Guessing and Derivatives

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I felt that my presentation of the connection between the derivative and our algorithm for finding the roots of numbers was confused. Here is another go.

We started with the definition.

Definition 1. $\frac{df(x)}{dy} = \lim_{x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

Then we wanted to think about how it relates to our case. We want to find that adjustment we need to make in our guess (the input x) so that we get the right answer. The outputs we get are the results of running our x through our f. So, if we actually knew the right number, if we adjusted our x by the right amount (Δx), then we would get the number we are aiming for. That means that $f(x + \Delta x)$ is our number that we are trying to find the cube root for. And the $x + \Delta x$ is the input we need, and x is the guess we are working with. We know, or we can figure out, the derivative using *sympy* or calculus or asking a friend in the math faculty.

The equation above then becomes:
$\frac{df(guess)}{dy} = \frac{128 - f(guess)}{Amount\ to\ adjust\ our\ guess}$

Which we can re-arrange using algebra (multiply both sides by the same thing). And becomes:
$Amount\ to\ adjust\ our\ guess = \frac{128 - f(guess)}{\frac{df(guess)}{dy}}$

But we were just using 128 as an example, so it really should be:

$$Amount\ to\ adjust\ our\ guess = \frac{Number\ we\ are\ trying\ to\ find\ root\ for - f(guess)}{\frac{df(guess)}{dy}} \tag{1}$$

You should now be able to see the last equation implemented in our Jupyter Notebook. It all relies on using the definition of the derivative and assuming that if we make our steps small enough (the changes in the guesses) we can ignore that limit.

What we are doing in our function is taking a current guess, and calculating by the above equation what we think we need to adjust our guess by using the straight-line slope. We then check if we are close enough to the right answer. If we are we stop. If not, we do the above again, and we continue getting closer and closer until we have what works.

Note that we don't actually have the formula for the function explicitly in our equations above. We just f. That is because whatever our f is, squares, roots, or anything like that, this basic process will work. It is akin to somthing called Newton-Raphson and Newton's method. Discovered by good old Isaac Newton himself.