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Effects of a Nambu–Goldstone boson on the polarization of radio galaxies and the cosmic microwave background

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A coupling $g\varphi \boldsymbol{E}\cdot\boldsymbol{B}$ to a pseudoscalar field φ affects the polarization properties of electromagnetic waves as they propagate through background fields. We analyse the bounds imposed on g from astronomical polarization measurements. If φ is the Nambu–Goldstone boson field of a global symmetry spontaneously broken at an energy scale v, then $g=N\alpha/\pi v$, with N a model-dependent numerical coefficient. For an exactly massless Nambu–Goldstone boson, we show that $N \gtrsim 50$ is incompatible with the observed correlation between polarization properties and shape of distant radio galaxies and quasars, unless there was inflation after the breakdown of the global symmetry. We also show that propagation through intervening magnetic fields induces a small degree of linear polarization in the cosmic microwave background.

Distant radio galaxies and quasars are often elongated along one axis. Those observed at large redshift emit radiation that, in most cases, has some degree of integrated linear polarization in a direction approximately perpendicular to the source axis. This correlation between polarization and shape was recently exploited to constrain parameters in a theory with a Chern–Simons term added to the Maxwell lagrangian [1] and in a theory with an electromagnetic coupling to gravity that violates the Einstein equivalence principle [2]. In both cases, the additional terms induce a rotation of the plane of polarization which must be small enough to be compatible with the observed polarization properties of distant radio sources.

In this paper we extend this analysis to the case of any massless or extremely light pseudoscalar field φ that couples to electromagnetism through a term

$$\mathscr{L}_{\varphi\gamma\gamma} = -g\varphi \boldsymbol{E}\cdot\boldsymbol{B} \tag{1}$$

in the lagrangian. We also discuss how this coupling induces a small degree of linear polarization in the cosmic microwave background. We consider two different mechanisms that potentially affect the observed polarization properties of distant sources when such a coupling exists: propagation along intervening magnetic fields and propagation through a classical, slowly varying background field φ. The existence of optical activity in these two cases was already discussed in different contexts. An external magnetic field prompts the mutual conversion of pseudoscalars and photons, providing an efficient mechanism for axion detection [3]. This same mechanism affects the polarization properties of electromagnetic waves that propagate across magnetic fields [4,5]. Optical activity due to a space-dependent background of φ was discussed in the context of domain wall [6] and vortex [7] configurations.

The complete electromagnetic and pseudoscalar lagrangian reads

$$\mathscr{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + \tfrac{1}{2}\partial_\mu\varphi\partial^\mu\varphi + \tfrac{1}{4}g\varphi F_{\mu\nu}\tilde{F}^{\mu\nu}\,, \tag{2}$$

where $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$ is the dual of the field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. We use natural Heaviside–Lorentz units ($\alpha = e^2/4\pi \approx \frac{1}{137}$ is the fine structure constant,

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$1\ \mathrm{G} \approx 1.95\times10^{-2}\ \mathrm{eV}^2$, $1\ \mathrm{cm} \approx 5.07\times10^4\ \mathrm{eV}^{-1}$). The field equations are

$$\nabla\cdot\boldsymbol{E}=g\nabla\varphi\cdot\boldsymbol{B}\,,\quad \nabla\times\boldsymbol{E}+\frac{\partial\boldsymbol{B}}{\partial t}=0\,,$$

$$\nabla\times\boldsymbol{B}-\frac{\partial\boldsymbol{E}}{\partial t}=g\left(\boldsymbol{E}\times\nabla\varphi-\boldsymbol{B}\frac{\partial\varphi}{\partial t}\right),$$

$$\nabla\cdot\boldsymbol{B}=0\,, \tag{3}$$

$$\Box\varphi=\frac{\partial^2\varphi}{\partial t^2}-\nabla^2\varphi=-g\boldsymbol{E}\cdot\boldsymbol{B}\,.$$

We first discuss optical activity in a background field φ. There is a simple way to evaluate the rotation of the plane of polarization of a wave that propagates in a background φ such that the length scale L over which $g\varphi$ changes appreciably is much larger than the wavelength λ of the electromagnetic wave. We use the geometrical optics approximation, in which all terms in the field equation containing second derivatives of φ, or first derivatives squared, are neglected since they are smaller than other terms by factors of order λ/L. One can then show that

$$\Box(\boldsymbol{E}+\tfrac{1}{2}g\varphi\boldsymbol{B})=\tfrac{1}{2}g\varphi\Box\boldsymbol{B}\,,$$

$$\Box(\boldsymbol{B}-\tfrac{1}{2}g\varphi\boldsymbol{E})=-\tfrac{1}{2}g\varphi\Box\boldsymbol{E}\,. \tag{4}$$

We now solve perturbatively. To lowest order, i.e. for constant φ, $\Box\boldsymbol{E}=\Box\boldsymbol{B}=0$. Thus, to first order the combinations $\boldsymbol{D}\equiv\boldsymbol{E}+\frac{1}{2}g\varphi\boldsymbol{B}$ and $\boldsymbol{H}\equiv\boldsymbol{B}-\frac{1}{2}g\varphi\boldsymbol{E}$ satisfy free wave equations (while $\boldsymbol{E}$ and $\boldsymbol{B}$ do not). This means that a solution which to lowest order is a linearly polarized wave, to first order is such that the vectors $\boldsymbol{D}$ and $\boldsymbol{H}$, rather than $\boldsymbol{E}$ and $\boldsymbol{B}$, oscillate along a constant direction. Now this implies that as φ changes by $\Delta\varphi$ along the trajectory of an electromagnetic wave, $\boldsymbol{E}$ (and $\boldsymbol{B}$ too of course) rotate by an angle $\Delta\phi$ given by

$$\Delta\phi=\tfrac{1}{2}g\Delta\varphi\,. \tag{5}$$

Positive $\Delta\phi$ corresponds to a counterclockwise rotation looking down the photon path. This effect is, in the approximation considered, independent of the frequency. Notice that $g\Delta\varphi$ in eq. (5) need not be small; that is, the polarization may rotate many turns along the trajectory. The only assumption is that $g\Delta\varphi$ is very small over a wavelength, not over the complete trajectory.

An alternative derivation of the effect can be obtained along the lines of refs. [1,2]. In the approximation of constant $\nabla\varphi$ and $\partial_t\varphi$, the field equations admit plane wave solutions for $\boldsymbol{E}$ and $\boldsymbol{B}$ that correspond to left and right circularly polarized waves to lowest order, but that have different dispersion relations to first order:

$$\omega_\pm\approx k\pm\tfrac{1}{2}g\left(\frac{\partial\varphi}{\partial t}+\nabla\varphi\cdot\frac{\boldsymbol{k}}{k}\right). \tag{6}$$

The different signs correspond to left and right circular polarization. Notice that the modification from the standard dispersion relation is just $\pm\frac{1}{2}g$ times the total derivative of φ along the wave trajectory. Now a linearly polarized wave is the superposition of left and right circular polarizations of the same amplitude. If these, however, propagate at different speed, they get out of phase or, equivalently, the direction of linear polarization rotates. The angle of rotation is given by $\Delta\phi=\frac{1}{2}\int(\omega_+-\omega_-)\mathrm{d}t$, with the integral evaluated along the wave trajectory. Since the integrand is g times the total derivative of φ, the result is the same as in (5).

We now take φ to be the Nambu–Goldstone boson of a global, chiral U(1) symmetry, spontaneously broken at a scale v, such that fermions get their mass through Yukawa couplings to the scalar field responsible for symmetry breaking. Then φ has an effective coupling to electromagnetism as in (1), with

$$g=\frac{N\alpha}{\pi v}\,. \tag{7}$$

Here α is the fine structure constant, and N is a numerical coefficient related to the number of fermion species that run around a loop in the anomalous triangle diagram.

A natural expectation is $N=\mathrm{O}(1)$. But one can also take N to be an arbitrary numerical coefficient, unconstrained by theoretical prejudice. If N is very large, strong electromagnetic effects occur on astronomical scales irrespective of the value of v, provided the Nambu–Goldstone (NG) boson involved is strictly massless or extremely light and provided there is no inflation after the phase transition during which the global symmetry becomes spontaneously broken. Indeed, in that case, the cosmic global strings formed during that phase transition are still present in the Universe today, radiating bosons very profusely.

Galactic and intergalactic magnetic fields then convert these bosons to photons, and the resulting photon flux is proportional to N^2, but independent of v except for some threshold effects (the v^2 in the cosmological NG boson production rate is cancelled by the v^{-2} in the NG boson to photon conversion rate). If global strings radiate with the spectrum proposed in ref. [8] (for an alternative scenario see ref. [9]), then the conversion process has observable consequences in cosmic ray and radio astronomy if the coefficient N is very large, of order 10^5 [10]. The direct electromagnetic radiation emitted by the currents induced onto global strings as the latter move through astrophysical magnetic fields is also proportional to N^2 and independent of v, and may yield a power as large as 10^{50} erg/s if $N \approx 10^5$ [11].

We now show that the polarization properties of distant radio sources impose the constraint $N \lesssim 50$ upon this scenario. Indeed, the NG boson field φ is the phase of the complex scalar field responsible for the global symmetry breakdown, properly normalized by the magnitude of the expectation value v of that field. No correlation is expected between values of the phase of the scalar field over distances comparable to the horizon scale (those we are probing at high redshift), unless there was inflation after the breakdown of the global symmetry. Hence, φ is expected to vary randomly by as much as $2\pi v$ as we look into different directions in the sky, at high redshift. This implies that when we observe objects at high redshift, the plane of polarization is rotated with respect to its direction at the source by random amounts, of order (taken $\Delta\varphi \approx \pi v$ as an average value)

$$\Delta\phi \approx \tfrac{1}{2} N\alpha . \tag{8}$$

If N were too large, the direction along which we observe high-redshift objects to be polarized would bear no relation with their intrinsic properties, something that is not the case, as we now discuss, summarizing the analysis of ref. [1].

The angle Δ between the direction of integrated linear polarization at the source and the direction along which the radio source is most elongated has been measured for over 200 objects [12,13] (for a review on polarization properties of extragalactic objects see ref. [14]). What is actually measured is the polarization angle at different wavelengths, from which the polarization at the source is derived by subtraction of Faraday rotation due to intervening magnetized plasmas. This subtraction is possible since Faraday rotation depends on wavelength as $\phi = (\mathrm{RM})\lambda^2$, where ϕ is the angle by which the polarization plane rotates, and (RM) stands for rotation measure, a quantity determined by the intergral along the line of sight of the electron density times the longitudinal component of the magnetic field. The uncertainty in Δ is believed to be of order $5°$ for sources with more than 5% of maximum degree of linear polarization, $p_{\max}$. The number of sources plotted as a function of Δ shows distinct peaks around both $90°$ and $0°$, if the data set is constrained to $p_{\max} > 11\%$. This means that most sources have integrated linear polarization either parallel or perpendicular to the source axis, a feature that can be accomodated by models based on synchrotron emission. This correlation becomes less strong as the threshold on $p_{\max}$ is lowered, the peak at $\Delta \approx 0°$ being the first to disappear. Carroll, Field and Jackiw [1] have also analyzed these data as a function of redshift. The peak at $\Delta \approx 0°$ is stronger at low redshifts, while the peak at $\Delta \approx 90°$ is stronger at large redshifts, something that may be due to a bias in the intrinsic luminosity of the sources observed at different redshifts [14]. The work of ref. [1] exploited the existence of a distinctive peak around $\Delta \approx 90°$, with about $10°$ of root mean square dispersion, for objects at redshift higher than $z = 0.4$. The existence of this correlation for high-redshift objects constrains mechanisms that rotate the polarization angle coherently over long distances.

One such mechanism is propagation in a background field φ. We have shown that over cosmological scales we expect random rotations of the plane of polarization of order $\Delta\phi \approx \frac{1}{2} N\alpha$, independent of frequency, in different directions in the sky. Demanding $\Delta\phi \lesssim 10°$, for this effect not to destroy the observed correlation between the polarization at the source and axis orientation, we get

$$N \lesssim 50 . \tag{9}$$

This bound is applicable only to massless or extremely light pseudoscalar fields, i.e. the mass must be less than the present Hubble constant. Also, the bound is valid only if there is no inflation after the breakdown of the global symmetry. Indeed, inflation makes φ uniform over distances that are huge compared to the present horizon, except for small fluctuations of quantum mechanical origin [15]. These fluctuations lead to $\Delta lg4 \approx H/2\pi$ over the present horizon scale, where H is the value of the Hubble constant during inflation. The rotation of the plane of polarization is then of order $N\alpha H/4\pi^2 v$, a very small value unless N were extremely large, since H/v is already small, at least smaller than v/M_{Planck}.

Now we turn our attention to the propagation of electromagnetic waves across a magnetic field. The effect is relevant for astronomical polarization measurements because galactic and extragalactic magnetic fields, although weak, extend over extremely large regions [16]. The effect under consideration is amenable to an entirely classical description, as a mixing between electromagnetic and pseudoscalar oscillations [3-5]. On the RHS of the field equations (3), one can neglect the $\boldsymbol{E}$ and $\boldsymbol{B}$ associated with the electromagnetic wave as compared with the external magnetic field $\boldsymbol{B}_0$. The electric and magnetic fields can be derived as usual from a vector potential $\boldsymbol{A}$. We will assume that the magnetic field $\boldsymbol{B}_0$ is transverse to the electromagnetic wave vector, i.e. $\boldsymbol{B}_0 \cdot \boldsymbol{k} = 0$. A component of $\boldsymbol{B}_0$ parallel to the direction in which the electromagnetic wave propagates has no significant effect. In that case the equations for $\boldsymbol{A}$ and φ read, neglecting terms of order $\varphi\boldsymbol{A}$ and A^2,

$$\Box \boldsymbol{A} = -g\boldsymbol{B}_0 \frac{\partial \varphi}{\partial t}, \quad \Box \varphi = g\boldsymbol{B}_0 \cdot \frac{\partial \boldsymbol{A}}{\partial t}. \tag{10}$$

Clearly, the component of the vector potential perpendicular to the magnetic field, $A_\perp$, is not affected by the external magnetic field. It has solutions of the form $A_\perp = A_\perp^0 \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{x})$, with the standard dispersion relation, $\omega^2 = k^2$. Only the parallel component, $A_\parallel$, mixes with the φ field. If we define a complex variable $\xi = A_\parallel + \mathrm{i}\varphi$, then

$$\Box \xi = \mathrm{i} g B_0 \frac{\partial \xi}{\partial t}. \tag{11}$$

This equation has solutions of the form $\xi \propto \exp(\mathrm{i}\omega' t) \cos \boldsymbol{k}\cdot\boldsymbol{x}$ with $\omega'^2 = k^2 + gB_0\omega'$. In the (realistic) approximation $gB_0/k \ll 1$, $\omega' \approx \pm k + \frac{1}{2} g B_0$. The solution representing an electromagnetic wave that at $t=0$ in $\boldsymbol{x}=0$ is unmixed with φ and has amplitude $A_\parallel^0$ is

$$A_\parallel = A_\parallel^0 \cos(\tfrac{1}{2} g B_0 t) \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}),$$
$$\varphi = A_\parallel^0 \sin(\tfrac{1}{2} g B_0 t) \cos(\omega t - \boldsymbol{k}\cdot\boldsymbol{x}). \tag{12}$$

The amplitude of the component of the electric field parallel to the external magnetic field is modulated over a length scale $L = \pi/gB_0$. As a linearly polarized wave propagates across a homogeneous transverse magnetic field, the direction along which it is polarized slowly oscillates between the direction of its initial polarization and the direction perpendicular to the magnetic field. The effect is independent of frequency (under the approximation we have been working, $\omega \gg gB_0$).

Propagation across a magnetic field, as described by eqs. (12), could in principle affect the observed polarization properties of radio galaxies and quasars, since the emission from distant sources is expected to traverse magnetic fields along our line of sight, with orientations that bear no relation to the orientation of the axis of the source galaxy. As conservative estimates for intervening magnetic fields we may take [16] $B_0 \approx 10^{-6}$ G, uniform over scales of order $L \approx 10^{23}$ cm ≈ 30 kpc. The effect described by eq. (12) would wash away the correlation between the polarization direction at the source and the source axis if $gB_0 L/\pi \gtrsim 1$. In the real world, however, the effect is much weaker, because plasma effects prevent the mixing of photons and pseudoscalars to occur coherently over such large distances. The limiting factor is the coherence length

$$l \approx 2 \frac{\omega}{\omega_{\mathrm{pl}}^2} \approx 2 \times 10^{15}\ \mathrm{cm} \frac{\nu}{\mathrm{GHz}} \frac{5 \times 10^{-5}\ \mathrm{cm}^{-3}}{n_{\mathrm{e}}}. \tag{13}$$

Here $\omega = 2\pi\nu$ is the electromagnetic wave angular frequency, $\omega_{\mathrm{pl}}^2 = 4\pi\alpha n_{\mathrm{e}}/m$ is the square of the plasma frequency, n_{e} the free electron density and m the electron mass. l is typically much shorter than the length scale over which we expect galactic magnetic fields to be approximately homogeneous. The effect of rotation of the plane of polarization is consequently reduced from what it would have been in vacuum. The relevant factor is the probability P for conversion of a photon into a pseudoscalar. It can be obtained in the limit $P \ll 1$ from the expansion of $\cos^2(gB_0 L)$ in eq. (12), with L^2 replaced by Ll [3,5,10]:

$$P \equiv \tfrac{1}{4}(gB_0)^2 Ll$$
$$\approx 6\times10^{-7}\ (g\cdot 10^{10}\ \mathrm{GeV})^2\left(\frac{B_0}{10^{-6}\ \mathrm{G}}\right)^2$$
$$\times \frac{L}{10^{23}\ \mathrm{cm}}\frac{\nu}{\mathrm{GHz}}\frac{5\times10^{-5}\ \mathrm{cm}^{-3}}{n_e}\,. \tag{14}$$

This factor would approach unity, and the effect under consideration would be in conflict with the polarization properties of distant radio galaxies and quasars, only if $g^{-1}\approx 10^7$ GeV (assuming other parameters as explicitly written). Also, this result applies to massive pseudoscalars only if their mass is smaller than the plasma frequency, $m \lesssim 10^{-13}$ eV. On the other hand, astrophysical bounds based on the lifetime of He-burning low mass stars imply $g^{-1}\gtrsim 10^{10}$ GeV [17,18], a bound that is applicable to any pseudoscalar mass below 30 keV. Thus, the rotation of the plane of polarization of distant radio galaxies and quasars due to mixing with pseudoscalars along intervening magnetic fields is typically very small.

The conversion probability P in eq. (14) is also the degree of linear polarization that originally unpolarized radiation acquires as it propagates through a medium permeated by a transverse homogeneus magnetic field. Indeed, P represents the attenuation factor for the component of the electric field parallel to the external magnetic field. This effect may show up, for instance, in the cosmic microwave background, where no linear polarization has so far been detected. The best model-independent current limit is $P<6\times10^{-5}$, from a fit to spherical harmonics of the data collected at 33 GHz through a 7° aperture horn oriented into 11 different declinations between $-27°$ and $+63°$ [19]. This limit improves by a factor of 2 for a fit to an anisotropic axisymmetric model for the expansion of the universe prior to decoupling as the source of the polarization [20].

Eq. (14) shows that if our galaxy had a halo with a large scale, ordered magnetic field of about 10^{-6} G and an electron density $n_e\approx 5\times10^{-5}$ cm^{-3}, the cosmic microwave background would have a degree of linear polarization of order 10^{-5}, close to the current limit, if $g^{-1}\approx 10^{10}$ GeV. There is no firm evidence for such a magnetic field in our galactic halo, although the numbers quoted seem appropriate for other galaxies like ours [16].

To place a firm bound on g from the lack of polarization of the microwave background we should invoke firmly established magnetic fields. Also, they should be aligned over large distances, and cover a significant fraction of the sky, if they are to affect the observations of ref. [19]. The best candidate is probably the magnetic field within a few kpc of our solar system. Observations of Faraday rotation measurements both of galactic pulsars as well as of extragalactic objects suggest a value $B_0\approx 3\times10^{-6}$ G with an electron density $n_e\approx 3\times10^{-2}$ cm^{-3} [21]. With $L\approx 1$ kpc to be conservative, the condition $P\lesssim 6\times10^{-5}$ amounts to $g^{-1}\gtrsim 1.3\times10^8$ GeV. This is two orders of magnitude below the bound from stellar evolution. Besides, it is only applicable to pseudoscalar masses smaller than the plasma frequency.

Although the current bound on g from the lack of polarization of the microwave background is less significant than the astrophysical bound based on stellar evolution, it could potentially improve, for instance with more detailed measurements of the polarization of the microwave background in regions of the sky where larger magnetic fields and smaller electron densities may be found.

Let us conclude by mentioning a general implication of a non-negligible conversion probability P. No astronomical object should be seen to have a degree of linear polarization smaller than P, when observed through regions permeated by magnetic fields. The galactic magnetic field in our neighborhood thus determines a minimum degree of linear polarization (albeit rather small) for astronomical observations.

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