**ATOC5860 – Application Lab #1**

**Significance Testing Using Bootstrapping and Z/T-tests**

**in class Thursday January 20 and Tuesday January 25, 2020**

**Notebook #1 – Statistical significance using Bootstrapping**

**ATOC7500\_applicationlab1\_bootstrapping.ipynb**

**LEARNING GOALS:**

1) Use an ipython notebook to read in csv file, print variables, calculate basic statistics, do a bootstrap, make histogram plot

2) Hypothesis testing and statistical significance testing using bootstrapping

**DATA and UNDERLYING SCIENCE:**

In this notebook, you will analyze the relationship between Tropical Pacific Sea Surface Temperature (SST) anomalies and Colorado snowpack. Specifically, you will test the hypothesis that December Pacific SST anomalies driven by the El Nino Southern Oscillation affect the total wintertime snow accumulation at Loveland Pass, Colorado. When SSTs in the central Pacific are anomalously warm/cold, the position of the mid-latitude jet shifts and precipitation in the United States shifts. This notebook will guide you through an analysis to investigate the connections between December Nino3.4 SST anomalies (in units of °C) and the following April 1 Loveland Pass, Colorado Snow Water Equivalence (in units of inches). Note that SWE is a measure of the amount of water contained in the snowpack. To convert to snow depth, you multiply by ~5 (the exact value depends on the snow density).

The Loveland Pass SWE data are from:

<https://www.wcc.nrcs.usda.gov/snow/>

The Nino3.4 data are from:

<https://www.esrl.noaa.gov/psd/gcos_wgsp/Timeseries/Nino34/>

**Questions to guide your analysis of Notebook #1:**

For full credit: write answers to the questions and then upload this document to your github along with notebook #1 (including any edits that you make).

1) Composite Loveland Pass, Colorado snowpack. Fill out the following table showing the April 1 SWE in all years, in El Nino years (conditioned on Nino3.4 being 1 degree C warmer than average), and in La Nina years (condition on Nino3.4 being 1 degree C cooler than average).

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Mean SWE** | **Std. Dev. SWE** | **N (# years)** |
| **All years** | **16.33** | **4.22** | **81** |
| **El Nino Years** | **15.29** | **4.0** | **16** |
| **La Nina Years** | **17.78** | **4.11** | **15** |

2) Use hypothesis testing to assess if the differences in snowpack are statistically significant. Write the 5 steps. Test your hypothesis using bootstrapping.

Steps:

1. State the significance level:

95% confidence -> alpha = 0.05

1. State the null hypothesis H0 and alternative hypothesis H1:
   1. H0 (null hypothesis): the mean SWE during El Nino years is the same as the SWE mean for all the years. (sample mean is equal to the population mean).
   2. H1 (alternative hypothesis): the mean SWE during El Nino years is NOT the same as the SWE mean for all the years. (sample mean is not equal to the population mean).
2. State the statistic to be used and the assumptions required to use it: we will use bootstrapping, so no distribution assumption is needed for bootstrap. Central limit theorem will be used.
3. State the critical region: Probability that the difference happened by chance less than 5%.
4. Evaluate the statistic and state the conclusion

Z = -0.97

This corresponds to a probability that differences between the El Nino composite and all years occurred by chance is about 16.5%, which is higher than our alpha value. Therefore, we cannot reject the null hypothesis.

Instructions for bootstrap: Say there are N years with El Nino conditions. Instead of averaging the Loveland SWE in those N years, randomly grab N Loveland SWE values and take their average. Then do this again, and again, and again 1000 times. In the end you will end up with a distribution of SWE averages in the case of random sampling, i.e., the distribution you would expect if there was no physical relationship between Nino3.4 SST anomalies and Loveland Pass SWE.

1. Plot a histogram of this distribution and provide basic statistics describing this distribution( (mean, standard deviation, minimum, and maximum).

Chart, histogram

Description automatically generated

Basic statistics describing the distribution:

Mean: 16.31

Stdev: 1.05

Min: 12.95

Max: 19.37

1. Quantify the likelihood of getting your value of mean SWE by chance alone using percentiles of this bootstrapped distribution. What is the probability that differences between the El Nino composite and all years occurred by chance? What is the probability that differences between the La Nina composite and all years occurred by chance?

For El Nino case, Z = -0.99, the probability that differences between the El Nino composite and all years occurred by chance is about 8%,

The probability that differences between the La Nina composite and all years occurred by chance is about 8%, higher than our significance level. Again, we cannot reject the null hypothesis.

For La Nina case, Z = 1.37.

The probability that differences between the La Nina composite and all years occurred by chance is about 8%, higher than our significance level. Again, we cannot reject the null hypothesis.

3) Test the sensitivity of the results obtained in 2) by changing the number of bootstraps, the statistical significance level, or the definition of El Nino/La Nina (e.g., change the temperature threshold so that El Nino is defined using a 0.5 degree C temperature anomaly or a 3 degree C temperature anomaly). In other words – play and learn something about the robustness of your conclusions.

If we do the bootstrapping 10,000 times, we see small different values for the population:

Mean: 16.33

Stdev: 1.05

Min: 11.99

Max: 21.02

For El-Nino case:

The Z-value now is Z = -0.99. The null hypothesis can’t be rejected.

Probability: 16.2%

4) Maybe you want to see if you get the same answer when you use a t-test… Maybe you want to set up the bootstrap in another way?? Another bootstrapping approach is provided by Vineel Yettella (ATOC Ph.D. 2018). Check these out and see what you find!!

**Notebook #2 – Statistical significance using z/t-tests**

**ATOC7500\_applicationlab1\_ztest\_ttest.ipynb**

**LEARNING GOALS:**

1) Use an ipython notebook to read in a netcdf file, make line plots and histograms, and calculate statistics

2) Calculate statistical significance of the changes in a standardized mean using a z-statistic and a t-statistic

3) Calculate confidence intervals for model-projected global warming using z-statistic and t-statistic.

**DATA and UNDERLYING SCIENCE:**

You will be plotting *munged* climate model output from the Community Earth System Model (CESM) Large Ensemble Project. The Large Ensemble Project includes a 42-member ensemble of fully coupled climate model simulations for the period 1920-2100 (*note: only the original 30 are provided here*). Each individual ensemble member is subject to the same radiative forcing scenario (historical up to 2005 and high greenhouse gas emission scenario (RCP8.5) thereafter), but begins from a slightly different initial atmospheric state (created by randomly perturbing temperatures at the level of round-off error). In the notebook, you will compare the ensemble remembers with a 2600-year-long model simulation having constant pre-industrial (1850) radiative forcing conditions (perpetual 1850). By comparing the ensemble members to each other and to the 1850 control, you can assess the climate change in the presence of internal climate variability.

**More information on the CESM Large Ensemble Project can be found at:**

<http://www.cesm.ucar.edu/projects/community-projects/LENS/>

**Questions to guide your analysis of Notebook #2:**

For full credit: write answers to the questions and then upload this document to your github along with notebook #1 (including any edits that you make).

1) Use the 2600-year long 1850 control run to calculate population statistics with constant forcing (in the absence of climate change). Find the population mean and population standard deviation for CESM1 global annual mean surface temperature. Standardize the data and again find the population mean and population standard deviation. Plot a histogram of the standardized data. Is the distribution Gaussian?

Population statistics with constant forcing:

Population mean: 287.1. K

Population standard deviation: 0.1 K

Statistics of normalized data:

Normalized mean: 1.9 10^-4 (~=0)

Normalized standard deviation: 0.99999999 (~=1)

The distribution is very close to a normalized Gaussian.

Histogram of standardized data:

Chart, histogram

Description automatically generated

2) Calculate global warming in the first ensemble member over a given time period defined by the startyear and endyear variables. Compare the warming in this first ensemble member with the 1850 control run statistics and assess if the warming is statistically significant. Use hypothesis testing and state the 5 steps. What is your null hypothesis? Try using a z-statistic (appropriate for N>30) and a t-statistic (appropriate for N<30). What is the probability that the warming in the first ensemble member occurred by chance? Change the startyear and endyear variables – When does global warming become statistically significant in the first ensemble member?

Analysis of first ensemble member with the 1850 control run - Hypothesis test:

1. State the significance level: alpha = 0.05 (95% confidence)
2. State the null hypothesis and the alternative hypothesis

H0 = the sample mean (temperature in the first ensemble member) is the same as the population mean (the mean temperature in the control run).

H1 = the sample mean (temperature in the first ensemble member) is NOT the same as the population mean (the mean temperature in the control run).

1. State assumptions and statistic to be used:

We assume that the data are normally distributed, and we use the z-statistic and a t-statistic (more appropriate for N < 30)

1. State the critical region:

We use a 1-tailed test.. For the t-test, we use a number of degrees of freedom = 10. The critical values are:

Zc = 1.960

tc = 2.262

1. Evaluate the statistic and state the conclusion:

z-test:

z = 35.36

Probability: 0

Since Z> Zc, we can reject the null hypothesis at 95% confidence.

t-test:

t = 37.12

Probability: 1.8 10^-9

Since t> tc, we can reject the null hypothesis at 95% confidence.

We tried with 1975 – 1985 and the differences are not significant, so we would not be able to reject the null hypothesis in this case.

3) Many climate modeling centers run only a handful of ensemble members for climate change projections. Given that the CESM Large Ensemble has lots of members, you can calculate the warming over the 21st century and place confidence intervals in that warming by assessing the spread across ensemble members. Calculate confidence intervals using both a z-statistic and a t-statistic. How different are they? Plot a histogram of global warming in the ensemble members – Is a normal distribution a good approximation? Re-do your confidence interval analysis by assuming that you only had 6 ensemble members or 3 ensemble members. How many members do you need? Look at the difference between a 95% confidence interval and a 99% confidence interval.

Confidence intervals (surface temperature increase) on all 30 ensemble members:

z-statistic:

3.61-3.65 (95%)

3.6-3.66 (99%)

t-statistic:

3.61-3.66 (95%)

3.6-3.67 (99%)

Chart, histogram

Description automatically generated

I don’t think that is a great approximation for a normal distribution, but we could use and f-statistic to test if we wanted.

With 6 ensemble members the confidence intervals widen significantly:

95% confidence limits - t-statistic

3.5952689933866235

3.681851119986179

99% confidence limits - t-statistic

3.5706549020445166

3.706465211328286

It is not clear to us how many members we would need for the warming to be normally distributed, but the confidence intervals narrow significantly with nmembers=15. We think that we could always have more members to try and improve the confidence intervals.