NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2022

TECHNICAL MATHEMATICS: PAPER I

MARKING GUIDELINES

Time: 3 hours | 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1 1.1.1

$$x(x-5)=5$$
$$x^2-5x=5$$
$$x^2-5x-5=0$$
$$x=\frac{-(-5)\pm\sqrt{(-5)^2-4(1)(-5)}}{2(1)}$$
$$=\frac{5\pm\sqrt{25+20}}{2}$$
$$=\frac{5\pm3\sqrt{5}}{2}$$

1.1.2

$$2x+6y=4 \text{ and } x^2+xy=4$$
$$2x+6y=4 \quad ① \qquad x^2+xy=4 \quad ②$$
From ① $2x=4-6y$
$$x=2-3y \quad ③$$
Subst. in ② $(2-3y)^2+(2-3y)y=4$
$$4-12y+9y^2+2y-3y^2-4=0$$
$$6y^2-10y=0$$
$$2y(3y-5)=0$$
$$y=0 \qquad y=\frac{5}{3}$$
Subst. in ③ $x=2 \quad x=2-5$
$$=-3$$

or

$$2x+6y=4 \quad ① \qquad x^2+xy=4 \quad ②$$
From ① $6y=4-2x$
$$3y=2-x$$
$$y=\frac{2-x}{3} \quad ③$$
Subst. in ② $x^2+x\left(\frac{2-x}{3}\right)=4$
$$x^2+\frac{2x-x^2}{3}=4$$
$$3x^2+2x-x^2=12$$
$$x^2+x-6=0$$
$$(x+3)(x-2)=0$$
$$x=-3 \text{ or } x=2$$
Subst. in ③ $y=0$
$$y=\frac{2-(-3)}{3}$$
$$=\frac{5}{3}$$

1.2 1.2.1 Expression is real if $\frac{-4}{2x+3} \geq 0$

i.e. $2x+3<0$

$x<-\frac{3}{2}$

1.2.2 $x^2-3x+9p-5=0$

$$\Delta=(-3)^2-4(1)(9p-5)$$
$$=9-36p+20$$
$$=36p+29$$

For real equal roots $36p+29=0$

$$p=-\frac{29}{36}$$

QUESTION 2

2.1 $\frac{5^{2x}+3}{5^{3x}+3.5^{x}}=\frac{5^{x}}{5^{x+2}}$

$$\frac{\cancel{(5^{2x}+3)}}{5^{x}\cancel{(5^{2x}+3)}}=5^{-2}$$

$$5^{-x}=5^{-2}$$
$$-x=-2$$
$$x=2$$

2.2 $(3\sqrt{5}-2\sqrt{2})^2$

$=9\times5-12\sqrt{5}\sqrt{2}+4x2$

$=45-12\sqrt{10}+8$

$=53-12\sqrt{10}$

2.3 $\sqrt{x-2}=x-4$

$(\sqrt{x-2})^2=(x-4)^2$

$$x-2=x^2-8x+16$$
$$0=x^2-9x+18$$
$$0=(x-3)(x-6)$$
$$x=3 \text{ or } x=6$$

Check solutions

$x=3$ is invalid

$\therefore x=6$

QUESTION 3

3.1 $\dfrac{2}{1-2i}$

$= \dfrac{2}{1-2i} \times \dfrac{1+2i}{1+2i}$

$= \dfrac{2+4i}{(1-2i)(1+2i)}$

$= \dfrac{2+4i}{1-4i^2}$

$= \dfrac{2+4i}{1+4}$

$= \dfrac{2+4i}{5}$

$= \dfrac{2}{5} + \dfrac{4}{5}i$

3.2 i^{2022}

$= \left(i^2\right)^{1011}$

$= (-1)^{1011}$

$= (-1)(-1)...(-1)$ 1011 times or $(-1)^{1010}(-1)^1$

$= -1$ $= (+1)(-1) = -1$

or

$i^{2022} = \left(i^4\right)^{505}\left(i^2\right)$

$= (1)(-1)$

$= -1$

3.3 3.3.1 $V = r\left(\cos 210^\circ + i\sin 210^\circ\right)$

3.3.2 $= r\left(-\cos 30^\circ - \sin 30^\circ\right)$

$= r\left(\dfrac{-\sqrt{3}}{2} - \dfrac{1}{2}i\right)$

$= \dfrac{-\sqrt{3}}{2}r - \dfrac{r}{2}i$

3.4 $\dfrac{1\times 2^5 + 1\times 2^4 + 1\times 2^2}{10^5}$

$= \dfrac{32+16+4}{100\ 000}$ or $\dfrac{32+16+4}{10^5}$

$= \dfrac{52}{100\ 000}$ $= \dfrac{52}{10^5}$

$5{,}2\times 10^{-4}$ $= 5{,}2\times 10^{-4}$

QUESTION 4

4.1 $A = P(1-0{,}06)^7$

$A = 12\,500\,(1-0{,}06)^7$

$\approx \text{R}8\,105{,}97$

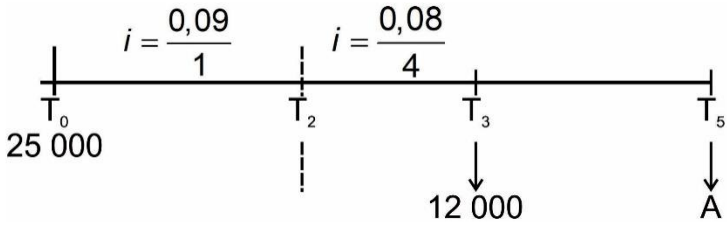
4.2 4.2.1 $1+i_{\text{eff}} = \left(1+\dfrac{i\text{ nom}}{t}\right)^t$

$i_{\text{eff}} = \left(1+\dfrac{0{,}08}{4}\right)^4 - 1$

$\approx 0{,}0824...$

eff rate $\approx 8{,}24\%$

4.2.2



OPTION 1

$$A = 25\,000(1+0,09)^2\left(1+\frac{0,08}{4}\right)^{12} - 12\,000\left(1+\frac{0,08}{4}\right)^{8}$$
$$\approx R23\,610,04$$

OPTION 2

$$A = \left[25\,000(1+0,09)^2\left(1+\frac{0,08}{4}\right)^{4\times1} - 12\,000\right]\left(1+\frac{0,08}{4}\right)^{4\times2}$$
$$\approx R23\,610,64$$

OPTION 3

$$A \text{ (at end of 3 years)} = 25\,000(1+0,09)^2\left(1+\frac{0,08}{4}\right)^{4}$$
$$\equiv R32\,150,94\ldots$$
$$A \text{ (after withdrawal)} \approx 32\,150,94\ldots - 12\,000$$
$$\approx R20\,150,94\ldots$$
$$A \text{ (at end of 5 years)} \approx 20\,150,94\ldots \times \left(1+\frac{0,08}{4}\right)^{4\times2}$$
$$\approx R23\,610,04$$

OPTION 4

$$A \text{ (at end of 5 years)} = 25\,000(1+0,09)^2\left(1+\frac{0,08}{4}\right)^{4\times3}$$
$$\equiv R37\,669,95\ldots$$
$$A \text{ (at end of 2 years)} \approx 12\,000\left(1+\frac{0,08}{4}\right)^{4\times2}$$
$$\approx R14\,059,91$$
$$\text{Value of investment} \approx 37\,669,95 - 14\,059,91$$
$$\approx R23\,610,04$$

4.3 $A = P(1+i)^n$

$$3P = P\left(1+\frac{0,0825}{12}\right)^n$$

$$3 = \frac{\cancel{P}\left(1+\frac{0,0825}{12}\right)^n}{\cancel{P}}$$

$$3 = \left(1+\frac{0,0825}{12}\right)^n$$

$$\log_{\left(1+\frac{0,0825}{12}\right)} 3 = n$$

i.e. 161 months

QUESTION 5

5.1 $y = 0$

5.2 $(0\,;-1)$

or

$y = -b^0$

$= -1$

i.e. $(0\,;-1)$

5.3 Subst. $(3\,;-8)$ in f :

$-8 = -b^3$

$b^3 = 8$

$b = 2$

QUESTION 6

6.1 6.1.1 A is $(0;4)$

B is $(0;2)$

6.1.2 At C & D

$$\frac{-x^2}{2}+x+4=0$$
$$x^2-2x-8=0 \qquad C \text{ is } (-2;0)$$
$$(x-4)(x+2)=0 \qquad D \text{ is } (4;0)$$
$$x=4 \text{ or } x=-2$$

6.1.3 A/s at $x=1$ (by symmetry) or $x=\frac{-b}{2a}$

$$y_E=\frac{-1^2}{2}+1+4$$
$$=4\frac{1}{2} E \text{ is } \left(1;4\frac{1}{2}\right)$$

or

$$f^{/}(x)=0$$
$$f^{/}(x)=\frac{-2x}{2}+1$$
$$=-x+1$$
$$-x+1=0$$
$$-x=-1$$
$$x=1$$
$$y=\frac{-x^2}{2}+x+4$$
$$=\frac{-1}{2}+1+4$$
$$=4\frac{1}{2}$$

E is $\left(1;4\frac{1}{2}\right)$

6.2 Range $y\in\left(-\infty;4\frac{1}{2}\right]$ or $y\le 4\frac{1}{2}$

6.3 Domain $x\in(-\infty;\infty)$

or

$x\in\mathbb{R}$

6.4 $f^{/}(x) = -x + 1$

$m_{\text{tangent}} = f^{/}(0) = 1$

$y - y_1 = m(x - x_1)$

$y - 4 = 1(x - 0)$

$y - 4 = x$

$y = x + 4$

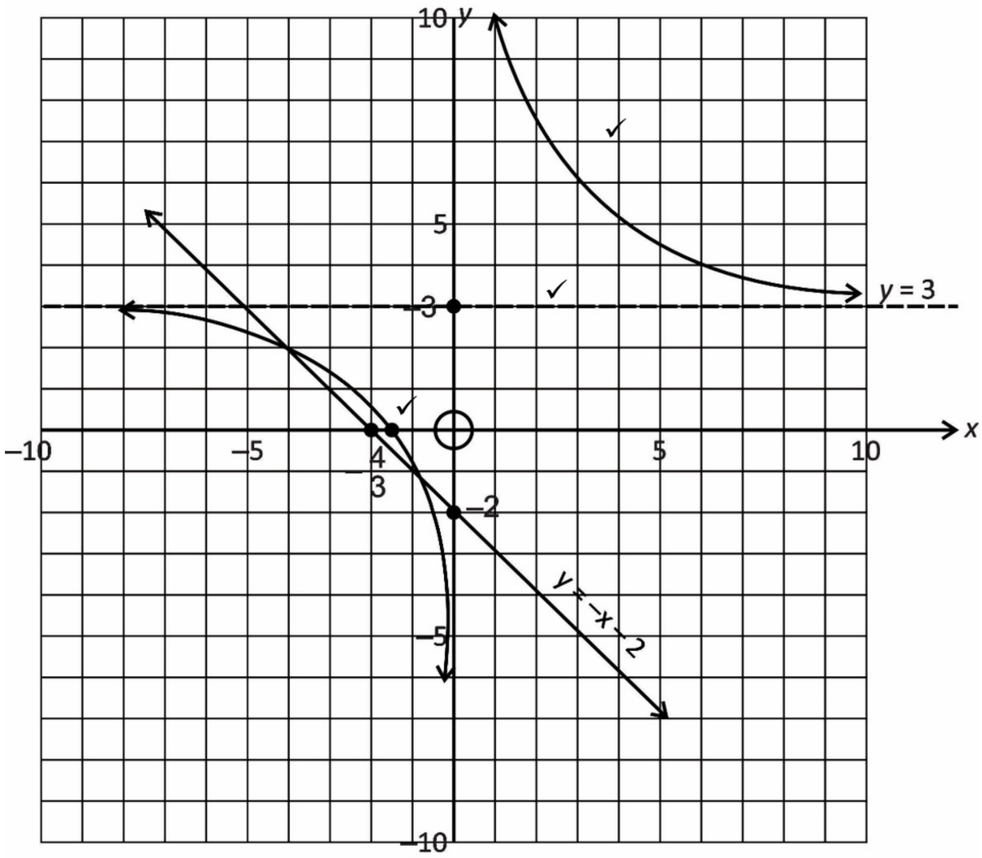
6.5 6.5.1 Turning points at $x = 4$ and $x = -2$

6.5.2 Gradient at $x = 0$ is 4

QUESTION 7

7.1 7.1.1 Asymptote $y = 3$

7.1.2



Put $y = 0$

$$0 = \frac{4}{x} + 3$$
$$-3 = \frac{4}{x}$$
$$x = -\frac{4}{3}$$

7.2

$$\frac{4}{x} + 3 = -x + k$$
$$4 + 3x = -x^2 + kx$$
$$x^2 + (3 - k)x + 4 = 0$$
$$3 - k = 5$$
$$-2 = k$$

i.e. $y = -x - 2$ (see graph)

7.3

$$(x + 4)(x + 1) = 0$$

$x = -4$ or $x = -1$

$y = 2$ or $y = -1$ (substitute $y = -x - 2$)

$(-4;2)$ $(-1;-1)$

QUESTION 8

8.1 $f'(x) \lim_{h \to 0}\left(\dfrac{f(x+h)-f(x)}{h}\right)$

$= \lim_{h \to 0}\left(\dfrac{\dfrac{3(x+h)}{2}+7-\left(\dfrac{3x}{2}+7\right)}{h}\right)$

$= \lim_{h \to 0}\left(\dfrac{\dfrac{3x}{2}+\dfrac{3h}{2}+7-\dfrac{3x}{2}-7}{h}\right)$

$= \lim_{h \to 0}\left(\dfrac{3\cancel{h}}{2}\times\dfrac{1}{\cancel{h}}\right)=\dfrac{3}{2}$

8.2 $f(x)=8.\left(x^2\right)^{\frac{1}{3}}+\dfrac{5}{2}x^{-3}$

$=8x^{\frac{2}{3}}+\dfrac{5}{2}x^{-3}$

$f'(x)=\dfrac{16}{3}x^{-\frac{1}{3}}-\dfrac{15}{2}x^{-4}$

or $\dfrac{16}{3\sqrt[3]{x}}-\dfrac{15}{2x^4}$

8.3 8.3.1 $2r+H=3$

$H=3-2r$

8.3.2 (a) $V=\dfrac{1}{3}\pi r^2(3-2r)$

$V(r)=\pi r^2-\dfrac{2}{3}\pi r^3$

(b) $V'(r)=2\pi r-2\pi r^2$

At max $2\pi r-2\pi r^2=0$

$2\pi r(1-r)=0$

$r=0$ OR $r=1$ m

N/A

$\therefore$ Max vol $=\pi(1)^2-\dfrac{2}{3}(\pi)(1)^3$

$=\pi-\dfrac{2}{3}\pi$

$=\dfrac{\pi}{3}$ m^3 or $V\approx 1{,}05$ m^3

or

$$V=\frac{1}{3}\pi r^{2}\left(3-2r\right)$$
$$=\frac{1}{3}\pi\left(1\right)^{2}\left(3-2\left(1\right)\right)$$
$$=\frac{\pi}{3}m^{3}$$

or

$$\approx\frac{3{,}14...}{3}$$
$$\approx 1{,}04\ \text{m}^{3}$$

QUESTION 9

9.1 9.1.1 Subst. $(3;0): 0 = 3(3-k)^2$

$k = 3$ OR $y = x(x-3)^2$ by inspection

9.1.2 $f(x) = x(x-3)^2$

$= x^3 - 6x^2 + 9x$

$f'(x) = 3x^2 - 12x + 9$

At st pts $3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-3)(x-1) = 0$

$x_Q = 3 \quad x_P = 1$

Subst. $x = 1: y = 1(1-3)^2$

$= 4$

P is $(1;4)$

9.1.3 At $R: y_R = y_P = 4$

$4 = x^3 - 6x^2 + 9x$

$0 = x^3 - 6x^2 + 9x - 4$

$0 = (x-1)(x-1)(x-4)$

R is $(4;4)$

9.1.4 Base of $\Delta = PR$

$= x_R - x_P$

$= 4 - 1 = 3$

Height $= y_R = y_P = 4$

Area $= \frac{1}{2}(PR).y_P$

$= \frac{1}{2} \times 3 \times 4$

$= 6 \text{ units}^2$

9.2 9.2.1 $= 0x + c$

$= c$

9.2.2 $= \frac{2x^3}{3} + \frac{3x^2}{2} + c$

$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + c$

9.3 Area $\int_0^4 \left(-x^2 + 4x\right) dx$

$$= \left[\frac{-x^3}{3} + \frac{{}^{2}\cancel{4}x^2}{\cancel{2}} \right]_0^4$$

$$= \left(\frac{-(4)^3}{3} + 2(4)^2 \right) - (0)$$

$$= -\frac{64}{3} + 32$$

$$= \frac{32}{3} \text{ units}^2$$

Total: 150 marks