TEGNIESE WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur   150 punte

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Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

VRAAG 1

1.1
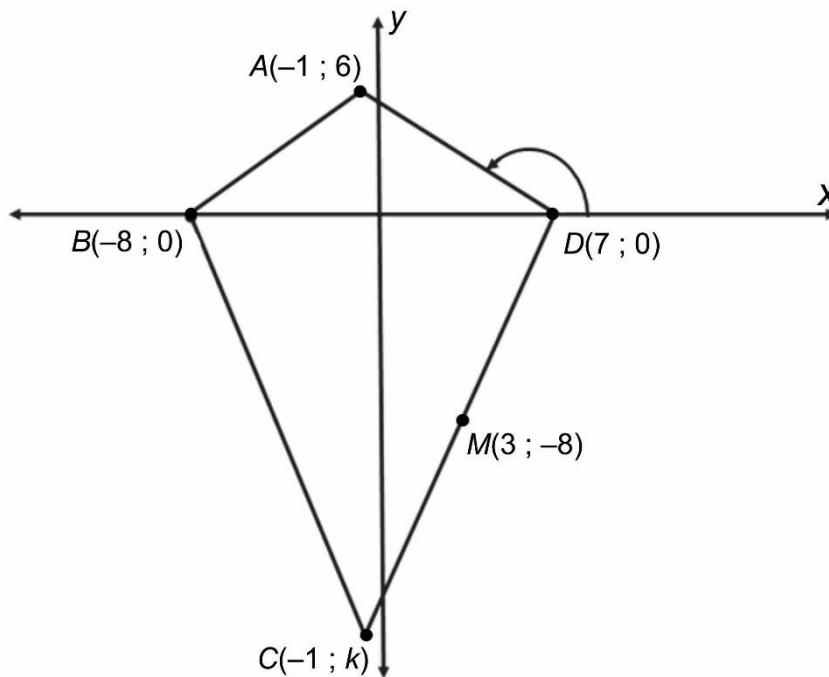
$$m_{AD} = \frac{y_D - y_A}{x_D - x_A} = \frac{0 - 6}{7 - (-1)} = \frac{-6}{8} = -\frac{3}{4}$$

1.2 $\tan\theta = m_{AD}$

$\tan\theta = -\dfrac{3}{4}$

$\therefore \theta \approx 180° - 36,87°$

$\therefore \theta \approx 143,13°$

1.3 $y = mx + c$ **OF** $y - y_1 = m(x - x_1)$

$0 = \left(-\dfrac{3}{4}\right)(7) + c$ $y - 0 = \left(-\dfrac{3}{4}\right)(x - 7)$

$c = \dfrac{21}{4}$ $y = -\dfrac{3}{4}x + \dfrac{21}{4}$

$\therefore y = -\dfrac{3}{4}x + \dfrac{21}{4}$

1.4 $M(3; -8) = \left(\dfrac{x_D + x_C}{2} ; \dfrac{y_D + y_C}{2}\right)$

$-8 = \dfrac{0 + k}{2}$

$k = -16$

1.5 $AC = 16 + 6 = 22$ eenhede

VRAAG 2

2.1
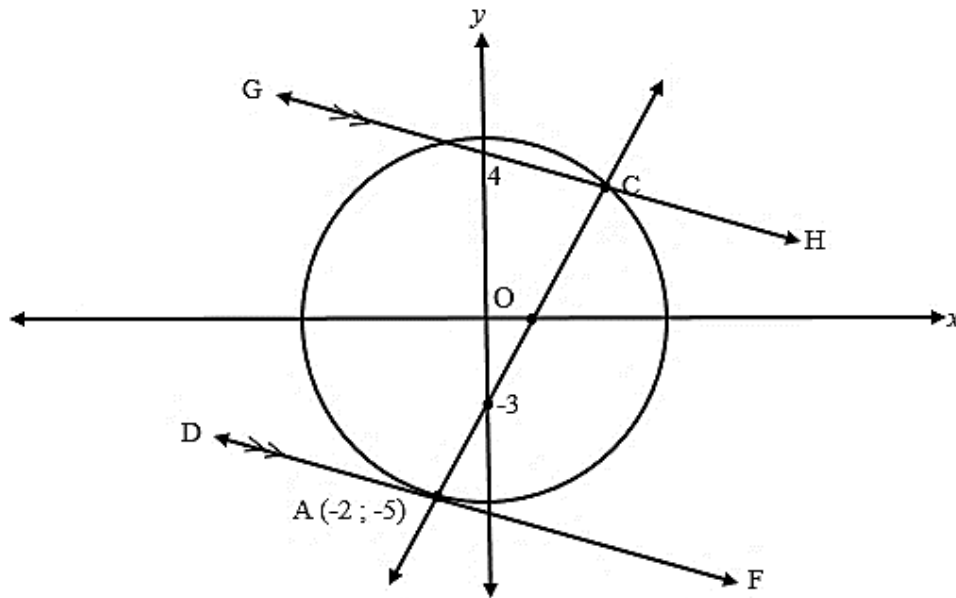
2.1.1 $r^2 = x^2 + y^2$

$= (-2)^2 + (-5)^2$

$r^2 = 29$

$\therefore x^2 + y^2 = 29$

2.1.2   Gradiënt van radius = $\dfrac{5}{2}$

Gradiënt van raaklyn = $\dfrac{-2}{5}$

$\therefore y = mx + c$ **OF** $y - y_1 = m(x - x_1)$

$\therefore -5 = \left(\dfrac{-2}{5}\right)(-2) + c$ $y - (-5) = \left(-\dfrac{2}{5}\right)(x - (-2))$

$\therefore c = \dfrac{-29}{5}$ $y + 5 = -\dfrac{2}{5}x - \dfrac{4}{5}$

$\therefore y = \dfrac{-2}{5}x - \dfrac{29}{5}$ $y = -\dfrac{2}{5}x - \dfrac{29}{5}$

2.1.3   Lyn AC: $\therefore y = x - 3$

Lyn GH: $y = -\dfrac{2}{5}x + 4$

Punt C: $x - 3 = -\dfrac{2}{5}x + 4$

$\dfrac{7}{5}x = 7$

$x = 5$
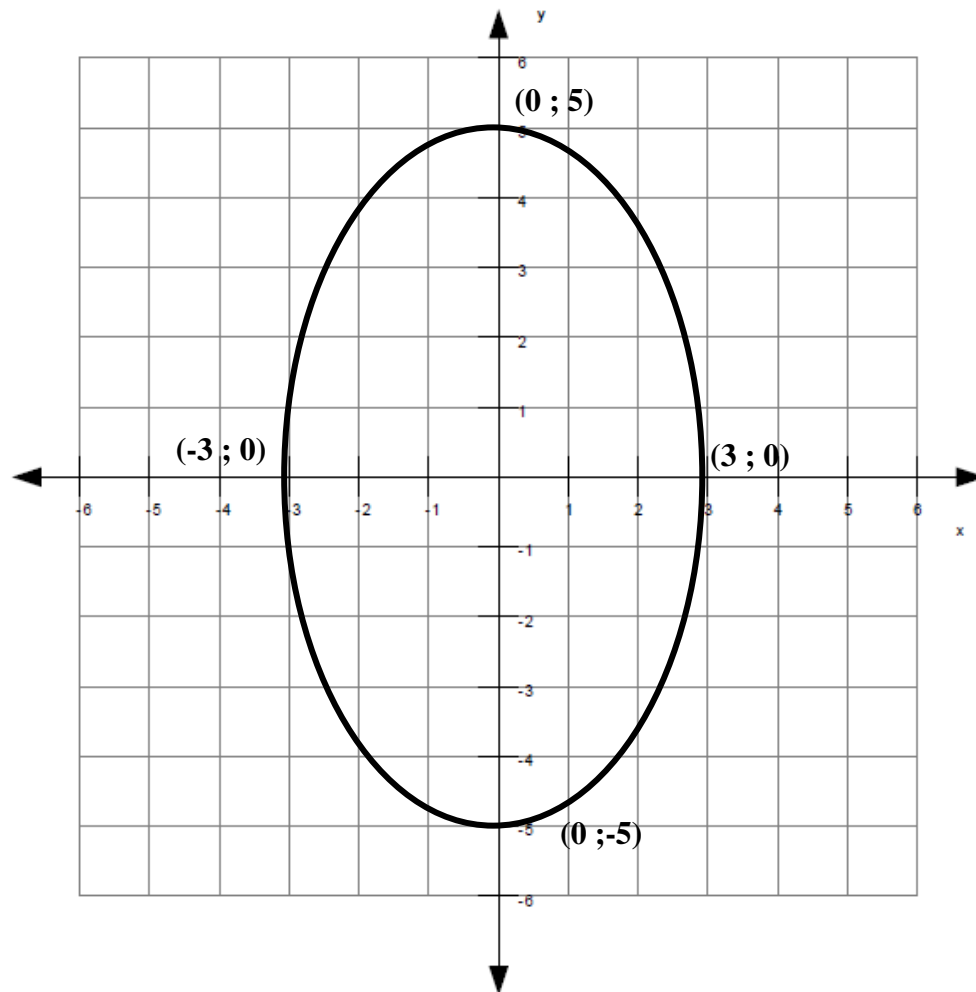
$y = 2$

C(5 ; 2)

2.2 $\dfrac{2x^2}{9} + \dfrac{2y^2}{25} - 2 = 0$

$\dfrac{x^2}{9} + \dfrac{y^2}{25} - 1 = 0$

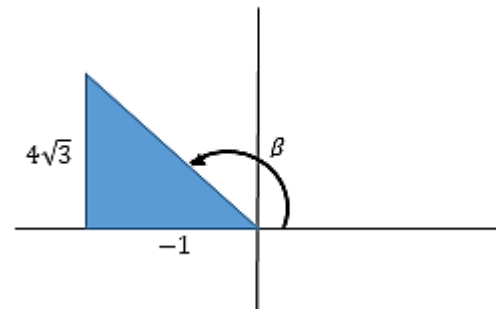$\dfrac{x^2}{9} + \dfrac{y^2}{25} = 1$

vorm, *x*-afsnitte, *y*-afsnitte

VRAAG 3

3.1   3.1.1
$$6^2 = p^2 + 4^2$$
$$36 = p^2 + 16$$
$$20 = p^2$$
$$p = -2\sqrt{5}$$

3.1.2 $\tan\theta = \dfrac{4}{-2\sqrt{5}}$ **OF** $\sin\theta = \dfrac{4}{6}$ **OF** $\cos\theta = \dfrac{2\sqrt{5}}{6}$

$\theta \approx 180° - 41,81°$ $\theta \approx 180° - 41,81°$ $\theta \approx 180° - 41,81°$

$\therefore \theta \approx 138,19°$ $\theta \approx 138,19°$ $\theta \approx 138,19°$

3.1.3
$$= \left(\frac{2\sqrt{5}}{6}\right)^2 - \left(\frac{4}{6}\right)^2$$
$$= \frac{20}{36} - \frac{16}{36}$$
$$= \frac{4}{36} = \frac{1}{9}$$

3.2
$$\tan\beta + 4\sqrt{3} = 0$$
$$\tan\beta = -4\sqrt{3}$$
$$r^2 = x^2 + y^2$$
$$= (-1)^2 + (4\sqrt{3})^2$$
$$= 49$$
$$r = 7$$

$$\frac{49(\cos\beta - \sin^2\beta)}{\sec 120° \cdot \tan 225°}$$

$$= \frac{49\left(\dfrac{-1}{7} - \left(\dfrac{4\sqrt{3}}{7}\right)^2\right)}{\sec(180° - 60°) \cdot \tan(180° + 45°)}$$

$$= \frac{49\left(\dfrac{-1}{7} - \dfrac{48}{49}\right)}{-\sec 60° \cdot \tan 45°}$$

$$= \frac{-7 - 48}{(-2)(1)}$$

$$= \frac{55}{2}$$

3.3 $\therefore \operatorname{cosec} \theta = \sqrt{2}$

$\therefore \sin \theta = \dfrac{1}{\sqrt{2}}$

verwysingshoek $= 45°$

$\theta = 45°$ of $\theta = 180° - 45° = 135°$

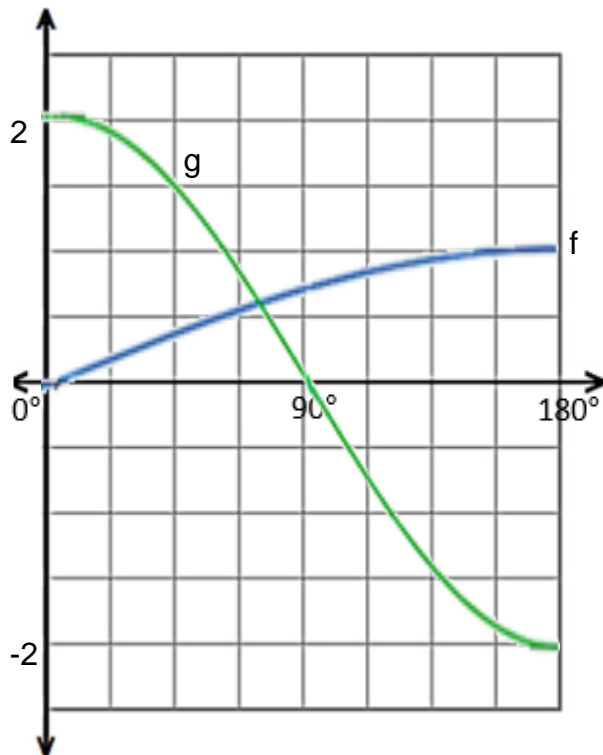3.4   LK: $\dfrac{\sin^2 \theta - 1}{\tan \theta \cdot \sin \theta - \tan \theta}$

$= \dfrac{(\sin \theta - 1)(\sin \theta + 1)}{\tan \theta (\sin \theta - 1)}$

$= \dfrac{\sin \theta + 1}{\tan \theta}$

$\therefore$ LK = RK

VRAAG 4

4.1
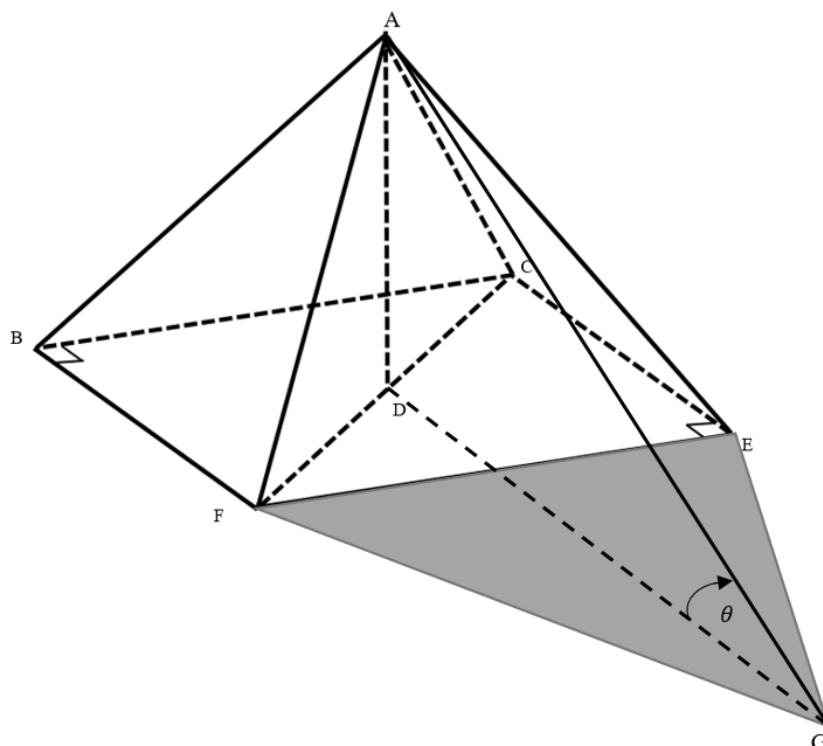

funksie f

afsnitte

draaipunte

vorm

funksie g

afsnitte

draaipunte

vorm

4.2   4.2.1 $\sin \dfrac{x}{2} \geq 2 \cos x$

$x \in [73° ; 180°]$ OF $73° \leq x \leq 180°$

4.2.2 $x = 0°$

4.2.3 $x \in (60° ; 180°]$ OF $60° < x \leq 180°$

VRAAG 5

5.1   In $\triangle AEF$: $2 \times F\hat{E}A + 51,8° = 180°$

$\therefore F\hat{E}A = 64,1°$

$$\frac{AF}{\sin E} = \frac{FE}{\sin A}$$

$$\frac{AF}{\sin(64,1°)} = \frac{2m}{\sin(51,8°)}$$

$$AF = 2,29 \text{ m}$$

5.2   Volume $= \frac{1}{3}$ (basisoppervlakte $\times$ $\perp$ hoogte)

$= \frac{1}{3}$ (2 m $\times$ 2 m $\times$ 1,8 m)

$= 2,4 \text{ m}^3$

5.3   Oppervlakte $\triangle EFG = \frac{1}{2} FG \times EG \times \sin G$

$= \frac{1}{2}(2,3) \times (2,3) \times \sin 51,5°$

$\approx 2,07 \text{ m}^2$

5.4   Hoogte van ΔEFG

$$\Delta EFG = \frac{1}{2}EF \times h$$

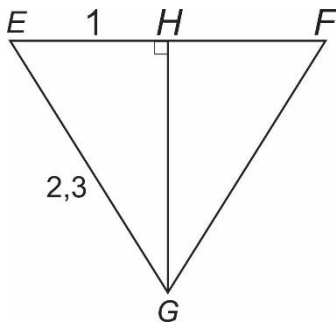Oppervlakte $2,07 = \frac{1}{2}(2) \times h$

$$2,07 \approx h$$

Lengte van $DG \approx 2,07 + 1 \approx 3,07$ m

Hoogtehoek

$$\tan\theta = \frac{1,8 \text{ m}}{3,07 \text{ m}}$$
$$\therefore \theta \approx 30,38°$$

OF

$\perp h : HG$ volgens Pythagoras
$$EG^2 = EH^2 + HG^2$$
$$(2,3)^2 = (1)^2 + HG^2$$
$$(2,3)^2 - (1)^2 = HG^2$$
$$HG = \sqrt{4,29}$$
$$2,07 \approx h \text{ m}$$

Lengte van $DG \approx 2,07 + 1 = 3,07$ m

Hoogtehoek

$$\tan\theta = \frac{1,8 \text{ m}}{3,07 \text{ m}}$$
$$\therefore \theta \approx 30,38°$$

VRAAG 6

6.1
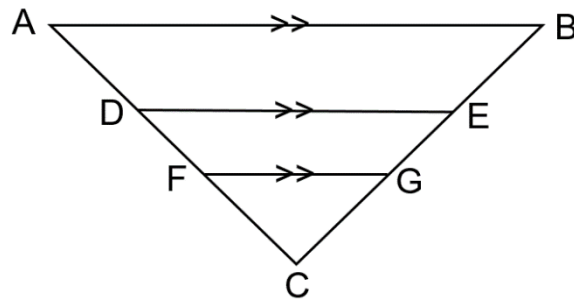
6.1.1 $\dfrac{CG}{EG} = \dfrac{4}{3}$

$\dfrac{4 \text{ m}}{EG} = \dfrac{4}{3}$

$\therefore EG = 3 \text{ m}$

6.1.2 $\dfrac{AB}{FG} = \dfrac{CB}{CG}$

$\dfrac{AB}{6 \text{ m}} = \dfrac{11}{4}$

$\therefore AB = 16,5 \text{ m}$

6.2
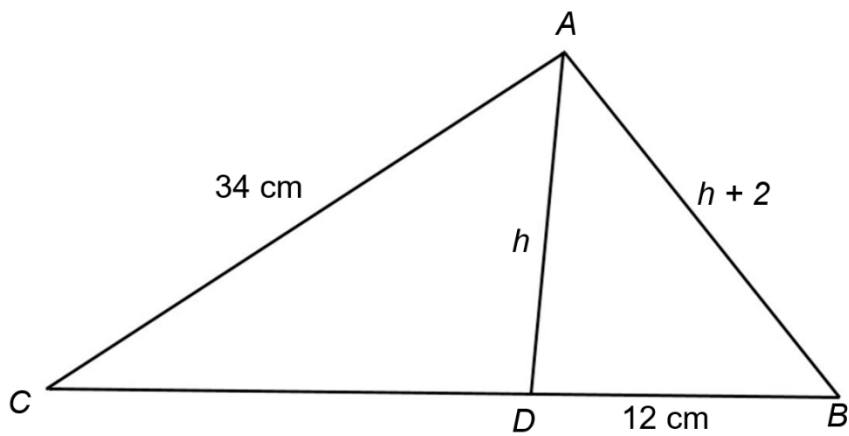
$\dfrac{AC}{DA} = \dfrac{AB}{DB}$

$\dfrac{34}{h} = \dfrac{h+2}{12}$

$408 = h^2 + 2h$

$0 = h^2 + 2h - 408$

$h = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$h = \dfrac{-2 \pm \sqrt{2^2 - 4(1)(-408)}}{2(1)}$

$h \approx 19,22 \quad \text{of} \quad h \approx -21,22 \text{ (NVT)}$

$\therefore AD \approx 19,22 \text{ cm}$

6.3
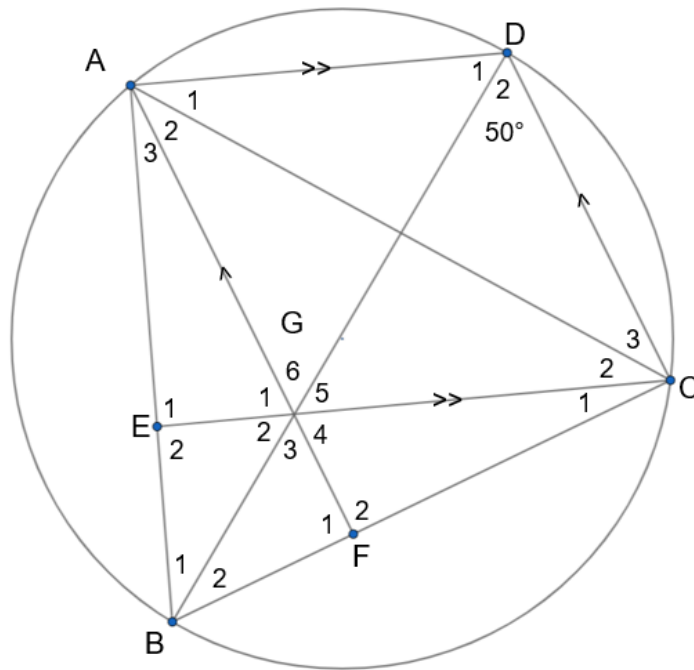
6.3.1 (a) $\hat{E}_2 = 90°$ (gegee)

$\therefore D\hat{A}B = 90°$ (ooreenk. hoeke OF Ko-binnehoeke; $AD//EC$)

(b) $\hat{D}_2 = 50°$ (gegee)

$\therefore \hat{A}_2 + \hat{A}_3 = \hat{D}_2 = 50°$ (hoeke in dieselfde segment)

en $\hat{A}_2 + \hat{A}_3 + \hat{A}_1 = 90°$ (bereken)

$\therefore \hat{A}_1 = 40°$

(c) $\hat{G}_6 = \hat{D}_2 = 50°$

(verw. hoeke, $DC//AG$)

6.3.2 $\hat{G}_6 = \hat{G}_3 = 50°$ (Regoorstaande hoeke)

en $\hat{G}_3 + \hat{B}_2 + \hat{F}_1 = 180°$ (Binnehoeke van driehoek)

$\hat{B}_2 = \hat{A}_1 = 40°$ (Hoeke in dieselfde segment)

$\therefore \hat{F}_1 = \hat{F}_2 = 90°$ (Hoeke op 'n reguitlyn)

$\therefore \hat{E}_1 = \hat{F}_2$

$\therefore$ ACFE is koordevierhoek   (Lyn onderspan gelyke hoeke OF omgekeerde hoeke in dieselfde segment)

OF

$\hat{G}_6 = \hat{G}_3 = 50°$ (Regoorstaande hoeke)

$\hat{B}_1 = \hat{A}_1 = 40°$ (Hoeke in dieselfde segment)

$\therefore \hat{F}_1 = 90°$ (Som van hoeke van driehoek)

$\therefore \hat{F}_1 + \hat{E}_2 = 90° + 90° = 180°$

$\therefore$ ACFE is koordevierh.   (Omgekeerde teenoorst. hoeke koordevierh.)

OF

$\hat{G}_6 = \hat{G}_6 = 50°$ (Regoorstaande hoeke)

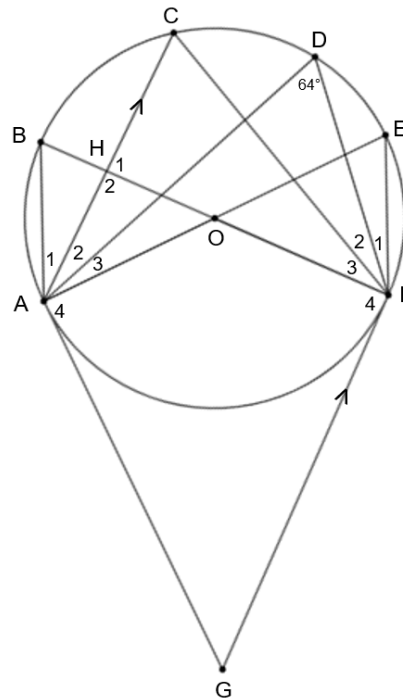$\hat{B}_1 = \hat{A}_1 = 40°$ (Hoeke in dieselfde segment)

$\therefore \hat{F}_2 = 40° + 50° = 90°$ (Buitehoek driehoek)

$\therefore \hat{F}_2 = \hat{E}_2 = 90°$

$\therefore$ ACFE is koordevierh.   (Omgekeerde buitehoek van koordevierh.)

6.4

6.4.1 $A\hat{O}F = 2\hat{D}$ (Hoek by middelpunt = 2 × hoek op omtrek)
        $\therefore A\hat{O}F = 128°$

6.4.2 $\hat{F}_4 = 90°$ (Radius loodreg op raaklyn)

6.4.3 $\hat{D} = \hat{C} = 64°$ (Hoeke in dieselfde segment)
            $\hat{H}_1 = \hat{F}_4 = 90°$ (Verw. hoeke; *CA*//*FG*)
        $\hat{H}_1 + \hat{F}_3 + \hat{C} = 180°$ (Binnehoeke van driehoek)
       $90° + \hat{F}_3 + 64° = 180°$
               $\therefore \hat{F}_3 = 26°$

OF

$\hat{D} = \hat{C} = 64°$ (Hoeke in dieselfde segment)
$\hat{F}_4 + \hat{F}_3 = 180° - 64°$ (Ko-binnehoeke, *AC*//*GF*)
         $= 116°$
      $\hat{F}_3 = 116° - 90°$
         $= 26°$

6.4.4 $AG = FG$ (Raaklyne van gemeenskaplike punt)
$AO = FO$ (Radii)
$AOFG$ is 'n vlieër   (Aangrensende pare sye gelyk)

6.4.5 $\hat{G} + \hat{A} + \hat{O} + \hat{F} = 360°$ (Binnehoeke van vlieër of vierhoek)
$\hat{G} + 90° + 128° + 90° = 360°$
$\therefore \hat{G} = 52°$

VRAAG 7

7.1   40 km $= 40 \times 1\ 000$ m $= 40\ 000$ m
1 uur $= 60 \times 60$ sek $= 3\ 600$ sek
$\therefore \dfrac{40\ \text{km}}{1\ \text{uur}} = \dfrac{40\ 000\ \text{m}}{3\ 600\ \text{s}} \approx 11{,}11\ \text{m}\cdot\text{s}^{-1}$

7.2 $v = \pi D n$
$11{,}11\ \text{m}\cdot\text{s}^{-1} = \pi\,(0{,}44\ m)\,n$
$n = \dfrac{25{,}25}{\pi}\ \text{rad/s} \approx 8{,}04\ \text{rad/s}$

7.3 $v = \pi D n$
$11{,}11\ \text{m}\cdot\text{s}^{-1} = \pi\,(0{,}1\ m)\,n$
$n = \dfrac{111{,}1}{\pi}\ \text{rad/s} \approx 35{,}36\ \text{rad/s}$

$\omega = 2\pi n$
$= 2\pi(35{,}36\ \text{rad/s})$
$\approx 222{,}2\ \text{rad/s}$

7.4 $222{,}2 \times \dfrac{180°}{\pi} \approx 12\ 731{,}12°$ per sekonde

7.5   Oppervlakte $= \dfrac{rs}{2}$
$= \dfrac{22\ \text{cm} \times 8{,}3\ \text{cm}}{2}$
$= 91{,}30\ \text{cm}^2$

VRAAG 8

8.1   8.1.1 $(XY)^2 = (XO)^2 + (YO)^2 - 2(XO)(YO)\cos(X\hat{O}Y)$

$(21)^2 = (14)^2 + (14)^2 - 2(14)(14)\cos(X\hat{O}Y)$

$\cos(X\hat{O}Y) = \dfrac{21^2 - 14^2 - 14^2}{-2(14)(14)}$

verw. $\angle \approx 82{,}82°$

stomphoek: $X\hat{O}Y \approx 180° - 82{,}82° \approx 97{,}18°$

8.1.2   **OPSIE 1**

$4h^2 - 4dh + x^2 = 0$

$4h^2 - 4(28)h + (21)^2 = 0$

$4h^2 - 112h + 441 = 0$

$h = \dfrac{-(-112) \pm \sqrt{(-112)^2 - 4(4)(441)}}{2(4)}$

$= \dfrac{112 \pm \sqrt{5\,488}}{8}$

$\therefore h = 23{,}26\ (NVT)$ of $h = 4{,}74$ cm

OPSIE 2

Teken punt Z in middel van lyn XY

dan $(OY)^2 = (OZ)^2 + (ZY)^2$

$(14)^2 = (OZ)^2 + (10{,}5)^2$

$85{,}75 = (OZ)^2$

$9{,}26 = OZ$

$\therefore$ hoogte van segment $= 14$ cm $- 9{,}26 = 4{,}74$ cm

8.1.3   Die oppervlakte van kleinsegment XY

Oppervlakte van segment XY = oppervlakte sektor – oppervlakte $\triangle$

Oppervlakte van segment XY

$= \left[\dfrac{r^2\theta}{2} \right] - \left[\dfrac{1}{2} r \times r \times \sin\theta \right]$

$= \left[\dfrac{(14)^2 \left(97{,}18° \times \dfrac{\pi}{180°}\right)}{2} \right] - \left[\dfrac{1}{2} \times 14 \times 14 \times \sin(97{,}18°) \right]$

$= \left[\dfrac{196(0{,}539\dot{8}\,\pi)}{2} \right] - \left[98 \times \sin(97{,}18°) \right]$

$\approx 68{,}9873525$

$= 69$ cm^2

8.2 $AC^2 = AB^2 + BC^2$ (Pythagoras)

$\therefore (12 \ m)^2 = AB^2 + (6 \ m)^2$

$108 = AB^2$

$AB = 6\sqrt{3}$

$AD^2 = \left(6\sqrt{3} + 6\right)^2 + \left(6\sqrt{3} + 6\right)^2$

$AD \approx 23,18 \text{ m}$

$\therefore \text{Radius} \approx 23,18 \text{ m} + 6 \text{ m} \approx 29,18 \text{ m}$

VRAAG 9

$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$

$= 50\left(\dfrac{0 + 190}{2} + \dfrac{190 + 220}{2} + \dfrac{220 + 290}{2} + \dfrac{290 + 290}{2} + \dfrac{290 + 210}{2} + \dfrac{210 + 95}{2} + \dfrac{95 + 0}{2}\right)$

$= 50\left(95 + 205 + 255 + 290 + 250 + 152,5 + 47,5\right)$

$= 64 \ 750 \text{ m}^2$

OF

$A_T = a\left(\dfrac{0_1 + 0_n}{2} + 0_2 + 0_3 + 0_4 + \ldots + 0_{n-1}\right)$

$= 50\left(\dfrac{0 + 0}{2} + 190 + 220 + 290 + 290 + 210 + 95\right)$

$= 50(1295)$

$= 64 \ 750 \text{ m}^2$

Totaal: 150 punte