# High Energy Experimental Astroparticle Physics

Jacopo Tissino

2021-12-20

## Introduction

Monday 2021-11-8, compiled 2021-12-20

This first part of this course is given by Ivan De Mitri.

This is not a course on particle detectors: the basics will be assumed, and there will be some short courses about the details. It is more about "experimental tools": how do we design an experiment?

"High energy", here, means roughly considering energies $E \gtrsim 1\,\text{GeV}$, but there will be some things in the MeV range as well. A more apt description of this field is that it is about observing things that come from outside the Earth: particles or high-energy radiation.

The other, "low-energy" course is about the search for rare events, where the issue is the presence of a large background, so we need to go underground.

Then, there is neutrino physics which is at the intersection between these two areas.

The part on gravitation and cosmology, on the other hand, is wholly distinct.

### Cosmic rays

We start with a review of some basic concepts.

As a first approximation, the spectrum of cosmic particles/radiation $\phi(E)$ is decreasing with $E$. From $E \sim 10^8\,\text{eV}$ to $E \sim 10^{20}\,\text{eV}$ there is roughly a powerlaw, $\phi \propto E^{-\gamma}$ with $\gamma \sim 2.7$ at first, then $\gamma \sim 3$ (as shown in figure 1).

With artificial particle accelerators we can probe up to roughly $10^{13}\,\text{eV}$ (since in the center of mass we have $(7+7)\text{TeV}$).

Since the spectrum is roughly a power-law with index 3 spanning 12 orders of magnitude, the number of observed events changes by roughly $12 \times 3$ orders of magnitude between its ends.

There is a "knee" in the energy spectrum, where the spectral index $\gamma$ goes from 2.7 to 3, is at about $3 \times 10^{15}\,\text{eV}$.

Typically, below the knee we can do direct, small experiments, while above it we need to do indirect measurements: they need to be very large, since the presence of a single event becomes very rare.

The particles which have been found to make up **cosmic rays** are protons p, Helium nuclei He, heavy nuclei such as Fe, $\gamma$ rays, electrons and positrons $e^{\pm}$, antiprotons $\overline{\text{p}}$. Antihelium still has to be discovered, but there has been some progress in this regard lately.

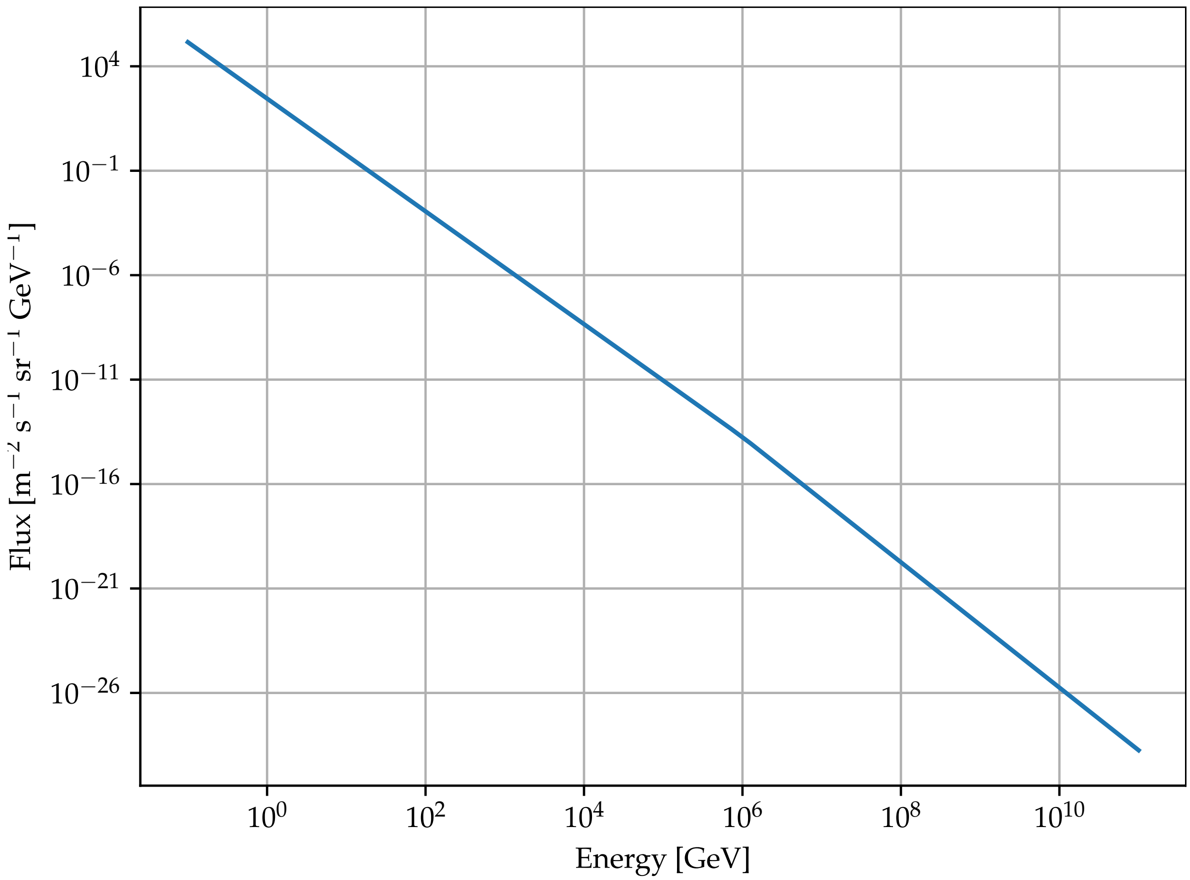


Figure 1: A rough sketch of the cosmic ray flux dependence on the energy.

Also, there are neutrinos: solar ones are on the scale of the MeV, up to 10 MeV. Protons come down the atmosphere and interact with it a few tens of km above, producing other things, such as neutrinos (these are "atmospheric neutrinos"). Alternatively, we can have a source producing neutrinos directly, "astrophysical neutrinos".

*Why* are we doing these experiments?

1. We can explore the astrophysical processes allowing for the production of such high-energy particles, and
2. we can probe high-energy particle physics.

In order to match the center-of-mass energy of the LHC collisions we'd need to have $10^{17}$ eV in a fixed-target experiment.

This is due to the fact that fixed-target experiments have much lower center-of-mass energetics in general.

Here is a quick sketch of why, in natural units, using the formalism detailed in the second half of this lecture. If we have two particles colliding and with equal and opposite momenta the total four-momentum will be $p_{\text{tot}} = (E, \vec{p}) + (E, -\vec{p}) = (2E, \vec{0})$, therefore the total center-of-mass energy will be $\sqrt{s} = 2E$.

On the other hand, in a fixed-target experiment we will have one of them being stationary, so the total four-momentum will be $p_{\text{tot}}(E, \vec{p}) + (m, \vec{0}) = (E+m, \vec{p})$, where the momentum $\vec{p}$ is determined by $m^2 = E^2 - m^2$, so the magnitude will be $\sqrt{s} = \sqrt{(E+m)^2 - (E^2 - m^2)} = \sqrt{2m(E+m)}$.

In terms of the Lorentz factor $\gamma$ the beam-beam collision is $\sqrt{s} = 2m\gamma$, while the fixed-target one is $\sqrt{s} = m\sqrt{2(1+\gamma)}$. For large values of $\gamma$, the first one clearly wins out, by a factor roughly $\sqrt{\gamma}$.

Specifically, if one wants $\sqrt{s} = 14\,\text{TeV}$, this can be achieved with $E \approx 7\,\text{TeV}$ in a beam-beam collision, or with $E \approx 1000\,\text{TeV}$ in a fixed-target collision. The ratio, as expected, is roughly $70 \div 80 \approx \sqrt{\gamma} = \sqrt{7\,\text{TeV}/1\,\text{GeV}}$ when accounting for the fact that we need to accelerate twice as many particles in the beam-beam scenario.

### History

Cosmic rays were discovered in the early 1900s, and up to the early 50s most particle physics was done through them.

In 1957 the antiproton was discovered by Segrè and Chamberlain. This was done with accelerators, thanks to proton collisions producing secondaries, in a process like

$$\mathrm{p} + \mathrm{A} \to \mathrm{p} + \mathrm{A} + \mathrm{p} + \overline{\mathrm{p}}\,. \tag{0.1}$$

The cosmic ray discovery of the antiproton was performed only a few months later.

### References

A good one is Aloisio et al. [Alo+18].

# 1 Review

## 1.1 Relativistic kinematics

Suppose we have a frame $O$ and a different frame $O'$ moving with constant velocity along the $x \sim x'$ direction.

If we define, in terms of the speed of light $c$ and the relative velocity $v$ between the two frames,

$$\beta = \frac{|\vec{v}|}{c} \qquad \text{and} \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}\,, \tag{1.1}$$

then all observations in one frame can be connected with ones in the other thanks to the Lorentz transformation law.

The way the four-vector $(ct, x, y, z)^\top$ transforms is

$$\begin{bmatrix} ct' \\ x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} ct \\ x \\ y \\ z \end{bmatrix}\,. \tag{1.2a}$$

This is actually a general law, since we can write the transformation decomposing the position vector into parallel and perpendicular components to the motion:

$$\begin{bmatrix} ct' \\ r'_{\parallel} \end{bmatrix} = \begin{bmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ r_{\parallel} \end{bmatrix} \tag{1.3a}$$

$$r'_{\perp} = r_{\perp} \,. \tag{1.3b}$$

We can denote $(ct, x, y, z)^{\top}$ as $(x_0, x_1, x_2, x_3) \equiv x$.

The scalar product used by these vectors is the mixed signature

$$x \cdot y = x^{\mu} \eta_{\mu\nu} x^{\nu} \,, \tag{1.4}$$

where

$$\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \,, \tag{1.5a}$$

because we are using the evil particle physicist's mostly-minus convention.

We choose this because of the invariance of the spacetime interval.

Lorentz transformations are "rotations" in the sense that they leave the magnitude of vectors unchanged.

We need to define a *velocity* four-vector. In the nonrelativistic case, $\vec{v} = \mathrm{d}\vec{r}/\mathrm{d}t$; how do we extend it to the relativistic case?

We could define this as $\mathrm{d}x_{\mu} \Big/ \mathrm{d}t$, but this is not invariant since time is not invariant (i.e. it is not a scalar).

Instead, we define the *proper time* as the time *as measured in the reference frame of the particle*. We know that $\Delta t = \gamma \Delta \tau$ by the Lorentz transformation law: therefore, we can define the relativistic velocity as

$$u_{\mu} = \lim_{\Delta\tau \to 0} \frac{\Delta x_{\mu}}{\Delta \tau} = \gamma \frac{\mathrm{d}x_{\mu}}{\mathrm{d}t} \,. \tag{1.6}$$

This is indeed a four-vector. Its components are $(\gamma c, \gamma v_x, \gamma v_y, \gamma v_z)^{\top}$, where $\vec{v}$ is the non-relativistic velocity $\mathrm{d}\vec{x}/\mathrm{d}t$.

The magnitude of this four-vector must be a Lorentz invariant: it comes out to be

$$u^2 = \gamma^2 \left( c^2 - |v|^2 \right) = c^2 \,. \tag{1.7}$$

In the nonrelativistic case the momentum is $\vec{p} = m\vec{v}$, while in the relativistic case we can define $p = mu$.

Its components will be

$$p^{\mu} = (\gamma mc, \gamma m\vec{v})^{\top} \,. \tag{1.8}$$

The term $\gamma mc$ is the (kinetic + rest) energy divided by the speed of light:

$$\gamma mc = \frac{mc}{\sqrt{1-\beta^2}} \approx mc\left(1 - \frac{1}{2}\beta^2\right) \tag{1.9a}$$

$$\approx \frac{1}{c}\left(mc^2 + \frac{1}{2}mv^2 + \mathcal{O}\left(\beta^4\right)\right). \tag{1.9b}$$

Therefore, we can write $p = (E/c, \vec{p})^\top$, where $\vec{p} = \gamma m \vec{v}$ and $E = \gamma mc^2$. This is a four-vector, and transforms as such, since the mass is a scalar. Its magnitude is $p^2 = m^2c^2$. This relation means that

$$E^2 = (mc^2)^2 + (\left|p\right|^2 c^2)^2 . \tag{1.10}$$

The conventional way to define the kinetic energy is $E_{\text{kin}} = E - mc^2$.

If we have a system of particles with four-momenta $p_i$ we can compute the total energy and the total three-momentum by adding them all together.

If these particles scatter the four-momentum is conserved. What is the variable associated with the modulus of the total momentum?

$$p^2_{\text{tot}} = \left(\frac{\sum_i E_i}{c}\right)^2 - \left|\sum_i \vec{p}_i\right|^2 \stackrel{\text{def}}{=} M^2c^2s . \tag{1.11}$$

The variable $M$ is called *invariant mass*, and it is also written in terms of the Mandelstam variable $s$ as $\sqrt{s}$ in the case of a two-two process.

## 1.2 The $J/\psi$ discovery

Professor Samuel Ting got the Nobel Prize in 1976. His experiments can be used as an illustration for the formalism outlined here.

They were doing fixed-target experiments in Brookhaven: protons against Beryllium targets. The process looked like

$$\mathrm{p} + \mathrm{N} \to x + e^+ + e^- , \tag{1.12}$$

where the kinematic properties of the electron and positron were measured by spectrometers.

So, they knew $E^\pm$ and $\vec{p}^\pm$ (where $+$ and $-$ denote the positron and electron respectively).

This allows one to compute

$$m_{e^+e^-} = \frac{1}{c}\sqrt{(p_{e^+} + p_{e^-})} \tag{1.13a}$$

$$= \frac{1}{c}\sqrt{\left(\frac{E^- + E^+}{c}\right)^2 - \left|\vec{p}_+ + \vec{p}_-\right|^2} . \tag{1.13b}$$

If we plot a histogram of these quantities, we get a big spike at $m_{e^+e^-}c^2 = 3.1\,\text{GeV}$.

Therefore, we have a good suggestion of the fact that there was an intermediary:

$$\mathrm{p} + \mathrm{N} \to \Upsilon + x \tag{1.14a}$$

$$\Upsilon \to e^{+} + e^{-} . \tag{1.14b}$$

Also, we know that $m_{\Upsilon} \approx 3.1\,\mathrm{GeV}/c^{2}$.

At the same time, on the opposite coast of the US, a different group was looking at $e^{+}e^{-}$ collisions, and considering the number of reactions as a function of the center-of-mass energy: they also saw a peak in the effective cross-section at 3.1 GeV.

This was happening in 1974, and this particle was dubbed the $J/\psi$.

For one event they saw a "$\psi$" shape. It was later discovered that the $J/\psi$ particle was a meson made of charm quarks, $c\bar{c}$.

The width of the peak is related to the decay time of the particle: $\tau \Delta E \sim \hbar$. Very narrow peaks mean that $\tau$ is quite large — there are kinematic reasons why this particle has a hard time decaying, which we will not go into here.

A similar experiment was done in Frascati a few months later, with ADONE (which is a larger ADA, "Anello di Accumulazione").

### 1.2.1 A $1 \to 2$ decay example

Suppose we have a particle with mass $M$ decaying into two particles with masses $m_1$ and $m_2$.

In the lab system $M$ will be moving, but we can look at the decay in the *center-of-mass* frame, in which $p_{\text{tot}}$ is purely timelike. This is typically denoted with a star: $p^{*}_{\text{tot}} = (Mc, \vec{0})^{\top}$.

After the decay, the two particles are produced with momenta $\vec{p}^{\,*}_1 = -\vec{p}^{\,*}_2$ by conservation of momentum.

The angle $\theta^{*}$ is the one made by the two particles 1 and 2 with respect to the propagation direction of $M$, as measured in the CoM frame.

The energy conservation law reads

$$E^{*}_1 + E^{*}_2 = Mc^2 \tag{1.15a}$$

$$E^{*}_1 = \sqrt{(m_1 c^2)^2 + (|p^{*}_1| c)^2} \tag{1.15b}$$

$$E^{*}_2 = \sqrt{(m_2 c^2)^2 + (|p^{*}_2| c)^2} \,, \tag{1.15c}$$

where $p^{*}_1 = -p^{*}_2$.

We can compute $p_1^2 = m_1^2 c^2$ as the square of $p - p_2$: this yields

$$(p - p_2)^2 = p^2 + p_2^2 - 2 p \cdot p_2 \tag{1.16a}$$

$$m_1^2 c^2 = M^2 c^2 + m_2^2 c^2 - 2 \left( \frac{E^{*}}{c} \frac{E^{*}_2}{c} - \vec{p}^{\,*} \cdot \vec{p}^{\,*}_2 \right) \tag{1.16b}$$

$$= M^2 c^2 + m_2^2 c^2 - 2 M E^{*}_2 \,, \tag{1.16c}$$

therefore

$$E^{*}_2 = \frac{M^2 c^4 + m_2^2 c^4 - m_1^2 c^4}{2 M c^2} \,. \tag{1.17}$$

We know that this will be $E_2^* \geq m_2 c^2$, which can be found to be equivalent to $M \geq m_1 + m_2$. The same reasoning applies for $E_1^*$, swapping $1 \leftrightarrow 2$.

This is independent of the center-of-mass emission angle $\theta^*$.

A two-output decay like this is a very "constrained" problem: in the CoM frame the energies of the particles are fully determined.

We can recover the lab-frame quantities by a Lorentz transformation with velocity $-\beta$ (opposite to the motion of $M$). The decomposition of the momentum in the CoM frame is

$$p_{\parallel}^* = p^* \cos\theta^* \tag{1.18a}$$

$$p_{\perp}^* = p^* \sin\theta^* \,. \tag{1.18b}$$

Consider a positively charged pion: it decays as

$$\pi^+ \to \mu^+ + \nu_\mu \,. \tag{1.19}$$

We can compute the value $E_\mu^*$ in this case, for example, approximating the mass of the neutrino as 0. In natural units, the mass of the pion is $m_{\pi^+} \approx 139.57\,\text{MeV}$, while the mass of the muon is $m_{\mu^+} \approx 105.66\,\text{MeV}$ [Gro+20]; this means that in the center of mass we will have

$$E_\mu^* \approx \frac{m_\pi^2 + m_\mu^2}{2m_\pi} \approx 109.78\,\text{MeV} \tag{1.20a}$$

$$E_\nu^* \approx \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \approx 29.79\,\text{MeV} \,. \tag{1.20b}$$

As expected, $m_\pi = E_\mu^* + E_\nu^*$.

As a reminder, the Lorentz factor can be written as $\gamma = E/mc^2$, while the factor $\beta$ can be written as $\beta = |\vec{p}|c/E$.

Tuesday 2021-11-9, compiled 2021-12-20

Now we move to natural units: $\hbar = c = 1$.

We come back to our decay $M \to m_1 + m_2$. We have computed the CoM energies and momenta, now we need to move back to the lab frame. The Lorentz transformation with $-\beta$ reads

$$\begin{bmatrix} E_i \\ p_{i,\parallel} \end{bmatrix} = \begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} E_i^* \\ p_i^* \cos\theta^* \end{bmatrix} \tag{1.21a}$$

$$= \begin{bmatrix} \gamma\left(E_i^* + \beta p^* \cos\theta^*\right) \\ \gamma\left(\beta E_i^* + p^* \cos\theta^*\right) \end{bmatrix} . \tag{1.21b}$$

The emission angles may change depending on the interaction, so the $\theta^*$ dependence is relevant.

We can recover the emission angle in the lab frame by

$$\tan\theta_i = \frac{p_{i,\perp}}{p_{i,\parallel}} = \frac{p_i^* \sin\theta^*}{\gamma\left(\beta E_i^* + p_i^* \cos\theta^*\right)} \,. \tag{1.22}$$

Even if the emission is isotropic in the CoM frame, it will not be in the lab frame; qualitatively, it will be peaked in the forward direction.

If $\theta^* = 0$, we get $\theta = 0$. If $\theta^* = \pi/2$, we get

$$\tan\theta = \frac{1}{\gamma}\frac{p^*}{\beta E_i^*} = \frac{1}{\gamma}\frac{\beta_i^*}{\beta^*}\,. \tag{1.23}$$

In the ultrarelativistic limit, this will be $\sim 1/\gamma$: therefore, we will get $\theta \sim \arctan(1/\gamma) \sim 1/\gamma$.

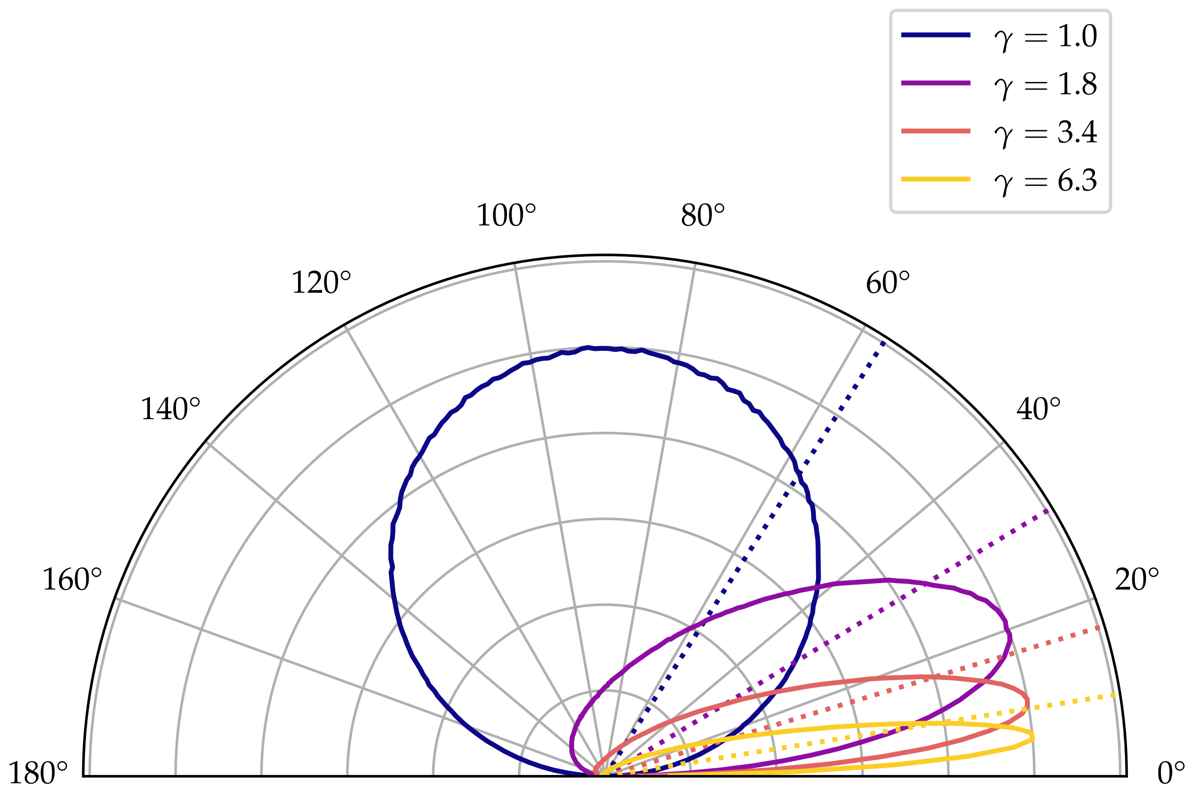


Figure 2: Angular distribution of the emitted radiation if the starting emission is isotropic, which means that the probability distribution for $\theta^*$ is $\mathrm{d}p = \sin\theta^*\,\mathrm{d}\theta^*$, which corresponds in the plot to the $\gamma = 1$ case. Here we are assuming that the decay looks like $1 \to 2 + 3$, where both 2 and 3 are massless. We also show the $\theta \sim 1/\gamma$ approximation as a dashed line. In the high $\gamma$ limit, this is the median of the distribution.

Let us consider the decay of a neutral pion:

$$\pi^0 \to \gamma + \gamma \,. \tag{1.24}$$

In the CoM system, both photons will get $E^* = M/2 = p^*$, where $M = m_{\pi^0} \approx 134.98\,\text{MeV}$.

The lab-frame energy of one of these is given by

$$\begin{aligned}
E &= \gamma\big(E^* + \beta p^* \cos\theta^*\big) && (1.25a)\\
&= \gamma E^* (1 + \beta \cos\theta^*) && (1.25b)\\
&= \frac{E_\pi}{m_\pi} \frac{m_\pi}{2} \big(1 + \beta\cos\theta^*\big) && (1.25c)\\
&= \frac{E_\pi}{2} \big(1 + \beta\cos\theta^*\big) \,. && (1.25d)
\end{aligned}$$

The maximum of this quantity is $E_\pi(1+\beta)/2$; its minimum is $E_\pi(1-\beta)/2$. This is just a bit smaller than the distribution $E \in [0, \pi]$.

What is the distribution of the photons we get in the lab frame? How wide will the detector need to be? Well, we can frame it by selecting the size we need to get a certain high percentage of the emitted photons.

An order-of-magnitude estimate is: half of the photons are emitted with $\theta^* < \pi/2$, and their angle will be $\lesssim 1/\gamma$.

The original $J/\psi$ papers are in the drive — add citation.

### 1.2.2 A $1 \to 3$ decay example

Now the three emitted particle momenta in the CoM frame will satisfy

$$\vec{p}^*_{\text{tot}} = p^*_1 + p^*_2 + p^*_3 = 0 \,. \tag{1.26}$$

We define $\theta^*$ as $\theta^*_1$; the trick to solving this problem is to "bunch" particles 2 and 3 into one, with momentum $\vec{p}^*_{23} = p^*_2 + p^*_3$. The invariant mass of this system will read

$$\begin{aligned}
m_{23} &= \sqrt{p_2^2 + p_3^2 + 2 p_2 \cdot p_3} && (1.27a)\\
&= \sqrt{m_2^2 + m_3^2 + 2E^*_2 E^*_3 - 2\big|p^*_2\big|\big|p^*_3\big| \cos\theta^*_{23}} \,, && (1.27b)
\end{aligned}$$

where $\theta^*_{23}$ is the angle between 2 and 3 in the CoM frame.

We can recurse back to the 2 body formulas for the behaviour of the 1 and 23 system: the energy of 1 will be

$$E^*_1 = \frac{M^2 + m_1^2 - m_{23}^2}{2M} \,. \tag{1.28}$$

Now, even in the CoM frame the energy of particle 1 (which is arbitrary) depends on the emission angle $\theta^*_{23}$. We can, however, compute the minimum and maximum values of $E^*_1$.

There is a "trick" which allows us to skip the whole computation. The idea is that $m_{23}$ will be maximum when the most energy will be given to the 23 system; so this happens when $m_1$ is produced at rest, while the other two are back-to-back.

Then, we will have $p_2^* = -p_3^*$.

In this case, we will have $m_{23}^{\max} = M - m_1$. It's as if both particles (1 and the "combined particle" 23) were produced at rest.

In this case, then,

$$E_{1,\min}^* = \frac{M^2 + m_1^2 - (M^2 + m_1^2 - 2Mm_1)}{2M} = m_1 \,, \tag{1.29}$$

as expected.

The other extreme happens when $m_{23} = m_2 + m_3$; in this case, then, there must exist a frame in which both particles are at rest. This means that they must travel with the same velocity. This does not mean that the momenta are equal, in general $p_2^* \neq p_3^*$, but $\vec{v}_2^* = \vec{v}_3^*$.

The maximum energy is therefore

$$E_{1,\max}^* = \frac{M^2 + m_1^2 - (m_2 + m_3)^2}{2M} \,. \tag{1.30}$$

The most famous example here is $\beta$ decay: from a nuclear perspective it is

$$N(A, Z) \to N(A, Z+1) + e^- + \overline{\nu}_e \,, \tag{1.31}$$

since from a microphysical perspective what happened was

$$\mathrm{n} \to \mathrm{p} + e^- + \overline{\nu}_e \,. \tag{1.32}$$

Proton decay has not been observed, but $\beta^+$ decay can happen since the nucleus is also involved.

In the lab system, which is also the CoM system, we will have a minimum energy of 511 keV, and a maximum energy of roughly 1.29 MeV, which comes from the approximation

$$E_{e,\max} \approx \frac{M^2(A, Z) - M^2(A, Z+1)}{2M(A, Z)} \,. \tag{1.33}$$

If the neutrino were massive, this maximum of the distribution will be shifted down a bit. This is an experimental challenge, since the mass of the neutrino is at most on the eV scale.

Let us consider the decay $\pi^- \to \mu^- + \overline{\nu}_\mu$.

The decay $\pi^- \to e^- + \overline{\nu}_e$ is forbidden by the handedness of weak interactions. The helicity is conventionally positive for antiparticles and negative for particles.

As a first approximation, if the antineutrino is going to the left and the electron is going to the right, both spins will be to the left, summing to 1 to the left, while the pion has spin 0.

This is also the case for decay towards muons, but $m_\mu \gg m_e$, therefore the probability for the muon to have a spin component to the right is much higher.

**A 2 → 3 process** Before looking at three-body decays, let us consider a 2 → 3 process. Consider

Thursday 2021-11-11

$$
\mathrm{K}^{-} + p \to \pi^{+} + \pi^{-} + \Lambda \,, \tag{1.34}
$$

where the masses are $m_\pi \approx 1\,\mathrm{GeV}$, $m_\pi \approx 140\,\mathrm{GeV}$, $m_K \approx 500\,\mathrm{MeV}$, and $m_\Lambda \approx 1.1\,\mathrm{GeV}$.

The experimental setup is a kaon beam impacting upon a proton target.

The invariant mass of the starting particles will be

$$
m_{Kp} = \frac{1}{c}\sqrt{(\sum p_i)^2} = \frac{1}{c}\sqrt{(p_k + p_p)^2} \,. \tag{1.35}
$$

We could also define a corresponding energy $m_{Kp}c^2$.

If we extend this sum to all the particles in the initial state, $m_{\text{initial}}c^2 = m_{\text{final}}c^2 = \sqrt{s}$ is the total invariant mass. The variable $s$ is written as $s = c^2 p_{\text{tot}}^2$. Its meaning is that it is the total energy in the center-of-mass frame.

We can then think of this as an equivalent decay of a particle with $m_{Kp}$, therefore the kinematic threshold is $m_{Kp} \geq m_{\pi^+} + m_{\pi^-} + m_\Lambda$. This is a general statement:

$$
\sqrt{s} \geq \sum_{i \in \text{products}} m_i c^2 \,. \tag{1.36}
$$

Let us approximate the protons as stationary, and we have a kaon with $E_K$.

The total four-momentum reads

$$
p_{\text{tot}} = (E_K + m_p, \vec{p}_k)^\top \,, \tag{1.37}
$$

so we get

$$
\sqrt{s} = \sqrt{(E_k + m_p)^2 - \left|\vec{p}_k\right|^2} \tag{1.38}
$$

$$
= \sqrt{m_K^2 + m_p^2 + 2E_k m_p} \,. \tag{1.39}
$$

We can then straightforwardly compute the energy threshold for $E_k$. In this case we have $E_k^{\text{threshold}} < m_k$, so we have no real lower limit.

This reasoning can be applied, for example, when looking at the reaction

$$
\mathrm{p} + \mathrm{p} \to 3\mathrm{p} + \overline{p} \,. \tag{1.40}
$$

The threshold for the energy of the single proton impacting on the target comes out to be $7m_p \approx 6.6\,\mathrm{GeV}$.

This is counterintuitive! It is due to the fact that we lose a lot of energy when boosting back to the CoM frame.

Let us go back to the 2 → 3 reaction. We can always compute the invariant mass of a subsystem such as $m_{\pi^\pm\Lambda}$:

$$
m_{\pi^\pm\Lambda} = \sqrt{(E_{\pi^\pm} + E_\Lambda)^2 - \left|\vec{p}_{\pi^\pm} + \vec{p}_\Lambda\right|^2} \,, \tag{1.41}
$$

which we can compute once we have the measurement from our detector.

See "dalitz-pi-lambda" figure in the drive.

We can look at the histogram of $m^2_{\pi^+\Lambda}$ versus $m^2_{\pi^-\Lambda}$.

There are peaks! This suggests the presence of something decaying into $\pi^\pm\Lambda$. This particle is a $\Sigma^\pm$, and what happened was

$$\mathrm{K}^- + p \to \Sigma^\pm + \pi^\mp \tag{1.42}$$

$$\Sigma^\pm \to \pi^\pm + \Lambda\,. \tag{1.43}$$

We have not detected the $\Sigma$ particle, which was short-lived.

We know that there should be a range $m_2 + m_2 \leq m_{23} \leq M - m_1$.

This can be extended to a constraint on the two invariant masses.

This is called a **Dalitz plot**, a common trick to find peaks for new particles.

The (true?) width of the peak is about 36 MeV, which corresponds to $1.8 \times 10^{-23}$ s. Is this a $\Sigma(1385)$?

But we should also consider experimental error in the determination of the energy...

An alternative way to measure lifetimes is to look at track length: the path length we expect is

$$\lambda = \beta c \tau \gamma\,, \tag{1.44}$$

where we need the Lorentz factor since the particle decays in time $\tau$ in its own rest frame.

In this case the track length would be about 5.5 fm; however we also have the $\gamma$ factor: if that is large enough we may get something measurable.

We'd want to see at least a few points. If $\sigma_x$ is our spatial resolution, we need a track length which is at least $\gtrsim 5\sigma_x$.

The best spatial trackers are silicon detectors or emulsion trackers, on the order of 10 µm. This means that we need to increase our path length by a factor $10^{10}$... The $\gamma$ factor is surely not that large: it would mean having a beam with energy on the order of $10^{19}$ eV.

Let us consider another example: $\mathrm{p} + \gamma \to \Delta^+$, which can decay into $\pi^+ + \mathrm{n}$ or $\pi^0 + \mathrm{p}$.

What is the kinematic threshold for this reaction? This is a cosmic ray proton interacting with a CMB photon.

These photons have an average energy of 2.7 K $\approx$ 230 µeV.

This does not precisely match $\Omega_{0\gamma}\rho_c c^2$ when multiplied by $n_{0\gamma}$, since we'd need to integrate the Planckian.

The value of $\sqrt{s}$ is

$$\sqrt{s} = \sqrt{m_p^2 + 2\Big(E_p E_\gamma - \vec{p}_p \cdot \vec{p}_\gamma\Big)} \tag{1.45}$$

$$= \sqrt{m_p^2 + 2E_p E_\gamma \Big(1 - \cos\theta_{p\gamma}\Big)} \geq m_{\Delta^+}\,. \tag{1.46}$$

This means that

$$E_p \geq \frac{m^2_{\Delta^+} - m_p^2}{2E_\gamma \Big(1 - \cos\theta_{p\gamma}\Big)}\,. \tag{1.47}$$

If the collision is head-on, we get $1 - \cos = 2$, while in the other case the threshold diverges since the proton cannot reach the photon.

We should average the formula over all values of $\theta_{p\gamma}$.

The threshold is about $10^{19}$ eV. This is the GZK cutoff, or GZK effect. In order to properly study this phenomenon we would need to also look at the probability: [Gro+20] says it is of the order of $10^{-1}$ mb for the $\gamma p$ process.

We can study this in the lab simply, we only need 200 MeV photons colliding on stationary protons.

With this, we can find that a typical proton above the threshold will travel for only tens of Mpc before losing energy to this process.

The products from protons being annihilated in this way will produce many secondary particles: photons, neutrinos, electrons, positrons, muons and more, all being very energetic.

Let us do another possibility: could photons from a high-energy source be absorbed by the CMB? For example, you can have pair production.

We need $\sqrt{s}$ for the process to be larger or equal than about 1 MeV. This comes out to be about $10^{14}$ eV.

Let's see how that works. We are looking at the process $\gamma + \gamma \to e^+ + e^-$; in the lab frame (which is the frame in which the CMB has vanishing dipole moment, the universal rest frame) one photon will have a typical energy of $E_{\gamma_{\text{CMB}}} \approx 235\,\mu\text{eV}$, while the other will have a large energy $E_\gamma$.

The invariant mass of the scattering will read

$$\sqrt{s} = \sqrt{(p_\gamma + p_{\gamma_{\text{CMB}}})^2} \tag{1.48}$$

$$= \sqrt{(E_\gamma + E_{\gamma_{\text{CMB}}})^2 - (\vec{p}_\gamma + \vec{p}_{\gamma_{\text{CMB}}})^2} \tag{1.49}$$

$$= \sqrt{2E_\gamma E_{\gamma_{\text{CMB}}}(1 - \cos\theta_{\gamma\gamma})}\,, \tag{1.50}$$

$E_\gamma^2 - \vec{p}_\gamma^{\,2} = 0$ for both photons.

which we want to be $\geq 2m_e \approx 1\,\text{MeV}$. This condition reads

$$E_\gamma \geq \frac{(1\,\text{MeV})^2}{2E_{\gamma_{\text{CMB}}}(1 - \cos\theta_{\gamma\gamma})} \approx \frac{2.2\,\text{PeV}}{(1 - \cos\theta_{\gamma\gamma})}\,. \tag{1.51}$$

Tuesday 2021-11-16

Last time we compute the threshold for reactions with a CMB photon interacting with some high-energy particles.

We can plot the mean free path with varying energy. It is roughly infinite below a certain threshold (we can have some interactions with the tails of the distribution near it), then it asymptotes to $\Lambda \sim 100\,\text{Mpc}$.

Around $5 \times 10^{19}$ eV the $\text{p} + \gamma_{\text{CMB}} \to \Delta^\pm$ reaction kicks in, while at around $10^{14}$ eV we get the $\gamma + \gamma_{\text{CMB}} \to e^+e^-$ reaction. There, however, the energy affects the cross-section of the reaction, so we expect to see a minimum.

The all-particle energy spectrum has an ultra-high-energy cutoff around $10^{20}$ eV because of these reactions.

At LHC we have center-of-mass energies of 14 TeV, which is found by adding together the energies of the bins. This is true in this particular case since we have a symmetric configuration, meaning that the total momentum in the lab frame as well vanishes.

In a fixed target context (such as a cosmic ray coming in) instead we get

$$\sqrt{s} = \sqrt{2m(E+m)}\,, \tag{1.52}$$

so the correct comparison reads

$$\sqrt{2m(E+m)} = 2E^*\,, \tag{1.53}$$

where $E^* = 7\,\mathrm{TeV}$. Therefore,

$$E = \frac{(2E^*)^2}{2m} - m\,. \tag{1.54}$$

It depends on $E^*$ quadratically! In our case, with protons, we get $E \sim 10^{17}\,\mathrm{eV}$.

How are the angles changed? The invariant mass of the total system is

$$m_{\mathrm{inv}} = \sqrt{\left(\sum_i E_i\right)^2 - \left|\sum_i p_i\right|^2}\,. \tag{1.55}$$

We also know that $\gamma = E/m$. The total $\gamma$ can be computed by looking at this expression with the total energy $E_{\mathrm{tot}}$ and $\sqrt{s} = m_{\mathrm{inv}}$ as the total mass.

In the case of our cosmic rays, this reads

$$\gamma_{\mathrm{CMS}} \sim \sqrt{\frac{10^{17}\,\mathrm{eV}}{2\times 10^9\,\mathrm{eV}}} \approx 7\times 10^3\,, \tag{1.56}$$

which corresponds to an angle of $1/\gamma_{\mathrm{CMS}} \sim 30\,''$.

This is a good thing, in a way: instead of having to make a detector all around the event, we can make a smaller one in the forward direction only.

We can define a new kinematic variable in order to better understand these angles.

Consider a Lorentz transformation with a velocity $v_0 = \beta_0 c$ along the $x$ axis. The transformation for a velocity $v$ reads

$$v = v' + v_0 \tag{1.57}$$

in the NR case, while for the relativistic case it will be

$$v = \frac{v' + v}{1 + v' v_0 / c^2}\,. \tag{1.58}$$

This only holds for the $x$ component, along the boost.

Therefore, the velocities are not additive. A trick is to introduce a new quantity, the *rapidity*:

$$y' = \frac{1}{2}\log\frac{1+\beta'}{1-\beta'} = \operatorname{arctanh}(\beta')\,, \tag{1.59}$$

and rapidities are indeed additive. These are typically considered only looking at the component parallel to the boost:

$$y = \operatorname{arctanh}(\beta_\parallel) \qquad \text{where} \qquad \beta_\parallel = \beta \cos\theta = \frac{p_\parallel}{E} \,. \tag{1.60}$$

This formulation holds in 3D as well. The rapidity can also be written as

$$y = \frac{1}{2} \log \left( \frac{E + p_\parallel}{E - p_\parallel} \right) . \tag{1.61}$$

The Lorentz parameters can be written in terms of hyperbolic functions:

$$\beta = \tanh(y) \tag{1.62}$$
$$\gamma = \cosh(y) \tag{1.63}$$
$$\beta\gamma = \sinh(y) \,, \tag{1.64}$$

so that the Lorentz matrix reads

$$\begin{bmatrix} \gamma & -\beta\gamma \\ -\beta\gamma & \gamma \end{bmatrix} = \begin{bmatrix} \cosh(y) & -\sinh(y) \\ -\sinh(y) & \cosh(y) \end{bmatrix} . \tag{1.65}$$

In the ultrarelativistic limit, the rapidity reads

$$y \approx \log \sqrt{\frac{1 + \cos\theta}{1 - \cos\theta}} \tag{1.66}$$
$$\approx \log \frac{\sqrt{(1 + \cos\theta)/2}}{\sqrt{(1 - \cos\theta)/2}} \tag{1.67}$$
$$\approx -\log \tan(\theta/2) = \eta \,, \tag{1.68}$$

where $\eta$ is called the pseudo-rapidity.

make figure

At LHC we can only detect particles which are not emitted very forward or very backward.

What about the rapidities of the particles for the case of a $10^{17}$ eV cosmic ray? They are additive, so $y_{\text{lab}} = y' + y_0$, where

$$y_0 = \frac{1}{2} \log \left( \frac{1 + \beta_{\text{CMS}}}{1 - \beta_{\text{CMS}}} \right) \tag{1.69}$$
$$\beta_{\text{CMS}} = \frac{|p'|}{E + m} \approx \frac{E}{E + m} \tag{1.70}$$
$$y_0 \approx \frac{1}{2} \log \left( \frac{2E + m}{m} \right) \approx \frac{1}{2} \log \frac{2E}{m} \,. \tag{1.71}$$

Let us consider an energy $E \approx 10^{15}$ eV. Then, $y_0 \approx 7$.

We can divide the CMS depending on the sign of $\eta^*$,the pseudo-rapidity in the CMS. Then the total rapidity will read $\eta = \eta^* + y_0$.

The boundary between these in the lab frame is found when $\eta^* = 0$, so we get $\eta = -\log\tan\theta/2$: $\theta$ is about 6 arcminutes for $\eta = 7$.

For a collision 20 km in the air, we expect about half of the particles to be found within a 40 m radius.

Collider experiments typically cover $|\eta| \lesssim 5$ (the figure refers to ATLAS); this only covers the central region, we do not get all of them.

The definition of the pseudo-rapidity is convenient because PDFs in the form $\mathrm{d}N/\mathrm{d}\eta$ are only shifted under Lorentz boosts.

The **transverse mass** can also be defined:

$$E^2 = m^2 + p^2 = \underbrace{m^2 + p_T^2}_{m_T^2} + p_\parallel^2 \,, \tag{1.72}$$

and $m_T^2 = m^2 + p_T^2$ defined as above is Lorentz invariant under boosts in the $\parallel$ direction. We can also write this relation like

$$\left(\frac{E}{m_T}\right)^2 - \left(\frac{p_\parallel}{m_T}\right)^2 = 1 \tag{1.73}$$

$$\cosh^2 y - \sinh^2 y = 1 \,. \tag{1.74}$$

The rapidity can also be expressed as

$$y = \frac{1}{2}\log\left(\frac{E + p_\parallel}{m_T}\right) . \tag{1.75}$$

Exercise: suppose we have an incoming particle with mass $M$ impacting upon a particle with mass $m$; what is the maximum transfer of energy from particle $M$ to particle $m$, if the collision is elastic?

The result should be

$$\Delta E = \frac{2m\gamma^2\beta^2}{1 + 2\gamma m/M + (m/M)^2} \,. \tag{1.76}$$

The case we are interested in will be a proton interacting with a medium and interacting with an electron. There, and for a relativistic proton, this will read $\Delta E \approx 2m\gamma^2$.

Next time, we will discuss some matter-radiation interactions.

A particle of mass $M$ with Lorentz factor $\gamma$ and velocity $\beta$ is impacting a stationary particle of mass $m$.
The total invariant mass will be

$$\sqrt{s} = \sqrt{(E_M + m)^2 - p_M^2} \tag{1.77}$$

$$= \sqrt{M^2 + m^2 + 2\gamma M m} \,, \tag{1.78}$$

where $E_M = \gamma M$ while $p_M = \beta E_M$.
The Lorentz factor $\gamma_{\text{CoM}}$ of the transformation required to move between the center of mass frame and the lab one can be computed from the total energy:

$$\gamma_{\text{CoM}} = \frac{E_{\text{tot}}}{\sqrt{s}} = \frac{\gamma M + m}{\sqrt{M^2 + m^2 + 2\gamma M m}} . \tag{1.79}$$

As one would expect, this approaches 1 as $\gamma \to 1$.
In the CoM frame we will have

$$\sqrt{s} = E_M^* + E_m^* = E_M^{*\prime} + E_m^{*\prime} , \tag{1.80}$$

where the prime denotes a value computed after the collision.
Let us denote the momentum of particle $m$ after the collision in the center of mass frame as $\vec{p}^*$, so that

$$p_M^{*\prime} = [E_M^{*\prime}, -\vec{p}^*] \tag{1.81}$$

$$p_m^{*\prime} = [E_m^{*\prime}, \vec{p}^*] , \tag{1.82}$$

the sum of which will equal $[\sqrt{s}, 0]^\top = p_M^{*\prime} + p_m^{*\prime}$. Isolating the $M$ momentum and computing the modulus of this equation yields the following expression for the energy of $m$ after the collision:

$$\begin{aligned} E_m^{*\prime} &= \frac{s + m^2 - M^2}{2\sqrt{s}} && (1.83) \\ &= \frac{M^2 + m^2 + 2\gamma M m + m^2 - M^2}{2\sqrt{M^2 + m^2 + 2\gamma M m}} && (1.84) \\ &= \frac{m^2 + \gamma M m}{\sqrt{M^2 + m^2 + 2\gamma M m}} . && (1.85) \end{aligned}$$

As expected, this equals $m$ when $\gamma = 1$. Also, this precisely equals $m\gamma_{\text{CoM}}$: this is correct, since what we have done is boosting particle $m$ from stationarity with $\gamma_{\text{CoM}}$; the kinematics before and after the collision are exactly the same, so the energy of particle $m$ in the center of mass frame will be the same before and after it.
In the CoM frame this is fixed, but the angle of emission is arbitrary, and can lead to differing values for the energy of $m$ in the lab frame.
Specifically, the expression for the energy and momentum along the boost direction of mass $m$ in the lab frame reads

$$\begin{bmatrix} E_m \\ p_{m\parallel} \end{bmatrix} = \begin{bmatrix} \gamma_{\text{CoM}} & -\beta_{\text{CoM}}\gamma_{\text{CoM}} \\ -\beta_{\text{CoM}}\gamma_{\text{CoM}} & \gamma_{\text{CoM}} \end{bmatrix} \begin{bmatrix} E_m^{*\prime} \\ p_\parallel^* \end{bmatrix} , \tag{1.86}$$

where $p_\parallel^* = p^* \cos\theta$ is the momentum along the boost direction for mass $m$. The value of $p^*$ is fixed: we know $\gamma_m^* = E_m^*/m$, and the corresponding value for $\beta_m^*$ is readily computed as $\beta_m^* = \sqrt{1 - (1/\gamma_m^*)^2}$. Then, we will have $p_m^* = \beta_m^* \gamma_m^* m$.

Further, we can simplify the expression by making use of the fact that $\gamma_{\text{CoM}} = \gamma_m^*$, and similarly for $\beta_{\text{CoM}}$. With all this, the energy $E_m$ in the lab frame reads

$$\begin{aligned} E_m &= \gamma_{\text{CoM}}\gamma_m^* m - \gamma_{\text{CoM}}\beta_{\text{CoM}}\gamma_m^*\beta_m^* m\cos\theta && (1.87) \\ &= \left(1 - \beta_{\text{CoM}}^2\cos\theta\right)\gamma_{\text{CoM}}^2 m && (1.88) \\ &= \left(1 - \beta_{\text{CoM}}^2\cos\theta\right)\gamma_{\text{CoM}}^2 m\,. && (1.89) \end{aligned}$$

This will be maximized when $\cos\theta$ is as small as possible, so we require $\cos\theta = -1$;[a] this means we get

$$\begin{aligned} E_m &= \left(1 + \beta_{\text{CoM}}^2\right)\gamma_{\text{CoM}}^2 m && (1.90) \\ &= \left(2 - \frac{1}{\gamma_{\text{CoM}}^2}\right)\gamma_{\text{CoM}}^2 m && (1.91) \\ &= \left(2\gamma_{\text{CoM}}^2 - 1\right) m\,. && (1.92) \end{aligned}$$

Before proceeding, we should remember that the quantity we are after is not the final energy $E_m$ but the *change* in energy of particle $m$, which starts out stationary, so $\Delta E = E_m - m$:

$$\begin{aligned} \Delta E &= 2\left(\gamma_{\text{CoM}}^2 - 1\right) m && (1.93) \\ &= 2\left(\frac{(\gamma M + m)^2}{M^2 + m^2 + 2\gamma M m} - 1\right) m && (1.94) \\ &= 2\frac{\left(\gamma^2 M^2 + m^2 + 2\gamma M m\right) - M^2 - m^2 - 2\gamma M m}{M^2 + m^2 + 2\gamma M m} m && (1.95) \\ &= 2\frac{M^2\left(\gamma^2 - 1\right)}{M^2 + m^2 + 2\gamma M m} m && (1.96) \\ &= 2\frac{\gamma^2\beta^2}{1 + 2\gamma m/M + (m/M)^2} m\,. && (1.97) \end{aligned}$$

[a] While setting $\theta = 0$ yields the minimum, $E_m = m$: the collision is perfectly elastic and time-symmetric, $m$ is bounced straight back in the CoM frame, so it returns stationary in the lab frame.

# 2 Matter-radiation interaction

Tuesday 2021-11-23

The binding energy of electrons to atoms is typically of the order of $1 \div 10\,\text{eV}$; the maximum change in energy of an electron struck by a high-energy particle is $\Delta E \approx 2 m_e \beta^2 \gamma^2$.

This is a lot for the electron, but a small amount compared to the initial energy of the high-energy particle.

The trajectory of the large particle is also deflected but only by a small amount. The quantity we are interested in computing is the average change in energy per unit length: $\mathrm{d}E/\mathrm{d}x$.

Suppose we have a beam of particles with a certain flux $\Phi$, defined as

$$\Phi = \frac{\text{\# incoming particles}}{\text{time} \times \text{surface}} . \tag{2.1}$$

After hitting a target, these particles will go along a trajectory defined by angles $\theta$, $\varphi$ from the interaction point.

The differential amount of particles emitted in a certain direction, $\Delta N_s(\theta, \varphi)$, will satisfy

$$\Delta \dot{N}_s(\theta, \varphi) \propto \Phi , \tag{2.2}$$

so we define

$$\frac{\Delta \dot{N}_s}{\Delta \Omega} = \Phi \frac{\Delta \sigma}{\Delta \Omega} , \tag{2.3}$$

where $\Delta\Omega$ is a small solid angle. This allows us to compute the differential cross-section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{\Phi} \frac{\mathrm{d}\dot{N}_s(\theta, \varphi)}{\mathrm{d}\Omega} . \tag{2.4}$$

We can further define the total, or integral cross-section:

$$\sigma = \int \frac{1}{\Phi} \frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} \,\mathrm{d}\Omega . \tag{2.5}$$

This is all written with respect to a single target, but in practice we will have a certain slab of matter with thickness $\delta x$, and a particle flux impacting on it within an area $A$. In this case, then, we also need to account for the number of targets $N_T$:

$$\frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} = \Phi N_T \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} . \tag{2.6}$$

If $n$ is the number density of the targets, and $\rho$ is the matter density there, the number of targets will read $N_T = nA\delta x$. With this, we find

$$\frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} = \underbrace{\Phi A}_{\dot{N}_b} n \delta x \frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} , \tag{2.7}$$

where we can identify the incoming beam particle rate $\dot{N}_b$. The definitions for the differential and total cross-sections will then read

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \frac{1}{\dot{N}_b} \frac{1}{n\delta x} \frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} \tag{2.8}$$

$$\sigma = \frac{1}{\dot{N}_b} \frac{1}{n\delta x} \underbrace{\int \frac{\mathrm{d}\dot{N}_s}{\mathrm{d}\Omega} \,\mathrm{d}\Omega}_{=\dot{N}_s} . \tag{2.9}$$

There is an alternative way to compute this in terms of QFT quantities, but we will not concern ourselves with it.

If we know the cross-section for, say, electron-proton interactions, we can compute the number of scatterings as

$$\dot{N}_s = \dot{N}_b (n \, \mathrm{d}x) \sigma \,. \tag{2.10}$$

These quantities all depend on $n\delta x$, never on the number density or the thickness singularly.

How does the intensity of a particle beam decrease as it travels through a medium? We know $I(x = 0)$, and we want to compute the dependence $I(x)$. In a certain thickness $\Delta x$, we will have a certain number of interactions $\Delta N_s = N_b(n\Delta x)\sigma$, so we can define the probability of interaction

$$\frac{\Delta N_s}{N_b} = \Delta P = n\sigma\Delta x \,. \tag{2.11}$$

The probability of *not* having an interaction is $1 - n\sigma\Delta x$. The survival probability $P(x)$ is the probability of a particle surviving at least until $x$. We can write a relation for $P(x + \Delta x)$:

$$P(x + \Delta) = P(x)(1 - n\sigma\Delta x) \,, \tag{2.12}$$

since this means that in this $\Delta x$ the particle still has not interacted. Then,

$$P(x + \Delta x) - P(x) = \Delta P = -P(x) n\sigma\Delta x \tag{2.13}$$

$$\frac{\mathrm{d}P}{\mathrm{d}x} = -n\sigma P \tag{2.14}$$

$$P(x) \propto \exp(-n\sigma x) \,. \tag{2.15}$$

We can also compute the average $x$ at which an interaction occurs:

$$\langle x \rangle = \frac{\int_0^\infty x \exp(-n\sigma x) \, \mathrm{d}x}{\int_0^\infty \exp(-n\sigma x) \, \mathrm{d}x} = \frac{1}{n\sigma} = \Lambda \,, \tag{2.16}$$

which can therefore be interpreted as the *mean free path*.[1]

Protons coming into a medium with atoms may interact with the electrons through EM interactions, but also through with nucleons through hadronic interactions.

Since these hadrons are very heavy, the deflection by hadronic interactions is much heavier.

What is the order of magnitude for the cross-section of proton-proton, or neutron-proton interactions?

Roughly, the radius of a nucleus is $R(A) \sim r_0 A^{1/3}$, where $r_0 \sim 1.2\,\mathrm{fm}$.

A rough estimate of the geometric cross-section is $\sigma \sim 4\,\mathrm{fm}^2$. This can be written as $40\,\mathrm{mb}$, where $1\,\mathrm{barn} = 10^{-24}\,\mathrm{cm}^2$.

[1] The mean free path before the *first* interaction - after that the particle may change, so this simple expression may not hold anymore.

We can look at the proton-proton cross-section plot from the PDG [Gro+20]. There is some structure at low energies, but between 1 and 100 GeV the cross-section is roughly 40 mb indeed.

At higher energies, roughly $\sqrt{s} \sim 10\,\text{TeV}$, this increases up to 100 mb.[2] These numbers have an order of magnitude which is a good approximation for all hadronic interactions.

A barn is a large cross-section!

Let us consider a beam of 5 GeV protons going toward a 10 cm thick block of graphite. The total number of particles going to the target is called protons-on-target, and in this case it is $10^{16}$.

How many interactions will we have? First, we need the $\sqrt{s}$ for the interaction, in order to compute $\sigma$. It comes out to $s = (E+m)^2 - |\vec{p}|^2 = 2m^2 + 2Em \approx 3.5\,\text{GeV}$ (the nucleons can be considered to be stationary: the order of magnitude of their kinetic energies is tens of MeV, so their momenta are $p^2/2m_p \sim 250\,\text{MeV}$).

What is $n$ for graphite? The molar mass is 12 g/mol, while $\rho \approx 2.2\,\text{g/cm}^3$, so we get

$$n = N_A \frac{A}{\text{molar mass}} \rho \approx 1.3 \times 10^{24}\,\text{cm}^{-3}\,. \tag{2.17}$$

This means that

$$n\sigma\delta x = \frac{N_{\text{interactions}}}{N_{\text{protons on target}}} \approx 0.53\,. \tag{2.18}$$

The mean free path is therefore roughly 20 cm. The target size is typically chosen so that its length is roughly $\Lambda$: this way, we often have one interaction but rarely two.

What happens if, instead, we have a neutrino beam? The cross-section is directly proportional to the energy of the beam, with

$$\sigma_{\nu N} \sim 0.6 \times 10^{-38} \left(\frac{E}{\text{GeV}}\right) \text{cm}^2\,. \tag{2.19}$$

At high energies the process is called *deep inelastic scattering*.

The charged current process for the interaction of a neutrino with a nucleus looks like

$$\nu_\mu + \text{n} \rightarrow \text{p} + \mu^-\,. \tag{2.20}$$

Unless the energy is extremely high, this cross-section is extremely low. This means that the mean free path is much higher. What is the probability of a neutrino being able to cross the Earth?

The density of the Earth is on average roughly $5\,\text{g/cm}^3$. It changes a lot through its volume, from $\sim 2.6\,\text{g/cm}^3$ to $\lesssim 15\,\text{g/cm}^3$.

For $E = 1\,\text{GeV}$, we get roughly one thousand times the Earth-Moon distance, $\Lambda = 1/n\sigma \approx 3 \times 10^8\,\text{km}$.

Thus, the Earth is almost transparent to neutrinos at these energies! For an energy of 25 TeV, though, the Earth's diameter is close to the mean free path of the neutrino.

[2] See figure 52.6 in `http://pdg.ge.infn.it/2021/reviews/rpp2020-rev-cross-section-plots.pdf`.

What about neutrions coming from pion decay, $\pi^+ \to \mu^+ + \nu_\mu$ and then $\mu^+ \to e^+ + \overline{\nu}_\mu + \nu_e$, in the atmosphere?

Typically, the order of magnitude of the energy of these secondary neutrinos is about a factor of 10 less than the energy of the proton initiating the shower.

So, these can be very high-energy neutrinos, up to $10^{19}\,\text{eV}$, and their spectrum will roughly mirror the $\sim E^{-3}$ powerlaw for cosmic rays.

The flux of neutrinos produced from astrophysical sources, on the other hand, scales like $E^{-2}$. At lower energies, therefore, we have atmospheric neutrino domination, while at high energies the astrophysical neutrinos dominate.

The crossing point happens at energies of roughly $1\,\text{TeV}$. Thursday 2021-12-2

We can approximate the interaction of a high energy particle with matter as a continuous energy loss.

There can also be radiative processes: a decelerated particle will also produce EM radiation.

This will also often produce secondary particles.

A standard, main feature is just Coulomb force, but there are many other things to consider if one wants to get a precise prediction.

This main contribution will be described by the notorious Bethe-Bloch formula.

We will write $\mathrm{d}E/\mathrm{d}x$ to mean $\left\langle \mathrm{d}E/\mathrm{d}x \right\rangle$: the mesoscopic average value of the energy loss per unit length.

The Bethe-Bloch formula tells us that this is proportional to certain quantities:

$$\frac{\mathrm{d}E}{\mathrm{d}x} \propto \rho \left( \frac{Z}{A} \right) \mathscr{Z}^2 \frac{1}{\beta^2} \left[ \log f(\beta, \gamma) + \dots \right] , \tag{2.21}$$

where $\rho$ is the mass density of the medium, $Z$ and $A$ refer to the nuclei in the medium, $\mathscr{Z}$ and $\beta$ are the charge and velocity of the particle.

The dependence on $\beta$ is relatively weak, since $\beta \sim 1$ will hold in most cases we are interested in.

At high energies, as a first approximation, $\mathrm{d}E/\mathrm{d}x \sim f(\beta\gamma)$, so we often plot this as a function of $\beta\gamma$. A minimum can be seen around $\beta\gamma \approx 3$, where the value of the energy loss per unit density is about $1.6\,\text{MeV}/(\text{g}\text{cm}^{-2})$.

We know, however, that the number of interactions is given by $N_{\text{int}} \propto (\rho \Delta x) \sigma$, so the relevant quantity is the *grammage* $\rho \Delta x$, measured in $\text{g}\text{cm}^{-2}$. Confusingly, this is often denoted as

$$\frac{1}{\rho} \frac{\mathrm{d}E}{\mathrm{d}x} \overset{\text{def}}{=} \frac{\mathrm{d}E}{\mathrm{d}X} . \tag{2.22}$$

The plateau at high $\beta\gamma$ is not much higher than the minimum: it is at around $2\,\text{MeV}\text{cm}^2\text{g}^{-1}$.

A *minimum ionizing particle* is one which is coming in at the minimum value for $\beta\gamma$.

Since $\gamma = E/m$ and $\beta = \left|\vec{p}\right|/E$, we have $\beta\gamma = \left|\vec{p}\right|/m$: this means that $\beta\gamma = 3$ corresponds to $\left|\vec{p}\right| \approx 3m$.

Integrating this we get a plot for $E$ against $x$: we expect a slow decrease and a sharp drop, but we must keep in mind the fact that this is just an average. The low part will not be as sharply decreasing, since the approximation of the BB formula gets worse as $\beta$ approaches 0.

We can also compute the *range* of the particle, defined as the average depth the particle reaches in the material.

The range of a particle with energy $E$ is given by

$$R(E) = \int_{E_0}^{mc^2} \frac{\mathrm{d}E}{-\,\mathrm{d}E/\mathrm{d}x} = \int_{mc^2}^{E_0} \frac{\mathrm{d}E}{\mathrm{d}E/\mathrm{d}x}\,. \tag{2.23}$$

Consider a cosmic muon at 10 GeV.

The atmospheric grammage is roughly $10^3\,\mathrm{g}\mathrm{cm}^{-2}$.

Protons produce pp $\to \pi^+\pi^0\pi^-$, the pions produce $\mu^+\overline{\nu}_\mu$, the muons produce $e^+\overline{\nu}_\mu\nu_e$.

At sea level, we have roughly $300\,\mathrm{muons}/\mathrm{m}^2\mathrm{s}$.

The muon flux increases rapidly in the upper atmosphere, and then stays roughly constant. Why is this? If we start with a TeV proton, we expect roughly a 100 GeV muon; they then lose about $2\,\mathrm{MeV}\mathrm{cm}^2/\mathrm{g}$, so over $10^3\,\mathrm{g}/\mathrm{cm}^2$ they lose about 2 GeV.

This does not take into account the fact that they might decay. A muon's decay time is 2.2 µs. It travels about 660 m in that time, but its Lorentz factor is about $\gamma \approx 1000$, so its decay time in our reference frame is about 2.2 ms, in which it travels 660 km.

Less energetic muons will also be part of the distribution, and those might start to decay within 20 km, but the effect as a whole is small at sea level.

The ranges of the particles can differ by a lot in certain ranges.

Suppose we have a beam with $|\vec{p}| = 200\,\mathrm{MeV}$ with both kaons and muons.

The range of kaons there is about $6\,\mathrm{g}/\mathrm{cm}^2$, while the range of muons is about $60\,\mathrm{g}/\mathrm{cm}^2$, while the density of lead is about $11\,\mathrm{g}/\mathrm{cm}^3$.

So, the range in lead for kaons is about 0.5 cm, while the range for muons is about 7 cm.

If we put a 1 cm barrier, the muons will lose about 12 MeV, so they will remain relativistic, while most muons will be stopped.

For electrons, radiative energy losses (bremsstrahlung) are very significant, since the energy emitted in that way has a strong inverse dependence on the mass.

In that case, we will have a dependence like

$$\left|\frac{\mathrm{d}E}{\mathrm{d}x}\right| \propto E\,, \tag{2.24}$$

therefore there will be some $x_0$, which we call the *radiation length*, such that

$$\frac{\mathrm{d}E}{\mathrm{d}x} = -\frac{E}{x_0} \implies E(x) = E_0 e^{-x/x_0}\,. \tag{2.25}$$

The constant $x_0$ only depends on the medium, and it scales like

$$x_0 = 180\frac{A}{Z^2}\mathrm{g}/\mathrm{m}^2\,. \tag{2.26}$$

In water, this is about $36\,\mathrm{g}/\mathrm{cm}^2$.

Adding this $\propto E$ term to the Bethe-Bloch formula means we will have a *critical energy* at which the two cross.

For electrons, this will be of the order of

$$E_c \approx \frac{600\,\mathrm{MeV}}{Z+1}\,. \tag{2.27}$$

For water, $E_c \approx 78\,\mathrm{MeV}$.

What happens to the photons? At low energies, we have the photoelectric effect. At few eV this is possible, it will then depend on the binding energies of the atoms there. This means there will be a few peaks in the $\sigma$ against $E$ plot in the eV to 10 eV range.

For Compton scattering, we can compute the Klein-Nishina cross-section, and we get a "shoulder" at mid-energies of 100 eV to a few keV.

After the 1 MeV kinematic threshold for pair production, the cross-section for that is roughly constant.

The typical length for pair production is $\Lambda_{pp} = 1/(n\sigma_{pp})$, which is roughly constant with energy when the photon energy is above a few MeV.

It comes out that this $\Lambda_{pp}$ is similar to the radiation length $x_0$ for electrons, even though they are describing very different physics. So, typically, the photons emitted by bremsstrahlung emit a second generation of electrons and positrons on the same length scale as the bremsstrahlung itself.

This is the process underlying an electromagnetic shower. In this toy model, we have $e^- \to \gamma e^- \to (e^+e^-)(e^-\gamma) \to \dots$, so typically the number of particles scales like

$$N(t = x/x_0) \sim 2^t \qquad \text{and} \qquad E(t = x/x_0) \sim \frac{E_0}{2^t}\,. \tag{2.28}$$

This is the **Heitler model**. After the energy drops too low the secondary particles will start all being absorbed, so the actual curve will have a maximum $N_{\max} = N(t_{\max})$.

A good approximation for this is

$$t_{\max} = \log_2\left(\frac{E}{E_c}\right)\,. \tag{2.29}$$

The logarithmic dependence is good if we want to build a detector: increasing the energy by an order of magnitude does not mean we need to increase the size of the detector by an order of magnitude!

This all holds both for a high energy $e^\pm$ and for a high energy $\gamma$, but it is a good model also for other cosmic rays. The qualitative behavior is the same for higher-mass particles, but the critical energy is much higher.

The radiation length scales like

$$x_0^\mu = x_0^e \left(\frac{m_\mu}{m_e}\right)^2, \tag{2.30}$$

and so does the critical energy.

The critical energy for muons is $4 \times 10^4$ times the one for electrons: roughly 4 TeV, which means we only rarely have to deal with bremsstrahlung induced by muons.

Friday 2021-12-17

We were dealing with the interaction of radiation with matter, and we saw the Bethe-Bloch formula.

We have found a way to determine the radiation length

$$x_0 = 180 \frac{A}{Z^2} \mathrm{g/cm^2} \,, \tag{2.31}$$

as well as seeing that the maximum penetration length is $x \propto \log(E_0/E_i)$.

The final angle of the particle after interaction with the medium will be distributed according to a Gaussian with:

$$\sigma_\theta \approx \frac{21\,\mathrm{MeV}}{p\beta} \sqrt{\frac{x}{x_0}} \left(1 + 2 \times 10^{-3} \log(\dots)\right) \,, \tag{2.32}$$

where the logarithmic correction is small, of the scale of its prefactor.

The Molière radius is defined as

$$R_{\text{Molière}} = \frac{21\,\mathrm{MeV}}{E_c} x_0 \,, \tag{2.33}$$

and it characterizes the transverse scale of the beam.

What happens if a proton is incoming? The length before it interacts will typically be

$$\Lambda \approx \frac{1}{n\sigma} \,, \tag{2.34}$$

and we have already seen what the plot for $\sigma_{pp}$ looks like. The oscillations mostly happen at low energies, ad high energies there is an increase, while in a wide region we have $\sigma_{pp} \approx 40\,\mathrm{mb}$.

The hadronic cross-section is quite universal: the length scale for the interaction of a pion is similar to that for a proton.

The longitudinal shower development is always peaked at few interaction lengths, like $5\Lambda$, always proportional to $\log E$.

The distribution of the angle of the products after an hadronic interaction will look like $\theta \approx p_T/p_\parallel$, with $e^{-p_T/p_0}$.

Let us assume that $p_0$ is close to $300\,\mathrm{MeV}$; the scale of the transverse momentum can be estimated with $\Delta x \Delta p \sim \hbar$, where $\Delta x$ is the length scale of the interaction cross-section.

Typically, in a material $\Lambda > x_0$ as long as $Z > 10$ in the medium. Therefore, the hadronic shower typically is deeper than the electromagnetic one.

Suppose we are in LNGS and we have cosmic rays in the atmosphere. What is the minimum energy the muons must have to reach the detector?

Gran Sasso rock has a density of roughly $2.3\,\mathrm{g/cm^3}$ and a depth of roughly $1500\,\mathrm{m}$. This is equivalent to about $3\,\mathrm{km}$ of water.

So, we want to require that $R_\mu(E) \geq 3200\,\mathrm{mwe}$. This is defined as

$$R_\mu(E) = \int_{m_\mu c^2}^{E} \frac{\mathrm{d}E}{\left|\mathrm{d}E/\mathrm{d}x\right|} \,. \tag{2.35}$$

How does the energy loss scale as a function of energy? We will have several contributions, so

$$\frac{\mathrm{d}E}{\mathrm{d}x} = f_{\text{bethe-bloch}}(E) + f_{\text{bremsstrahlung}}(E)\,, \tag{2.36}$$

but the bremsstrahlung for muons will be suppressed by $(m_\mu/m_e)^2$ compared to that of electrons: the threshold for electrons is roughly 100 MeV, so that for muons is 4 TeV.

So, unless we deal with very high-energy muons the contribution can be neglected; we will be able to write the radiative energy loss by bremsstrahlung as $bE$ if needed (above a few hundred GeV).

The Bethe-Bloch term is roughly constant for a very large energy range, so the integral is rather easy. In general, if

$$\left|\frac{\mathrm{d}E}{\mathrm{d}x}\right| = a + bE \tag{2.37}$$

then we will have

$$R(E) = \int_{mc^2}^{E} \frac{\mathrm{d}E}{a+bE} \approx \int_{mc^2}^{E_c} \frac{\mathrm{d}E}{a} + \int_{E_c}^{E} \frac{\mathrm{d}E}{a+bE}\,, \tag{2.38}$$

where the $bE$ term is dominant in the high-energy range. This means that when $E < E_c$ we simply have $R(E) = (E - mc^2)/a$, while for $E > E_c$ we get

$$R(E) \approx \frac{E_c - mc^2}{a} + \frac{1}{b}\log\frac{E}{E_c} \approx \left(2.5 + 1.2\log\frac{1+4E/\text{TeV}}{3}\right)\,. \tag{2.39}$$

The value of $b$ when $E$ is expressed in MeV is approximately $b \approx 8 \times 10^{-6}\,\text{MeV}\text{g}^{-1}\text{cm}^2$. This means that the minimum energy for muons is about 1.4 TeV. Not very many muons have such an energy. Suppose we have a pion with energy 10 GeV.

The minimum energy loss rate is $2\,\text{MeV}\text{g}^{-1}\text{cm}^2$, this means that the range is

$$R = \frac{E}{\Delta E/\Delta x} \approx 5 \times 10^3\,\text{g}/\text{cm}^2\,, \tag{2.40}$$

which translates to 50 m in water, 7 m in iron, 50 km in air.

These are generally quite long; what about the hadronic interaction length $\Lambda$? For the cross-section we can use $\sigma \sim 40\,\text{mb}$, for the number density we have

$$n = N_A \frac{\text{atoms}}{\text{mol}} \frac{A\text{nucleons / atom}}{A\text{grams/mol}} \rho \approx N_A \frac{\rho}{\text{g}/\text{cm}^3} \text{targets / cm}^3\,. \tag{2.41}$$

This comes out to

$$\Lambda \approx \frac{0.4\,\text{m}}{\rho/(\text{g}/\text{cm}^3)}\,. \tag{2.42}$$

When looking up the proper numbers, we have that interaction dominates: in iron, it happens after only 1.6 cm, in water after 8.6 cm, in air after about 700 m.

We also need to look at decay! The decay time is 26 ns for a pion, meaning that the decay length is $c\tau\gamma\beta \approx \gamma 7.8\,\text{m} \approx 540\,\text{m}$.

This scales linearly with the energy!

# 3 Detectors

One of the simplest detectors one can make is a Geiger counter: a chamber with gas and a wire.

A particle comes through and generates ionization; since there is an electric field the electrons and ions are separated from each other.

The velocity of the drift is typically plotted as a function of $E/P$; it saturates to a certain plateau value, typically of the order of a few cm/µs for electrons, while ions are a factor 1000 slower.

The variation in the voltage is related to a variation in the charge by $\delta V = \delta q / C$. The rise time of the voltage is roughly $v_{\text{drift}} R$, where $R$ is the length scale of the detector.

The HV of the detector is quite large compared to $\delta V$; so we need a high-pass filter, such that $RC \lesssim v_{\text{drift}} R$.

Typically one uses $RC = (50\,\Omega)(\dots)$. Also, one puts a strong resistor before the detector, to reduce the current.

The value of $\delta q$ will be $N_{\text{pairs}} e$; the number of pairs can be computed as

$$N_{\text{pairs}} = \frac{\Delta E}{W}\,, \tag{3.1}$$

where $W$ is the energy needed to release one pair, while $\Delta E$ is the total energy lost in the detector. Typically, in gas detectors one has $W \approx 10\,\text{eV}$.

What is $\Delta E$? Since $\mathrm{d}E/\mathrm{d}x \approx 2\,\text{MeV}\text{g}^{-1}\text{cm}^2$ we have $\Delta E = 2\,\text{keV}$ in a cm of material. This means that we release about 200 pairs, or $3 \times 10^{-17}\,\text{C}$.

What is the capacitance of a cylinder with a given length and a radius of 1 cm? Monday 2021-12-20

> What is the capacitance of a cylinder with a wire?
>
> $$C = \frac{2\pi\epsilon L}{\log R_{\text{cylinder}} / R_{\text{wire}}}\,. \tag{3.2}$$

Last time we were looking at some particle detectors. We are looking at a detector with a cylindrical shape — a wire within a cylinder.

We saw that for a $\sim 2\,\text{MeV}$ particle there will be roughly 200 emitted electrons.

The drift time for a 1 cm detector will roughly be 200 ns, since the drift time is roughly 5 cm/µs.

This means that the shape of the pulse, after an RC high-pass filter, is

$$\overline{V}(t) = \overline{RC}\frac{\mathrm{d}V(t)}{\mathrm{d}t} \approx \overline{RC}\frac{\delta V}{T_D} = \overline{R}\frac{\overline{C}}{C}\frac{\delta q}{T_D}\,. \tag{3.3}$$

The current will roughly be $\delta q / T_D \approx 0.16\,\text{nA}$. The change in tension is roughly $\Delta V \sim IR \approx 8\,\text{nV}$.

So, we need an amplification mechanism.

The anodic wire needs to be very thin, so that we have a large electric field, which means that we get amplification by the emission of secondary electrons. This gain can be a factor $10^5$! This means we get a charge $\delta q \approx N_{\text{pair}} e \times \text{gain}$.

We get $\Delta V \sim 0.8\,\mathrm{mV}$; with better systems we typically get millivolt currents.

We should also aim to reduce the capacitance of the detector!

At low values for the high voltage we are in the ionization regime; this means that we see exactly the number of charges produced by the particle. This is the ionization regime. $\delta q(V) = \text{constant}$.

At higher voltages, we will be getting a certain gain: this is the proportional regime. $\delta q(V) = \alpha \delta q$.

At even higher voltages, we will get a nonlinear profile, with saturation. This is the limited proportional regime.

At even higher voltages, we fully saturate: this is the Geiger-Muller regime. Here we do not have any information about the particle, we just know that something has passed through.

This all depends on the charge of the particle: for $\alpha$ particle the $\delta q$ is always $Z^2 = 4$ times higher.

We can use many of these gas detectors to figure out where a particle passed through.

If this is the case, we can also get trajectory information! The delay between the arrival time of the particle ionization in different wires tells us about how close to the center of each of them it passed. If we have a star, trigger that's even better!

This kind of thing is used in LHC.

An alternative is to have many HV wires in the same chamber. This is called a *wire chamber*.

The spatial resolution can be computed as $\sigma_x \approx v_{\text{drift}} \sigma_t$, so if we have millisecond timing we can get a few hundreds of microns.

Another component is a *quencher*. Some electrons and ions produced might recombine, producing UV photons. Those can then ionize the gas again, in a place which is unrelated. So, we need to quench these UV photons (absorb them) so that they do not mess up our measurement.

If we also have a magnetic field, the trajectory of the particle passing through will be bent; this allows us to measure the momentum and charge of the particle, but it will also curve the trajectories of the electrons.

What to do when the interaction cross-section is very low? We can increase the density of the gas, or even make it a liquid.

Those can be for example liquid Argon or liquid Xenon Time Projection Chambers.

### 3.1 Scintillators

A fraction of the energy from a particle is emitted as light. Atoms are ionized and then recombine; the UV light they emit thus is called *scintillation light*.

If the total energy lost is $\Delta E$, and $W$ is the energy required to produce a scintillation photon ($W \approx 100\,\mathrm{eV}$).

For a $2\,\mathrm{MeV}$ particle, we get about 20000 photons. The wavelength of this light depends on the energy states of the material. Typically the emission of these happens within a few nanoseconds.

We need the material to be transparent to the scintillation photons, but we need to contain it within a reflective material so that the photons are not going everywhere.

The mechanism for the absorption of these visible or UV photons will be the photoelectric effect.

The quantum efficiency of the detector describes the fraction of the photons which are detected.

We need to give energy to the electron emitted by the photon! It will have low, order-eV energy. The way this is typically done is through a series of dynodes. We connect the HV supply to a series of resistors, and have a uniformly decreasing voltage between the dynodes. The gain will depend on the number of secondary electrons produced by the dynode chain per incident electron, which is denoted as $\alpha$.

The dependence is roughly a powerlaw as a function of the HV. The system is roughly saturated when the current due to the secondary electrons roughly equals the HV/$R$ current.

These are *photomultipliers*.

We will have a certain efficiency for our collection $\epsilon_{\text{coll}}$, as well as a quantum efficiency $\epsilon_q$.

The signal will be

$$N_\gamma \epsilon_q \epsilon_{\text{coll}} \times \text{gain}\,. \tag{3.4}$$

Typical numbers are about $N_\gamma \approx 2 \times 10^4$, and both efficiencies at 20 %.

With this, we get about $N = 800$ photoelectrons.

The time required for the detection is a few nanoseconds for the emission, a few nanoseconds for the propagation, and a further few nanoseconds for the collection by the photomultiplier. In the end, the current will be roughly $I \approx eN/(100\,\text{ns})$;

With a $50\,\Omega$ resistor, this means a voltage of $70\,\text{nV}$ times the gain.

In order to get to the millivolt regime, we need a gain on the order of a million. So, if each dynode allows for a doubling of the electrons we need about 20.

With these kinds of detectors we can do nuclear spectroscopy.

## References


[Alo+18] Roberto Aloisio et al. "Selected Topics in Cosmic Ray Physics". In: *Multiple Messengers and Challenges in Astroparticle Physics*. Ed. by Roberto Aloisio, Eugenio Coccia, and Francesco Vissani. Cham: Springer International Publishing, 2018, pp. 1–95. ISBN: 978-3-319-65423-2 978-3-319-65425-6. DOI: 10.1007/978-3-319-65425-6_1. URL: http://link.springer.com/10.1007/978-3-319-65425-6_1 (visited on 2021-11-08).

[Gro+20] Particle Data Group et al. "Review of Particle Physics". In: *Progress of Theoretical and Experimental Physics* 2020.8 (Aug. 14, 2020). DOI: 10.1093/ptep/ptaa104. URL: https://academic.oup.com/ptep/article/2020/8/083C01/5891211 (visited on 2021-11-08).