

# The FLP Theorem

Jacopo Notarstefano

`jacopo.notarstefano [at] gmail.com`

# The Distributed Consensus Problem

## Definition

# Consensus Protocol

# Message System

# Partial correctness

A configuration  $C$  has **decision value**  $v$  if some process  $p$  is in a decision state with  $y_p = v$ .

## Definition (Partial correctness)

A consensus protocol is **partially correct** if:

- 1 No accessible configuration has more than one decision value.
- 2 For each  $v \in \{0, 1\}$ , some accessible configuration has decision value  $v$ .

# Total correctness in spite of one fault

A process  $p$  is **nonfaulty** in run if it takes infinitely many steps, otherwise it is **faulty**.

A run is **admissible** if at most one process is faulty and all messages sent to nonfaulty processes are eventually received.

A run is **deciding** if some process reaches a decision state.

## Definition (Total correctness in spite of one fault)

A consensus protocol  $P$  is **totally correct in spite of one fault** if it is partially correct and every admissible run is deciding.

# Main result

Theorem (Fischer, Lynch, Paterson 1985)

*No consensus protocol is totally correct in spite of one fault.*

A configuration is **bivalent** if the set of decision values of configurations reachable from it has 2 elements. It is instead **0-valent** or **1-valent** according to the corresponding value.

Proof (sketch).

Given an initial bivalent configuration, we construct an admissible run that at each stage results in another bivalent configuration. □

# Lemma 1

Lemma

Proof.





# Lemma 2

Lemma

Proof.



# Lemma 3

Lemma

Proof.



# Proof of main result

Proof.

