



NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2022

TEGNIIESE WISKUNDE: VRAESTEL I
NASIENRIGLYNE

Tyd: 3 uur

150 punte

Hierdie nasienriglyne word voorberei vir gebruik deur eksaminatore en hulpeksaminatore. Daar word van alle nasieners vereis om 'n standaardiseringsvergadering by te woon om te verseker dat die nasienriglyne konsekwent vertolk en toegepas word tydens die nasien van kandidate se skrifte.

Die IEB sal geen gesprek aanknoop of korrespondensie voer oor enige nasienriglyne nie. Daar word toegegee dat verskillende menings rondom sake van beklemtoning of detail in sodanige riglyne mag voorkom. Dit is ook voor die hand liggend dat, sonder die voordeel van bywoning van 'n standaardiseringsvergadering, daar verskillende vertolkings mag wees oor die toepassing van die nasienriglyne.

VRAAG 1

1.1 1.1.1 $x(x-5) = 5$

$$x^2 - 5x = 5$$

$$x^2 - 5x - 5 = 0$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(-5)}}{2(1)}$$

$$= \frac{5 \pm \sqrt{25 + 20}}{2}$$

$$= \frac{5 \pm 3\sqrt{5}}{2}$$

1.1.2 $2x + 6y = 4$ en $x^2 + xy = 4$

$$2x + 6y = 4 \quad \textcircled{1} \quad x^2 + xy = 4 \quad \textcircled{2}$$

$$\text{Vanaf } \textcircled{1}: 2x = 4 - 6y$$

$$x = 2 - 3y \quad \textcircled{3}$$

$$\text{Vervang in } \textcircled{2}: (2 - 3y)^2 + (2 - 3y)y = 4$$

$$4 - 12y + 9y^2 + 2y - 3y^2 - 4 = 0$$

$$6y^2 - 10y = 0$$

$$2y(3y - 5) = 0$$

$$y = 0 \quad y = \frac{5}{3}$$

$$\text{Vervang in } \textcircled{3}: x = 2 \quad x = 2 - 5 = -3$$

of

$$2x + 6y = 4 \quad \textcircled{1} \quad x^2 + xy = 4 \quad \textcircled{2}$$

$$\text{Vanaf } \textcircled{1}: 6y = 4 - 2x$$

$$3y = 2 - x$$

$$y = \frac{2 - x}{3} \quad \textcircled{3}$$

$$\text{Vervang in } \textcircled{2}: x^2 + x\left(\frac{2 - x}{3}\right) = 4$$

$$x^2 + \frac{2x - x^2}{3} = 4$$

$$3x^2 + 2x - x^2 = 12$$

$$x^2 + x - 6 = 0$$

$$(x + 3)(x - 2) = 0$$

$$x = -3 \text{ of } x = 2$$

$$\text{Vervang in } \textcircled{3} \quad y = 0$$

$$y = \frac{2 - (-3)}{3}$$

$$= \frac{5}{3}$$

1.2 1.2.1 Uitdrukking is reëel indien $\frac{-4}{2x+3} \geq 0$

d.w.s. $2x+3 < 0$

$$x < -\frac{3}{2}$$

1.2.2 $x^2 - 3x + 9p - 5 = 0$

$$\begin{aligned}\Delta &= (-3)^2 - 4(1)(9p-5) \\ &= 9 - 36p + 20 \\ &= 36p + 29\end{aligned}$$

Vir gelyke reële wortels $36p + 29 = 0$

$$p = -\frac{29}{36}$$

VRAAG 2

2.1 $\frac{5^{2x} + 3}{5^{3x} + 3 \cdot 5^x} = \frac{5^x}{5^{x+2}}$

$$\frac{\cancel{(5^{2x} + 3)}}{5^x \cancel{(5^{2x} + 3)}} = 5^{-2}$$

$$5^{-x} = 5^{-2}$$

$$-x = -2$$

$$x = 2$$

2.2 $(3\sqrt{5} - 2\sqrt{2})^2$

$$\begin{aligned}&= 9 \times 5 - 12\sqrt{5}\sqrt{2} + 4 \times 2 \\ &= 45 - 12\sqrt{10} + 8 \\ &= 53 - 12\sqrt{10}\end{aligned}$$

2.3 $\sqrt{x-2} = x-4$

$$(\sqrt{x-2})^2 = (x-4)^2$$

$$x-2 = x^2 - 8x + 16$$

$$0 = x^2 - 9x + 18$$

$$0 = (x-3)(x-6)$$

$$x = 3 \text{ of } x = 6$$

Kontroleer oplossings

$x = 3$ is ongeldig

$\therefore x = 6$

VRAAG 3

$$\begin{aligned}
 3.1 \quad & \frac{2}{1-2i} \\
 &= \frac{2}{1-2i} \times \frac{1+2i}{1+2i} \\
 &= \frac{2+4i}{(1-2i)(1+2i)} \\
 &= \frac{2+4i}{1-4i^2} \\
 &= \frac{2+4i}{1+4} \\
 &= \frac{2+4i}{5} \\
 &= \frac{2}{5} + \frac{4}{5}i
 \end{aligned}$$

$$\begin{aligned}
 3.2 \quad & i^{2022} \\
 &= (i^2)^{1011} \\
 &= (-1)^{1011} \\
 &= (-1)(-1)\dots(-1) \quad 1011 \text{ keer} \quad \text{of} \quad (-1)^{1010}(-1)^1 \\
 &= -1 \quad \quad \quad = (+1)(-1) = -1
 \end{aligned}$$

of

$$\begin{aligned}
 i^{2022} &= (i^4)^{505} (i^2) \\
 &= (1)(-1) \\
 &= -1
 \end{aligned}$$

$$3.3 \quad 3.3.1 \quad V = r(\cos 210^\circ + i \sin 210^\circ)$$

$$\begin{aligned}
 3.3.2 \quad &= r(-\cos 30^\circ - \sin 30^\circ) \\
 &= r\left(\frac{-\sqrt{3}}{2} - \frac{1}{2}i\right) \\
 &= \frac{-\sqrt{3}}{2}r - \frac{r}{2}i
 \end{aligned}$$

$$\begin{aligned}
 3.4 \quad & \frac{1 \times 2^5 + 1 \times 2^4 + 1 \times 2^2}{10^5} \\
 &= \frac{32 + 16 + 4}{100\,000} \quad \text{of} \quad \frac{32 + 16 + 4}{10^5} \\
 &= \frac{52}{100\,000} \quad \quad \quad = \frac{52}{10^5} \\
 &= 5,2 \times 10^{-4} \quad \quad \quad = 5,2 \times 10^{-4}
 \end{aligned}$$

VRAAG 4

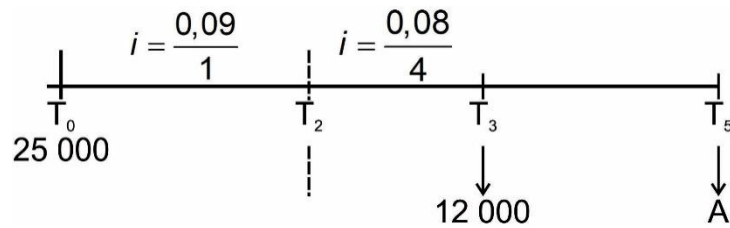
$$4.1 \quad A = P(1 - 0,06)^7$$

$$A = 12\,500(1 - 0,06)^7$$
$$\approx R8\,105,97$$

$$4.2 \quad 4.2.1 \quad 1 + i_{\text{eff}} = \left(1 + \frac{i_{\text{nom}}}{t}\right)^t$$
$$i_{\text{eff}} = \left(1 + \frac{0,08}{4}\right)^4 - 1$$
$$\approx 0,0824\dots$$

effektiewe koers $\approx 8,24\%$

4.2.2



OPSIE 1

$$A = 25\,000(1+0,09)^2 \left(1+\frac{0,08}{4}\right)^{12} - 12\,000 \left(1+\frac{0,08}{4}\right)^8$$

$$\approx R23\,610,04$$

OPSIE 2

$$A = \left[25\,000(1+0,09)^2 \left(1+\frac{0,08}{4}\right)^{4 \times 1} - 12\,000 \right] \left(1+\frac{0,08}{4}\right)^{4 \times 2}$$

$$\approx R23\,610,64$$

OPSIE 3

$$A \text{ (aan einde van 3 jaar)} = 25\,000(1+0,09)^2 \left(1+\frac{0,08}{4}\right)^4$$

$$\equiv R32\,150,94 \dots$$

$$A \text{ (na onttrekking)} \approx 32\,150,94 \dots - 12\,000$$

$$\approx R20\,150,94 \dots$$

$$A \text{ (aan einde van 5 jaar)} \approx 20\,150,94 \dots \times \left(1+\frac{0,08}{4}\right)^{4 \times 2}$$

$$\approx R23\,610,04$$

OPSIE 4

$$A \text{ (aan einde van 5 jaar)} = 25\,000(1+0,09)^2 \left(1+\frac{0,08}{4}\right)^{4 \times 3}$$

$$\equiv R37\,669,95 \dots$$

$$A \text{ (aan einde van 2 jaar)} \approx 12\,000 \left(1+\frac{0,08}{4}\right)^{4 \times 2}$$

$$\approx R14\,059,91$$

$$\text{Waarde van belegging} \approx 37\,669,95 - 14\,059,91$$

$$\approx R23\,610,04$$

4.3

$$A = P(1+i)^n$$

$$3P = P\left(1 + \frac{0,0825}{12}\right)^n$$

$$3 = \frac{\cancel{P}\left(1 + \frac{0,0825}{12}\right)^n}{\cancel{P}}$$

$$3 = \left(1 + \frac{0,0825}{12}\right)^n$$

$$\log_{\left(1 + \frac{0,0825}{12}\right)} 3 = n$$

d.w.s. 161 maande

VRAAG 5

5.1 $y = 0$

5.2 $(0; -1)$

of

$$y = -b^0$$

$$= -1$$

bv. $(0; -1)$

5.3 Vervang $(3; -8)$ in f :

$$-8 = -b^3$$

$$b^3 = 8$$

$$b = 2$$

VRAAG 6

6.1 6.1.1 A is $(0; 4)$
 B is $(0; 2)$

6.1.2 By C en D $\frac{-x^2}{2} + x + 4 = 0$
 $x^2 - 2x - 8 = 0$ C is $(-2; 0)$
 $(x - 4)(x + 2) = 0$ D is $(4; 0)$
 $x = 4$ of $x = -2$

6.1.3 Simmetrie-as by $x = 1$ (volgens simmetrie) of $x = \frac{-b}{2a}$

$$y_E = \frac{-1^2}{2} + 1 + 4$$

$$= 4\frac{1}{2} \quad E \text{ is } \left(1; 4\frac{1}{2}\right)$$

of

$$f'(x) = 0$$

$$f'(x) = \frac{-2x}{2} + 1$$

$$= -x + 1$$

$$-x + 1 = 0$$

$$-x = -1$$

$$x = 1$$

$$y = \frac{-x^2}{2} + x + 4$$

$$= \frac{-1}{2} + 1 + 4$$

$$= 4\frac{1}{2}$$

$$E \text{ is } \left(1; 4\frac{1}{2}\right)$$

6.2 Waardeversameling (terrein) $y \in \left(-\infty; 4\frac{1}{2}\right]$ of $y \leq 4\frac{1}{2}$

6.3 Definisieversameling (gebied) $x \in (-\infty; \infty)$

of

$$x \in \mathbb{R}$$

$$6.4 \quad f'(x) = -x + 1$$

$$m_{\text{raaklyn}} = f'(0) = 1$$

$$y - y_1 = m(x - x_1)$$

$$y - 4 = 1(x - 0)$$

$$y - 4 = x$$

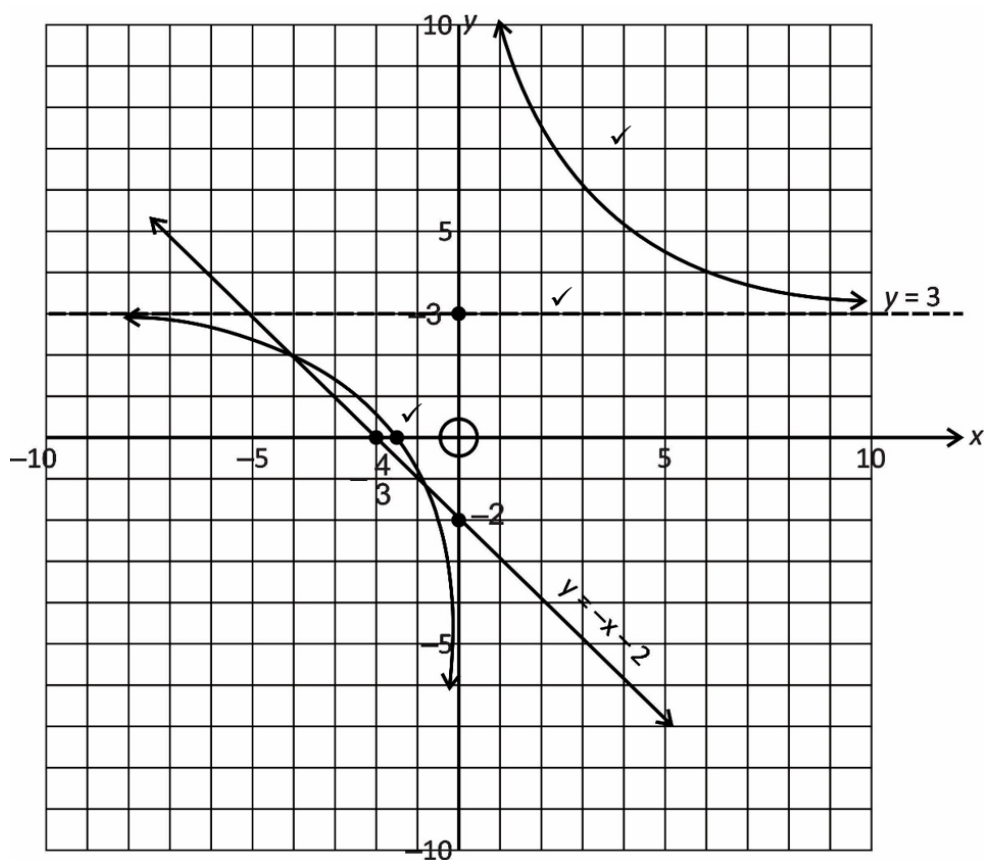
$$y = x + 4$$

6.5 6.5.1 Draaipunte by $x = 4$ en $x = -2$

6.5.2 Gradiënt by $x = 0$ is 4

VRAAG 77.1 7.1.1 Asimptoot $y = 3$

7.1.2



$$\begin{aligned} \text{Laat } y &= 0 & 0 &= \frac{4}{x} + 3 \\ & & -3 &= \frac{4}{x} \\ & & x &= -\frac{4}{3} \end{aligned}$$

$$\begin{aligned} 7.2 \quad \frac{4}{x} + 3 &= -x + k \\ 4 + 3x &= -x^2 + kx \\ x^2 + (3-k)x + 4 &= 0 \\ 3-k &= 5 \\ -2 &= k \\ \text{d.w.s. } y &= -x - 2 \text{ (sien grafiek)} \end{aligned}$$

$$\begin{aligned} 7.3 \quad (x+4)(x+1) &= 0 \\ x &= -4 \text{ of } x = -1 \\ y &= 2 \text{ of } y = -1 \text{ (vervang } y = -x - 2) \\ (-4; 2) \quad (-1; -1) \end{aligned}$$

VRAAG 8

$$\begin{aligned}
 8.1 \quad f'(x) &= \lim_{h \rightarrow 0} \left(\frac{f(x+h) - f(x)}{h} \right) \\
 &= \lim_{h \rightarrow 0} \left(\frac{\frac{3(x+h)}{2} + 7 - \left(\frac{3x}{2} + 7 \right)}{h} \right) \\
 &= \lim_{h \rightarrow 0} \left(\frac{\frac{3x}{2} + \frac{3h}{2} + 7 - \frac{3x}{2} - 7}{h} \right) \\
 &= \lim_{h \rightarrow 0} \left(\frac{3h}{2} \times \frac{1}{h} \right) = \frac{3}{2}
 \end{aligned}$$

$$\begin{aligned}
 8.2 \quad f(x) &= 8 \cdot (x^2)^{\frac{1}{3}} + \frac{5}{2} x^{-3} \\
 &= 8x^{\frac{2}{3}} + \frac{5}{2} x^{-3} \quad \text{of} \quad \frac{16}{3\sqrt[3]{x}} - \frac{15}{2x^4} \\
 f'(x) &= \frac{16}{3} x^{-\frac{1}{3}} - \frac{15}{2} x^{-4}
 \end{aligned}$$

$$\begin{aligned}
 8.3 \quad 8.3.1 \quad 2r + H &= 3 \\
 H &= 3 - 2r
 \end{aligned}$$

$$\begin{aligned}
 8.3.2 \quad (a) \quad V &= \frac{1}{3} \pi r^2 (3 - 2r) \\
 V(r) &= \pi r^2 - \frac{2}{3} \pi r^3
 \end{aligned}$$

$$\begin{aligned}
 (b) \quad V'(r) &= 2\pi r - 2\pi r^2 \\
 \text{By maksimum } 2\pi r - 2\pi r^2 &= 0 \\
 2\pi r(1 - r) &= 0 \\
 r = 0 \quad \text{OF} \quad r &= 1 \text{ m} \\
 \text{NVT}
 \end{aligned}$$

$$\begin{aligned}
 \therefore \text{Maks vol} &= \pi(1)^2 - \frac{2}{3}(\pi)(1)^3 \\
 &= \pi - \frac{2}{3}\pi \\
 &= \frac{\pi}{3} \text{ m}^3 \quad \text{of} \quad V \approx 1,05 \text{ m}^3
 \end{aligned}$$

of

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 (3 - 2r) \\ &= \frac{1}{3} \pi (1)^2 (3 - 2(1)) \\ &= \frac{\pi}{3} m^3 \end{aligned}$$

of

$$\begin{aligned} &\approx \frac{3,14...}{3} \\ &\approx 1,04 \text{ m}^3 \end{aligned}$$

VRAAG 9

9.1 9.1.1 Vervang $(3;0): 0 = 3(3-k)^2$

$$k = 3 \text{ OF } y = x(x-3)^2 \text{ deur inspeksie}$$

9.1.2 $f(x) = x(x-3)^2$

$$= x^3 - 6x^2 + 9x$$

$$f'(x) = 3x^2 - 12x + 9$$

By stasionêre punte $3x^2 - 12x + 9 = 0$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x_Q = 3 \quad x_P = 1$$

Vervang $x = 1: y = 1(1-3)^2$

$$= 4$$

P is $(1;4)$

9.1.3 By $R: y_R = y_P = 4$

$$4 = x^3 - 6x^2 + 9x$$

$$0 = x^3 - 6x^2 + 9x - 4$$

$$0 = (x-1)(x-1)(x-4)$$

R is $(4;4)$

9.1.4 Basis van $\Delta = PR$

$$= x_R - x_P$$

$$= 4 - 1 = 3$$

Hoogte $= y_R = y_P = 4$

$$\text{Oppervlakte} = \frac{1}{2}(PR) \cdot y_P$$

$$= \frac{1}{2} \times 3 \times 4$$

$$= 6 \text{ eenhede}^2$$

9.2 9.2.1 $= 0x + c$

$$= c$$

9.2.2 $= \frac{2x^3}{3} + \frac{3x^2}{2} + c$

$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + c$$

9.3 Oppervlakte $\int_0^4 (-x^2 + 4x) dx$

$$= \left[\frac{-x^3}{3} + \frac{2 \cdot 4 x^2}{2} \right]_0^4$$

$$= \left(\frac{-(4)^3}{3} + 2(4)^2 \right) - (0)$$

$$= -\frac{64}{3} + 32$$

$$= \frac{32}{3} \text{ eenhede}^2$$

Totaal: 150 punte