Times, Clocks, and the Ordering of Events in a Distributed System

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Main objectives and outline

In this paper Lamport:

1. Discusses the partial ordering defined by the "happened before" relation,
2. Gives a distributed algorithm for extending it to a consistent total ordering of all the events,
3. Uses this algorithm to solve a mutual exclusion problem.

Distributed system

Definition (Distributed system)

A *distributed system* consists of a collection of distinct processes which are spatially separated, and which communicate with one another by exchanging messages.
A system is distributed if the message transmission delay is not negligible compared to the time between events in a single process.

Examples of distributed systems

- A worldwide network of interconnected computers
- A cluster of workstation in a data center
- Processes on a single computer

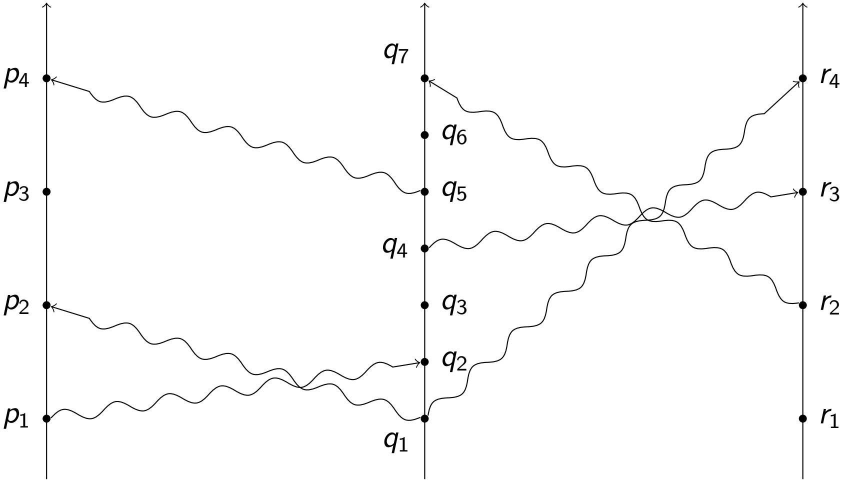
The "$\rightarrow$" relation

Definition (The "$\rightarrow$" relation)

The "$\rightarrow$" relation on the set of events of a system is the smallest relation satisfying the following three conditions:

1. If a and b are events in the same process, and a comes before b, then $a \rightarrow b$.
2. If a is the sending of a message by one process, and b is the receipt of the same message by another process, then $a \rightarrow b$.
3. If $a \rightarrow b$, and $b \rightarrow c$, then $a \rightarrow c$.

The "space-time diagram"



Clocks

Definition (Clock)

For each process P_i we define a *clock* C_i to be a function that assigns a number $C_i\langle a\rangle$ to each event a in the process.

Definition (System of clocks)

A *system of clocks* is a function C that assigns to the event b in process P_j the time $C\langle b\rangle = C_j\langle b\rangle$.

The clock condition

Definition (The clock condition)

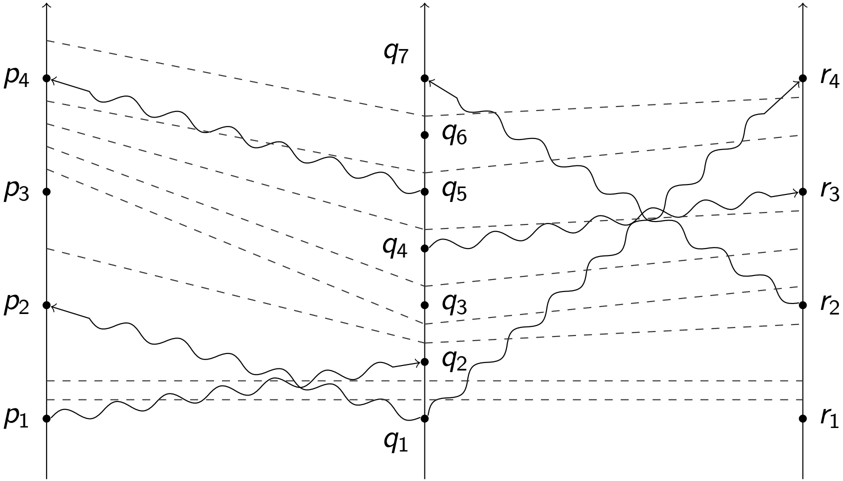
We say that a system of clocks satisfies the *clock condition* if, for any events a and b, we have: if $a \rightarrow b$ then $C\langle a\rangle < C\langle b\rangle$.

Lemma

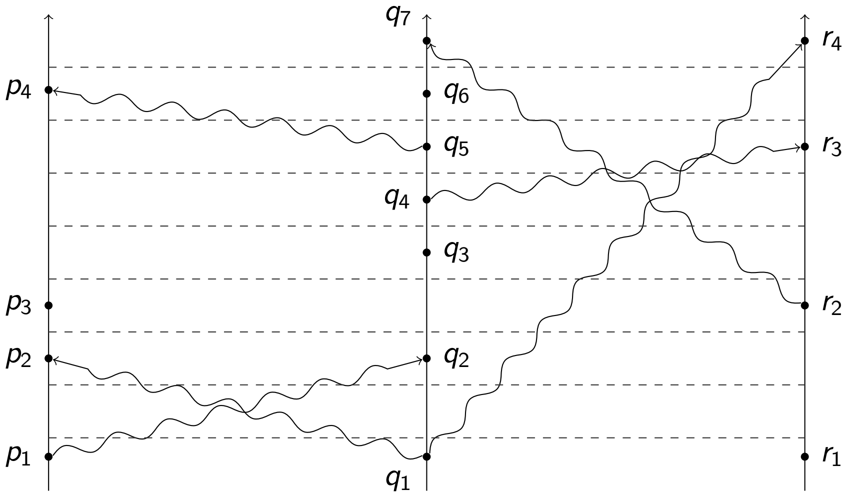
The clock condition is satisfied if the following conditions hold:

1. *If a and b are events in process P_i and a comes before b, then $C_i\langle a\rangle < C_i\langle b\rangle$.*
2. *If a is the sending of a message by process P_i and b is the receipt of that message by process P_j, then $C_i\langle a\rangle < C_i\langle b\rangle$.*

The “space-time diagram”, revisited



The "space-time diagram", rearranged



Implementation of the clock condition

Lemma

To guarantee that the system of clocks satisfies the clock condition we need to obey the following implementation rules:

1. *Each process* P_i *increments* C_i *between any two successive events.*
2. *If event* a *is the sending of a message* m *by process* P_i*, then the message* m *contains a timestamp* $T_m = C_i\langle a\rangle$.
3. *Upon receiving a message* m*, process* P_j *sets* C_j *greater than or equal to its present value and greater than* T_m.

The "$\Rightarrow$" relation

Definition (The "$\Rightarrow$" relation)

Let $\prec$ be a total ordering on the processes. If a is an event in process P_i and b is an event in process P_j, then $a \Rightarrow b$ if and only if either

1. $C_i\langle a\rangle < C_j\langle b\rangle$ or
2. $C_i\langle a\rangle = C_j\langle b\rangle$ and $P_i \prec P_j$.

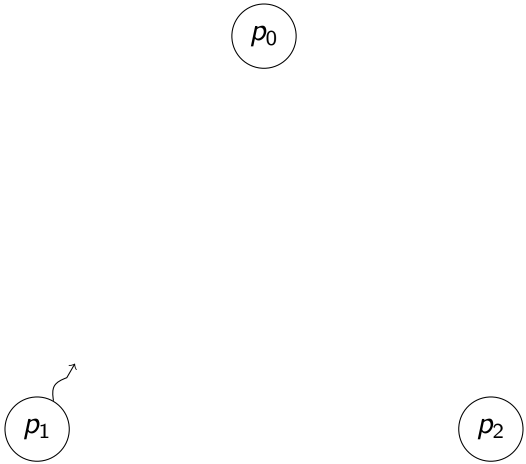
A mutual exclusion problem

A fixed collection of processes share a single resource, which can be used by one process at a time. We want to find an algorithm that satisfies the following conditions:

1. A process which has been granted the resource must release it before it can be granted to another process.
2. Different requests for the resource must be granted in the order in which they are made.
3. If every process which is granted the resource eventually releases it, then every request is eventually granted.

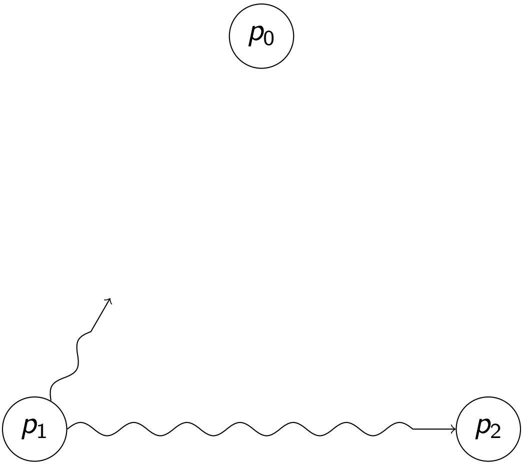
A wrong solution, 1/3

We can't use a central scheduling process which grants requests in the order they are received.



A wrong solution, 2/3

We can't use a central scheduling process which grants requests in the order they are received.



A wrong solution, 3/3

We can't use a central scheduling process which grants requests in the order they are received.