NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2023

WISKUNDE: VRAESTEL II

NASIENRIGLYNE

Tyd: 3 uur | 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulp-eksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

LET WEL:

- Indien 'n kandidaat 'n vraag meer as een keer beantwoord, sien slegs die EERSTE poging na.
- Deurlopende akkuraatheid geld vir alle aspekte van die nasienmemorandum.

AFDELING A

VRAAG 1

(a)(1)	$95 - a = 80$ $a = 15$	$a = 15$
(a)(2)	$b - 40 = 30$ $b = 70$	$b = 70$
(b)	$Q_1 - 1{,}5 \times \text{IKV}$ $= 40 - 1{,}5 \times 30$ $= -5$ Aangesien die minimumwaarde groter is as –5, is dit nie 'n uitskieter nie.	–5 nie 'n uitskieter nie
(c)(1)	$y = a + bx$ $y = 19{,}259 + 0{,}552x$ $y = 19{,}259 + 0{,}552(180)$ $y = 118{,}603$ Die leerder sal 100% behaal.	19,259 0,552 Leerder sal 100% behaal.
(c)(2)	Nee 180 min. is buite die datastel, dus ekstrapolasie. Impliseer dat enigiemand wat vir meer as 180 min. studeer, 100% sal behaal.	Nee Verduideliking

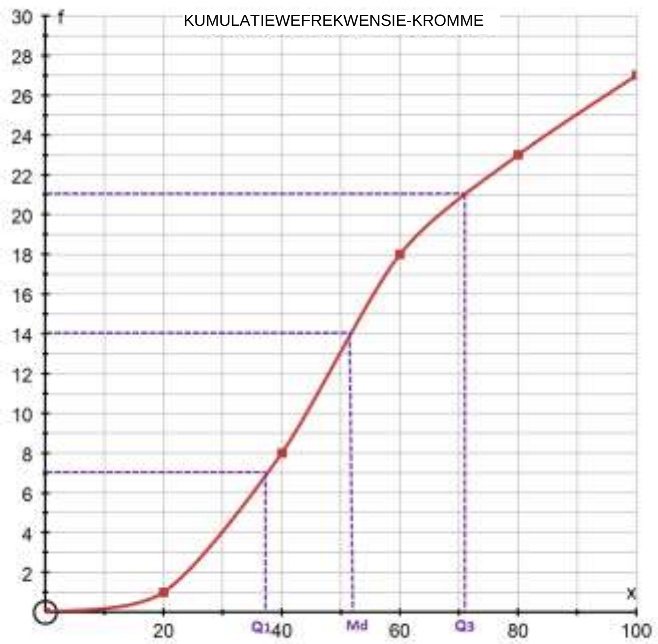
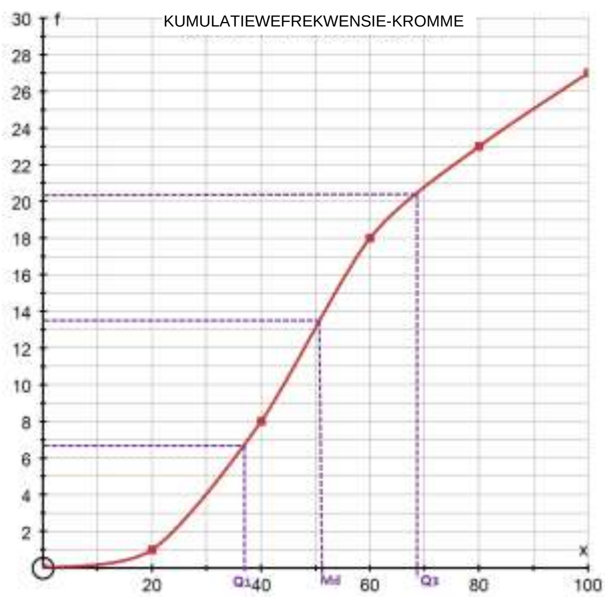
VRAAG 2

(a)(1)	$m_{BC} = \frac{7-4}{6-0}$ $\therefore m_{BC} = \frac{1}{2}$	$m_{BC} = \frac{7-4}{6-0}$ $m_{BC} = \frac{1}{2}$
(a)(2)	$m_{CD} = -2$ Vergelyking van CD: $y = -2x + c$ ... vervang $(6;7)$ $c = 19$ $\therefore y = -2x + 19$ Alternatief: $m_{CD} = -2$ $y - 7 = -2(x - 6)$ $y = -2x + 19$	$m_{CD} = -2$ vervang (6; 7) $y = -2x + 19$
(b)	Vergelyking AD: $y = \frac{1}{2}x + c$ ... vervang $(4;1)$ $c = -1$ $\therefore y = \frac{1}{2}x - 1$ Snypunt van CD en AD: $-2x + 19 = \frac{1}{2}x - 1$ $-5x = -40$ $x = 8$ $\therefore y = -2(8) + 19$ $y = 3$ $\therefore D(8;3)$	$y = \frac{1}{2}x + c$ $c = -1$ $-2x + 19 = \frac{1}{2}x - 1$ $x = 8$ $D(8;3)$

	Alternatief: Vergelyking AD: vervang (4;1) $y-1=\frac{1}{2}(x-4)$ $y=\frac{x}{2}-1$ Snypunt van CD en AD: $-2x+19=\frac{x}{2}-1$ $-5x=-40$ $x=8$ $\therefore y=-2(8)+19$ $y=3$ $\therefore D(8;3)$	
(c)(1)	$\text{E}\left(\frac{6+4}{2};\frac{7+1}{2}\right)$ $\therefore \text{E}(5;4)$ Middelpunt AE $\left(\frac{9}{2};\frac{5}{2}\right)$ $\text{F}\left(\frac{x+8}{2};\frac{y+3}{2}\right)$ $\therefore \text{F}(1;2)$	$\text{E}(5;4)$ Middelpunt AE $\left(\frac{9}{2};\frac{5}{2}\right)$ $\text{F}(1;2)$
(c)(2)	Uit V2b: $m_{AD}=\frac{1}{2}$ $\tan\theta_1=\frac{1}{2}$ $\theta_1=26,6^\circ$ Gradiënt AC: $m_{AC}=\frac{7-1}{6-4}$ $\therefore m_{AC}=3$ $\tan\theta_2=3$ $\therefore \theta_2=71,6^\circ$ $\text{E}\hat{\text{A}}\text{D}=71,6^\circ-26,6^\circ$ $\therefore \text{E}\hat{\text{A}}\text{D}=45^\circ$	$\theta_1=26,6^\circ$ $m_{AC}=3$ $\theta_2=71,6^\circ$ $\text{E}\hat{\text{A}}\text{D}=45^\circ$

(d)(2)	Oppervlakte EAD $= \frac{1}{2} EA \times AD \times \sin E\hat{A}D$ Afstand EA $= \sqrt{10}$ Afstand AD $= 2\sqrt{5}$ Oppervlakte EAD $= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{5} \times \sin 45°$ $\therefore$ Oppervlakte EAD $= 5$ eenhede^2 $\therefore$ Opp EFA $= 5$ eenhede^2 ... hoeklyne //^{m} halveer opp $\therefore$ Opp ADEF $= 10$ eenhede^2	Afstand EA $= \sqrt{10}$ Afstand AD $= 2\sqrt{5}$ Oppervlakte EAD $= \frac{1}{2} \times \sqrt{10} \times 2\sqrt{5} \times \sin 45°$ Opp EAD $= 5$ eenhede^2 Opp ADEF $= 10$ eenhede^2

VRAAG 3

(a)	$\bar{x} = \frac{(10\times1)+(30\times7)+(50\times10)+(70\times5)+(90\times4)}{27}$ $\bar{x} = 53$	middelpunt × frekwensie deel deur 27 $\bar{x} = 53$
(b)	$\frac{5}{27}\times100$ $=18{,}5\%$	$=18{,}5\%$
(c)	 Alternatief: 	(0;0) Stip eindpunte Kumulatiewe frekwensie

(d)(1)(i)		sien grafiek
(d)(1)(ii)		sien grafiek
(d)(2)	Gemiddelde > Mediaan Positief skeef OF Gemiddelde = Mediaan Simmetries	gemiddelde > mediaan positief skeef

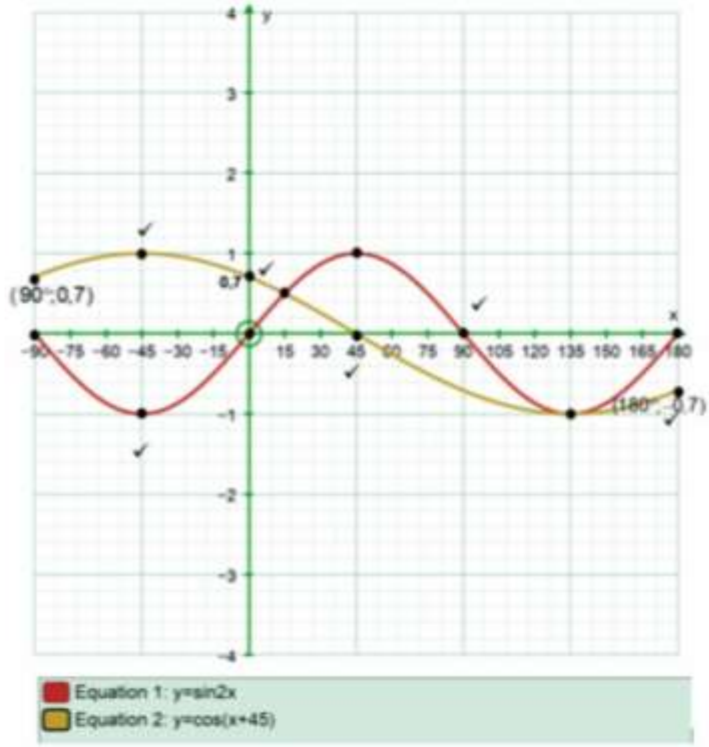
VRAAG 4

(a)	$\hat{C}_1 = 25°$ … hoeke teenoor = sye/radii $C\hat{O}A = 180° - 50°$ … binnehoeke van driehoek $= 130°$	$\hat{C}_1 = 25°$ $= 130°$
(b)	$\hat{C}_2 = 25°$ … verw. hoeke OA // CB $\hat{C}_1 + \hat{C}_2 = \hat{B}$ … hoeke teenoor = sye/radii $\therefore \hat{B} = 50°$ $\hat{O}_1 = 180° - 100°$ … binnehoeke van driehoek $\hat{O}_1 = 80°$	$\hat{C}_2 = 25°$ $\hat{C}_1 + \hat{C}_2 = \hat{B}$ $\hat{O}_1 = 80°$

VRAAG 5

(a)	Konstruksie: Verbind DC en BE. $\dfrac{\text{Opp } \Delta ADE}{\text{Opp } \Delta BDE} = \dfrac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$ $\dfrac{\text{Opp } \Delta ADE}{\text{Opp } \Delta BDE} = \dfrac{AD}{BD}$ ... (1) $\dfrac{\text{Opp } \Delta ADE}{\text{Opp } \Delta DEC} = \dfrac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG}$ $\dfrac{\text{Opp } \Delta ADE}{\text{Opp } \Delta DEC} = \dfrac{AE}{CE}$ ... (2) Maar: $\text{Opp } \Delta BDE = \text{Opp } \Delta DEC$ … dieselfde basis, tussen // lyne $\therefore \dfrac{AD}{BD} = \dfrac{AE}{CE}$	konstruksie $\dfrac{\text{Opp } \Delta ADE}{\text{Opp } \Delta BDE} = \dfrac{\frac{1}{2} \times AD \times EF}{\frac{1}{2} \times DB \times EF}$ $\dfrac{\text{Opp } \Delta ADE}{\text{Opp } \Delta DEC} = \dfrac{\frac{1}{2} \times AE \times DG}{\frac{1}{2} \times EC \times DG}$ $\text{Opp } \Delta BDE = \text{Opp } \Delta DEC$ rede $\therefore \dfrac{AD}{BD} = \dfrac{AE}{CE}$
(b)	In ΔHIJ: $(HI)^2 = (29)^2 - (21)^2$ … Pythagoras $HI = 20$ eenhede $\dfrac{IL}{IJ} = \dfrac{IK}{IH}$ ... lyn // een sy van Δ $\dfrac{12}{21} = \dfrac{IK}{20}$ $\therefore IK = 11\frac{3}{7}$ of IK = 11,4 eenhede	$(HI)^2 = (29)^2 - (21)^2$ $HI = 20$ eenhede $\dfrac{IL}{IJ} = \dfrac{IK}{IH}$ rede $IK = 11\frac{3}{7}$

VRAAG 6

(a)		Grafiek van *f* amplitude frekwensie draaipunte, eindpunte en afsnitte en vorm korrek Grafiek van *g* draaipunte frekwensie eindpunte en afsnitte en vorm korrek
(b)	$\sin 2x = \cos(x+45°)$ $\cos(90° - 2x) = \cos(x+45°)$ $90° - 2x = x + 45° + 360°k \;\; ; \;\; k \in Z$ $x = 15° + 120°k$ $\therefore x = 15°$ OF $90° - 2x = -x - 45° + 360°k$ $x = 135° + 360°k \;\; ; \;\; k \in Z$ $\therefore x = 135°$	$\cos(90° - 2x)$ $\cos(90° - 2x) = \cos(x+45°)$ $90° - 2x = x + 45°$ $\therefore x = 15°$ $\therefore x = 135°$
(c)(1)	$x \in [15°;180°]$ Alternatief: $15° \leq x \leq 180°$	$x \in [15°;180°]$
(c)(2)	$x \in (-90°;0°) \cup (45°;90°)$ Alternatief: $-90° < x < 0$ of $45° < x < 90°$	$x \in (-90°;0°)$ $\cup (45°;90°)$

AFDELING B

VRAAG 7

	In ΔABC: $\hat{B} = 90°$ … raaklyn loodreg op radius AO = 7 … radius $\therefore \cos 38° = \frac{14}{AC}$ $AC = \frac{14}{\cos 38°}$ $AC = 17,766...$ Konstrueer: DB $A\hat{D}B = 90°$ … hoek in halfsirkel $\cos 38° = \frac{AD}{14}$ $\therefore AD = 11,032...$ $\therefore CD = 17,766... - 11,032...$ $\therefore CD \approx 6,7$ eenhede Alternatief: $\hat{B} = 90°$ … raaklyn loodreg op radius AO = 7 …radius In ΔABC: $\tan 38° = \frac{CB}{14}$ $\therefore CB = 10,9$ eenhede Konstrueer: DB $A\hat{D}B = 90°$ … hoek in halfsirkel In ΔDCB: $\hat{C} = 52°$ $\therefore \cos 52° = \frac{CD}{CB}$ $\therefore CD = 6,7$ eenhede	$\hat{B} = 90°$ en rede $\cos 38° = \frac{14}{AC}$ $AC = 17,766...$ $A\hat{D}B = 90°$ rede $\cos 38° = \frac{AD}{14}$ $\therefore AD = 11,032...$ $\therefore CD \approx 6,7$ eenhede

VRAAG 8

(a)	$a > b > 0$ en $\sin\theta < 0$ $\therefore$ Kwadrant 4 $x = a^2 - b^2$ $r = a^2 + b^2$ $\therefore y^2 = \left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2$ … Pythagoras $y^2 = a^4 + 2a^2b^2 + b^4 - a^4 + 2a^2b^2 - b^4$ $\therefore y^2 = 4a^2b^2$ $y = \pm\sqrt{4a^2b^2}$ $\therefore y = -2ab$ … kwadrant 4 $\therefore \tan\theta = -\dfrac{2ab}{a^2 - b^2}$	$\therefore$ Kwadrant 4 $\therefore y^2 = \left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2$ $\therefore y^2 = 4a^2b^2$ $\therefore \tan\theta = -\dfrac{2ab}{a^2 - b^2}$
(b)	$\tan\theta = -\dfrac{2ab}{a^2 - b^2}$ $\tan\theta = -\dfrac{2\times(3)\times(2)}{(3)^2 - (2)^2}$ Verwysingshoek: $-67{,}4°$ $\theta = -67{,}4° + 360°(k)$ ... $k \in Z$ $\theta \in \{292{,}6°\,;\,652{,}6°\}$ **Alternatief:** $\cos\theta = \dfrac{(3)^2 - (2)^2}{(3)^2 + (2)^2}$ $\therefore \cos\theta = \dfrac{5}{13}$ Verwysingshoek: $67{,}4°$ $\theta = 292{,}6° + 360°(k)$ ... $k \in Z$ $\theta \in \{292{,}6°\,;\,652{,}6°\}$	$\tan\theta = -\dfrac{2\times(3)\times(2)}{(3)^2 - (2)^2}$ Verwysingshoek $\theta \in \{652{,}6°\}$

VRAAG 9

(a)	$\dfrac{\frac{1}{2}\cos(90°+\theta)-\sin\theta.\sin(\theta-90°)}{\cos^2(180°-\theta)-2\cos(-\theta)+\cos^2(\theta+90°)}$ $=\dfrac{-\frac{1}{2}\sin(\theta)-(\sin\theta).(-\cos(\theta))}{\cos^2(\theta)-2\cos(\theta)+\sin^2(\theta)}$ $=\dfrac{-\frac{1}{2}\sin\theta+\sin\theta.\cos\theta}{1-2\cos\theta}$ $=\dfrac{-\frac{1}{2}\sin\theta(1-2\cos\theta)}{1-2\cos\theta}$ $=-\dfrac{1}{2}\sin\theta$	$-\frac{1}{2}\sin(\theta)$ $(-\cos(\theta))$ $\cos^2(\theta)$ $-2\cos(\theta)$ $\sin^2\theta+\cos^2\theta=1$ $-\frac{1}{2}\sin\theta(1-2\cos\theta)$ $=-\frac{1}{2}\sin\theta$
(b)(1)	$LK=\sin\theta\times\dfrac{\sin\theta}{\cos\theta}\div\left[\dfrac{\sin 2\theta}{\cos 2\theta}\times\left(1-\dfrac{\sin^2\theta}{\cos^2\theta}\right)\right]$ $=\dfrac{\sin^2\theta}{\cos\theta}\div\left[\dfrac{\sin 2\theta}{\cos 2\theta}\times\left(\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\right)\right]$ $=\dfrac{\sin^2\theta}{\cos\theta}\div\left[\dfrac{\sin 2\theta}{\cos 2\theta}\times\left(\dfrac{\cos 2\theta}{\cos^2\theta}\right)\right]$ $=\dfrac{\sin^2\theta}{\cos\theta}\div\left[\dfrac{2\sin\theta.\cos\theta}{\cos^2\theta}\right]$ $=\dfrac{\sin^2\theta}{\cos\theta}\times\left[\dfrac{\cos^2\theta}{2\sin\theta.\cos\theta}\right]$ $=\dfrac{\sin\theta}{2}$ = RK	$\dfrac{\sin\theta}{\cos\theta}$ $(\cos^2\theta-\sin^2\theta)$ $\cos 2\theta$ $2\sin\theta.\cos\theta$ $=\dfrac{\sin^2\theta}{\cos\theta}\times\left[\dfrac{\cos^2\theta}{2\sin\theta.\cos\theta}\right]$ = RK
(b)(2)	**Berekening: Nie geldig nie vir:** $\sin 2\theta=0$ $\therefore\theta=90°k \quad ... k\in Z$ $\cos 2\theta=0$ $\therefore\theta=45°+90°k \quad ... k\in Z$ $\cos\theta=0 \quad \therefore\theta=\pm 90°+360°k \quad ... k\in Z$ $\tan^2\theta=1$ $\therefore\theta=\pm 45°+180°k \quad ... k\in Z$ **Dus nie geldig nie vir** $\theta=45°k \quad ... k\in Z$	$\tan 2\theta=0$ Algemene oplossing $1-\tan^2\theta=0$ Algemene oplossing $\theta=45°k \quad ... k\in Z$

	Alternatief: $\tan 2\theta = 0$ $\therefore 2\theta = 180°k$ $\therefore \theta = 90°k$ en $\tan 2\theta$ is ongedefinieerd vir: $\theta = 45° + 90°k$ Dus: $\theta = 45°k$ ... $k \in Z$	

VRAAG 10

(a)	$\hat{A}_1 = 90°$ … raaklyn loodreg op radius $\hat{D}_3 = 90°$ … raaklyn loodreg op radius $\therefore$ AODE is koordevierh. … teenoorst. hoeke suppl.	$\hat{A}_1 = 90°$ raaklyn loodreg op radius $\hat{D}_3 = 90°$ teenoorst. hoeke suppl.
(b)	$\hat{O}_2 = 2(68°)$ … hoek by middelpunt $\hat{O}_2 = 136°$ $\therefore \hat{E} = 44°$ … teenoorst. hoeke koordevierhoek	$\hat{O}_2 = 136°$ rede $\therefore \hat{E} = 44°$ rede

VRAAG 11

(a)	$x^2 - 12x + y^2 - 4y = -p$ $(x-6)^2 + (y-2)^2 = -p + 36 + 4$ $(x-6)^2 + (y-2)^2 = -p + 40$ $C(6;2) \quad \therefore \text{radius} = 2$ $-p + 40 = 2^2$ $\therefore p = 36$	$(x-6)^2$ $(y-2)^2$ Radius = 2 $-p + 40 = 2^2$ $\therefore p = 36$
(b)	Trek CB $m_{CB} = \frac{0-2}{12-6}$ $m_{CB} = -\frac{1}{3}$ Trek CD: radius loodreg op raaklyn OB by D $\tan C\hat{B}D = \frac{1}{3}$ $\therefore C\hat{B}D = 18,4349...$ $\Delta ECB \equiv \Delta DCB$ … RHS $\therefore D\hat{B}A = 2 \times 18,4349...$ $D\hat{B}A = 36,8698...$ $m_{AB} = \tan(180° - 36,8689...)$ $m_{AB} \approx -0,75 = -\frac{3}{4}$ $y = -\frac{3}{4}x + c$ …vervang (12 ; 0) $c = 9$ $\therefore y = -\frac{3}{4}x + 9$	$m_{CB} = -\frac{1}{3}$ $\therefore C\hat{B}D = 18,4349...$ $\Delta ECB \equiv \Delta DCB$ … RHS $D\hat{B}A = 36,8698...$ $m_{AB} = -\frac{3}{4}$ $c = 9$ $\therefore y = -\frac{3}{4}x + 9$

(c)	Tweede sirkel: $x^2+(y-9)^2=r^2$ … vervang (2;3) $r^2=40$ Alternatief: $r=\sqrt{(2-0)^2+(3-9)^2}=\sqrt{40}$ Afstand tussen middelpunte $=\sqrt{(6-0)^2+(2-9)^2}$ $=\sqrt{85}$ $\approx 9{,}2$ Som van radii $=\sqrt{40}+2$ $\approx 8{,}3$ Hulle sny nie, aangesien die afstand tussen middelpunte groter is as die som van die radii.	$x^2+(y-9)^2=r^2$ $r^2=40$ $\approx 9{,}2$ $\approx 8{,}3$ Hulle sny nie. Die afstand tussen middelpunte is groter as die som van die radii

VRAAG 12

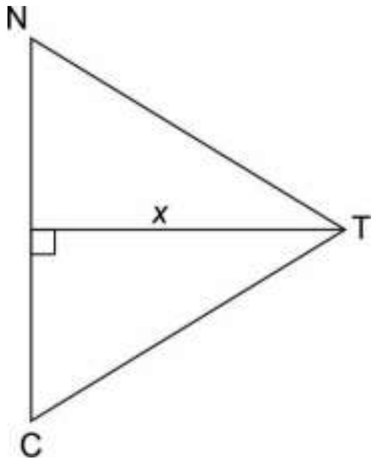
(a)	In ΔABC en ΔOFC: $\hat{C}_1 = \hat{C}_2$ … gegee $\hat{B} = 90°$ … hoek in halfsirkel $\therefore \hat{B} = \hat{F}_1$ $\hat{A}_2 = \hat{O}_2$ … derde hoek $\therefore \Delta ABC \parallel\!\mid \Delta OFC$ … gelykhoekig	$\hat{C}_1 = \hat{C}_2$ $\hat{B} = 90°$ … hoek in halfsirkel $\therefore \Delta ABC \parallel\!\mid \Delta OFC$ … gelykhoekig
(b)	$\frac{BC}{FC} = \frac{AC}{OC}$ … /// driehoeke; sye eweredig Laat: $OC = x$ $\therefore AC = 2x$ …radii $\therefore BC : FC = 2:1$	$\frac{BC}{FC} = \frac{AC}{OC}$ $\therefore BC : FC = 2:1$
(c)	$LK = \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2$ $LK = \frac{(AC)^2 - (AB)^2}{4}$ $= \frac{(BC)^2}{4}$ Pythagoras $\frac{(BC)^2}{4} = (CF)^2$ uit 12(b) Te bewys: $(FC)^2 = DF \times FA$ In ΔCFA en ΔDFE $\hat{E} = \hat{A}$ … hoek in dieselfde segment $\hat{C}_1 = \hat{D}$ … hoek in dieselfde segment $\therefore \Delta CFA \parallel\!\mid \Delta DFE$ … gelykhoekig $\therefore \frac{CF}{DF} = \frac{FA}{FE}$ $CF = FE$ … lyn van middelpunt loodreg op koord $\therefore \frac{CF}{DF} = \frac{FA}{CF}$ $\therefore (FC)^2 = DF \times FA$	$\frac{(BC)^2}{4}$ $\frac{(BC)^2}{4} = (CF)^2$ $\hat{E} = \hat{A}$ en $\hat{C}_1 = \hat{D}$ rede $\therefore \Delta CFA \parallel\!\mid \Delta DFE$ $\therefore \frac{CF}{DF} = \frac{FA}{FE}$ $CF = FE$ $\therefore (CF)^2 = DF \times FA$

	Alternatief: In ΔOFC: $(OC)^2 = (CF)^2 + (OF)^2$ ... Pythag $(CF)^2 = (OC)^2 - (OF)^2$ Aangesien AC = 2.OC $(CF)^2 = \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2$ Te bewys: $(CF)^2 = DF \times FA$ In ΔCFA en ΔDFE $\hat{E} = \hat{A}$ … hoek in dieselfde segment $\hat{C}_1 = \hat{D}$ … hoek in dieselfde segment $\therefore \Delta CFA \text{ ///} \Delta DFE$ … gelykhoekig $\therefore \frac{CF}{DF} = \frac{FA}{FE}$ $CF = FE$ … lyn van middelpunt loodreg op koord $\therefore \frac{CF}{DF} = \frac{FA}{CF}$ $\therefore (CF)^2 = DF \times FA$ $\therefore \left(\frac{AC}{2}\right)^2 - \left(\frac{AB}{2}\right)^2 = DF \times FA$	

VRAAG 13

<table>
<tr>
<td></td>
<td>
$N\hat{T}C = 46°$ ... binne$\angle$ van Δ

Vir CT:
$\frac{10}{\sin 46°} = \frac{CT}{\sin 69°}$
$CT = 12,978295...$

In ΔATC:
$\tan 43,5° = \frac{\text{Hoogte van boom}}{12,978...}$
$\therefore$ Hoogte $= 12,3$ m


$\frac{x}{12,978295} = \sin 65°$
$\therefore x = 11,7623...$
$\therefore$ Die boom die huis tref.
</td>
<td>
$N\hat{T}C = 46°$ ... binne$\angle$ van Δ

$\frac{10}{\sin 46°} = \frac{CT}{\sin 69°}$
$CT = 12,978295...$
$\tan 43,5° = \frac{\text{Hoogte van boom}}{12,978...}$
$\therefore$ Hoogte = 12,3 m
$\frac{x}{12,978295} = \sin 65°$
$\therefore x = 11,7623...$
gevolgtrekking
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Totaal: 150 punte