NATIONAL SENIOR CERTIFICATE EXAMINATION
NOVEMBER 2022

TECHNICAL MATHEMATICS: PAPER II

MARKING GUIDELINES

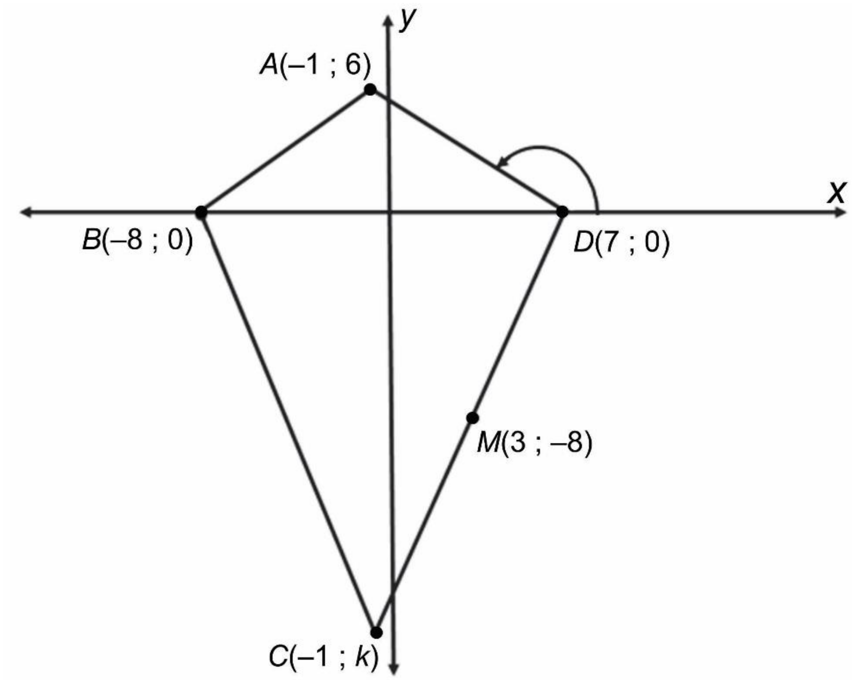
Time: 3 hours 150 marks

These marking guidelines are prepared for use by examiners and sub-examiners, all of whom are required to attend a standardisation meeting to ensure that the guidelines are consistently interpreted and applied in the marking of candidates' scripts.

The IEB will not enter into any discussions or correspondence about any marking guidelines. It is acknowledged that there may be different views about some matters of emphasis or detail in the guidelines. It is also recognised that, without the benefit of attendance at a standardisation meeting, there may be different interpretations of the application of the marking guidelines.

QUESTION 1

1.1



$$m_{AD} = \frac{y_D - y_A}{x_D - x_A} = \frac{0-6}{7-(-1)} = \frac{-6}{8} = -\frac{3}{4}$$

1.2 $\tan\theta = m_{AD}$

$$\tan\theta = -\frac{3}{4}$$
$$\therefore \theta \approx 180° - 36{,}87°$$
$$\therefore \theta \approx 143{,}13°$$

1.3

$$y = mx + c$$
$$0 = \left(-\frac{3}{4}\right)(7) + c$$
$$c = \frac{21}{4}$$
$$\therefore y = -\frac{3}{4}x + \frac{21}{4}$$

OR

$$y - y_1 = m(x - x_1)$$
$$y - 0 = \left(-\frac{3}{4}\right)(x - 7)$$
$$y = -\frac{3}{4}x + \frac{21}{4}$$

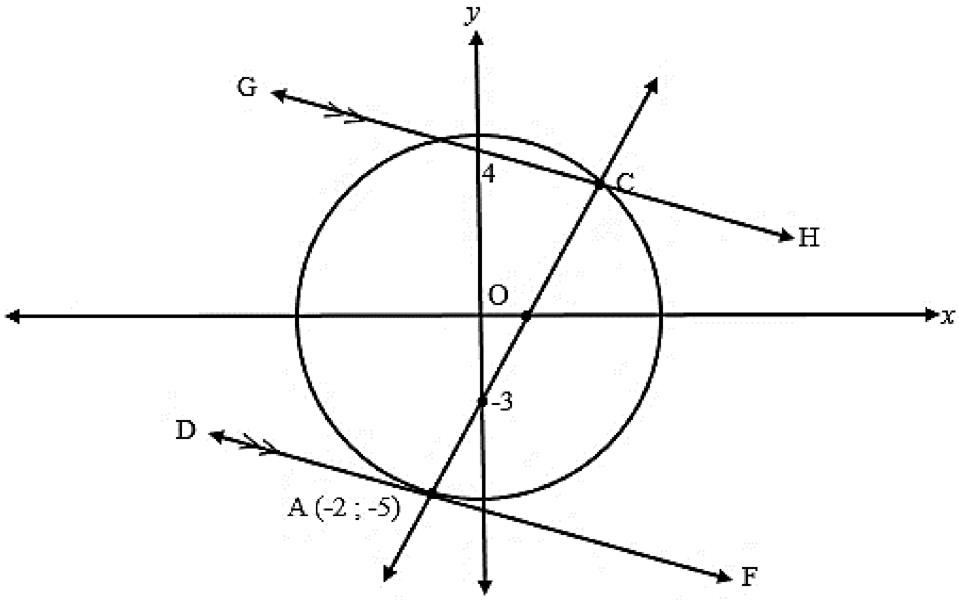
1.4 $M(3;-8) = \left(\frac{x_D + x_C}{2} ; \frac{y_D + y_C}{2}\right)$

$$-8 = \frac{0+k}{2}$$
$$k = -16$$

1.5 AC = 16 + 6 = 22 units

QUESTION 2

2.1



2.1.1

$$r^2 = x^2 + y^2$$
$$= (-2)^2 + (-5)^2$$
$$r^2 = 29$$
$$\therefore x^2 + y^2 = 29$$

2.1.2 Gradient of radius = $\frac{5}{2}$

Gradient of tangent = $\frac{-2}{5}$

$$\therefore y = mx + c$$
$$\therefore -5 = \left(\frac{-2}{5}\right)(-2) + c$$
$$\therefore c = \frac{-29}{5}$$
$$\therefore y = \frac{-2}{5}x - \frac{29}{5}$$

OR

$$y - y_1 = m(x - x_1)$$
$$y - (-5) = \left(-\frac{2}{5}\right)(x - (-2))$$
$$y + 5 = -\frac{2}{5}x - \frac{4}{5}$$
$$y = -\frac{2}{5}x - \frac{29}{5}$$

2.1.3 Line AC: $\therefore y = x - 3$

Line GH: $y = -\frac{2}{5}x + 4$

Point C: $x - 3 = -\frac{2}{5}x + 4$

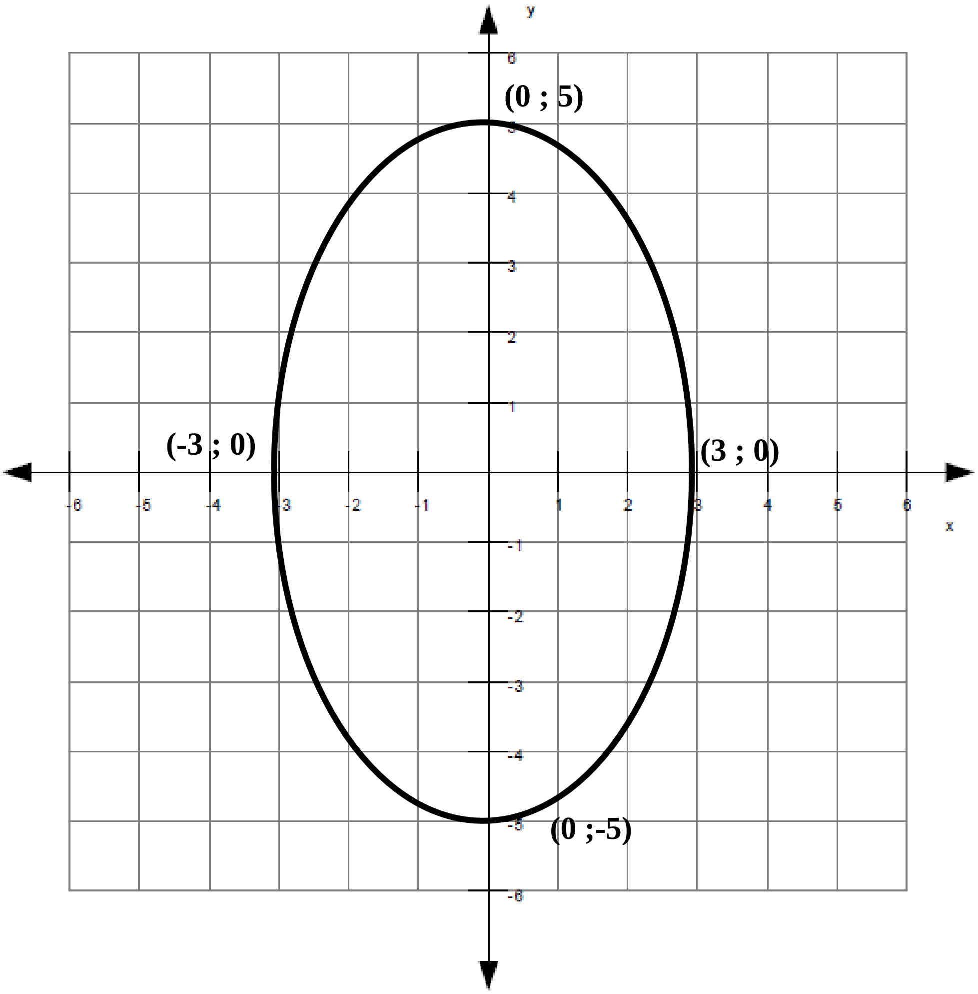
$$\frac{7}{5}x = 7$$
$$x = 5$$
$$y = 2$$

C(5 ; 2)

2.2 $$\frac{2x^2}{9}+\frac{2y^2}{25}-2=0$$

$$\frac{x^2}{9}+\frac{y^2}{25}-1=0$$

$$\frac{x^2}{9}+\frac{y^2}{25}=1$$



shape, *x*-intercepts, *y*-intercepts

QUESTION 3

3.1 3.1.1 $6^2 = p^2 + 4^2$

$36 = p^2 + 16$

$20 = p^2$

$p = -2\sqrt{5}$

3.1.2 $\tan\theta = \dfrac{4}{-2\sqrt{5}}$ **OR** $\sin\theta = \dfrac{4}{6}$ **OR** $\cos\theta = \dfrac{2\sqrt{5}}{6}$

$\theta \approx 180° - 41{,}81°$ $\theta \approx 180° - 41{,}81°$ $\theta \approx 180° - 41{,}81°$

$\therefore \theta \approx 138{,}19°$ $\theta \approx 138{,}19°$ $\theta \approx 138{,}19°$

3.1.3 $= \left(\dfrac{2\sqrt{5}}{6}\right)^2 - \left(\dfrac{4}{6}\right)^2$

$= \dfrac{20}{36} - \dfrac{16}{36}$

$= \dfrac{4}{36} = \dfrac{1}{9}$

3.2 $\tan\beta + 4\sqrt{3} = 0$

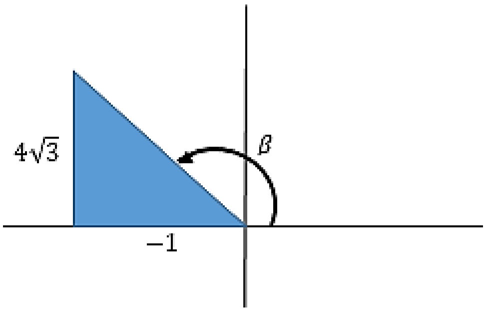
$\tan\beta = -4\sqrt{3}$

$r^2 = x^2 + y^2$

$= (-1)^2 + (4\sqrt{3})^2$

$= 49$

$r = 7$



$$\frac{49(\cos\beta - \sin^2\beta)}{\sec 120° \cdot \tan 225°}$$

$$= \frac{49\left(\dfrac{-1}{7} - \left(\dfrac{4\sqrt{3}}{7}\right)^2\right)}{\sec(180° - 60°) \cdot \tan(180° + 45°)}$$

$$= \frac{49\left(\dfrac{-1}{7} - \dfrac{48}{49}\right)}{-\sec 60° \cdot \tan 45°}$$

$$= \frac{-7 - 48}{(-2)\,(1)}$$

$$= \frac{55}{2}$$

3.3 $\therefore \text{cosec}\,\theta = \sqrt{2}$

$\therefore \sin\theta = \dfrac{1}{\sqrt{2}}$

ref angle $= 45°$

$\theta = 45°$ or $\theta = 180° - 45° = 135°$

3.4 $LHS: \dfrac{\sin^2\theta - 1}{\tan\theta \cdot \sin\theta - \tan\theta}$

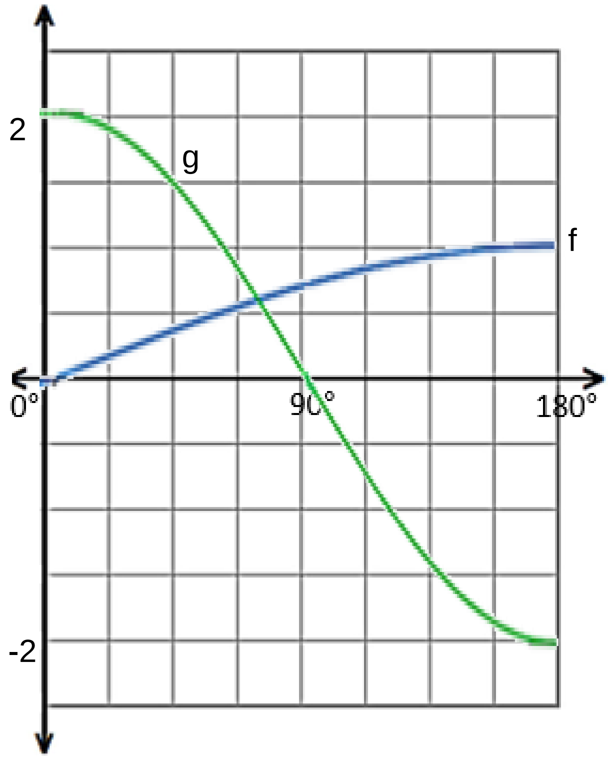
$= \dfrac{(\sin\theta - 1)(\sin\theta + 1)}{\tan\theta(\sin\theta - 1)}$

$= \dfrac{\sin\theta + 1}{\tan\theta}$

$\therefore LHS = RHS$

QUESTION 4

4.1



function f

intercepts

turning points

shape

function g

intercepts

turning points

shape

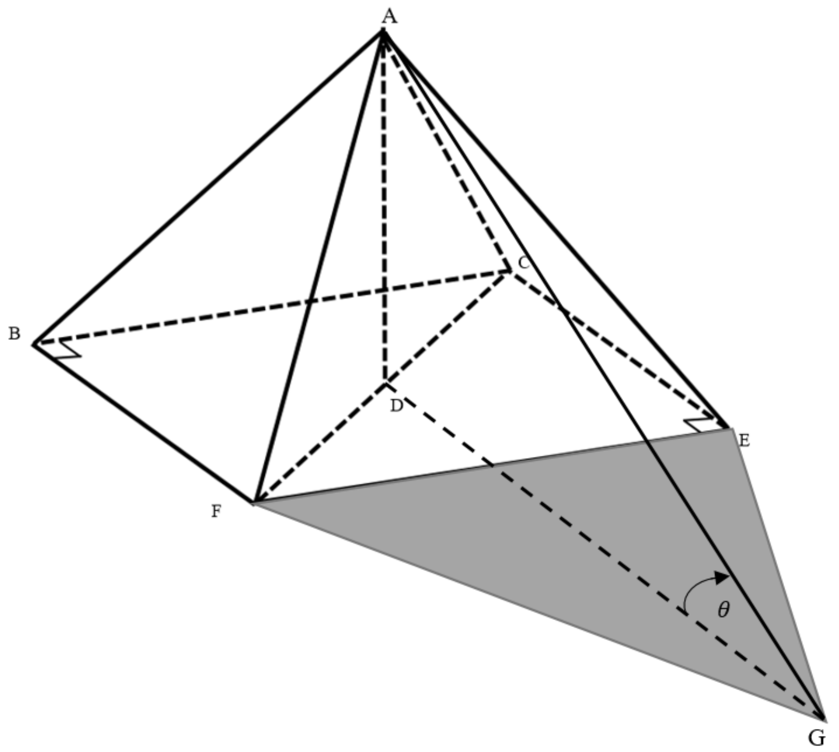
4.2 4.2.1 $\sin\dfrac{x}{2} \geq 2\cos x$

$x \in [73° \,;\, 180°]$ OR $73° \leq x \leq 180°$

4.2.2 $x = 0°$

4.2.3 $x \in (60° \,;\, 180°]$ OR $60° < x \leq 180°$

QUESTION 5



5.1 In Δ AEF: $2 \times \hat{FEA} + 51{,}8° = 180°$

$\therefore \hat{FEA} = 64{,}1°$

$$\frac{AF}{\sin E} = \frac{FE}{\sin A}$$

$$\frac{AF}{\sin(64{,}1°)} = \frac{2\,m}{\sin(51{,}8°)}$$

$$AF = 2{,}29 \text{ m}$$

5.2 Volume $= \frac{1}{3}(\text{base area} \times \perp \text{height})$

$= \frac{1}{3}(2 \text{ m} \times 2 \text{ m} \times 1{,}8 \text{ m})$

$= 2{,}4 \text{ m}^3$

5.3 Area$\Delta EFG = \frac{1}{2} FG \times EG \times \sin G$

$= \frac{1}{2}(2{,}3) \times (2{,}3) \times \sin 51{,}5°$

$\approx 2{,}07 \text{ m}^2$

5.4 Height of ΔEFG

$$\Delta EFG = \frac{1}{2} EF \times h$$

Area $2{,}07 = \frac{1}{2}(2) \times h$

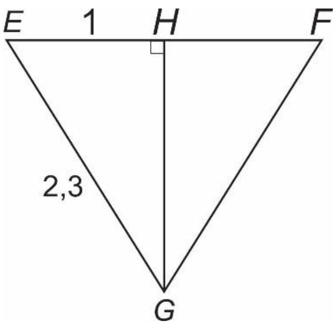
$2{,}07 \approx h$

Length of $DG \approx 2{,}07 + 1 \approx 3{,}07$ m

Angle of elevation

$$\tan\theta = \frac{1{,}8 \text{ m}}{3{,}07 \text{ m}}$$
$$\therefore \theta \approx 30{,}38^\circ$$

OR



$\perp h : HG$ by Pythagoras

$$EG^2 = EH^2 + HG^2$$
$$(2{,}3)^2 = (1)^2 + HG^2$$
$$(2{,}3)^2 - (1)^2 = HG^2$$
$$HG = \sqrt{4{,}29}$$
$$2{,}07 \approx h \text{ m}$$

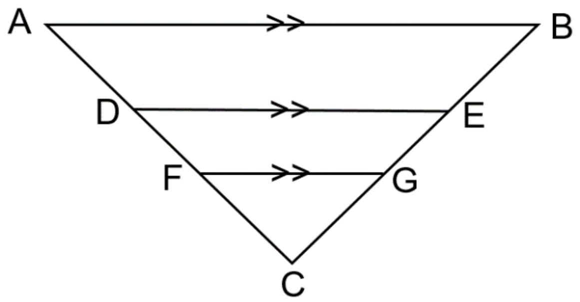
Length of $DG \approx 2{,}07 + 1 = 3{,}07$ m

Angle of elevation

$$\tan\theta = \frac{1{,}8 \text{ m}}{3{,}07 \text{ m}}$$
$$\therefore \theta \approx 30{,}38^\circ$$

QUESTION 6

6.1



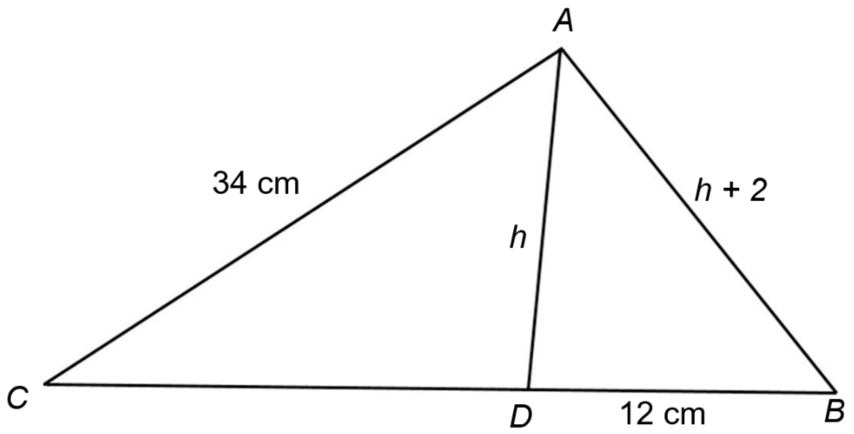
6.1.1

$$\frac{CG}{EG}=\frac{4}{3}$$
$$\frac{4\text{ m}}{EG}=\frac{4}{3}$$
$$\therefore EG=3\text{ m}$$

6.1.2

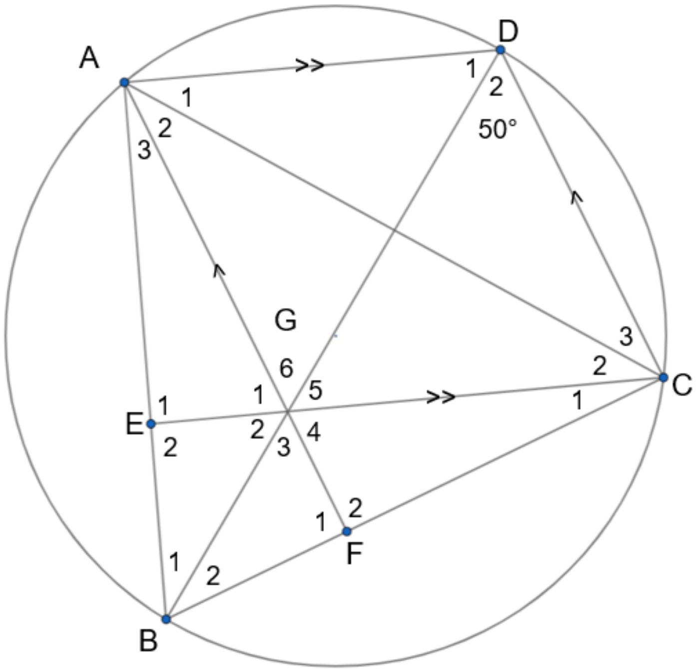
$$\frac{AB}{FG}=\frac{CB}{CG}$$
$$\frac{AB}{6\text{ m}}=\frac{11}{4}$$
$$\therefore AB=16{,}5\text{ m}$$

6.2



$$\frac{AC}{DA}=\frac{AB}{DB}$$
$$\frac{34}{h}=\frac{h+2}{12}$$
$$408=h^2+2h$$
$$0=h^2+2h-408$$
$$h=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$h=\frac{-2\pm\sqrt{2^2-4(1)(-408)}}{2(1)}$$
$$h\approx 19{,}22 \quad\text{or}\quad h\approx -21{,}22\ \text{(N/A)}$$
$$\therefore AD\approx 19{,}22\ \text{cm}$$

6.3



6.3.1 (a) $\hat{E}_2 = 90^\circ$ (given)

$\therefore D\hat{A}B = 90^\circ$ (corresp. angles OR Co-int angles; $AD//EC$)

(b) $\hat{D}_2 = 50^\circ$ (given)

$\therefore \hat{A}_2 + \hat{A}_3 = \hat{D}_2 = 50^\circ$ (angles in same segment)

and $\hat{A}_2 + \hat{A}_3 + \hat{A}_1 = 90^\circ$ (calculated)

$\therefore \hat{A}_1 = 40^\circ$

(c) $\hat{G}_6 = \hat{D}_2 = 50^\circ$

(Alt. int. angles, $DC//AG$)

6.3.2 $\hat{G}_6 = \hat{G}_3 = 50^\circ$ (Vert. opp angles)

and $\hat{G}_3 + \hat{B}_2 + \hat{F}_1 = 180^\circ$ (Int. angles of triangle)

$\hat{B}_2 = \hat{A}_1 = 40^\circ$ (Angles in same segment)

$\therefore \hat{F}_1 = \hat{F}_2 = 90^\circ$ (Angles on a straight line)

$\therefore \hat{E}_1 = \hat{F}_2$

$\therefore$ *ACFE* is a cyclic quad (Line subtends equal anles OR converse angles in the same segment))

OR

$\hat{G}_6 = \hat{G}_3 = 50^\circ$ (Vert. opp angles)

$\hat{B}_1 = \hat{A}_1 = 40^\circ$ (Angles in same segment)

$\therefore \hat{F}_1 = 90^\circ$ (Sum of angles of triangle)

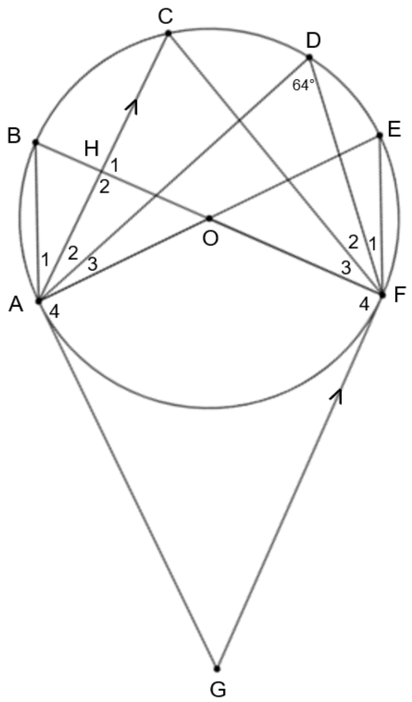
$\therefore \hat{F}_1 + \hat{E}_2 = 90^\circ + 90^\circ = 180^\circ$

$\therefore$ ACFE is a cyclic quad (Converse of opp angles of cyclic quad)

OR

$\hat{G}_6 = \hat{G}_6 = 50°$	(Vert. opp angles)
$\hat{B}_1 = \hat{A}_1 = 40°$	(Angles in same segment)
$\therefore \hat{F}_2 = 40° + 50° = 90°$	(Ext angle of triangle)
$\therefore \hat{F}_2 = \hat{E}_2 = 90°$	
$\therefore$ ACFE is a cyclic quad	(Converse of ext angle of cyclic quad)

6.4



6.4.1 $A\hat{O}F = 2\hat{D}$ (Angle at centre = 2 × angle on circumference)

$\therefore A\hat{O}F = 128°$

6.4.2 $\hat{F}_4 = 90°$ (Radius perpendicular to tangent)

6.4.3

$\hat{D} = \hat{C} = 64°$	(Angles in same segment)
$\hat{H}_1 = \hat{F}_4 = 90°$	(Alternate angles; *CA*//*FG*)
$\hat{H}_1 + \hat{F}_3 + \hat{C} = 180°$	(Int angles of triangle)
$90° + \hat{F}_3 + 64° = 180°$	
$\therefore \hat{F}_3 = 26°$	

OR

$\hat{D} = \hat{C} = 64°$	(Angles in same segment)
$\hat{F}_4 + \hat{F}_3 = 180° - 64°$	(Co. int. angles, *AC*//*G)*
$= 116°$	
$\hat{F}_3 = 116° - 90°$	
$= 26°$	

6.4.4 $AG = FG$ (Tangents from common point)
$AO = FO$ (Radii)
$AOFG$ is a kite (Adj. pairs of sides equal)

6.4.5 $\hat{G}+\hat{A}+\hat{O}+\hat{F}=360^\circ$ (Int angles of kite or quad)
$\hat{G}+90^\circ+128^\circ+90^\circ=360^\circ$
$\therefore \hat{G}=52^\circ$

QUESTION 7

7.1 $40 \text{ km} = 40 \times 1\,000 \text{ m} = 40\,000 \text{ m}$
$1 \text{ hour} = 60 \times 60 \text{ sec} = 3\,600 \text{ sec}$
$\therefore \dfrac{40 \text{ km}}{1 \text{ hour}} = \dfrac{40\,000 \text{ m}}{3\,600 \text{ s}} \approx 11{,}11 \text{ m}\cdot\text{s}^{-1}$

7.2 $v = \pi D n$
$11{,}11 \text{ m}\cdot\text{s}^{-1} = \pi(0{,}44\ m)n$
$n = \dfrac{25{,}25}{\pi} \text{ rad/s} \approx 8{,}04 \text{ rad/s}$

7.3 $v = \pi D n$
$11{,}11 \text{ m}\cdot\text{s}^{-1} = \pi(0{,}1\ m)n$
$n = \dfrac{111{,}1}{\pi} \text{ rad/s} \approx 35{,}36 \text{ rad/s}$

$\omega = 2\pi n$
$= 2\pi(35{,}36 \text{ rad/s})$
$\approx 222{,}2 \text{ rad/s}$

7.4 $222{,}2 \times \dfrac{180^\circ}{\pi} \approx 12731{,}12^\circ$ per second

7.5 Area $= \dfrac{rs}{2}$
$= \dfrac{22 \text{ cm} \times 8{,}3 \text{ cm}}{2}$
$= 91{,}30 \text{ cm}^2$

QUESTION 8

8.1 8.1.1 $(XY)^2 = (XO)^2 + (YO)^2 - 2(XO)(YO)\cos\left(X\hat{O}Y\right)$

$(21)^2 = (14)^2 + (14)^2 - 2(14)(14)\cos\left(X\hat{O}Y\right)$

$\cos\left(X\hat{O}Y\right) = \dfrac{21^2 - 14^2 - 14^2}{-2(14)(14)}$

ref $\angle \approx 82{,}82°$

obtuse angle: $X\hat{O}Y \approx 180° - 82{,}82° \approx 97{,}18°$

8.1.2 **OPTION 1**

$4h^2 - 4dh + x^2 = 0$

$4h^2 - 4(28)h + (21)^2 = 0$

$4h^2 - 112h + 441 = 0$

$$h = \frac{-(-112) \pm \sqrt{(-112)^2 - 4(4)(441)}}{2(4)}$$

$$= \frac{112 \pm \sqrt{5\ 488}}{8}$$

$\therefore h = 23{,}26\ (N.A.)$ or $h = 4{,}74$ cm

OPTION 2

Draw point Z in centre of line XY

then $(OY)^2 = (OZ)^2 + (ZY)^2$

$(14)^2 = (OZ)^2 + (10{,}5)^2$

$85{,}75 = (OZ)^2$

$9{,}26 = OZ$

$\therefore$ height of segment $=$ 14 cm – 9,26 $=$ 4,74 cm

8.1.3 The area of minor segment XY

Area of segment XY = area sector – area Δ

Area of segment XY

$= \left[\dfrac{r^2\theta}{2}\right] - \left[\dfrac{1}{2} r \times r \times \sin\theta\right]$

$= \left[\dfrac{(14)^2\left(97{,}18° \times \dfrac{\pi}{180°}\right)}{2}\right] - \left[\dfrac{1}{2} \times 14 \times 14 \times \sin(97{,}18°)\right]$

$= \left[\dfrac{196\left(0{,}539\dot{8}\pi\right)}{2}\right] - \left[98 \times \sin(97{,}18°)\right]$

$\approx 68{,}9873525$

$= 69\ \text{cm}^2$

8.2 $AC^2 = AB^2 + BC^2$ (Pythagoras)

$$\therefore (12\ m)^2 = AB^2 + (6\ m)^2$$
$$108 = AB^2$$
$$AB = 6\sqrt{3}$$

$$AD^2 = \left(6\sqrt{3}+6\right)^2 + \left(6\sqrt{3}+6\right)^2$$
$$AD \approx 23{,}18 \text{ m}$$

$\therefore$ Radius $\approx$ 23,18 m + 6 m $\approx$ 29,18 m

QUESTION 9

$$A_T = a\left(m_1 + m_2 + m_3 + \ldots + m_n\right)$$
$$= 50\left(\frac{0+190}{2} + \frac{190+220}{2} + \frac{220+290}{2} + \frac{290+290}{2} + \frac{290+210}{2} + \frac{210+95}{2} + \frac{95+0}{2}\right)$$
$$= 50\left(95+205+255+290+250+152{,}5+47{,}5\right)$$
$$= 64\ 750 \text{ m}^2$$

OR

$$A_T = a\left(\frac{0_1 + 0_n}{2} + 0_2 + 0_3 + 0_4 + \ldots + 0_{n-1}\right)$$
$$= 50\left(\frac{0+0}{2} + 190 + 220 + 290 + 290 + 210 + 95\right)$$
$$= 50(1295)$$
$$= 64\ 750 \text{ m}^2$$

Total: 150 marks