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TOTAL MARKS

NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2022

TECHNICAL MATHEMATICS: PAPER I

EXAMINATION NUMBER

Time: 3 hours 150 marks

PLEASE READ THE FOLLOWING INSTRUCTIONS CAREFULLY

1. This question paper consists of 28 pages and an Information Sheet of 2 pages (i–ii). Please check that your question paper is complete.
2. Read the questions carefully.
3. **Answer ALL the questions on the question paper and hand this in at the end of the examination. Remember to write your examination number in the space provided.**
4. Diagrams are not necessarily drawn to scale.
5. You may use an approved non-programmable and non-graphical calculator, unless otherwise stated.
6. Round off your answers to one decimal digit where necessary, unless otherwise stated.
7. All the necessary working details must be clearly shown.
8. It is in your own interest to write legibly and to present your work neatly.
9. FOUR blank pages (page 25–28) are included at the end of the paper. If you run out of space for a question, use these pages. Clearly indicate the question number of your answer should you use this extra space.

FOR OFFICE USE ONLY: MARKER TO ENTER MARKS

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	TOTAL
19	14	15	18	4	19	17	18	26	150

QUESTION 1

1.1 1.1.1 Solve for x, leaving answers in simplified surd form:

$x(x - 5) = 5$

(4)

1.1.2 Solve the following simultaneous equations for x and y:

$2x + 6y = 4$ and $x^2 + xy = 4$

(7)

1.2 1.2.1 For which real value(s) of x will $\sqrt{\frac{-4}{2x+3}}$ be real?

(3)

1.2.2 Determine the numerical value(s) of p so that the equation $x^2 - 3x + 9p = 5$ will have equal real roots.

(5)
[19]

QUESTION 2

2.1 If $\frac{5^{2x}+3}{5^{3x}+3.5^{x}}=\frac{5^{x}}{5^{x+2}}$, determine the value of x.

(4)

2.2 Prove, without the use of a calculator, that $\left(3\sqrt{5}-2\sqrt{2}\right)$ is a square root of the number $\left(53-12\sqrt{10}\right)$.

(4)

2.3 Solve for x: $\sqrt{x-2}+4=x$

(6)
[14]

QUESTION 3

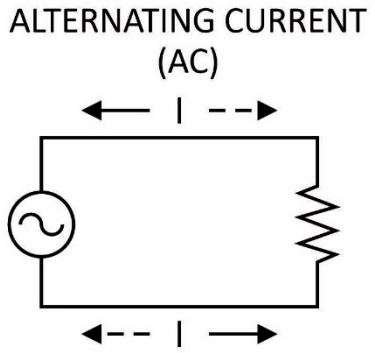
3.1 Write $\frac{2}{1-2i}$ in the form $a + bi$, without using a calculator. Show all workings.

(5)

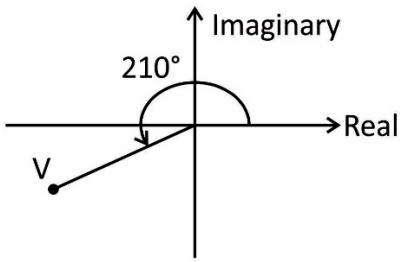
3.2 Evaluate without using a calculator, i^{2022}. Show all workings.

(2)

3.3 The image below shows an alternating current circuit.



Study the following argand diagram which represents the voltage (V) of the alternating current circuit.



3.3.1 Use the diagram to write V in the form $V = r(\cos\theta + i\sin\theta)$.

(1)

3.3.2 Hence, write V in rectangular form.

(3)

3.4 Express $\frac{110100_2}{10^5}$ in scientific notation. Show all working.

(4)
[15]

QUESTION 4

4.1 The laptop below was purchased at the beginning of 2015 for R12 500 and depreciated at an annual rate of 6% according to the reducing-balance method. Calculate the depreciated value of the laptop at the beginning of 2022.

Cost Price: R12 500

(3)

4.2 Piet won R25 000 in the Powerball draw, and invested his winnings in an account paying 9% interest per annum compounded annually. At the end of two years, the interest rate changed to 8% per annum, compounded quarterly.

4.2.1 Calculate the effective annual interest rate equivalent to the 8% interest rate compounded quarterly.

(3)

4.2.2 He made a withdrawal of R12 000 at the end of the third year to cover repairs to his car. Calculate the amount that remains in the account at the end of a 5-year investment period.

(7)

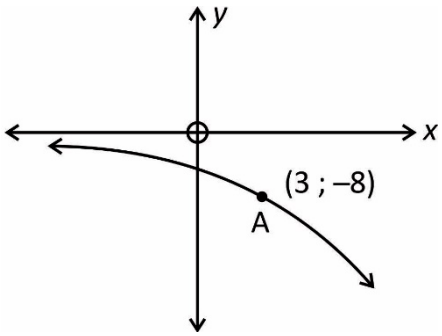
4.3 Calculate how long it would take an investment to treble in value at an interest rate of 8,25% per annum compounded monthly. Give your answer correct to the nearest month.

(5)
[18]

QUESTION 5

Given below is the graph of f defined by $f(x) = -b^x$

$A(3;-8)$ is a point on the curve.



Determine:

5.1 The equation of the horizontal asymptote

(1)

5.2 The coordinate of the *y*-intercept

(1)

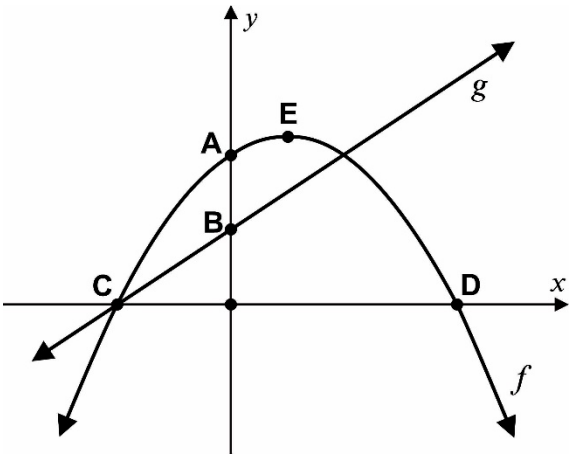
5.3 The numerical value of *b*

(2)
[4]

QUESTION 6

Given below are the graphs of g and f defined by the equations $g(x) = x + 2$ and $f(x) = -\frac{x^2}{2} + x + 4$

f and g intersect at point C on the x-axis.



6.1 Determine the coordinates of:

6.1.1 A and B, the y-intercepts of the graphs

(2)

6.1.2 C and D, the x-intercepts of the graphs

(4)

6.1.3 E, the turning point of f

(3)

6.2 Write down the range of f.

(2)

6.3 Write down the domain of f.

(1)

6.4 Determine the tangent to f at point A.

(4)

6.5 It is further given that $f(x) = h'(x)$, where $h(x)$ defines a cubic function.

Use the graph to write down:

6.5.1 The x-coordinates of the turning points of h

(2)

6.5.2 The gradient of the tangent to h at $x = 0$

(1)
[19]

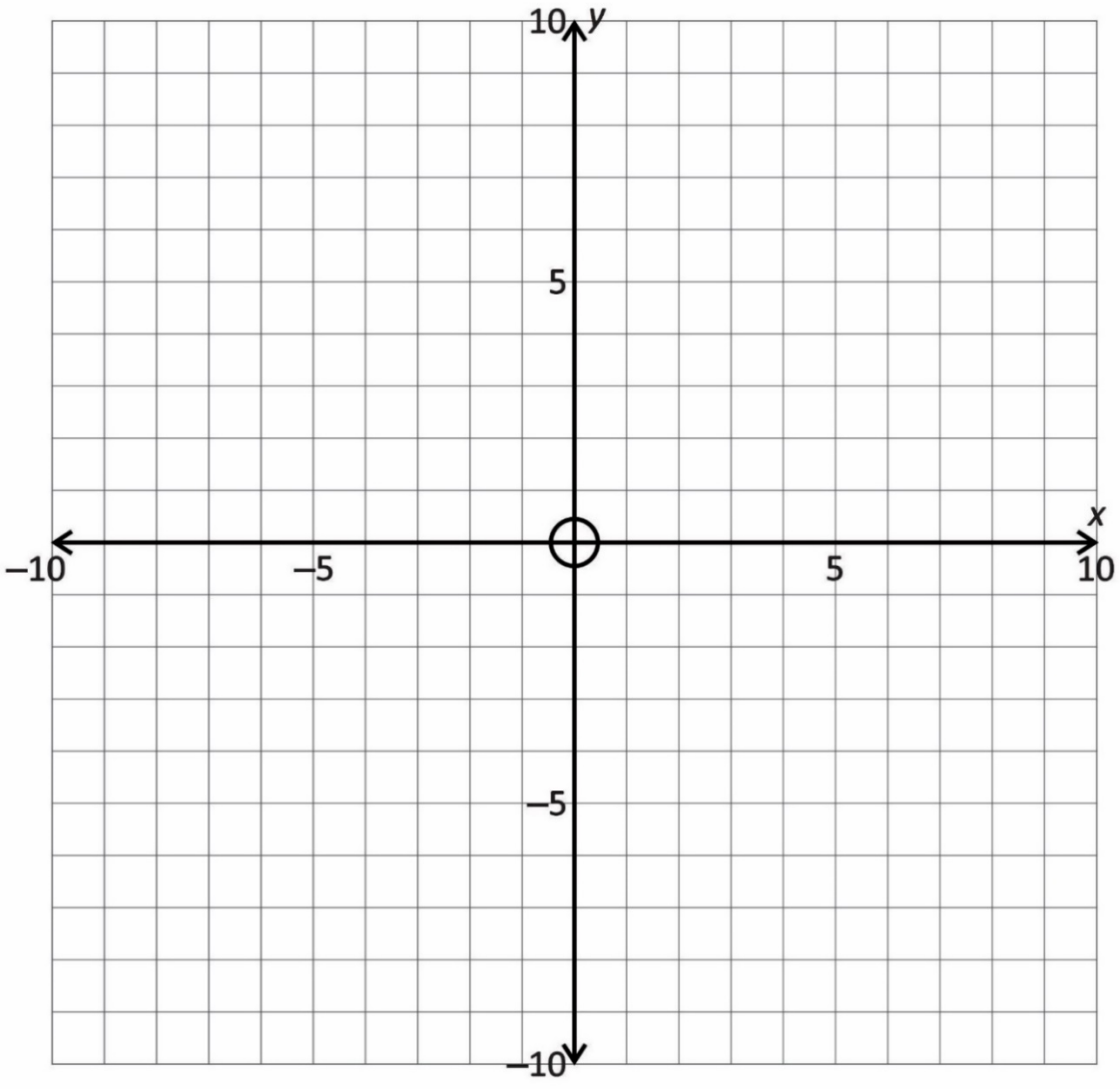
QUESTION 7

7.1 The graph of function f is defined by $f(x) = \frac{4}{x} + 3$

7.1.1 Determine the equation of the horizontal asymptote.

(1)

7.1.2 Sketch the graph of f on the system of axes below. Clearly indicate all asymptotes and intercepts with the axes.



(5)

7.2 On the same system of axes, draw a straight line of the form $g(x) = -x + k$ which would assist you to solve the equation $x^2 + 5x + 4 = 0$ graphically.

HINT: Use $f(x) = g(x)$

(7)

7.3 Hence, determine the points of intersection of *f* and *g*.

(4)
[17]

QUESTION 8

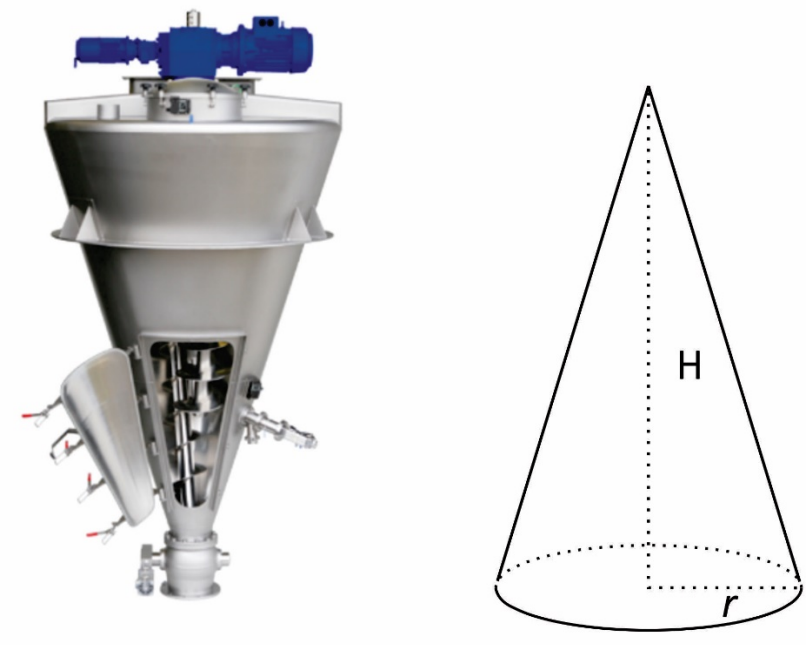
8.1 If $f(x)=\frac{3x}{2}+7$, determine $f'(x)$ from first principles.

(5)

8.2 Given $f(x)=8\sqrt[3]{x^2}+\frac{5}{2x^3}$, determine $f'(x)$.

(4)

8.3 A company uses a right cone-shaped double screw mixer for their chemical mixture reaction.



The sum of the base diameter and the height of the cone-shaped double screw mixer is equal to 3 m.

(Useful formula: Volume of cone $= \frac{1}{3}\pi r^2 \text{H}$)

8.3.1 Write H in terms of *r*.

(2)

8.3.2 (a) Show that the volume of the cone can be expressed as $V(r) = \pi r^2 - \frac{2}{3}\pi r^3$

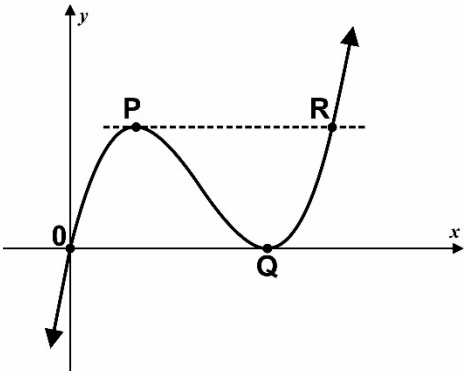
(1)

(b) Hence, determine the maximum possible volume of the cone.

(6)
[18]

QUESTION 9

9.1 The graph of f is defined by $f(x) = x(x - k)^2$



f has a local maximum at P and touches the x-axis at Q (3 ; 0), PR is a tangent at P and intersects f at R.

Determine:

9.1.1 The numerical value of k

(2)

9.1.2 Hence, the coordinates of P

(6)

9.1.3 Hence, the coordinates of R

(4)

9.1.4 The area of ΔPQR

(4)

9.2 Simplify:

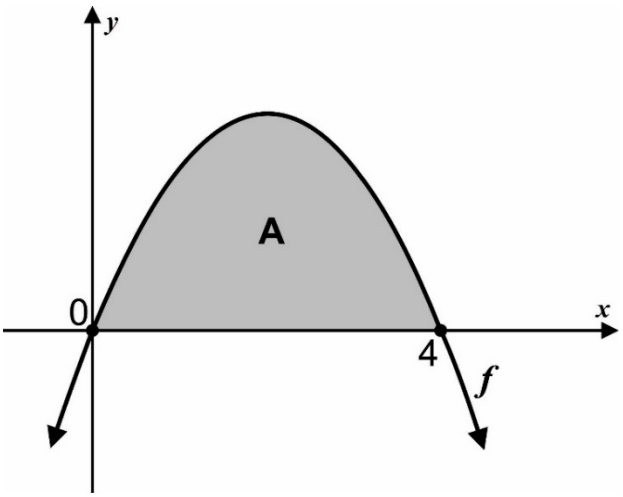
9.2.1 $\int 0dx$

(1)

9.2.2 $\int \left(2x^2 + 3x\right)dx$

(3)

9.3 Determine the shaded area (A) between the curve $y = -x^2 + 4x$ and the x-axis with $(0\,;0)$ and $(4\,;0)$ the x-intercepts as shown in the diagram below. Show all calculations.



(6)
[26]

Total: 150 marks

ADDITIONAL SPACE (ALL questions)

CLEARLY INDICATE AT THE QUESTION THAT YOU USED THE ADDITIONAL SPACE TO ENSURE THAT ALL ANSWERS ARE MARKED.

