NASIONALE SENIOR SERTIFIKAAT-EKSAMEN
NOVEMBER 2023

WISKUNDE: VRAESTEL I

NASIENRIGLYNE

Tyd: 3 uur | 150 punte

Hierdie nasienriglyne is opgestel vir gebruik deur eksaminators en hulpeksaminators van wie verwag word om almal 'n standaardiseringsvergadering by te woon om te verseker dat die riglyne konsekwent vertolk en toegepas word by die nasien van kandidate se skrifte.

Die IEB sal geen bespreking of korrespondensie oor enige nasienriglyne voer nie. Ons erken dat daar verskillende standpunte oor sommige aangeleenthede van beklemtoning of detail in die riglyne kan wees. Ons erken ook dat daar sonder die voordeel van die bywoning van 'n standaardiseringsvergadering verskillende vertolkings van die toepassing van die nasienriglyne kan wees.

AFDELING A

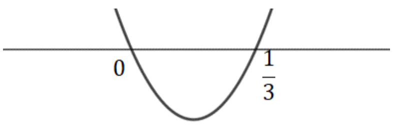
VRAAG 1

(a) (1) $x = \log_3 2$

(2)
$$x + 7 = (x+1)^2$$
$$x^2 + x - 6 = 0$$
$$(x+3)(x-2) = 0$$
$$x \neq -3 \text{ of } x = 2$$

(b)
$$y = 2x + 7$$
$$x^2 - x - (2x+7) - 3 = 0$$
$$x^2 - 3x - 10 = 0$$
$$(x-5)(x+2) = 0$$
$$x = 5 \text{ of } x = -2$$
$$(5\,;17) \text{ of } (-2\,;3)$$

(c)
$$3x^2 - x \leq 0$$
$$x(3x-1) \leq 0$$



$$0 \leq x \leq \frac{1}{3}$$

VRAAG 2

(a) $\left(0\,;32\right)$

(b) $f\left(x\right)=\left(x+2\right)\left(x^2-8x+16\right)$

$f\left(x\right)=x^3-6x^2+32$

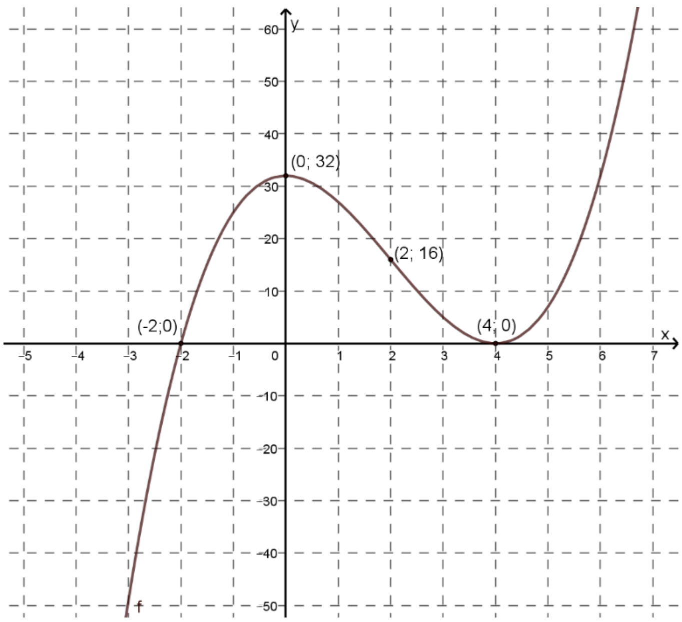
$f'\left(x\right)=3x^2-12x$

(c) $3x^2-12x=0$ (geen punt nie, aangesien dit in vraag gegee word)

$3x\left(x-4\right)=0$

$x=0$ of $x=4$

(d) Draaipunte
x-afsnit
Buigpunt
Vorm



(e) $x\geq-2$

VRAAG 3

(a) $1 = \log_m (3+2)$

$m = 5$

(b) $y = \log_5 (0+2)$

$C(0\,;0,4)$

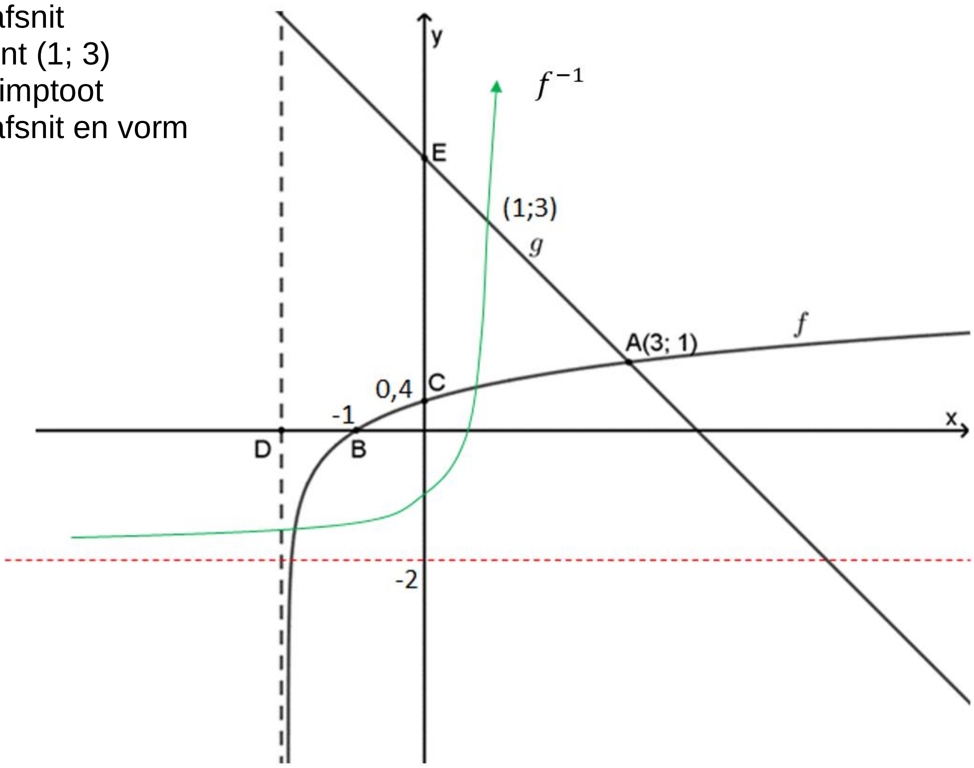
$0 = \log_5 (x+2)$

$x = -1$

$B(-1;0)$

(c) $x \in (-2\,;3]$ notasie Alternatief: $-2 < x \leq 3$

(d) *y*-afsnit
Punt (1; 3)
Asimptoot
x-afsnit en vorm



VRAAG 4

(a) (1)

$$f(x) = \sqrt[5]{x^2} - \frac{3}{x^3}$$

$$f(x) = x^{\frac{2}{5}} - 3x^{-3}$$

$$f'(x) = \frac{2}{5}x^{\frac{-3}{5}} + 9x^{-4}$$

(2)

$$g(x) = \frac{(x-1)(x-1)}{4(x-1)}$$

$$g(x) = \frac{x}{4} - \frac{1}{4}$$

$$g'(x) = \frac{1}{4}$$

(b) $y = 7x + c$ Alternatief: $y - 5 = 7(x-4)$

$5 = 7(4) + c$

$c = -23$

$y = 7x - 23$

(c)

$$h'(x) = \lim_{h \to 0} \frac{5(x+h) - 8 - (5x-8)}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{5x + 5h - 8 - 5x + 8}{h}$$

$$h'(x) = \lim_{h \to 0} \frac{5h}{h}$$

$$h'(x) = 5$$

VRAAG 5

(a)
$$T_n = 22 + (n-1)(3)$$
$$T_n = 3n + 19$$
$$3n + 19 = 262$$
$$3n = 243$$
$$n = 81$$

(b)
$$\sum_{2=1}^{81} 3n + 19$$

(c)
$$\frac{n}{2}\left(2(22) + (n-1)(3)\right) > 15\,000$$
$$3n^2 + 41n - 30\,000 > 0$$
$$n < -107{,}1 \text{ or } n = 93{,}4$$
$$n = 94 - 81 = 13$$

VRAAG 6

(a)

Belegging 1: $\dfrac{3\,500\left(\left(1+\dfrac{0,15}{12}\right)^{60}-1\right)}{\dfrac{0,15}{12}}=\text{R}310\,010,78$

Belegging 2: $24\,000\left(1+\dfrac{0,20}{4}\right)^{20}+7\,000\left(1+\dfrac{0,20}{4}\right)^{8}=\text{R}74\,021,33$

Die totale enkelbedrag is R384 032,11

(b) (1)

$$250\,000=\frac{x\left(1-\left(1+\dfrac{0,10}{12}\right)^{-120}\right)}{\dfrac{0,10}{12}}$$

$x=\text{R}3\,303,77$ ✓

(2)

$$250\,000\left(1+\frac{0,10}{12}\right)=\frac{6\,000\left(1-\left(1+\dfrac{0,10}{12}\right)^{-n}\right)}{\dfrac{0,10}{12}}$$

$$\frac{1123}{1\,728}=\left(1+\frac{0,10}{12}\right)^{-n}$$

$$-n=\log_{\left(1+\frac{0,10}{12}\right)}\frac{1123}{1\,728}$$

$$-n=-51,93$$

$$n=52\text{ maande}$$

AFDELING B

VRAAG 7

(a) $x^2 - 16x + 64 + 9$

$(x-8)^2 + 9$

$m = 8$ en $p = 9$

(b) (1) k^2

(2) $k^{\frac{1}{3}}$ of $\sqrt[3]{k}$

(3) $\dfrac{2}{k}$ of $2k^{-1}$

(c)

$$\frac{3^{\log_p 2}}{3^{\log_p 8}} = 9$$

$$3^{\log_p 2} = 3^{\log_p 8}.3^2$$

$$\log_p 2 = \log_p 8 + 2$$

$$\log_p 2 - \log_p 8 = 2$$

$$\log_p \frac{1}{4} = 2$$

$$p = \frac{1}{2}$$

Alternative:

$$3^{\log_p 2 - \log_p 2^3} = 3^2$$

$$\therefore \log_p 2 - 3\log_p 2 = 2$$

$$-2\log_p 2 = 2 \quad \therefore \log_p 2 = -1 \quad \therefore p^{-1} = 2 \qquad \therefore p = \frac{1}{2}$$

VRAAG 8

(a) $y = x$

(b) $x = \dfrac{12}{x}$

$x^2 = 12$

$x = \pm\sqrt{12}$

$B\left(\sqrt{12}\,;\sqrt{12}\right)$

(c) $f'(x) = -12x^{-2}$ of $f'(x) = \dfrac{-12}{x^2}$

$\dfrac{-12}{x^2} = \dfrac{-1}{3}$

$-36 = -x^2$

$x^2 = 36$

$x = \pm 6$

$x = 6$

$C(6\,;2)$

$y = -\dfrac{1}{3}x + c$ of alternatief vir vergelyking van lyn: $y - 2 = -\dfrac{1}{3}(x-6)$ $\quad \therefore y = -\dfrac{1}{3}x + 4$

$2 = -\dfrac{1}{3}(6) + c$

$c = 4$

Oppervlakte van $\Delta ACO = \dfrac{1}{2} \times 4 \times 6$

Oppervlakte van $\Delta ACO = 12$ eenhede^2

VRAAG 9

(a)

$$y = a(x-1)^2 + 2$$
$$D(7;5)$$
$$5 = a(7-1)^2 + 2$$
$$3 = 36a$$
$$a = \frac{1}{12}$$

(b)

$$y = \frac{1}{100}(x-p)^2 + 4$$
$$B(-5;5)$$
$$1 = \frac{1}{100}(-5-p)^2$$
$$100 = 25 + 10p + p^2 \quad \text{of} \quad (p+5)^2 = 100 \quad \therefore p+5 = \pm 10$$
$$p^2 + 10p - 75 = 0$$
$$(p+15)(p-5) = 0$$
$$p \neq -15 \quad \text{of} \quad p = 5$$

Die afstand tussen die pale moet 20 meter wees.

VRAAG 10

(a) (1) $T_n = 1.(1,1)^{24-1}$

$T_n = 8{,}95$ of 9,0 liter/minuut ***9,0 is die afgeronde waarde***

(2) $S_n = \dfrac{60.(1{,}1^{24} - 1)}{1{,}1 - 1}$

$S_n = 5\ 309{,}8$ liter

(b) $T_2 - T_1 = 4\,; T_3 - T_2 = 7$ and $T_{10} = 0$

$3a + b = 4$

$2a = 3$

$a = \dfrac{3}{2}$ and $b = -\dfrac{1}{2}$

$0 = \dfrac{3}{2}(10)^2 - \dfrac{1}{2}(10) + c$

$c = -145$

$T_n = \dfrac{3}{2}n^2 - \dfrac{1}{2}n - 145$

(c) $\dfrac{r}{1-r} = \dfrac{1}{5}$ Alternatiewe oplossing: $\dfrac{a}{1-a} = \dfrac{1}{5}$ $\therefore 5a = 1 - a$ $\therefore a = \dfrac{1}{6}$

$5r = 1 - r$

$r = \dfrac{1}{6}$

maar $a = r$, dus $a = \dfrac{1}{6}$

(d) $x\,;y\,;x+y\,;...$

$\frac{x+y}{y}=\frac{y}{x}$ ALTERNATIEF $a=x$ en $r=\frac{y}{x}$ dus $T_3=ar^2=x\left(\frac{y}{x}\right)^2=\frac{y^2}{x}$

Maar $T_3=\frac{y^2}{x}=x+y$ en $y^2=x^2+xy$

$x^2+xy=y^2$

$x^2+xy-y^2=0$

$$x=\frac{-(y)\pm\sqrt{(y)^2-4(1)(-y^2)}}{2(1)}$$

$$x=\frac{-(y)\pm\sqrt{5}y}{2(1)}$$

$$x=\frac{y\left(-1\pm\sqrt{5}\right)}{2}$$

$\frac{x}{y}=\frac{-1+\sqrt{5}}{2}$ aangesien x en y albei positief is

VRAAG 11

(a) (1) (i) 9^6 (eerste wagwoord) of 531 441

(ii) $9^6 \times 8 \times 7 \times 6 \times 4 = 714\ 256\ 704$
(Die viersyfer- ewe wagwoord × eerste wagwoord)

(2) $4 \times 7! \times 1$ (Eindig met 3 en begin met 6, 7, 8 of 9)
$3 \times 7! \times 2$ (Eindig met 6 of 9, begin met 7, 8 en óf 6 óf 9)
$= 4 \times 7! \times 1 + 3 \times 7! \times 2 = 50\ 400$
ALT: $10 \times 7!$

(b) (1) $\frac{3}{36}$ of $\frac{1}{12}$

Alle *y*-afsnitte is negatief.
Slegs drie waardes kleiner as –29.

	1	2	3	4	5	6
1	-1	-2	-3	-4	-5	-6
2	-2	-4	-6	-8	-10	-12
3	-3	-6	-9	-12	-15	-18
4	-4	-8	-12	-16	-20	-24
5	-5	-10	-15	-20	-25	-30
6	-6	-12	-18	-24	-30	-36

(2) 1 (Hulle sal altyd reëel, rasionaal en ongelyk wees.)

(3) Die vorm van die grafiek is

Vir niereële wortels moet die draaipunt onder die *x*-as wees.

$= \frac{2}{6}$ of $\frac{1}{3}$

Alternatiewe oplossing:
$h(x) = -x^2 + 2gx + g^2 + 4 - r$

$\Delta = (-2g)^2 - 4(-1)(-g^2 + 4 - r)$

$= 4g^2 - 4g^2 + 16 - 4r$

$\therefore \Delta = 16 - 4r < 0 \qquad \therefore > 4$

$\therefore 5$ of 6 dus $P(\text{niereële wortels}) = \frac{2}{6} = \frac{1}{3}$

VRAAG 12

(a) $y = -\frac{2}{5}x + 6$

(b)
$$\text{Volume} = x(x-2)\left(\frac{-2}{5}x + 6\right)$$

$$\text{Volume} = x\left(\frac{-2}{5}x^2 + 6x + \frac{4}{5}x - 12\right)$$

$$\text{Volume} = \frac{-2}{5}x^3 + 6x^2 + \frac{4}{5}x^2 - 12x$$

$$\frac{Dv}{dx} = -\frac{6}{5}x^2 + 12x + \frac{8}{5}x - 12$$

$$-\frac{6}{5}x^2 + 12x + \frac{8}{5}x - 12 = 0$$

$$-6x^2 + 60x + 8x - 60 = 0$$

$$-6x^2 + 68x - 60 = 0$$

$$3x^2 - 34x + 30 = 0$$

$$x = 0{,}96 \text{ of } x = 10{,}3689$$

Vertikale balk

Hoogte van vertikale balk $= -\frac{2}{5}(10{,}3689\ldots) + 6$

Hoogte van vertikale balk vir maksimum volume is 1,9 meter.

Totaal: 150 punte