

Experimental Gravitation and Cosmology

Jacopo Tissino

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Wednesday
2021-11-17

GW detection! This part is given by Jan Harms.

Marica will talk more about GW science and the multimessenger approach.
The first slide is the north arm of Virgo, Einstein Telescope and the LGWA.

Outline:

1. overview of GW detection;
2. detector responses to GW;
3. noise spectra, filtering, SNR, transfer functions;
4. fluctuation-dissipation, thermal noise;
5. quantum noise & squeezing;
6. passive and active seismic isolation, Newtonian noise;
7. GW detector concepts.

The interferometer is operated at the dark fringe

but not exactly there?

At a frequency scale of $\sim H_0$ we can do measurements thanks to CMB observations.

At the nHz scale (corresponding to lyr wavelengths) we can use Pulsar Timing Arrays, with radio telescopes. Pulsars being the “best clocks in the universe” is not really true. For now, they’ve discovered red noise, which could potentially be a GW signal.

At the mHz scale we might have space detectors such as LISA: here the GW wavelengths are of the order of 1 AU.

At the 100 Hz scale we have ground-based detectors.

Current infrastructure is expected to reach $z \sim 2$; we expect future infrastructure to reach $z \sim 100$. This goes all the way back to the dark ages in the Universe’s history.

This means that ET + CE would see something like 90 % of BNS mergers, and basically all BBH mergers.

It is good to have a long baseline.

Jan disses Cosmic Explorer a bit. 40 km is ambitious because of the cross-couplings due to the fact that the mirrors will not be perpendicular to the suspension system, which will

point straight down at both edges. Also, with such lengths there is a loss in sensitivity in the kHz band.

High performance sensing will require cold temperatures, which however also means that we need to redesign basically all the infrastructure.

The dissipation in the system dictates the thermal noise.

We need in-vacuum optical systems; people were extremely worried before the construction of LIGO about this. Now in Livingston they found a hole. The idea to find it is like finding a hole in a bike tire, with liquid helium around the detector and helium sensors inside.

Current detectors have a finite lifetime. Virgo is slowly sinking into the ground, since it is built on soft soil.

There are disturbances caused by powerful lightning strikes. It is crucial to monitor the environment. In the future, there will be online background removal.

Einstein Telescope needs very big underground chambers, with roofs on the order of 25 m. Do we have enough people in Europe to build ET? Not clear.

Kagra taught us some lessons. The spring melt filled up water reservoirs there, so there was a waterfall inside the arms there. That means a lot of humidity. The water stream near the test masses was gravitationally coupled to them; Newtonian noise is bad.

If ET is built in Sardinia it will be really dry; maybe not so for the Netherlands. We need to be careful that the ventilation and cryogenic systems do not create noise.

Ventilation is needed for both humidity and radioactivity management. The experiments in LNGS, say, are much more sensitive.

LISA will be able to locate sources thanks to their amplitude modulation during the year. At these frequencies going underground would not help.

There is a requirement for drag-free navigation, though.

The beam to another satellite is on the order of 25 km in size.

Now we talk about LGWA. Weber invented everything when it comes to GW detection technologies. He invented the resonant bar detectors.

He tried detecting GWs from quadrupolar seismic vibrations on the Earth! We now know that their amplitude is much too small for that. They did, however, put the first upper limits on the amplitude of GWs.

Apollo 17 brought the Lunar Surface Gravimeter. A design flaw limited its sensitivity. The problem of having 14 days of lunar night without solar power is large.

A Russian team is probing microwave beaming for the transmission of power. Nuclear power is possible, but how do we shield from it? They used plutonium there. However, nobody's producing plutonium anymore... But it might be produced again thanks to a decision by the Trump administration.

That gravimeter failed on the Moon because of an arithmetic mistake which failed to account for the decreased gravity there.

One can bring stuff to the Moon at a price of about $\$10^6/\text{kg}$. This might decrease in the future.

LGWA: four seismometers at \sim km separation.

LSGA: interferometers connected to the ground, measuring the deformation of the sur-

face of the Moon. They'd need to be deployed at 10 km distance, but the Moon curves: maybe put them at the rims of a crater? this is very difficult.

GLOC: basically Cosmic Explorer on the Moon. By Jani and Loeb, two theorists. These detectors do require a lot of maintenance.

The spectrum of noise on the Moon is on the order of $10^{-10} \text{ m}/\sqrt{\text{Hz}}$ between 0.1 Hz and 1 Hz.

This is much lower than what is observed on the Earth, and which is mostly due to the ocean.

Micro-meteoroid impacts have a small impact.

The Moon might be the quietest place in the Solar System: it being tidally locked helps a lot. It is also near the coldest: there are permanently shadowed regions at the poles. These have never seen sunlight, neither direct nor indirect, for a billion years or more.

It might even be colder than Uranus or Neptune: the absence of radioactivity helps a lot.

We can use superconductors for free there.

How do we get power there? An option is beaming from solar panels at the rim. Another option is beaming from satellites in orbit.

Nuclear power? Europe will not do it, since ESA does not launch for Europe (as opposed to NASA and the Chinese space agency): therefore, any issues would be dumped onto a South American country, a huge political issue.

The presence of gravitational waves from inflation, at the tensor-to-scalar ratio given by simple, single-field inflationary models, would be incredibly important.

A direct observation for this kind of thing would be the Big Bang Observer. This would be a proof of quantum gravity!

This would be a 12-satellite configuration, two triangles and a hexagon, smaller than LISA, sensitive to the deciHertz regime. Why not the milliHertz band? LISA is also limited by GW foregrounds!

Foreground removal is computationally difficult.

1 GW detectors

Monday

The basic design of a GW detector is that found in LIGO Scientific Collaboration and Virgo Collaboration et al. [LIG+16, fig. 3].

It is roughly a Michelson-Morley interferometer, with 10 kW coming in from a power-recycling mirror plus a Fabry-Perot cavity amplifying that power to 100 kW.

These cavities are roughly 3 km long.

The frequency of the laser is on the order of $f \sim 10^{15} \text{ Hz}$, and it has a small dispersion in frequency space.

What then happens is that when the length of the cavities is perturbed, some power in each cavity is shifted into the *sidebands* $f_0 \pm \Delta f$ of the carrier frequency f_0 , where $\Delta f = f_{\text{GW}} \ll f_0$.

We can fully control the length of the arms, to get our preferred interference condition.

What can happen to laser light? It can be dissipated, it can come back to the mirror, but we need a device to dump the beam and prevent it from coming back to the laser.

A DC offset: we want a small fraction of the light, roughly 0.1 %, to get to the photodetector. We add a bit of the laser light to the sidebands.

This is a homodyne detection scheme, and the laser is called a local oscillator.

Correlated signals go back to the power recycling mirror, while anticorrelated signals go to the detector.

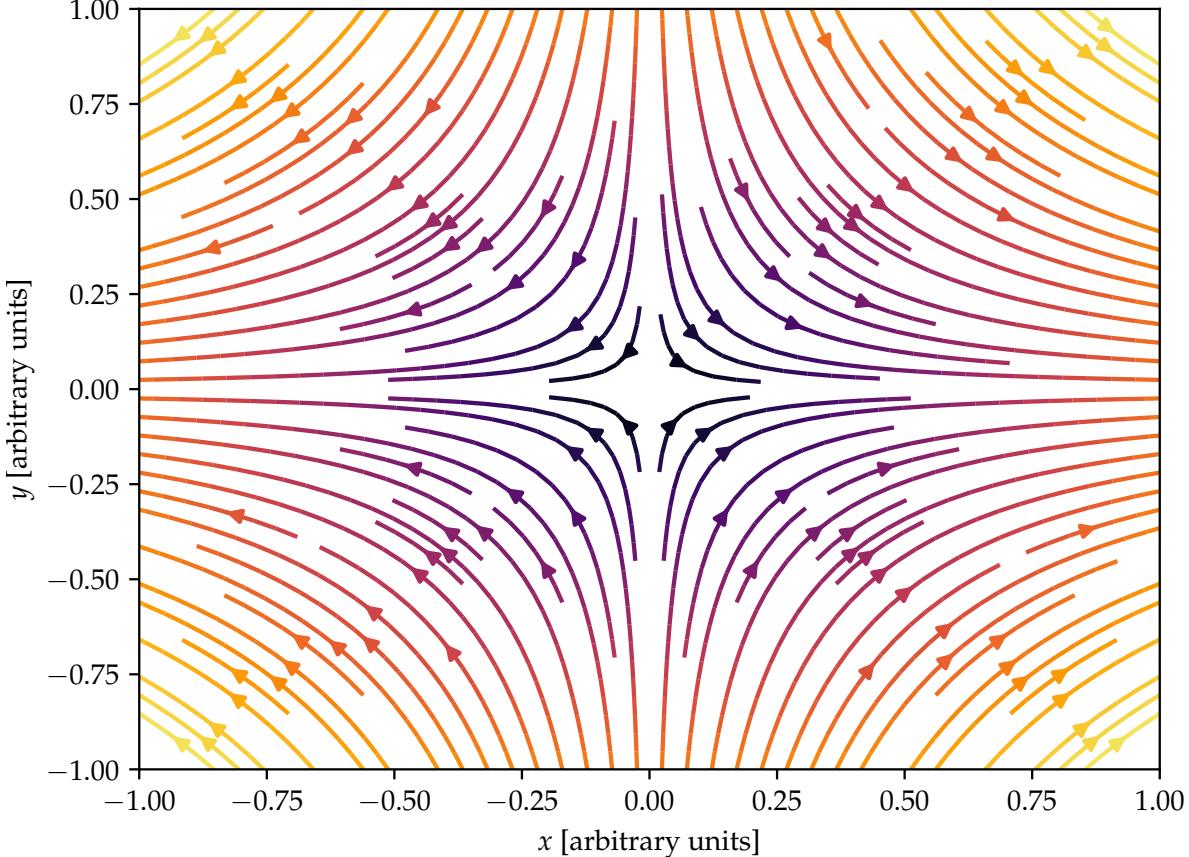


Figure 1: The effect of the h_+ polarization of gravitational waves. Color indicates the magnitude of the field (weaker in the middle).

Noise sources

The main *environmental* noise sources are

1. scattered light;
2. seismic / vibrational noise;
3. Newtonian noise;
4. electromagnetic noise.

Basically all GW detectors have a strain sensitivity curve, which is roughly speaking the noise referred to the quantity “strain”: $h = 2\Delta L/L$. The “bucket” is centered around 100 Hz.

The true signal we measure is the current from the photodiode. The noise in the detector could be reported in Ampères, but plotting things in this way allows us to include the response.

A rough way to say this is

$$\text{sensitivity} = \frac{\text{noise}}{\text{response}}. \quad (1.1)$$

The laser is tuned to the resonant frequency of the cavity, but the same does not hold for the sidebands, which are therefore not amplified to the same amount!

However, we can do resonant amplification of the sideband signal as well!

There are around a hundred different relevant noise sources. There are “fundamental” noises and “technical” noises. The former are a limit set by the detector configuration, nothing can be done about them. The technical noises can be gradually reduced by the crew working on the detector.

At high frequencies, the problem is quantum noise; at mid-low frequencies we have thermal noise, while at low frequencies the dominant contribution is environmental noise.

Isolation from the environment means:

1. seismic isolation;
2. reducing susceptibility to EM fields;
3. picking a quiet environment;
4. dealing with scattered light;
5. vacuum system.

Building underground helps! The seismic fields are quieter there, and there is more insulation.

Why do we need to build an interferometer in Europe? We (Europeans) have optical telescopes in Chile, for example.

Telescopes only need a small amount of people to man them, while GW interferometers need tens of people, and not that many people want to work in an extremely remote location.

We only want the zeroth-order, Gaussian mode in the spatial cross-section of the decomposition of the beam.

However, since the mirrors are not perfectly spherical higher modes are also excited, with magnitudes of the order of 20 ppm. Out of the fraction, then, a tiny fraction scatters on the edge of the vacuum tube (10 ppm) and re-enters the beam. The vacuum tube is not seismically isolated! This means that that light picks up a huge amount of noise.

We therefore need to insert a *baffle*, which absorbs any light which hits it. What people now do is make detailed models of the system, use raytracers to figure out where the light

is going and block it. People have also thought about just coating the whole interior — then, the issue becomes maintenance and coating lifetime.

Thermal noise: it is mainly about thermal vibrations of our suspensions, our mirrors (coating and substrate), and the electronics.

Changing the mirror's thermal noise is hard, electronics could be made superconductive...

Quantum noise is quite simple: it has only two components,

1. shot noise;
2. radiation pressure noise.

Once we know how the fluctuations enter our system, we can control them! What defines the quantum state of the detector?

The scaling of the high-frequency part is mostly shot noise, RP noise has a lower frequency, and we can currently manage to make it negligible.

There are methods to manipulate quantum states in order to reduce quantum noise. The broad topic here is “quantum nondemolition techniques”.

There are all kinds of other noises which enter our system, but the most important ones are the ones we mentioned.

How do we cool our experiment? We have roughly 0.5 ppm absorption, which means about a Watt of power going to the optics! A thermal link is dangerous, since it can introduce vibrations. Voyager wants to do radiative cooling, since it introduces no extra vibrations. However, getting to very low temperatures is basically impossible because of the $\sim T^4$ temperature.

So, ET needs a thermal link to get to 10 K to 20 K. Maybe superfluid helium could work for this purpose...

Timeseries analysis

The basic tenet of timeseries analysis is to work with the spectral representation of the data.

This timeseries will typically be uniformly sampled.

We define the autocorrelation as

$$C(y; t, t') = \langle y(t)y(t') \rangle , \quad (1.2)$$

where the brackets denote an ensemble average. If the noise is stationary, this will just be a function of $\tau = t' - t$, so we get $C(y; \tau) = \langle y(t)y(t + \tau) \rangle$.

Even things like thermal and quantum noise are not really stationary, while environmental noise is *definitely* not stationary.

Fourier transforms diverge for infinitely-extending timeseries, but we can take an alternative approach, which is consistent with the fact that our signals are finite in time.

We can only transform a section of length T ; its transform will be $\tilde{y}_T(f)$. In the case of stationary noise, we can define the single-sided Power Spectral Density as

$$\text{PSD}(f) = S(f) = \lim_{T \rightarrow \infty} \frac{1}{T} |\tilde{y}_T(f)| . \quad (1.3)$$

This converges for stationary noise. We can also include a 2 in the definition, it is a matter of convention. This is the standard way to represent timeseries in the frequency domain.

In terms of units, $[\tilde{y}] = [y]/\text{frequency}$; therefore, $[S] = [y]^2/\text{frequency}$.

If we plot \sqrt{S} , this will have units of $[\sqrt{S}] = [y]/\sqrt{\text{Hz}}$.

The integral of the PSD over all frequencies yields

$$\int_{-\infty}^{\infty} df S(f) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt y_T(t) \int_{-T/2}^{T/2} dt' y_T(t') \underbrace{\int_{-\infty}^{\infty} df e^{2\pi i(t-t')f}}_{\delta(t-t')} \quad (1.4)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} dt y_T^2(t) = \langle y^2 \rangle = \sigma_y^2. \quad (1.5)$$

We can also compute a *bandlimited* variance:

$$\sigma_{\text{bandlimited}}^2 = \int_{f_1}^{f_2} df S(f). \quad (1.6)$$

We have the following relation for the power spectral density:

$$\langle \tilde{y}(f) \tilde{y}^*(f') \rangle = \int dt e^{2\pi ift} \int dt' e^{-2\pi if't'} \underbrace{\langle y(t)y(t') \rangle}_{C(y,\tau)} \quad (1.7)$$

$$= \int dt e^{2\pi i(f-f')t} \int d\tau e^{-2\pi if\tau} C(y, \tau) \quad (1.8)$$

$$= \int dt e^{2\pi i(f-f')t} S(y, f) \quad (1.9)$$

$$= \delta(f - f') S(f). \quad (1.10)$$

Tuesday
2021-11-23
 $\tau = t' - t$

The fact that if $f \neq f'$ this vanishes is a crucial property of stationary noise.

Any linear transformation of such a stationary-noise timeseries will leave it stationary.

If we have a force timeseries $F(t)$ applied to a pendulum, we will get a response like

$$\tilde{x}(f) = \frac{\tilde{F}(f)/m}{(2\pi)^3(f_0^2 - f^2)} \quad \text{where} \quad f_0 = \frac{1}{2\pi} \sqrt{\frac{g}{L}}, \quad (1.11)$$

but this *linear* approximation will fail if the force is very strong. Nonlinear dynamics can turn stationary noise into nonstationary noise. There is a deep connection between linearity and stationarity.

In the time domain, the application of a linear filter looks like

$$y'(t) = \int dt' K(t - t') y(t'), \quad (1.12)$$

which modifies the Fourier transform as

$$\tilde{y}'(f) = \tilde{K}(f) \tilde{y}(f). \quad (1.13)$$

Therefore, the spectral density of the transformed signal reads

$$\tilde{S}(y', f) = |\tilde{K}(f)|^2 S(y, f). \quad (1.14)$$

Matched filtering

How do we define the best linear statistic? Specifically, we'd want to have something which has the largest amplitude variation between the presence of a signal and the absence of it.

If the system is linear, we can approximate the data as

$$d(t) = s(t) + n(t), \quad (1.15)$$

and in the operation of the detector one tries very hard to make this as close to true as possible.

Suppose we define a filter and apply it to the noise and signal:

$$s_F(t) = \int dt' F(t - t') s(t') \quad (1.16)$$

$$n_F(t) = \int dt' F(t - t') n(t'). \quad (1.17)$$

The variance of the filtered noise will then read

$$\langle n_F^2(t) \rangle = \int df |\tilde{F}(f)|^2 S(n, f). \quad (1.18)$$

The filtered signal timeseries can also be written as

$$s_F(t) = \int df e^{2\pi i f t} \tilde{s}(f) \tilde{K}(f). \quad (1.19)$$

We define the SNR timeseries through

$$\text{SNR}^2 = \frac{s_F^2(t_0)}{\langle n_F^2 \rangle} \quad (1.20)$$

$$= \frac{\left| \int df e^{2\pi i f t_0} \tilde{s}(f) \tilde{K}(f) \right|^2}{\int df |\tilde{K}|^2 S(n, f)} \quad (1.21)$$

$$\leq \int df \frac{|\tilde{s}|^2}{S_n}. \quad (1.22)$$

We exploited the Schwarz inequality:

$$\left| \int df e^{2\pi i f t_0} \tilde{s} \tilde{K} \right|^2 = \left| \int df e^{2\pi i f t_0} \tilde{K} \sqrt{S_n} \frac{\tilde{s}}{\sqrt{S_n}} \right|^2 \quad (1.23)$$

$$\leq \int df |\tilde{K}|^2 S_n \times \int df \frac{|\tilde{s}|^2}{S_n}. \quad (1.24)$$

What this means is that any linear filter can reach, at a maximum, this value. If we can then write a filter which reaches this value we know it is the best!

The optimal filter reads

$$\tilde{K}(f) = e^{-2\pi i f t_0} \frac{\tilde{s}(f)}{S_n(f)}. \quad (1.25)$$

Intuitively, one puts less weight on the region of higher noise.

The kernel looks a bit like the elephant in the Little Prince.

2 Perspectives for GW

GW echoes: if the horizon is quantized, the GWs can be reflected if they are not at the right frequencies. For any known quantum gravity theories, the frequencies at which these are reflected are precisely the ones we can probe.

NS equations of states can be probed by the high-frequency region of the waveform.

SN explosions can emit GWs if they have anisotropies.

Stochastic backgrounds can be emitted by cosmological processes.

How do we calibrate our detector? Blind injections were done by moving the test masses. The first GW signal was thought to be a blind injection at first, but then we found that nobody had done it.

The issue is that we cannot continuously characterize the detector in this way. The alternative is modelling the detector.

There are controls which keep the masses stationary at low frequencies. So, at low frequencies we look at the *control signal*!

The problem is most difficult at low frequencies. The control interferes with the GW response below 50 Hz.

The equation for the strain, including this, is

$$h(t) = \frac{1}{L} \left[\mathcal{C}^{-1} d_{\text{err}}(t) + \mathcal{A} d_{\text{ctrl}}(t) \right]. \quad (2.1)$$

The inverse of the SNR gives roughly an order of magnitude for the required calibration error. Currently, the LIGO-Virgo collaboration has reached roughly 2 to 3 %.

There are very few places which can give a laser to be used in pushing to calibrate...

Unmodelled signals

Modelling a SN GW signal is very hard, simulations do not agree with each other.

The signal roughly increases in frequency from a hundred to a thousand Hz within about a second.

The first signal we detected was detected through a burst search: templates with masses as high as $30M_{\odot}$ were not used because those masses were not though to be likely.

Convolving a signal with wavelets allows for a time-frequency plot.

A type II supernova would have to happen in our galaxy for us to be able to see it with current instruments, and future detectors will not improve this by much.

See [gwburst.gitlab.io](https://gitlab.io/gwburst).

Continuous signals

A pulsar which is not axisymmetric will emit GWs. These are signals which are definitely modelled: we know basically everything.

Computational cost scales with T^6 .

Glitches are a problem in this case.

Why is there such a large cluster of millisecond pulsars?

We know from these GW observations that most of the braking in the NS spin-down is *not* due to GW emission. We have bounds on the order of $h \sim 10^{-26}$.

Quantum noise

Friday
2021-12-3

The measurement process can be thought of as “counting photons”, although we cannot really determine their exact number, but this is relevant since it means the measured intensity fluctuates with \sqrt{N} .

If $\Delta N / \Delta t$ is constant (we get a fixed number of photons per unit time), we have a **Fock state** $\hat{n} |n\rangle = n |n\rangle$, an eigenvector of the photon number operator $\hat{n} = a^\dagger a$.

If this were the case, we would have no fluctuations in the readout. But, besides the fact that photodetectors do not count photons, we do not produce Fock states.

It is convenient to describe this process in the Heisenberg picture. If we use it, the operators we use are all functions of t while the states are unchanged. The state we will use will always just be the vacuum state $|0\rangle$.

Classically, the output field will be given by a certain linear transformation of the output field:

$$\vec{E}_{\text{out}}(\vec{x}, t) = \mathcal{L}[\vec{E}_{\text{in}}(\vec{x}, t)], \quad (2.2)$$

but this will also be what we do in our quantum-mechanical, Heisenberg-picture treatment.

The difficulty comes from the fact that there are actually many input fields: at various places, like the mirrors, we do not have “nothing” since there is always at least the vacuum.

The transmissivity τ of a mirror is the ratio between the incoming and transmitted field magnitudes. All mirrors have a transmissivity $\neq 0$, so we always have a coupling to the vacuum state on the other side of the mirror.

If there are losses, we cannot have a unitary process: so, there will never be “one-way” losses, and a loss will always be associated with an extraneous input from the environment.

The transmissivity and reflectivity satisfy $\tau^2 + \rho^2 = 1$; the transmissivity can be interpreted as a loss parameter, so if $\tau = 0$ all the field is reflected back into the system, but we will never have this condition.

So, if A is our input, some τA is leaving our system; because of what we were saying above the field going back will not be just ρA but instead $\rho A + \tau V$, where V denotes the vacuum state on the other side of the port.

A very important field is the one coming from the photodiode, which will often look like a thermal state. However, its temperature will be very low compared to the laser light: that is near-infrared, 100 to 1000 THz, corresponding to $\gtrsim 700$ K of thermal temperature. So, we can take it to be the vacuum.

Only about 0.5 % of the laser light at the beamsplitter reaches the photodiode. Therefore, changing the field at the laser will have little effect on the diode; on the other hand, 99.5 % of the field from the photodiode will come back to the laser.

Only in the eighties someone properly described an interferometer in QFT. Before, people just used Poissonian statistics to calculate the quantum noise.

The first important insight is that we need to act on the field at the photodiode.

Let us neglect the vector character of the field for simplicity; the fields are all linearly polarized, and the quantum mechanics for the electric and magnetic fields are the same. The field will be

$$\hat{E}(x, t) = \sqrt{\frac{2\pi\hbar}{Ac}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \sqrt{\omega} \left(\hat{a}(\omega) e^{-i\omega(t-\vec{x}\cdot\vec{e}/c)} + \hat{a}^\dagger e^{i\omega(t-\vec{x}\cdot\vec{e}/c)} \right), \quad (2.3)$$

where we are fixing the propagation direction by fixing $k = \omega/c$; the creation and annihilation operators will satisfy

$$[\hat{a}(\omega), \hat{a}^\dagger(\omega')] = 2\pi\delta(\omega - \omega'). \quad (2.4)$$

If we apply the annihilation operator to the vacuum we get $\hat{a}|0\rangle = 0|0\rangle$; since the photon number operator is $\hat{n} = \hat{a}^\dagger \hat{a}$ we also have $\hat{n}|0\rangle = 0|0\rangle$.

A **coherent state** is an eigenstate of the annihilation operator: $\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$, where the eigenvalue α is a generic complex number. This is “close to classical”, it describes the output of a laser quite well, and it has Poissonian statistics.

What is the distribution of the number of photons in a coherent state?

$$p(n) = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}. \quad (2.5)$$

This is a Poissonian with average photon number $\langle n \rangle = |\alpha|^2$.

The energy in the field is given by $E = \hbar\omega_0 n$, so this value also corresponds to the mean energy in the field.

In the Heisenberg picture, one finds that the laser is mapping $\hat{a} \rightarrow \hat{a} + \alpha$. In a sense, all “classical” things are fixed complex numbers as opposed to operators.

The full operator is $\hat{a} + h + \alpha$, where \hat{a} describes the quantum fluctuations, h describes the GW signal, while α describes the laser.

We have not yet defined what is our canonical pair of observables. We can write

$$\hat{E}(x, t) = E_1(x, t) \cos(\omega_0 t - kx) + E_2(x, t) \sin(\omega_0 t - kx). \quad (2.6)$$

Why are we picking a single frequency ω_0 , while our field has many? It is convenient since the laser frequency (ω_0) is the main one, but there are also things fluctuating at all other frequencies. We expect quantum fluctuations to have a white spectrum, so for those it is the same; however the sidebands from the GW signal will be close to ω_0 .

In the photocurrent we measure, $I_{\text{ph}}(t) = |E_{\text{photodiode}}(t)|^2$, we would have product terms between the laser frequency and the GW frequency.

The Fourier transform of the photocurrent allows us to compute a Power Spectral Density $S(I_{\text{ph}}, \omega)$.

The Fourier transform looks like

$$\tilde{I}_{\text{ph}}(\omega) \sim \underbrace{|\alpha|^2}_{\omega_0, \omega_0} + \underbrace{\text{Re}(\alpha\hat{a})}_{\omega_0, \omega} + \underbrace{\text{Re}(\alpha h)}_{\omega_0 \pm \Omega, \omega_0} + \underbrace{\text{Re}(h\hat{a})}_{\omega_0 \pm \Omega, \omega}. \quad (2.7)$$

The term which does not fluctuate, $|\alpha|^2$, is often called a DC component or DC offset: since we bandpass the photocurrent signal, any low-frequency component like that will vanish.

The noise term will oscillate at $\omega_0 - \omega = \Omega$; in the end the photocurrent we will actually have left after the bandpassing we get

$$\hat{I}_{\text{ph}}(\omega) \sim h(\Omega) + \hat{a}(\Omega). \quad (2.8)$$

What we are basically doing is extracting out the fast-oscillating term, and we can focus on the slow, audio-band oscillations we care about.

The fluctuations in the photocurrent will contain the square moduli of the *quadratures* E_1 and E_2 , which form a Heisenberg pair.

How do we actually manipulate the vacuum field at the photodiode? A squeezer is introduced, and it passes a polarizing beamsplitter, which is always either fully transmissive or fully reflective depending on the polarization of the light.

The unpolarized vacuum passes the polarizer in some fraction, but when it comes back it is fully reflected.

The squeezer emits little power, and some of it is in green light as opposed to infrared!

The power in the Fabry-Perot cavity is very high ($\sim 200 \text{ kW}$), but a small amount of squeezing power is enough to improve the sensitivity significantly.

As mentioned last time, the measured intensity is proportional to the *low-pass* filtered square of the electric field: $\hat{I} \propto |E|_{LP}^2$.

The two quadrature fields E_1 and E_2 can be written in terms of the $\hat{a}_{1,2}$ creation and annihilation operators:

$$\hat{E}_{1,2}(\vec{r}, t) = \sqrt{\frac{4\pi\hbar}{\mathcal{A}c}} \int \frac{d\Omega}{2\pi} \left[\hat{a}_{1,2}(\Omega) e^{-i\Omega + i\vec{k}\cdot\vec{r}} + \hat{a}_{1,2}^\dagger(\Omega) e^{+i\Omega - i\vec{k}\cdot\vec{r}} \right], \quad (2.9)$$

where

$$\hat{a}_1(\Omega) = \underbrace{\sqrt{\frac{\omega_0 + \Omega}{2\omega_0}}}_{\approx 1/2 \text{ when } \Omega \ll \omega_0} \hat{a}(\omega_0 + \Omega) + \sqrt{\frac{\omega_0 - \Omega}{2\omega_0}} \hat{a}(\omega_0 - \Omega) \quad (2.10)$$

$$\hat{a}_2(\Omega) = -i \underbrace{\sqrt{\frac{\omega_0 + \Omega}{2\omega_0}}}_{\approx 1/2 \text{ when } \Omega \ll \omega_0} \hat{a}(\omega_0 + \Omega) + i \sqrt{\frac{\omega_0 - \Omega}{2\omega_0}} \hat{a}(\omega_0 - \Omega). \quad (2.11)$$

When we compute the modulus $|E|^2$ we get a $\sim |E_1|^2 \cos^2(\omega_0 t)$ term, a $|E_2|^2 \sin^2(\omega_0 t)$ term and a $2E_1 E_2 \sin(2\omega_0 t)$ term. The last of these averages to zero over a few periods the laser pulsation $2\pi/\omega_0$ (so, a very short time) while the square cosines and sines both average to 1/2. This means we get

$$|E|^2 = \frac{1}{2} \left(|E_1|^2 + |E_2|^2 \right). \quad (2.12)$$

Tuesday
2021-12-7
(missed lesson)

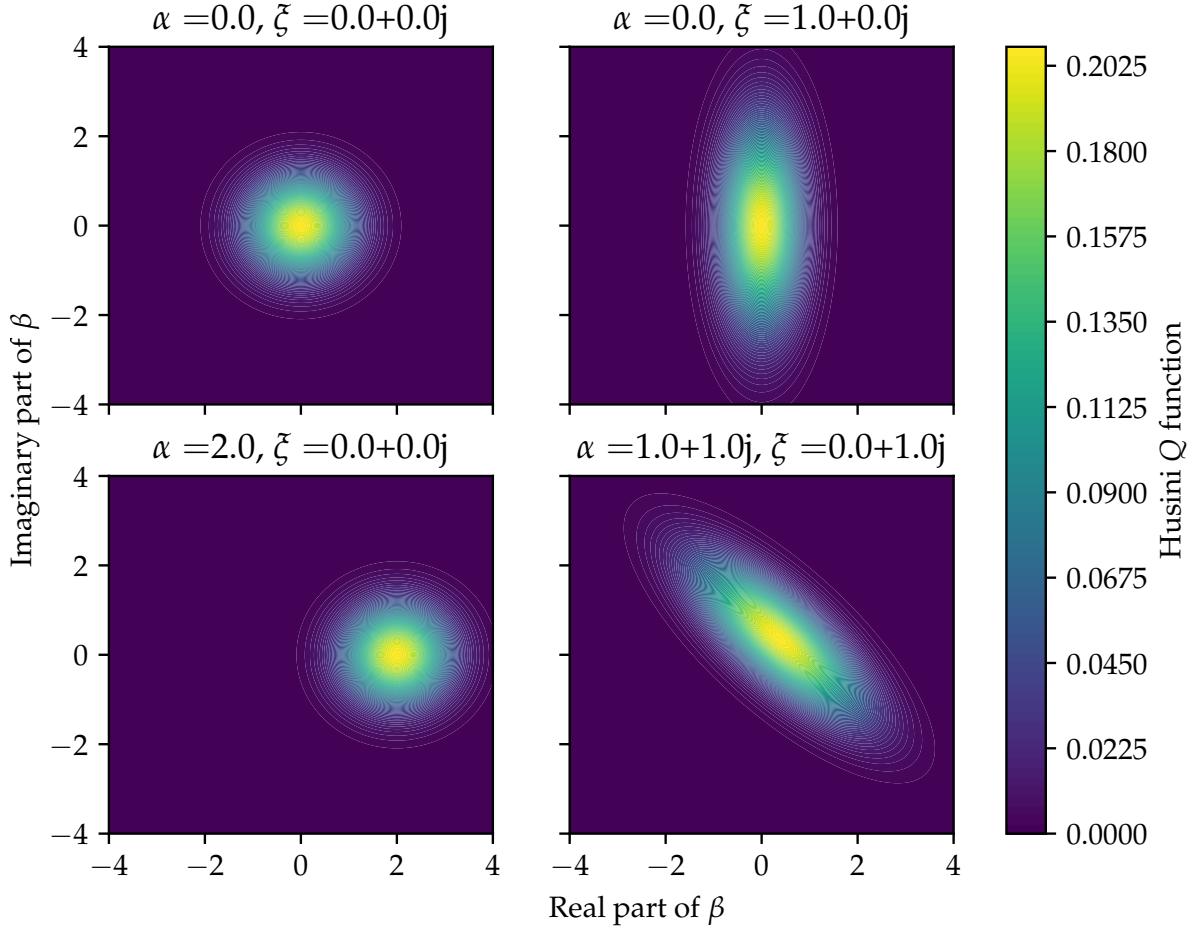


Figure 2: Husini Q -function, $Q(\beta) = \langle \beta | \hat{\rho} | \beta \rangle / \pi$, for squeezed vacuum states [GK04, eqs. 3.112 and 7.82].

A field quadrature can be written in terms of a baseline A plus some oscillations with period Ω :

$$\hat{E}_1 = A + \sqrt{\frac{4\pi\hbar}{\mathcal{A}c}} \int \frac{d\Omega}{2\pi} \left[\underbrace{\hat{a}_1 e^{-i\Omega t + i\vec{k} \cdot \vec{r}}}_{\hat{c}_1} + h. c. \right]. \quad (2.13)$$

This quantum noise is constant in Ω :

$$S(\hat{a}_1, \Omega) = \lim_{T \rightarrow \infty} \frac{\langle \hat{a}_1 \hat{a}_1^\dagger \rangle}{T} = 1(?). \quad (2.14)$$

The Heisenberg principle for these is $S(a_1)S(a_2) \geq 1$.

When we do homodyne detection we are measuring a product in the form $I(\Omega) \propto E_{\text{LO}} \cdot E_{\text{signal}}$; we can determine amplitude and phase of the local oscillator field, then we can squeeze the vacuum in the direction parallel to it and allow it to expand in the other.

Typically, the wavelength corresponding to the pump laser, at $2\omega_0$, is green light ($\lambda = 532 \text{ nm}$). This is then passed to an Optical Parametric Amplification crystal which is basically an SPDC crystal plus amplification.

The squeezing procedure can be schematically represented as

$$\begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix}}_{\Sigma(\sigma, \varphi)} \begin{bmatrix} e^{-\sigma} & 0 \\ 0 & e^{\sigma} \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix}. \quad (2.15)$$

The transformation matrix Σ determines the power spectral density:

$$S(s_1, s_2, \omega) = \text{Re}(\Sigma \Sigma^\dagger) \underbrace{S(a_1, a_2, \Omega)}_{\equiv \mathbb{1}_2}, \quad (2.16)$$

where these power spectral densities are to be interpreted as covariance matrices.

Monday

At high frequency we also lose sensitivity because our detector responds less well to GWs, as well as shot noise.

At high frequency we also lose sensitivity because our detector responds less well to GWs, as well as shot noise.

In the simplest version, squeezing reduces shot noise but increases radiation pressure noise.

RP noise is at low frequency mostly. The resonant frequency of the suspensions for our detectors is typically at a few Hz; therefore for practical purposes we are above it and we can treat the mirror as a free mass.

The radiation pressure noise can be described by relating the fluctuation in applied power to the fluctuation in position:

$$-m\omega^2 \delta x = 2 \frac{\delta P}{c} \quad (2.17)$$

$$\delta x = -\frac{2\delta P}{mc\omega^2} = -2 \frac{\sqrt{E\omega\delta P}}{mc\omega^2} \hat{a}_1(\omega). \quad (2.18)$$

We suppose that the amplitude quadrature contains the power, as $E_1 = \sqrt{P} + \dots$, while the phase quadrature E_2 contains the GW signature.

The expression for the fluctuation is due to the fact that the quadrature also contains an integral over \hat{a}_1 .

The phase of the reflected signal is modulated by the change in position of the mirror:

$$\frac{\delta\phi}{2\pi} = 2 \frac{\delta x}{\lambda} = 2 \frac{\delta x \omega_0}{c} \quad (2.19)$$

$$\delta\phi = \frac{4\pi\omega_0}{c} \delta x, \quad (2.20)$$

where the factor 2 is due to the fact that the new path is added both forward and back.

The quadrature reads

$$\sqrt{P} \cos(\omega_0 t + \delta\phi) = \sqrt{P} \left(\cos(\omega_0 t) \underbrace{\cos(\delta\phi)}_{\sim 1} - \sin(\omega_0 t) \delta\phi \right). \quad (2.21)$$

Therefore, the modulation is roughly $E_1 \approx \sqrt{P}\delta\phi$.

The incoming field will be described by $\vec{a} = (a_1, a_2)^\top$, while the outgoing one will be

$$\vec{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ -k & 0 \end{bmatrix}}_T \vec{a}, \quad (2.22)$$

where $k = 4\omega_0 P / mc^2 \omega^2$. The spectral density of b will read

$$S(\vec{b}) = \text{Re}(TT^\dagger) \underbrace{S(\vec{a})}_1. \quad (2.23)$$

The real part of TT^\dagger reads

$$\text{Re}(TT^\dagger) = \begin{bmatrix} 1 & -k \\ -k & 1+k^2 \end{bmatrix}. \quad (2.24)$$

The off-diagonal terms mean that the noise in the amplitude and phase quadratures is correlated.

We can plot $1+k^2$ against f with the expression we have for k . It is roughly $1/f^2$ and then flat.

How can we show that this is still a minimum uncertainty state? We need to see that the determinant of the transfer matrix is 1, which holds.

There is a quadrature which has reduced quantum noise with respect to the vacuum, at the expense of another which has larger noise.

People tried to do squeezing in this way; the main thing to do is to use tiny mirrors so that k becomes very large.

This effect will rotate the ellipse, even if we already send in squeezed light!

This is the reason why the RP noise worsens the noise curve when we send in squeezed light!

The solution to this problem is to pre-rotate the ellipse so that this effect is compensated.

At the quantum level, this is analogous to thinking of regularizing photon arrival times; the statistics of this squeezing operator should be the same as what we get with an SPDC. Of course, that's not really the physics of the measurement we make.

Some really big names were working on quantum technologies GW detection. Braginsky, Khalili, Kip Thorne, Alessandra Buonanno.

The things we did today appeared in a paper "Ponderomotive effects in electromagnetic radiation".

A detuned power recycling cavity allows for moving the resonance frequency of the Fabry-Perot cavity into the observation band, which means we have very low noise there.

Squeezed noise is not stationary in the photocurrent!

When we squeeze there is also an addition of some extra noise, we do not stay minimum-uncertainty.

Squeezing and changing the power are equivalent in the respect of overcoming the Standard Quantum Limit!

Frequency-dependent squeezing is achieved through the ponderomotive effect: an input cavity rotates the ellipse by 90° between high and low frequency.

One can do an output filter, alternatively: doing the same thing after the round-trip in the arms. This turns out to be much better, but it is more sensitive to losses in the filtering cavity.

What does the configuration look like there?

So, the idea is that the mirror's fluctuation will rotate the noise among the quadratures, so we do that in the opposite direction to the squeezed light (?).

Mirror coatings can create issues! Even if a requirement on the amplitude is satisfied, if the pattern is regular there can be problems: it acts like a diffraction grid.

Where does the $\Delta \sim \text{MHz}$ frequency shift come from?

We can do measurements of entangled photons coming from down-conversion to reduce noise.

The OPA is where we get a pump with an offset. This allows us to make an EPR filter with frequency-dependent filtering!

The MHz photons experience no frequency offset: there is no radiation pressure there! Tuesday
2021-12-21

The radiation pressure term is written as

$$\text{RP} = \frac{1}{c} \left(P + 2\sqrt{P} \sqrt{\hbar\omega_0} \hat{a}_1(\Omega) \right), \quad (2.25)$$

with quadratic, $\mathcal{O}(\hat{a}^2)$ corrections.

This corresponds to a displacement $\text{RP}/m\Omega^2 = \delta x(\Omega)$.

This displacement is connected to a phase fluctuation

$$\delta\phi(\Omega) = 2 \frac{\delta x(\Omega)\omega_0}{c}. \quad (2.26)$$

The thing is then that the modulation can be written as

$$E_1 = \sqrt{P} \cos(\omega_0 t + \delta\phi(\Omega)) \quad (2.27)$$

$$= \sqrt{P} \cos(\omega_0 t) - \delta\phi(\Omega) \sqrt{O} \sin(\omega_0 t), \quad (2.28)$$

while the phase quadrature reads

$$E_2 = \sqrt{P} \delta\phi(\Omega) \propto -\frac{\sqrt{P}}{\Omega^2}. \quad (2.29)$$

This is the reason why the MHz sidebands are not affected by radiation pressure noise.

Wiener filtering

If we have an estimated signal $\hat{y} = F(x)$ and an observed signal y , we want to minimize $\langle (y - \hat{y})^2 \rangle = e^2$.

We choose this by determining a maximum, with $\partial e^2 / \partial F = 0$ and $\partial e^2 / \partial F < 0$. The optimal choice is then $F = \langle xy \rangle / \langle x^2 \rangle$.

In the frequency domain, a filter looks like $\hat{y}(\Omega) = F(\Omega)x(\Omega)$.

There are ways to implement this and keep track of continuous variations of the environment (Kanman filtering?).

Environmental noise

Part of the site selection process is looking at seismic noise. We need to have noise globally below a certain RMS value, even microseismic peaks at a few Hz create issues since they can throw the interferometer out of alignment, even though we do not detect signals at those frequencies.

A big issue is also Newtonian gravitational noise, gravitational coupling of the environment to the test mass, which scales like f^{-2} .

Temperature gradients of sufficiently small scales in the atmosphere can also create issues.

The acoustic noise in the cave also creates issues. One can subtract the measurement of noise actively, but this is very expensive for ET since we would need many new boreholes!

Measuring acoustic fields in the atmosphere is hard: eddies can form around the microphone if the wind is too high. Lasers are quite good for this purpose.

There are magnetic disturbances correlating all the way around the world. These are Schumann resonances.

Thermal noise

Material science is very poorly understood. We are not able to compute things like Young's modulus from first principles.

The fundamental theorem describing this is the fluctuation-dissipation theorem:

$$S_x(\Omega) = \frac{8\pi k_B T}{\Omega^2} \frac{W_{\text{diss}}}{F_p^2}. \quad (2.30)$$

This describes the thermal noise spectrum due to mechanically dissipated power. The thermal noise we measure is average over the beam, so low-wavelength thermal noise are not really a problem.

Some high-order Laguerre-Gauss modes have been proposed as a way to moderate this issue.

Heat links can be a shortcut for vibrations! The way coating of the mirror is done is relevant. Tunneling is a big source of dissipation in materials.

The study of internal friction of materials is often studied by perturbing them and looking at the ringdown.

Exam: one can say which part they want to focus on. There is a short, 20-minute presentation to give.

3 Multimessenger Astrophysics

Wednesday
2021-12-22

This course is more on the astrophysics side.

Marica worked on radio, AGNs, and then moved to gravitational waves.

Overview:

1. main GW discoveries;
2. astrophysical sources of GWs;
3. modelling of EM emission of GW transients;
4. observations and data analysis;
5. astrophysics of compact objects;
6. future perspectives.

Neutrinos will mostly be in a short course, here we discuss GW and EM emission.

The LIGO-Virgo-KAGRA collaboration is thinking to start observations in December 2022.

Gravitational Wave discoveries

Having a network is important to do localization!

History of GW150914, GW170817.

A self-gravitating system has a natural frequency

$$f_{\text{dyn}} = \sqrt{\frac{G\bar{\rho}}{4\pi}}, \quad (3.1)$$

where $\bar{\rho}$ is the mean density of the object.

This yields 1.9 kHz for $1.4M_{\odot}$ within 10 km (BNS), 1 kHz for $10M_{\odot}$ within 30 km (BBH), while for a SMBH we get about 4 mHz.

The GW amplitude scales like

$$h \sim 2 \left(\frac{GM}{Rc^2} \right) \left(\frac{GM}{rc^2} \right), \quad (3.2)$$

where R is the system scale, r is the distance to the source.

We measure the *amplitude* of GWs! not the flux.

We can make a plot of lines in the mass-radius plane. Observation bands with fixed-frequency, lifetimes left for binary systems...

The energy emitted in a BNS merger is around $10^{-2}M_{\odot}c^2$.

The waveforms for supernovae and for NS instabilities are not known. There are no simulations that can make a supernova explode completely.

GW150914 was detected with CWB!

Unmodelled searches just search for correlated excess power.

Is there a systematic review of GW modelling degeneracies?

The chirp mass has an analytic dependence on the frequency and frequency derivative, we can do this calculation for 150914 to get the chirp mass, which translates to an upper bound on the total mass. This yields a Schwarzschild radius of 210 km; but the orbital frequency reached 75 Hz which corresponds, with Kepler's law, to 350 km! These objects must have been compact.

Compactness C is the Newtonian orbital separation divided by the sum of their smallest possible (Schwarzschild) radii.

In the non-spinning, circular orbit case we get $C \approx 1.7$.

For NSs we get a number between 2 and 5 (accounting for the uncertainty on the NS mass). With a line of reasoning accounting for the mass ratio, we can also exclude the BHNS case.

As we discussed, from f and \dot{f} in a GW signal we can extract the chirp mass.

Monday
2022-1-10

The maximum orbital frequency of GW150914 was 75 Hz; using Kepler's law we get a separation of 350 km.

The compactness of the system is defined as the ratio between the Newtonian orbital separation divided by the sum of the possible radii (Schwarzschild radii).

For unequal masses we have a compactness of the order of

$$C \approx \frac{3.0q^{2/5}}{(1+q)^{4/5}}. \quad (3.3)$$

We can plot C as a function of mass ratio q and eccentricity, then using the $C = 1$ line as a boundary we get a limit beyond which the object would be inside their respective Schwarzschild radii.

Plot of $\sqrt{S(f)}$ against $2|h(f)|/\sqrt{f}$? from Abbott 2016?

We can measure the source-frame masses, chirp mass, effective aligned spin, radiated energy, distance and redshift, but the redshift is not estimated directly.

What people thought from EM observations was that stellar-mass black holes would be below 20 solar masses.

The first evidence for BH existence was in the 1970s, when the mass of the X-ray binary Cygnus X-1 exceeded the maximum mass of a neutron star. We see about 22 of these systems we detect in X-rays; we can look at their orbital motion.

The formula is

$$\frac{PK^3}{2\pi G} = \frac{M \sin^3 \iota}{(1+q)^2}, \quad (3.4)$$

where P is the orbital period, which we measure directly, K is the radial velocity amplitude of the companion, ι is the binary inclination and q is the mass ratio.

Doing these estimates we get 5 to 20 solar masses.

Between 8 and 25 solar masses for the progenitor we have NS formation; above 40 solar masses we can have direct collapse to a BH, "failed supernovae".

Roughly, NS masses are constrained to lie somewhere between 1 and 3 solar masses. The lower limit is the Chandrasekhar mass (for NS formation), the upper limit comes from the Oppenheimer-Volkoff equation (above a certain mass pressure cannot counteract gravity).

Theoretically, knowing the mass distribution of BH masses is hard, because of the uncertainties in the mass loss with stellar winds and supernova explosions.

A plot of remnant mass against ZAMS mass of the progenitor; varying the metallicity makes a huge difference. For $Z = Z_\odot = 0.02$ we always get remnants below $20M_\odot$; the other extreme $Z = Z_\odot/100$ yields prompt collapse, while in an intermediate region $Z \sim Z_\odot/10$ we get BHs compatible with the observed population by LVC.

Metallicity affects stellar winds due to line-driven winds.

The mass loss scales roughly like $\dot{M} \propto Z^\alpha$, with α around 0.5 to 0.9.

Heavy BH formation is complicated, but the main idea is to have low metallicity ($< Z_\odot/2$), meaning that stellar winds are quenched leading to larger pre-collapse mass, and then a good probability of direct collapse.

What are the mechanisms for SN formation? One is a *pair-production supernova*: if the Helium core is larger than $64M_\odot$, there is efficient production of 1 MeV or harder photons, which produce pairs leading to a decrease of photon pressure.

This leads to a collapse during oxygen burning, a runaway thermonuclear reaction without an iron core. No remnant is left.

A lighter version of this is a pulsation-pair instability supernova, in which the star oscillates due to the same mechanism.

This leads to a *mass gap*, between 60 to $120M_\odot$.

In O3 we saw GW190521: a component had $85M_\odot$, in the mass gap!

Massive black holes can form in the galaxy field or, more probably, in a globular cluster, young star cluster or near an AGN.

Dynamical interactions favor the formation of lower mass BHs!

Globular clusters are older, more compact, typically observed in the bulk of the galaxy, and most stars are of the same age, in elliptical galaxies. Young star clusters are less gravitationally bound, found in both spiral and elliptical galaxies, and they typically evolve. Nuclear star clusters are the ones which are closer to the center of the galaxy.

BBHs are typically ejected from the cluster at the beginning.

The BBHs we are looking at could come from isolated binaries, or from dynamical interactions in a dense environment: which is the main formation channel?

It is very common to have stars in a binary system. In order to form an original binary we need two massive stars; about 70 % of massive stars have a companion.

Many evolutionary processes can destroy our binary!

One very hard thing to model is the common envelope phase. This happens when the separation is about 1000 to 10000 solar radii; the mass transfer becomes unstable forming the common envelope.

What about dynamical binaries? This becomes important when we have more than 1000 stars per cubic parsec.

A star can do a flyby of a binary, acquiring kinetic energy and thereby hardening the binary. Further, we can have exchanges! More than 90 % of binaries in young star clusters

are formed by exchanges.

How can we characterize the exchange-origin population? These will have

1. very massive BHs;
2. high eccentricity;
3. misaligned BH spins.

The first detections of BBHs had χ_{eff} compatible with zero, which could both mean small aligned spins or large misaligned spins.

Now we can do population studies!

Tuesday

We were discussing where a single massive BH can form — something with a mass larger than $25M_{\odot}$.

In a globular cluster, for energetic reasons there starts to be a separation between BHs and stars — BHs cluster towards the center.

We have observed BBH mergers, so heavier BHs coming from lighter ones are a possibility.

The ingredients which are not fully understood are:

1. SN kick;
2. mass transfer;
3. common envelope.

What is the order of magnitude of the SN kick? we don't know!

When is the common envelope ejected? This depends on how efficient the transfer of orbital energy to it is. This has a large effect on the BBH merger timescale.

What are the population observables we can look at?

1. Mass distribution;
2. spin distribution (aligned or antialigned / misaligned), for isolated binaries we expect the former and for dynamical binaries we expect the latter;
3. eccentricity distribution;
4. astrophysical rate with z ;
5. host galaxies.

Knowing the host galaxy would be great, but it's very hard. Orbits are circularized by GW emission, but it is feasible that we might detect a slight nonzero eccentricity as we go to lower frequencies, which would be a smoking gun signature for dynamical formation.

The BBH mass spectrum, as measured now (post-O3), is not well described by a power law. There seems to be a feature at $40M_{\odot}$, which might be caused by PPISNe.

There is a dearth of low-mass BHs, in the $2.6M_{\odot}$ to $6M_{\odot}$.

There is a slight indication of another peak at high mass.

The effective spin seems to be negative for a significant fraction of the population.

The high-mass sector seems to have more precessing spin, but we don't really have good statistics to split the population in two.

We have decent estimates for the astrophysical rate for BBH, BNS and NSBH.

The merger rate seems to increase going to redshift ~ 2 , following the star formation rate. If we can find a delay between the two peaks, we can directly estimate the BBH formation timescale!

BBHs from AGN disks are a good candidate for an electromagnetic counterpart, since there is a lot of baryonic mass around!

Observation strategies for an EM counterpart

LVC developed a very fast data analysis pipeline. Current pipelines can identify a statistically significant signal and estimate its localization, within 1 minute.

The slow step is human validation — it should take about 30 minutes. So, in O3 this was removed: the telegram to the telescopes is sent automatically, and then possibly retracted if it is not actually from data.

A single GW detector has a broad antenna pattern — good for detecting many signals, terrible for localization!

The time delay between two interferometers yields an annulus; when we have three we get the intersection of three annuli. Three will always have a degeneracy between two regions. With four, we remove this degeneracy.

The localization area scales with $1/\text{SNR}^2$.

The sky localization algorithm BAYESTAR allows for an estimate within 1 minute.

With the addition of Virgo we decreased the localization area by a factor 20.

These are still of the order of hundreds of square degrees currently; even “wide-field telescopes” do not have such an observing area.

Also, looking at such a wide area we may have several contaminants!

We need to narrow down to very few sources (on the order of ten) which we can then take spectra of with large Earth-bound telescopes like VLT, or space-bound ones like JWST or Hubble.

Infrared emission is interesting for nucleosynthesis in kilonovae.

In 100 square degrees we typically have 10^4 to 10^5 variable objects within 100 square degrees!

A very important thing is to have surveys to compare to the after-merger object.

These contaminants are typically M-dwarf flares, supernovae. There is a lot of ML done to remove these contaminants.

The X-ray sky is quite empty, but there is no wide-field telescope. The γ -ray sky is also quite empty and we have all-sky monitors in that band; the problem here is the emission, since the γ emission is very beamed, unlikely to get to us on average.

Radio is good in that there are few contaminants, we have wide-field telescopes at $\sim \text{MHz}$, however GW sources are quite faint in radio, and also there is a long delay between GW and radio emission.

The False Alarm Rate is the rate of noise events louder than the candidate event. The typical threshold to have a candidate is less than 1 event per month.

The sky localization is typically given as a HEALPix FITS file: for each pixel we have the probability that that pixel contains the source.

The luminosity distance is also estimated; this allows for the selection of candidate host galaxies according to their known distances, as well as choosing the band in which to observe.

The prompt pipeline also gives an estimate of the potential type of object: BNS, BBH, NSBH, MassGap.

A parameter given to the astronomer is the quantity P_{astro} : based on some assumptions about the sources, we can give an estimate that the signal is not terrestrial noise.

Some probabilities also given are “HasNS”, whether a NS is contained in the merger (only estimated through mass). Also, there is an estimate of “HasRemnant”: when there is a NS in the merger, part of its material is tidally ejected. This is the probability that there is still leftover material after the merger beyond the compact remnant. This is required for the presence of an electromagnetic counterpart.

All this is described in the LVC Public Alerts User Guide.

The FAR allows us to determine whether a signal has low significance (between 1 per month and 1 per year), it is significant (between 1 per year and 1 per hundred years) or very significant (less than 1 per hundred years).

Typically, we do a mosaic search starting from the highest probability region and moving outward.

After a burst signal picked up by an unmodelled search, people thought it was a failed SN, so they started to look for missing supergiants in the Large Magellanic Cloud.

A smart way to do the search is to weigh the galaxies based on the probability that they are the host galaxies based on their known spectra, their star formation rate and so on.

Last time we discussed how to make an effective search for an EM counterpart.

Thursday
2022-1-13

The search for the most likely galaxies within the uncertainty in volume was used already in GW170817.

The ways to rank galaxies are various: one is the blue luminosity; another is the K-band luminosity. Blue luminosity is a proxy for young stars and star formation, K-band luminosity is a proxy for stellar mass.

How do we rank galaxies? One way is through theoretical simulations: combining population synthesis models with cosmological simulations.

Simulations show that, roughly, the trend for BNS is of a strong correlation between total mass and number of binary systems; there is a dependence on metallicity, but it is weak.

For BHNS, lower-mass galaxies seem to have a higher number of systems per unit mass.

One of the typical counterparts is a short GRB, and there is also a dependence on the mass of the rate of short GRBs... this approach might be slightly flawed.

How do we proceed?

We take the 3D sky localization map, compute the localization probability for each pixel,

and combine it with a luminosity-derived probability estimate in the form

$$\mathbb{P}_{\text{lum}} = \frac{L_{K,b}}{\sum L_{K,b}}. \quad (3.5)$$

We can also optimize the observation strategy by taking into account the estimated masses, spins, equations of state!

We can even try to estimate the best *time* at which to observe, since different lightcurves peak at different times.

We can evaluate a posterior detectability distribution: what are the fraction of fluxes based on the posterior distribution which exceed the limit flux for this telescope?

(See Salafia et al 2017)

The observation strategy also includes the amount of exposure time to use — short exposures at first, longer exposures later.

Detectable sources

We have already seen the three combinations of NSs and BHs; the other interesting objects in this GW band are CCSNe and SN instabilities.

What is a short Gamma Ray Burst? It is a very energetic, beamed emission, due to an ultra-relativistic jet.

For a kilonova, we also expect to see sub-relativistic dynamical effects, connected to dynamical outflows, as well as disk-wind outflow and spin-down luminosity.

What is the EM emission of a supernova? The short breakout is typically seen in X-ray and UV. Then, there is emission in all bands.

What are the EM counterparts to isolated NS instabilities? They could be “soft gamma-ray repeaters”...

Magnetar flares were also associated with the newly-discovered fast radio bursts.

After a BNS merger, an accretion disk is formed!

There are full GR simulation by the Jena group which treat well the microphysics. There is a large fraction of unbound mass; the geometry of this mass heavily determines the EM emission.

For a NSBH binary, the interesting quantity is the tidal disruption radius to the ISCO of the BH; if this is less than 1 the NS is swallowed by the BH; if it is larger than 1 the NS is tidally disrupted, and we have long spiral arms.

Before the merger, the BH is fully defined by its mass and spin, which determine the ISCO, going from 9 with maximal antialigned spin to 1 with maximal aligned spin to the orbital angular momentum.

The spin of the NS is expected to be small; what matters is the BH spin.

Tidal disruption is defined as happening when the tidal force of the BH is stronger than the self-gravity of the NS: this yields

$$d_{\text{tidal}} \sim R_{\text{NS}} \left(\frac{3M_{\text{BH}}}{M_{\text{NS}}} \right)^{1/3}. \quad (3.6)$$

We can make a bivariate plot with χ_{BH} and q , and color it according to the fraction of ejected material.

The mass-radius relation is uniquely given by the underlying EoS.

Stiff EoS means the NS is *less* compact. These are harder to compress, and easier to disrupt.

Using spin, mass from the GW signal and amount of ejecta from the EM counterpart, we can constrain the equation of state!

For the EM emission, it is also important to know: what is the remnant? If the mass is relatively small, we can have unstable supermassive (lifetime of an hour) or hypermassive (lifetime of a second) neutron stars.

Electromagnetic emission

Some basic relations: the total luminosity L is given in terms of the monochromatic luminosity $L(\nu)$ as

$$L = \int L(\nu) d\nu . \quad (3.7)$$

The most relevant processes are blackbody, bremsstrahlung, synchrotron, and inverse Compton.

We can define a brightness temperature: the temperature of a blackbody which would emit that amount of radiation at that frequency.

this never exceeds the kinetic temperature of the source

Synchrotron emission yields a powerlaw.

Particle energy is $E = mc^2\gamma^2$?

The energy radiated scales with $\gamma^2 B$. The superposition of the synchrotron spectra from electrons at various energies gives the slope of the overall spectrum.

Synchrotron self-absorption also happens at low frequencies: at low frequency the electron medium is optically thick, since the brightness temperature is close to the electron temperature.

The history of gamma ray bursts

To know the intrinsic luminosity of the source of a GRB, we should know what is the width of the cone.

The emission of GRBs really starts in the X-ray band, near 10 keV.

They were discovered serendipitously in the 1960s, since the US people were looking for nuclear tests of the Russians and found these flashes happening every 2 or 3 days.

The satellite BATSE was the first to be sent to study them; and it showed an isotropic distribution.

This was an indication that these objects were extragalactic, cosmological. It could also correspond to nearby stars, which are also nearly isotropic.

A new satellite was BeppoSAX; it showed an afterglow emission in the X-rays, so they were able to also identify the emission galaxy and associate it to a cosmological redshift.

The first observation of the GRB optical afterglow was done in Campo Imperatore! But, the first paper which was published was by Dutch people.

SWIFT is a current telescope, and it is very important for X-ray observation. We also have XRT with a sky localization of arcseconds.

An ultraviolet optical telescope can also observe the optical afterglow.

Friday

2022-1-14

The particle energy distribution from synchrotron emission has a spectral index δ , while the index of the spectral distribution is $\alpha = (\delta - 1)/2$. This is due to the superposition of many emission spectra.

The frequency of the emitted energy scales like $v \propto \gamma^2 B$.

The cutoff at low frequencies, $S_v \sim v^{5/2}$, happens when the electron spectral temperature gets close to the electron kinetic temperature. There, the plasma is optically thick, while in the high-energy regime the plasma is optically thin and we get a powerlaw $S_v \sim v^{-\alpha}$.

Roughly, we observe a GRB every couple of days, and as we discussed earlier they were found to be cosmological.

If we plot the distribution of GRB durations, it is bimodal; there is a peak at just less than a second, and a peak of 10 to 100 s.

The distribution of the hardness ratio (hard photons vs soft photons) is also bimodal: short GRBs also have a higher hardness ratio.

Long, soft GRBs are connected to CCSNe. Short, hard GRBs are not connected to SNe, and they typically have a larger distance from their host galaxy center (5 to 10 kpc, but even more).

They seemed to be connected with the timescale of accretion after a binary merger with a NS. This is all consistent with what we expect for a binary system of NSs.

The fact that these are further from the galaxy center is consistent with the possibility that they drifted away from the star-forming galactic center due to the SN kick.

In optical, the flux seems to decrease over time according to a powerlaw. For IGRBs, we then have a bump at the end, corresponding to a SN.

Why is the bump there in IGRBs? is the SN not the start?

It is interesting to study the galaxies the GRBs are coming from.

The distribution of sGRBs seems to match up with field galaxies, while IGRBs typically are fainter, more star-forming and with lower metallicity than field galaxies.

Certain GRBs seem to be hostless: they can go as far as 1 Mpc. We can have an independent estimation of distance with the evaluation of the absorption.

Only 20% of sGRBs are in early type galaxies. Many more are in late-type, spiral galaxies.

Elliptical galaxies are early-type. They have older stars, maybe they formed with the mergers of spiral galaxies.

When we go distant, we use color: spiral galaxies are bluer.

We observe sGRBs mostly at low redshift, this is partly expected because it takes time to make a compact binary, and partly because of observational bias.

GRB formation

No simulation is currently able to produce jets. The current best scenario is to produce a jet thanks to a high magnetic field, which allows for the extraction of angular momentum.

We have prompt emission in γ within seconds, while the afterglow in lower-energy bands can last much longer. The scenario with the most consensus is the “fireball” model.

We get a jet with bulk Lorentz factor $\Gamma \gtrsim 100$, so that particles undergo Fermi acceleration, leading to prompt emission. When the jet interacts with the medium we have a shock.

We also have a reverse shock moving backwards.

The idea is that particles are initially very relativistic, but as the jet decelerates in the ISM we find the lower-energy afterglow.

Typically, the decay of the optical lightcurve is like $t^{-\alpha}$ with α between 1 and 1.5.

Long GRBs have higher energy, their lightcurves are higher up.

The relativistic beaming is of the order of $\theta \sim 1/\Gamma$ where Γ is the jet bulk Lorentz factor. However, the jet decelerates in the ISM and its Lorentz factor decreases.

An off-axis observer cannot observe the prompt emission, but as the jet slows down the angle increases.

People tried to observe off-axis GRBs, but it's hard! There are not gamma-rays to guide us.

Off-axis means that even the highest-energy photons we observe are not in the gamma range.

A lower ISM density means the emission is fainter and peaks later, even though the total energy is the same.

We only have about 30 sGRBs with an associated redshift.

Before SWIFT, people looked at the prompt emission, at the X-ray, but not in the middle! When it came out it showed that some sources can have flares in the range of a few minutes to a few hours, or a plateau.

We don't know why there is a plateau in X-rays but not in the optical!

The plateau might originate in the slow region of the jet! GW170817 showed us that jets have structure.

Thermal emission

The thermal emission associated to a BNS is called the *kilonova emission*. The almost-relativistic material which gets unbound in the merger is very neutron-rich: it is the perfect place for *r*-process nucleosynthesis to occur.

We have expectations about which elements may form — lanthanides and actinides are very opaque, and they may emit in the infrared.

In the interface between the two neutron stars we have shock-heated ejecta. These are typically orthogonal to the tidal tail.

In the ejecta we have weak interactions which increase the electron fraction; this decreases the neutron fraction and prevents the formation of very heavy elements. The emission of these is on the bluer side of the spectrum.

What is *r*-process nucleosynthesis?

How do we form elements higher than iron? *s* (slow) -process happens when the neutron capture timescale is faster than the decay time.

The basic reactions are β -decay and neutron capture.

This might happen in supernovae, but simulations show it will not happen a lot, while in BNS mergers we have very dense environments.

Some peaks we observe in the Sun are only compatible with *r*-process, some others are only compatible with *s*-process.

The electron fraction, $Y_e = n_p / (n_n + n_p)$, is inversely correlated with the neutron fraction.

The number of neutrons is damped by positron capture, which yields protons, as well as by neutrino capture, which yields protons and electrons.

In the end, we don't have enough neutrons to form really heavy elements.

We have different ejecta types:

1. tidal — equatorial, cold, with low $Y_e \lesssim 0.1$, and with low mass;
2. shock-heated — polar; hot, with high $Y_e \gtrsim 0.1$, and with low mass;
3. neutrino-driven winds — polar, with high $Y_e \gtrsim 0.1$, and with low mass;
4. secular — isotropic, with a broad range of Y_e , and with high mass.

Low mass means about 1 % of the disk mass, which is about $0.3M_\odot$ typically.

The total mass of these components, for GW170817, is quite well known, while its breakdown is more uncertain.

If the electron fraction is $Y_e \lesssim 0.15$ we can form actinides, with $0.15 \lesssim Y_e \lesssim 0.25$ we can form lanthanides, with more we cannot go beyond iron. These heavy elements are opaque since they have a lot of atomic lines.

The main parameters are the opacity κ , the mass of the ejecta m_{ej} , the velocity of the ejecta v_{ej} .

We have empirical fitting formulas connecting the peak time of the lightcurve and its luminosity to these quantities.

Which come from simulations, right?

A further important parameter is the radioactive heating rate.

r-process kilonovae have suppression of the UV curves, and they have more broad spectra.

Decay for nuclei formed in *r*-process can happen through α , β decay or fission, the radioactive heating rate is the rate of energy deposited by these.

Also, a problem is due to the fact that we don't know all the energy levels of very heavy elements.

The remnant type is also very important: if we have a direct BH collapse the spectra are quite short-lived, while if we form a magnetar we get longer-lived spectra.

People started to talk about kilonovae after 2010! The first lightcurve only came in 2013.

Are the ejecta then still orbiting the remnant?

The material propagating outward can then have an effect in the radio band. However, the long-lasting radio remnant was not really observed for 170817.

What about accretion of the ejecta?

Monday
2022-1-17

We were talking about the various types of ejected mass in compact binary mergers which contain a neutron star.

The ejected mass leaves a rather empty region near the remnant; as it goes out it shocks the ISM, leading to a radio remnant. Its peak energy is heavily dependent on the ejecta energy, which is thought to be of the order of magnitude of 10^{50} erg to 10^{51} erg.

If a jet is more energetic, it is also more collimated, so an off-axis observer will see it later.

Another interesting type of emission is X-ray emission from a long-lived NS remnant: this can be stable or unstable. This emission is expected to be very bright, in the soft X-rays (half to 10 keV), and close to isotropic. It should be peaked 10^2 to 10^4 s after the merger.

The plateau we observe in the time evolution of GRBs could be due to the presence of a neutron star. What we know for sure is that a NS which formed will emit GWs, although we currently do not have detectors which can detect them.

There was a nomenclature discussion about the term "kilonova": before GW170817 people thought that their energies were not a thousand times that of a nova, so the term "macronova" was preferred.

The moral of the story is this: we have emission in every band and at different timescales, so we need a network of detectors!

For a BBH, we do not expect to see a detectable EM counterpart; however Fermi observed some data associated with one GW event, so people have started to think about scenarios.

There could be emission

1. from the remnants of the stellar progenitors;
2. from the tidal disruption of a star in a triple system;
3. from the environment of binaries in AGNs.

The sky localization was around 30 square degrees. There were only about 50 galaxies to be observed. A signal, blue at first and then red 10 days later, was observed!

The signal was in a blind spot of Virgo.

For almost three months every instrument was pointing at that galaxy.

Then, astronomers worked in small groups, but now European astronomers decided to associate into ENGRAVE.

The source galaxy was already catalogued, and it was exactly at the distance predicted by GWs.

The mass distribution predicted for 170817 heavily depends on the prior spin distribution we assume.

We can also measure the tidal polarizability of the stars.

A supermassive NS is sustained by uniform rotation and lasts for about an hour; a hypermassive NS is sustained by differential rotation and lasts for about a second.

With the same total mass, but a soft EOS, we can have direct collapse into a BH; however with a hard EOS we can have a higher likelihood of a NS remnant. This is because a hard EOS means a higher maximum mass threshold.

The postmerger signal, in any case, is emitted at a few kHz; too much for current GW detectors.

GRB 170817A was precisely consistent with a short GRB; however if we look at it compared to the population of sGRBs we see something interesting.

We look at the isotropic energy emission — the luminosity we would have if the emission was isotropic — and we see that the luminosity of this event was very faint compared to the sGRB population.

Is it an off-axis GRB, or an intrinsically faint emission of another kind? Initially people thought of the first, but there are alternatives.

An isotropic outflow would also explain the data: if a jet is formed but it is choked.

In any case, people expected a decline in the flux; however what was observed was a rise in flux for several months!

The spectrum observed was perfectly consistent with a powerlaw (if the kilonova was removed). Powerlaw means nonthermal synchrotron: again, there are two scenarios.

In the jet case, we have a structured jet with a fast core and slower outer parts; in the case of isotropic outflow we also have a layered structure with a faster outer part.

The rise of luminosity is shallow, roughly with $t^{0.8}$.

The way to resolve this degeneracy was *radio high resolution imaging*! The VLBI network looked at the size of the source. The measured size ruled out mildly isotropic outflow: it was very small, meaning that there was a jet.

It seemed that the viewing angle was about 15° , outside the jet core.

The point which is still a bit controversial is the GRB observation itself; is it the shock breakout or the slow part of the jet?

Next time we will look at the thermal, kilonova emission.

Friday

2022-1-21

VLBI works with resolution $\phi \sim \lambda/D$ where D is the distance among the antennas. Quasars are typically discovered in the optical band, but then radio observations can allow us to measure distances.

A source moving with angle θ at speed βc will have moved by a displacement $\Delta x = \beta c \Delta t \sin \theta$; the time interval on the other hand is $\Delta t(1 - \beta \cos \theta)$, so the apparent velocity is given by

$$v_{\text{app}} = \frac{\beta \sin \theta}{1 - \beta \cos \theta}. \quad (3.8)$$

Thermal emission

The red kilonova is expected to peak around 1 week after the merger, and it is due to tidal ejecta; the blue kilonova is given by shock-heated ejecta, and it peaks sooner, 1–2 days after merger.

The fraction of lanthanides determines the color and timeseries — they absorb a lot, so a lot of them means later and redder.

The spectrum is initially very close to a (blue-ish) blackbody, then absorption spectral features appear after a few days!

The temperature decay for a kilonova is much faster than that of a supernova.

Chemical signatures are hidden by the velocity distribution. It is therefore not possible to identify single elements.

There were, however, predictions for a kilonova spectrum, which made some assumptions about the spectral lines and these were validated by 170817!

Enrico Cappellaro in Padua said he'd never seen something like that in his life!

The ejected mass can be found through a fit; it is on the order of 0.03 to $0.05 M_{\odot}$.

Bolometric luminosity as a function of time depends on the r -process mass: we observe two different components at different time, the blue and red kilonova.

The temperature initially decreases from 10^4 K, but after a few days it stabilizes to 2500 K. This is due to the fact that the photosphere moves inward in the ejecta.

Models are currently not able to consistently reproduce the observed spectral features.

It is hard to identify single elements, but people thought they found Cesium and Tellurium.

Nowadays people are seeing strontium lines.

We can constrain the BNS rate both from GW observations, and from heavy elements' abundance!

EM observations exclude very soft EoS, $\tilde{\Lambda} \lesssim 400$! GW observations exclude $\tilde{\Lambda} \gtrsim 800$, so this is great!

In the case of prompt collapse we expect a red KN powered by the tidal ejecta; For a supermassive NS we expect it to impart a lot of energy to the jet. In the end, therefore, we think that the middle situation, a HMNS is the one which is consistent with observation.

The host galaxy, NGC4993, showed some spiral characteristics even though it was mostly lenticular. It was consistent with the merger of two galaxies less than 1 Gyr ago.

There is no evidence of YSC or GSC there.

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