Jacqueline Garrahan
Final Project Proposal

For this project, I would like to build upon the results presented by Ahn, Choi and colleagues in "Modeling stock return distribution using the quantum harmonic oscillator". The paper applies a quantum harmonic oscillator to modeling stock returns and compares the results to the geometric Brownian motion model. The motivation to use the quantum harmonic oscillator was the ability to capture oscillatory behaviors of restorative market forces.

However, improvements may be made on the traditional GBM equations to capture the restorative dynamics that make the QHM attractive for this application. This may be accomplished by combining features of the Ornstein-Uhlenbeck Process and Geometric Brownian Motion. Notably, the Ornstein-Uhlenbeck Process is a mean reverting process, meaning that it demonstrates a tendency to drift towards its long-term mean. The mean restorative attraction is greater at larger distances from the mean

Geometric Brownian Motion:

$$dX_t = aX_t dt + \sigma X_t dW_t$$

Ornstein-Uhlenbeck process:
$$dX_t = a(\mu - X_t)dt + \sigma dW_t$$

For my model, I will modify the GBM equation to include a mean-reversion term, giving an equation of the form:

$$dX_t = a(\mu - X_t)dt + \sigma_1 dW_{1t} + \sigma_2 X_t dW_{2t}$$

For this project, I will use the original data set from the original paper, the FTSE All Share Index from 15 November, 2007 to 21 September 2014. I have chosen this set because I agree with the original author's claim that the set spans interesting periods in financial history and because my model offers an improvement. I will provide a solution to the combined Ornstein-Uhlenbeck and GBM equations (this should be possible because it is linear). Finally, I will compare the simulated behavior of my model, GBM, and the QHO. I will perform a goodness of fit test on each of the simulated trajectories and compare the results and parameters will be chosen to minimize simulate error against the empirical data. The optimal results for each model will then be compared.