The FLP Theorem

Jacopo Notarstefano
jacopo.notarstefano [at] gmail.com

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The Distributed Consensus Problem

Let's consider the problem of reaching agreement on a single value in a distributed, asynchronous system of processes.

For example, in a distributed database, all entities that have partecipated in the processing of a particular transaction need to agree wether to commit or rollback.

Whatever decision is made, all entities must make the same decision in order to preserve the consistency of the database.

Faults

In this course we usually assumed that the partecipating processes and the network were completely reliable.

This is not the case with real systems, which are subject to a number of possible **faults**, such as process crashes, network partitioning, and lost, distorted, duplicated messages.

One can even consider more **Byzantine** types of failures, in which faulty processes might go completely haywire, perhaps even sending messages according to some malevolent plan.

The FLP Theorem

In 1985, Michael Fischer, Nancy Lynch, and Michael Paterson proved the surprising result that no completely asynchronous consensus protocol can tolerate even a single unannounced process death.

Their proof did not consider Byzantine failures, and assumed that the message system is reliable — it delivers all messages correctly and exactly once.

Nevertheless, even with these assumptions, the stopping of a single process at an inopportune time can cause any distributed consensus protocol to fail to reach agreement.

Consensus protocol

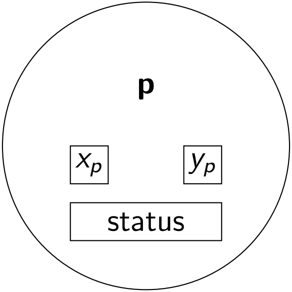
A **consensus protocol** is an asynchronous system of N processes ($N \geq 2$). Each process p has a one-bit **input register** x_p, an **output register** y_p with values in $\{\perp, 0, 1\}$ and an unbounded amount of internal storage.

Initial states prescribe fixed starting values for all but the input register; in particular, the output register starts with value $\perp$.

p acts deterministically according to a **transition** function.

Decision states

The states in which the output register has value 0 or 1 are distinguished as being **decision states**. The transition function cannot change the value of the output register once the process has reached a decision state; that is, the output register is "write-once".



Message system

A **message** is a pair (p, m), where p is the name of the destination process and m is a "message value" from a fixed universe M.

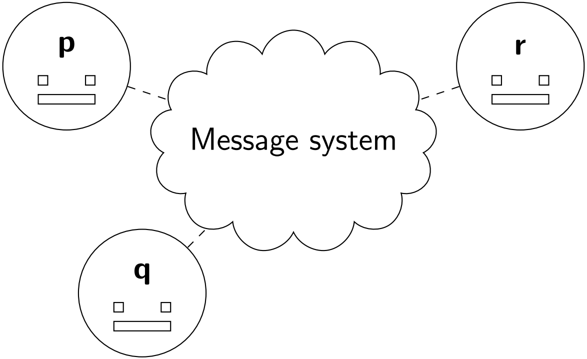
The **message system** mantains a multiset, called the **message buffer**, of messages that have been sent but not yet delivered. It supports two abstract operations:

`send(p,m)`: Places (p, m) in the message buffer.

`receive(p)`: Deletes some message (p, m) from the buffer and returns m, in which case we say (p, m) is **delivered**, or returns the special null marker $\varnothing$ and leaves the buffer unchanged.

Nondeterministic messaging

The message system acts nondeterministically, subject only to the condition that if `receive(p)` is performed infinitely many times, then every message (p, m) in the message buffer is eventually delivered.



Configurations and steps

A **configuration** of the system consists of the internal state of each process, together with the contents of the message buffer. An **initial configuration** is one in which each process starts at an initial state and the message buffer is empty.

A **step** takes one configuration to another and consists of a primitive step by a single process p.

Primitive steps and applicable events

Let C be a configuration. A step consists of two phases:

1. A `receive(p)` is performed on the message buffer in C to obtain a value $m \in M \cup \{\varnothing\}$.
2. Depending on p's internal state in C and on m, p enters a new internal state and sends a finite set of messages to other processes.

The step is completely determined by the pair $e = (p, m)$, which we call an **event**.

An event e that *could happen* at configuration C is called **applicable**, and the resulting configuration is denoted $e(C)$.

Schedules and runs

A **schedule** from C is a finite or infinite sequences σ of applicable events, in turn, starting from C. The associated sequence of steps is called a **run**.

If σ is finite, we let $\sigma(C)$ denote the resulting configuration, which is said to be **reachable** from C.

A configuration reachable from some initial configuration is said to be **accessible**.

Partial correctness

A configuration C has **decision value** v if some process p is in a decision state with $y_p = v$.

Definition (Partial correctness)

A consensus protocol is **partially correct** if:

1. No accessible configuration has more than one decision value.
2. For each $v \in \{0, 1\}$, some accessible configuration has decision value v.

Total correctness in spite of one fault

A process p is **nonfaulty** in run if it takes infinitely many steps, otherwise it is **faulty**.

A run is **admissible** if at most one process is faulty and all messages sent to nonfaulty processes are eventually received.

A run is **deciding** if some process reaches a decision state.

Definition (Total correctness in spite of one fault)

A consensus protocol P is **totaly correct in spite of one fault** if it is partially correct and every admissibile run is deciding.

Main result

Theorem (Fischer, Lynch, Paterson [FLP85])

No consensus protocol is totally correct in spite of one fault.

A configuration is **bivalent** if the set of decision values of configurations reachable from it has 2 elements. It is instead 0-**valent** or 1-**valent** according to the corresponding value.

Proof (sketch).

Given an initial bivalent configuration, we construct an admissible run that at each stage results in another bivalent configuration. □

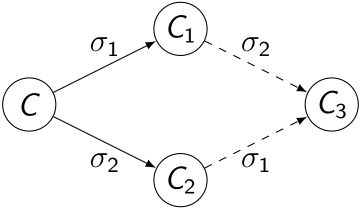
Lemma 1

Lemma

Suppose that from some configuration C, the schedules σ_1, σ_2 lead to configurations C_1, C_2 respectively. If the sets of processes taking steps in σ_1 and σ_2, respectively, are disjoint, then σ_2 can be applied to C_1 and σ_1 can be applied to C_2, and both lead to the same configuration C_3.

In other words: **schedules about disjoint processes commute with each other**.

Proof of Lemma 1



Because the sets of processes are disjoint, an event in σ_1 applicable to C is applicable to C_2 as well.

Because of determinism, after all events are processed they must end up in the same state. □

Lemma 2

Lemma

P has a bivalent initial configuration.

Assume by contradiction that there isn't. Then, by partial correctness, P must have both 0-valent and 1-valent initial configurations.

Let's call two initial configurations **adjacent** if they differ only in the initial value x_p of a single process p. Any two initial configurations are joined by a chain of inital configurations, each adjacent to the next.

Proof of Lemma 2

Hence, there must exist a 0-valent initial configuration C_0 adjacent to a 1-valent configuration C_1. Let p be the process in whose initial value they differ.

Consider some admissibile deciding run from C_0 in which process p takes no steps, and let σ be the associated schedule. Then σ can be applied to C_1 as well, and the corresponding configurations will be identical except for p's internal state.

Both runs reach the same decision value; if it is 1 then C_0 was bivalent, if it is 0 then C_1 was bivalent. $\square$

Lemma 3

Lemma

Let C be a bivalent configuration of P, and let $e = (p, m)$ be an event that is applicable to C. Let $\mathcal{C}$ be the set of configurations reachable from C without applying e, and let $\mathcal{D} = e(\mathcal{C}) = \{e(E) \mid E \in \mathcal{C}$ and e is applicable to $E\}$. Then, $\mathcal{D}$ contains a bivalent configuration.

In other words: given a bivalent configuration and an event e applicable to it, **we construct another bivalent configuration having e as the last applied event**.

Proof of Lemma 3

Assume by contradiction that $\mathcal{D}$ contains no bivalent configuration. We first show that $\mathcal{D}$ contains both 0-valent and 1-valent configurations.

Let E_i be an i-valent configuration from C. If $E_i \in \mathcal{C}$, let $F_i = e(E_i) \in \mathcal{D}$. Otherwise e was applied in reaching E_i, and so there exists $F_i \in \mathcal{D}$ from which E_i is reachable.

In either case, F_i is i-valent since it is in $\mathcal{D}$, which contains no bivalent configuration by hypothesis.

Proof of Lemma 3

TODO

Proof of main result

TODO

Conclusions

TODO

Bibliography

Michael J. Fischer, Nancy A. Lynch, and Michael S. Paterson, *Impossibility of distributed consensus with one faulty process*, Journal of the ACM (JACM) **32** (1985), no. 2, 374–382.