, it is a matter of convention. This is the standard way to represent timeseries in the frequency domain.

In terms of units, $[\widetilde{y}] = [y]/\text{frequency}$; therefore, $[S] = [y]^2/\text{frequency}$.

If we plot $\sqrt{S}$, this will have units of $[\sqrt{S}] = [y]/\sqrt{\text{Hz}}$.

The integral of the PSD over all frequencies yields

$$\int_{-\infty}^{\infty} \mathrm{d}f\, S(f) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathrm{d}t\, y_T(t) \int_{-T/2}^{T/2} \mathrm{d}t'\, y_T(t') \underbrace{\int_{-\infty}^{\infty} \mathrm{d}f\, e^{2\pi i (t-t')f}}_{\delta(t-t')} \tag{1.4}$$

$$= \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} \mathrm{d}t\, y_T^2(t) = \left\langle y^2 \right\rangle = \sigma_y^2\,. \tag{1.5}$$

We can also compute a *bandlimited* variance:

$$\sigma^2_{\text{bandlimited}} = \int_{f_1}^{f_2} \mathrm{d}f\, S(f)\,. \tag{1.6}$$

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$\tau = t' - t$.

We have the following relation for the power spectral density:

$$\left\langle \widetilde{y}(f) \widetilde{y}^*(f') \right\rangle = \int \mathrm{d}t\, e^{2\pi i f t} \int \mathrm{d}t'\, e^{-2\pi i f' t'} \underbrace{\left\langle y(t) y(t') \right\rangle}_{C(y,\tau)} \tag{1.7}$$

$$= \int \mathrm{d}t\, e^{2\pi i (f-f')t} \int \mathrm{d}\tau\, e^{-2\pi i f \tau} C(y,\tau) \tag{1.8}$$

$$= \int \mathrm{d}t\, e^{2\pi i (f-f')t} S(y,f) \tag{1.9}$$

$$= \delta(f-f') S(f)\,. \tag{1.10}$$

The fact that if $f \neq f'$ this vanishes is a crucial property of stationary noise.

Any linear transformation of such a stationary-noise timeseries will leave it stationary.

If we have a force timeseries $F(t)$ applied to a pendulum, we will get a response like

$$\widetilde{x}(f) = \frac{\widetilde{F}(f)/m}{(2\pi)^3 (f_0^2 - f^2)} \qquad \text{where} \qquad f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{L}}\,, \tag{1.11}$$

but this *linear* approximation will fail if the force is very strong. Nonlinear dynamics can turn stationary noise into nonstationary noise. There is a deep connection between linearity and stationarity.

In the time domain, the application of a linear filter looks like

$$y'(t) = \int \mathrm{d}t'\, K(t-t') y(t')\,, \tag{1.12}$$

which modifies the Fourier transform as

$$\widetilde{y}'(f) = \widetilde{K}(f) \widetilde{y}(f)\,. \tag{1.13}$$

Therefore, the spectral density of the transformed signal reads

$$\widetilde{S}(y', f) = \left| \widetilde{K}(f) \right|^2 S(y,f)\,. \tag{1.14}$$

Matched filtering

How do we define the best linear statistic? Specifically, we'd want to have something which has the largest amplitude variation between the presence of a signal and the absence of it.

If the system is linear, we can approximate the data as

$$d(t) = s(t) + n(t)\,, \tag{1.15}$$

and in the operation of the detector one tries very hard to make this as close to true as possible.

Suppose we define a filter and apply it to the noise and signal:

$$s_F(t) = \int \mathrm{d}t'\, F(t-t')s(t') \tag{1.16}$$

$$n_F(t) = \int \mathrm{d}t'\, F(t-t')n(t')\,. \tag{1.17}$$

The variance of the filtered noise will then read

$$\left\langle n_F^2(t) \right\rangle = \int \mathrm{d}f \left| \widetilde{F}(f) \right|^2 S(n,f)\,. \tag{1.18}$$

The filtered signal timeseries can also be written as

$$s_F(t) = \int \mathrm{d}f\, e^{2\pi i f t} \widetilde{s}(f) \widetilde{K}(f)\,. \tag{1.19}$$

We define the SNR timeseries through

$$\begin{aligned}
\mathrm{SNR}^2 &= \frac{s_F^2(t_0)}{\langle n_F^2 \rangle} && (1.20)\\
&= \frac{\left| \int \mathrm{d}f\, e^{2\pi i f t_0} \widetilde{s}(f) \widetilde{K}(f) \right|^2}{\int \mathrm{d}f \left| \widetilde{K} \right|^2 S(n,f)} && (1.21)\\
&\leq \int \mathrm{d}f\, \frac{|\widetilde{s}|^2}{S_n}\,. && (1.22)
\end{aligned}$$

We exploited the Schwarz inequality:

$$\begin{aligned}
\left| \int \mathrm{d}f\, e^{2\pi i f t_0} \widetilde{s} \widetilde{K} \right|^2 &= \left| \int \mathrm{d}f\, e^{2\pi i f t_0} \widetilde{K} \sqrt{S_n} \frac{\widetilde{s}}{\sqrt{S_n}} \right| && (1.23)\\
&\leq \int \mathrm{d}f \left| \widetilde{K} \right|^2 S_n \times \int \mathrm{d}f\, \frac{|\widetilde{s}|^2}{S_n}\,. && (1.24)
\end{aligned}$$

What this means is that any linear filter can reach, at a maximum, this value. If we can then write a filter which reaches this value we know it is the best!

The optimal filter reads

$$\widetilde{K}(f) = e^{-2\pi i f t_0} \frac{\widetilde{s}(f)}{S_n(f)}\,. \tag{1.25}$$

Intuitively, one puts less weight on the region of higher noise.

The kernel looks a bit like the elephant in the Little Prince.

2 Perspectives for GW

GW echoes: if the horizon is quantized, the GWs can be reflected if they are not at the right frequencies. For any known quantum gravity theories, the frequencies at which these are reflected are precisely the ones we can probe.

NS equations of states can be probed by the high-frequency region of the waveform.

SN explosions can emit GWs if they have anisotropies.

Stochastic backgrounds can be emitted by cosmological processes.

How do we calibrate our detector? Blind injections were done by moving the test masses. The first GW signal was thought to be a blind injection at first, but then we found that nobody had done it.

The issue is that we cannot continuously characterize the detector in this way. The alternative is modelling the detector.

There are controls which keep the masses stationary at low frequencies. So, at low frequencies we look at the *control signal*!

The problem is most difficult at low frequencies. The control interferes with the GW response below 50 Hz.

The equation for the strain, including this, is

$$h(t) = \frac{1}{L}\left[\mathcal{C}^{-1} d_{\text{err}}(t) + \mathcal{A} d_{\text{ctrl}}(t)\right] . \tag{2.1}$$

The inverse of the SNR gives roughly an order of magnitude for the required calibration error. Currently, the LIGO-Virgo collaboration has reached roughly 2 to 3 %.

There are very few places which can give a laser to be used in pushing to calibrate. . .

Unmodelled signals

Modelling a SN GW signal is very hard, simulations do not agree with each other.

The signal roughly increases in frequency from a hundred to a thousand Hz within about a second.

The first signal we detected was detected through a burst search: templates with masses as high as $30M_{\odot}$ were not used because those masses were not though to be likely.

Convolving a signal with wavelets allows for a time-frequency plot.

A type II supernova would have to happen in our galaxy for us to be able to see it with current instruments, and future detectors will not improve this by much.

See `gwburst.gitlab.io`.

Continuous signals

A pulsar which is not axisymmetric will emit GWs. These are signals which are definitely modelled: we know basically everything.

Computational cost scales with T^6.

Glitches are a problem in this case.

Why is there such a large cluster of millisecond pulsars?

We know from these GW observations that most of the braking in the NS spin-down is *not* due to GW emission. We have bounds on the order of $h \sim 10^{-26}$.

Quantum noise

Friday 2021-12-3

The measurement process can be thought of as "counting photons", although we cannot really determine their exact number, but this is relevant since it means the measured intensity fluctuates with $\sqrt{N}$.

If $\Delta N / \Delta t$ is constant (we get a fixed number of photons per unit time), we have a **Fock state** $\hat{n} |n\rangle = n |n\rangle$, an eigenvector of the photon number operator $\hat{n} = a^\dagger a$.

If this were the case, we would have no fluctuations in the readout. But, besides the fact that photodetectors do not count photons, we do not produce Fock states.

It is convenient to describe this process in the Heisenberg picture. If we use it, the operators we use are all functions of t while the states are unchanged. The state we will use will always just be the vacuum state $|0\rangle$.

Classically, the output field will be given by a certain linear transformation of the output field:

$$\vec{E}_{\text{out}}(\vec{x}, t) = \mathscr{L}\left[\vec{E}_{\text{in}}(\vec{x}, t)\right], \tag{2.2}$$

but this will also be what we do in our quantum-mechanical, Heisenberg-picture treatment.

The difficulty comes from the fact that there are actually many input fields: at various places, like the mirrors, we do not have "nothing" since there is always at least the vacuum.

The transmissivity τ of a mirror is the ratio between the incoming and transmitted field magnitudes. All mirrors have a transmissivity $\neq 0$, so we always have a coupling to the vacuum state on the other side of the mirror.

If there are losses, we cannot have a unitary process: so, there will never be "one-way" losses, and a loss will always be associated with an extraneous input from the environment.

The transmissivity and reflectivity satisfy $\tau^2 + \rho^2 = 1$; the transmissivity can be interpreted as a loss parameter, so if $\tau = 0$ all the field is reflected back into the system, but we will never have this condition.

So, if A is our input, some τA is leaving our system; because of what we were saying above the field going back will not be just ρA but instead $\rho A + \tau V$, where V denotes the vacuum state on the other side of the port.

A very important field is the one coming from the photodiode, which will often look like a thermal state. However, its temperature will be very low compared to the laser light: that is near-infrared, 100 to 1000 THz, corresponding to $\gtrsim 700\,\text{K}$ of thermal temperature. So, we can take it to be the vacuum.

Only about 0.5 % of the laser light at the beamsplitter reaches the photodiode. Therefore, changing the field at the laser will have little effect on the diode; on the other hand, 99.5 % of the field from the photodiode will come back to the laser.

Only in the eighties someone properly described an interferometer in QFT. Before, people just used Poissonian statistics to calculate the quantum noise.

The first important insight is that we need to act on the field at the photodiode.

Let us neglect the vector character of the field for simplicity; the fields are all linearly polarized, and the quantum mechanics for the electric and magnetic fields are the same. The field will be

$$\hat{E}(x,t) = \sqrt{\frac{2\pi\hbar}{\mathcal{A}c}} \int_{-\infty}^{\infty} \frac{\mathrm{d}\omega}{2\pi} \sqrt{\omega} \Big(\hat{a}(\omega) e^{-i\omega(t - \vec{x}\cdot\vec{\epsilon}/c)} + \hat{a}^\dagger e^{i\omega(t-\vec{x}\cdot\vec{\epsilon}/c)} \Big) , \tag{2.3}$$

where we are fixing the propagation direction by fixing $k = \omega/c$; the creation and annihilation operators will satisfy

$$\Big[\hat{a}(\omega), \hat{a}^\dagger(\omega')\Big] = 2\pi\delta(\omega - \omega') . \tag{2.4}$$

If we apply the annihilation operator to the vacuum we get $\hat{a}\,|0\rangle = 0\,|0\rangle$; since the photon number operator is $\hat{n} = \hat{a}^\dagger \hat{a}$ we also have $\hat{n}\,|0\rangle = 0\,|0\rangle$.

A **coherent state** is an eigenstate of the annihilation operator: $\hat{a}\,|\alpha\rangle = \alpha\,|\alpha\rangle$, where the eigenvalue α is a generic complex number. This is “close to classical”, it describes the output of a laser quite well, and it has Poissonian statistics.

What is the distribution of the number of photons in a coherent state?

$$p(n) = \left|\langle n|\alpha\rangle\right|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!} . \tag{2.5}$$

This is a Poissonian with average photon number $\langle n\rangle = |\alpha|^2$.

The energy in the field is given by $E = \hbar\omega_0 n$, so this value also corresponds to the mean energy in the field.

In the Heisenberg picture, one finds that the laser is mapping $\hat{a} \to \hat{a} + \alpha$. In a sense, all “classical” things are fixed complex numbers as opposed to operators.

The full operator is $\hat{a} + h + \alpha$, where $\hat{a}$ describes the quantum fluctuations, h describes the GW signal, while αdescribes the laser.

We have not yet defined what is our canonical pair of observables. We can write

$$\hat{E}(x,t) = E_1(x,t)\cos(\omega_0 t - kx) + E_2(x,t)\sin(\omega_0 t - kx) . \tag{2.6}$$

Why are we picking a single frequency ω_0, while our field has many? It is convenient since the laser frequency (ω_0) is the main one, but there are also things fluctuating at all other frequencies. We expect quantum fluctuations to have a white spectrum, so for those it is the same; however the sidebands from the GW signal will be close to ω_0.

In the photocurrent we measure, $I_{\mathrm{ph}}(t) = \Big|E(x_{\mathrm{photodiode}}, t)\Big|^2$, we would have product terms between the laser frequency and the GW frequency.

The Fourier transform of the photocurrent allows us to compute a Power Spectral Density $S(I_{\mathrm{ph}}, \omega)$.

The Fourier transform looks like

$$\widetilde{I}_{\mathrm{ph}}(\omega) \sim \underbrace{|\alpha|^2}_{\omega_0,\omega_0} + \underbrace{\mathrm{Re}(\alpha\hat{a})}_{\omega_0,\omega} + \underbrace{\mathrm{Re}(\alpha h)}_{\omega_0\pm\Omega,\omega_0} + \underbrace{\mathrm{Re}(h\hat{a})}_{\omega_0\pm\Omega,\omega} . \tag{2.7}$$

The term which does not fluctuate, $|\alpha|^2$, is often called a DC component or DC offset: since we bandpass the photocurrent signal, any low-frequency component like that will vanish.

The noise term will oscillate at $\omega_0 - \omega = \Omega$; in the end the photocurrent we will actually have left after the bandpassing we get

$$\hat{I}_{\text{ph}}(\omega) \sim h(\Omega) + \hat{a}(\Omega) \,. \tag{2.8}$$

What we are basically doing is extracting out the fast-oscillating term, and we can focus on the slow, audio-band oscillations we care about.

The fluctuations in the photocurrent will contain the square moduli of the *quadratures* E_1 and E_2, which form a Heisenberg pair.

How do we actually manipulate the vacuum field at the photodiode? A squeezer is introduced, and it passes a polarizing beamsplitter, which is always either fully transmissive or fully reflective depending on the polarization of the light.

The unpolarized vacuum passes the polarizer in some fraction, but when it comes back it is fully reflected.

The squeezer emits little power, and some of it is in green light as opposed to infrared!

The power in the Fabry-Perot cavity is very high ($\sim 200\,\text{kW}$), but a small amount of squeezing power is enough to improve the sensitivity significantly.

Tuesday 2021-12-7 (missed lesson) notes by Silvia

As mentioned last time, the measured intensity is proportional to the *low-pass* filtered square of the electric field: $\hat{I} \propto |E|^2_{LP}$.

The two quadrature fields E_1 and E_2 can be written in terms of the $\hat{a}_{1,2}$ creation and annihilation operators:

$$\hat{E}_{1,2}(\hat{r}, t) = \sqrt{\frac{4\pi\hbar}{\mathcal{A}c}} \int \frac{\mathrm{d}\Omega}{2\pi} \left[\hat{a}_{1,2}(\Omega) e^{-i\Omega + i\vec{k}\cdot\vec{r}} + \hat{a}^\dagger_{1,2}(\Omega) e^{+i\Omega - i\vec{k}\cdot\vec{r}} \right] , \tag{2.9}$$

where

$$\hat{a}_1(\Omega) = \underbrace{\sqrt{\frac{\omega_0 + \Omega}{2\omega_0}}}_{\approx 1/2 \text{ when } \Omega \ll \omega_0} \hat{a}(\omega_0 + \Omega) + \sqrt{\frac{\omega_0 - \Omega}{2\omega_0}} \hat{a}(\omega_0 + \Omega) \tag{2.10}$$

$$\hat{a}_2(\Omega) = -i \underbrace{\sqrt{\frac{\omega_0 + \Omega}{2\omega_0}}}_{\approx 1/2 \text{ when } \Omega \ll \omega_0} \hat{a}(\omega_0 + \Omega) + i \sqrt{\frac{\omega_0 - \Omega}{2\omega_0}} \hat{a}(\omega_0 + \Omega) \,. \tag{2.11}$$

When we compute the modulus $|E|^2$ we get a $\sim |E_1|^2 \cos^2(\omega_0 t)$ term, a $|E_2|^2 \sin^2(\omega_0 t)$ term and a $2E_1E_2 \sin(2\omega_0 t)$ term. The last of these averages to zero over a few periods the laser pulsation $2\pi/\omega_0$ (so, a very short time) while the square cosines and sines both average to $1/2$. This means we get

$$|E|^2 = \frac{1}{2} \left(|E_1|^2 + |E_2|^2 \right) \,. \tag{2.12}$$

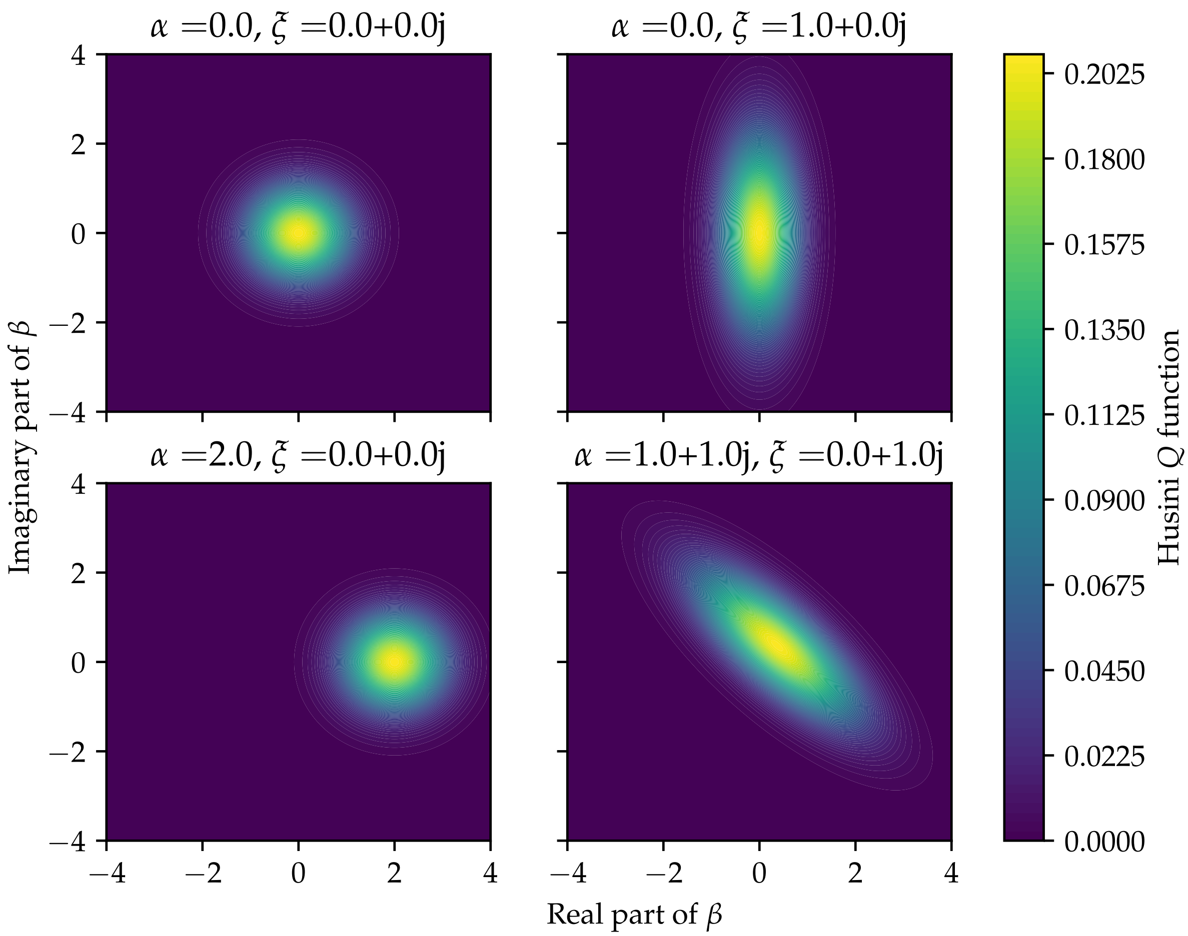

Figure 2: Husini Q-function, $Q(\beta) = \langle \beta | \hat{\rho} | \beta \rangle / \pi$, for squeezed vacuum states [GK04, eqs. 3.112 and 7.82].

A field quadrature can be written in terms of a baseline A plus some oscillations with period Ω:

$$\hat{E}_1 = A + \sqrt{\frac{4\pi\hbar}{\mathcal{A}c}} \int \frac{\mathrm{d}\Omega}{2\pi} \left[\underbrace{\hat{a}_1 e^{-i\Omega t + i\vec{k}\cdot\vec{r}}}_{\hat{c}_1} + \text{h. c.} \right] . \tag{2.13}$$

This quantum noise is constant in Ω:

$$S(\hat{a}_1, \Omega) = \lim_{T\to\infty} \frac{\left\langle \hat{a}_1 \hat{a}_1^\dagger \right\rangle}{T} = 1(?) . \tag{2.14}$$

The Heisenberg principle for these is $S(a_1)S(a_2) \geq 1$.

When we do homodyne detection we are measuring a product in the form $I(\Omega) \propto E_{\mathrm{LO}} \cdot E_{\mathrm{signal}}$; we can determine amplitude and phase of the local oscillator field, then we can squeeze the vacuum in the direction parallel to it and allow it to expand in the other.

Typically, the wavelength corresponding to the pump laser, at $2\omega_0$, is green light ($\lambda = 532\,\text{nm}$). This is then passed to an Optical Parametric Amplification crystal which is basically an SPDC crystal plus amplification.

The squeezing procedure can be schematically represented as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} e^{-\sigma} & 0 \\ 0 & e^{\sigma} \end{bmatrix}}_{\Sigma(\sigma,\varphi)} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix} . \tag{2.15}$$

The transformation matrix Σ determines the power spectral density:

$$S(s_1, s_2, \omega) = \operatorname{Re}(\Sigma\Sigma^\dagger) \underbrace{S(a_1, a_2, \Omega)}_{\equiv \mathbb{1}_2} , \tag{2.16}$$

where these power spectral densities are to be interpreted as covariance matrices. Monday 2021-12-20

At high frequency we also lose sensitivity because our detector responds less well to GWs, as well as shot noise.

In the simplest version, squeezing reduces shot noise but increases radiation pressure noise.

RP noise is at low frequency mostly. The resonant frequency of the suspensions for our detectors is typically at a few Hz; therefore for practical purposes we are above it and we can treat the mirror as a free mass.

The radiation pressure noise can be described by relating the fluctuation in applied power to the fluctuation in position:

$$-m\omega^2 \delta x = 2\frac{\delta P}{c} \tag{2.17}$$

$$\delta x = -\frac{2\delta P}{mc\omega^2} = -2\frac{\sqrt{E\omega\delta P}}{mc\omega^2} \hat{a}_1(\omega) . \tag{2.18}$$

We suppose that the amplitude quadrature contains the power, as $E_1 = \sqrt{P} + \dots$, while the phase quadrature E_2 contains the GW signature.

The expression for the fluctuation is due to the fact that the quadrature also contains an integral over $\hat{a}_1$.

The phase of the reflected signal is modulated by the change in position of the mirror:

$$\frac{\delta\phi}{2\pi} = 2\frac{\delta x}{\lambda} = 2\frac{\delta x \omega_0}{c} \tag{2.19}$$

$$\delta\phi = \frac{4\pi\omega_0}{c}\delta x , \tag{2.20}$$

where the factor 2 is due to the fact that the new path is added both forward and back.

The quadrature reads

$$\sqrt{P}\cos(\omega_0 t + \delta\phi) = \sqrt{P}\left(\cos(\omega_0 t) \underbrace{\cos(\delta\phi)}_{\sim 1} - \sin(\omega_0 t)\delta\phi \right) . \tag{2.21}$$

Therefore, the modulation is roughly $E_1 \approx \sqrt{P}\delta\phi$.

The incoming field will be described by $\vec{a} = (a_1, a_2)^\top$, while the outgoing one will be

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -k & 0 \end{bmatrix}}_{T} \vec{a}\,, \tag{2.22}$$

where $k = 4\omega_0 P / mc^2\omega^2$. The spectral density of b will read

$$S(\vec{b}) = \mathrm{Re}(TT^\dagger)\, \underbrace{S(\vec{a})}_{\mathbb{1}}\,. \tag{2.23}$$

The real part of $TT^\dagger$ reads

$$\mathrm{Re}(TT^\dagger) = \begin{bmatrix} 1 & -k \\ -k & 1+k^2 \end{bmatrix}. \tag{2.24}$$

The off-diagonal terms mean that the noise in the amplitude and phase quadratures is correlated.

We can plot $1+k^2$ against f with the expression we have for k. It is roughly $1/f^2$ and then flat.

How can we show that this is still a minimum uncertainty state? We need to see that the determinant of the transfer matrix is 1, which holds.

There is a quadrature which has reduced quantum noise with respect to the vacuum, at the expense of another which has larger noise.

People tried to do squeezing in this way; the main thing to do is to use tiny mirrors so that k becomes very large.

This effect will rotate the ellipse, even if we already send in squeezed light!

This is the reason why the RP noise worsens the noise curve when we send in squeezed light!

The solution to this problem is to pre-rotate the ellipse so that this effect is compensated.

At the quantum level, this is analogous to thinking of regularizing photon arrival times; the statistics of this squeezing operator should be the same as what we get with an SPDC. Of course, that's not really the physics of the measurement we make.

Some really big names were working on quantum technologies GW detection. Braginsky, Khalili, Kip Thorne, Alessandra Buonanno.

The things we did today appeared in a paper "Ponderomotive effects in electromagnetic radiation".

A detuned power recycling cavity allows for moving the resonance frequency of the Fabry-Perot cavity into the observation band, which means we have very low noise there.

Squeezed noise is not stationary in the photocurrent!

When we squeeze there is also an addition of some extra noise, we do not stay minimum-uncertainty.

Squeezing and changing the power are equivalent in the respect of overcoming the Standard Quantum Limit!

Frequency-dependent squeezing is achieved through the ponderomotive effect: an input cavity rotates the ellipse by $90°$ between high and low frequency.

One can do an output filter, alternatively: doing the same thing after the round-trip in the arms. This turns out to be much better, but it is more sensitive to losses in the filtering cavity.

What does the configuration look like there?

So, the idea is that the mirror's fluctuation will rotate the noise among the quadratures, so we do that in the opposite direction to the squeezed light (?).

Mirror coatings can create issues! Even if a requirement on the amplitude is satisfied, if the pattern is regular there can be problems: it acts like a diffraction grid.

Where does the $\Delta \sim$ MHz frequency shift come from?

We can do measurements of entangled photons coming from down-conversion to reduce noise.

The OPA is where we get a pump with an offset. This allows us to make an EPR filter with frequency-dependent filtering!

The MHz photons experience no frequency offset: there is no radiation pressure there!

References

[GK04] Christopher Gerry and Peter Knight. *Introductory Quantum Optics*. Illustrated Edition. Cambridge, UK ; New York: Cambridge University Press, Nov. 22, 2004. 332 pp. ISBN: 978-0-521-52735-4.

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