

The FLP Theorem

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The Distributed Consensus Problem

TODO

Faults

In this course we usually assumed that the participating processes and the network were completely reliable.

This is not the case with real systems, which are subject to a number of possible **faults**, such as process crashes, network partitioning, and lost, distorted, duplicated messages.

One can even consider more **Byzantine** types of failures, in which faulty processes might go completely haywire, perhaps even sending messages according to some malevolent plan.

The FLP Theorem

TODO

Consensus protocol

Definition (Consensus protocol)

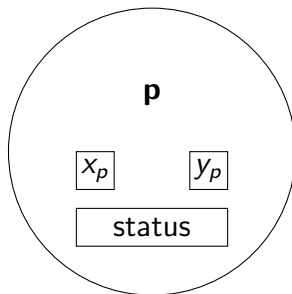
A **consensus protocol** is an asynchronous system of N processes ($N \geq 2$). Each process p has a one-bit **input register** x_p , an **output register** y_p with values in $\{\perp, 0, 1\}$ and an unbounded amount of internal storage.

Initial states prescribe fixed starting values for all but the input register; in particular, the output register starts with value \perp .

p acts deterministically according to a **transition** function.

Decision states

The states in which the output register has value 0 or 1 are distinguished as being **decision states**. The transition function cannot change the value of the output register once the process has reached a decision state; that is, the output register is “write-once”.



Message system

A **message** is a pair (p, m) , where p is the name of the destination process and m is a “message value” from a fixed universe M .

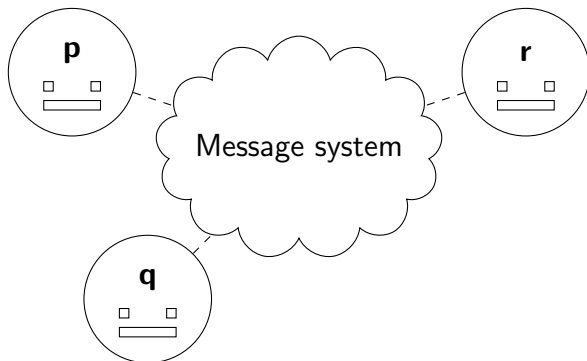
Definition (Message system)

The **message system** maintains a multiset, called the **message buffer**, of messages that have been sent but not yet delivered. It supports two abstract operations:

- $\text{send}(p, m)$: Places (p, m) in the message buffer.
- $\text{receive}(p)$: Deletes some message (p, m) from the buffer and returns m , in which case we say (p, m) is **delivered**, or returns the special null marker \emptyset and leaves the buffer unchanged.

Nondeterministic messaging

The message system acts nondeterministically, subject only to the condition that if `receive(p)` is performed infinitely many times, then every message (p, m) in the message buffer is eventually delivered.



Configurations and steps

TODO

Schedules and runs

TODO

Partial correctness

A configuration C has **decision value** v if some process p is in a decision state with $y_p = v$.

Definition (Partial correctness)

A consensus protocol is **partially correct** if:

- 1 No accessible configuration has more than one decision value.
- 2 For each $v \in \{0, 1\}$, some accessible configuration has decision value v .

Total correctness in spite of one fault

A process p is **nonfaulty** in run if it takes infinitely many steps, otherwise it is **faulty**.

A run is **admissible** if at most one process is faulty and all messages sent to nonfaulty processes are eventually received.

A run is **deciding** if some process reaches a decision state.

Definition (Total correctness in spite of one fault)

A consensus protocol P is **totally correct in spite of one fault** if it is partially correct and every admissible run is deciding.

Main result

Theorem (Fischer, Lynch, Paterson [FLP85])

No consensus protocol is totally correct in spite of one fault.

A configuration is **bivalent** if the set of decision values of configurations reachable from it has 2 elements. It is instead **0-valent** or **1-valent** according to the corresponding value.

Proof (sketch).

Given an initial bivalent configuration, we construct an admissible run that at each stage results in another bivalent configuration. □

Lemma 1

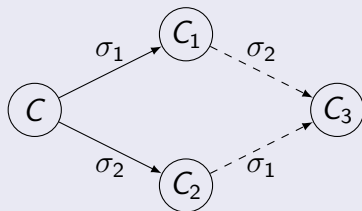
Lemma

Suppose that from some configuration C , the schedules σ_1, σ_2 lead to configurations C_1, C_2 respectively. If the sets of processes taking steps in σ_1 and σ_2 , respectively, are disjoint, then σ_2 can be applied to C_1 and σ_1 can be applied to C_2 , and both lead to the same configuration C_3 .

In other words: **schedules about disjoint processes commute with each other.**

Proof of Lemma 1

Proof (Lemma 1).



Because the sets of processes are disjoint, an event in σ_1 applicable to C is applicable to C_2 as well.

Because of determinism, after all events are processed they must end up in the same state. □

Lemma 2

Lemma

P has a bivalent initial configuration.

Proof (Lemma 2).



Lemma 3

Lemma

Let C be a bivalent configuration of P , and let $e = (p, m)$ be an event that is applicable to C . Let \mathcal{C} be the set of configurations reachable from C without applying e , and let $\mathcal{D} = e(\mathcal{C}) = \{e(E) \mid E \in \mathcal{C} \text{ and } e \text{ is applicable to } E\}$. Then, \mathcal{D} contains a bivalent configuration.

In other words: given a bivalent configuration and an event e applicable to it, **we construct another bivalent configuration having e as the last applied event.**

Proof of Lemma 3

Proof (Lemma 3).

Assume by contradiction that \mathcal{D} contains no bivalent configuration. We first show that \mathcal{D} contains both 0-valent and 1-valent configurations. □

Proof of main result

Proof (main result).



Conclusions

TODO

Bibliography



Michael J. Fischer, Nancy A. Lynch, and Michael S. Paterson, *Impossibility of distributed consensus with one faulty process*, Journal of the ACM (JACM) **32** (1985), no. 2, 374–382.